

AUSTRALIAN RESEARCH COUNCIL
ARC Future Fellowships
Application for Funding Commencing in 2024

FT

Project ID: FT240100267

First Investigator: Dr Dmitriy Zanin

Admin Org: The University of New South Wales

Total number of sheets contained in this Application: 35

Information on this form and its attachments is collected to make recommendations to the Minister on the allocation of financial assistance under the Australian Research Council Act 2001 (Cth) and for due diligence and post award reporting. The information collected may be passed to third parties, including being sent to overseas parties for assessment purposes. It may also be passed to any other Australian Government Department or Agency, where required, and noting information contained in this application can be disclosed without consent where authorised or required by law.

Part A - Administrative Summary (FT240100267)

A1. Application Title

(Provide a short title (up to 75 characters, approximately 10 words).)

Non-commutative Laplacians, quantum symmetries, and the Chern character

A2. Person Participant Summary

(Add the Future Fellowship candidate participating in this application.)

| Number | Name | Participant Type | Current Organisation(s) |
|--------|------------------|-------------------|-----------------------------------|
| 1 | Dr Dmitriy Zanin | Future Fellowship | The University of New South Wales |

A3. Organisation Participant Summary

(Add the Administering Organisations participating in this application. Refer to the Instructions to Applicants for further information.)

| Number | Name | Participant Type |
|--------|-----------------------------------|----------------------------|
| 1 | The University of New South Wales | Administering Organisation |

A4. Application Summary

(Provide an Application Summary, focusing on the aims, significance, expected outcomes and benefits of this project. Write the Application Summary simply, clearly and in plain English. If the application is successful, the Application Summary, along with the National Interest Test, will be used to give the general community an understanding of the research. Avoid the use of acronyms, quotation marks and upper-case characters. Refer to the Instructions to Applicants for further information (up to 750 characters, approximately 100 words).)

Non-commutative geometry is an exciting new way to think of quantization of classical systems from the geometric point of view. This project aims to develop a new approach to computing geometric and topological invariants of non-commutative manifolds. The project expects to generate new mathematical methods, results and examples in the field of non-commutative geometry and to promote Australian research in non-commutative analysis at the international level. The project includes building an international collaboration with research groups in USA and Germany. We apply this approach to a range of examples including quantum groups and non-flat non-commutative manifolds. This will establish Australia as a world leader in modern geometry.

A5. National Interest Test Statement

(See the Instructions to Applicants for addressing the National Interest Test and 'How do I write a National Interest Test Statement' available on the ARC website.)

Modern mathematics holds incredible power to describe the characteristics of the world around us, such as understanding patterns, quantifying relationships, and predicting the future. Pure mathematics builds the theoretical frameworks or language from which a clearer understanding of real world scenarios may emerge, and underpins all fundamental physical sciences and their application. This fundamental research project is focussed on the development of a new mathematical theory of the structure of objects. These novel theories will have future applications across a broad spectrum of sectors, including the emerging quantum computing industry, in which Australia is at the forefront, as well as in materials science, theoretical mathematics, physics and structural engineering.

Part B - Participant Details including ROPE (Dr Dmitriy Zanin)

B1. Personal Details

(To update any Personal Details, click on the 'Manage Personal Details' link below. Note this will open a new browser tab. When returning to the form ensure to 'Refresh' the page to capture the changes made to the participant's profile.

Note: The date of birth, country of birth, citizenship, material personal interests and Indigenous status section will not appear in the PDF version of the form and will not be visible to assessors.

Data may be shared with other Commonwealth Entities.

All information contained in Part B is visible to the Administering Organisation on this application.)

Participation Type

Future Fellowship

Title

Dr

First Name

Dmitriy

Middle Name

Zanin

B3. Qualifications

(To update any qualifications, click on the 'Manage Qualifications' link below. Note this will open a new browser tab. When returning to the form ensure to 'Refresh' the page to capture the changes made to the Future Fellowship candidate's profile.)

| Conferral Date | AQF Level | Degree/Award Title | Discipline/Field | Awarding Organisation | Country of Award |
|----------------|-----------------|--------------------|------------------|-----------------------------------|------------------|
| 02/06/2011 | Doctoral Degree | PhD | Mathematics | Flinders University | Australia |
| 26/08/2006 | Masters Degree | MSc | Mathematics | National University of Uzbekistan | Uzbekistan |
| 07/06/2004 | Bachelor Degree | BSc | Mathematics | National University of Uzbekistan | Uzbekistan |

B4. Research Load (non-ARC Grants and Research)

(Provide details of research funding from non-ARC sources (in Australia and overseas). For research funding from non-ARC sources, list all projects/applications/awards/fellowships awarded or requests submitted involving the Future Fellowship candidate for funding for the years 2023 to 2028 inclusive.)

Uploaded PDF file follows on next page.

B4 - Research Load (non-ARC Grants and Research)

| Description (All named investigators on any application or grant/fellowship in which a participant is involved, project title, source of support, scheme and round) | Same Research Area (Yes/No) | Support Status (Requested/Current/Past) | Application/ Project ID (for NHMRC applications only) | 2023 \$'000 | 2024 \$'000 | 2025 \$'000 | 2026 \$'000 | 2027 \$'000 | 2028 \$'000 |
|---|------------------------------------|---|---|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| N/A | | | | N/A | N/A | N/A | N/A | N/A | N/A |

B5. Research Opportunity and Performance Evidence (ROPE) - Current and previous appointment(s) / position(s) - during the past 10 years

(To update any details in this table, click on the 'Manage Employment Details' link in this question. Note this will open in a new browser tab. 'Refresh' the application page when returning to the form to capture changes made to the Future Fellowship candidate's profile.)

| Description | Department | Contract Type | Employment Type | Start Date | End Date | Organisation |
|----------------------|--|---------------|-----------------|------------|------------|-----------------------------------|
| Senior Lecturer | School of Mathematics and Statistics | Permanent | Full Time | 01/01/2022 | | The University of New South Wales |
| UNSW Scientia Fellow | School of Mathematics and Statistics, Department of Pure Mathematics | Permanent | Full Time | 01/07/2018 | 31/12/2021 | The University of New South Wales |
| DECRA Fellow | School of Mathematics and Statistics | Contract | Full Time | 01/07/2015 | 30/06/2018 | The University of New South Wales |
| Postdoctoral Fellow | School of Mathematics and Statistics | Contract | Full Time | 01/01/2012 | 30/06/2015 | The University of New South Wales |

B6. Research Opportunity and Performance Evidence (ROPE) - Career Interruptions

(You must read the ROPE Statement <https://www.arc.gov.au/about-arc/program-policies/research-opportunity-and-performance-evidence-rope-statement> before filling out this section.)

Has the Future Fellowship candidate experienced a significant interruption that has impacted on research opportunity?

No

Total Period of Career Interruptions

B7. Research Opportunity and Performance Evidence (ROPE) - Career Highlights

(Include up to 10 career highlights including a short context statement for each highlight, where relevant (up to 1500 characters, approximately 200 words).)

In 2016, I was invited to Oberwolfach to participate in their prestigious Research in Pairs program.

I am regularly invited to deliver keynote research talks in the top international conferences. In 2017, I delivered a plenary talk at Fields medallist Alain Connes' anniversary conference in Fudan (China). In 2019, I delivered an invited talk at International Workshop on Operator Theory and Applications. In 2021, I was invited by Professor Connes to deliver a plenary lecture concerning my recent results at the international conference "Cyclic cohomology at 40" at Fields Institute. In 2023 I delivered a plenary lecture in the international conference "Noncommutative Geometry, Index Theory and Representation Theory" in Kyoto.

In 2017 I received a Scientia Fellowship from UNSW (being the first Scientia Fellow in the field of Mathematics). This highly selective award was designed to promote and develop the most talented early to mid-career staff at UNSW, and helped to foster my development as a researcher.

Finally, I am very proud to have co-supervised the Ph.D.s - Galina Levitina, Edward McDonald, Jinghao Huang, and Thomas Sheckter. All of them received the dean's award for outstanding PhD theses, and have since gone on to become talented young researchers. Levitina has since gone on to be awarded both an ARC DECRA and Discovery Project. Huang was awarded a prestigious "Thousand Talents" grant in China.

B8. Research Opportunity and Performance Evidence (ROPE) - Details of the Future Fellowship candidate's career, evidence of research impact and contributions to the field, including those most relevant to this application and evidence of leadership, mentoring and research training or supervision.

(Provide details of the Future Fellowship candidate's research impact and contributions. This should not include information provided elsewhere in the application (upload a PDF of 1 A4 page).)

Uploaded PDF file follows on next page.

B8. Research Opportunity and Performance Evidence (ROPE) - Details of the Future Fellowship candidate's career, evidence of research impact and contributions to the field, including those most relevant to this application and evidence of leadership, mentoring and research training or supervision.

Amount of time as an active researcher I was awarded my PhD in Mathematics 12 years ago in 2011 at Flinders University of South Australia. During those 12 years, I have had no interruptions in research opportunity.

Research opportunities I am employed at UNSW as a Senior Lecturer in the School of Mathematics and Statistics.

Currently, I hold ARC DP 230100434 (\$ 415 000, started in 2023, resulted in 14 publications). In 2018-2021, I held UNSW Scientia Fellowship (\$ 160,000, resulted in 35 publication publications and promotion to a permanent position). In 2015-2018, I held ARC DECRA (\$ 300 000, resulted in 29 publications).

From 2011 until June 2015 I was employed as a researcher under DP grants DP110100064 and DP120103263 awarded to F. Sukochev (resulted in 17 publications). From 2007 until December 2010, I was a PhD student at Flinders University.

Research impact and contributions to the field I work in a broad area of Functional Analysis, more precisely, in (a) theory of singular traces (b) operator theory (c) probability theory (d) harmonic analysis (e) applications to mathematical physics. Below is an account of my research history and some of the most important contributions made in my career.

In 2021-2023, I published (in collaboration with S. Lord, E. McDonald and F. Sukochev) two-volume book "Singular traces" (vol. 1 "Theory" and vol. 2 "Trace Formulas"). This book is based on my previous journal publications and constitutes my contribution to the *Theory of Singular Traces*. In this area, I solved a long-standing problem by A. Pietsch on the spectrality of traces. My works in the area are published in e.g. in Crelle's Journal and Advances in Mathematics.

Starting from 2015, I published a number of articles in *Non-commutative Probability Theory*. In this area, I developed a new framework of distributional inequalities which allowed final solution to problems proposed by M. Junge and Q. Xu. Publication in this area appear e.g. in the Journal of Functional Analysis.

I intensively work in the field of *Non-commutative Geometry* (collaborating with A. Connes and N. Higson). My publications in the area appear e.g. in Communications in Mathematical Physics and the Journal of Functional Analysis. Among my achievements in this direction is the resolution of the problem proposed by A. Connes in 1994. This area of my research interests is closely related to the proposed Future Fellowship project.

Another area of my interest is *Operator Theory*. I obtained an ultimate resolution of the 60-year old problem by M. Krein on operator Lipschitz functions. My works in that area are published e.g. in the American Journal of Mathematics, Crelle's Journal and Proceedings of the London Mathematical Society.

I am also interested in the *Non-commutative Harmonic Analysis* (collaborating with R. Frank). My papers in this area are published e.g. in Transactions of the American Mathematical Society and in Mathematische Annalen. Some of my works in that direction are related to the proposed Future Fellowship project.

Another direction of my research is *Mathematical Physics* (collaborating with F. Gesztesy). My works in that area are published e.g. in the Memoirs of the European Mathematical Society.

I am one of the top contributors to the Journal of Functional Analysis (second in the last 3 years, top 10 in all times).

I am regularly invited to deliver keynote research talks in the top international conferences. In 2021, I was invited by Professor Connes to deliver a plenary lecture concerning my recent results at the international conference "Cyclic cohomology at 40" at Fields Institute. In 2023 I delivered a plenary lecture in the international conferences "Noncommutative Geometry, Index Theory and Representation Theory" in Kyoto.

I am regular assessor for the submissions to the best international journals such as Journal of Functional Analysis, Advances in Mathematics, Duke Mathematical Journal and Transactions of the American Mathematical Society.

I frequently collaborate with a number of the highest calibre researchers in Australia and overseas (Europe, US, China). My list of collaborators includes Alain Connes (Paris), Kenneth Dykema (Texas A&M), Rupert Frank (Munich), Friedrich Gesztesy (Vienna), Nigel Higson (Penn State), Marius Junge (Illinois), Albrecht Pietsch (Jena), Fedor Sukochev (UNSW) and Quanhua Xu (Harbin).

Many of these research achievements underpin the research proposed for this Future Fellowship, and demonstrate my insight and ability in solving hard problems in mathematics using novel methods that often extend to new results in considerably more general contexts.

Leadership and mentoring I mentor the group of postdocs at UNSW: (a) Thomas Scheckter (Non-commutative Harmonic Analysis), (b) Teun van Nuland (Non-commutative Geometry and Mathematical Physics), (c) Shiqi Liu (Semi-classical Analysis). (d) Dejian Zhou (Non-commutative Probability) (e) Edward McDonald (Non-commutative analysis). I have helped these postdocs to develop individual research programmes, and guided them in their own grant applications. Edward McDonald has gone on to receive a prestigious postdoc at Penn State University, for three years, and Teun van Nuland has gone on to work on a postdoc at Delft University.

B9. Research Opportunity and Performance Evidence (ROPE) - How many PhDs, Masters and Honours students that the Future Fellowship candidate has supervised have completed their degree?

(Provide total numbers under each category for completions where you have been the principal supervisor.)
PhD student completions as principal supervisor:

4

Masters student completions as principal supervisor:

0

Honours student completions as principal supervisor:

0

B10. Research Opportunity and Performance Evidence (ROPE) - Research Output Context

(The total word count available in this section is approximately 500 words (up to 3750 characters))

Research context – provide clear information that explains the relative importance of different research outputs and expectations in the Future Fellowship candidate’s discipline/s (up to 1,500 characters, or approximately 200 words). In addition, this section should also provide clear information regarding the research impact of the Future Fellowship candidate’s chosen ten career-best research outputs (listed in B11) (up to 30 words for each of the ten career-best research outputs, up to 2,250 characters, or approximately 300 words in total).)

The best measures of the quality of the mathematical research is feedback or acclaim by peers, including invitations to collaborate and present, and quality of the journals where publications appear.

In mathematics, the list of authors is always arranged alphabetically and does not indicate contribution. Citation rates in pure mathematics are lower than other fields, >400 citations for a Level C Research Fellow is outstanding, a typical Level E Professor in an Australian university will have >500 citations and >1000 citations for a Level E Professor is outstanding. The bibliometric database universally accepted in mathematics is MathSciNet, which includes citations from book and journal publications only and does not include arXiv citations. Typically it provides much lower citation rates than e.g. Google Scholar.

Journal quality: I consistently publish influential articles in high-profile mathematical journals. These journals are typically ranked A* or A by the Australian Mathematical Society. I have published 39 articles in A* ranked journals. Examples of these journals are: Crelle's Journal (the oldest mathematical journal still in publishing) where I have published 2 articles, Advances in Mathematics (among the top five journals in Pure Mathematics) where I have published 5 articles, Communications in Mathematical Physics (the most authoritative journal in the area) where I have published 2 articles, Journal of Functional Analysis (the primary journal in Functional Analysis) where I have published 22 articles, Ergodic Theory and Dynamical Systems, American Journal of Mathematics, Transactions of the AMS, Proceedings of the London Mathematical Society, and other excellent journals.

Output: According to MathSciNet, I have >100 articles published in international research journals and 1 monograph. In addition to that, there are 3 accepted articles and a large number of submissions. 52 of my publications are in the last 5 years. Having completed my PhD 12 years ago, many of my publications were published only a few years ago. I currently have 786 citations from >350 different authors. As a measure of the quantity and quality of my research collaborators, I have 44 co-authors who, between them, have >40000 citations on MathSciNet. Among these co-authors are luminaries such as Alain Connes (Fields laureate), Nigel Kalton and Kenneth Dykema as mentioned who have won numerous international prizes. Collaboration with mathematicians of such calibre is the highest honour and recognition in the field.

According to MathSciNet, my monograph has 186 citations. This is an excellent rate for a specialist reference published decade ago. As a comparison, the nearest text in terms of content, Barry Simon's "Trace ideals and their applications", has 680 citations over 45 years. 24 of my papers were cited at least 10 times. My citation rate is increasing each year.

B11-3 (American Journal of Mathematics, among top 5 journals in US) develops the framework of Double Operator Integrals. B11-2 (with Alain Connes) expands the interplay between the action of groups and the

Quantised Calculus. B11-1 (Mathematische Annalen, among best European journals) establishes the framework of the principal symbol. B11-4 (Proc. of the London Mathematical Society, the best journal in the UK) provides estimates which allow to transfer the methods designed for compact non-commutative spaces into non-compact ones. B11-7 (Letters in Mathematical Physics, one of the leading journals in Mathematical Physics) develops spectral asymptotic methods. B11-10 (Communications in Mathematical Physics) demonstrates the applicability of the CI's approach to the density of states, the central theme in Solid State Physics.

B11. Research Opportunity and Performance Evidence (ROPE) – 10 Career-Best Research Outputs

(Provide a list of 10 career best research outputs marking those that are most relevant to this application categorised under the following headings: Authored books; Edited books; Book chapters; Refereed Journal articles; Fully refereed conference proceedings; Additional research outputs (including non-traditional research outputs and preprints or comparable resources). CVs and theses should not be included in this list.)

Research Outputs Listing

Generated research output document follows on the next page

Ten Career-Best Research Outputs

- [1] * McDonald, Edward, Sukochev, Fedor & Zanin, Dmitriy 2019, 'A C^* -algebraic approach to the principal symbol II', *Math. Ann.*, vol. 374, no. 1-2, pp. 273–322, doi:10.1007/s00208-019-01822-7, FL170100052 (2018-2023) (Refereed Journal Article)
- [2] * Connes, A., Sukochev, F. A. & Zanin, D. V. 2017, 'Trace theorem for quasi-Fuchsian groups', *Mat. Sb.*, vol. 208, no. 10, pp. 59–90, doi:10.4213/sm8794, DP150100920 (2015-2019) (Refereed Journal Article)
- [3] * Caspers, M., Potapov, D., Sukochev, F. & Zanin, D. 2019, 'Weak type commutator and Lipschitz estimates: resolution of the Nazarov-Peller conjecture', *Amer. J. Math.*, vol. 141, no. 3, pp. 593–610, doi:10.1353/ajm.2019.0019, FL170100052 (2018-2023) (Refereed Journal Article)
- [4] * Levitina, Galina, Sukochev, Fedor & Zanin, Dmitriy 2020, 'Cwikel estimates revisited', *Proc. Lond. Math. Soc.* (3), vol. 120, no. 2, pp. 265–304, doi:10.1112/plms.12301, FL170100052 (2018-2023) (Refereed Journal Article)
- [5] Jiao, Yong, Sukochev, Fedor, Wu, Lian & Zanin, Dmitriy 2023, 'Distributional inequalities for noncommutative martingales', *J. Funct. Anal.*, vol. 284, no. 5, pp. Paper No. 109798, doi:10.1016/j.jfa.2022.109798 (Refereed Journal Article)
- [6] Sukochev, F. & Zanin, D. 2014, 'Which traces are spectral?', *Adv. Math.*, vol. 252, pp. 406–428, doi:10.1016/j.aim.2013.10.028, DP140100906 (2014-2016) (Refereed Journal Article)
- [7] * McDonald, E., Sukochev, F. & Zanin, D. 2022, 'A noncommutative Tauberian theorem and Weyl asymptotics in noncommutative geometry', *Lett. Math. Phys.*, vol. 112, no. 4, pp. Paper No. 77, 45, doi:10.1007/s11005-022-01568-5 (Refereed Journal Article)
- [8] Junge, M., Sukochev, F. & Zanin, D. 2017, 'Embeddings of operator ideals into L_p -spaces on finite von Neumann algebras', *Adv. Math.*, vol. 312, pp. 473–546, doi:10.1016/j.aim.2017.03.031, DP150100920 (2015-2019), DE150100030 (2015-2018) (Refereed Journal Article)
- [9] Dykema, Kenneth, Sukochev, Fedor & Zanin, Dmitriy 2015, 'A decomposition theorem in II_1 -factors', *J. Reine Angew. Math.*, vol. 708, pp. 97–114, doi:10.1515/crelle-2013-0084, DP150100920 (2015-2019) (Refereed Journal Article)
- [10] * Azamov, N., McDonald, E., Sukochev, F. & Zanin, D. 2020, 'A Dixmier trace formula for the density of states', *Comm. Math. Phys.*, vol. 377, no. 3, pp. 2597–2628, doi:10.1007/s00220-020-03756-7 (Refereed Journal Article)

B12. Currently held ARC Projects

(This information is automatically populated. If you have any concerns with the information recorded here, please contact your Administering Organisation's Research Office.)

| Identifier | Investigators | Admin Organisation | Project Title | Funding | End Date | Final Report Due Date | Final Report Status |
|-------------|--|-----------------------------------|---|-----------|------------|-----------------------|---------------------|
| DP230100434 | Dr Dmitriy Zanin ; Prof Fedor Sukochev ; Prof Nigel Higson | The University of New South Wales | A Functional Analysis of the Hypoelliptic Laplacian | \$415,000 | 31/12/2025 | 31/12/2026 | Draft |

B13. Relevant Qualification

(To update any qualifications, click on the 'Manage Qualifications' link in this question. This will open in a new browser tab. When returning to the form ensure to 'Refresh' the page to capture changes made to the candidate's profile.)

| Degree/Award Title | Awarding Organisation | Conferral Date |
|--------------------|-----------------------|----------------|
| PhD | Flinders University | 02/06/2011 |

B14. Has the Future Fellowship candidate been granted an extension by the Administering Organisation, to the eligibility period due to a significant career interruption as outlined in the Grant Guidelines?

(If the Future Fellowships candidate's qualification relevant to this application (listed in Question B13) was awarded prior to 1 March 2009 and they have had a significant career interruption (as listed in the Grant Guidelines), the candidate will need to seek an extension to the eligibility period through their Deputy Vice-Chancellor (Research).

If 'Yes' Questions B15 and B16 will be enabled and you will be required to specify the type(s) of career interruption claimed and the total period of the extension claimed.)

| |
|----|
| No |
|----|

B15. Select the category of career interruption claimed (more than one may be selected)

((Choose all types of career interruptions which have been claimed in the application for extension to the Future Fellowship candidate's qualification as certified by the Deputy Vice-Chancellor (Research) or equivalent.

Select a type of interruption and click 'Add'.))

| |
|--|
| |
|--|

B16. What is the total period of extension that the Future Fellowship candidate has claimed?

(Select the period of time which most closely equals the total period of extension claimed.)

| |
|--|
| |
|--|

B17. Does the Future Fellowship candidate hold a professional equivalent to a PhD as certified by the Administering Organisation?

(Where the Future Fellowship candidate does not hold a PhD, evidence must be provided to the Administering Organisation, and the Administering Organisation must certify that the candidate holds a professional equivalent to a PhD.)

No

B18. What is the Future Fellowship candidate's current academic level?

(Select the Future Fellowship candidate's current academic level from the menu below. If the Future Fellowship candidate is applying from an academic position outside Australia, please select the Australian academic level that corresponds to the Future Fellowship level of this application.)

Select the Future Fellowship candidate's current academic level from the drop-down below.

Level C

Part C - Project Description (FT240100267)

C1. Project Description

(Upload a Project Description as detailed in the Instructions to Applicants and in the required format. (Up to 7 A4 pages including references))

Uploaded PDF file follows on next page.

Project quality and innovation

Brief outline Differential equations describe the world around us, from the behaviour of black holes and supernovae (Einstein field equations) to subatomic particles (Schrödinger's equation), and from evolutionary biology (replicator equations) to how the leopard got its spots (reaction-diffusion systems). Given the diversity of differential equations, one may be surprised to find that only a small handful of mathematical tools are necessary to build up this remarkable field. Amongst such tools, the Laplacian is arguably the most important, describing the movement of heat and fluids, the behaviour of electric fields, and more recently having found application in image processing and machine learning. Here our interest lies in heat kernels, which describe the spectrum (resonant frequencies) of the Laplacian and its generalisations.

Unfortunately, the Laplacian is not panacea, and understanding the behaviour of quantum mechanics suggests to us that more involved mathematical tools inspired by these simple ideas are necessary. In the same way that you should not put on your shoes before your socks, Heisenberg's uncertainty principle tells us that you get different results depending on the order in which you measure momentum and position at the level of the sub-atomic. Mathematically, this is governed by the field of "Non-commutative Analysis", the study of objects in which changing the order changes the outcome.

The field arose in response to the then nascent field of quantum mechanics, seeking to establish a mathematical framework which could describe these strange new behaviours. In 1930, John von Neumann introduced "Rings of Operators", now called von Neumann algebras, to provide a tool with which we could describe quantum systems. This was followed by the development of non-commutative topology and integration in the '40s, '50s, and '60s, which sought to understand this strange new mathematical landscape, and culminated in Alain Connes' theory of non-commutative geometry in the '80s, which extended the purely commutative world of differential geometry to the non-commutative.

Connes' non-commutative geometry sets the scene for the project at hand, wherein we study the mathematical properties of non-commutative extensions of the Laplacian.

Our first objective is to develop a Connes Character Formula, which states an equality between cocycles in non-commutative Riemannian geometry and conformal geometry. While the formula was first stated in Connes' book "Noncommutative Geometry", he provided no proof, leading to a flurry of research in trying to fill this gap, with mixed success. For compact non-commutative manifolds, the final word is due to the CI and co-authors [4], and here we seek to extend these results to quantum groups.

Our second objective is to extend the Minakshisundaram–Plejel theorem to the non-commutative, providing a suitable analogue for the heat kernel expansion. In the commutative world, this describes the eigenvalues (which one may think of as resonant frequencies) of the Laplacian, and we seek to do the same for the non-commutative.

Connes and collaborators managed to calculate the curvature term of the non-commutative heat expansion for the simplest non-flat non-commutative manifold, but only through tremendously difficult effort. Appealing to the theory of Double Operator Integrals, the CI and co-authors have developed a breakthrough method to calculate the curvature term for more general non-commutative manifolds, whilst greatly simplifying the complexity of the work.

Our final objective is to construct a spectral triple (the suitable description of non-commutative geometries) from a given quantum group. While Lie groups describe the geometrical structure for much of classical mechanics, quantum groups provide a non-commutative generalisation suitable for the structure of quantum mechanics. Prior attempts to build such spectral triples have been met with two key failures — the dimension drop phenomena, in which spectral dimension is no longer equal to homological dimension, and that the first term of the heat kernel expansion does not recover the Haar state as it should. We propose a new construction which rectifies both these issues.

The CIs believes (and his publication list supports this) that all the stated objectives are plausible.

National/international progress in the field of research and its relationship to this Proposal At the heart of non-commutative geometry lies the Gelfand–Naimark theorem, which gives a categorical equivalence between locally compact Hausdorff spaces, and commutative C^* -algebras. This means that we are able to recover all of the information about topology and geometry in the setting of operator algebras, and naturally leads one to ask what happens when we perform the same calculations for non-commutative algebras. In much the same way, the results of von Neumann and Irving Segal allow us to translate measure and integration theory to operator algebras. Using these ideas, we may study geometry and integration in the non-commutative realm, giving us a mathematical framework in which to properly understand and describe the quantum world around us.

While these ideas are fairly simply stated, the mathematics is difficult and subtle. Attempts to describe non-commutative geometry stem back to von Neumann's original research in the '30s, but only successfully manifested with Connes' development of non-commutative geometry in the '80s. This gave a suitable non-commutative extension of differential geometry, built out of spectral triples, which encode the information carried by Riemannian manifolds.

How do we replace a Riemannian manifold and its metric tensor g ? Take a Hilbert space H , and let \mathcal{A} be a $*$ -algebra represented on H . The three components (\mathcal{A}, H, D) form a (compact) spectral triple if:

1. D is a self-adjoint bounded operator on H ;
2. For any operator $a \in \mathcal{A}$, the commutator $[D, a] = Da - aD$ has bounded extension;
3. And lastly, the operator $(D + i)^{-1}$ must be compact.

Here, the operator D (called the Dirac operator) stands in place of the metric tensor g . If the singular values of $(D + i)^{-1}$ decay proportionally to the sequence $(k^{-\frac{1}{d}})_{k=1}^{\infty}$, then the spectral triple is d -dimensional.

Given a Riemannian manifold (M, g) , we can build a spectral triple (\mathcal{A}, H, D) to recover its geometry as follows.

1. Let $\mathcal{A} = C^\infty(M)$ be the algebra of all smooth functions on M .
2. Let $H = L^2\Omega(M)$ be the space of all square integrable functions on M . The representation $\pi : \mathcal{A} \rightarrow B(H)$ is then given by pointwise multiplication.
3. The Dirac operator D is then given by the classical Hodge–Dirac operator (see [2], for example).

This gives a d -dimensional spectral triple, where d is the dimension of the original manifold M (as one should expect). Conversely, Connes' Reconstruction theorem [10] tells us that every spectral triple over a commutative algebra \mathcal{A} corresponds to some Riemannian manifold (M, g) . In light of this, spectral triples form non-commutative manifolds, and set the stage for a geometry of the quantum world.

In typical applications to harmonic analysis [23] and mathematical physics [20], a manifold is equipped with some isometric action of a Lie group. Moreover, in examples of serious interest and importance, the manifold will actually be a homogeneous space of some Lie group. For example, the sphere \mathbb{S}^{n-1} is a homogeneous space of the Lie group $SO(n)$. Hence it is natural to consider not just a manifold, but the manifold equipped with an isometric group action. In parallel, we may consider the action of a Lie group or a quantum group on a spectral triple. In this setting, it is most sensible to consider the actions of quantum groups, the natural non-commutative extension of Lie groups. The need for such groups arose in the early 1980s, trying to find a suitable notion of duality to extend the Pontrjagin duality theorem to non-abelian groups. In response to these questions, Woronowicz developed a general theory of quantum compact groups, including an extension of the Peter–Weyl theory. The most famous example of one of these quantum groups is the group $SU_q(n)$, the q -deformation of the Lie group $SU(n)$.

The rich and extensive interplay between Lie groups and differential geometry naturally begs the question — what is the interaction between quantum groups and spectral triples? Initial attempts to reconcile these structures, such as [5, 28], drew the attention of Connes (see [6]), who further developed the results of Chakraborty and Pal. These developments help to motivate the aims of this proposal.

In this project, we aim to construct spectral triples for certain quantum groups (e.g. compact quantum groups like $SU_q(n)$ and $SO_q(n)$), as well as non-compact quantum groups like $SL_q(n)$, and their homogeneous spaces. This would allow us to study the extension of major results in Non-commutative Geometry (such as Connes' Character Formula) for quantum groups, and even to be able to compute topological invariants (such as the K -theory) for these examples.

Returning to our starting point, we see that a spectral triple is naturally equipped with a semi-group action, given by the heat semigroup. If (M, g) is a d -dimensional Riemannian manifold, and (\mathcal{A}, H, D) the associated spectral triple, then D^2 is the Hodge–Laplace operator, denoted by Δ_g [31]. This suggests to us how we can use the Dirac operator for a given spectral triple to build a *non-commutative Laplacian*, the object which lies at the heart of this proposal. In this setting, the heat semi-group is now defined by $t \mapsto e^{-t\Delta_g}$, for any $t > 0$.

In his seminal work [35], Weyl proved that, for a compact manifold,

$$\lim_{t \downarrow 0} (4\pi t)^{\frac{d}{2}} \text{Tr}(e^{-t\Delta_g}) = \text{Vol}(M). \quad (1)$$

Following Weyl's work, it became an established custom to measure various geometric (and often topological) quantities associated with a Riemannian manifold M in terms of its heat semi-group expansion $t \mapsto e^{-t\Delta_g}$, $t > 0$. The mere existence of such an expansion is a famous theorem of Minakshisundaram and Plejel (among all approaches to that theorem, a particularly detailed account is given in [31]).

For every $f \in C^\infty(M)$, the Minakshisundaram–Plejel theorem asserts an existence of an asymptotic expansion

$$\text{Tr}(M_f e^{-t\Delta_g}) \approx (4\pi t)^{-\frac{d}{2}} \cdot \sum_{n \geq 0} a_n(f) t^n, \quad t \downarrow 0. \quad (2)$$

Here, d is the dimension of M and $M_f : L_2(M) \rightarrow L_2(M)$ is the operator of pointwise multiplication by f . Moreover, there exist functions $A_k \in C^\infty(M)$ such that

$$a_k(f) = \int_M A_k \cdot f d\text{vol}_g, \quad (3)$$

where vol_g is the Riemannian volume on M .

As follows from (1), $A_0 = 1$. Further computations (see e.g. Proposition 3.29 in [31]) show that $A_1 = \frac{1}{6}R$, where R is the scalar curvature. In particular, $a_1(1)$ is the Einstein–Hilbert action (see e.g. [8]).

Note that a_0 extends to a normal state h on $L_\infty(M)$ by the obvious formula

$$h(f) = \int_M f d\text{vol}_g, \quad f \in L_\infty(M).$$

Equation (3) can be re-written as

$$a_k(f) = h(A_k \cdot f), \quad f \in C^\infty(M).$$

One of the *primary targets* of this project is to find suitable extensions of the Minakshisundaram–Plejel theorem (and, consequently, of the Weyl theorem) for a vast class of non-commutative manifolds (such as non-commutative tori with generic, non-flat, metric tensor). This grand program began with [17] (although this was only published in 2011, the concepts and theorems therein stretch back to the '90s), where special 2–dimensional non-commutative manifolds (conformal deformations of a flat non-commutative torus) were considered. The authors of [17] proved that Euler characteristic of such manifold is 0 by means of the Gauss–Bonnet theorem (which asserts that the Euler characteristic of the 2–dimensional Riemannian manifold equals the average of its scalar curvature). Subsequently, the authors were able to explicitly calculate the scalar curvature for these manifolds in [15] and [19]. Later, the term a_2 (the first place where the Riemann curvature tensor manifests itself beyond the scalar curvature) was computed in [12] (intermediate computations include about a million terms!).

The process for these calculations, as in [17], is as follows. Let (\mathcal{A}, H, D) be a d -dimensional spectral triple, and for simplicity we assume \mathcal{A} to be unital.

Step 1: Verify that the following generalisation of (2) (the asymptotic heat expansion) holds:

$$\text{Tr}(xe^{-tD^2}) \approx (4\pi t)^{-\frac{d}{2}} \sum_{n \geq 0} a_n(x) t^n, \quad t \downarrow 0, \quad x \in \mathcal{A}''.$$

Step 2: Verify the normality of coefficients as in (3) with respect to the volume state, or, more precisely, to show that

$$a_k(x) = h(xA_k), \quad x \in \mathcal{A}'' , \text{ for some } A_k \in \mathcal{A}'', \quad k \geq 0.$$

Step 3: Compute A_k explicitly. (Far more easily said than done.)

When this mission is accomplished, one can *define* a scalar curvature of a non-commutative manifold by setting $R = 6A_1$.

We acknowledge that this program sets reasonable objectives but we claim that the tools based on pseudo-differential calculus in various guises applied up-to-date to accomplish them have been inadequate. Our main technical innovation which we bring here is based on the novel Double Operator Integration techniques developed by the CI recently in close collaboration with Alain Connes and Fedor Sukochev. The particular integral representations (see Approach to Aims 1,2,3 below) arose in [16] when geometric measures on limit sets of Quasi-Fuchsian groups were recovered by means of singular traces. They form the core contribution of UNSW team (including the CI) to [16].

One of the fundamental tools in noncommutative geometry is the Chern character. The Connes Character Formula provides an expression for the class of the Chern character in Hochschild cohomology, and it is an important tool in the computation of the Chern character. The formula has been applied to many areas of noncommutative geometry and its applications: such as the local index formula [14], the spectral characterisation of manifolds [10] and recent work in mathematical physics [11]. We point out to its applications in [10] as particularly relevant to the theme of this application.

In its original formulation, [7], the Character Formula is stated as follows: Let (\mathcal{A}, H, D) be a p -dimensional compact spectral triple with (possibly trivial) grading Γ . By the definition of a spectral triple, for all $a \in \mathcal{A}$ the commutator $[D, a]$ has an extension to a bounded operator $\partial(a)$ on H . Assume for simplicity that $\ker(D) = \{0\}$ and set $F = \text{sgn}(D)$. For all $a \in \mathcal{A}$ the commutator $[F, a]$ is a compact operator in the weak Schatten ideal $\mathcal{L}_{p,\infty}$ (see e.g. [8, 26]).

Consider the following two linear maps on the algebraic tensor power $\mathcal{A}^{\otimes(p+1)}$, defined on an elementary tensor $c = a_0 \otimes a_1 \otimes \cdots \otimes a_p \in \mathcal{A}^{\otimes(p+1)}$ by setting

$$\text{Ch}(c) := \frac{1}{2} \text{Tr}(\Gamma F [F, a_0] [F, a_1] \cdots [F, a_p]), \quad \Omega(c) := \Gamma a_0 \partial a_1 \partial a_2 \cdots \partial a_p.$$

Then the Connes Character Formula states that if c is a Hochschild cycle then

$$\mathrm{Tr}_\omega(\Omega(c)(1 + D^2)^{-p/2}) = \mathrm{Ch}(c)$$

for every Dixmier trace Tr_ω . In other words, the multilinear maps Ch and $c \mapsto \mathrm{Tr}_\omega(\Omega(c)(1 + D^2)^{-p/2})$ define the same class in Hochschild cohomology. We mention, in passing, that the generality of the result just stated was achieved fairly recently by the CI and his co-authors, see [4].

There has been great interest in generalising the tools and results of noncommutative geometry to the “non-compact” (i.e., non-unital) setting. The definition of a spectral triple associated to a non-unital algebra originates with Connes [9], was furthered by the work of Rennie [30] and Gayral, Gracia-Bondía, Iochum, Schücker and Varilly [21]. Earlier, similar ideas appeared in the work of Baaj and Julg [1]. Additional contributions to this area were made by Carey, Gayral, Rennie and Sukochev [3]. The conventional definition of a non-compact spectral triple is to replace the condition that $(D + i)^{-1}$ be compact with the assumption that for all $a \in \mathcal{A}$ the operator $a(D + i)^{-1}$ is compact. This raises an important question: is the Connes Character Formula true for locally compact spectral triples? This question was suggested to the CI by Professor Connes himself during Shanghai conference celebrating the 70-th anniversary of A. Connes (held at Fudan University). During lengthy discussions covering a substantial range of topics of importance in non-commutative geometry, Professor Connes, in particular had emphasized to the CI that such an extension could be an excellent starting point for several new directions in non-commutative geometry. Here are two such directions. Firstly, the Connes Character Formula plays an important role in the reconstruction theorem for closed Riemannian manifolds, and it is natural to expect that its extension would play a similar role for locally compact Riemannian manifolds. Secondly, developing suitable new techniques needed for a self-contained theory of locally compact spectral triples should also open an avenue for treating the case of noncommutative manifolds with boundary or even incomplete (e.g. punctured) manifolds. This suggestion by Professor Connes has been taken seriously by the CI and preliminary work in this direction has already brought substantial fruits. The ground-breaking manuscript co-authored by Professor Connes and the CI (and a number of collaborators from UNSW) the new approach to spectral triples involving *symmetric, non-self-adjoint* operators has been developed [13]. These are precisely the required tools allowing the possibility to develop a new theory for non-commutative Riemannian manifolds with boundary. This development indicates the importance and timeliness of the present proposal which has already achieved substantial progress and leads to a unified theory of locally compact non-commutative manifolds with boundary and incomplete manifolds. We emphasize that the progress achieved involves joint work with luminaries like Alain Connes, and local top experts in noncommutative analysis and geometry like Alan Carey, Adam Rennie and Fedor Sukochev.

Significance of the project This project intertwines the small scale geometry (studies in terms of calculus and differential equations) with global geometry (which aims to comprehend the shape of a manifold). The significance of the specific Aims proposed below is to establish links between different fields of mathematics (in particular, between Operator Algebras and Differential Geometry).

Heat semi-group in the classical Differential Geometry provides a solution of a parabolic PDE. For non-commutative manifold, heat semi-group essentially delivers the time evolution of an irreversible quantum system whose Hamiltonian is a (non-commutative) Laplace-Beltrami operator.

The project will concentrate on non-commutative geometries with high degree of symmetry provided by quantum groups, the feature which is paramount both in Mathematics and Physics. The research impact of this project will be enhancement of the Australian profile in the crucial area of Quantised Calculus and its applications to Non-commutative Geometry. The project will maintain and foster international collaborative links already built by the CI (Connes, Junge, Higson, Dykema) simultaneously contributing to training the next generation of Australian mathematicians. Certain parts of this proposal were discussed in the past 3 years with each of just listed collaborators in order to ensure that Aims below are cutting edge research.

Generally speaking, the development of noncommutative geometry and its applications is hindered by the paucity of non-trivial examples demonstrating its richness and ability to *compute* quantities of geometric significance for genuinely non-commutative manifolds. The project will generate an original approach to this computational task which will interact with and improve on the work of top class practitioners of Non-commutative Analysis/Geometry such as Marius Junge (University of Illinois) and Nigel Higson (Penn State University). The CI is in regular contact with both (they both visited UNSW during last years) and this collaboration is to continue unabated.

The project will have scholarly impact on the crucial parts of Non-commutative Differential Geometry with links to several other areas of Mathematics.

Aims of the project There are 7 specific aims.

Aim 1: Investigate when Chern character provides an asymptotic expansion for the heat semi-group (arising from a locally compact spectral triple). More precisely, when

$$\mathrm{Tr}(\Omega(c)e^{-s^2 D^2}) = \mathrm{Ch}(c)s^{-p} + O(s^{1-p}), \quad s \downarrow 0, \quad (4)$$

for every Hochschild cycle $c \in \mathcal{A}^{\otimes(p+1)}$? It seems plausible that Hochschild cochain on the left hand side is cohomologous to the one in the right hand side (modulo $O(s^{1-p})$). We expect to achieve Aim 1 during Year 1.

Aim 2: Aim 1 above is closely related (albeit, not equivalent) to a question concerning analyticity of a suitable ζ -function. The latter is an analytic function defined by the formula

$$z \rightarrow \text{Tr}(\Omega(c)(1 + D^2)^{-\frac{z}{2}}), \quad \Re(z) > p.$$

We aim to find an analytic extension to the half-plane $\Re(z) > p - 1$ so that

$$\lim_{z \rightarrow p} (z - p) \text{Tr}(\Omega(c)(1 + D^2)^{-\frac{z}{2}}) = p \text{Ch}(c).$$

Aim 3: To compute the Hochschild class of the Chern character by a “local” formula, which is customarily stated in terms of singular traces on the ideal $\mathcal{L}_{1,\infty}$ ($\mathcal{L}_{1,\infty}$ is the principal ideal in $B(H)$ generated by an operator with singular values $(\frac{1}{k})_{k \geq 1}$). Here, trace $\varphi : \mathcal{L}_{1,\infty} \rightarrow \mathbb{C}$ is a unitarily invariant linear functional; it can be seen from CI’s results in [26] that such a functional is automatically singular. Our third aim is to show that

$$\varphi(\Omega(c)(1 + D^2)^{-\frac{p}{2}}) = \text{Ch}(c). \quad (5)$$

for every (normalised) trace φ on $\mathcal{L}_{1,\infty}$ and for every Hochschild cycle $c \in \mathcal{A}^{\otimes(p+1)}$.

Aims 2 and 3 are intimately connected with Aim 1, however, the amount of analytical complications which arise when one navigates between formulas stated in all three aims is enormous and requires a very careful treatment and certainly warrants a separation of these aims.

We rely on the deep theory of singular traces and its connections with operator ζ -functions which the CI (with various collaborators) has been developing since 2009 [26,32]. The CI is in the unique position to apply numerous and well-developed techniques from that theory in order to achieve this aim. We expect to achieve Aim 2 during Year 2.

Aim 4: Introduce a generic (i.e., not necessarily a conformal deformation of a flat one) Laplace-Beltrami operator

$$\Delta_g x = M_{G^{-\frac{1}{2}}} \sum_{i,j=1}^d D_i M_{G^{\frac{1}{4}}(g^{-1})_{ij} G^{\frac{1}{4}}} D_j \text{ where } G^{-\frac{1}{2}} = (\det(g_{ij}))^{-\frac{1}{2}} \stackrel{\text{def}}{=} \pi^{-\frac{d}{2}} \int_{\mathbb{R}^d} e^{-\sum_{i,j} g_{ij} t_i t_j} dt,$$

on the non-commutative torus and non-commutative Euclidean space. Prove a non-commutative version of Minakshisundar Plejel theorem (for these manifolds) as outlined above.

Aim 4, in turn, depends on Aims 1 and 2, or rather on the technical instruments which are to be developed to deal with those aims in full generality.

Aim 5: Compute explicitly the curvature for a generic Riemannian metric on the non-commutative torus and non-commutative Euclidean space.

By using Double Operator Integration technique as developed in [29], we aim to prove analyticity of the right hand side and, hence, of the left hand side. The application of these techniques is our trump card in the joint work with Professor Connes [16], in which we have completed the work started by such giants as Connes and Sullivan, and which left dormant for more than 20 years, until our new techniques were brought to bear on that problem.

Aim 6: Construct a spectral triple (or possibly, a twisted spectral triple) on quantum groups (like $\text{SU}_q(n)$) and on their homogeneous spaces (like Podleś sphere). Previous attempts [5] suffer from a “dimension drop” pathology (that is, where spectral dimension differs from cohomological dimension). We aim to have an equivariance property for the Dirac operator in a way that prevents “dimension drop” pathology.

The spectral triple constructed in [5] is equivariant and its spectral dimension is 3. However, every 3-cocycle is cohomologous to 0 and, therefore, the homological dimension is strictly less than 3.

We expect that the reason for this phenomenon is the wrong choice of q -deformed left regular representation. Our aim is to find a suitable q -deformation which permit the spectral triple to be equivariant, 3-dimensional and, at the same time to allow certain 3-cocycles to be non-trivial. A natural candidate for such a 3-cocycle is the Hochschild class of the Chern Character. Successful resolution of Aims 1,2,3 will deliver the required technical tools to determine the “correct representation” and such cocycles.

Aim 7: Design a version of Connes Character Formula for the above spectral triples which recovers a non-trivial cocycle.

It is extremely probable that ordinary spectral triples in this setting should be replaced by twisted ones (where commutators $[D, a]$ may be unbounded, but a certain “twisted” commutator is bounded). At the moment, no satisfactory theory is available which allows the derivation of a Connes Character formula for such triples. The only attempt is made in [18]. However, we expect that a combination of approaches from [18] and [4] would yield (at least a germ of) such a theory.

Benefit

The CI expects the project to produce significant results and to publish them in the most reputed journals. In addition, the CI expects the project to be beneficial in the following ways.

a. Australia has strong research profile in Non-Commutative Geometry and Operator Algebras. This area develops rapidly and the CI enthusiastically contributes to this development. The outcomes of this project will therefore be of interest to a number of research groups in Australia (Wollongong, Adelaide, Sydney and Canberra). The project will broaden existing strengths and introduce new directions in a highly active and internationally competitive area of research endeavour.

b. The project will enhance international collaboration in research. The CI is collaborating with highly-distinguished international experts (Alain Connes, Kenneth Dykema, Nigel Higson, Marius Junge). This collaboration already resulted in a number of papers in prestigious journals. Involvement of mathematicians of such a calibre would be beneficial not only for this project, but also to other directions of mathematical research in UNSW.

It should be pointed out that Professor Connes is very enthusiastic about the suggested direction of research and this strong endorsement from the world leading expert and his on-going commitment to the joint research efforts is a strong acknowledgement of the depth and importance of the current research proposal.

c. Improve the international competitiveness of Australian research. The area of Quantised Calculus is the cutting edge research toolkit in the modern analysis. This proposal is on par with efforts of top world experts in the area and thus will strengthen the Australian leadership in this area.

d. Due to the breadth and depth of the proposal which involves groundbreaking research in several mathematical disciplines, the CI expects to attract the top Australian students for postgraduate studies who otherwise might be heading overseas.

Mentoring and capacity building The CI has already demonstrated his capacity to make significant, original and innovative contributions to wide range of Non-commutative Analysis and Non-commutative Geometry. He has co-authored the very first monograph on singular traces [26], which has albeit in embryonic form, some necessary components to attack Aim 3. The project presents a realistic timeframe as seen from above. Confidence in the feasibility is enhanced by the following:

1. The CI has already demonstrated his research credentials in the field of Noncommutative Analysis. His numerous papers in this area are published in the highly ranked journals like Crelle's Journal, Advances in Mathematics, Journal of Functional Analysis.

2. The CI wrote a number of publications in the field of Mathematical Physics. For example, paper [34] is published in prestigious journal Communications in Mathematical Physics.

3. The CI has co-authored a number of publications in the field of Classical Analysis.

4. The CI has authored a research monograph [26] jointly written with S. Lord and F. Sukochev. The substantial portion of this monograph describes CI's contribution to the field of Non-commutative Geometry and related parts of Quantised Calculus. At the moment, a second edition of the book is in preparation.

5. The CI can rely on support and expert advice from the members of the Noncommutative Analysis group at UNSW (M. Cowling, I. Doust, D. Potapov, F. Sukochev).

It should also be pointed out that some parts of the current proposal have been thoroughly discussed with Professor Alain Connes (Fields medalist) who is the originator of (and leading expert in) Non-commutative Geometry. Professor Connes strongly and enthusiastically endorsed the ideas and methods underlying this proposal and the approach.

Communication of results Results of this project will be published in peer-reviewed journals (as well as on arxiv). CI and collaborators will also deliver the results in thematic international conferences.

References

- [1] Baaj S., Julg P. *Théorie bivariante de Kasparov et opérateurs non bornés dans les C^* -modules hilbertiens*. C. R. Acad. Sci. Paris Sér. I Math. **296** (1983), no. 21, 875–878.
- [2] Berline N., Getzler E., Vergne M. *Heat kernels and Dirac operators*. Grundlehren der Mathematischen Wissenschaften, **298**. Springer-Verlag, Berlin, 1992.
- [3] Carey A., Gayral V., Rennie A., Sukochev F. *Integration on locally compact noncommutative spaces*. J. Funct. Anal. **263** (2012), no. 2, 383–414; Carey A., Gayral V., Rennie A., Sukochev F. *Index theory for locally compact noncommutative geometries*. Mem. Amer. Math. Soc. **231** (2014), no. 1085, vi+130 pp.
- [4] Carey A., Rennie A., Sukochev F., Zanin D. *Universal measurability and the Hochschild class of the Chern character*. J. Spectr. Theory **6** (2016), no. 1, 1–41.
- [5] Chakraborty P., Pal A. *Equivariant spectral triples on the quantum $SU(2)$ group*. K-Theory **28** (2003), no. 2, 107–126; Chakraborty P., Pal A. *Spectral triples and associated Connes-de Rham complex for the quantum $SU(2)$ and the quantum sphere*. Comm. Math. Phys. **240** (2003), no. 3, 447–456.

- [6] Connes A. *Cyclic cohomology, quantum group symmetries and the local index formula for $SU_q(2)$* . J. Inst. Math. Jussieu **3** (2004), no. 1, 17–68.
- [7] Connes A. *Geometry from the spectral point of view*. Lett. Math. Phys., **34** (3) 203–238, 1995.
- [8] Connes A. *Noncommutative geometry*. Academic Press, Inc., San Diego, CA, 1994.
- [9] Connes A., *Noncommutative geometry and reality*. J. Math. Phys. **36** (1995), 6194–6231.
- [10] Connes A. *On the spectral characterization of manifolds*. J. Noncommut. Geom. **7** (2013), no. 1, 1–82.
- [11] Chamseddine, A., Connes, A., Mukhanov, V. *Quanta of geometry: noncommutative aspects*. Phys. Rev. Lett. **114** (2015), no. 9, 091302, 5pp.
- [12] Connes A., Fathizadeh F. *The term a_4 in the heat kernel expansion of noncommutative tori*. Münster J. Math. **12** (2019), no.2, 239–410.
- [13] Connes A., Levitina G., McDonald E., Sukochev F., Zanin D. *Noncommutative geometry for symmetric non-self-adjoint operators*. J. Funct. Anal. **277** (2019), no.3, 889–936.
- [14] Connes A., Moscovici H. *The local index formula in noncommutative geometry*. Geom. Funct. Anal. **5** (1995), no. 2, 174–243.
- [15] Connes A., Moscovici H. *Modular curvature for noncommutative two-tori*. J. Amer. Math. Soc. **27** (2014), no. 3, 639–684.
- [16] Connes A., Sukochev F., Zanin D. *Trace theorem for quasi-Fuchsian groups*. Mat. Sb. **208** (2017); Connes A., McDonald E., Sukochev F., Zanin D. *Conformal trace theorem for Julia sets of quadratic polynomials*. Ergodic Theory Dynam. Systems **39** (2019), no. 9, 2481–2506.
- [17] Connes A., Tretkoff P. *The Gauss-Bonnet theorem for the noncommutative two torus*. Noncommutative geometry, arithmetic, and related topics, 141–158, Johns Hopkins Univ. Press, Baltimore, MD, 2011.
- [18] Fathizadeh F., Khalkhali M. *Twisted spectral triples and Connes’ character formula*. Perspectives on noncommutative geometry, 79–101, Fields Inst. Commun., **61**, Amer. Math. Soc., Providence, RI, 2011.
- [19] Fathizadeh F., Khalkhali M. *Scalar curvature for the noncommutative two torus*. J. Noncommut. Geom. **7** (2013), no. 4, 1145–1183; Fathizadeh F., Khalkhali M. *Scalar curvature for noncommutative four-tori*. J. Noncommut. Geom. **9** (2015), no. 2, 473–503.
- [20] di Francesco P., Mathieu P., Senechal D. *Conformal Field Theory*. Springer-Verlag, New York, 1997.
- [21] Gayral V., Gracia-Bondia J., Iochum B., Schücker T., Varilly J. *Moyal planes are spectral triples*. Comm. Math. Phys. **246** (2004), no. 3, 569–623.
- [22] Gracia-Bondia J., Varilly J., Figueroa H. *Elements of noncommutative geometry*. Birkhäuser Advanced Texts: Basler Lehrbücher. Birkhäuser Boston, Inc., Boston, MA, 2001.
- [23] Helgason S., *Differential geometry, Lie groups, and symmetric spaces*. Graduate Studies in Mathematics, **34**. American Mathematical Society, Providence, RI, 2001.
- [24] Junge M., Sukochev F., Zanin D. *Embeddings of operator ideals into L_p -spaces on finite von Neumann algebras*. Adv. Math. **312** (2017), 473–546.
- [25] Lesch M. *Divided differences in noncommutative geometry: rearrangement lemma, functional calculus and expansional formula*. J. Noncommut. Geom. **11** (2017), no. 1, 193–223.
- [26] Lord S., Sukochev F., Zanin D. *Singular traces. Theory and applications*. De Gruyter Studies in Mathematics, **46**. De Gruyter, Berlin, 2013.
- [27] Loreaux J., Weiss G. *Traces on ideals and the commutator property*, Operator theory: themes and variations: conference proceedings, Timisoara, June 27 - July 2, 2016 (Hari Bercovici, ed.), Theta series in advanced mathematics, vol. 20, Theta Society, Bucharest, 2018, p. 145–153.
- [28] Neshveyev S., Tuset L. *The Dirac operator on compact quantum groups*. J. Reine Angew. Math. **641** (2010), 1–20; Dabrowski L., Landi G., Sitarz A., van Suijlekom W., Varilly J. *The Dirac operator on $SU_q(2)$* . Comm. Math. Phys. **259** (2005), no. 3, 729–759.
- [29] Potapov D., Sukochev F. *Operator-Lipschitz functions in Schatten-von Neumann classes*. Acta Math. **207** (2011), no. 2, 375–389; Potapov D., Sukochev F. *Unbounded Fredholm modules and double operator integrals*. J. Reine Angew. Math. **626** (2009), 159–185.
- [30] Rennie A. *Smoothness and locality for nonunital spectral triples*. K-Theory **28** (2003), no. 2, 127–165; Rennie A. *Summability for nonunital spectral triples*. K-Theory **31** (2004), no. 1, 71–100.
- [31] Rosenberg S. *The Laplacian on a Riemannian manifold. An introduction to analysis on manifolds*. London Mathematical Society Student Texts, **31**. Cambridge University Press, Cambridge, 1997.
- [32] Sukochev F., Usachev A., Zanin D. *Singular traces and residues of the ζ -function*. Indiana U. Math. J., **66** (2017), no. 4, 1107–1144.
- [33] Sukochev F., Zanin D. *Which traces are spectral?*, Adv. Math. **252** (2014), 406–428.
- [34] Sukochev F., Zanin D. *Connes integration formula for the noncommutative plane*. Comm. Math. Phys. **359** (2018), no. 2, 449–466.
- [35] Weyl H. *Das asymptotische Verteilungsgesetz der Eigenwerte linearer partieller Differentialgleichungen (mit einer Anwendung auf die Theorie der Hohlraumstrahlung)*. Math. Ann. **71** (1912), no. 4, 441–479.

C2. Administering Organisation Letter of Support

(Provide a letter of support from the Administering Organisation confirming its commitment to the appointment of the Future Fellow, and the application. Refer to the Instructions to Applicants for the submission requirements. (Upload a PDF of up to 2 A4 pages).)

Uploaded PDF file follows on next page.

C2 Statement by the Administering Organisation

The University of New South Wales (UNSW Sydney) fully supports the application **FT240100267** “Non-commutative Laplacians, quantum symmetries, and the Chern character” by Dr Zanin for a Future Fellowship, Level 2.

Dr. Zanin is a Senior Lecturer in the School of Mathematics and Statistics, and one of the foremost researchers Mathematical Analysis in Australia. His research predominantly focuses on the field of Functional Analysis (with applications to the fields of Geometry and Mathematical Physics). This research project deals with subtle issues concerning the foundations of Non-commutative Geometry and notions paramount to the modern development of Non-commutative Analysis. Dr. Zanin is well prepared to attack these important problems: during 2018-2022, he held a UNSW Scientia Fellowship and developed deep results in Noncommutative Analysis which substantially improved our understanding of the theory, its problems, and approaches. It should be pointed out that a number of the most distinguished mathematicians (such as Fields medal laureate Professor Alain Connes) are very enthusiastic about the suggested direction of research, and this strong endorsement from world leading experts, along with their commitment to joint research efforts is a strong acknowledgement that the candidate has the capacity to undertake the proposed research, as well as to the importance of such work.

Alignment with core or emerging strengths at UNSW

UNSW Sydney is one of Australia’s leading research and teaching universities, renowned for the quality of its graduates and its commitment to new and creative approaches to education and research. UNSW Sydney is one of the founding members of the Group of Eight, a coalition of Australian research-intensive universities, and of Universitas 21, a global network of research universities. In the most recent 2018 national ERA and its companion Engagement and Impact exercise, UNSW Sydney had more two-digit FoR units assessed as well above world average and of high impact than any other Australian university.

The research proposed in Dr Zanin’s application is closely aligned with UNSW Sydney’s research strength in ‘Fundamental and Enabling Sciences’, particularly our growing research activity in ‘Mathematics’.

Dr Zanin’s research program will be conducted in the School of Mathematics and Statistics within the UNSW Faculty of Science.

The School of Mathematics and Statistics at the University of New South Wales offers an excellent environment for this project. It is one of the largest and strongest departments of mathematics and statistics in Australia and is highly ranked. It has grade 5 (the highest rank) according to the most recent 2018 ERA ranking in both mathematics as well as in the research area of Dr Zanin. UNSW is the only Australian university to feature in the top 75 in the world for Mathematics in the three major and influential world university rankings in 2023 simultaneously: 2023 QS World University Rankings (48), the 2022-2023 US News Global Universities Ranking (73) and the 2023 Times Higher Education (71). During the past 5 years, UNSW is also consistently ranked as the top three universities in Australia in mathematics according to the US News Global Universities and the QS World University Rankings.

The School of Mathematics and Statistics in UNSW has a strong and growing Noncommutative Analysis group, several of whom work in Quantum Calculi and Pseudodifferential Operators. Among other areas the group is well versed in noncommutative harmonic analysis, noncommutative ergodic theory and dynamical systems, and noncommutative stochastic processes. In particular, the research of Prof. Fedor Sukochev, Dr. Denis Potapov, and Dr. Shiqi Liu aligns closely with Dr. Zanin’s work on the analysis of non-commutative Laplacians and the associated pseudodifferential calculi.

Dr. Zanin has a strong and expanding network of collaborators, both in the fields of quantum calculus and mathematical physics. Dr. Zanin has ongoing projects with Prof Fedor Sukochev and Dr. Shiqi Liu at UNSW, as well as with Dr. Ji Li at Macquarie University, Prof. Nigel Higson at Penn State University, Prof. Marius Junge at

University of Illinois Urbana-Champaign, Prof. Quanhua Xu at Harbin Institute of Technology, and Prof. Yong Jiao at Central Southern University.

The project provides an opportunity to further expand the collaborative network for Dr. Zanin. The topic of the proposed project has received significant attention during the last couple of years, presenting several new opportunities for collaboration. This will provide opportunity for Dr. Zanin to establish collaborations with Prof. Matthias Lesch at the University of Bonn, Prof. Elmar Schrohe at Leibniz University.

The project would contribute to the excellence in research, as part of the UNSW 2025 strategy, Strategic Priority A Theme 01.

Level of resources to be provided to support the Future Fellow

As per the requirements of the Scheme, UNSW Sydney will provide Dr. Zanin with an appointment at UNSW Sydney for the duration of the Future Fellowship and will fund the salary gap between that of the ARC salary rate and the appointed UNSW Sydney salary rate. This additional commitment will depend on the level and step of the Future Fellowship appointment.

The School of Mathematics and Statistics will provide an additional \$10,000 per year in research support to the Fellow. These funds may be utilised to support a specialist conference that the Fellow aims to organise during the Fellowship, for conference travel (including travel of PhD students associated with the project), collaborative visits, computing hardware, journal publication costs and any other reasonable expenses occurred in carrying out the project. The school will also provide Dr Zanin with the usual office and School facilities.

In addition to the resources provided by the School, Dr. Zanin will also have access to the wide-ranging facilities of the university, including computing and the library. Since 2023, UNSW provides a higher level of higher degree research scholarships at the rate of \$35,000 per annum, which is well above the Department of Education's standard rate of \$29,863 for 2023. The amount of this stipend will be further increased to \$37,684 in 2024. The school will additionally support any PhD students associated with the project with \$5,000 p.a. top-up to their scholarships. The UNSW Graduate Research School also supports \$3,000 for HDR students for conferences during their candidature via the UNSW development and research training grant (DRTG) scheme.

The University assists its researchers in developing and maintaining pathways for their ongoing development and has established several initiatives that provide research staff with professional support in planning and developing their careers. Formal performance appraisals are undertaken in all Faculties and researchers are proactively mentored through an innovative Researcher Development Framework program.³

Integration of the Future Fellow into UNSW research activities after the fellowship

The goal of this work is to establish and nurture a strong research group in quantum calculi and analysis of noncommutative Laplacians. In doing so, Dr. Zanin will mentor a number of young postdoctoral researchers, as well as doctoral candidates. This will develop exciting new research opportunities for UNSW, in a rapidly developing field with deep connections to problems in mathematical physics, and expand the research capabilities of the school of mathematics and statistics, as well as establishing UNSW as an international centre of expertise in modern operator theoretic methods in the study of these problems.

After the end of the Fellowship, Dr Zanin will return to his continuing appointment at the School of Mathematics and Statistics.

In closing, UNSW Sydney strongly supports the application of Dr. Zanin for a Future Fellowship and welcomes the opportunity provided by the Australian Research Council to promote and support research both for the benefit of Australia and the University.

Part D - Project Cost (FT240100267)

D1. What is the proposed budget for your project?

(There are rules around what funds can be requested from the ARC. You must adhere to the scheme specific requirements listed in the Grant Guidelines. Refer to the Instructions to Applicants for detailed instructions on how to fill out the budget section.)

Remunerated Participants

| | | |
|-----------------|-------------------|--|
| Dr Dmitry Zanin | Future Fellowship | Level 2 from year 1 annually for 4 years |
|-----------------|-------------------|--|

Total requested budget: \$901,372

Year 1

| Description | ARC | Admin Org |
|-------------------------------------|---------|-----------|
| | Cash | Cash |
| Total | 225,718 | 10,000 |
| Personnel | 206,818 | |
| Dr Dmitry Zanin (Future Fellowship) | 206,818 | |
| Travel | 18,900 | 10,000 |
| Travel costs | 18,900 | 10,000 |

Year 2

| Description | ARC | Admin Org |
|-------------------------------------|---------|-----------|
| | Cash | Cash |
| Total | 224,218 | 10,000 |
| Personnel | 206,818 | |
| Dr Dmitry Zanin (Future Fellowship) | 206,818 | |
| Travel | 17,400 | 10,000 |
| Travel costs | 17,400 | 10,000 |

Year 3

| Description | ARC | Admin Org |
|-------------------------------------|---------|-----------|
| | Cash | Cash |
| Total | 227,218 | 10,000 |
| Personnel | 206,818 | |
| Dr Dmitry Zanin (Future Fellowship) | 206,818 | |
| Travel | 20,400 | 10,000 |
| Travel costs | 20,400 | 10,000 |

Year 4

| Description | ARC | Admin Org |
|-------------|------|-----------|
| | Cash | Cash |

| | | |
|--------------------------------------|---------|--------|
| Total | 224,218 | 10,000 |
| Personnel | 206,818 | |
| Dr Dmitriy Zanin (Future Fellowship) | 206,818 | |
| Travel | 17,400 | 10,000 |
| Travel costs | 17,400 | 10,000 |

D2. Justification of non-salary funding requested from the ARC

(Fully justify, in terms of need and cost, each budget item requested from the ARC. Use the same headings as in the Description column in the Budget Table of this application. (Upload a PDF of up to 3 A4 pages and within the required format))

Budget Justification

Uploaded PDF file follows on next page.

D2. Justification of non-salary funding requested from the ARC

Travel

Collaboration is a central part of the development, production and impact of research in mathematics. In particular the majority of the ideas in Section C1 were developed as a result of interactions with overseas colleagues. The technical expertise of colleagues accelerates the writing of quality publications and the progress toward genuinely deep results. It is through joint publications and networks that the impact of the project and the ARC's investment is maximised.

The project is designed for a high level of interaction between the CI Zanin and other international collaborators in the US and Europe. UNSW is providing additional travel contributions in recognition of the project's potential. The UNSW contribution will be utilised for conference attendance primarily.

ARC travel funding is requested to enable CI to collaborate with Prof. Nigel Higson and Prof. Marius Junge in the US and engage with North American networks in geometry.

The opportunity for face-to-face collaboration over several weeks is essential for advanced mathematical research. The exchange of ideas and the intense interaction over hand-written notes and boards is still indispensable to mathematics research, and it is not yet replicable by Zoom, emails, or other methods of remote collaboration. Indeed, any serious collaboration requires many hours of intense discussion each day, for weeks, or sometimes even months. The ability to clearly and effectively communicate in front of a board is a powerful catalyst for the completion of substantial collaborations in a timely manner. Additional outcomes of travel will be enhanced training and experience for the PDRA and PGRs, extension of UNSW and Australian impact, and investment in greater international collaboration.

All travel costs are estimated from previous experience and web travel sites such as Google Flights and Booking.com. However, the uncertainty resulting from the ongoing COVID-19 pandemic may lead these numbers to change.

Year 1: (Total 18,900 AUD)

Reciprocal visit to the US (Research visit to Penn. State and Nigel Higson) 30 days with PGR student. (2 people)

Airfare US (Penn. State)-Sydney Return: AU\$2500 x 2 (AU\$5000)

Accommodation, Meals and Incidentals: 30 days @ AU\$100 + AU\$80 (AU\$3000 + AU\$2400)

Reciprocal visit to Germany (Research visit to Elmar Schrohe) 30 days (1 person)

Airfare Germany (Frankfurt)-Sydney Return: AU\$2500

Accommodation: 30 days @ AU\$100 (AU\$3000)

Meals and Incidentals / Per Diem: 30 days @ AU\$100 (AU\$3000)

Year 2: (Total 17,400 AUD)

Reciprocal visit to the US (Research visit to Penn. State and Nigel Higson) 30 days with PGR student. (2 people)

Airfare US (Penn. State)-Sydney Return: AU\$2500 x 2 (AU\$5000)

Accommodation, Meals and Incidentals: 30 days @ AU\$100 + AU\$80 (AU\$3000 + AU\$2400)

Reciprocal visit by Nigel Higson (30 days research visit to UNSW Sydney)

Airfare US (Penn. State)-Sydney Return: AU\$2500

Per Diem: 30 days @ AU\$150 (AU\$4500)

Year 3: (Total 20,400 AUD)

Reciprocal visit to Germany (Research visit to Elmar Schrohe) 30 days (1 person)

Airfare Germany (Frankfurt)-Sydney Return: AU\$2500

Accommodation: 30 days @ AU\$100 (AU\$3000)

Meals and Incidentals / Per Diem: 30 days @ AU\$100 (AU\$3000)

Reciprocal visit by Elmar Schrohe (30 days research visit to UNSW Sydney)

Airfare Germany (Frankfurt)-Sydney Return: AU\$2500

Per Diem: 30 days @ AU\$150 (AU\$4500)

Year 4: (Total 17,400 AUD)

Reciprocal visit to the US (Research visit to Marius Junge at UIUC) 30 days with PGR student. (2 people)

Airfare US (UIUC)-Sydney Return: AU\$2500 x 2 (AU\$5000)

Accommodation, Meals and Incidentals: 30 days @ AU\$100 + AU\$80 (AU\$3000 + AU\$2400)

Reciprocal visit by Marius Junge (30 days research visit to UNSW Sydney)

Airfare US (Chicago)-Sydney Return: AU\$2500

Per Diem: 30 days @ AU\$150 (AU\$4500)

Travel costs are calculated using estimates of present-day domestic and international economy airfares, accommodation and a modest per-diem allowance. These figures add up to AU\$68,100 over Years 1-4.

D3. Details of non-ARC contributions

(Provide an explanation of how non-ARC contributions will support the proposed project. Use the same headings as in the Description column in the above Budget Table of this application. (Upload a PDF of up to 2 A4 pages and within the required format))

Details of non-ARC Contributions

Uploaded PDF file follows on next page.

D3 Details of non-ARC contributions

Personnel - CI Zanin (UNSW)

CI Zanin is currently employed as a senior lecturer at UNSW, with 40% research, 40% teaching, and 20% administration. Under the Future Fellowship, CI Zanin would change to 80% research, 20% administration, allowing for 40% of his time to be dedicated to the proposed project, with another 40% dedicated to a current ARC Discovery Project (DP230100434). The funding would cover CI Zanin's base salary at UNSW Level C5 as of January 1st 2024, including 29.42% on-costs. This would be \$206,818 in years 1 to 4.

Travel

Conferences form an essential part of the dissemination and communication of new results and research. Attending international conferences are also fundamental for PGRs, both to establish their profile as young researchers, and to allow them to see the state of the art in the field. Moreover, conferences serve as a platform to establish new collaborations, as well as to engage in face-to-face discussions with other researchers. This interaction allows for both the refinement of existing results, and the further development of new techniques and ideas.

UNSW School of Mathematics and Statistics will provide a supplementary 10,000 AUD in each of years 1 – 4 for travel expenses.

All travel costs are estimated from previous experience and web travel sites such as Google Flights and Booking.com. However, the uncertainty resulting from the ongoing COVID-19 pandemic may lead these numbers to change.

Year 1: (Total 10,000 AUD)

Conference visit to the US (Great Plains Operator Theory Seminar) Visit with PGR student. (2 people)

Airfare Sydney-US Return: AU\$2500 x 2 (AU\$5000)

Conference visit to the Netherlands (International Workshop on Operator Theory and its Applications) Visit with PGR student. (2 people)

Airfare Sydney-Netherlands Return: AU\$2500 x 2 (AU\$5000)

Year 2: (Total 10,000 AUD)

Conference visit to the US (International Congress of Mathematicians) Visit with PGR student. (2 people)

Airfare Sydney-US Return: AU\$2500 x 2 (AU\$5000)

Conference visit to France (Mathematical aspects of the physics with non-self-adjoint operators) Visit with PGR student. (2 people)

Airfare Sydney-France Return: AU\$2500 x 2 (AU\$5000)

Year 3: (Total 10,000 AUD)

Conference visit to Canada (Canadian Operator Theory Seminar) Visit with PGR student. (2 people)

Airfare Sydney-Canada Return: AU\$2500 x 2 (AU\$5000)

Conference visit to France (Franco-american conference in analysis and PDE) Visit with PGR student. (2 people)

Airfare Sydney-France Return: AU\$2500 x 2 (AU\$5000)

Year 4: (Total 10,000 AUD)

Conference visit to the US (Annual Spring Institute Noncommutative Geometry and Operator Algebras) Visit with PGR student. (2 people)

Airfare Sydney-US Return: AU\$2500 x 2 (AU\$5000)

Conference visit to Austria (Operator Theory and Approximation) Visit with PGR student. (2 people)

Airfare Sydney-Austria Return: AU\$2500 x 2 (AU\$5000)

Travel costs are calculated using estimates of present-day domestic and international economy airfares, accommodation and a modest per-diem allowance. These figures add up to AU\$40,000 over Years 1-4.

Part E - Classifications and Other Statistical Information (FT240100267)

E1. Australian Government priority areas

(Does this application align with an announced Australian Government policy? For reporting purposes, the ARC is capturing relevant Australian Government policies for your application. If your application does not align with an announced Australian Government policy, please enter No.)

No

E2. Field of Research

(Select up to 3 FoR classification codes that relate to the application. Note that the percentages must total 100.)

| Code | Percentage |
|--|------------|
| 490408 - Operator algebras and functional analysis | 50 |
| 490406 - Lie groups, harmonic and Fourier analysis | 40 |
| 490412 - Topology | 10 |

E3. Socio-Economic Objective (SEO-2020)

(Select up to 3 SEO classification codes that relate to the application. Note that the percentages must total 100.)

| Code | Percentage |
|---|------------|
| 280118 - Expanding knowledge in the mathematical sciences | 100 |

E4. Interdisciplinary Research

(This is a 'Yes' or 'No' question. If you select 'Yes' two additional questions will be enabled:

1. Specify the ways in which the research is interdisciplinary by selecting one or more of the options below and click 'Add'.

2. Indicate the nature of the interdisciplinary research involved. (Up to 375 characters, approximately 50 words))

Does this application involve interdisciplinary research?

No

Specify the ways in which the research is interdisciplinary by selecting one or more of the options below.

Indicate the nature of the interdisciplinary research involved. (Up to 375 characters, approximately 50 words)

E5. Does the proposed research involve international collaboration?

(This is a 'Yes' or 'No' question. If you select 'Yes' you will need to answer: If the proposed research involves international collaboration, specify the countries involved.)

Yes

E6. If the proposed research involves international collaboration, please specify the country/ies involved.

(Commence typing in the search box and select from the drop-down list the name of the country/ies of collaborators)

who will be involved in the proposed project. Note that Australia is not to be listed and is not available to be selected from the drop-down list.)

| |
|--------------------------|
| United States of America |
| Germany |

E7. How many PhDs, Masters and Honours positions will be filled as a result of this project?

(For reporting purposes, the ARC is capturing the number of Research Students that would be involved if the application is funded. Enter the number of all student places (full-time equivalent - FTE) that will be filled as a result of this project, not just those requested in the budget for funding in the application form.)

Number of Research Student Places (FTE) - PhD

2

Number of Research Student Places (FTE) - Masters

1

Number of Research Student Places (FTE) - Honours

2

Part F - Project Eligibility (FT240100267)

F1. Medical Research

(This is a 'Yes' or 'No' question. Does this application contain content which requires a statement to demonstrate that it complies with the eligible research requirements set out in the ARC Medical Research Policy located on the ARC website?)

No

F2. Medical Research Statement

(Justify why this application complies with the eligible research requirements set out in the ARC Medical Research Policy located on the ARC website. Eligibility will be based solely on the information contained in this application. This is the only chance to provide justification, the ARC will not seek further clarification. (Up to 750 characters, approximately 100 words))

Certification

Certification by the Deputy/Pro Vice-Chancellor (Research) or their delegate or equivalent in the Administering Organisation

I certify that—

- I have read, understood, and complied with the *Discovery Program Grant Guidelines - Fellowships (2023 edition)*, (Grant Guidelines) and, to the best of my knowledge all details provided in this application form and in any supporting documentation are true and complete in accordance with the Grant Guidelines.
- Proper enquiries were made and I am satisfied that the candidate and the organisation listed in this application meet the requirements specified in the Grant Guidelines, including the candidate having been awarded a PhD on or between 1 March 2009 and 1 March 2019. Where the candidate has allowable career interruptions, sufficient evidence has been provided to the Administering Organisation and based on this evidence, I certify that the candidate has an award of PhD date together with an allowable period of career interruption (as listed in the Grant Guidelines) that would be commensurate with an award of PhD date on or between 1 March 2009 and 1 March 2019.
- Where the candidate holds a higher research degree that is not a PhD, sufficient evidence was provided to the Administering Organisation and based on this evidence, I certify that the candidate's qualification meets the level 10 criteria of the *Australian Qualifications Framework Second Edition* or is a professional equivalent to a PhD.
- I certify that the Future Fellowship level selected for the candidate corresponds to the relevant academic level of the candidate.
- Upon request from the ARC, this organisation will provide evidence to support a career interruption justification in relation to the PhD award date.
- In certifying the National Interest Test statement, I have considered the requirements detailed in the Instructions to Applicants, including whether the National Interest Test statement is written in plain English and for the audience – the general public.
- The ARC reserves the right to audit any evidence on which an application is based
- I have received confirmation that the candidate will not be undertaking any Higher Degree by Research during the project activity.
- I will notify the ARC if there are changes to the candidate, participants and the organisation listed after the submission of this application.
- The candidate and listed participants are responsible for the authorship and intellectual content of this application, and have appropriately cited sources and acknowledged significant contributions to this application.
- To the best of my knowledge, all material personal and financial interests and Conflicts of Interest relating to parties involved in or associated with this application are disclosed to the Administering Organisation, and, if the application is successful, I agree to manage all Conflicts of Interest relating to this application in accordance with the *Australian Code for the Responsible Conduct of Research (2018)*, the *ARC Conflict of Interest and Confidentiality Policy* located on the ARC website and any relevant successor documents.
- I have obtained the agreement, attested to by written evidence, of all the relevant persons and organisations necessary to allow the project to proceed. This written evidence is retained and will be provided to the ARC if requested.
- The application, including all parties involved in or associated with the application, have undergone due diligence to assess risks from foreign interference in line with the *Guidelines to Counter Foreign Interference in the Australian University Sector (2019)* developed by the University Foreign Interference Taskforce.
- This application complies with the eligible research requirements set out in the *ARC Medical Research Policy*, located on the ARC website.
- This application does not request funding for the same research activities, infrastructure, or project previously funded or currently being funded through any other Commonwealth funding.
- This application complies with the requirements to manage other similar or linked research applications by the candidate and management is in place to avoid duplication of Australian Government funding if all applications are funded.

- If this application is successful, I am prepared to have the project carried out as set out in this application and agree to abide by the terms and conditions of the Grant Guidelines and the relevant Grant Agreement.
- The project can be accommodated within the general facilities of this organisation and if applicable, within the facilities of other relevant organisations specified in this application and sufficient working and office space is available for any proposed additional staff.
- All funds for this project will only be spent for the purpose for which they are provided.
- The project cannot commence until there is an ethics plan in place.
- I consent, or where necessary, I have obtained the necessary consent(s) concerning all the parties, to this application being referred to third parties, including to overseas parties, who will remain anonymous, for assessment purposes.
- I consent, or where necessary, I have obtained the necessary consent(s) concerning all the parties, to the ARC copying, modifying and otherwise dealing with information contained in this application.
- I acknowledge, or where necessary have informed all the parties, that information from this application may be provided to other Commonwealth agencies to seek advice on national security or other matters.
- I confirm that potential risks, are factored into the proposed project and if awarded a risk management plan will be in place before the project can commence.
- To the best of my knowledge, the Privacy Notice appearing at the top of this form was drawn to the attention of the Future Fellowship candidate whose personal details are provided in the Participant section of the application.