

Задание 10.

1. Вариант.

$$1) \text{a) } Z = \frac{3\sqrt{xy}}{2x-5y}$$

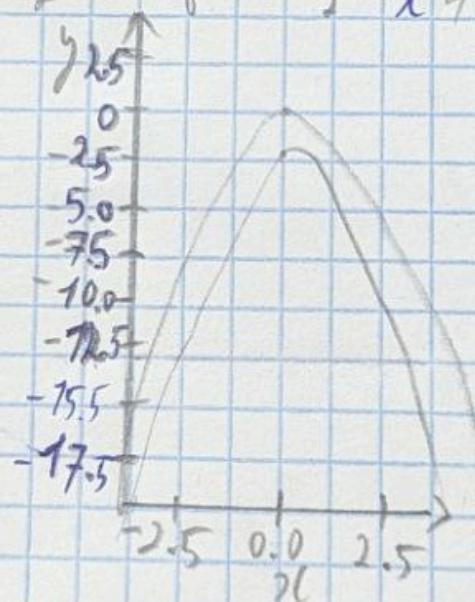
$$2x-5y \neq 0$$

$$2x-5y=0$$



$$\text{б) } |Z| = \sqrt{1-(x^2+y)^2}$$

$$-1-x^2 \leq y \leq 1-x^2$$



$$2) \quad Z = \ln(y^2 - e^{-x})$$

$$Z_x = \frac{d}{dx} \ln(\dots) = \frac{1}{y^2 - e^{-x}} \cdot \frac{d}{dx} (y^2 - e^{-x}) = \frac{e^{-x}}{y^2 - e^{-x}}$$

$$Z_y = \frac{2y}{y^2 - e^{-x}}$$

$$dz = Z_x dx + Z_y dy = \frac{e^{-x}}{y^2 - e^{-x}} dx + \frac{2y}{y^2 - e^{-x}} dy$$

$$3) \frac{dZ}{dt} \text{ npa } Z = e^{x-2y}, x = \sin^2 t, y = t^3, \text{ nput} = 0$$

$$\frac{dZ}{dt} = e^{x-2y} \left(\frac{dx}{dt} - 2 \frac{dy}{dt} \right), \frac{dx}{dt} = \sin 2t, \frac{dy}{dt} = 3t^2$$

$$\text{npa } t=0: x=0, y=0, e^{x-2y}=1, \frac{dx}{dt}=0, \frac{dy}{dt}=0 \Rightarrow$$

$$\Rightarrow \left. \frac{dZ}{dt} \right|_{t=0} = 0$$

$$4) \frac{dU}{dZ} \text{ qua } U = x \cot(Z^2 + x), x = e^{\frac{z}{2}}$$

$$\frac{dU}{dt} = u_x \frac{dx}{dz} + u_z$$

$$\text{Tge } Z = z^2 + x$$

$$u_x = \cot Z - x \csc^2 Z, u_z = -2xz \csc^2 Z,$$

$$\frac{dx}{dz} = \frac{x}{z^2} \quad (x = e^{\frac{z}{2}})$$

$$\frac{du}{dz} = [\cot Z - x \csc^2 Z] \left(-\frac{x}{z^2} \right) - 2xz \csc^2 Z, Z = z^2 + x$$

$$5.) Z_u, Z_v \text{ qua } Z = x \cos(1+y^2), x = u+v, y = u-v$$

$$C = \cos(1+(u-v)^2), S = \sin(1+(u-v)^2).$$

$$Z_u = C(-2(u+v)(u-v))S, Z_v = C+2(u+v)(u-v)S.$$

$$6) \text{ Käymä } \frac{dy}{dx} \text{ eellä } y = y(x)$$

$$\arctan(x+y) = x, \frac{1}{1+(x+y)^2} (1+y') = 1 \Rightarrow 1+y' = 1+(x+y)^2$$

$$y' = (x+y)^2$$

$$7.) x^3 + y^3 + z^3 - 3xyz = 48 \text{ morske } M_0(2, 1, 1)$$

$$zx = \frac{y^2 - x^2}{z^2 - xy}, \quad zy = \frac{xz - y^2}{z^2 - xy}$$

$$M_0: zx = 3, zy = -1$$

$$8.) u = \frac{y}{x}$$

$$x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0$$

$$u = \frac{y}{x}, \quad u_{xx} = \frac{2y}{x^3}, \quad u_{xy} = -\frac{1}{x^2}, \quad u_{yy} = 0.$$

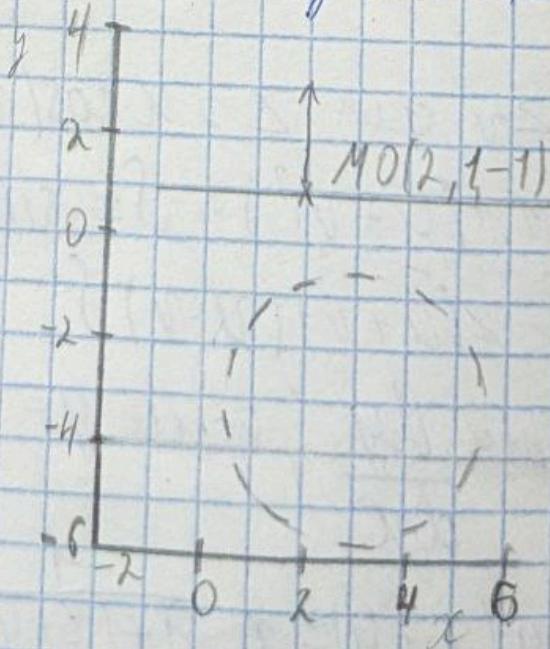
$$\frac{2y}{x} - \frac{2y}{x} = 0$$

$$9.) S: x^2 + y^2 + z^2 - 4x + 6z + 8 = 0 \text{ morske } M_0(2, 1, -1)$$

$$F_x = 2x - 4, F_y = 2y, F_z = 2z + 6. \quad F_x = 0, F_y = 2, F_z = 4$$

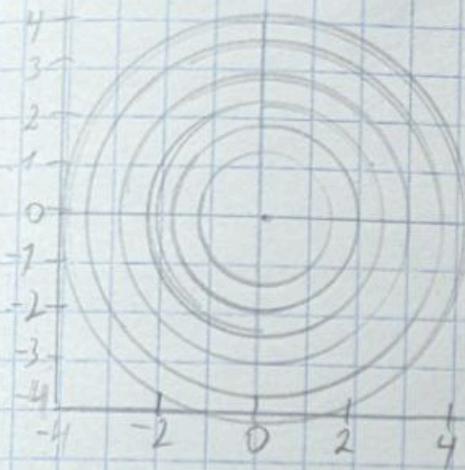
$$0 \cdot (x-2) + 2(y-1) + 4(z+1) = 0 \Rightarrow y + 2z + 1 = 0$$

$$h = [0, 2, 1]$$



$$10) S : z = x^2 + y^2$$

$$2x + 2y - 2 = 0$$

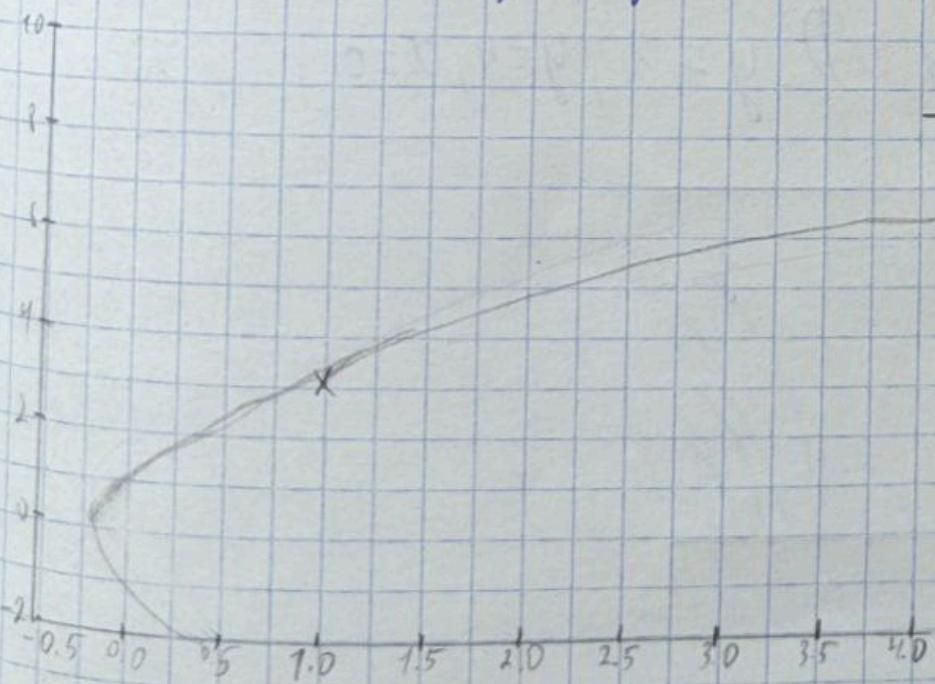


$$11.) z = \ln(x+y) \text{ für } M_0(1;3)$$

$$y^2 = 9x \quad \nabla z = \left(\frac{1}{x+y}, \frac{1}{x+y} \right)$$

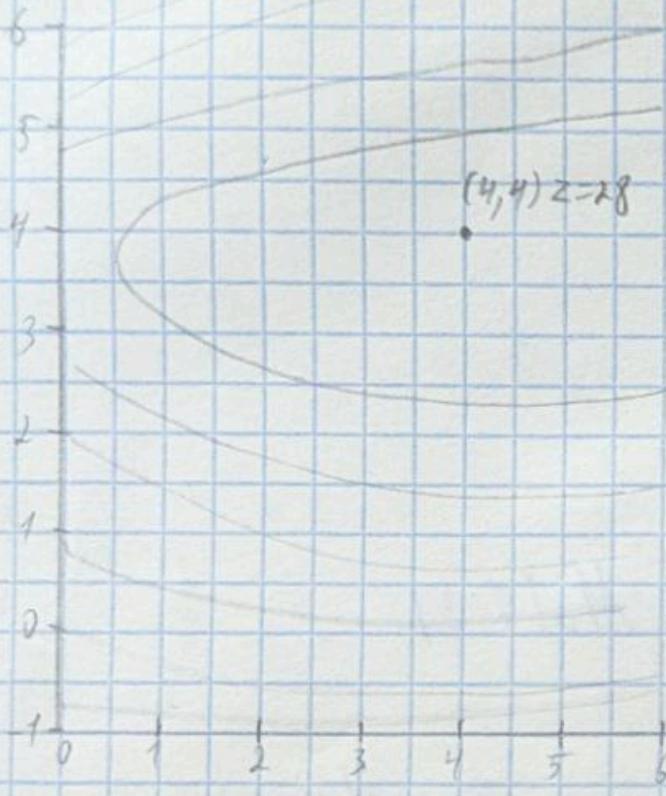
$\times M_0(1,3)$

$$y^2 = 9x$$



$$12.) Z = y\sqrt{x} - 2y^2 - x + 14y, \quad x \geq 0$$

$$\sqrt{x} = 2 \Rightarrow x = 4, y = 4$$



$$13.) Z = 3x + y - xy \quad D: y = x, y = 4, x = 0$$

