Rayleigh Bénard Convection **Dmitriy Kats**

Finite Difference Solution of Rayleigh Benard Convection 12/02/16

Rayleigh Bénard Convection is a natural-convection phenomenon, which occurs in the Earth's mantle. This phenomenon is well studied and in this report, I will simulate it using computational fluid dynamics.

Problem Statement

Rayleigh Bénard Convection is simplified as an infintie thin strip of fluid between two parallel plates. The top plate is cold while the bottom plate is heated. The problem setup and boundary conditions are included in Figure 1.

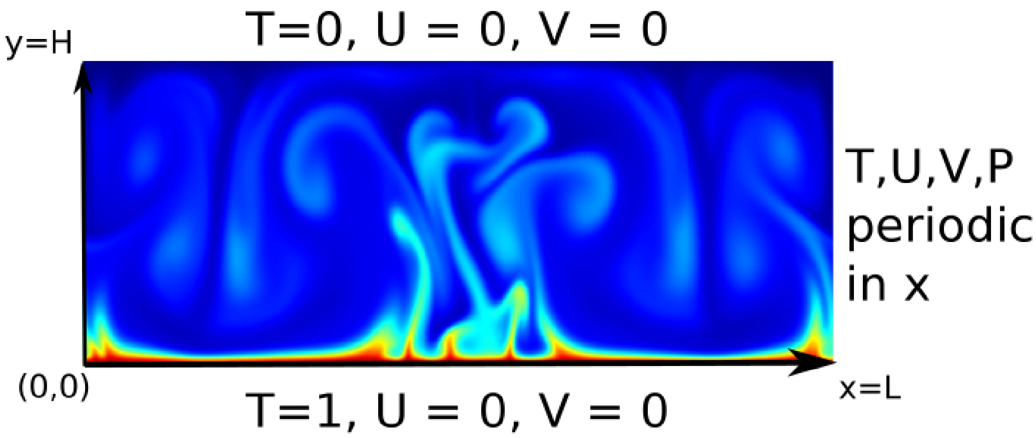

Figure 1: Rayleigh Bénard Convection Simplification and BCs

Governing Equations

The governing equations are a set of coupled equations including the Navier-Stokes equations and the energy equation. The momentum equation is:

$$\frac{1}{Pr}\left(\frac{\partial}{\partial t}\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = \nabla^2\mathbf{u} - \nabla P + RaT\mathbf{j} \tag{1}$$

and the energy equation is:

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot T = \nabla^2 T \tag{2}$$

and the mass conservation equation is:

$$\nabla\cdot\mathbf{u} = 0 \tag{3}$$

where Pr is the Prandtl number and Ra is the Rayleigh number.

Numerical Implementation

The Chorin Projection Scheme is used with different grids for the temperature and velocity. The numerical steps are described below.

1. Solve the momentum equation without the pressure term:

$$\frac{\mathbf{u}^* - \mathbf{u^n}}{dt} + \mathbf{u^n} \cdot \nabla \mathbf{u}^* = \nabla^2 \mathbf{u}^* + RaT^n \mathbf{j}$$

2. Solve for pressure and update velocity. The divergence of this equation is taken:

$$\frac{\mathbf{u^{n+1}} - \mathbf{u}^*}{dt} = -\nabla P^{n+1}$$

   and using mass conservation leads to:

$$\nabla^2 P^{n+1} = \frac{1}{dt} \nabla \cdot \mathbf{u}^*$$

   which is used to solve for the new pressure. This new pressure is then used to update the velocity with this equation:

$$\mathbf{u^{n+1}} = \mathbf{u}^* - dt \nabla P^{n+1}$$

3. Use the updated velocity in the energy equation:

$$\frac{T^{n+1} - T^n}{dt} + \mathbf{u^{n+1}} \cdot \nabla T^n = \nabla^2 T^{n+1}$$

Case Study

The case of $H = 0.5$ and $L = 1$ with $Ra = 5 \times 10^4$ is explored.

The evolution of the temperature field is shown in Figure 2.

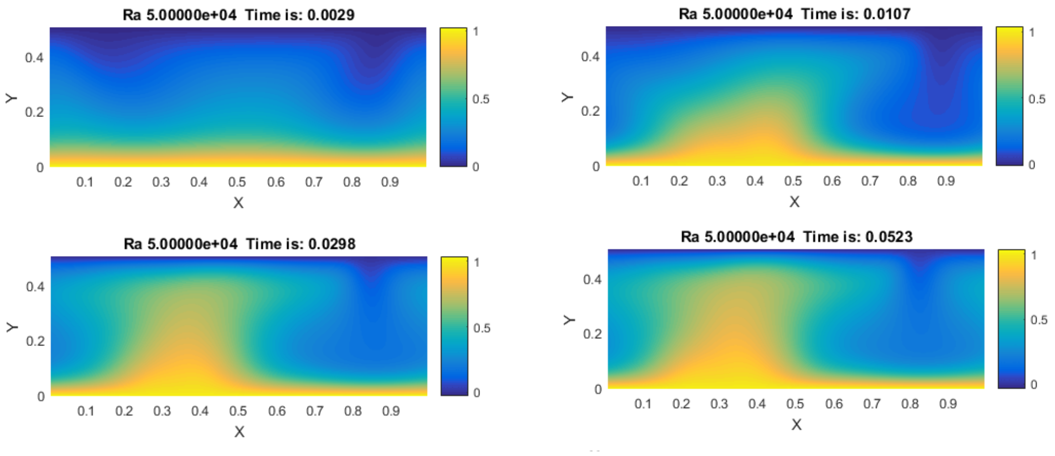

Figure 2: Rayleigh Bénard Temperature Evolution

The average Nusselt number is plotted below as well.

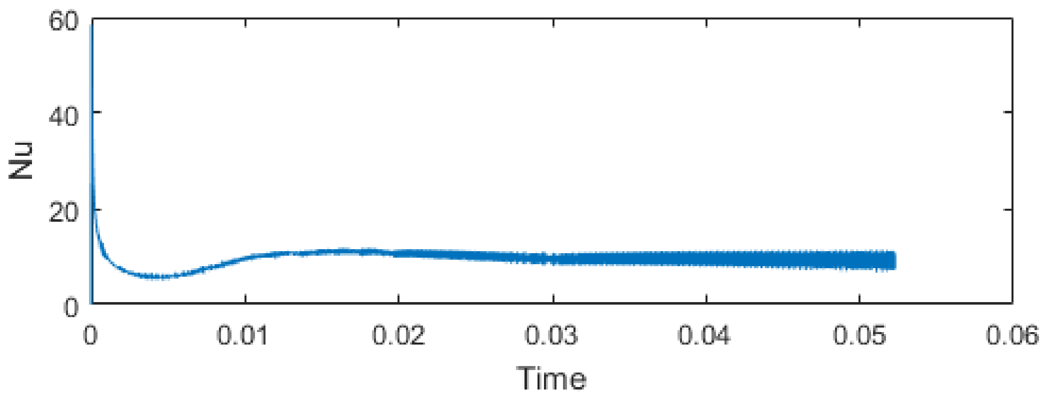

Figure 3: Rayleigh Bénard Average Nusselt Number