

Задача 10.

1. Вспомогательная.

$$1) Z = \frac{3\sqrt{xy}}{2x-5y}$$

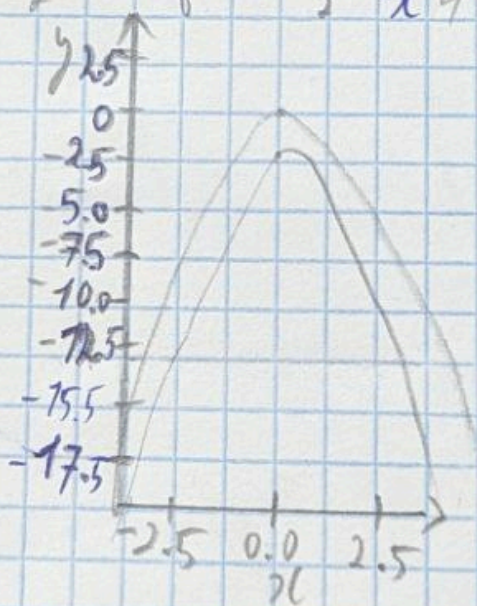
$$2x-5y \neq 0$$

$$2x-5y=0$$



$$2) Z = \sqrt{1-(x^2+y^2)}$$

$$-1-x^2 \leq y \leq 1-x^2$$



$$2) Z = \ln(y^2 - e^{-x})$$

$$Z_x = \frac{d}{dx} \ln(\dots) = \frac{1}{y^2 - e^{-x}} \cdot \frac{d}{dx} (y^2 - e^{-x}) = \frac{e^{-x}}{y^2 - e^{-x}}$$

$$Z_y = \frac{2y}{y^2 - e^{-x}}$$

$$dz = Z_x dx + Z_y dy = \frac{e^{-x}}{y^2 - e^{-x}} dx + \frac{2y}{y^2 - e^{-x}} dy$$

3) $\frac{dz}{dt}$ при $z = e^{x-2y}$, $x = \sin^2 t$, $y = t^3$, при $t = 0$

$$\frac{dz}{dt} = e^{x-2y} \left(\frac{dx}{dt} - 2 \frac{dy}{dt} \right), \frac{dx}{dt} = \sin 2t, \frac{dy}{dt} = 3t^2$$

при $t = 0$: $x = 0$, $y = 0$, $e^{x-2y} = 1$, $\frac{dx}{dt} = 0$, $\frac{dy}{dt} = 0 \Rightarrow$
 $\Rightarrow \frac{dz}{dt} \Big|_{t=0} = 0$

4) $\frac{du}{dz}$ при $u = x \cot(z^2 + x)$, $x = e^{\frac{1}{z}}$

$$\frac{du}{dz} = u_x \frac{dx}{dz} + u_z$$

где $Z = z^2 + x$

$$u_x = \cot Z - x \csc^2 Z, u_z = -2xz \csc^2 Z,$$

$$\frac{dx}{dz} = -\frac{x}{z^2} \quad (x = e^{\frac{1}{z}})$$

$$\frac{du}{dz} = \left(\cot Z - x \csc^2 Z \right) \left(-\frac{x}{z^2} \right) - 2xz \csc^2 Z, Z = z^2 + x$$

5) Z_u, Z_v при $Z = x \cos(1+y^2)$, $x = u+v$, $y = u-v$
 $C = \cos(1+(u-v)^2)$, $S = \sin(1+(u-v)^2)$.

$$Z_u = (-2(u+v)(u-v)S), Z_v = (+2(u+v)(u-v)S).$$

6) Найти $\frac{dy}{dx}$ если $y = y(x)$

$$\arctan(x+y) = x. \quad \frac{1}{1+(x+y)^2} [1+y'] = 1 \Rightarrow 1+y' = 1+(x+y)^2$$

$$y' = (x+y)^2$$

$$7.) x^3 + y^3 + z^3 - 3xyz = 4 \text{ в точке } M_0(2, 1, 1)$$

$$z_x = \frac{yz - x^2}{z^2 - xy}, \quad z_y = \frac{xz - y^2}{z^2 - xy}$$

$$M_0: z_x = 3, z_y = -1$$

$$8.) u = \frac{y}{x}$$

$$x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0$$

$$u = \frac{y}{x}, \quad u_{xx} = \frac{2y}{x^3}, \quad u_{xy} = -\frac{1}{x^2}, \quad u_{yy} = 0.$$

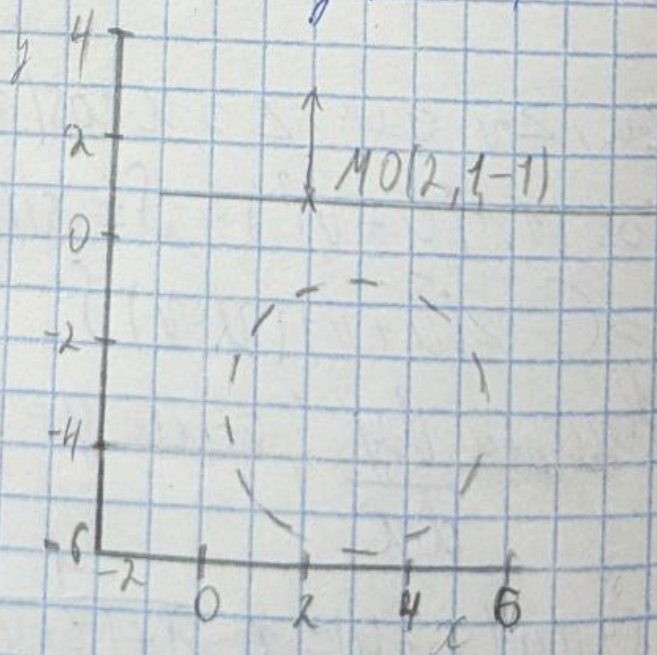
$$\frac{2y}{x} - \frac{2y}{x} = 0$$

$$9.) S: x^2 + y^2 + z^2 - 4x + 6z + 8 = 0 \text{ в точке } M_0(2, 1, -1)$$

$$F_x = 2x - 4, F_y = 2y, F_z = 2z + 6. \quad F_x = 0, F_y = 2, F_z = 4$$

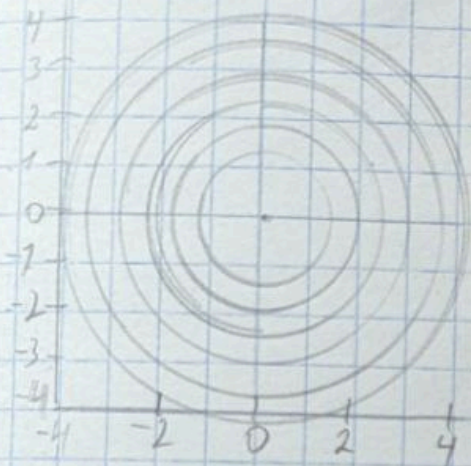
$$0 \cdot (x - 2) + 2(y - 1) + 4(z + 1) = 0 \Rightarrow y + 2z + 1 = 0$$

$$h = (0, 2, 4)$$



$$10) \int : z = x^2 + y^2$$

$$2x + 2y - z = 0$$

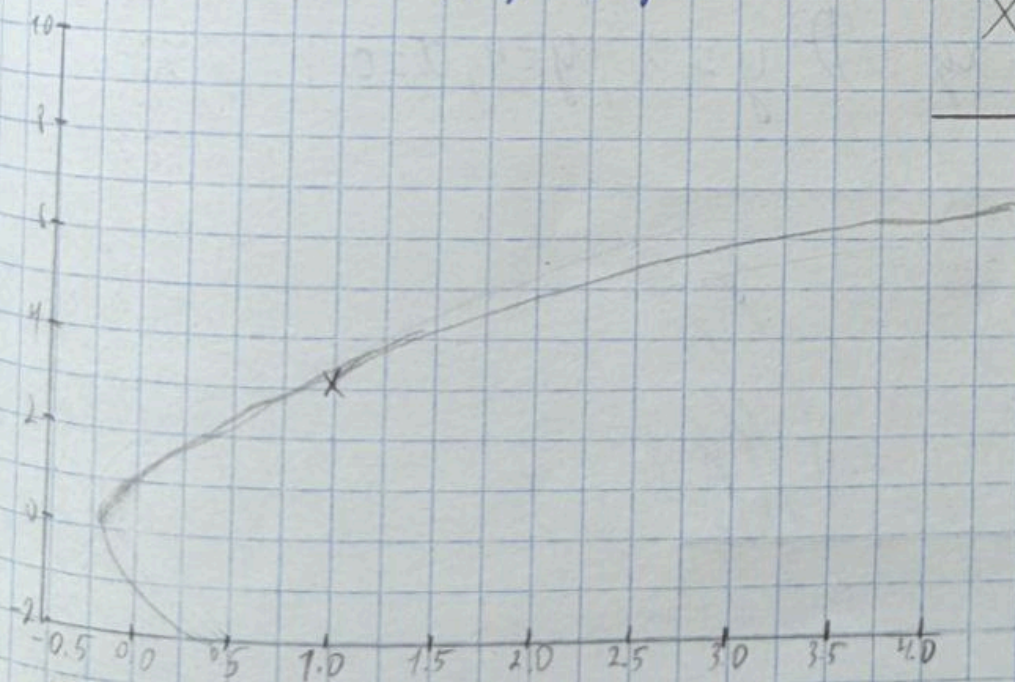


$$11.) z = \ln(x+y) \text{ в точке } M_0(1;3)$$

$$y^2 = 9x \quad \nabla z = \left(\frac{1}{x+y}, \frac{1}{x+y} \right)$$

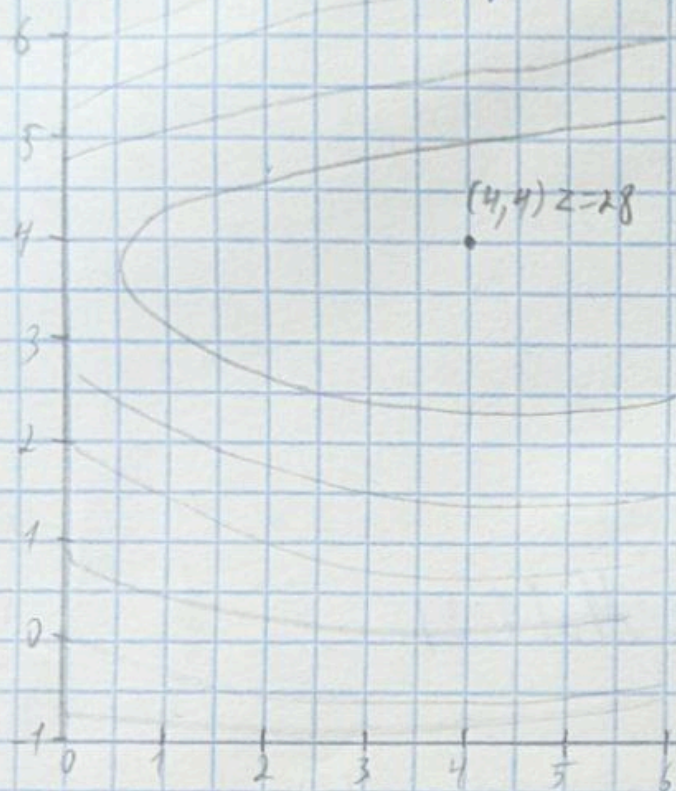
$\times M_0(1,3)$

— $y^2 = 9x$



$$12.) Z = y\sqrt{x} - 2y^2 - x + 14y, \quad x \geq 0$$

$$\sqrt{x} = 2 \Rightarrow x = 4, y = 4$$



$$13.) Z = 3x + y - xy \quad D: y = x, y = 4, x = 0$$

