AUSTRALIAN RESEARCH COUNCIL
ARC Future Fellowships
Proposal for Funding Commencing in 2019

FT

PROJECT ID: FT190100442

First Investigator: Dr Dmitriy Zanin

Admin Org: The University of New South Wales

Total number of sheets contained in this Proposal: 47

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Part A - Administrative Summary (FT190100442)

A1. Application Title

(Provide a short title. (No more than 75 characters, approximately ten words))

Non-commutative Laplacians, quantum symmetries, and the Chern character

A2. Person Participant Summary

(Add the Future Fellowship candidate participating in this application.)

Number	Name	Participant Type	Current Organisation(s)
1	Dr Dmitriy Zanin	Future Fellowship	The University of New South Wales

A3. Organisation Participant Summary

(Add the organisations participating in this application. Refer to the Instructions to Applicants for further information.)

Number	Name	Participant Type
1	The University of New South Wales	Administering Organisation

A4. Application Summary

(Provide an Application Summary (which is used by the Minister to consider the application), focusing on the aims, significance, expected outcomes and benefits of this project. Write the Application Summary simply, clearly and in plain English. If the application is successful, the Application Summary is used to give the general community an understanding of the research. Avoid the use of acronyms, quotation marks and upper case characters. Refer to the Instructions to Applicants for further information. (No more than 750 characters, approximately 100 words))

Non-commutative geometry is an exciting new way to think of quantization of classical systems from the geometric point of view. This project aims to develop a new approach to computing geometric and topological invariants of non-commutative manifolds. The project expects to generate new mathematical methods, results and examples in the field of non-commutative geometry and to promote Australian research in non-commutative analysis at the international level. The project includes building an international collaboration with research groups in USA, China and France. We apply this approach to a range of examples including quantum groups and non-flat non-commutative manifolds.

A5. List the objectives of the Future Fellowship candidate's proposed project.

*(List each objective separately by clicking 'add answer' to add the next objective. This information will be used for future reporting purposes if this application is funded. (No more than 500 characters, approximately 70 words per objective)
)*

Objective

Prove a non-commutative version of a partition function expansion of a Laplacian associated to a noncommutative metric. This allows a definition and explicit computation of noncommutative analogues of the most important geometric invariants like curvature and topological invariants like the Euler characteristic.

Objective

Prove a version of Connes Character Formula for locally compact non-commutative manifolds. This allows a local expression for the Chern Character linking the Riemannian and conformal geometry and sets a lower obstruction for the order of the partition function expansion.

Objective

Build spectral triples associated to the actions of quantum unitary or quantum affine groups. In analogy with Lie groups this will present the quantum group as a non-commutative manifold equipped with the quantum group as its isometries. This spectral triple should avoid dimension drop pathology (that is, spectral dimension should equal the homological one).

A6. Benefit and Impact Statement

(Outline the intended benefit and impact of the project. Write the Benefit and Impact Statement simply, clearly and in plain English. Refer to the Instructions to Applicants for further information. (No more than 750 characters, approximately 100 words))

The outcomes are expected to develop research at the forefront of non-commutative mathematics and impact research in mathematical physics and other sciences which will strengthen the reputation of Australian science. This proposal aims at establishing UNSW at the centre of a highly active international research field and bringing prominent international researchers in mathematics to Australia to teach and research. The project will produce highly trained personnel leading in an international network in a fundamental science.

Part B - Classifications and Other Statistical Information (FT190100442)

B1. Does this application fall within one of the Science and Research Priorities?

No

Science and Research Priority	Practical Research Challenge

B2. Field of Research (FOR)

(Select up to three classification codes that relate to the Future Fellowship candidate's application. Note that the percentages must total 100.)

Code	Percentage
010108 - Operator Algebras and Functional Analysis	70
010106 - Lie Groups, Harmonic and Fourier Analysis	30

B3. Socio-Economic Objective (SEO-08)

(Select up to three classification codes that relate to the Future Fellowship candidate's application. Note that the percentages must total 100.)

Code	Percentage
970101 - Expanding Knowledge in the Mathematical Sciences	90
970102 - Expanding Knowledge in the Physical Sciences	10

B4. Interdisciplinary Research

(This is a 'Yes' or 'No' question. If You select 'Yes' two additional questions will be enabled:
1. Specify the ways in which the research is interdisciplinary by selecting one or more of the options below.
2. Indicate the nature of the interdisciplinary research involved. (No more than 375 characters, approximately 50 words))

Does this application involve interdisciplinary research?

No

Specify the ways in which the research is interdisciplinary by selecting one or more of the options below.

Indicate the nature of the interdisciplinary research involved. (No more than 375 characters, approximately 50 words)

B5. Does the proposed research involve international collaboration?

(This is a 'Yes' or 'No' question. If You select 'Yes' two additional questions will be enabled:
1. Specify the nature of the proposed international collaboration by selecting one or more of the options below.
2. Specify the countries which are involved in the international collaboration.)

Yes

B6. What is the nature of the proposed international collaboration activities?

(Select all options from the drop down list which apply to this application by clicking on the 'Add' button each time an option is selected.)

Correspondence: eg email; telephone; or video-conference
Attendance at and/or hosting of workshop or conference
Hosting international Partner Investigator: short-term (less than 4 weeks)
Hosting international Partner Investigator: long-term (more than 4 weeks)
Travel to international collaborator: short-term (less than 4 weeks)
Travel to international collaborator: long-term (more than 4 weeks)

B7. If the proposed research involves international collaboration, specify the country/ies involved.

(Commence typing in the search box and select from the drop-down list the name of the country/ies of collaborators who will be involved in the proposed project. Note that Australia is not to be listed and is not available to be selected from the drop-down list.)

United States of America
China (excludes SARs and Taiwan)
France
Russian Federation
Korea, Republic of (South)

B8. How many PhD, Masters and Honours places will be filled as a result of this project?

(The ARC is capturing the number of Research Students that would be involved in this application if it is funded. Enter the number of student places (full-time equivalent - FTE) that will be filled as a result of this project.)
Number of Research Student Places (FTE) - PhD

2

Number of Research Student Places (FTE) - Masters

1

Number of Research Student Places (FTE) - Honours

2

Part C - Project Description (FT190100442)

C1. Project Description

(Upload a Project Description as detailed in the Instructions to Applicants and in the required format. Ensure that the Project Description responds to the Assessment Criteria listed in the grant guidelines. (No more than ten A4 pages))

Uploaded PDF file follows on next page.

Project Title

Asymptotics of non-commutative Laplacians, quantum symmetries and the Chern character.

Aims and Background

Brief outline of the aims and background of this Proposal. The project belongs to the broad field of Non-commutative Analysis. This area appeared from foundational questions in Quantum Physics. While classically, observables are functions, in the quantum world observables are operators. In abstract terms, observables in a classical system form a commutative von Neumann algebra, while the ones in a quantum system form a non-commutative von Neumann algebra. Noncommutative topology and integration theory were developed in 1940-1960. Non-commutative Geometry, the more specific field of this project, takes its origin in 1980's in the works of Alain Connes.

One of the objectives of this project is to provide a non-commutative version of the Minakshisundaram-Plejel theorem, that is, to find an analogue for the heat kernel expansion of the Laplace-Beltrami operator. In this expansion, the coefficient in the most important term is a constant, while the one in the second term is scalar curvature. Higher order terms can be expressed via higher order differential invariants. Heuristically, such an expansion is strongly related with singular traces: the most important term should be equal to the value of the singular trace. This heuristic was recently verified by the CI in a quite general setting [34] (which includes, e.g. compact manifolds). For the simplest non-flat non-commutative manifold (a conformal deformation of a 2-dimensional noncommutative torus), curvature was computed by Connes and collaborators [17] (e.g. Tretkoff, Moscovici, Fathizadeh, Khalkhali). In this project, we propose a new technique to compute the curvature (and higher order coefficients) by using the theory of Double Operator Integrals. This theory, developing since the 1970's, has recently experienced a breakthrough by the CI and co-authors (to appear in American Journal of Mathematics).

Another objective in this project is to create a spectral triple from a given quantum group. Quantum groups are certain deformations of well known Lie groups. The latter are, by definition, smooth manifolds. It is of crucial importance that Dirac operators on such manifolds commute with the action of the group on itself, which relates to symmetries of actions in mathematical physics. A similar property for quantum groups is called equivariance. A number of attempts were made to construct an equivariant spectral triple for the simplest possible example, the quantum unitary group [5, 6, 28]. These attempts suffer from 2 drawbacks: one is dimension drop phenomenon (i.e. the spectral dimension of the triple does not equal the homological one), the other is that the first term in the heat kernel expansion does not recover the Haar state (as it should). The CI proposes to construct an equivariant spectral triple on quantum groups which is free from these drawbacks.

The third objective in this project is Connes' Character Formula for noncompact non-commutative manifolds. In short, it declares the equality between 2 cocycles: one appears from (non-commutative) Riemannian differential geometry, while another appears from conformal geometry. The Character Formula expresses the Chern character in Non-commutative Geometry as the analogue of a de Rham current and can be used to compute index pairings with the K-theory of the non-commutative manifold. Such an index theory has new applications beyond classical geometry as observed in Bellisard's non-commutative geometric model of the quantum Hall effect [1]. The Character Formula is given (without any proof) in the book "Noncommutative geometry" by Connes [8]. A number of authors tried to prove it with various degrees of success. For compact non-commutative manifolds, the final word is due to the CI and co-authors [4]. The proof used double operator integrals in a novel integral representation of the products of complex powers of positive operators and deep results on singular traces. Connes' Character Formula for noncompact non-commutative manifolds is considerably more difficult.

National/international progress in the field of research and its relationship to this Proposal. Operator algebras provide a natural edifice to many areas of classical and modern mathematics, and also play a paramount role in the present proposal. A particular role is played by the class of C^*-algebras (uniformly closed $*-$subalgebras in the $*-$algebra $B(H)$ of all bounded operators on a Hilbert space H) and that of von Neumann algebras (weakly closed unital $*-$subalgebras of $B(H)$). The topological requirements imposed on these algebras make them a suitable foundation for developing noncommutative analysis.

The Gelfand-Naimark theorem provides an anti-equivalence $X \to C_0(X)$, where $C_0(X)$ is the algebra of all continuous complex valued functions which vanish at infinity, between the category of locally compact Hausdorff spaces and the category of commutative C^*-algebras. Under this equivalence all of the topological information about a space is actually encoded in the algebra of continuous complex valued functions on that space. It allows us to express numerous topological properties of topological spaces in the language of C^*-algebras, e.g. point sets as maximal ideals and so on. Thinking of noncommutative C^*-algebras as noncommutative topological spaces and attempts to apply topological methods to understand these algebras has proven to be well justified, e.g. K-theory of C^*-algebras as an extension of topological K-theory.

Another fundamental result due to von Neumann and Segal provides a duality between a certain category of measure spaces and that of commutative von Neumann algebras. This fact provided impetus to view classical measure theory as a part of von Neumann algebra theory and led to the development of noncommutative integration theory.

Conventional wisdom suggests further analogies between the classical and quantum worlds. While topology and measure theory were supplied with quantum counterparts in the 1950s, the development of geometric concepts were lacking until the 1980s. So far, the most successful attempt to quantise differential geometry is due to Connes who introduced the notion of a spectral triple and promoted it as a non-commutative analogue of differential geometry.

Let H be a (separable) Hilbert space, and let $\mathscr{A}$ be an algebra equipped with an involution $*$. We say that $(\mathscr{A},H,D)$ is a spectral triple if:

1. There is a representation $\pi:\mathscr{A}\to B(H)$ of the $*-$algebra $\mathscr{A}$ on the Hilbert space H. For brevity we omit π from the formulation below.
2. D is a self-adjoint unbounded operator on H.
3. For every $a\in\mathscr{A}$, the commutator $[D,a]$ has an extension to a bounded operator.
4. For every $a\in\mathscr{A}$, the operator $\pi(a)(D+i)^{-1}$ is a compact operator.
5. If, for every $a\in\mathscr{A}$, the eigenvalues of the operator $\pi(a)(D+i)^{-1}$ arranged in decreasing order with mulitplicity decay like the sequence $k^{-\frac{1}{d}},k\geq 1$ the spectral triple is called $d-$dimensional.

A Riemannian manifold (a paracompact topological space X where every point admits a neighborhood which is diffeomorphic to Euclidean space) equipped with a smooth metric tensor g can be associated to a spectral triple $(\mathscr{A},H,D)$ as follows: (i) $\mathscr{A}$ is the algebra $C_c^\infty(X)$ of all compactly supported smooth functions on X; (ii) H is the Hilbert space $L_2\Omega(X)$ of all square-integrable differential forms on X and π is the representation of $\mathscr{A}$ on H by pointwise multiplication; (iii) D is the Hodge-Dirac operator (see e.g. [2]) on $L_2\Omega(X)$. For compact manifolds it is a consequence of Weyl's celebrated theorem of the asymptotics of the eigenvalues Lapacian on bounded Euclidean domains that the just constructed spectral triple $(\mathscr{A},H,D)$ is $d-$dimensional, where $d=\dim(X)$.

Connes' Reconstruction Theorem [10] stipulates that every spectral triple where the $*$-algebra $\mathscr{A}$ is commutative (satisfying a few natural conditions) comes from a $d-$dimensional Riemannian manifold (X,g). Connes' Theorem is the equivalent for geometry of the Gelfand-Naimark theorem. Thus it makes good sense to think of spectral triples as non-commutative manifolds. We adopt this point of view, and this proposal aims to develop it further by: (a) using heat trace expansions of the operator D^2 to obtain curvature; (b) develop the geometry of quantum groups in analogy to "Lie groups are smooth manifolds"; and (c) completing the proof of Connes' Character Formula.

We describe existing progress toward these aims.

When (X,g) is a d-dimensional Riemannian manifold, then D^2 is the Hodge-Laplace operator and its component acting on 0 order differential forms is minus the Laplace-Beltrami operator (denoted by Δ_g) [31]. For brevity we discuss only the zero-order component and define the heat semi-group by the formula

$$t\to e^{t\Delta_g},\quad t>0.$$

If X is compact, then the resolvent of the Laplace-Beltrami operator Δ_g is compact. Hence, $e^{t\Delta_g}$ is compact for $t>0$. In fact, the operator $e^{t\Delta_g}$ is trace class for $t>0$.

In his seminal work [36], Weyl proved that, up to fixed constants depending only on d, for a compact manifold,

$$\lim_{t\downarrow 0}(4\pi t)^{\frac{d}{2}}\mathrm{Tr}(e^{t\Delta_g})=\mathrm{Vol}(X),\quad t\downarrow 0. \tag{1}$$

Following Weyl's work, it became an established custom to measure various geometric (and often topological) quantities associated with a Riemannian manifold X in terms of its heat semi-group expansion $t\to e^{t\Delta_g}$, $t>0$. The mere existence of such an expansion is a famous theorem of Minakshisundaram and Plejel (among all approaches to that theorem, a particularly detailed account is given in [31]; even though Theorem 3.24 there concerns only a special case $f=1$, the proof of the formula stated below in the general case is very similar).

For every $f\in C^\infty(X)$, the Minakshisundaram-Plejel theorem asserts the existence of an asymptotic expansion

$$\mathrm{Tr}(M_f e^{t\Delta_g})\approx(4\pi t)^{-\frac{d}{2}}\cdot\sum_{n\geq 0}a_n(f)t^n,\quad t\downarrow 0. \tag{2}$$

Here, d is the dimension of X and $M_f:L_2(X)\to L_2(X)$ is the operator of pointwise multiplication by f. Moreover, there exist functions $A_k\in C^\infty(X)$ such that

$$a_k(f)=\int_X A_k\cdot f d\mathrm{vol}_g, \tag{3}$$

where vol_g is the standard volume element on X given in local coordinates by the formula

$$d\mathrm{vol}_g=(\det(g))^{\frac{1}{2}}(x)dx.$$

As follows from (1), $A_0 = 1$. Further computations (see e.g. Proposition 3.29 in [31]) show that

$$A_1 = \frac{1}{6}R,$$

where R is the scalar curvature. In particular, $a_1(1)$ expresses the Einstein-Hilbert action (see e.g. [8]).

Note that a_0 extends to a normal state h on $L_\infty(X)$ by the obvious formula

$$h(f) = \int_X f d\mathrm{vol}_g, \quad f \in L_\infty(X).$$

Equation (3) can be re-written as

$$a_k(f) = h(A_k \cdot f), \quad f \in C^\infty(X).$$

One of the objectives of this project is to find suitable extensions of the Minakshisundaram-Plejel theorem (and, consequently, of the Weyl theorem — see formula (1)) for a large class of non-commutative manifolds (such as non-commutative tori with generic, non-flat, metric tensor). This grand program began in [17] (published only in 2011, but the main concepts and techniques were developed in the 1990's), where special $2-$dimensional non-commutative manifolds (conformal deformations of a flat non-commutative torus) were considered. The authors of [17] proved that the Euler characteristic of such a manifold is 0 by means of the Gauss-Bonnet theorem (recall that the classical Gauss-Bonnet theorem asserts that Euler characterstic of a $2-$dimensional Riemannian manifold is equal to the average of its scalar curvature). Subsequently, the scalar curvature (for the conformal deformation of the $2-$dimensional non-commutative torus) was explicitly computed in [15] and [19] and, later, the term a_2 (the first place where the Riemann curvature tensor manifests itself beyond the scalar curvature) was further computed in [12] (intermediate computations include about a million terms!). Below, we briefly restate the whole programme as it can be surmised from [17].

For a given $d-$dimensional spectral triple $(\mathscr{A}, H, D)$, the steps needed to be done in the program are as follows (for brevity we assume $\mathscr{A}$ to be unital):

1. to prove a non-commutative version of Weyl's theorem, that is, to find a normal state h on $\mathscr{A}''$ such that

$$\mathrm{Tr}(xe^{-tD^2}) \approx (4\pi t)^{-\frac{d}{2}} h(x), \quad t \downarrow 0, \tag{4}$$

   for every $x \in \mathscr{A}''$;
2. to prove a non-commutative version of the Minakshisundaram-Plejel theorem or, equivalently, to verify that the following strong generalization of (2) holds

$$\mathrm{Tr}(xe^{-tD^2}) \approx (4\pi t)^{-\frac{d}{2}} \sum_{n \geq 0} a_n(x) t^n, \quad t \downarrow 0,$$

   for every $x \in \mathscr{A}''$;
3. to verify the normality of coefficients as in (3) with respect to the volume state, or, more precisely, to show that

$$a_k(x) = h(xA_k), \quad x \in \mathscr{A}'',$$

   for every $k \geq 0$ and for some $A_k \in \mathscr{A}''$;
4. to compute A_k explicitly.

When this mission is accomplished, one can *define* scalar curvature of a non-commutative manifold by setting $R = 6A_1$.

This program sets reasonable objectives but we claim that the tools based on pseudo-differential calculus in various guises applied up-to-date to accomplish them have been inadequate. Our approach is discussed in the Section "Project Quality and Innovation" below.

Now we discuss geometry of quantum groups. The Hodge-Dirac operator D describes the isometries of the manifold X. If $\gamma : X \to X$ is an isometry and if $U : L_2\Omega(X) \to L_2\Omega(X)$ is the operator of composition with γ, then (i) U is unitary on $L_2\Omega(X)$; (ii) U preserves $\pi(\mathscr{A})$ (that is, $U^{-1}\pi(\mathscr{A})U = \pi(\mathscr{A})$); (iii) U commutes with D. Conversely, every diffeomorphism $\gamma : X \to X$ which commutes with D is an isometry. This simple but revealing result can be found in [23]. It is typical in applications to harmonic analysis [23] and mathematical physics [20] that manifolds are equipped with isometric actions of Lie groups. Moreover, most examples of manifolds which are of interest in applications are homogeneous spaces of some Lie group (e.g., the sphere $\mathbb{S}^{n-1}$ is the homogeneous space of the Lie group $SO(n)$). In such a situation, it does not make sense to consider the manifold alone, but rather a manifold equipped with an isometric group action.

We propose to investigate spectral triples which arise from the action of a suitable group-like object. The class which is most suitable for this task are quantum groups, whose theory has been under development since the early 1980's. The theory of quantum groups has its origin in attempts to find a good duality theorem, analogous to the Pontryagin duality theorem, for general locally compact non-Abelian groups. In late 1980's, Woronowicz developed a general theory of compact quantum groups and developed a Peter-Weyl theory for them. One of the main examples in Woronowicz's theory is the quantum group $SU_q(n)$ (a natural $q-$deformation of the compact Lie group $SU(n)$).

The rich interplay between Lie groups and differential geometry naturally raises the question of understanding the interaction between quantum groups and Non-commutative Geometry. The papers [5], [28] (among many others) attempt to put quantum groups within the framework of Non-commutative Geometry. These attempts drew the attention of Alain Connes (see [6]) who developed further results by Chakraborty and Pal [5].

Given a d-dimensional spectral triple $(\mathscr{A}, H, D)$ define the functional

$$h_\varphi : x \to \varphi(x(1+D^2)^{-\frac{d}{2}}), \quad x \in \mathscr{A}, \tag{5}$$

where φ is a singular trace on the ideal $\mathscr{L}_{1,\infty}$. Here $\mathscr{L}_{1,\infty}$ is the principal ideal in $B(H)$ generated by an operator with singular values $(\frac{1}{k})_{k\geq 1}$ (see [26] for the definition and basic properties of singular traces on ideals of compact operators). It happens that h_φ possesses the tracial property. The functional in (5) should coincide with the Haar state on the quantum group, which is known to fail the tracial property in all non-trivial examples. This discrepancy demonstrates that the definition of spectral triple given in [5] for $SU_q(2)$ needs to be adjusted.

We propose to remedy this issue by considering *twisted* spectral triples for the quantum groups (twisted spectral triples were introduced by Connes and Moscovici in 2006). When the spectral triple is twisted, h_φ is no longer tracial. Instead, its tracial property is distorted by the same twist as in a KMS condition. By choosing a suitable twist, Connes' Trace Theorem should equate h_φ with the Haar state.

We now discuss the Chern character. It is one of the fundamental tools in Non-commutative Geometry. The Connes Character Formula (also known as the Hochschild character theorem) provides an expression for the class of the Chern character in Hochschild cohomology, and it is an important tool for computation. The formula has been applied to many areas of Non-commutative Geometry and its applications: such as the local index formula [14], the spectral characterisation of manifolds [10] and recent work in mathematical physics [11].

In its original formulation, [7], the Character Formula is stated as follows: Let $(\mathscr{A}, H, D)$ be a p-dimensional compact spectral triple with (possibly trivial) grading Γ. By the definition of a spectral triple, for all $a \in \mathscr{A}$ the commutator $[D, a]$ has an extension to a bounded operator $\partial(a)$ on H. Assume for simplicity that $\ker(D) = \{0\}$ and set $F = \mathrm{sgn}(D)$. For all $a \in \mathscr{A}$ the commutator $[F, a]$ is a compact operator in the weak Schatten ideal $\mathscr{L}_{p,\infty}$ (see e.g. [8, 26]).

Consider the following two linear maps of the algebraic tensor power $\mathscr{A}^{\otimes(p+1)}$, defined on an elementary tensor $c = a_0 \otimes a_1 \otimes \cdots \otimes a_p \in \mathscr{A}^{\otimes(p+1)}$ by

$$\mathrm{Ch}(c) := \frac{1}{2}\mathrm{Tr}(\Gamma F[F, a_0][F, a_1]\cdots[F, a_p]), \quad \Omega(c) := \Gamma a_0 \partial a_1 \partial a_2 \cdots \partial a_p.$$

Connes defined the non-commutative integral of a bounded operator A on H in Non-commutative Geometry by the formula

$$\int_\omega A := \mathrm{Tr}_\omega(A(1+D^2)^{-p/2})$$

where Tr_ω is a singular trace called the Dixmier trace (that this integral is an extension of the state described in both (4) and (5) is known through the CI's work [26, Part III] building on earlier work of Connes and Carey-Phillips-Sukochev).

Connes Character Formula states that if c is a Hochschild cycle then

$$\mathrm{Ch}(c) = \int_\omega \Omega(c)$$

for every Dixmier trace Tr_ω. The multilinear maps Ch and $c \mapsto \int_\omega \Omega(c)$ define the same class in Hochschild cohomology. The generality of the result just stated was achieved recently by the CI and his co-authors, see [4].

The definition of a non-compact spectral triple originated with Connes [9], and was furthered in [3, 21, 30]. The conventional definition of a non-compact spectral triple is to replace the condition that $(1+D^2)^{-1/2}$ be compact with the assumption that for all $a \in \mathscr{A}$ the operator $a(1+D^2)^{-1/2}$ is compact. This raises the question of whether the Character Formula is true for locally compact spectral triples? This question was suggested to the CI by Prof. Connes.

Future Fellowship Candidate

The proposed research uses fundamental tools in non-commutative analysis to approach new problems in Non-commutative Geometry. The CI is well equipped to attack this research problem: during 2015–2018, the candidate held the ARC DECRA DE150100030 titled "New concept of independence in non-commutative probability theory" and achieved deep results which substantially improved understanding of the theory, its problems and approaches. Over the last 5 years, since 2013, in the area of the DECRA and while contributing substantially to the ARC Discover Projects DP110100064, DP120103263, DP14010096 and DP150100920 in non-commutative analysis and Non-commutative Geometry, the CI has published 40 articles in peer-reviewed international research journals and 1 monograph. 19 of those articles are in journals ranked A* by the Australian Mathematical Society. For additional evidence and measures of the CI's research performance, see Section D9(i). The CI received his PhD in 2011 and was employed in full time research under DP projects DP110100064 and DP120103263 from 2011 to 2015.

In [24], published in Advances in Mathematics, a long standing problem about the non-commutative analogue of the Poisson process (which had defied the efforts of top experts in the area) was resolved. The CI's solution in [33] of spectrality of traces, an open problem since 1981, has been called a "tour de force" [27, p. 149]. The CI's solution with Caspers of the Nazarov-Peller conjecture on operator Lipschitz continuity of trace class operators is accepted for publication in the American Journal of Mathematics. More examples of the CI's impact in the area of the proposal are described in Section D8.

The research performance of the CI has been recognised in the School of Mathematics and Statistics at UNSW where he was appointed in 2018 as the inaugural Scientia Fellow in the School, overcoming stiff competition from top national and international candidates. Amongst the strongest evidence of research standing and capacity in mathematics is the calibre of a researcher's collaborators. Since 2016 the CI has collaborated with Prof. Alain Connes (Fields Medal, Crafoord Prize), founder of the area of Non-commutative Geometry. The collaboration has yielded three publications [13,16] which are partly based on the theory of singular traces developed in the CI's works and monograph [26] on the subject. The CI was among 24 invited speakers at the international conference "Noncommutative Geometry: State of the Art and Future Prospects" celebrating Prof. Connes' 70-th birthday March 29-April 2 2017 at the Fudan Insitute for Advanced Study in Shanghai. Other invitees included Sir Michael Atiyah (Fields Medal, De Morgan Medal, Abel Prize), Pierre Cartier (Ampere Prize), Joachim Cuntz (Gottfried Wilhelm Leibniz Prize), Sorin Popa (Ostrowski Prize), Graeme Segal (Sylvester Prize), Dennis Sullivan (Wolf Prize, Oswald Veblen Prize, National Medal of Science), and Dan-Virgil Voiculescu (NAS Award in Mathematics).

The CI's current employment as UNSW Scientia Fellow involves 75% research and 25% teaching. The CI teaches high-level courses for UNSW's best students and is passionate about introducing them to research level mathematics (the proposed project contains a number of sub-tasks suitable for a PhD and honours students). Currently, the CI co-supervises 3 research PhD students working in the area of Non-commutative Probability, Analysis and Geometry; two of these students will submit their theses in 2019.

Project quality and innovation

Significance

During discussions covering a range of topics of importance in Non-commutative Geometry, Prof. Connes, in particular had emphasized to the CI that an extension of the Character Formula could be an excellent starting point for several new directions in Non-commutative Geometry. Here are two such directions. Firstly, the Character Formula plays an important role in the reconstruction theorem for closed Riemannian manifolds, and it is natural to expect that its extension would play a similar role for noncompact Riemannian manifolds. The reconstruction theorem for noncompact manifolds is, as yet, unproven. Secondly, developing suitable new techniques needed for a self-contained theory of locally compact spectral triples should also open an avenue for treating the case of noncommutative manifolds with boundary or even incomplete (e.g. punctured) manifolds.

This suggestion by Prof. Connes has been taken seriously by the CI and preliminary work in this direction has already brought substantial results. Firstly, jointly with Prof. Sukochev, the CI has achieved substantial progress in treating locally compact spectral triples using novel analytic methods (this work has not yet been published). Secondly, in a manuscript co-authored by Prof. Connes and the CI (and a number of collaborators from UNSW) a new approach to spectral triples involving *symmetric, non-self-adjoint* operators has been proposed [13]. These are precisely the required tools allowing the possibility to develop a new theory for non-commutative Riemannian manifolds with boundary. This development, alongside the present proposal, leads to a unified theory of noncompact non-commutative manifolds with boundary and incomplete manifolds.

Generally speaking, the development of Non-commutative Geometry and its applications is hindered by the paucity of non-trivial examples demonstrating its richness and ability to *compute* quantities of geometric significance for genuinely non-commutative manifolds. The project will generate a highly original approach to this computational task. By developing non-commutative versions of actions of symmetries on non-commutative differential manifolds, and non-commutative versions of the Minakshisundaram and Plejel asymptotic expansion of the heat kernel partition function, one of the most central tools in differential geometry and mathematical physics, the project would provide a breakthrough to definitions of fundamental geometric and topological invariants for non-commutative manifolds and will have direct applications to new models of the action principles that unify gravitation and the standard model of particle physics. The heat semi-group in the classical differential geometry provides a solution of a parabolic PDE. For a non-commutative manifold, the heat semi-group delivers the time evolution of an irreversible quantum system whose Hamiltonian is a (non-commutative) Laplace-Beltrami operator.

Constructing spectral triples for certain quantum groups (e.g. compact quantum groups like $SU_q(n)$ and $SO_q(n)$ as well as non-compact quantum groups like $SL_q(n)$) and their homogeneous spaces, the project would provide a new class of examples for Connes' Character Formula and for numerical computation of geometric invariants from kernel expansion and topological invariants from $K-$theory. Present constructions for quantum groups fail to provide such examples, resulting in a trivial Character Formula and trivial invariants.

One of the main technical innovation of the project is based on Double Operator Integration techniques developed by the CI recently in close collaboration with Alain Connes and Fedor Sukochev. The particular integral representations (see Approach to Aims 1,2,3 below) arose in [16] when geometric measures on limit sets of Quasi-Fuchsian groups were recovered by means of singular traces. They form the core contribution of a UNSW team (including the CI) to [16]. Achieving a proof of the Character Formula and heat trace expansions of non-commutative Laplacians using these techniques will have a marked scholarly impact on Non-commutative Geometry, demonstrating the utility of a major new technical device from another area of Mathematics to Non-commutative Geometry and enhancing the capacity of other researchers.

Conceptual/theoretical framework. Here we describe the design and methods which we are going to employ in order to resolve the problems stated above.

Our design involves breaking down the objectives into 7 specific aims.

Aim 1: Investigate when the Chern character provides a first order asymptotic expansion for the heat semi-group. More precisely, when

$$\mathrm{Tr}(\Omega(c)e^{-s^2D^2}) = \mathrm{Ch}(c)s^{-p} + O(s^{1-p}), \quad s \downarrow 0, \tag{6}$$

for every Hochschild cycle $c \in \mathscr{A}^{\otimes(p+1)}$? This aim is in "high completion status" due to unpublished joint work with Fedor Sukochev. We need, however, to substantially revise and strengthen that work and take into account deep connections with the Local Index Formula as in [3, 14] ignored in the first version.

Aim 2: Aim 1 above is related to a question concerning analyticity of a suitable $\zeta-$function. The latter is an analytic function defined by the formula

$$z \to \mathrm{Tr}(\Omega(c)(1+D^2)^{-\frac{z}{2}}), \quad \Re(z) > p.$$

We aim to find an analytic extension to the half-plane $\Re(z) > p-1$ so that

$$\lim_{z\to p}(z-p)\mathrm{Tr}(\Omega(c)(1+D^2)^{-\frac{z}{2}}) = p\mathrm{Ch}(c).$$

Aim 3: The purpose of Connes' Character Formula is to compute the Hochschild class of the Chern character by a "local" formula, which is customarily stated in terms of singular traces on the ideal $\mathscr{L}_{1,\infty}$. Here, a trace $\varphi : \mathscr{L}_{1,\infty} \to \mathbb{C}$ is a unitarily invariant linear functional; it can be seen from CI's results in [26] that such a functional is automatically singular, meaning that it vanishes on trace class operators. Our third aim is to show that

$$\varphi(\Omega(c)(1+D^2)^{-\frac{p}{2}}) = \mathrm{Ch}(c). \tag{7}$$

for every (normalised) trace φ on $\mathscr{L}_{1,\infty}$ and for every Hochschild cycle $c \in \mathscr{A}^{\otimes(p+1)}$, i.e. that any trace computes the leading term in Aim 1 and the residue in Aim 2.

Aims 2 and 3 are intimately connected with Aim 1 and together provide the Character Formula. However, the amount of analytical complications which arise when one navigates between formulas stated in all three aims for the non-compact case is enormous. It requires careful treatment and warrants a separation of these aims. Results from each aim are also of independent interest and warrant separate exploration and publication in terms of the theory of singular traces, analyticity of zeta functions in Non-commutative Geometry, and lower order heat trace expansions.

Aim 4: Introduce a generic (i.e., not necessarily a conformal deformation of a flat) Laplace-Beltrami operator Δ on the non-commutative torus and non-commutative Euclidean space. Prove a non-commutative version of the Minakshisundaram-Plejel theorem (for these manifolds) as outlined above.

Aim 4, in turn, depends on Aims 1 and 2, or rather on the technical instruments which are to be developed to deal with those aims in full generality.

Aim 5: Compute explicitly the curvature for a generic Riemannian metric on the non-commutative torus and non-commutative Euclidean space.

Completion of Aims 4 and 5 amounts to a completion of the program started by [5, 6, 28] to define approriate heat kernel co-efficients for these non-commutative manifolds. It opens many exciting new directions of research and generalisation in Non-commutative Geometry and enables genuine quantisation of an array of features of classical algebraic and differential geometry. Not in the least, it provides a new form of quantisation of the Einstein-Hilbert action that could have a profound influence on spectral action principles attempting to unify gravity and particle physics.

Aim 6: Construct a spectral triple (or more likely, a twisted spectral triple) on quantum groups (like $SU_q(n)$) and on their homogeneous spaces (like Podleŝ sphere). Previous attempts [5] suffer from a "dimension drop" pathology (that is, where spectral dimension differs from cohomological dimension). We aim to have an equivariance property for the Dirac operator in a way that prevents "dimension drop" pathology.

Aim 7: Design a version of Connes Character Formula for the above spectral triples on quantum groups which recover a non-trivial cocycle.

Our approach utilises new techniques developed by the CI and collaborators in Double Operator Integrals and singular trace theory. Techniques which have already enabled the CI to overcome technical challenges that have defeated others and resulted in breakthroughs in non-commutative probability theory and analysis as described in Section D8.

Approach to Aim 1: Our computations show, for a chain $c = a_0 \otimes \cdots \otimes a_p \in \mathscr{A}^{\otimes(p+1)}$, that

$$\mathrm{Ch}(c) := \frac{1}{2}\mathrm{Tr}(\Gamma F[F,a_0][F,a_1]\cdots[F,a_p]e^{-s^2D^2}) + O(s), \quad s \downarrow 0.$$

It seems plausible that Hochschild cochain on the right hand side is cohomologous to the one in the left hand side in (6). In fact, we have already made initial computations confirming this guess.

The method proposed is a more reliable than approach than [22] with far less assumptions. Their method provides a cluster of mutually cohomologous Hochschild cocycles. The key obstruction is that in our setting the corresponding cocycles are not exactly cohomologous. Hence, it is necessary to measure how far the cocycles in this cluster are from being cohomologous to each other.

Approach to Aim 2: The $\zeta-$function is of the form $z \to \mathrm{Tr}(CB^z)$. We know that $C = AC$ (hence $C = A^zC$) for a suitable operator A and, therefore,

$$\mathrm{Tr}(CB^z) = \mathrm{Tr}(CB^zA^z).$$

It is desirable to replace B^zA^z with $(A^{\frac{1}{2}}BA^{\frac{1}{2}})^z$ so that existing residue formulas, e.g. [26, Part III], for positive operators can be utilised. For this purpose, we use the integral representation (co-discovered by Connes, Sukochev and the CI [16])

$$B^zA^z - (A^{\frac{1}{2}}BA^{\frac{1}{2}})^z = T_z(0) - \int_{\mathbb{R}} T_z(s)\widehat{g}_z(s)\,ds,$$

where $(s,z) \to \widehat{g}_z(s)$ is a sufficiently good scalar-valued function and where

$$T_z(s) = B^{z-1+is}[BA^{\frac{1}{2}}, A^{z-\frac{1}{2}+is}]Y^{-is} + B^{is}[BA^{\frac{1}{2}}, A^{\frac{1}{2}+is}]Y^{z-1-is}.$$

By using Double Operator Integration techniques as developed in [29] for the commutators on the right hand side, we aim to prove analyticity of the right hand side and, hence, of the left hand side. The application of these techniques has been our trump card in joint work with Prof. Connes [16], in which we completed the proof of results conjectured by Connes and Sullivan left dormant for more than 20 years, until these new techniques were brought to bear.

Strong empirical evidence suggests the equivalence of Aims 1 and 2. Namely, the more is known about poles of the $\zeta-$function, the better asymptotic for the heat semi-group can be derived. In reality, the situation is not 100% clear – it looks like no information about the poles outside of the real line can be acquired from the heat semi-group asymptotic. We expect to show that one-side of the implication is correct, that is Aim 2 should actually follow from Aim 1.

Approach to Aim 3: Having established analyticity of the function

$$z \to \mathrm{Tr}(C(A^{\frac{1}{2}}BA^{\frac{1}{2}})^z), \quad \Re(z) > p-1,$$

with a simple pole at $z = p$, we expect that methods from [32] will lead us to the formula

$$\varphi(CA^{\frac{1}{2}}BA^{\frac{1}{2}}) = \frac{1}{p}\lim_{z\to p}(z-p)\mathrm{Tr}(C(A^{\frac{1}{2}}BA^{\frac{1}{2}})^z)$$

for every (normalised) trace on $\mathscr{L}_{1,\infty}$. Previous tailor-made approaches [3] are not sufficiently strong to derive Connes' Character formula in its full generality. We shall draw on the the deep theory of singular traces and its connections with operator $\zeta-$functions which the CI (with various collaborators) has been developing since 2009 [26, 32]. The CI is in a unique position to apply well-developed techniques from that theory in order to achieve this aim.

Approach to Aim 4: The definition of the Laplace-Beltrami operator involves a determinant of a matrix-valued function. On matrices, the determinant is defined as (practically unique) homomorphism into a field of scalars. On matrix-valued functions, it is defined pointwise.

However, on a non-commutative manifold, a matrix-valued function is replaced with a matrix whose entries belong to a von Neumann algebra. Though a surrogate notion of determinant (due to Fuglede and Kadison) exists for such matrices, our computations show an inconsistency in the formula (4) above. That is, instead of the volume state, a different functional appears on the right hand side of (4).

We propose to sacrifice the "homomorphism" property of the determinant for being able to perform computations making the formula (4) consistent with the definition of a Laplace-Beltrami operator. Precisely, we set

$$G^{-\frac{1}{2}} = (\det(g_{ij}))^{-\frac{1}{2}} \stackrel{def}{=} \pi^{-\frac{d}{2}} \int_{\mathbb{R}^d} e^{-\sum_{i,j} g_{ij}t_it_j}dt, \quad g = (g_{ij})_{i,j=1}^d \in GL_d(\mathscr{M}),$$

where $\mathscr{M}$ is the von Neumann algebra of the non-commutative torus (or non-commutative Euclidean space). Now, define the volume state h by setting $h(x) = \tau(xG^{\frac{1}{2}})$ and consider an inner product $(x,y) \to h(xy^*)$ on the non-commutative torus. We may define a Laplace-Beltrami operator by the formula

$$-\Delta_g x = M_{G^{-\frac{1}{2}}} \sum_{i,j=1}^{d} D_i M_{G^{\frac{1}{4}}(g^{-1})_{ij} G^{\frac{1}{4}}} D_j.$$

Here, M_x is the operator of multiplication by x on the left. Operators D_i, $1 \leq i \leq d$, are partial derivation operators (defined as in the commutative setting).

Our computations with a PhD student at UNSW, Edward McDonald, show that the so-defined operator is self-adjoint (and positive) and that the formula (4) becomes consistent. It should be also stated that this particular problem has been discussed in depth with Prof. Raphaël Ponge, who is to take a part time position on an ARC funded grant (with CI Fedor Sukochev) in UNSW for the next few years. Prof. Ponge (who is a former PhD student of Alain Connes and a top class expert in non-commutative differential geometry) is actively involved in the discussions with CI concerning this plan of action.

Approach to Aim 5: In the special case of a conformal deformation of a flat metric on the non-commutative torus, the Laplace-Beltrami operator is (unitarily equivalent to) $M_h \Delta M_h$, where Δ is the flat Laplacian and $h \in C(\mathbb{T}^d_\theta)$ is a positive invertible element. According to the asymptotics in Aim 3, for $x \in L_\infty(\mathbb{T}^d_\theta)$, the function

$$z \to \mathrm{Tr}(M_x(-M_h \Delta M_h)^{-z})$$

admits an analytic extension with at most simple poles at $z = \frac{d}{2}, \frac{d}{2} - 1, \cdots$. The curvature term is provided (for $d > 2$) by the residue of this function at the point $\frac{d}{2} - 1$. Our function has the shape

$$z \to \mathrm{Tr}(C(A^{\frac{1}{2}} B A^{\frac{1}{2}})^z).$$

It is desirable to replace $(A^{\frac{1}{2}} B A^{\frac{1}{2}})^z$ with $B^z A^z$ in order to obtain candidates for the co-efficients A_k described in "Aims and Background". Here, we propose to use an integral representation similar to (but more complicated than) the one specified in the Approach to Aim 2. Despite the technical difficult of the resultant terms the double operator integration techniques we have developed over the last 10 years are still able to be used.

We expect that the Laplace-Beltrami operator introduced above is compatible with heat semi-group asymptotics up to terms of arbitrarily high order. In the case of a general metric tensor, the formulae become much harder and more investigation is required. One needs to investigate conditions under which such an asymptotic exists. We plan to refine methods in [25] by the use of double and multiple operator integrals.

Approach to Aim 6: The spectral triple constructed in [5] for $SU_q(2)$ is equivariant and its spectral dimension is 3. However, every $3-$cocycle is cohomologous to 0 and, therefore, the homological dimension is strictly less than 3.

We expect that the reason for this phenomenon is the wrong choice of $q-$deformed left regular representation. Our aim is to find a suitable $q-$deformation which permit the spectral triple to be equivariant, $3-$dimensional and, at the same time to allow certain $3-$cocycles to be non-trivial. A natural candidate for such a $3-$cocycle is the Hochschild class of the Chern Character. Successful resolution of Aims 1,2,3 will deliver the required technical tools to determine the "correct representation" and such cocycles. We expect the approach for $SU_q(2)$ to generalise to other quantum groups.

Approach to Aim 7: Ordinary spectral triples in this setting should be replaced by twisted ones (where commutators $[D,a]$ may be unbounded, but a certain "twisted" commutator is bounded). At the moment, no satisfactory theory is available which allows the derivation of Connes Character formula for such triples. The only attempt is made in [18]. However, we expect that a combination of approaches from [18] and [4] provide the germ of such a theory.

Connes Character Formula is one of the central components of Connes' Reconstruction Theorem. Based on our firm belief in the validity of Connes Character Formula in this setting, we intend to analyse the assumptions on the operator D in a spectral triple for $SU_q(n)$ required to make a proof of such a formula work. We expect these assumptions, together with the equivariance property, to define the operator D uniquely.

Feasibility and strategic alignment

UNSW's Strategy 2025 goal of Research Excellence aims to foster researchers of the growing impact and calibre of the CI in Fundamental and Enabling Sciences. The CI's research in Mathematics, and the topic of this Fellowship, is directly aligned with the UNSW School of Mathematics and Statistics' world-class profile in non-commutative analysis; a strength the CI has helped to build.

The research facilities are ideal. The School has been ranked number one in Mathematics in Australia for each of the past six years in the Academic Ranking of World Universities. It has an extensive and well supported program of colloquia and international visits. It has two current Laureate Fellows, including Prof. Sukochev in non-commutative analysis. Recognising the importance of the contribution of Prof. Sukochev and the CI to the School's success, the area of the project (non-commutative analysis and geometry) has been named as one of the core strengths of the School.

The School has a Distinguished Researcher Support Program that will provide $10K support for one international visitor for a visit of up to four weeks in each year of the Fellowship. The UNSW Scientia Fellowship Program will top up the difference between the Future Fellowship and the CI's current Scientia Fellowship. The School will use salary savings to employ a fixed term Lecturer in the area of the Fellowship (i.e. functional analysis). Strong preference will be given to an appointment that can work with the CI on research directly related to the project. The CI will sit on the search committee. As one of UNSW's top young researchers, the CI will be proactively mentored through UNSW's Research Development Framework program.

At the conclusion of the Fellowship, the CI will return to a continuing appointment in the School of Mathematics and Statistics. The CI's current 4 year Scientia Fellowship will be paused for the period of the Fellowship and available for the CI to resume after the completion of the Fellowship. This will afford the CI additional opportunity and resources to maximise the outcomes and benefit of the Fellowship.

Benefit and collaboration

The outcomes are expected to develop research at the forefront of non-commutative mathematics and ultimately impact research in mathematical physics and other sciences. The collaborations enhanced and established during this project, the interest generated by contributing to fundamental non-commutative geometric analogues of objects central to modern mathematics such as Laplacians and symmetries, and the quality of the journals where results from the project can be published, will strengthen Australian science.

The project will enhance international collaboration in a highly active international research field. The CI is collaborating with highly-distinguished international experts (Alain Connes, Kenneth Dykema, Nigel Higson, Marius Junge). This collaboration has already resulted in a number of papers in prestigious journals. Involvement of mathematicians of such calibre would be beneficial not only for this project, but also to other directions of mathematical research in UNSW. Alain Connes, the leading world expert in the area and one of the most outstanding mathematicians of the last fifty years, is very enthusiastic about the suggested direction of research. This endorsement and his on-going commitment to the joint research effort is a strong acknowledgment of the depth and importance of the current proposal.

The research impact of this project will enhance Australia's profile in the crucial area of Quantised Calculus and its applications to Non-commutative Geometry and improve the international competitiveness of Australian research. The project will maintain and foster international collaborations as mentioned and simultaneously contribute to training the next generation of Australian mathematicians. This proposal is on par with efforts of top world experts in the area and thus will strengthen Australian leadership in this area.

Australia has a strong research profile in Non-commutative Geometry and Operator Algebras broadly. The outcomes of this project will therefore be of interest to a number of research groups in Australia (Wollongong, Adelaide, Sydney and Canberra). The project will broaden existing Australian strengths and introduce new directions in this active and internationally competitive area of research.

Due to the breadth and depth of the proposal across three cutting-edge topics in Non-commutative Geometry, the CI expects to attract top Australian students for postgraduate studies that might otherwise head overseas.

Communication of results

Results of this project will be published in peer-reviewed journals (as well as on arxiv). CI and collaborators will also deliver the results in thematic international conferences.

Management of data

UNSW has implemented a data storage solution for every stage in the life cycle of a research project. The data management plan for the project will be established using UNSW's resources when applicable. Data will be archived using UNSW's Long-term Data archive (or other archive mechanism as applicable). Data will made discoverable by registration on discipline-specific registries and indexation services.

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C2. Statement by the Administering Organisation

(Provide a Statement that addresses the relevant criteria as set out in the grant guidelines. The Statement must be signed by the Deputy Vice-Chancellor (Research) or equivalent. (Upload a PDF of no more than three A4 pages))

Uploaded PDF file follows on next page.

C2 Statement by the Administering Organisation outlining Strategic Alignment

ARC *Future Fellowships* Applicant Dr Dmitriy Zanin [FT190100442]

UNSW, one of Australia's leading research intensive universities, fully supports the application by Dr Dmitriy Zanin for a Future Fellowship, Level 1.

The research proposed in Dr Zanin’s application is closely aligned with the UNSW Research Strength[1] of ‘Fundamental and Enabling Sciences’. Dr Zanin’s research in Mathematics, and the topic of this Fellowship will have direct applications to new models of the action principles that unify gravitation and the standard model of particle physics.

Contribution of the Future Fellow and Project to Building these Research Strengths at UNSW

Dr Zanin currently holds a UNSW Scientia Fellowship in the School of Mathematics and Statistics, within the UNSW Faculty of Science.

The UNSW Scientia Fellowship program is one of the cornerstones of UNSW’s 2025 Strategy[2]. The primary goal of the program is to enhance UNSW research performance by attracting and retaining exceptional researchers at the highest level relative to career stage and supporting them with a unique development and collaboration package and career pathway commitment.

The UNSW School of Mathematics and Statistics provides an ideal intellectual environment for the successful execution of Dr Zanin’s research program. UNSW has been ranked number one in Mathematics in Australia for each of the past six years in the Academic Ranking of World Universities and received the highest ranking of 5 (‘well above world standard’) in Pure Mathematics in the 2015 ERA report. Furthermore, this proposed research is strongly aligned with the School’s core research strengths in operator algebras and noncommutative analysis.

UNSW Support for the Future Fellow

The UNSW Scientia Fellowship Program comprises a salary, as per the UNSW Academic salary rates, and a career development and collaboration support package of additional $40,000 per annum. The UNSW Scientia Fellowship Program will top up the difference between the Future Fellowship and the Scientia Fellowship.

The UNSW School of Mathematics and Statistics has a Distinguished Researcher Support Program that will provide $10K support (return air-fare, accommodation allowance and living allowance) for one international visitor for a visit of up to four weeks in each year of this Fellowship. The international visits will bring expertise to the project and help to build enduring international research links that will last beyond the term of the project.

The School recognises the high value and likely impact of this Fellowship, and will use salary savings to employ a fixed term Lecturer in the area of the Fellowship (i.e. functional analysis). Strong preference will be given to an appointment that can work with Dr Zanin on research directly related to the project. Dr Zanin will sit on the search committee to ensure that an appropriate voice is given to this selection criterion.

The University also assists its researchers in developing and maintaining pathways for their ongoing development. As such, UNSW has established several initiatives that provide research staff with professional support in planning and developing their careers. Formal performance appraisals are performed in all Faculties and researchers are proactively mentored through an innovative Researcher Development Framework program.

Integration of the Future Fellow into UNSW Research Activities after Fellowship

At the conclusion of the Fellowship, Dr Zanin will return to a continuing appointment in the UNSW School of Mathematics and Statistics.

Dr Zanin’s current four year UNSW Scientia Fellowship, which will be paused for the period of the ARC Fellowship, will be available for him to resume. This will afford him additional opportunity to focus on research and additional resources to continue his international research collaboration after the Future Fellowship. He will have a key role in the growth and further development of the School’s research group and world-leading profile in noncommutative analysis.

In closing, I reiterate UNSW Australia's strongest support for the application by Dr Zanin for a Future Fellowship in 2019 and welcome the opportunity provided by the Australian Research Council to promote and support research both for the benefit of Australia and the University.

Yours sincerely

Professor Nicholas Fisk
Deputy Vice-Chancellor (Research)
UNSW Sydney

[1] UNSW Research Strengths (https://research.unsw.edu.au/unsw-areas-research-strength)
[2] UNSW 2025 Strategy (https://www.2025.unsw.edu.au/)

C3. Medical Research

(Does this project contain content which requires a statement to demonstrate that it complies with the eligible research requirements set out in the ARC Medical Research Policy located on the ARC website?)

No

C4. Medical Research Statement

(If applicable, justify why this project complies with the eligible research requirements set out in the ARC Medical Research Policy located on the ARC website. Eligibility will be based solely on the information contained in this application. This is the only chance to provide justification and the ARC will not seek further clarification. (No more than 750 characters, approximately 100 words))

Part D - Personnel and ROPE (Dr Dmitriy Zanin)

D1. Personal Details

(To update personal details, click the ‘Manage Personal Details’ link below. Note this will open a new browser tab. When returning to the form ensure to ‘Refresh’ the page to capture the changes made to the Future Fellowship candidate's profile.)

Participation Type

Future Fellowship

Title

Dr

First Name

Dmitriy

Family Name

Zanin

Citizenship

Australia

Australian Permanent Resident

N/A

Australian Temporary Resident

N/A

D4. Current country of residence

(If the Future Fellowship candidate is a Foreign National, they must obtain a legal right to work and reside in Australia.)

Australia

D5. Qualifications

(To update any details in this table, the details in the 'Qualifications' section of the Future Fellowship candidate's profile must be updated.)

Conferral Date	AQF Level	Degree/Award Title	Discipline/Field	Awarding Organisation	Country of Award
02/06/2011	Doctoral Degree	PhD	Mathematics	Flinders University	Australia

D6. Research Opportunity and Performance Evidence (ROPE) – Current and previous appointment(s) / position(s) - during the past 10 years

(To update any details in this table, the details in the 'Employment' section of the participant's profile must be updated. Refer to the Instructions to Applicants for more information.)

Description	Department	Contract	Employment	Start Date	End Date	Organisation

		Type	Type			
UNSW Scientia Fellow	School of Mathematics and Statistics, Department of Pure Mathematics	Permanent	Full Time	01/07/2018		The University of New South Wales
DECRA Fellow	School of Mathematics and Statistics	Contract	Full Time	01/07/2015	30/06/2018	The University of New South Wales
Postdoctoral Fellow	School of Mathematics and Statistics	Contract	Full Time	01/01/2012	30/06/2015	The University of New South Wales
Postdoctoral Fellow	School of Mathematics and Statistics	Contract	Part Time	01/01/2011	31/12/2011	The University of New South Wales
PhD student	School of Computer Science, Engineering and Mathematics	Contract	Full Time	01/01/2008	31/12/2011	Flinders University

D7. Research Opportunity and Performance Evidence (ROPE) - Academic Interruptions

(You and the Future Fellowship candidate must read the ROPE Statement http://www.arc.gov.au/arc-research-opportunity-and-performance-evidence-rope-statement before filling out this section.)
Has the Future Fellowship candidate experienced an interruption that has impacted on their academic record?

No

D8. Research Opportunity and Performance Evidence (ROPE) - Details of the Future Fellowship candidate's academic career and opportunities for research, evidence of research impact and contributions to the field, including those most relevant to this application

(Provide details of the Future Fellowship candidate's academic career and opportunities. This should not include information presented in the following sections. (Upload a PDF of no more than five A4 pages))

Uploaded PDF file follows on next page.

D8. Research Opportunity and Performance Evidence (ROPE) - Details of the Future Fellowship candidates academic career and opportunities for research, evidence of research impact and contributions to the field, including those most relevant to this application

Amount of time as an active researcher

I was awarded my PhD in Mathematics 7 years ago in 2011 at Flinders University of South Australia. During those 7 years, I have had no interruptions in research opportunity.

Research opportunities

Currently I am employed at UNSW as Scientia Fellow in the School of Mathematics and Statistics. It is a research intensive position that began July 2018 and will last 4 years. My workload is 75% research and 25% teaching including high-level courses for honours students. At the end of the Scientia Fellowship, my position becomes permanent.

Previously, I was employed as an ARC DECRA Fellow at UNSW (July 2015 – June 2018).

From 2011 until June 2015 I was employed as a researcher under DP grants DP110100064 and DP120103263 awarded to Prof. Sukochev. I contributed subsequently to both DP110100064 and DP120103263. The positions under Prof. Sukochev afforded me the opportunity to extend my areas of interest and expertise in noncommutative probability and functional analysis to noncommutative geometry, and engage internationally (Germany, USA, Russia, China, Poland) through invited visits and attendance at workshops such as Oberwolfach and Bedlewo. Without a permanent position I have been unable to apply for funding under the ARC Discover Project scheme directly.

A number of brilliant mathematicians have made, and continue to make, a significant impact on my research career. While completing my undergraduate study in Tashkent, Uzbekistan, I was impressed by Professor Chilin's attitude to Mathematics. It would be fair to say that following his example I decided to be a mathematician. My PhD supervisor Professor Sukochev taught me the foundations of non-commutative integration theory, but also an approach to Mathematics involving strict work ethics, enthusiasm and perseverance. From 2009, I was introduced to the problems on singular traces and was fortunate to work with Professor Nigel Kalton, a giant of functional analysis and Banach Medal winner. I was positively shocked by the depth and speed of this great thinker, whose insight and technical ability dwarfed everyone else I had encountered. The impression is with me still of the modesty of his genius, he was gentle and patient with those who were not grasping the details as quickly as himself. The technical devices introduced by Professor Kalton subsequently inspired my own inventions in this area. Since 2013, I had an opportunity to collaborate with Professor Dykema during my visits to Texas A&M University and his reciprocal visits to UNSW. Professor Dykema introduced me to the advanced techniques of non-commutative probability which helped me a lot during my DECRA project. From 2015, I regularly collaborate with Professor Connes, a Fields Medal and Crafoord Prize winner, both are equivalents to the Nobel Prize in Mathematics, so needless to say this collaboration has impressed and inspired me deeply.

Research achievements and contributions

I was awarded ARC DECRA Fellowship in 2014. My DECRA project titled "On the new concept of independence in non-commutative probability theory" was ranked highly — the top ranking in the area of Pure Mathematics. The research income generated by this Fellowship equals \$ 300 000. In the duration of the Fellowship, 29 publications in high ranked international journals (see the list of publications in Section D9 below) were produced.

In 2017 I have gained the hightly competitive position of Scientia Fellow in UNSW (the first Fellow appointed in the School of Mathematics and Statistics). This position also entailed research funding up to \$ 160 000 for the period of 4 years.

I am regularly invited to deliver keynote research talks in the top international conferences. For example, in 2017, I was invited by Professor Connes to deliver a plenary lecture concerning my recents at his 70th anniversary conference. In 2018 I delivered 3 keynote plenary lectures in the international conferences "Noncommutative Geometry and Representation Theory " in Chengdu, "Banach Space Theory and Applications" in Sanya and "Operator Theory 27" in Timisoara.

I am regular assessor for the submissions to the top international journals such as Journal of Functional Analysis, Advances in Mathematics and Journal of Operator Theory.

Below I will take the opportunity and present a detailed account of my research history and most important contributions made in my career.

I started my research in non-commutative functional analysis at Flinders University as a PhD student. Since then my contribution to this research field include my PhD Thesis and over 50 peer-reviewed publications including a monograph "Singular traces: Theory and Applications" in the De Gruyter series "Studies in Mathematics". My research articles are published in high ranked journals such as Journal für die Reine und Angewandte Mathematik (Crelle's Journal), Advances in Mathematics, Journal of Functional Analysis, Journal of Spectral Theory, Pacific Journal of Mathematics as described in Section D9. All numbered references in this Section refer to the numbered publications in Section D9. My PhD Thesis was highly regarded by the referees, one of them wrote "this is the best PhD thesis I ever refereed during my whole career". My subsequent work spans non-commutative analysis, non-commutative geometry and non-commutative probability theory.

Some of the principal achievements of my work now follow. The primary objective of my PhD study was two-fold. One direction was studying orbits in symmetric function and sequence spaces. The particular question of interest was whether an orbit of a semi-group of bicontractions is a norm-closed convex hull of its extreme points. While answering this question, I established a collaboration with Nigel Kalton. I published a number of research papers on orbits; one of them with Professor Kalton. The other direction concerned Khinchine-type inequalities. The principal tool in the studies of orbits involves Khinchine-type inequalities; a deep extension of the Birkhoff theorem on doubly stochastic matrices. By employing this technique in my PhD Thesis I was able to (i) develop a unified approach to orbits on symmetric function and sequence spaces; (ii) significantly broaden the setting of applicability of earlier results; (iii) to determine the conditions that are necessary and sufficient for the affirmative answer to the above question. Central to the study of probabilistic inequalities (such as Khinchine inequalities) is the so-called Kruglov operator (an integration over a Poisson process). Using the Kruglov operator, I radically extended the area where Khinchine-type inequalities hold true.

Based on the advances in the theory of symmetric function spaces made in my PhD Thesis I obtained several profound results in non-commutative analysis. One of them resolves a long standing question of existence of continuous traces on Banach ideals in $\mathscr{L}(H)$ (the $*-$algebra of all bounded operators on a separable Hilbert space H). It was already known to experts that traces (even without the continuity assumption) do not exist on an ideal which can be obtained from the couple $(\mathscr{L}_p, \mathscr{L}_\infty)$, $p > 1$, by interpolation. Using ideas from my PhD thesis, I proposed to study a weaker condition:

$$\frac{1}{n}\|A^{\oplus n}\|_{\mathscr{I}} \to 0 \text{ as } n \to \infty \text{ for every operator } A \text{ in the ideal } \mathscr{I}. \quad (1)$$

The condition (1) holds for every interpolation space as above and prevents the existence of a continuous trace on the Banach ideal $\mathscr{I}$. In my paper [10] (joint with my former supervisor F. Sukochev, published in Crelle's Journal), the converse assertion is proved; namely, condition (1) holds for every Banach ideal $\mathscr{I}$ without continuous traces. In the same paper, I constructed (under the assumption that (1) fails) a positive continuous trace which respects the Hardy-Littlewood submajorization (and also the one that does not).

It is a standard result in linear algebra that a trace of a matrix depends only on its eigenvalues. A corresponding result for the standard trace on the trace class ideal $\mathscr{L}_1$ is radically harder (it was resolved in the affirmative by V. Lidskii only in 1959). More precisely, the following formula holds.

$$\mathrm{Tr}(A) = \mathrm{Tr}(\mathrm{diag}(\lambda(A))) \text{ for every operator } A \text{ in the ideal } \mathscr{L}_1.$$

Here, $\mathrm{diag}(\lambda(A))$ is a diagonal operator with eigenvalues of A on the diagonal (repeated with multiplicities). In his famous 1990 paper, Pietsch asked whether the same is true for an arbitrary trace on an arbitraryHe characterized this question as "extremely difficult". A partial answer for countably generated ideals was given by Kalton in his 1998 paper. In my paper [9] (joint with F. Sukochev, published in Advances in Mathematics), a class of ideals closed with respect to the logarithmic submajorization is introduced. In this paper, it is proved that if an ideal $\mathscr{I}$ is closed with respect to the logarithmic submajorization, then

$$\varphi(A) = \varphi(\mathrm{diag}(\lambda(A)))$$

for every operator $A \in \mathscr{I}$ and for every trace φ on $\mathscr{I}$. Hence, for the mentioned class of ideals Pietsch's question is answered in the affirmative. Conversely, if the ideal $\mathscr{I}$ is not closed with respect to the logarithmic submajorization, then there exists an operator $A \in \mathscr{I}$ such that $\mathrm{diag}(\lambda(A)) \notin \mathscr{I}$ and the question is answered in the negative.

A classical result due to Schur states that every matrix is unitarily conjugate to an upper-triangular one. Thus, it is a sum of a normal matrix (diagonal part) and a nilpotent matrix (strictly upper-triangular part). A corresponding result for compact operators in $\mathscr{L}(H)$ is proved by Ringrose in his 1962 paper. Namely, he proved that every compact operator is a sum of a normal one and a quasi-nilpotent one. It is natural to ask whether a similar result holds in the setting of type II von Neumann algebras. In 2013, I worked with Ken Dykema and we managed to solve this question by using techniques from Free Probability Theory. In my paper [8] (joint with K. Dykema and F. Sukochev, published in Crelle's Journal), it is proved that every operator T in a type II_1 von Neumann algebra can be written as $T = N + Q$, where N is normal and Q is "almost"nilpotent in the following sense

$$|Q^n|^{\frac{1}{n}} \to 0 \text{ as } n \to \infty \text{ in the strong operator topology.}$$

Connes Character Theorem is of paramount importance in non-commutative geometry. It expresses a certain cocycle (called the Chern character) in terms of Dixmier traces. This cocycle is then used to explicitly compute indices, Euler characteristics and other topological invariants via smooth expressions. In my paper [7] (joint with A. Rennie, A. Carey and F. Sukochev, published in Journal of Spectral Theory), a completely new approach to Connes Character Theorem is proposed. In a holistic way, motivation came from (Kalton et al., Adv. in Math., 2013). The result is now proved for an arbitrary trace (not only a Dixmier trace as in Connes' original approach).

A 50-year old problem due to Krein asks whether Lipschitz functions are operator Lipschitz on a given Banach ideal. For Schatten ideals $\mathscr{L}_p$, $1 < p < \infty$, the assertion is known to be true. Namely, Potapov and Sukochev (Acta Math., 2011) proved that

$$\|f(A) - f(B)\|_p \leq c_p \|f'\|_\infty \|A - B\|_p$$

for every couple of self-adjoint operators (A,B) such that $A - B \in \mathscr{L}_p$. For $p = 1$ (or for $p = \infty$), a similar result is false. Kato and Davies proved that even absolute value fails to be operator Lipschitz in $\mathscr{L}_1$. Nazarov and Peller asked whether the following replacement

$$\|f(A) - f(B)\|_{1,\infty} \leq c_{abs} \|f'\|_\infty \|A - B\|_1, \tag{2}$$

holds, where $\mathscr{L}_{1,\infty}$ is the principal ideal generated by the operator $\mathrm{diag}(\{1, \frac{1}{2}, \frac{1}{3}, \cdots\})$ equipped with a natural quasi-norm.

To solve this problem, I co-operated with Martijn Caspers and we jointly created a transference method. This method reduces boundedness of certain Double Operator Integrals (which naturally arise in the setting above) to complete boundedness of certain Fourier multipliers. Using this method, we proved (in collaboration with D. Potapov and F. Sukochev) that, indeed, (2) holds true. The manuscript [1] is accepted for publication in the American Journal of Mathematics. The result has been verified by leading experts G. Pisier, Q. Xu, V. Peller, M. Junge and others and has now become accepted as an outstanding achievement in operator calculus. Some of the key machinery of our proof was developed in our earlier publications.

Inequalities proved by Johnson and Schechtman relate the norm of the sum of independent random variables to that of their disjoint copies. Namely, if $E \supset L_p$, $p < \infty$, is a symmetric function space on $(0,1)$ and if $x_k \in E$ are mean zero independent random variables, then

$$\|\sum_{k\geq 0} x_k\|_E \approx_E \|\bigoplus_{k\geq 0} x_k\|_{Z_E^2},$$

where Z_E^2 is a certain symmetric function space on $(0,\infty)$. Astashkin and Sukochev (Isr. J. Math., 2005) proposed another method of proving Johnson-Schechtman inequalities by means of the so-called Kruglov operator (which is an integration with respect to the Poisson process). In my paper (joint with S. Astashkin and F. Sukochev, published in Pacific Journal of Mathematics), it is proved that Johnson-Schechtman inequalities hold in a symmetric (quasi-Banach) function space if and only if the Kruglov operator is bounded on that space. It is tempting to extend the technique related to the Kruglov operator to the domain of the non-commutative probability. In my paper (joint with F. Sukochev, published in Journal of Functional Analysis), I constructed an analogue of the Kruglov operator in the Free Probability Theory. It appears that Kruglov operator behaves better in the setting of free probability than in the setting of a classical probability; and that Johnson-Schechtman inequalities are always true.

This breakthrough inspired a question: what sort of independence suffices to guarantee the Johnson-Schechtman inequalities for a sufficiently rich class of symmetric spaces? I proposed this investigation as the subject for my DECRA project approved by the ARC in November 2014. We shared this question with a well-known Chinese probabilist Yong Jiao. Professor Jiao than came to UNSW for a long-term research visit, during which we obtained a significant achievement: if a symmetric space E is an interpolation space for the couple (L_p, L_q), $1 < p < q < \infty$, then Johnson-Schechtman inequalities hold true in E for an arbitrary sequence of mean zero random variables independent in the sense of Junge and Xu. This paper [6] is published in the Journal of the London Mathematical Society. In 2016, it was presented on the Symposium on Modern Analysis and Applications in Harbin.

In 2016, Alain Connes suggested to me the conjecture from his book "Noncommutative geometry". This conjecture due to Connes and Sullivan stood for more than 20 years. There are 2 versions of the conjecture (for Julia sets as stated below) and for Kleinian groups. Let f_c be a quadratic polynomial $f : z \to z^2 + c$. Julia set of f is a Jordan curve if and only if c is in the main cardioid of the Mandelbrot set. For such c, there exists an analytic mapping Z with the property

$$Z \circ f_0 = f_c \circ Z.$$

The conjecture asserts that

$$\varphi(M_h |[F, M_Z]|^p) = \int_{J(f_c)} h(z) d\nu(z)$$

for every singular trace φ on $\mathscr{L}_{1,\infty}$. Here, F is the Hilbert transform and ν is the $p-$dimensional geometric measure on the Julia set $J(f_c)$. In other words, one can recover the geometric measure via singular traces.

Jointly with Alain Connes, Edward McDonald and Fedor Sukochev, we solved this conjecture (see [3] and [13]). I delivered a keynote talk about resolution of this conjecture in the international conference "Noncommutative Geometry: State of the Art and Future Prospects" celebrating Connes's 70'th birthday March 29-April 2 2017 at the Fudan Insitute for Advanced Study in Shanghai. I was among two-dozen invited speakers including Sir Michael Atiyah (Fields Medal, De Morgan Medal, Abel Prize), Pierre Cartier (Ampere Prize), Alain Connes (Fields Medal, Grafoord Prize), Joachim Cuntz (Gottfried Wilhelm Leibniz Prize), Sorin Popa (Ostrowski Prize), Graeme Segal (Sylvester Prize), Dennis Sullivan (Wolf Prize, Oswald Veblen Prize, National Medal of Science), and Dan-Virgil Voiculescu (NAS Award in Mathematics).

Many of these research achievements underpin the research proposed for this Future Fellowship, and demonstrate my insight and ability in solving hard problems in mathematics using novel methods that often extend to new results in considerably more general contexts.

D9. Research Opportunity and Performance Evidence (ROPE) - Research Outputs

i. Research context: Provide clear information that explains the relative importance of different research outputs and expectations in the Future Fellowship candidate’s discipline/s. The information should help assessors understand the Future Fellowship candidate’s academic research achievements and should include the importance/esteem of specific journals in the Future Fellowship candidate’s field; specific indicators of recognition within the Future Fellowship candidate’s field such as first authorship/citations, or the significance of non-traditional research outputs. (No more than 3,750 characters, approximately 500 words)

The best measures of the quality of the mathematical research is feedback or acclaim by peers, including invitations to collaborate and present, and quality of the journals where publications appear.

In mathematics, the list of authors is always arranged alphabetically and does not indicate contribution. Citation rates in pure mathematics are lower than other fields, >200 citations for a Level B Research Fellow is outstanding, a typical Level E Professor in an Australian university will have >500 citations and >1000 citations for a Level E Professor is outstanding.

The bibliometric database universally accepted in mathematics is MathSciNet, which includes citations from book and journal publications only and does not include arXiv citations. Typically it provides much lower citation rates than, say, Google Scholar.

Journal quality: I consistently publish influential articles in high-profile mathematical journals. These journals are typically ranked A* or A by the Australian Mathematical Society. I have published 19 articles in A* ranked journals in the last 5 years. Examples of these journals are: Journal für die reine und angewandte Mathematik (the oldest mathematical journal still in publishing) where I have published 2 articles, Advance in Mathematics (among the top five journals in Pure Mathematics) where I have published 5 articles, Communications in Mathematical Physics (the most authoritative journal in the area) where I have published 1 article, Journal of Functional Analysis (the primary journal in Functional Analysis) where I have published 10 articles, Ergodic Theory and Dynamical Systems, American Journal of Mathematics, Transactions of the AMS, Journal of the London Mathematical Society where I have published 2 articles, and other excellent journals.

Output: According to MathSciNet, I have 47 articles published in international research journals and 1 monograph. In addition to that, there are 3 accepted articles and a large number of submissions. 41 of those publications are in the last 5 years. Having only completed my PhD 7 years ago, many of my publications were published only a few years ago. However, I currently have 204 citations from 81 different authors.

As a measure of the quantity and quality of my research collaborators, I have 31 co-authors who, between them, have 31212 citations on MathSciNet. Among these co-authors are "luminaries" such as Alain Connes, Nigel Kalton and Kenneth Dykema as mentioned who have won numerous international prizes. Collaboration with mathematicians of such calibre is the highest honour and recognition in the field.

According to MathSciNet, my monograph has 59 citations. This is an excellent rate for a specialist reference published 5 years ago. As a comparison, the nearest text in terms of content is Barry Simon’s “Trace ideals and their applications.” LMS Lecture Note Series, 35, Cambridge University Press, Cambridge-New York, 1979 which has 467 citations over 39 years. 5 more of my papers were cited at least 10 times. My citation rate is increasing each year.

ii. Research output list: List the research outputs most relevant to this application categorised under the following headings: Ten career-best research outputs; Authored books; Edited books; Book chapters; Referred Journal articles; Fully refereed conference proceedings; Additional research outputs (including non-traditional research outputs). CVs and theses should not be included in this list. The Future Fellowship candidate's ten career-best research outputs should not be repeated under subsequent headings. (Upload a PDF of no more than five A4 pages)

Uploaded PDF file follows on next page.

Ten career-best research outputs

[1] Caspers M., Potapov D., Sukochev F., Zanin D. *Weak type commutator and Lipschitz estimates: resolution of the Nazarov-Peller conjecture.* Amer. J. Math., to appear.

[2] Sukochev, F.; Zanin, D. *Connes integration formula for the noncommutative plane.* Comm. Math. Phys. **359** (2018), no. 2, 449–466.

[3] Connes A., Sukochev F., Zanin D. *Trace Theorem for quasi-Fuchsian groups.* Mat. Sb. **208** (2017), no. 10, 59–90.

[4] Junge M., Sukochev F., Zanin D. *Embeddings of symmetric operator spaces into $\mathscr{L}_p-$spaces on finite von Neumann algebras.* Adv. Math. **312** (2017), 473–546.

[5] Lord S., McDonald E., Sukochev F., Zanin D. *Quantum differentiability of essentially bounded functions on Euclidean space.* J. Funct. Anal. **273** (2017), no. 7, 2353–2387.

[6] Jiao Y., Sukochev F., Zanin D. *Johnson-Schechtman and Khinchine inequalities in noncommutative probability theory.* J. Lond. Math. Soc. (2) **94** (2016), no. 1, 113–140.

[7] Carey A., Rennie A., Sukochev F., Zanin D. *Universal measurability and the Hochschild class of the Chern character.* J. Spectr. Theory **6** (2016), 1–41.

[8] Dykema K., Sukochev F., Zanin D. *A decomposition theorem in II_1-factors.* J. Reine Angew. Math. **708** (2015), 97–114.

[9] Sukochev F., Zanin D. *Which traces are spectral?* Adv. Math. **252** (2014), 406–428.

[10] Sukochev F., Zanin D. *Traces on symmetrically normed operator ideals.* J. Reine Angew. Math. **678** (2013), 163–200.

Authored books

[11] Lord S., Sukochev F., Zanin D. *Singular traces: Theory and Applications.* De Gruyter Studies in Mathematics. Walter de Gruyter, Berlin, first edition, 2013.

Edited books

Book chapters

Refereed journal articles

[12] Ber A., Sukochev F., Zanin D. *Heisenberg relation for locally measurable operators.* Adv. Math. **335** (2018), 211–230.

[13] Connes A., McDonald E., Sukochev F., Zanin D. *Conformal trace theorem for Julia sets of quadratic polynomials.* Ergodic Theory Dynam. Systems. (published online).

[14] Potapov D., Sukochev F., Tomskova A., Zanin D. *Frechet differentiability of the norm of L_p-spaces associated with arbitrary von Neumann algebras.* Trans. AMS, to appear.

[15] Dykema K., Noles J., Zanin D. *Decomposability and norm convergence properties in finite von Neumann algebras.* Integral Equations Operator Theory **90** (2018), no. 5, Art. 54, 32 pp.

[16] Jiao Y., Zhou D., Wu L., Zanin D. *Noncommutative dyadic martingales and Walsh-Fourier series.* J. Lond. Math. Soc. (2) **97** (2018), no. 3, 550–574.

[17] Levitina G., Sukochev F., Vella D., Zanin D. *Schatten class estimates for the Riesz map of massless Dirac operators.* Integral Equations Operator Theory **90** (2018), no. 2, Art. 19, 36 pp.

[18] Ber A., Chilin V., Sukochev F., Zanin D. *Fuglede-Putnam theorem for locally measurable operators.* Proc. Amer. Math. Soc. **146** (2018), no. 4, 1681–1692.

[19] Dykema K., Sukochev F., Zanin D. *An upper triangular decomposition theorem for some unbounded operators affiliated to II_1-factors.* Israel J. Math. **222** (2017), no. 2, 645–709.

[20] Sukochev F., Usachev A., Zanin D. *Singular traces and residues of the $\zeta-$function.* Indiana Univ. Math. J. **66** (2017), no. 4, 1107–1144.

[21] Dykema K., Sukochev F., Zanin D. *Determinants associated to traces on operator bimodules.* J. Oper. Th. **78**:1 (2017), 119–134.

[22] Sukochev F., Zanin D. *Fubini theorem in noncommutative geometry.* J. Funct. Anal. **272** (2017), no. 3, 1230–1264.

[23] Sukochev F., Tulenov K., Zanin D. *Nehari type theorem for non-commutative Hardy spaces.* J. Geom. Anal. **27** (2017), no. 3, 1789–1802.

[24] Jiao Y., Sukochev F., Zanin D., Zhou D. *Noncommutative martingale inequalities in symmetric operator spaces.* J. Funct. Anal. **272** (2017), no. 3, 976–1016.

[25] Carey A., Gesztesy F., Grosse H., Levitina G., Potapov D., Sukochev F., Zanin D. *Trace formulas for a class of non-Fredholm operators: a review.* Reviews in Mathematical Physics, Vol. 28, No. 10 (2016) 1630002.

[26] Dykema K., Noles J., Sukochev F., Zanin D. *On reduction theory and Brown measure for closed unbounded operators.* J. Funct. Anal. **271** (2016), no. 12, 3403–3422.

[27] Dykema K., Sukochev F., Zanin D. *Algebras of Log-Integrable Functions and Operators.* Complex Anal. Oper. Theory. **10** (2016), no. 8, 1775–1787.

[28] Jiao Y., Sukochev F., Xie G., Zanin D. Φ-moment inequalities for independent and freely independent random variables. J. Funct. Anal. 270 (2016), no. 12, 4558–4596.

[29] Carey A., Gesztesy F., Levitina G., Potapov D., Sukochev F., Zanin D. *On index theory for non-Fredholm operators: a* $(1+1)-$*dimensional example.* Math.Nachr. **289** (2016), no. 5-6, 575–609.

[30] Aubrunn G., Sukochev F., Zanin D. *Catalysis in the trace class and weak trace class ideals.* Proc. Amer. Math. Soc. **144** (2016), no. 6, 2461–2471.

[31] Dykema K., Sukochev F., Zanin D. *Holomorphic functional calculus on upper triangular forms in finite von Neumann algebras.* Illinois J. Math. **59** (2015), no. 3, 819–824.

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Fully refereed conference proceedings

Additional research outputs (including non-traditional research outputs)

D10. Research Opportunity and Performance Evidence (ROPE) - Currently held ARC projects

(This information is auto-populated from the Future Fellowship candidate's RMS profile and will include any active project which has not yet had a Final Report approved and the project file closed by the ARC. You will not be able to submit an application to the ARC that involves a researcher who has an overdue Final Report on an ARC-funded project. If there are any concerns with the information recorded here, contact Your organisation's Research Office.)

Identifier	Investigators	Admin Organisation	Project Title	Funding	End Date	Final Report Due Date	Final Report Status
DE150100030	Dr Dmitriy Zanin	The University of New South Wales	A new concept of independence in noncommutative probability theory.	$300,000	29/06/2018	29/06/2019	Submitted to ARC

D11. Research Opportunity and Performance Evidence (ROPE) - Detail the number of students the Future Fellowship candidate has supervised over the last five years

(Provide the details of students the Future Fellowship candidate has supervised over the last five years. (No more than 350 characters, approximately 50 words))

PhD students: Edward Mcdonald (completion in 3 months), Dominic Vella (completion in 6 months), Jinghao Huang (completion in 1 year), Nathan Jackson (just started). Master student: Georges Nader (just started). Edward is an outstanding mathematician and is actively involved into this project. Jinghao and Nathan can also be involved.

D12. Eligibility - Relevant Qualification

(Select the qualification which is most relevant to the application.)

Degree/Award Title	Awarding Organisation	Conferral Date
PhD	Flinders University	02/06/2011

D13. Eligibility - Does the Future Fellowship candidate hold a professional equivalent to a PhD as certified by You?

(Where the Future Fellowship candidate does not hold a PhD, evidence must be provided to You, and You must certify that the Future Fellowship candidate holds a professional equivalent to a PhD.)

No

D14. Eligibility - Has the Future Fellowship candidate been granted an extension by You, to the eligibility period due to a significant career interruption as outlined in subsection B3.13 of the grant guidelines?

(If the Future Fellowship candidate's qualification relevant to this application (listed in question D12) was awarded prior to 1 March 2004 and they have had a significant career interruption (as listed in subsection B3.13 of the grant guidelines), the Future Fellowship candidate will need to seek an extension to the eligibility period through their Deputy Vice-Chancellor (Research).)

No

D15. Eligibility - Select the category of career interruption claimed (more than one may be selected)

(Choose all types of career interruptions which have been claimed and granted by the Future Fellowship candidate's Deputy Vice-Chancellor (Research).

Select a type of interruption and click 'Add'.)

D16. Eligibility - What is the total period of extension that the Future Fellowship candidate has claimed?

(Select the period of time which most closely equals the total period of extension claimed.)

D17. Eligibility - What is the Future Fellowship candidate's current academic level?

(Select the Future Fellowship candidate's current academic level from the menu below. If the Future Fellowship candidate is not employed at an Australian university, or is an international researcher, select "Other" and upload a letter from the DVCR or equivalent justifying the salary level requested in the Project Cost Part of the application.)

Select the Future Fellowship candidate's current academic level from the drop-down below.

Level B

D18. Eligibility - Academic level justification

(Upload a letter from the DVCR or equivalent justifying the salary level requested in the Project Cost Part of the application. This question is only mandatory if you have selected 'Other' because the Future Fellowship candidate is not employed at an Australian university, is an international researcher OR has chosen a Salary level which does not align with their academic level because they have experienced significant interruptions to their academic career, due to family responsibilities as primary care giver and/or due to working with a relevant industry. (No more than one A4 page))

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D19. Eligibility - What will the Future Fellowship candidate's time commitment (percentage of their time) be to the Administering Organisation?

(It is a requirement under the grant guidelines that the Future Fellow spend a minimum of 0.2 FTE annually located at Your organisation (the Administering Organisation).)

100

D20. Eligibility - What will the Future Fellowship candidate's time commitment (percentage of time) be to research activities related to this project?

(It is a requirement under the grant guidelines that the Future Fellow spend a minimum of 80 per cent of full time equivalent (0.8 FTE) (or pro-rata part-time equivalent) of their time on research and research capacity-building activities related to the proposed Future Fellowship.)

90

D21. Eligibility - Current Research Fellowship or Award funded by other Australian Government agencies

(Do not list Fellowships and Awards granted by the ARC. Only list Fellowships and Awards from other agencies.)
Does the Future Fellowship candidate hold a current Research Fellowship or Award funded by other Australian Government agencies?

No

D22. Project/Role relinquishment or application withdrawal

(Named participants on successful applications for Australian Laureate Fellowships, Future Fellowships, Centres of Excellence or Special Research Initiative projects must meet the project limit requirements. If required, this may be achieved by relinquishing existing project(s), or relinquishing role(s) on existing projects, or withdrawing application(s), where allowed, that would exceed the project limits. The Future Fellowship candidate must nominate and adequately justify the proposed relinquishment(s) if these applications were successful. We will determine the outcome of the Future Fellowship candidate's nominated relinquishment(s).
Failure to provide this information will jeopardise the eligibility of their applications. Provide project/application ID(s) and the justification for each separated by a comma.
(No more than 100 characters))

Part E - Project Cost (FT190100442)

E1. What is the proposed budget for the project?

(There are rules around what funds can be requested from the ARC. You must adhere to the scheme specific requirements listed in the grant guidelines. Refer to the Instructions to Applicants for detailed instructions on how to fill out the budget section.)

Remunerated Participants

Dr Dmitriy Zanin	Future Fellowship	Level 1 from year 1 annually for 4 years

Total requested budget: $678,680

Year 1

Description	ARC	Admin Org
	Cash	Cash
Total	170,720	10,000
Personnel	154,920	
Dr Dmitriy Zanin (Future Fellowship)	154,920	
Travel	15,800	10,000
CI Zanin airfares Sydney-New York	2,500	
PhD Student airfares Sydney-New York	2,500	
per diem CI Zanin in USA, $100 per day for 30 days	3,000	
per diem PhD student in USA, $80 per day for 30 days	2,400	
CI Zanin airfares Sydney-Ufa and local transportation	3,000	
CI Zanin accommodation in Ufa, $50 per day for 30 days	1,500	
per diem CI Zanin in Ufa, $30 per day for 30 days	900	
Admin Org contribution		10,000

Year 2

Description	ARC	Admin Org
	Cash	Cash
Total	170,820	10,000
Personnel	154,920	
Dr Dmitriy Zanin (Future Fellowship)	154,920	
Travel	15,900	10,000
accommodation collaborator Higson in Sydney, $150 per day for 30 days	4,500	
Per diem collaborator Higson in Sydney, $100 per day for 30 days	3,000	
CI Zanin airfares, Sydney-Seoul	1,500	
PhD student airfares Sydney-Seoul	1,500	
per diem CI Zanin in Seoul, $100 per day for 30 days	3,000	
per diem PhD student, $80 per day for 30 days	2,400	

Admin Org contribution		10,000

Year 3

Description	ARC	Admin Org
	Cash	Cash
Total	167,920	10,000
Personnel	154,920	
Dr Dmitriy Zanin (Future Fellowship)	154,920	
Travel	13,000	10,000
accommodation for collaborator Ponge in Sydney, $150 per day for 30 days	4,500	
per diem collaborator Ponge, $100 per day for 30 days	3,000	
CI Zanin airfares Sydney-Amsterdam and local transportation	2,500	
per diem CI Zanin, $100 per day for 30 days	3,000	
Admin Org contribution		10,000

Year 4

Description	ARC	Admin Org
	Cash	Cash
Total	169,220	10,000
Personnel	154,920	
Dr Dmitriy Zanin (Future Fellowship)	154,920	
Travel	14,300	10,000
per diem CI Zanin in USA, $100 per day for 30 days	3,000	
CI Zanin airfares Sydney-College Station	2,500	
partial airfares collaborator Caspers Amsterdam-Sydney	1,300	
accommodation collaborator Caspers in Sydney, $150 per day for 30 days	4,500	
per diem collaborator Caspers in Sydney, $100 per day for 30 days	3,000	
Admin Org contribution		10,000

Part F - Budget Justification (FT190100442)

F1. Justification of Future Fellowship non-salary funding requested from the ARC

(Fully justify, in terms of need and cost, each budget item requested from the ARC. Use the same headings as in the Description column in the Project Cost Part of this application. (Upload a PDF of no more than four A4 pages and within the required format))

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F1. Justification of Future Fellowship non-salary funding requested from the ARC.

Travel

Collaboration is a central part of the development, conduct and impact of Mathematics research. The ideas in Section C1 were developed as a result of conversations with overseas colleagues, the technical expertise of colleagues will accelerate the achievement of results, and it is through joint publication and networks that the impact of the Fellowship is maximised.

Below we describe and justify funds for travel and collaboration requested from the ARC. We indicate where our collaborators are contributing external funding. The UNSW School of Mathematics and Statistics will provide the Fellow with additional travel funds. Section F2 describes the contribution of UNSW funds to collaboration during the Fellowship.

Year 1: 15800 AUD in total. Costs are estimated.

(a) Professor Nigel Higson from Penn State University, USA, is a leading expert in non-commutative geometry and group representations. The Fellow discussed with him Aims 6 and 7 during a conference in Chengdu, China (May 2018) and during his short-term visit to UNSW (Dec 2018). Collaboration with Professor Higson is important for accelerating progress on Aims 6 and 7 of the project and will greatly increase the impact of achieved results. Professor Higson invited the Fellow to his home institution for the duration of up to 1 month. The Fellow and one of the PhD students plan to visit him in January 2020. It is an excellent opportunity for a PhD student in operator algebras and non-commutative analysis; the techniques in group representations he will learn from Professor Higson will allow him to contribute to Aim 6 of the project. Funding requested: return economy airfare Sydney-New York 2x2500 AUD; per diem 30 days x 100 AUD/day + 30 days x 80 AUD/day; 10400 AUD in total. Accommodation for the Fellow and for the PhD student and local transportation will be provided by Professor Higson.

(b) Professor Kordyukov from Ufa University, Russia, is known for his work in the geometry of the manifolds and foliation theory. The Fellow has continuous e-mail contact with him regarding Aims 1, 2 and 3 and his involvement is required for the success of these aims. For example, building a spectral triple from compact manifold is simple, but in the non-compact case this is a substantial result communicated to us by Kordyukov. Professor Kordyukov invited the Fellow to Ufa for the duration up to 1 month. The Fellow plans to visit him in May or June 2020. Funding requested: return economy airfare Sydney-Ufa and local transportation 3000 AUD; accommodation 30 days x 50 AUD/day; per diem 30 days x 30 AUD/day; 5400 AUD in total.

Year 2: 15900 AUD in total. Details are provided below. Costs are estimated.

(a) Reciprocal visit of Professor Higson to UNSW will ensure the success of Aims 6 and 7. The Fellow invited him to visit UNSW in 2021 (precise dates to be determined at a later stage) for the duration up to 1 month. Funding requested: accommodation 30 days x 150 AUD/day; per diem 30 days x 100 AUD/day; 7500 AUD in total. Airfare will be paid by Professor Higson.

(b) Professor Ponge from Seoul National University, South Korea, was a PhD student of Alain Connes and is the top specialist in non-commutative conformal geometry. The computations on the Minakshisundaram-Plejel theorem mentioned in Section C1 will be investigated in collaboration with Professor Ponge. His contribution and expertise are central for Aims 4 and 5 of the project. Professor Ponge invited the Fellow to visit him at Seoul for the duration up to 1 month. The Fellow and one of the PhD students will visit him in 2021 (precise dates to be determined at a later stage). The PhD student will learn Professor Ponge's developments on the relation between heat expansion and conformal geometry and work intesively on calculations for the noncommutative torus and noncommutative plane examples. Funding requested: return economy airfare Sydney-Seoul 2x1500 AUD; per diem 30 days x 100 AUD/day + 30 days x 80 AUD/day; 8400 AUD in total. Accommodation for the Fellow and for the PhD student will be paid by Professor Ponge.

Year 3: 13000 AUD in total. Details are provided below. Costs are estimated.

(a) Reciprocal visit of Professor Ponge to UNSW should accelerate the finalisation of Aims 4 and 5. The Fellow invited him to visit UNSW in 2022 (precise dates to be determined at a later stage) for the duration up to 1 month. Funding requested: accommodation 30 days x 150 AUD/day; per diem 30 days x 100 AUD/day; 7500 AUD in total. Airfare will be paid by Professor Ponge.

(b) Dr Caspers from Delft University of Technology, Netherlands, is a top specialist in both Double Operator Integrals and in quantum groups and also a long term collaborator of the Fellow. His expertise in Double Operator Integrals will be beneficial for Aim 5 as the transference principle invented by Dr Caspers may yield new integral formulae similar to the ones in the Approach to Aim 5. On the other hand, Dr Caspers knowledge of quantum groups is extremely relevant for Aim 6 (his unpublished note shows some progress on spectral triples for certain quantum groups). Dr Caspers invited the Fellow to visit him in Delft for the duration up to 1 month. The Fellow plans to visit him in 2022 (precise dates to be determined at a later stage). Funding requested: return economy airfare Sydney-Amsterdam and local transportation 2500 AUD; per diem 30 days x

100 AUD/day; 5500 AUD in total. Accommodation for the Fellow will be paid by Dr Caspers.

Year 4: 14300 AUD in total. Details are provided below. Costs are estimated.

(a) Professor Dykema from Texas A&M University, USA, is one of the creators of free Probability Theory and a world leading expert in Operator Theory. He is a long-term collaborator of the Fellow and conversations with him were important for the foundation of this project. Collaboration with Professor Dykema is invaluable in the later stages of the project to maximise impact and look for wider applications. He invited the Fellow to visit his home institution for the duration up to 1 month. The Fellow will visit him in 2023 (precise dates to be determined at a later stage). Funding requested: return economy Sydney-College Station 2500 AUD; per diem 30 days x 100 AUD/day; 5500 AUD in total. Accommodation for the Fellow will be paid by Professor Dykema.

(b) Reciprocal visit of Dr Caspers to UNSW serves the purpose of developing future applications (mostly, to representations of quantum groups) and strategy. Dr Caspers will visit the Fellow in 2023 (precise dates to be determined at a later stage). Funding requested: 50% of the return economy airfare Amsterdam-Sydney 1300 AUD; accommodation 30 days x 150 AUD/day; per diem 30 days x 100 AUD/day; 8800 AUD in total. Airfare will be partially paid by Dr Caspers.

F2. Details of Administering Organisation contributions

(Provide an explanation of how Your organisation's contributions will support the proposed project. Use the same headings as in the Description column in the Project Cost Part of this application. (Upload a PDF of no more than one A4 page and within the required format))

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F2. Details of Administering Organisation contributions.

Travel

UNSW School of Mathematics and Statistics will provide the Fellow with travel funds (10000 AUD annually). Using these funds, the Fellow and PhD students will attend international and domestic conferences which reduces funds requested from the ARC and increases collaboration and impact. The travel plan below does not interfere with the research visits specified in Section F1. All costs are estimated.

Year 1: International. 6000 AUD in total.

(a) Conference "Interactions of non-commutative analysis and quantum information theory" at Harbin Institute of Technology, China, June 2020. The Fellow will deliver preliminary results on Aims 1 and 2. Funding utilised: return economy airfare Sydney-Harbin 1500 AUD; accommodation 5 days x 80 AUD/day; per diem 5 days x 80 AUD/day; 2300 AUD in total.

(b) International Workshop on Operator Theory and Applications at Lancaster University, UK, July 2020. The Fellow will deliver preliminary results on Aim 3. Funding utilised: return economy airfare Sydney-London and local transportation 2500 AUD; accommodation 5 days x 120 AUD/day; per diem 5 days x 120 AUD/day; 3700 AUD in total.

Year 2: International. 10000 AUD in total.

(a) Great Plains Operator Theory Symposium (USA, location to be determined). Attendees: Fellow and one of the PhD students. The Fellow will deliver results on Aims 1, 2 and 3. Funding utilised: return economy airfare 2x2500 AUD; accommodation 2 x 5 days x 80 AUD/day; per diem 5 days x 100 AUD/day + 5 days x 80 AUD/day; 6700 AUD in total.

(b) Conference "Non-commutative harmonic analysis and non-commutative probability theory" at Bedlewo, Poland. Attendees: Fellow. The Fellow will deliver preliminary results on Aim 6. Funding utilised: return economy airfare Sydney-Warsaw and local transportation 2500 AUD; conference fee 300 AUD; per diem 7 days x 70 AUD/day; 3290 AUD in total.

Year 3: International. 10000 AUD in total.

(a) Canadian Operator Symposium (Canada, location to be determined). Attendees: Fellow and one of the PhD students. The Fellow will deliver results on Aims 4 and 5. Funding utilised: return economy airfare and local transportation 2x2500 AUD; accommodation 2 x 5 days x 100 AUD/day; per diem 5 days x 100 AUD/day + 5 days x 80 AUD/day; 6900 AUD in total.

(b) Conference "Quantum groups in non-commutative geometry" in Oberwolfach. Attendees: Fellow. The Fellow will deliver preliminary results on Aim 7. Funding utilised: return economy airfare Sydney-Frankfurt 2500 AUD; per diem 6 days x 100 AUD/day; 3100 AUD in total.

Year 4: International. 10000 AUD in total.

(a) Conference "Operator Theory 33" (Romania, location to be determined). Attendees: Fellow and both PhD students. The Fellow will deliver the final results on Aims 6 and 7. Funding utilised: return economy airfare Sydney-Bucharest and local transportation 3x2500 AUD; accommodation 3 x 5 days x 100 AUD/day; per diem 5 days x 80 AUD/day + 2 x 5 days x 60 AUD/day; 10000 AUD in total.

(b) Conference travel at this stage of the project should target international conferences that would maximise impact of the results. This may result from invitations or conferences not yet scheduled four years in advance. Administering Organisation funds will be utilised for these opportunities and may adjust conference plans as above.

Years 1-4: Domestic. 4000 AUD in total.

The outcomes of this project will be of interest in Australia, and broaden existing research strengths in new directions with the development of asymptotics for noncommutative Laplacians as mentioned under "Benefit and Collaboration" in Section C1. UNSW Funding to an estimated amount of 4000 AUD will be utilised as appropriate during the Fellowship to allow the Fellow and PhD students to communicate the progress and results of the Fellowship at Australian meetings and interact with other research groups within Australia (e.g. ANU, Adelaide, Sydney, Wollongong).

F3. Does this application request funding for research activities, infrastructure or a project previously funded, or currently being funded, with Australian Government funding (from the ARC or elsewhere)?

(This is a 'Yes' or 'No' question. If 'Yes' provide the project ID and outline the similarities of the research and explain how it will be managed.)

No

Funded Project ID

Outline the similarities and explain how these similarities will be managed if this application is funded. (No more than 2000 characters, approximately 285 words)

F4. Does this application request funding for research activities or infrastructure which are the subject of an application already submitted to the ARC?

(This is a 'Yes' or 'No' question. If 'Yes' provide the application ID and outline the similarities of the research.)

No

Provide the application ID

Outline the similarities and explain why more than one application has been submitted for the same research. (No more than 2000 characters, approximately 285 words)

Part G - Research Support and Statements on Progress (FT190100442)

G1. Research support

(For the Future Fellowship candidate on this application, provide details of:
i) current submitted ARC applications (i.e. for which the outcome has not yet been announced);
ii) any newly funded ARC projects which are not yet showing in the Future Fellowship candidate's question (Currently held ARC projects); and
iii) research funding from non-ARC sources (in Australia and overseas). For research funding from non-ARC sources, list all projects/proposals/awards/fellowships awarded or requests submitted involving the Future Fellowship candidate for funding for the years 2018 to 2023 inclusive.)

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G1: Research Support

Current submitted ARC Proposals

Description (All named investigators on any proposal or grant/ project/ fellowship in which a participant is involved, project title, scheme and round)	**Same Research Area**	**Support status** (Requested, Current, Past)	**Project ID**	**2018** $'000	**2019** $'000	**2020** $'000	**2021** $'000	**2022** $'000	**2023** $'000
Dr Dmitriy Zanin, Non-commutative Laplacians, quantum symmetries and the Chern character, ARC, Future Fellowship 2019.	yes	R	FT190100442	0	0	171	171	168	170

1

G2. Statements on Progress for ARC-funded projects

(A progress statement must be provided for any currently funded ARC project that involves the Future Fellowship candidate named on this application. This requirement applies to all ARC funding with the exception of ARC Centres of Excellence, Supporting Responses to Commonwealth Science Council Priorities, Learned Academies Special Projects and Special Research Initiatives schemes. Refer to the Instructions to Applicants for further information. (Upload a PDF of no more than one A4 page for each project))

Project ID

DE150100030

First Named Investigator

Dmitriy Zanin

Scheme

Discovery Early Career Researcher Award

Statement

Uploaded PDF file follows on next page.

DE150100030, ”A new concept of independence in noncommutative probability theory”, CI D. Zanin

Final report was submitted to the ARC on 18.12.18.

Certification

Certification by the Deputy/Pro Vice-Chancellor (Research) or their delegate or equivalent in the Administering Organisation

I certify that—

- I have read, understood and complied with the *Grant Guidelines for the Discovery Program (2018)*, (the grant guidelines) and, to the best of my knowledge all details provided in this application form and in any supporting documentation are true and complete in accordance with the grant guidelines.
- Proper enquiries have been made and I am satisfied that the Future Fellowship candidate meets the requirements specified in the grant guidelines, including having been awarded a PhD between 1 March 2004 and 1 March 2014. Where the Future Fellowship candidate has allowable career interruptions, sufficient evidence has been provided to the Administering Organisation and based on this evidence, I certify that the candidate's PhD award date together with allowable career interruptions (as listed under subsection B2.6 of the grant guidelines) would be commensurate with being awarded a PhD on or between 1 March 2004 and 1 March 2014.
- Where the Future Fellowship candidate holds a research higher degree, which is not a PhD, sufficient evidence has been provided to the Administering Organisation and based on this evidence, I certify that the candidate's qualification meets the level 10 criteria of the *Australian Qualifications Framework Second Edition January 2013*, or is a professional equivalent to a PhD.
- Upon request from the ARC, this organisation will provide evidence to support a career interruption justification in relation to the PhD Award date.
- The ARC reserves the right to audit any evidence on which an application is based.
- I will notify the ARC if there are any changes to the Future Fellowship candidate after the submission of this application.
- The Future Fellowship candidate is responsible for the authorship and intellectual content of this application, and has appropriately cited sources and acknowledged significant contributions to this application.
- To the best of my knowledge, all Conflicts of Interest relating to parties involved in or associated with this application have been disclosed to the Administering Organisation, and, if the application is successful, I agree to manage all Conflicts of Interest relating to this application in accordance with the *Australian Code for the Responsible Conduct of Research* (2018), the *ARC Conflict of Interest and Confidentiality Policy* and any relevant successor documents.
- I have obtained the agreement, attested to by written evidence, of all the relevant persons and organisations necessary to allow the project to proceed. This written evidence has been retained and will be provided to the ARC if requested.
- This application complies with the eligible research requirements set out in the *ARC Medical Research Policy*, located on the ARC website.
- This application does not request funding for the same research activities, infrastructure or project previously funded or currently being funded through any other Commonwealth funding.
- If this application is successful, I am prepared to have the project carried out as set out in this application and agree to abide by the terms and conditions of the grant guidelines and the *Grant Agreement for the Discovery Program (2018)*.
- The project can be accommodated within the general facilities of this organisation and if applicable, within the facilities of other relevant organisations specified in this application, and sufficient working and office space is available for any proposed additional staff.
- This organisation supports this application and, if successful, will provide the Future Fellowship candidate with an appropriate appointment for the duration of the Fellowship.
- All funds for this project will only be spent for the purpose for which they are provided.
- The project will not be permitted to commence until appropriate ethical clearance(s) has/have been obtained and all statutory requirements have been met.
- I consent, on behalf of all the parties, to this application being referred to third parties, including to overseas

parties, who will remain anonymous, for assessment purposes.

- I consent, on behalf of all the parties, to the ARC copying, modifying and otherwise dealing with information contained in this application.

- To the best of my knowledge, the Privacy Notice appearing at the top of this application has been drawn to the attention of the Future Fellowship candidate whose personal details have been provided in the Personnel section.