$$u_{i-1/2,j+1} \qquad u_{i+1/2,j+1}$$

$$v_{i,j+1/2} \qquad v_{i+1,j+1/2}$$

$$u_{i-1/2,j} \qquad p_{ij} \qquad u_{i+1/2,j}$$

$$v_{i,j-1/2} \qquad v_{i+1,j-1/2}$$

$$\frac{u^*_{i+1/2,j} - u^n_{i+1/2,j}}{dt} = -\left(u^n u^*_x + v^n u^*_y\right)_{i+\frac{1}{2},j} + \left(u^*_{xx} + u^*_{yy}\right)_{i+1/2,j}$$

$$\left(u^*_x\right)_{i+1/2,j} = \frac{u^*_{i+3/2,j} - u^*_{i-1/2,j}}{2\Delta x}; \quad \left(u^*_y\right)_{i+1/2,j} = \frac{u^*_{i+1/2,j+1} - u^*_{i+1/2,j-1}}{2\Delta y}$$

$$\left(u^*_{xx}\right)_{i+1/2,j} = \frac{u^*_{i+3/2,j} - 2u^*_{i+1/2,j} + u^*_{i-1/2,j}}{\Delta x^2}; \quad \left(u^*_{yy}\right)_{i+1/2,j} = \frac{u^*_{i+1/2,j+1} - 2u^*_{i+1/2,j} + u^*_{i+1/2,j-1}}{\Delta y^2}$$

$$\left(v^n\right)_{i+\frac{1}{2},j} = \frac{1}{4}\left(v^n_{i,j-1/2} + v^n_{i+1,j-1/2} + v^n_{i,j+1/2} + v^n_{i+1,j+1/2}\right)$$
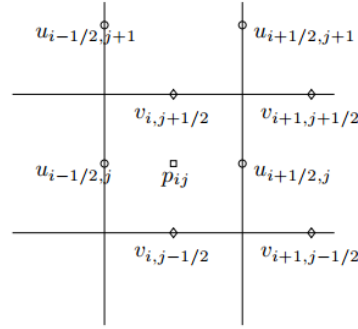
Let $b = \dfrac{u^n_{i+1/2,j}}{dt}$

$$\frac{u^*_{i+\frac{1}{2},j}}{dt} + \left(u^n_{i+1/2,j}\frac{u^*_{i+3/2,j} - u^*_{i-1/2,j}}{2\Delta x} + \frac{1}{4}\left(v^n_{i,j-1/2} + v^n_{i+1,j-1/2} + v^n_{i,j+1/2} + v^n_{i+1,j+1/2}\right)\frac{u^*_{i+\frac{1}{2},j+1} - u^*_{i+\frac{1}{2},j-1}}{2\Delta y}\right) - \frac{u^*_{i+\frac{3}{2},j} - 2u^*_{i+\frac{1}{2},j} + u^*_{i-\frac{1}{2},j}}{\Delta x^2} - \frac{u^*_{i+\frac{1}{2},j+1} - 2u^*_{i+\frac{1}{2},j} + u^*_{i+\frac{1}{2},j-1}}{\Delta y^2} = b$$

$$a_w u^*_{i-1/2,j} + a_e u^*_{i+\frac{1}{2},j} + a_{ee} u^*_{i+\frac{3}{2},j} + a_{en} u^*_{i+\frac{1}{2},j+1} + a_{es} u^*_{i+\frac{1}{2},j-1} = b$$

$$a_w = -\frac{u^n_{i+\frac{1}{2},j}}{2\Delta x} - \frac{1}{\Delta x^2}; \quad a_e = \frac{1}{dt} + \frac{2}{\Delta x^2} + \frac{2}{\Delta y^2}; \quad a_{ee} = \frac{u^n_{i+\frac{1}{2},j}}{2\Delta x} - \frac{1}{\Delta x^2}$$

$$a_{en} = \frac{\left(v^n_{i,j-1/2} + v^n_{i+1,j-1/2} + v^n_{i,j+1/2} + v^n_{i+1,j+1/2}\right)}{8\Delta y} - \frac{1}{\Delta y^2}$$

$$a_{es} = -\frac{\left(v^n_{i,j-1/2} + v^n_{i+1,j-1/2} + v^n_{i,j+1/2} + v^n_{i+1,j+1/2}\right)}{8\Delta y} - \frac{1}{\Delta y^2}$$

The grid diagram shows nodes: $u_{i-1/2,j+1}$, $u_{i+1/2,j+1}$, $v_{i,j+1/2}$, $v_{i+1,j+1/2}$, $u_{i-1/2,j}$, p_{ij}, $u_{i+1/2,j}$, $v_{i,j-1/2}$, $v_{i+1,j-1/2}$

$$\frac{v^*_{i,j+1/2} - v^n_{i,j+1/2}}{dt} = -\left(u^n v^*_x + v^n v^*_y\right)_{i,j+1/2} + \left(v^*_{xx} + v^*_{yy}\right)_{i,j+1/2} + \left(\frac{Ra}{2}\right)\left(T^n_{i,j} + T^n_{i,j+1}\right)$$

$$\left(v^*_x\right)_{i,j+1/2} = \frac{v^*_{i+1,j+1/2} - v^*_{i-1,j+1/2}}{2\Delta x}; \quad \left(v^*_y\right)_{i,j+1/2} = \frac{v^*_{i,j+3/2} - v^*_{i,j-1/2}}{2\Delta y}$$

$$\left(v^*_{xx}\right)_{i,j+1/2} = \frac{v^*_{i+1,j+1/2} - 2v^*_{i,j+1/2} + v^*_{i-1,j+1/2}}{\Delta x^2}; \quad \left(v^*_{yy}\right)_{i,j+1/2} = \frac{v^*_{i,j+3/2} - 2v^*_{i,j+1/2} + v^*_{i,j-1/2}}{\Delta y^2}$$

$$(u^n)_{i,j+1/2} = \frac{1}{4}\left(u^n_{i-1/2,j} + u^n_{i+1/2,j} + u^n_{i-1/2,j+1} + u^n_{i+1/2,j+1}\right)$$

Let $b = \dfrac{v^n_{i,j+\frac{1}{2}}}{dt} + \left(\dfrac{Ra}{2}\right)\left(T^n_{i,j} + T^n_{i,j+1}\right)$

$$\frac{v^*_{i,j+1/2}}{dt} + \left(\left(u^n_{i-\frac{1}{2},j} + u^n_{i+\frac{1}{2},j} + u^n_{i-\frac{1}{2},j+1} + u^n_{i+\frac{1}{2},j+1}\right)\frac{v^*_{i+1,j+\frac{1}{2}} - v^*_{i-1,j+\frac{1}{2}}}{8\Delta x} + v^n_{i,j+\frac{1}{2}}\frac{v^*_{i,j+\frac{3}{2}} - v^*_{i,j-\frac{1}{2}}}{2\Delta y}\right) - \frac{v^*_{i+1,j+\frac{1}{2}} - 2v^*_{i,j+\frac{1}{2}} + v^*_{i-1,j+\frac{1}{2}}}{\Delta x^2} - \frac{v^*_{i,j+\frac{3}{2}} - 2v^*_{i,j+\frac{1}{2}} + v^*_{i,j-\frac{1}{2}}}{\Delta y^2} = b$$
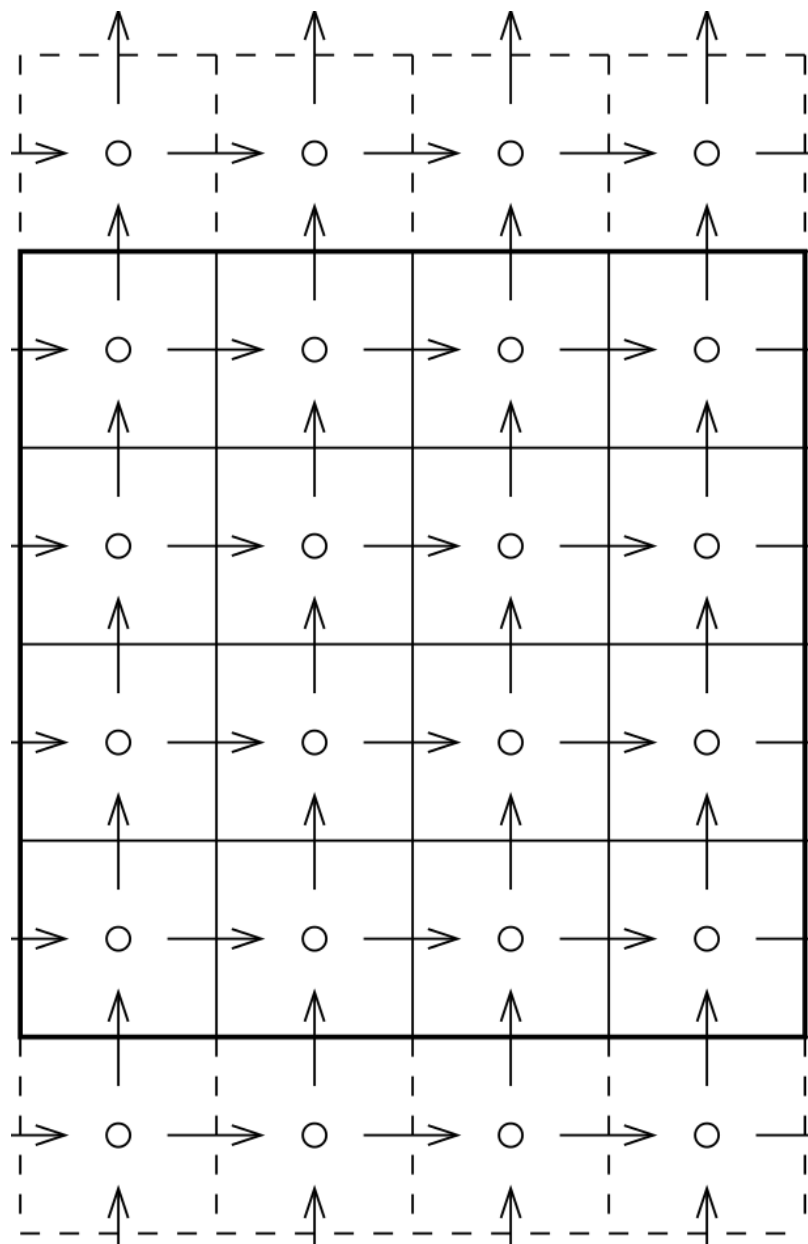
$$a_s v^*_{i,j-1/2} + a_n v^*_{i,j+1/2} + a_{nn} v^*_{i,j+3/2} + a_{ne} v^*_{i+1,j+1/2} + a_{nw} v^*_{i-1,j+1/2} = b$$

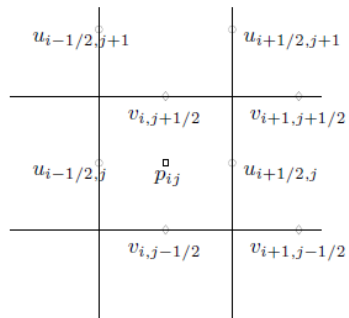$$a_s = -\frac{1}{\Delta y^2} - \frac{v^n_{i,j+\frac{1}{2}}}{2\Delta y}; \quad a_n = \frac{1}{dt} + \frac{2}{\Delta x^2} + \frac{2}{\Delta y^2}$$

$$a_{nn} = \frac{v^n_{i,j+\frac{1}{2}}}{2\Delta y} - \frac{1}{\Delta y^2};$$

$$a_{ne} = \frac{\left(u^n_{i-\frac{1}{2},j} + u^n_{i+\frac{1}{2},j} + u^n_{i-\frac{1}{2},j+1} + u^n_{i+\frac{1}{2},j+1}\right)}{8\Delta x} - \frac{1}{\Delta x^2}; \quad a_{nw} = -\frac{\left(u^n_{i-\frac{1}{2},j} + u^n_{i+\frac{1}{2},j} + u^n_{i-\frac{1}{2},j+1} + u^n_{i+\frac{1}{2},j+1}\right)}{8\Delta x} - \frac{1}{\Delta x^2}$$

$u_{i-1/2,j+1}$ $u_{i+1/2,j+1}$

$v_{i,j+1/2}$ $v_{i+1,j+1/2}$

$u_{i-1/2,j}$ $p_{ij}^{\square}$ $u_{i+1/2,j}$

$v_{i,j-1/2}$ $v_{i+1,j-1/2}$
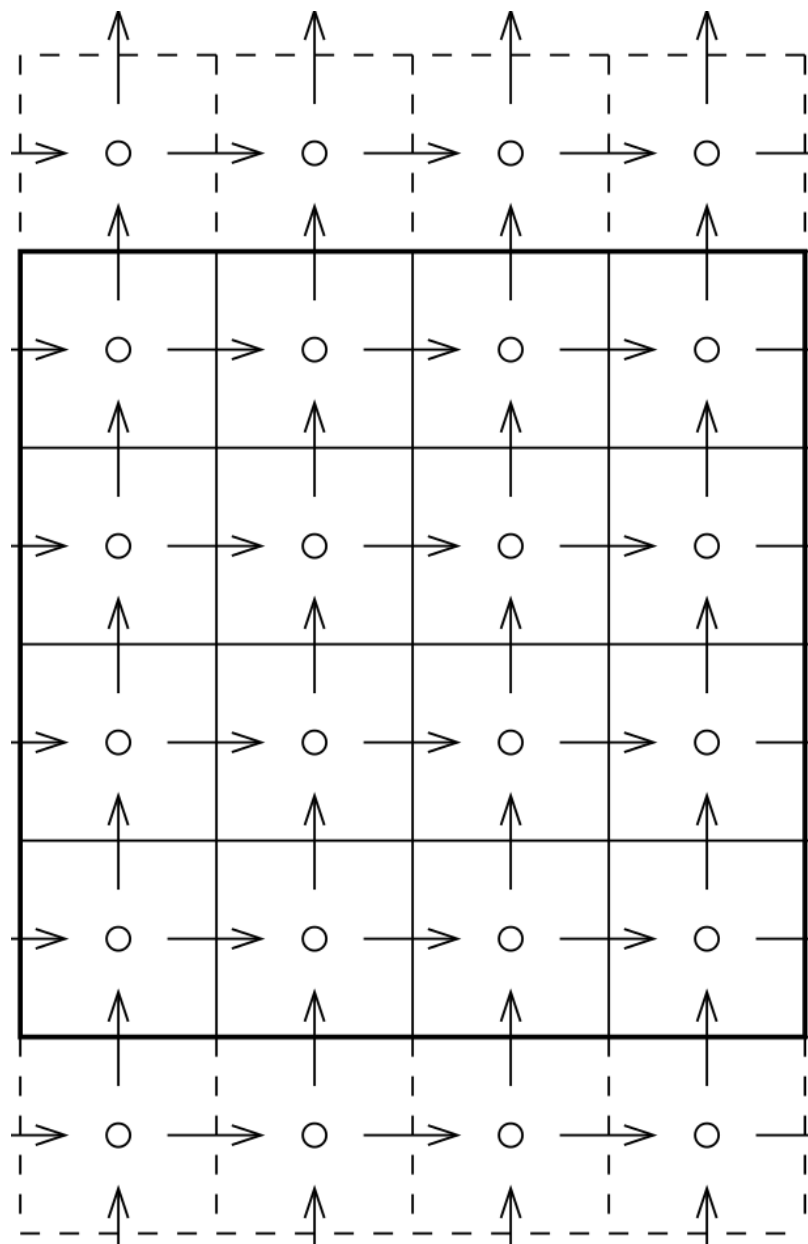
$$dt\left(P_{xx}^{n+1} + P_{yy}^{n+1}\right) = u_x^* + v_y^*$$

$$\frac{P_E - 2P_P + P_W}{\Delta x^2} + \frac{P_N - 2P_P + P_S}{\Delta y^2} = \frac{1}{dt}\left(\frac{u_e^* - u_w^*}{\Delta x} + \frac{v_n^* - v_s^*}{\Delta y}\right)$$

$$u^{n+1} = u^* - dt(P_x^{n+1})$$

$$u_{i+1/2,j}^{n+1} = u_{i+1/2,j}^* - \frac{dt}{\Delta x}(P_{i+1,j}^{n+1} - P_{i,j}^{n+1})$$

$$v^{n+1} = v^* - dt(P_y^{n+1})$$

$$v_{i+1/2,j}^{n+1} = v_{i+1/2,j}^* - \frac{dt}{\Delta y}(P_{i,j+1}^{n+1} - P_{i,j}^{n+1})$$

$$\frac{T_P^{n+1} - T_P^n}{dt} + u_{i,j}\frac{T_E^{n+1} - T_W^{n+1}}{2\Delta x} + v_{i,j}\frac{T_N^{n+1} - T_S^{n+1}}{2\Delta y} = \frac{T_E^{n+1} - 2T_P^{n+1} + T_W^{n+1}}{\Delta x^2} + \frac{T_N^{n+1} - 2T_P^{n+1} + T_S^{n+1}}{\Delta y^2}$$

$$(\frac{u_{i,j}}{2\Delta x} - \frac{1}{\Delta x^2})T_E^{n+1} + T_P^{n+1}\left(\frac{1}{dt} + \frac{2}{\Delta x^2} + \frac{2}{\Delta y^2}\right) + (-\frac{u_{i,j}}{2\Delta x} - \frac{1}{\Delta x^2})T_W^{n+1} + \left(\frac{v_{i,j}}{2\Delta y} - \frac{1}{\Delta y^2}\right)T_N^{n+1} + \left(-\frac{v_{i,j}}{2\Delta y} - \frac{1}{\Delta y^2}\right)T_S^{n+1} = T_P^n/dt$$