Задание 8.

53.

1) $\int_1^2 \left(2x^2+\frac{2}{x^4}\right)dx = \int 2x^2dx = \frac{2}{3}x^3, = \int 2x^{-4}dx = 2\cdot\frac{x^{-3}}{-3} = -\frac{2}{3}x^{-3} =$
$\left(\frac{2}{3}\cdot 8 - \frac{2}{3}\cdot\frac{1}{8}\right) - \left(\frac{2}{3}-\frac{2}{3}\right) = \frac{16}{3} - \frac{1}{12} = \frac{63}{12} = \frac{21}{4}$

2) $\int_0^1 \sqrt{1+x}\,dx = \int_0^1 (1+x)^{1/2}dx = \frac{2}{3}\left((1+1)^{3/2} - (1+0)^{3/2}\right) = \frac{2}{3}(2^{3/2}-1) = \frac{2}{3}(2\sqrt{2}-1)$

3) $\int_0^{16} \frac{dx}{\sqrt{x+9}-\sqrt{x}} = \frac{\sqrt{x+9}+\sqrt{x}}{(x+9)-x} = \frac{\sqrt{x+9}+\sqrt{x}}{9} = \frac{1}{9}\int_0^{16}\left((x+9)^{1/2}+x^{1/2}\right)dx$
$= \frac{1}{9}\left[\frac{2}{3}(x+9)^{3/2}+\frac{2}{3}x^{3/2}\right]_0^{16} = \frac{2}{27}\left[(25)^{3/2}+(16)^{3/2}-9^{3/2}-0\right] = \frac{2}{27}$
$[125+64-27] = \frac{2}{27}\cdot 162 = 12$

4) $\int_1^{e^3} \frac{dx}{x\sqrt{1+\ln x}} = \int_1^4 \frac{dt}{\sqrt{t}} = \int_1^4 t^{-1/2}dt = 2t^{1/2}\Big|_1^4 = 2(2-1) = 2$

5) $\int_0^1 \frac{dx}{x^2+4x+5} = \int_0^1 \frac{dx}{(x+2)^2+1} = \int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\frac{u}{a}+C =$
$\int \frac{dx}{(x+2)^2+1} = \arctan(x+2)+C = \arctan(x+2)\Big|_0^1 = \arctan(3) -$
$\arctan(2)$

6) $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\,dx = 2\int_1^0 u^{1/2}(-du) = 2\int_0^1 u^{1/2}du = 2\cdot\frac{2}{3}u^{3/2}\Big|_0^1 = \frac{4}{3}$

7) $\cancel{\int_0^{\pi/2}\frac{dx}{2+\cos x}}$ $\int_0^4 \frac{dx}{1+\sqrt{2x+1}} = \int_1^3 \frac{t\,dt}{1+t} = \int_1^3\left(1-\frac{1}{1+t}\right)dt = [t-\ln|1+t|]_1^3$

$= [3 - \ln 4] - [1 - \ln 2] = 2 - \ln 4 + \ln 2 = 2 - \ln 2$

8.) $\int_0^{\pi/2} x \cos x \, dx = x \sin x \Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x \, dx = \frac{\pi}{2} \cdot 1 - [-\cos x]_0^{\pi/2} =$

$\frac{\pi}{2} - [0 - 1] = \frac{\pi}{2} + 1$

12.) $\int_0^{e-1} \ln(x+1) dx = \int_0^{e-1} \ln(x+1) dx = [x \ln(x+1)]_0^{e-1} \left(\frac{x+1-1}{x+1}\right) dx =$

$= \int_0^{e-1} \ln(x+1) dx = [x \ln(x+1) - (x - \ln|x+1|)]_0^{e-1} = \int_0^{e-1} \ln(x+1) dx =$

$= [(x+1) \ln(x+1) - x]_0^{e-1} = \int_0^{e-1} \ln(x+1) dx = 1 - 0 = 1$

13.) $\int_1^3 \frac{dx}{x\sqrt{x^2+5x+1}} = \ln\left(t + \frac{5}{2} + \sqrt{t^2+5t+1}\right)\Big|_{t=\frac{1}{3}}^{1} = \ln \frac{\frac{7}{2} + \sqrt{7}}{\frac{9}{3}} =$

$\ln \frac{7 + 2\sqrt{7}}{9}$

14.) $\int_3^4 \frac{x^2 - x + 2}{x^4 - 5x^2 + 4} dx = \frac{x^2 - x + 2}{x^4 - 5x^2 + 4} = -\frac{2}{3(x+2)} + \frac{2}{3(x+1)} -$

$\frac{1}{3(x-1)} + \frac{1}{3(x-2)} = \frac{1}{3}\left(\ln \frac{x-2}{x-1} + 2 \ln \frac{x+1}{x+2}\right)\Big|_3^4 = \frac{1}{3} \ln \frac{625}{432}$

15) $\int_{\pi/6}^{\pi/3} \frac{x\,dx}{\cos^2 x} = \int_{\pi/6}^{\pi/3} x \sec^2 x \, dx = [x \tan x]_{\pi/6}^{\pi/3} \tan x \, dx = [x \tan$

$x + \ln|\cos x|]_{\pi/6}^{\pi/3} = \frac{5\pi\sqrt{3}}{18} - \frac{1}{2} \ln 3$

54.

$$f(x) = \frac{1}{1+2\sin^2 x} \quad \text{Отрезок}: \left[0; \frac{\pi}{4}\right]$$

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx = f_{avg} = \frac{1}{\frac{\pi}{4}-0}\int_0^{\frac{\pi}{4}} \frac{1}{1+2\sin^2 x}\,dx = \frac{4}{\pi}\int_0^{\frac{\pi}{4}}$$

$$\frac{1}{1+2\sin^2 x}\,dx = \int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{\sec^2 x + 2\tan^2 x}\,dx = \int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1+\tan^2 x + 2\tan^2 x}\,dx =$$

$$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1+3\tan^2 x}\,dx = \int_0^{\sqrt{3}} \frac{1}{1+u^2}\cdot\frac{1}{\sqrt{3}}\,du = \frac{1}{\sqrt{3}}\left[\arctan(u)\right]_0^{\sqrt{3}} =$$

$$= \frac{1}{\sqrt{3}}\left(\arctan(\sqrt{3}) - \arctan(0)\right) = \frac{1}{\sqrt{3}}\left(\frac{\pi}{3}-0\right) = \frac{\pi}{3\sqrt{3}} = f_{avg}$$

$$= \frac{4}{\pi}\cdot\left(\frac{\pi}{3\sqrt{3}}\right) = \frac{4}{3\sqrt{3}} = \frac{4\sqrt{3}}{9}$$

55.

$$F(x) = \int_0^x (t-1)(t-2)^2\,dt,\ x \in i$$

$$\int_0^{\frac{\pi}{4}} \frac{1}{1+2\sin^2 x}\,dx = \int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{\sec^2 x + 2\tan^2 x}\,dx = \int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1+\tan^2 x + 2\tan^2 x}\,dx =$$

$$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1+3\tan^2 x}\,dx = \int_0^{\sqrt{3}} \frac{1}{1+u^2}\cdot\frac{1}{\sqrt{3}}\,du = \frac{1}{\sqrt{3}}\left[\arctan(u)\right.$$

$$\left.\right]_0^{\sqrt{3}} = \frac{1}{\sqrt{3}}\left(\arctan(\sqrt{3}) - \arctan(0)\right) = \frac{1}{\sqrt{3}}\left(\frac{\pi}{3}-0\right) = \frac{\pi}{3\sqrt{3}}$$

$$= f_{avg} = \frac{4}{\pi}\cdot\left(\frac{\pi}{3\sqrt{3}}\right) = \frac{4}{3\sqrt{3}} = \frac{4\sqrt{3}}{9}.$$