9. Задание.

5.) $x_{n+1} - x_n = 2^{-n}$ $\quad \left[x_n = x_0 + 2\left(1 - \frac{1}{2^n}\right)\right]$

$$\sum_{k=0}^{n-1} 2^{-k} = \frac{1-\left(\frac{1}{2}\right)^n}{1+\frac{1}{2}} = 2(1-2^{-n})$$

"$x_n = x_0 + 2(1-2^{-n})$."

$$x_{n+1} - x_n = 2(1-2^{-(n+1)}) - 2(1-2^{-n}) = 2(-2^{-(n+1)} + 2^{-n}) = 2\cdot 2^{-(n+1)}$$

$= 2^{-n}$ $\quad$ 1 порядок

6.) $x_{n+1} - x_n = 2x_n$ $\quad$ рекуррентная формула с постоянным множителем,

$x_{n+1} = x_n + 2x_n = 3x_n$

$x_1 = 3x_0$, $\quad x_2 = 3x_1 = 3^2 x_0, \ldots \; x_n = 3^n x_0$ $\quad$ "$x_n = 3^n x_0$"

7.) $x_{n+1} = \frac{n+5}{n+3} x_n$ $\quad \prod_{k=0}^{n-1}(k+5) = (5)(6)\ldots(n+4) = \frac{(n+4)!}{4!}$

$x_n = x_0 \prod_{k=0}^{n-1} \frac{k+5}{k+3}$ $\quad \prod_{k=0}^{n-1}(k+3) = (3)(4)\ldots(n+2) = \frac{(n+2)!}{2!}$

$$\prod_{k=0}^{n-1} \frac{k+5}{k+3} = \frac{(n+4)!/4!}{(n+2)!/2!} = \frac{2!\,(n+4)}{4!\,(n+2)} \quad \frac{2}{4!} = \frac{2}{24} = \frac{1}{12}$$

$\frac{(n+4)!}{(n+2)!} = (n+4)(n+3)$ $\quad \prod_{k=0}^{n-1} \frac{k+5}{k+3} = \frac{(n+4)(n+3)}{12}$

$$x_n = x_0 \frac{(n+4)(n+3)}{12}$$

8.) $x_{n+1} = n x_n$

$x_1 = 0 \cdot x_0 = 0, \quad x_2 = 1 \cdot x_1 = 0, \ldots$

$x_2 = 1 \cdot x_1, x_3 = 2 \cdot x_2 = 2 \cdot 1 \cdot x_1, \ldots, x_n = x_1 \prod_{k=1}^{n-1} k = x_1 (n-1)!$

9.) $x_{n+1} = -x_n$

$x_1 = -x_0, \quad x_2 = -x_1 = (-1)^2 x_0 \ldots, x_n = (-1)^n x_0$

$x_n = (-1)^n x_0$

10.) $2x_{n+1} = x_n$

$x_{n+1} = \frac{1}{2} x_n \Rightarrow x_n = x_0 \left(\frac{1}{2}\right)^n = x_0 2^{-n}$

11.) $x_{n+1} = x_n + e^{-n}$

$x_n = x_0 + \sum_{k=0}^{n-1} e^{-k} = x_0 + \frac{1 + e^{-n}}{1 - e^{-1}}$

$x_n = 1 + \frac{1 + e^{-n}}{1 - e^{-1}}$

12.) $(n+1) x_{n+1} = (n+2) x_n \quad x_{n+1} = \frac{n+2}{n+1} x_n$

$x_n = x_0 \prod_{k=0}^{n-1} \frac{k+2}{k+1} = x_0 \frac{(n+1)!/1!}{n!/0!} = x_0 (n+1)$

13.) $x_{n+1} + x_n = 2x_n + 2$

$x^{(h)}_{n+1} = 3x^{(h)}_n \Rightarrow x^{(h)}_n = C \cdot 3^n \quad p = 3p + 2 \Rightarrow -2p = 2 \Rightarrow p = -1$

$x_n = C \cdot 3^n - 1 \quad$ "$x_n = (x_0 + 1) 3^n - 1$"

14.) $x_{n+2} = x_n + x_{n+1}$

$r^2 + r - 1 = 0 \Rightarrow r_{1,2} = \frac{1 \pm \sqrt{5}}{2}$

$x_n = k_1 \left(\frac{1+\sqrt{5}}{2}\right)^2 + k_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$

1.) а) $x_n = x_{n+1} + x_{n+3}$ порядок 3.

б) $x_{n+1} = (x_n)^2 + (x_{n-1})^3$ порядок 2.

в) $\cancel{x_{n+2} + 4x_{n+1}}$ $x_n + n x_{n-1} = n^2$ порядок 1

г) $x_{n+2} + 4x_{n+1} + 4x_n = 2^n$ порядок 2

д) $x_{n+1} = x_n + 1$ порядок 1

е) $x_{n+1} = x_n + 1$ порядок 3

а) 3; в) 1; д) 1;

б) 2; г) 2; е) 3;

2) $\cancel{x_n = x_n - n}$

$x_n = n^2 + n$

$x_{n+1} = (n+1)^2 + (n+1) = n^2 + 3n + 2$

$x_n + 2n + 2 = (n^2 + n) + 2n + 2 = n^2 + 3n + 2$

$x_n + 2n + 2 = (n^2 + n + k + 2n + 2 = n^2 + 3n + 2 + k$

Ответ: Да

3) $x_2 = 3$, $x_3 = 5$

$x_2 = Ca^2 = 3$

$x_3 = Ca^3 = 5$

$\frac{x^3}{x^2} = \frac{Ca^3}{Ca^2} = a = \frac{5}{3}$

$C = \frac{x_2}{a^2} = \frac{3}{\left(\frac{5}{3}\right)^2} = \frac{3 \cdot 9}{25} = \frac{27}{25}$

4) $x_{n+1} = 1.25 x_n$ 1 порядок

Если $x_0 = 1600$

$x_2 = 1.25^2 \cdot 1600 = 1.5625 \cdot 1600 = 2500$

$x_4 = 1.25^4 \cdot 1600 = 2.4414 \cdot 1600 = 3906.25$

~~5)~~ Ответ: а) $x_{n+1} = 1.25 x_n$, б) 1 порядок, в) $x_2 = 2500$, $x_4 \approx 3906$

5) $x_{n+1} - x_n = 2^{-n}$

$x_{n+1} - x_n = 2^{-n}$

$x_n - x_0 = \sum_{k=0}^{n-1} 2^{-k}$

$\sum_{k=0}^{n-1} 2^{-k} = \frac{1-2^{-n}}{1-\frac{1}{2}} = 2(1-2^{-n})$

$x_n = x_0 + 2(1-2^{-n}) = x_0 + 2 - 2^{1-n}$.

$x_n = x_0 + 2\left(1 - \frac{1}{2^n}\right.$