AUSTRALIAN RESEARCH COUNCIL
ARC Future Fellowships
Proposal for Funding Commencing in 2019

FT

PROJECT ID: FT190100442

First Investigator: Dr Dmitriy Zanin

Admin Org: The University of New South Wales

Total number of sheets contained in this Proposal: 46

Part A - Administrative Summary (FT190100442)

A1. Application Title

(Provide a short title. (No more than 75 characters, approximately ten words))

Non-commutative Laplacians, quantum symmetries, and the Chern character

A2. Person Participant Summary

(Add the Future Fellowship candidate participating in this application.)

Number	Name	Participant Type	Current Organisation(s)
1	Dr Dmitriy Zanin	Future Fellowship	The University of New South Wales

A3. Organisation Participant Summary

(Add the organisations participating in this application. Refer to the Instructions to Applicants for further information.)

Number	Name	Participant Type
1	The University of New South Wales	Administering Organisation

A4. Application Summary

(Provide an Application Summary (which is used by the Minister to consider the application), focusing on the aims, significance, expected outcomes and benefits of this project. Write the Application Summary simply, clearly and in plain English. If the application is successful, the Application Summary is used to give the general community an understanding of the research. Avoid the use of acronyms, quotation marks and upper case characters. Refer to the Instructions to Applicants for further information. (No more than 750 characters, approximately 100 words))

Non-commutative geometry is an exciting new way to think of quantization of classical systems from the geometric point of view. This project aims to develop a new approach to computing geometric and topological invariants of non-commutative manifolds. The project expects to generate new mathematical methods, results and examples in the field of non-commutative geometry and to promote Australian research in non-commutative analysis at the international level. The project includes building an international collaboration with research groups in USA, China and France. The expected benefits are leading research which will impact research in mathematical physics and other sciences and strengthen the reputation of Australian science.

A5. List the objectives of the Future Fellowship candidate's proposed project.

*(List each objective separately by clicking 'add answer' to add the next objective. This information will be used for future reporting purposes if this application is funded. (No more than 500 characters, approximately 70 words per objective)
)*

Objective

Prove a non-commutative version of a partition function expansion of a Laplacian associated to a noncommutative metric. This allows a definition and explicit computation of noncommutative analogues of the most important geometric invariants like curvature and topological invariants like the Euler characteristic.

Objective

Prove a version of Connes Character Formula for locally compact non-commutative manifolds. This allows a local expression for the Chern Character linking the Riemannian and conformal geometry and sets a lower obstruction for the order of the partition function expansion.

Objective

Build spectral triples associated to the actions of quantum unitary or quantum affine groups. In analogy with Lie groups this will present the quantum group as a non-commutative manifold equipped with the quantum group as its isometries. This spectral triple should avoid dimension drop pathology (that is, spectral dimension should equal the homological one).

A6. Benefit and Impact Statement

(Outline the intended benefit and impact of the project. Write the Benefit and Impact Statement simply, clearly and in plain English. Refer to the Instructions to Applicants for further information. (No more than 750 characters, approximately 100 words))

The outcomes are expected to develop research at the forefront of non-commutative mathematics and impact research in mathematical physics and other sciences which will strengthen the reputation of Australian science. This proposal aims at establishing UNSW at the centre of a highly active international research field and bringing prominent international researchers in mathematics to Australia to teach and research. The project will produce highly trained personnel leading in an international network in a fundamental science.

Part B - Classifications and Other Statistical Information (FT190100442)

B1. Does this application fall within one of the Science and Research Priorities?

No

Science and Research Priority	Practical Research Challenge

B2. Field of Research (FOR)

(Select up to three classification codes that relate to the Future Fellowship candidate's application. Note that the percentages must total 100.)

Code	Percentage
010106 - Lie Groups, Harmonic and Fourier Analysis	40
010108 - Operator Algebras and Functional Analysis	60

B3. Socio-Economic Objective (SEO-08)

(Select up to three classification codes that relate to the Future Fellowship candidate's application. Note that the percentages must total 100.)

Code	Percentage
970101 - Expanding Knowledge in the Mathematical Sciences	90
970102 - Expanding Knowledge in the Physical Sciences	10

B4. Interdisciplinary Research

(This is a 'Yes' or 'No' question. If You select 'Yes' two additional questions will be enabled:
1. Specify the ways in which the research is interdisciplinary by selecting one or more of the options below.
2. Indicate the nature of the interdisciplinary research involved. (No more than 375 characters, approximately 50 words))

Does this application involve interdisciplinary research?

No

Specify the ways in which the research is interdisciplinary by selecting one or more of the options below.

Indicate the nature of the interdisciplinary research involved. (No more than 375 characters, approximately 50 words)

B5. Does the proposed research involve international collaboration?

(This is a 'Yes' or 'No' question. If You select 'Yes' two additional questions will be enabled:
1. Specify the nature of the proposed international collaboration by selecting one or more of the options below.
2. Specify the countries which are involved in the international collaboration.)

Yes

B6. What is the nature of the proposed international collaboration activities?

(Select all options from the drop down list which apply to this application by clicking on the 'Add' button each time an option is selected.)

Correspondence: eg email; telephone; or video-conference
Attendance at and/or hosting of workshop or conference
Hosting international Partner Investigator: short-term (less than 4 weeks)
Hosting international Partner Investigator: long-term (more than 4 weeks)
Travel to international collaborator: short-term (less than 4 weeks)
Travel to international collaborator: long-term (more than 4 weeks)

B7. If the proposed research involves international collaboration, specify the country/ies involved.

(Commence typing in the search box and select from the drop-down list the name of the country/ies of collaborators who will be involved in the proposed project. Note that Australia is not to be listed and is not available to be selected from the drop-down list.)

United States of America
China (excludes SARs and Taiwan)
France
Russian Federation

B8. How many PhD, Masters and Honours places will be filled as a result of this project?

(The ARC is capturing the number of Research Students that would be involved in this application if it is funded. Enter the number of student places (full-time equivalent - FTE) that will be filled as a result of this project.)
Number of Research Student Places (FTE) - PhD

2

Number of Research Student Places (FTE) - Masters

1

Number of Research Student Places (FTE) - Honours

2

Part C - Project Description (FT190100442)

C1. Project Description

(Upload a Project Description as detailed in the Instructions to Applicants and in the required format. Ensure that the Project Description responds to the Assessment Criteria listed in the grant guidelines. (No more than ten A4 pages))

Uploaded PDF file follows on next page.

Project Title Asymptotics of non-commutative Laplacians, quantum symmetries and the Chern character.

Aims and Background

Broad Aim This proposal covers a novel viewpoint on non-commutative (analogies of) Riemannian manifolds and their study through recently developed methods in non-commutative analysis. It aims to bring together an international team of experts (Professor Ponge, Professor Higson and the CI) with new approaches to fundamental and long-standing problems in geometry (see Aims 1-5 below) and harmonic analysis (see Aims 6 and 7 below).

Broadly speaking there exist two approaches to the study of Riemannian manifolds: the traditional one which forms the basis of classical differential geometry (which treats a manifold X as a locally Euclidean space) and a more recent perspective proposed by A. Connes (which treats a manifold as a spectral triple). A spectral triple consists of $*-$algebra $\mathcal{A}$ represented on a Hilbert space H and an unbounded self-adjoint operator D on H and satisfying a number of properties spelled out below in the General Background section below. Connes' beautiful idea [8] allowing us to compare both theories may be described as follows: let $\mathcal{A} = C^\infty(X)$ be the algebra of all smooth functions on X, let $H = L_2\Omega(X)$ be the Hilbert space of all square-integrable forms on X and let D be a Hodge-Dirac operator on H (see e.g. [2]). A fundamental Connes Reconstruction Theorem [10] tells us that so-defined spectral triple captures all the geometric information about a manifold X available from the classical approach. This result confirms the efficacy of this new approach based on spectral triples.

Classical Riemannian manifolds are often equipped with a Lie group action. For instance, if X is the real sphere $\mathbb{S}^{n-1}$, then an orthogonal group $\mathrm{SO}(n)$ acts on it by rotations. Moreover, isometries (i.e. rotations possibly composed with reflections) are exactly those automorphisms of the sphere which commute with Laplace-Beltrami operator Δ_g (see the definition in the General Background section below). If, in general, a Lie group G acts on a manifold X, then one wants to take this action into account by constructing a spectral triple compatible with this action. A natural way to achieve this objective is to request that the Dirac-type operator featuring in the definition of the spectral triple commutes with the action of the group (see e.g. a construction of a Dirac operator on the sphere given in [22]).

In the noncommutative realm, when we move from classical to noncommutative manifolds and replace the commutative algebra $\mathcal{A} = C^\infty(X)$ with a noncommutative C^* or pre-C^*-algebra $\mathcal{A}$ in practice there is often a group G (or even a quantum group) of symmetries of $\mathcal{A}$. This noncommutative analogue of the property "the operator D commutes with a group action" is called an *equivariance property*. If, for example, we wish to construct a spectral triple for an algebra $\mathcal{A}$ posessing an action of a q-deformed Lie group, then we should seek a Dirac-type operator D which commutes with that action.

Our proposal is centred around the new method of studying of non-commutative manifolds which are equipped with a natural (quantum) group action. The CI will investigate actions of groups (more broadly, quantum groups) on noncommutative manifolds using novel methods recently developed in non-commutative analysis with his immediate participation. The program which we outline below will benefit both Non-commutative Geometry and Non-commutative Analysis in general. In particular, our proposal envisages a strong contribution to a grand program started by Connes and Tretkoff [17] concerning curvature and higher order smooth invariants and suggests a completely different perspective on Connes' Reconstruction Theorem. Simultaneously with our study of quantum groups actions we shall introduce and study equivariant Dirac operators on a variety of noncommutative manifolds which capture both geometric information about the manifold and the quantum group, thus adding new features to the latter theory. More importantly, endowing Connes Reconstruction Theorem with these new features will make it amenable for a vast generalization of this theorem suitable for non-commutative manifolds with structure that is equivariant under a quantum group action.

General Background Operator algebras provide a natural edifice to many areas of classical and modern mathematics, and also play a paramount role in the present proposal. A particular role is played by the class of C^*-algebras (uniformly closed $*-$subalgebras in the $*-$algebra $B(H)$ of all bounded operators on a Hilbert space H) and that of von Neumann algebras (weakly closed unital $*-$subalgebras of $B(H)$). The topological requirements imposed on these algebras make them a suitable foundation for developing noncommutative analysis.

The starting point of noncommutative geometry can be traced back to the Gelfand-Naimark theorem which delivers a duality between the category of locally compact topological spaces and that of commutative C^*-algebras. Briefly speaking, it allows us to express numerous topological properties of topological spaces in the language of C^*-algebras. This theorem can be viewed as an anti-equivalence between a category of locally compact Hausdorff spaces and the category of commutative C^*-algebras. The corresponding anti-equivalence is given by the map $X \to C_0(X)$, where $C_0(X)$ is the algebra of all continuous complex valued functions which vanish at infinity. This may be interpreted as the statement that all of the information about a space is actually encoded in the algebra of continuous complex valued functions on that space. Thus one may think of non-commutative C^*-algebras as noncommutative topological spaces and attempts to apply topological methods to understand these algebras are well justified.

Another fundamental result due to von Neumann and Segal provides a duality between a certain category of measure spaces and that of commutative von Neumann algebras. This fact provided impetus to view classical measure theory as a part of von Neumann algebra theory led to the development of noncommutative integration theory.

Conventional wisdom suggests further analogies between the classical and quantum worlds. While topology and measure theory were supplied with natural quantum counterparts in the 1950s, a development of further geometric concepts were lacking until the 1980s. In fact, starting from von Neumann himself, a number of researchers attempted to find a quantum analogue of geometry (that is, to construct a functor from useful geometric categories to treatable quantum categories), however the success of their attempts was questionable.

Let us now briefly review the foundational features of classical differential geometry. We start with the notion of a Riemannian manifold which is its central concept. Usually, a manifold is defined as a topological space in which every point admits a neighborhood which is homeomorphic to Euclidean space. A manifold (denoted further by X) equipped with a smooth metric tensor (denoted further by g) is called Riemannian.

So far, the most successful attempt to quantise the notion of Riemannian manifold is due to Connes who introduced the notion of a spectral triple and promoted it as a non-commutative analogue of a Riemannian manifold and further as a convenient vehicle for a new (non-commutative) differential geometry encompassing closed Riemannian manifolds as a very special example.

Let H be a (separable) Hilbert space, and let $\mathcal{A}$ be a $*$-algebra. We say that $(\mathcal{A}, H, D)$ is a spectral triple if:

1. There is a representation $\pi : \mathcal{A} \to B(H)$ of the $*-$algebra $\mathcal{A}$ on the Hilbert space H.
2. D is a self-adjoint unbounded operator on H.
3. For every $a \in \mathcal{A}$, the commutator $[D, a]$ is has bounded extension.
4. For every $a \in \mathcal{A}$, the operator $\pi(a)(D+i)^{-1}$ is compact.
5. If, for every $a \in \mathcal{A}$, the singular values of the operator $\pi(a)(D+i)^{-1}$ decay like the sequence $k^{-\frac{1}{d}}, k \geq 1$ the spectral triple is called $d-$dimensional.

Associated to every Riemannian manifold X, there is a spectral triple $(\mathcal{A}, H, D)$ defined as follows: (i) $\mathcal{A}$ is the algebra $C_c^\infty(X)$ of all compactly supported smooth functions on X; (ii) H is the Hilbert space $L_2\Omega(X)$ of all square-integrable forms on X and π is the representation of $\mathcal{A}$ on H by pointwise multiplication; (iii) D is the Hodge-Dirac operator (see e.g. [2]) on $L_2\Omega(X)$. It is folklore (at least for compact manifolds) that the just constructed spectral triple $(\mathcal{A}, H, D)$ is $d-$dimensional, where $d = \dim(X)$.

As we already stated earlier, the celebrated Reconstruction Theorem due to Connes [10] stipulates that every spectral triple with commutative $\mathcal{A}$ (satisfying a few natural conditions) comes from a $d-$dimensional Riemannian manifold X. Thus it makes good sense to think of spectral triples as noncommutative manifolds. We wholeheartedly adopt this point of view in this proposal and shall develop it much further.

A Hodge-Dirac operator D is a convenient tool to identify and describe the isometries of the manifold X. If $\gamma : X \to X$ is an isometry and if $U : L_2\Omega(X) \to L_2\Omega(X)$ is the operator of composition with γ, then (i) U is unitary on $L_2\Omega(X)$; (ii) U preserves $\pi(\mathcal{A})$ (that is, $U^{-1}\pi(\mathcal{A})U = \pi(\mathcal{A})$); (iii) U commutes with D. Conversely, every diffeomorphism $\gamma : X \to X$ which commutes with D is an isometry. This simple but revealing result can be found in [23].

It is typical in applications to harmonic analysis [23] and mathematical physics [20] that manifolds are equipped with isometric action of Lie groups. Moreover, most examples of manifolds which are of serious interest and importance in applications are actually homogeneous spaces of some Lie group (e.g., the sphere $\mathbb{S}^{n-1}$ is a homogeneous space of the Lie group $SO(n)$). In such a situation, it does not make sense to consider the manifold alone, but rather only a manifold equipped with an isometric group action. We propose to investigate spectral triples which arise from an action of a Lie group (or some other group-like object, such as a quantum group) on a manifold (with a particular focus on generalisations to non-commutative manifolds).

The class of group-like objects which is most suitable for this task are quantum groups, whose theory has been under development since the early 1980's. The theory of quantum groups has its origin in attempts to find a good duality theorem, analogous to Pontryagin duality theorem, for general locally compact non-Abelian groups. In late 1980's, Woronowicz developed a general theory of compact quantum groups and developed a

Peter-Weyl theory for them. One of the main examples in Woronowicz's theory is the quantum group $SU_q(n)$ (a natural $q-$deformation of the compact Lie group $SU(n)$).

The rich interplay between Lie groups and differential geometry naturally raises the question of understanding the interaction between quantum groups and non-commutative geometry. Papers [5], [27] (among many others) attempt to put quantum groups within the framework of non-commutative Geometry. These attempts drew the attention of Alain Connes (see [6]) who developed further results by Chakraborty and Pal [5]. Our proposal is in fact partly motivated by these developments.

In this project, we aim to construct spectral triples for certain quantum groups (e.g. compact quantum groups like $SU_q(n)$ and $SO_q(n)$ as well as non-compact quantum groups like $SL_q(n)$) and their homogeneous spaces, to investigate the validity of major results in Non-commutative Geometry (such as Connes' Character Formula) for these examples and to compute numerically those topological invariants (such as their $K-$theory) which follow from these results.

We complete this section by explaining another important notion in this area, which is that a spectral triple $(\mathcal{A}, H, D)$ is equipped with a natural (semi)-group action, the heat semigroup. In the particular case where (X, g) is a d-dimensional Riemannian manifold, then $-D^2$ is the Hodge-Laplace operator (denoted further by Δ_g) and its component acting on 0 order forms being the Laplace-Beltrami operator (also denoted by Δ_g) [30]. The heat semi-group is now defined by the formula

$$t \to e^{t\Delta_g}, \quad t > 0.$$

If X is compact, then the resolvent of the Laplace-Beltrami operator Δ_g is compact. Hence, $e^{t\Delta_g}$ is compact for $t > 0$. In fact, it happens that $e^{t\Delta_g}$ belongs to the trace class for $t > 0$.

In his seminal work [33], Weyl proved that, for a compact manifold,

$$\lim_{t\downarrow 0}(4\pi t)^{\frac{d}{2}}\mathrm{Tr}(e^{t\Delta_g}) = \mathrm{Vol}(X), \quad t \downarrow 0. \tag{1}$$

Following Weyl's work, it became an established custom to measure various geometric (and often topological) quantities associated with a Riemannian manifold X in terms of its heat semi-group expansion $t \to e^{t\Delta_g}, \quad t > 0$. The mere existence of such expansion is a famous theorem of Minakshisundaram and Plejel (among all approaches to that theorem, a particularly detailed account is given in [30]; even though Theorem 3.24 there concerns only a special case $f = 1$, the proof of the formula stated below in the general case is very similar).

For every $f \in C^\infty(X)$, the Minakshisundaram-Plejel theorem asserts an existence of an asymptotic expansion

$$\mathrm{Tr}(M_f e^{t\Delta_g}) \approx (4\pi t)^{-\frac{d}{2}} \cdot \sum_{n\geq 0} a_n(f)t^n, \quad t \downarrow 0. \tag{2}$$

Here, d is the dimension of X and $M_f : L_2(X) \to L_2(X)$ is the operator of pointwise multiplication by f. Moreover, there exist functions $A_k \in C^\infty(X)$ such that

$$a_k(f) = \int_X A_k \cdot f d\mathrm{vol}_g, \tag{3}$$

where vol_g is the standard volume element on X given in local coordinates by the formula

$$d\mathrm{vol}_g = (\det(g))^{\frac{1}{2}}(x)dx.$$

As follows from (1), $A_0 = 1$. Further computations (see e.g. Proposition 3.29 in [30]) show that

$$A_1 = \frac{1}{6}R,$$

where R is the scalar curvature. In particular, $a_1(1)$ is the Einstein-Hilbert action (see e.g. [8]).

Note that a_0 extends to a normal state h on $L_\infty(X)$ by the obvious formula

$$h(f) = \int_X f d\mathrm{vol}_g, \quad f \in L_\infty(X).$$

Equation (3) can be re-written as

$$a_k(f) = h(A_k \cdot f), \quad f \in C^\infty(X).$$

One of the *primary targets* of this project is to find suitable extensions of the Minakshisundaram-Plejel theorem (and, consequently, of the Weyl theorem — see formula (1)) for a vast class of non-commutative manifolds (such as non-commutative tori with generic, non-flat, metric tensor). This grand program began in [17] (published only in 2011, but the main concepts and techniques were developed yet in the 1990's), where special 2−dimensional non-commutative manifolds (conformal deformations of a flat non-commutative torus) were considered. The authors of [17] proved that Euler characteristic of such manifold is 0 by means of Gauss-Bonnet theorem (recall that the classical Gauss-Bonnet theorem asserts that Euler characterstic of the 2−dimensional Riemannian manifold equals to the average of its scalar curvature). Subsequently, the scalar curvature (for the conformal deformation of the 2−dimensional non-commutative torus) was explicitly computed in [15] and [19] and, later, the term a_2 (the first place where the Riemann curvature tensor manifests itself beyond the scalar curvature) was further computed in [12] (intermediate computations include about a million terms!). Below, we briefly restate the whole programme as it can be surmised from [17].

For a given $d-$dimensional spectral triple $(\mathcal{A}, H, D)$, the steps needed to be done in the program are as follows (for brevity we assume $\mathcal{A}$ to be unital and omit π from the formulae below):

1. to prove a non-commutative version of Weyl theorem, that is, to find a normal state h on $\mathcal{A}''$ such that
$$\mathrm{Tr}(xe^{-tD^2}) \approx (4\pi t)^{-\frac{d}{2}} h(x), \quad t \downarrow 0, \tag{4}$$
for every $x \in \mathcal{A}''$;
2. to prove a non-commutative version of the Minakshisundaram-Plejel theorem or, equivalently, to verify that the following strong generalization of (2) holds
$$\mathrm{Tr}(xe^{-tD^2}) \approx (4\pi t)^{-\frac{d}{2}} \sum_{n\geq 0} a_n(x) t^n, \quad t \downarrow 0,$$
for every $x \in \mathcal{A}''$;
3. to verify the normality of coefficients as in (3) with respect to the volume state, or, more precisely, to show that
$$a_k(x) = h(xA_k), \quad x \in \mathcal{A}'',$$
for every $k \geq 0$ and for some $A_k \in \mathcal{A}''$;
4. to compute A_k explicitly.

When this mission is accomplished, one can *define* a scalar curvature of a non-commutative manifold by setting $R = 6A_1$.

We acknowledge that this program sets reasonable objectives but we claim that the tools based on on pseudo-differential calculus in various guises applied up-to-date to accomplish them have been inadequate. Our main technical innovation which we bring here is based on the novel Double Operator Integration techniques developed by the CI recently in close collaboration with Alain Connes and Fedor Sukochev. The particular integral representations (see Approach to Aims 1,2,3 below) arose in [16] when geometric measures on limit sets of Quasi-Fuchsian groups were recovered by means of singular traces. They form the core contribution of UNSW team (including the CI) to [16].

One of the fundamental tools in noncommutative geometry is the Chern character. The Connes Character Formula (also known as the Hochschild character theorem) provides an expression for the class of the Chern character in Hochschild cohomology, and it is an important tool in the computation of the Chern character. The formula has been applied to many areas of noncommutative geometry and its applications: such as the local index formula [14], the spectral characterisation of manifolds [10] and recent work in mathematical physics [11]. We point out to its applications in [10] as particularly relevant to the theme of this application.

In its original formulation, [7], the Character Formula is stated as follows: Let $(\mathcal{A}, H, D)$ be a p-dimensional compact spectral triple with (possibly trivial) grading Γ. By the definition of a spectral triple, for all $a \in \mathcal{A}$ the commutator $[D, a]$ has an extension to a bounded operator $\partial(a)$ on H. Assume for simplicity that $\ker(D) = \{0\}$ and set $F = \mathrm{sgn}(D)$. For all $a \in \mathcal{A}$ the commutator $[F, a]$ is a compact operator in the weak Schatten ideal $\mathcal{L}_{p,\infty}$ (see e.g. [8,26]).

Consider the following two linear maps on the algebraic tensor power $\mathcal{A}^{\otimes(p+1)}$, defined on an elementary tensor $c = a_0 \otimes a_1 \otimes \cdots \otimes a_p \in \mathcal{A}^{\otimes(p+1)}$ by setting
$$\mathrm{Ch}(c) := \frac{1}{2}\mathrm{Tr}(\Gamma F[F, a_0][F, a_1] \cdots [F, a_p]), \quad \Omega(c) := \Gamma a_0 \partial a_1 \partial a_2 \cdots \partial a_p.$$
Then the Connes Character Formula states that if c is a Hochschild cycle then
$$\mathrm{Tr}_\omega(\Omega(c)(1 + D^2)^{-p/2}) = \mathrm{Ch}(c)$$

for every Dixmier trace Tr_ω. In other words, the multilinear maps Ch and $c \mapsto \mathrm{Tr}_\omega(\Omega(c)(1+D^2)^{-p/2})$ define the same class in Hochschild cohomology. We mention, in passing, that the generality of the result just stated was achieved fairly recently by the CI and his co-authors, see [4].

There has been great interest in generalising the tools and results of noncommutative geometry to the "non-compact"(i.e., non-unital) setting. The definition of a spectral triple associated to a non-unital algebra originates with Connes [9], was furthered by the work of Rennie [29] and Gayral, Gracia-Bondía, Iochum, Schücker and Varilly [21]. Earlier, similar ideas appeared in the work of Baaj and Julg [1]. Additional contributions to this area were made by Carey, Gayral, Rennie and Sukochev [3]. The conventional definition of a non-compact spectral triple is to replace the condition that $(1+D^2)^{-1/2}$ be compact with the assumption that for all $a \in \mathcal{A}$ the operator $a(1+D^2)^{-1/2}$ is compact. This raises an important question: is the Connes Character Formula true for locally compact spectral triples? This question was suggested to the CI by Professor Connes himself during Shanghai conference celebrating the 70-th anniversary of A. Connes (held at Fudan University). During lengthy discussions covering a substantial range of topics of importance in non-commutative geometry, Professor Connes, in particular had emphasized to the CI that such an extension could be an excellent starting point for several new directions in non-commutative geometry. Here are two such directions. Firstly, the Connes Character Formula plays an important role in the reconstruction theorem for closed Riemannian manifolds, and it is natural to expect that its extension would play a similar role for locally compact Rimannian manifolds. Secondly, developing suitable new techniques needed for a self-contained theory of locally compact spectral triples should also open an avenue for treating the case of noncommutative manifolds with boundary or even incomplete (e.g. punctured) manifolds. This suggestion by Professor Connes has been taken seriously by the CI and preliminary work in this direction has already brought substantial fruits. Firstly, jointly with Professor Sukochev, the CI has achieved a substantial progress in treating locally compact spectral triples using novel analytic methods (this work has not yet been published). Secondly, in the ground-breaking manuscript co-authored by Professor Connes and the CI (and a number of collaborators from UNSW) the new approach to spectral triples involving *symmetric, non-self-adjoint* operators has been proposed [13]. These are precisely the required tools allowing the possibility to develop a new theory for non-commutative Riemannian manifolds with boundary. This development indicates the importance and timeliness of the present proposal which has already achieved substantial progress and leads to a unified theory of locally compact non-commutative manifolds with boundary and incomplete manifolds. We emphasize that the progress achieved involves joint work with luminaries like Alain Connes, and local top experts in noncommutative analysis and geometry like Alan Carey, Adam Rennie and Fedor Sukochev.

Specific aims There are 7 specific aims.

Aim 1: Investigate when Chern character provides an asymptotic expansion for the heat semi-group. More precisely, when

$$\mathrm{Tr}(\Omega(c)e^{-s^2D^2}) = \mathrm{Ch}(c)s^{-p} + O(s^{1-p}), \quad s \downarrow 0, \tag{5}$$

for every Hochschild cycle $c \in \mathcal{A}^{\otimes(p+1)}$? This aim is in "high completion status" due to the yet unpublished joint work with Fedor Sukochev. We need, however, to subtantially revise and stregthen that work and take into account serious connections with Local Index Formula as in [3, 14] ignored in the first version.

Aim 2: Aim 1 above is closely related (albeit, not equivalent) to a question concerning analyticity of a suitable $\zeta-$function. The latter is an analytic function defined by the formula

$$z \to \mathrm{Tr}(\Omega(c)(1+D^2)^{-\frac{z}{2}}), \quad \Re(z) > p.$$

We aim to find an analytic extension to the half-plane $\Re(z) > p-1$ so that

$$\lim_{z\to p}(z-p)\mathrm{Tr}(\Omega(c)(1+D^2)^{-\frac{z}{2}}) = p\mathrm{Ch}(c).$$

Aim 3: The purpose of Connes' Character Formula is to compute the Hochschild class of the Chern character by a "local" formula, which is customarily stated in terms of singular traces on the ideal $\mathcal{L}_{1,\infty}$ ($\mathcal{L}_{1,\infty}$ is the principal ideal in $B(H)$ generated by an operator with singular values $(\frac{1}{k})_{k\geq 1}$). Here, trace $\varphi : \mathcal{L}_{1,\infty} \to \mathbb{C}$ is a unitarily invariant linear functional; it can be seen from CI's results in [26] that such a functional is automatically singular. Our third aim is to show that

$$\varphi(\Omega(c)(1+D^2)^{-\frac{p}{2}}) = \mathrm{Ch}(c). \tag{6}$$

for every (normalised) trace φ on $\mathcal{L}_{1,\infty}$ and for every Hochschild cycle $c \in \mathcal{A}^{\otimes(p+1)}$. Equivalently, we aim to show that

$$\sum_{k=0}^{n} \lambda(k, \Omega(c)(1+D^2)^{-p/2}) = \mathrm{Ch}(c)\log(n) + O(1), \quad n \to \infty.$$

Here, $\lambda(k,T)$ means the $k-$th eigenvalue (counted with algebraic multiplicity) of a compact operator T.

Aims 2 and 3 are intimately connected with Aim 1, however, the amount of analytical complications which arise when one navigates between formulas stated in all three aims is enormous and requires a very careful treatment and certainly warrants a separation of these aims.

Aim 4: Introduce a generic (i.e., not necessarily a conformal deformation of a flat one) Laplace-Beltrami operator Δ on the non-commutative torus and non-commutative Euclidean space. Prove a non-commutative version of Minakshisundaram-Plejel theorem (for these manifolds) as outlined above.

Aim 4, in turn, depends on Aims 1 and 2, or rather on the technical instruments which are to be developed to deal with those aims in full generality.

Aim 5: Compute explicitly the curvature for a generic Riemannian metric on the non-commutative torus and non-commutative Euclidean space.

This aim, if achieved (and we are in a very strong position to do so) amounts to completion of the program started by [5, 6, 27].

Aim 6: Construct a spectral triple (or possibly, a twisted spectral triple) on quantum groups (like $\mathrm{SU}_q(n)$) and on their homogeneous spaces (like Podleś sphere). Previous attempts [5] suffer from a "dimension drop" pathology (that is, where spectral dimension differs from cohomological dimension). We aim to have an equivariance property for the Dirac operator in a way that prevents "dimension drop" pathology.

Aim 7: Design a version of Connes Character Formula for the above spectral triples which recovers a non-trivial cocycle.

Generally speaking, the development of noncommutative geometry and its applications is hindered by the paucity of non-trivial examples demosntrating its richness and ability to *compute* quantities of geometric significance for (genuine) noncommutative manifolds. This direction of study also involves top class practitioners of Noncommutative Analysis and Quantum Probablity such as Marius Junge (University of Illinois) and Nigel Higson (Penn State University). The CI is in regular contact with both (they both are to visit UNSW in 2019) and this collaboration is to continue unabated.

Future Fellowship Candidate

The proposed research deals with subtle issues concerning axioms and basic tools of Non-commutative Geometry and notions paramount to the development of Non-commutative Analysis. The CI is well prepared to attack this important problem: during 2015–2018, the candidate held an ARC DECRA titled "New concept of independence in non-commutative probability theory" and achieved deep results which substantially improved understanding of the theory, its problems and approaches. An acknowledgement of this assertion can be seen from the CI's paper [24] published in Advances in Mathematics. In this publication a long standing problem about non-commutative analogue of the Poisson process (which had defied the efforts top experts in the area) has been resolved. In the process of work on these deep problems, CI has acquired his ideas concerning the notion of non-commutative manifolds. Furthermore, the candidate has also established strong research collaboration with a number of world leading experts in the area (including a genuine legend of modern mathemaitcs Professor Alain Connes). The collaboration with Professor Connes has yielded recently publications [16] containing important results in Non-commutative Geometry based on the theory of singular traces (this theory was largely developed in CI's works and CI co-authored the world first monograph [26] on the subject). It should be pointed out that Professor Connes is very enthusiastic about the suggested direction of research and this strong endorsement from the world leading expert and his on-going commitment to the joint research efforts is a strong acknowledgement that the candidate has the capacity to undertake the proposed research.

The CI is currently employed as UNSW Scientia Fellow (75% research load, 25 % teaching load). CI plans to spend 100% of his research time on this project. CI also teaches high-level courses for UNSW's best students and is passionate about introducing them to research level mathematics (the proposed project contains a number of sub-tasks suitable for a PhD and honours students).

Project quality and innovation

Significance The problems to be considered are fundamental and are at the forefront of modern Non-commutative Analysis and are of substantial importance for Noncommutative Geometry. The complete resolution of Aims 1–7 will be important for the development of both disciplines, especially for Non-commutative Geometry. This is a mature project with an experienced CI who has already made significant progress and with a star-line of current and potential collaborators (Connes, Junge, Higson, Ponge, Carey, Rennie, Sukochev). Other researchers actively work in the field, so this is becoming a very competitive area of investigation, however, we are in the unique vaulting position to achieve success in the next four years.

Approach to specific aims Here we describe methods in our possession which we are going to employ in order to resolve the problems stated above.

Approach to Aim 1: Our computations show, for a chain $c = a_0 \otimes \cdots \otimes a_p \in \mathcal{A}^{\otimes(p+1)}$, that

$$\mathrm{Ch}(c) := \frac{1}{2}\mathrm{Tr}(\Gamma F[F,a_0][F,a_1]\cdots[F,a_p]e^{-s^2D^2}) + O(s), \quad s \downarrow 0.$$

It seems plausible that Hochschild cochain on the right hand side is cohomologous to the one in the left hand side in (5). In fact, we have already made initial computations confirming this guess in the already mentioned unpublished joint work with Professor Sukochev.

Approach to Aim 2: The $\zeta-$function, whose analyticity should be proved in Aim 1 is of the shape $z \to \mathrm{Tr}(CB^z)$. We have $C = AC$ (hence $C = A^zC$) for a suitable A and, therefore,

$$\mathrm{Tr}(CB^z) = \mathrm{Tr}(CB^zA^z).$$

It is desirable to replace B^zA^z with $(A^{\frac{1}{2}}BA^{\frac{1}{2}})^z$. For this purpose, we use the integral representation

$$B^zA^z - (A^{\frac{1}{2}}BA^{\frac{1}{2}})^z = T_z(0) - \int_{\mathbb{R}} T_z(s)\widehat{g}_z(s)\,ds,$$

where $(s,z) \to \widehat{g}_z(s)$ is a sufficiently good scalar-valued function and where

$$T_z(s) = B^{z-1+is}[BA^{\frac{1}{2}}, A^{z-\frac{1}{2}+is}]Y^{-is} + B^{is}[BA^{\frac{1}{2}}, A^{\frac{1}{2}+is}]Y^{z-1-is}.$$

By using Double Operator Integration technique as developed in [28], we aim to prove analyticity of the right hand side and, hence, of the left hand side. The application of these techniques is our trump card in the joint work with Professor Connes [16], in which we have completed the work started by such giants as Connes and Sullivan, and which left dormant for more than 20 years, until our new techniques were brought to bear on that problem.

Approach to Aim 3: Having established analyticity of the function

$$z \to \mathrm{Tr}(C(A^{\frac{1}{2}}BA^{\frac{1}{2}})^z), \quad \Re(z) > p-1,$$

with a simple pole at $z = p$, we expect that methods from [31] will lead us to the formula

$$\varphi(CA^{\frac{1}{2}}BA^{\frac{1}{2}}) = \frac{1}{p}\lim_{z\to p}(z-p)\mathrm{Tr}(C(A^{\frac{1}{2}}BA^{\frac{1}{2}})^z)$$

for every (normalised) trace on $\mathcal{L}_{1,\infty}$. We shall draw on the the deep theory of singular traces and its connections with operator $\zeta-$functions which the CI (with various collaborators) has been developing since 2009 [26, 31]. The CI is in the unique position to apply numerous and well-developed techniques from that theory in order to achieve this aim.

Approach to Aim 4: The very definition of Laplace-Beltrami operator, involves a determinant of a matrix-valued function. On matrices, determinant is defined as (practically unique) homomorphism into a field of scalars. On matrix-valued functions, it is defined pointwise.

However, on a non-commutative manifold, a matrix-valued function is replaced with a matrix whose entries belong to a von Neumann algebra. Though a surrogate notion of determinant (due to Fuglede and Kadison) exists for such matrices, our computations show an inconsistency in the formula (4) above. That is, instead of the volume state, a different functional appears on the right hand side of (4).

We propose to sacrifice the "homomorphism" property of the determinant for being able to perform computations making the formula (4) consistent with the definition of Laplace-Beltrami operator. Precisely, we set

$$G^{-\frac{1}{2}} = (\det(g_{ij}))^{-\frac{1}{2}} \stackrel{def}{=} \pi^{-\frac{d}{2}} \int_{\mathbb{R}^d} e^{-\sum_{i,j} g_{ij}t_it_j}dt.$$

Now, define the volume state h by setting $h(x) = \tau(xG^{\frac{1}{2}})$ and consider an inner product $(x,y) \to h(xy^*)$ on the non-commutative torus. We may define a Laplace-Beltrami operator by the formula

$$-\Delta_g x = M_{G^{-\frac{1}{2}}} \sum_{i,j=1}^{d} D_i M_{G^{\frac{1}{4}}(g^{-1})_{ij}G^{\frac{1}{4}}} D_j.$$

Our computations show that the so-defined operator is self-adjoint (and positive) and that the formula (4) becomes consistent. It should be also stated that this particular problem has been discussed in depth with

Professor Raphaël Ponge, who is to take a part time position on an ARC funded grant (with CI Fedor Sukochev) in UNSW for the next few years. Professor Ponge (who is a former PhD student of Alain Connes and a top class expert in noncommutative differential geometry) is actively involved in the discussions with CI concerning this plan of action. Again, our preliminary discussion with Professor Ponge as well as with a PhD student at UNSW, Edward McDonald, make us confident that this aim is achievable. The presense of Professor Ponge at UNSW will be a bonus and will stregthen the cooperation between him, the CI and McDonald, which the CI has already initiated.

Approach to Aim 5: In the special case of a conformal deformation of a flat metric, the Laplace-Beltrami operator is (unitarily equivalent to) $M_h \Delta M_h$, where Δ is the flat Laplacian. According to the asymptotics in Aim 3, the function

$$z \to \mathrm{Tr}(M_x(-M_h\Delta M_h)^{-z})$$

admits an analytic extension with at most simple poles at $z = \frac{d}{2}, \frac{d}{2} - 1, \cdots$. The curvature term is provided (for $d > 2$) by the residue of this function at the point $\frac{d}{2} - 1$. Our function has the shape

$$z \to \mathrm{Tr}(C(A^{\frac{1}{2}}BA^{\frac{1}{2}})^z).$$

It is desirable to replace $(A^{\frac{1}{2}}BA^{\frac{1}{2}})^z$ with B^zA^z. Here, we propose to use an integral representation similar to (but more complicated than) the one specified in the Approach to Aim 2. Again, despite numerous technical setbacks which are encountered by anyone who tries to perform the step outlined above, we are confident that our novel double operator integration techniques is the “magic wand” which will open this serious technical problem for successful resolution.

In the case of a general metric tensor, the formulae become much harder and more investigation is required.

Approach to Aim 6: The spectral triple constructed in [5] is equivariant and its spectral dimension is 3. However, every 3−cocycle is cohomologous to 0 and, therefore, the homological dimension is strictly less than 3.

We expect that the reason for this phenomenon is the wrong choice of $q-$deformed left regular representation. Our aim is to find a suitable $q-$deformation which permit the spectral triple to be equivariant, 3−dimensional and, at the same time to allow certain 3−cocycles to be non-trivial. A natural candidate for such a 3−cocycle is the Hochschild class of the Chern Character. Successful resolution of Aims 1,2,3 will deliver the required technical tools to determine the “correct representation” and such cocycles.

Approach to Aim 7: It is extremely probable that ordinary spectral triples in this setting should be replaced by twisted ones (where commutators $[D, a]$ may be unbounded, but a certain ”twisted” commutator is bounded). At the moment, no satisfactory theory is available which allows the derivation of a Connes Character formula for such triples. The only attempt is made in [18]. However, we expect that a combination of approaches from [18] and [4] would yield (at least a germ of) such a theory.

The expected results and timeframe Below we schedule the tasks for the next 4 years.

Character formula via heat semi-group, year 1. The method proposed in [4] seems more reliable than the approache in [22]. This method provides a cluster of mutually cohomologous Hochschild cocycles. The key obstruction is that in our setting the corresponding cocycles are not exactly cohomologous. Hence, it is necessary to measure how far the cocycles in this cluster are from being cohomologous to each other. This requires certain commutator estimates to be developed during the first year.

Character formula via $\zeta-$function residue, year 1. Strong empirical evidence suggests the equivalence of Aims 1 and 2. Namely, the more is known about poles of the $\zeta-$function, the better asymptotic for the heat semi-group can be derived. In reality, the situation is not 100% clear — it looks like no information about the poles outside of the real line can be acquired from the heat semi-group asymptotic. We expect, however, that one-side of the implication is correct, that is Aim 2 should actually follow from Aim 1.

Character formula via singular traces, year 2. Previous tailor-made approaches [3] are not sufficiently strong to derive Connes' Character formula in its full generality. The first task in Aim 3 is to build a theoretical framework which allows us to derive (6) from the formulae in Aims 1 and 2. We expect the integral representations specified in the Approach to Aim 3 (see above) to form the core of this framework. The precise conditions under which the methodology works well are yet unknown and one needs an honest theoretical investigation of the limitations of the methodology.

Minakshisundaram-Plejel theorem for non-commutative manifolds, year 3. We expect that Laplace-Beltrami operator introduced in Approach to Aim 4 (see above) is compatible with heat semi-group asymptotic up to terms of arbitrarily high order. One needs to investigate conditions under which such an asymptotic exists.

Computation of the curvature term, year 3. We plan to refine methods in [25] by the use of double and multiple operator integrals.

Equivariant Dirac operator on quantum groups, year 4. Based on our firm belief in the validity of Connes Character Formula in this setting, we intend to analyse the assumptions on the operator D required to make a proof of such a formula work. We expect these assumptions, together with equivariance property, to define the operator D uniquely.

Feasibility and Strategic Alignment The CI has already demonstrated his capacity to make significant, original and innovative contributions to wide range of Non-commutative Analysis and Non-commutative Geometry. He has co-authored the very first monograph on singular traces [26], which has albeit in embrionic form, some necessary components to attack Aim 3. The project presents a realistic timeframe as seen from above. Confidence in the feasibility is enhanced by the following:

1. The CI has already demonstrated his research credentials in the field of Noncommutative Analysis. His numerous papers in this area are published in the highly ranked journals like Crelle's Journal, Advances in Mathematics, Journal of Functional Analysis.

2. The CI wrote a number of publications in the field of Mathematical Physics. For example, paper [32] is published in prestigious journal Communications in Mathematical Physics.

3. The CI has co-authored a number of publications in the field of Classical Analysis.

4. The CI has authored a research monograph [26] jointly written with S. Lord and F. Sukochev. The substantial portion of this monograph describes CI's contribution to the field of Non-commutative Geometry and related parts of Quantised Calculus. At the moment, a second edition of the book is in preparation.

5. The CI can rely on support and expert advice from the members of the Noncommutative Analysis group at UNSW (M. Cowling, I. Doust, D. Potapov, F. Sukochev).

It should also be pointed out that some parts of the current proposal have been thoroughly discussed with Professor Alain Connes (Fields medalist) who is the originator of (and leading expert in) Non-commutative Geometry. Professor Connes strongly and enthusiastically endorsed the ideas and methods underlying this proposal and the approach.

Benefit and collaboration

The CI expects the project to produce significant results and to publish them in the most reputed journals. In addition, the CI expects the project to be beneficial in the following ways.

a. Australia has strong research profile in geometry, specifically geometric and homological results many of which are based on the properties of the partition function associated to the heat kernel asymptotic expansion on classical Riemannian manifolds and the spectral and algebraic properties of the Laplacian and associated operators. Australian mathematics also has a strong groups in symmetries and Lie groups and operator algebras and non-commutative geometry. The outcomes of this project will therefore be of interest in Australia, and broaden existing strengths in new directions with the development of asymptotics for non-commutative Laplacians and geometry associated to quantum groups.

b. Enhance international collaboration in research. The CI is collaborating with highly-distinguished international experts (Alain Connes, Kenneth Dykema, Nigel Higson, Marius Junge). This collaboration already resulted in a number of papers in prestigious journals. Involvement of mathematicians of such a calibre would be beneficial not only for this project, but for all the mathematical research in UNSW.

c. Improve the international competitiveness of Australian research. The area of free probability is the cutting edge research in the modern probability theory. This proposal is on par with efforts of top world experts in the area and thus enhances the Australian position in research.

Communication of results Results of this project will be published in peer-reviewed journals (as well as on arxiv). CI and collaborators will also deliver the results in thematic international conferences.

Management of data UNSW has implemented a data storage solution for every stage in the life cycle of a research project. The data management plan for the project will be established using UNSW's resources when applicable. Data will be archived using UNSW's Long-term Data archive (or other archive mechanism as applicable). Data will made discoverable by registration on discipline-specific registries and indexation services.

References

[1] Baaj, S., Julg, P. *Théorie bivariante de Kasparov et opérateurs non bornés dans les C^*-modules hilbertiens.* C. R. Acad. Sci. Paris Sér. I Math. **296** (1983), no. 21, 875–878.

[2] Berline N., Getzler E., Vergne M. *Heat kernels and Dirac operators.* Grundlehren der Mathematischen Wissenschaften, **298**. Springer-Verlag, Berlin, 1992.

[3] Carey A., Gayral V., Rennie A., Sukochev F. *Integration on locally compact noncommutative spaces.* J. Funct. Anal. **263** (2012), no. 2, 383–414; Carey A., Gayral V., Rennie A., Sukochev F. *Index theory for locally compact noncommutative geometries.* Mem. Amer. Math. Soc. **231** (2014), no. 1085, vi+130 pp.

[4] Carey A., Rennie A., Sukochev F., Zanin D. *Universal measurability and the Hochschild class of the Chern character.* J. Spectr. Theory **6** (2016), no. 1, 1–41.

[5] Chakraborty P., Pal A. *Equivariant spectral triples on the quantum $SU(2)$ group.* K-Theory **28** (2003), no. 2, 107–126; Chakraborty P., Pal A. *Spectral triples and associated Connes-de Rham complex for the quantum $SU(2)$ and the quantum sphere.* Comm. Math. Phys. **240** (2003), no. 3, 447–456.

[6] Connes A. *Cyclic cohomology, quantum group symmetries and the local index formula for $SU_q(2)$.* J. Inst. Math. Jussieu **3** (2004), no. 1, 17–68.

[7] Connes A. *Geometry from the spectral point of view.* Lett. Math. Phys., **34** (3) 203–238, 1995.

[8] Connes A. *Noncommutative geometry.* Academic Press, Inc., San Diego, CA, 1994.

[9] Connes A., *Noncommutative geometry and reality.* J. Math. Phys. **36** (1995), 6194-6231 .

[10] Connes A. *On the spectral characterization of manifolds.* J. Noncommut. Geom. **7** (2013), no. 1, 1–82.

[11] Chamseddine, A., Connes, A., Mukhanov, V. *Quanta of geometry: noncommutative aspects.* Phys. Rev. Lett. **114** (2015), no. 9, 091302, 5pp.

[12] Connes A., Fathizadeh F. *The term a_4 in the heat kernel expansion of noncommutative tori.* arXiv:1611.09815

[13] Connes A., Levitina G., McDonald E., Sukochev F., Zanin D. *Noncommutative Geometry for Symmetric Non-Self-Adjoint Operators.* arXiv:1808.01772

[14] Connes A., Moscovici H. *The local index formula in noncommutative geometry.* Geom. Funct. Anal. **5** (1995), no. 2, 174–243.

[15] Connes A., Moscovici H. *Modular curvature for noncommutative two-tori.* J. Amer. Math. Soc. **27** (2014), no. 3, 639–684.

[16] Connes A., Sukochev F., Zanin D. *Trace theorem for quasi-Fuchsian groups.* Mat. Sb. **208** (2017); Connes A., McDonald E., Sukochev F., Zanin D *Conformal trace theorem for Julia sets of quadratic polynomials.* Ergodic Theory Dyn. Syst. Published online: 04 December 2017. https://doi.org/10.1017/etds.2017.124

[17] Connes A., Tretkoff P. *The Gauss-Bonnet theorem for the noncommutative two torus.* Noncommutative geometry, arithmetic, and related topics, 141–158, Johns Hopkins Univ. Press, Baltimore, MD, 2011.

[18] Fathizadeh F., Khalkhali M. *Twisted spectral triples and Connes' character formula.* Perspectives on noncommutative geometry, 79–101, Fields Inst. Commun., **61**, Amer. Math. Soc., Providence, RI, 2011.

[19] Fathizadeh F., Khalkhali M. *Scalar curvature for the noncommutative two torus.* J. Noncommut. Geom. **7** (2013), no. 4, 1145–1183; Fathizadeh F., Khalkhali M. *Scalar curvature for noncommutative four-tori.* J. Noncommut. Geom. **9** (2015), no. 2, 473–503.

[20] di Francesco P., Mathieu P., Senechal D. *Conformal Field Theory.* Springer-Verlag, New York, 1997.

[21] Gayral V., Gracia-Bondia J., Iochum B., Schücker T., Varilly J. *Moyal planes are spectral triples.* Comm. Math. Phys. **246** (2004), no. 3, 569–623.

[22] Gracia-Bondia J., Varilly J., Figueroa H. *Elements of noncommutative geometry.* Birkhäuser Advanced Texts: Basler Lehrbücher. Birkhäuser Boston, Inc., Boston, MA, 2001.

[23] Helgason S., *Differential geometry, Lie groups, and symmetric spaces.* Graduate Studies in Mathematics, **34**. American Mathematical Society, Providence, RI, 2001.

[24] Junge M., Sukochev F., Zanin D. *Embeddings of operator ideals into Lp-spaces on finite von Neumann algebras.* Adv. Math. **312** (2017), 473–546.

[25] Lesch M. *Divided differences in noncommutative geometry: rearrangement lemma, functional calculus and expansional formula.* J. Noncommut. Geom. **11** (2017), no. 1, 193–223.

[26] Lord S., Sukochev F., Zanin D. *Singular traces. Theory and applications.* De Gruyter Studies in Mathematics, **46**. De Gruyter, Berlin, 2013.

[27] Neshveyev S., Tuset L. *The Dirac operator on compact quantum groups.* J. Reine Angew. Math. **641** (2010), 1–20; Dabrowski L., Landi G., Sitarz A., van Suijlekom W., Varilly J. *The Dirac operator on $SU_q(2)$.* Comm. Math. Phys. **259** (2005), no. 3, 729–759.

[28] Potapov D., Sukochev F. *Operator-Lipschitz functions in Schatten-von Neumann classes.* Acta Math. **207** (2011), no. 2, 375–389; Potapov D., Sukochev F. *Unbounded Fredholm modules and double operator integrals.* J. Reine Angew. Math. **626** (2009), 159–185.

[29] Rennie A. *Smoothness and locality for nonunital spectral triples.* K-Theory **28** (2003), no. 2, 127-165; Rennie A. *Summability for nonunital spectral triples.* K-Theory **31** (2004), no. 1, 71–100.

[30] Rosenberg S. *The Laplacian on a Riemannian manifold. An introduction to analysis on manifolds.* London Mathematical Society Student Texts, **31**. Cambridge University Press, Cambridge, 1997.

[31] Sukochev F., Usachev A., Zanin D. *Singular traces and residues of the ζ-function.* Indiana U. Math. J., **66** (2017), no. 4, 1107–1144.

[32] Sukochev F., Zanin D. *Connes integration formula for the noncommutative plane.* Comm. Math. Phys. **359** (2018), no. 2, 449–466.

[33] Weyl H. *Das asymptotische Verteilungsgesetz der Eigenwerte linearer partieller Differentialgleichungen (mit einer Anwendung auf die Theorie der Hohlraumstrahlung).* Math. Ann. **71** (1912), no. 4, 441–479.

C2. Statement by the Administering Organisation

(Provide a Statement that addresses the relevant criteria as set out in the grant guidelines. The Statement must be signed by the Deputy Vice-Chancellor (Research) or equivalent. (Upload a PDF of no more than three A4 pages))

Uploaded PDF file follows on next page.

C2 Statement by the Administering Organisation outlining Strategic Alignment

- This letter must be approved by your head of school and an email from your head of school approving the content of this letter must be forwarded to RSO (Grants.RSO@unsw.edu.au) in order for us to process it. It is your responsibility to ensure this is provided to us.

ARC *Future Fellowships* Applicant Dr Dmitriy Zanin [FT190100442]

Dr Dmitriy Zanin is a gifted researcher. In the 7 years at UNSW since obtaining his Doctorate in 2011 Dr Zanin has published over 50 peer reviewed journal articles in Mathematics, over half of which are in the highest rank of Mathematics journals according to the ERA classification, and published 1 monograph that is rapidly becoming the standard reference for its area. His potential as a future research leader has been recognised by the award of an ARC DECRA for the period 2015-2018 titled "A new concept of independence in non-commutative probability theory" and, currently, a Scientia Fellowship in the School of Mathematics and Statistics at UNSW.

His research achievements span several fields within Pure Mathematics and involve an impressive range of co-authors. Outside of his collaboration with his former supervisor, Prof. Fedor Sukochev, who is Chair of Pure Mathematics at UNSW and current Australian Laureate Fellow, Dr Zanin has established research collaborations and authored multiple publications with the foremost international experts in his research areas: in functional analysis Prof. Nigel Kalton (Banach Medal), noncommutative probability Prof. Ken Dykema (co-author of the monograph founding free probability theory), and noncommutative geometry Prof. Alain Connes (Fields Medal and Crafoord Prize Winner). Dr Zanin's PhD research resulted in a considerable extension of the Birkhoff theorem on doubly stochastic matrices and Khinchine-type and Johnson-Schechtman inequalities on norms of sums of independent random variables in noncommutative probability theory. He has solved outstanding open questions from A. Pietsch, N. Kalton and K. Dykema, leaders in functional analysis, on spectrality and existence of traces on trace ideals that are used extensively in perturbation and scattering theory. With K. Dykema he developed a generalisation to type II von Neumann operator algebras of the Schur decomposition of matrices which is used and taught daily by mathematicians in linear algebra courses. With M. Caspers and D. Potapov he developed a reformulation and solution of a 50-year old problem of the famous Russian mathematician M. Krien on Lipschitz continuity of Lipschitz functions of trace class operators. Dr Zanin's research output is prolific, a testament to his dedication and work ethic. His ability to master and contribute to the forefront of multiple areas of mathematics is very rare and a talent recognised by other mathematicians of the highest calibre. His growing international profile is a testament to his mathematical insight and his ability to overcome technical challenges that have defeated others.

UNSW, one of Australia's leading research intensive universities, fully supports the application by Dr Zanin for a Future Fellowship Level 1. UNSW's Strategy 2025 goal of Research Excellence aims to foster researchers of the growing impact and calibre of Dr Zanin in Fundamental and Enabling Sciences. His research in Mathematics, and the topic of this Fellowship, is directly aligned with the School of Mathematics and Statistics' international strength and world-class profile in noncommutative analysis; a strength Dr Zanin has helped to build. By developing noncommutative versions of actions of symmetries on noncommutative differential manifolds, and noncommutative versions of the Minakshisundaram and Plejel asymptotic expansion of the heat kernel partition function, one of the most central tools in differential geometry and mathematical physics, Dr Zanin's technical work in his Future Fellowship will have direct applications to new models of the action principles that unify gravitation and the standard model of particle physics, continuing his collaboration with Prof. Connes, one of the most outstanding mathematicians of the last fifty years and winner of the Fields Medal and the Crafoord Prize, both considered equivalents of the Nobel Prize for Mathematics.

The School of Mathematics and Statistics in the Faculty of Science at UNSW has been ranked number one in Mathematics in Australia for each of the past six years in the Academic Ranking of World Universities, and it has received the most ARC Discovery Project funding in Mathematics and Statistics codes totalled over the past five years. The School has the highest ERA rank of 5 in Pure Mathematics. It has two current Laureate Fellows, including Prof. Sukochev in noncommutative analysis. Recognising the importance of the contribution of Prof. Sukochev and Dr Zanin to the School's success, the area of this Fellowship (operator algebras and noncommutative analysis) has been named as one of the core strengths of the School.

The UNSW School of Mathematics and Statistics has a Distinguished Researcher Support Program that will provide $10K support (return air-fare, accommodation allowance and living allowance) for one international visitor for a visit of up to four weeks in each year of the Fellowship. The international visits will bring expertise to the project and help to build enduring international research links that will last beyond the term of the project.

The School of Mathematics and Statistics at UNSW recognises the high value and likely impact of this Fellowship, and will use salary savings to employ a fixed term Lecturer in the area of the Fellowship (i.e. functional analysis). Strong preference will be given to an appointment that can work with the Future Fellow on research directly related to the project. The Fellow will sit on the search committee to ensure that an appropriate voice is given to this selection criterion.

At the conclusion of the Fellowship, the Fellow will return to a continuing appointment in the School of Mathematics and Statistics at UNSW. Dr Zanin's current 4 year Scientia Fellowship at UNSW, which will be paused for the period of the ARC Fellowship, will be available for Dr Zanin to resume after the completion of the ARC Future Fellowship. This will afford him additional opportunity to focus on research and additional resources to continue his international research collaboration after the Future Fellowship; and is further evidence of UNSW's investment in Dr Zanin and belief in the quality and potential of his research. He will have a key role in the growth and further development of the School's research group and world-leading profile in noncommutative analysis.

UNSW assists its researchers in developing and maintaining pathways for their ongoing development. As such, UNSW has several initiatives that provide research staff and students with professional support in planning and developing their careers. These will be available to Dr Zanin. Formal performance appraisals are conducted in all faculties, and our top researchers are proactively mentored through an innovative Research Development Framework program.

In closing, I reiterate UNSW Australia's strongest support for the application by Dr Zanin for a Future Fellowship in 2019 and welcome the opportunity provided by the Australian Research Council to promote and support research both for the benefit of Australia and the University.

Yours sincerely

Professor Nicholas Fisk
Deputy Vice-Chancellor (Research)
UNSW Sydney

C3. Medical Research

(Does this project contain content which requires a statement to demonstrate that it complies with the eligible research requirements set out in the ARC Medical Research Policy located on the ARC website?)

No

C4. Medical Research Statement

(If applicable, justify why this project complies with the eligible research requirements set out in the ARC Medical Research Policy located on the ARC website. Eligibility will be based solely on the information contained in this application. This is the only chance to provide justification and the ARC will not seek further clarification. (No more than 750 characters, approximately 100 words))

Part D - Personnel and ROPE (Dr Dmitriy Zanin)

D1. Personal Details

(To update personal details, click the ‘Manage Personal Details’ link below. Note this will open a new browser tab. When returning to the form ensure to ‘Refresh’ the page to capture the changes made to the Future Fellowship candidate's profile.)

Participation Type

Future Fellowship

Title

Dr

First Name

Dmitriy

Family Name

Zanin

Citizenship

Australia

Australian Permanent Resident

N/A

Australian Temporary Resident

N/A

D4. Current country of residence

(If the Future Fellowship candidate is a Foreign National, they must obtain a legal right to work and reside in Australia.)

Australia

D5. Qualifications

(To update any details in this table, the details in the 'Qualifications' section of the Future Fellowship candidate's profile must be updated.)

Conferral Date	AQF Level	Degree/Award Title	Discipline/Field	Awarding Organisation	Country of Award
02/06/2011	Doctoral Degree	PhD	Mathematics	Flinders University	Australia

D6. Research Opportunity and Performance Evidence (ROPE) – Current and previous appointment(s) / position(s) - during the past 10 years

(To update any details in this table, the details in the 'Employment' section of the participant's profile must be updated. Refer to the Instructions to Applicants for more information.)

Description	Department	Contract	Employment	Start Date	End Date	Organisation

		Type	Type			
UNSW Scientia Fellow	School of Mathematics and Statistics, Department of Pure Mathematics	Permanent	Full Time	01/07/2018	30/06/2022	The University of New South Wales
DECRA Fellow	School of Mathematics and Statistics	Contract	Full Time	01/07/2015	30/06/2018	The University of New South Wales
Postdoctoral Fellow	School of Mathematics and Statistics	Contract	Full Time	01/01/2012	30/06/2015	The University of New South Wales
Postdoctoral Fellow	School of Mathematics and Statistics	Contract	Part Time	01/01/2011	31/12/2011	The University of New South Wales
PhD student	School of Computer Science, Engineering and Mathematics	Contract	Full Time	01/01/2008	31/12/2011	Flinders University

D7. Research Opportunity and Performance Evidence (ROPE) - Academic Interruptions

(You and the Future Fellowship candidate must read the ROPE Statement http://www.arc.gov.au/arc-research-opportunity-and-performance-evidence-rope-statement before filling out this section.)
Has the Future Fellowship candidate experienced an interruption that has impacted on their academic record?

No

D8. Research Opportunity and Performance Evidence (ROPE) - Details of the Future Fellowship candidate's academic career and opportunities for research, evidence of research impact and contributions to the field, including those most relevant to this application

(Provide details of the Future Fellowship candidate's academic career and opportunities. This should not include information presented in the following sections. (Upload a PDF of no more than five A4 pages))

Uploaded PDF file follows on next page.

D8. Research Opportunity and Performance Evidence (ROPE) - Details of the Future Fellowship candidates academic career and opportunities for research, evidence of research impact and contributions to the field, including those most relevant to this application

Amount of time as an active researcher

I was awarded my PhD in Mathematics 7 years ago in 2011 at Flinders University of South Australia. During those 7 years, I have had no interruptions in research opportunity.

Research opportunities

Currently I am employed at UNSW as Scientia Fellow in the School of Mathematics and Statistics. It is a research intensive position that began July 2018 and will last 4 years. My workload is 75% research and 25% teaching including high-level courses for honours students. At the end of the Scientia Fellowship, my position becomes permanent.

Previously, I was employed as an ARC DECRA Fellow at UNSW (July 2015 – June 2018). My DECRA project titled "On the new concept of independence in non-commutative probability theory" was ranked highly and produced 29 publications in high ranked international journals (see the list of publications in Section D9 below).

From 2011 until June 2015 I was employed as a researcher under ARC Discovery Project grants DP110100064 and DP120103263 awarded to Prof. Sukochev. My research and publications contributed to the outcomes of DP110100064, DP120103263 and have also contributed to DP14010096 and DP150100920. The positions under Prof. Sukochev afforded me the opportunity to extend my areas of interest and expertise in noncommutative probability and functional analysis to noncommutative geometry, and engage internationally (Germany, USA, Russia, China, Poland) through invited visits and attendance at workshops such as Oberwolfach and Bedlewo. Without a permanent position I have been unable to apply for funding under the ARC Discover Project scheme directly.

A number of brilliant mathematicians have made, and continue to make, a significant impact on my research career. While completing my undergraduate study in Tashkent, Uzbekistan, I was impressed by Professor Chilin's attitude to Mathematics. It would be fair to say that following his example I decided to be a mathematician. My PhD supervisor Professor Sukochev taught me the foundations of non-commutative integration theory, but also an approach to Mathematics involving strict work ethics, enthusiasm and perseverance. From 2009, I was introduced to the problems on singular traces and was fortunate to work with Professor Nigel Kalton, a giant of functional analysis and Banach Medal winner. I was positively shocked by the depth and speed of this great thinker, whose insight and technical ability dwarfed everyone else I had encountered. The impression is with me still of the modesty of his genius, he was gentle and patient with those who were not grasping the details as quickly as himself. The technical devices introduced by Professor Kalton subsequently inspired my own inventions in this area. Since 2013, I had an opportunity to collaborate with Professor Dykema during my visits to Texas A&M University and his reciprocal visits to UNSW. Professor Dykema introduced me to the advanced techniques of non-commutative probability which helped me a lot during my DECRA project. In 2017, I was invited by Professor Connes to talk at his 70th anniversary conference and this developed into a collaboration. Professor Connes is a Fields Medal and Crafoord Prize winner, both are equivalents to the Nobel Prize in Mathematics, so needless to say this collaboration has impressed and inspired me deeply.

Research achievements and contributions

I started my research in non-commutative functional analysis at Flinders University as a PhD student. Since then my contribution to this research field include my PhD Thesis and over 50 peer-reviewed publications including a monograph "Singular traces: Theory and Applications" in the De Gruyter series "Studies in Mathematics". My research articles are published in high ranked journals such as Journal für die Reine und Angewandte Mathematik (Crelle's Journal), Advances in Mathematics, Journal of Functional Analysis, Journal of Spectral Theory, Pacific Journal of Mathematics as described in Section D9. All numbered references in this Section refer to the numbered publications in Section D9. My PhD Thesis was highly regarded by the referees, one of them wrote "this is the best PhD thesis I ever refereed during my whole career". My subsequent work spans non-commutative analysis, non-commutative geometry and non-commutative probability theory.

Some of the principal achievements of my work now follow. The primary objective of my PhD study was two-fold. One direction was studying orbits in symmetric function and sequence spaces. The particular question of interest was whether an orbit of a semi-group of bicontractions is a norm-closed convex hull of its extreme points. While answering this question, I established a collaboration with Nigel Kalton. I published a number of research papers on orbits; one of them with Professor Kalton. The other direction concerned Khinchine-type inequalities. The principal tool in the studies of orbits involves Khinchine-type inequalities; a deep extension of the Birkhoff theorem on doubly stochastic matrices. By employing this technique in my PhD Thesis I was able to (i) develop a unified approach to orbits on symmetric function and sequence spaces; (ii) significantly broaden the setting of applicability of earlier results; (iii) to determine the conditions that are necessary and sufficient for the affirmative answer to the above question. Central to the study of probabilistic inequalities (such as Khinchine inequalities) is the so-called Kruglov operator (an integration over a Poisson process). Using the Kruglov operator, I radically extended the area where Khinchine-type inequalities hold true.

Based on the advances in the theory of symmetric function spaces made in my PhD Thesis I obtained several profound results in non-commutative analysis. One of them resolves a long standing question of existence of continuous traces on Banach ideals in $\mathcal{L}(H)$ (the $*-$algebra of all bounded operators on a separable Hilbert space H). It was already known to experts that traces (even without the continuity assumption) do not exist on an ideal which can be obtained from the couple $(\mathcal{L}_p, \mathcal{L}_\infty)$, $p > 1$, by interpolation. Using ideas from my PhD thesis, I proposed to study a weaker condition:

$$\frac{1}{n}\|A^{\oplus n}\|_{\mathcal{I}} \to 0 \text{ as } n \to \infty \text{ for every operator } A \text{ in the ideal } \mathcal{I}. \tag{1}$$

The condition (1) holds for every interpolation space as above and prevents the existence of a continuous trace on the Banach ideal $\mathcal{I}$. In my paper [10] (joint with my former supervisor F. Sukochev, published in Crelle's Journal), the converse assertion is proved; namely, condition (1) holds for every Banach ideal $\mathcal{I}$ without continuous traces. In the same paper, I constructed (under the assumption that (1) fails) a positive continuous trace which respects the Hardy-Littlewood submajorization (and also the one that does not).

It is a standard result in linear algebra that a trace of a matrix depends only on its eigenvalues. A corresponding result for the standard trace on the trace class ideal $\mathcal{L}_1$ is radically harder (it was resolved in the affirmative by V. Lidskii only in 1959). More precisely, the following formula holds.

$$\mathrm{Tr}(A) = \mathrm{Tr}(\mathrm{diag}(\lambda(A))) \text{ for every operator } A \text{ in the ideal } \mathcal{L}_1.$$

Here, $\mathrm{diag}(\lambda(A))$ is a diagonal operator with eigenvalues of A on the diagonal (repeated with multiplicities). In his famous 1990 paper, Pietsch asked whether the same is true for an arbitrary trace on an arbitraryHe characterized this question as "extremely difficult". A partial answer for countably generated ideals was given by Kalton in his 1998 paper. In my paper [9] (joint with F. Sukochev, published in Advances in Mathematics), a class of ideals closed with respect to the logarithmic submajorization is introduced. In this paper, it is proved that if an ideal $\mathcal{I}$ is closed with respect to the logarithmic submajorization, then

$$\varphi(A) = \varphi(\mathrm{diag}(\lambda(A)))$$

for every operator $A \in \mathcal{I}$ and for every trace φ on $\mathcal{I}$. Hence, for the mentioned class of ideals Pietsch's question is answered in the affirmative. Conversely, if the ideal $\mathcal{I}$ is not closed with respect to the logarithmic submajorization, then there exists an operator $A \in \mathcal{I}$ such that $\mathrm{diag}(\lambda(A)) \notin \mathcal{I}$ and the question is answered in the negative.

A classical result due to Schur states that every matrix is unitarily conjugate to an upper-triangular one. Thus, it is a sum of a normal matrix (diagonal part) and a nilpotent matrix (strictly upper-triangular part). A corresponding result for compact operators in $\mathcal{L}(H)$ is proved by Ringrose in his 1962 paper. Namely, he proved that every compact operator is a sum of a normal one and a quasi-nilpotent one. It is natural to ask whether a similar result holds in the setting of type II von Neumann algebras. In 2013, I worked with Ken Dykema and we managed to solve this question by using techniques from Free Probability Theory. In my paper [8] (joint with K. Dykema and F. Sukochev, published in Crelle's Journal), it is proved that every operator T in a type II_1 von Neumann algebra can be written as $T = N + Q$, where N is normal and Q is "almost"nilpotent in the following sense

$$|Q^n|^{\frac{1}{n}} \to 0 \text{ as } n \to \infty \text{ in the strong operator topology.}$$

Connes Character Theorem is of paramount importance in non-commutative geometry. It expresses a certain cocycle (called the Chern character) in terms of Dixmier traces. This cocycle is then used to explicitly compute indices, Euler characteristics and other topological invariants via smooth expressions. In my paper [7] (joint with A. Rennie, A. Carey and F. Sukochev, published in Journal of Spectral Theory), a completely new approach to Connes Character Theorem is proposed. In a holistic way, motivation came from (Kalton et al., Adv. in Math., 2013). The result is now proved for an arbitrary trace (not only a Dixmier trace as in Connes' original approach).

A 50-year old problem due to Krein asks whether Lipschitz functions are operator Lipschitz on a given Banach ideal. For Schatten ideals $\mathcal{L}_p$, $1 < p < \infty$, the assertion is known to be true. Namely, Potapov and Sukochev (Acta Math., 2011) proved that

$$\|f(A) - f(B)\|_p \le c_p \|f'\|_\infty \|A - B\|_p$$

for every couple of self-adjoint operators (A, B) such that $A - B \in \mathcal{L}_p$. For $p = 1$ (or for $p = \infty$), a similar result is false. Kato and Davies proved that even absolute value fails to be operator Lipschitz in $\mathcal{L}_1$. Nazarov and Peller asked whether the following replacement

$$\|f(A) - f(B)\|_{1,\infty} \leq c_{abs}\|f'\|_\infty\|A - B\|_1, \tag{2}$$

holds, where $\mathcal{L}_{1,\infty}$ is the principal ideal generated by the operator $\mathrm{diag}(\{1, \frac{1}{2}, \frac{1}{3}, \cdots\})$ equipped with a natural quasi-norm.

To solve this problem, I co-operated with Martijn Caspers and we jointly created a transference method. This method reduces boundedness of certain Double Operator Integrals (which naturally arise in the setting above) to complete boundedness of certain Fourier multipliers. Using this method, we proved (in collaboration with D. Potapov and F. Sukochev) that, indeed, (2) holds true. The manuscript [1] is accepted for publication in the American Journal of Mathematics. The result has been verified by leading experts G. Pisier, Q. Xu, V. Peller, M. Junge and others and has now become accepted as an outstanding achievement in operator calculus. Some of the key machinery of our proof was developed in our earlier publications.

Inequalities proved by Johnson and Schechtman relate the norm of the sum of independent random variables to that of their disjoint copies. Namely, if $E \supset L_p$, $p < \infty$, is a symmetric function space on $(0, 1)$ and if $x_k \in E$ are mean zero independent random variables, then

$$\|\sum_{k\geq 0} x_k\|_E \approx_E \|\bigoplus_{k\geq 0} x_k\|_{Z_E^2},$$

where Z_E^2 is a certain symmetric function space on $(0, \infty)$. Astashkin and Sukochev (Isr. J. Math., 2005) proposed another method of proving Johnson-Schechtman inequalities by means of the so-called Kruglov operator (which is an integration with respect to the Poisson process). In my paper (joint with S. Astashkin and F. Sukochev, published in Pacific Journal of Mathematics), it is proved that Johnson-Schechtman inequalities hold in a symmetric (quasi-Banach) function space if and only if the Kruglov operator is bounded on that space. It is tempting to extend the technique related to the Kruglov operator to the domain of the non-commutative probability. In my paper (joint with F. Sukochev, published in Journal of Functional Analysis), I constructed an analogue of the Kruglov operator in the Free Probability Theory. It appears that Kruglov operator behaves better in the setting of free probability than in the setting of a classical probability; and that Johnson-Schechtman inequalities are always true.

This breakthrough inspired a question: what sort of independence suffices to guarantee the Johnson-Schechtman inequalities for a sufficiently rich class of symmetric spaces? I proposed this investigation as the subject for my DECRA project approved by the ARC in November 2014. We shared this question with a well-known Chinese probabilist Yong Jiao. Professor Jiao than came to UNSW for a long-term research visit, during which we obtained a significant achievement: if a symmetric space E is an interpolation space for the couple (L_p, L_q), $1 < p < q < \infty$, then Johnson-Schechtman inequalities hold true in E for an arbitrary sequence of mean zero random variables independent in the sense of Junge and Xu. This paper [6] is published in the Journal of the London Mathematical Society. In 2016, it was presented on the Symposium on Modern Analysis and Applications in Harbin.

In 2016, Alain Connes suggested to me a conjecture from his book "Noncommutative geometry". This conjecture due to Connes and Sullivan stood for more than 20 years. There are 2 versions of the conjecture (for Julia sets as stated below) and for Kleinian groups. Let f_c be a quadratic polynomial $f : z \to z^2 + c$. Julia set of f is a Jordan curve if and only if c is in the main cardioid of the Mandelbrot set. For such c, there exists an analytic mapping Z with the property

$$Z \circ f_0 = f_c \circ Z.$$

The conjecture asserts that

$$\varphi(M_h|[F, M_Z]|^p) = \int_{J(f_c)} h(z)d\nu(z)$$

for every singular trace φ on $\mathcal{L}_{1,\infty}$. Here, F is the Hilbert transform and ν is the $p-$dimensional geometric measure on the Julia set $J(f_c)$. In other words, one can recover the geometric measure via singular traces.

Jointly with Alain Connes, Edward McDonald and Fedor Sukochev, we solved this conjecture (see [3] and [13]). I delivered a keynote talk about resolution of this conjecture in the international conference "Noncommutative Geometry: State of the Art and Future Prospects" celebrating Connes's 70'th birthday March 29-April 2 2017 at the Fudan Insitute for Advanced Study in Shanghai. I was among two-dozen invited speakers including Sir Michael Atiyah (Fields Medal, De Morgan Medal, Abel Prize), Pierre Cartier (Ampere Prize), Alain Connes (Fields Medal, Grafoord Prize), Joachim Cuntz (Gottfried Wilhelm Leibniz Prize), Sorin Popa (Ostrowski Prize), Graeme Segal (Sylvester Prize), Dennis Sullivan (Wolf Prize, Oswald Veblen Prize, National Medal of Science), and Dan-Virgil Voiculescu (NAS Award in Mathematics).

Many of these research achievements underpin the research proposed for this Future Fellowship, and demonstrate my insight and ability in solving hard problems in mathematics using novel methods that often extend to new results in considerably more general contexts.

D9. Research Opportunity and Performance Evidence (ROPE) - Research Outputs

i. Research context: Provide clear information that explains the relative importance of different research outputs and expectations in the Future Fellowship candidate's discipline/s. The information should help assessors understand the Future Fellowship candidate's academic research achievements and should include the importance/esteem of specific journals in the Future Fellowship candidate's field; specific indicators of recognition within the Future Fellowship candidate's field such as first authorship/citations, or the significance of non-traditional research outputs. (No more than 3,750 characters, approximately 500 words)

The best measures of the quality of the mathematical research is feedback or acclaim by peers, including invitations to collaborate and present, and quality of the journals where publications appear.

In mathematics, the list of authors is always arranged alphabetically and does not indicate contribution. Citation rates in pure mathematics are lower than other fields, >200 citations for a Level B Research Fellow is outstanding, a typical Level E Professor in an Australian university will have >500 citations and >1000 citations for a Level E Professor is outstanding.

The bibliometric database universally accepted in mathematics is MathSciNet, which includes citations from book and journal publications only and does not include arXiv citations. Typically it provides much lower citation rates than, say, Google Scholar.

Journal quality: I consistently publish influential articles in high-profile mathematical journals. These journals are typically ranked A* or A by the Australian Mathematical Society. I have published 19 articles in A* ranked journals in the last 5 years. Examples of these journals are: Journal für die reine und angewandte Mathematik (the oldest mathematical journal still in publishing) where I have published 2 articles, Advance in Mathematics (among the top five journals in Pure Mathematics) where I have published 5 articles, Communications in Mathematical Physics (the most authoritative journal in the area) where I have published 1 article, Journal of Functional Analysis (the primary journal in Functional Analysis) where I have published 10 articles, Ergodic Theory and Dynamical Systems, American Journal of Mathematics, Transactions of the AMS, Journal of the London Mathematical Society where I have published 2 articles, and other excellent journals.

Output: According to MathSciNet, I have 47 articles published in international research journals and 1 monograph. In addition to that, there are 3 accepted articles and a large number of submissions. 41 of those publications are in the last 5 years. Having only completed my PhD 7 years ago, many of my publications were published only a few years ago. However, I currently have 204 citations from 81 different authors.

As a measure of the quantity and quality of my research collaborators, I have 31 co-authors who, between them, have 31212 citations on MathSciNet. Among these co-authors are "luminaries" such as Alain Connes, Nigel Kalton and Kenneth Dykema as mentioned who have won numerous international prizes. Collaboration with mathematicians of such calibre is the highest honour and recognition in the field.

According to MathSciNet, my monograph has 59 citations. This is an excellent rate for a specialist reference published 5 years ago. As a comparison, the nearest text in terms of content is Barry Simon's "Trace ideals and their applications." LMS Lecture Note Series, 35, Cambridge University Press, Cambridge-New York, 1979 which has 467 citations over 39 years. 5 more of my papers were cited at least 10 times. My citation rate is increasing each year.

ii. Research output list: List the research outputs most relevant to this application categorised under the following headings: Ten career-best research outputs; Authored books; Edited books; Book chapters; Referred Journal articles; Fully refereed conference proceedings; Additional research outputs (including non-traditional research outputs). CVs and theses should not be included in this list. The Future Fellowship candidate's ten career-best research outputs should not be repeated under subsequent headings. (Upload a PDF of no more than five A4 pages)

Uploaded PDF file follows on next page.

Ten career-best research outputs

[1] Caspers M., Potapov D., Sukochev F., Zanin D. *Weak type commutator and Lipschitz estimates: resolution of the Nazarov-Peller conjecture.* Amer. J. Math., to appear.

[2] Sukochev, F.; Zanin, D. *Connes integration formula for the noncommutative plane.* Comm. Math. Phys. **359** (2018), no. 2, 449–466.

[3] Connes A., Sukochev F., Zanin D. *Trace Theorem for quasi-Fuchsian groups.* Mat. Sb. **208** (2017), no. 10, 59–90.

[4] Junge M., Sukochev F., Zanin D. *Embeddings of symmetric operator spaces into $\mathcal{L}_p-$spaces on finite von Neumann algebras.* Adv. Math. **312** (2017), 473–546.

[5] Lord S., McDonald E., Sukochev F., Zanin D. *Quantum differentiability of essentially bounded functions on Euclidean space.* J. Funct. Anal. **273** (2017), no. 7, 2353–2387.

[6] Jiao Y., Sukochev F., Zanin D. *Johnson-Schechtman and Khinchine inequalities in noncommutative probability theory.* J. Lond. Math. Soc. (2) **94** (2016), no. 1, 113–140.

[7] Carey A., Rennie A., Sukochev F., Zanin D. *Universal measurability and the Hochschild class of the Chern character.* J. Spectr. Theory **6** (2016), 1–41.

[8] Dykema K., Sukochev F., Zanin D. *A decomposition theorem in II_1-factors.* J. Reine Angew. Math. **708** (2015), 97–114.

[9] Sukochev F., Zanin D. *Which traces are spectral?* Adv. Math. **252** (2014), 406–428.

[10] Sukochev F., Zanin D. *Traces on symmetrically normed operator ideals.* J. Reine Angew. Math. **678** (2013), 163–200.

Authored books

[11] Lord S., Sukochev F., Zanin D. *Singular traces: Theory and Applications.* De Gruyter Studies in Mathematics. Walter de Gruyter, Berlin, first edition, 2013.

Edited books

Book chapters

Refereed journal articles

[12] Ber A., Sukochev F., Zanin D. *Heisenberg relation for locally measurable operators.* Adv. Math. **335** (2018), 211–230.

[13] Connes A., McDonald E., Sukochev F., Zanin D. *Conformal trace theorem for Julia sets of quadratic polynomials.* Ergodic Theory Dynam. Systems. (published online).

[14] Potapov D., Sukochev F., Tomskova A., Zanin D. *Frechet differentiability of the norm of L_p-spaces associated with arbitrary von Neumann algebras.* Trans. AMS, to appear.

[15] Dykema K., Noles J., Zanin D. *Decomposability and norm convergence properties in finite von Neumann algebras.* Integral Equations Operator Theory **90** (2018), no. 5, Art. 54, 32 pp.

[16] Jiao Y., Zhou D., Wu L., Zanin D. *Noncommutative dyadic martingales and Walsh-Fourier series.* J. Lond. Math. Soc. (2) **97** (2018), no. 3, 550–574.

[17] Levitina G., Sukochev F., Vella D., Zanin D. *Schatten class estimates for the Riesz map of massless Dirac operators.* Integral Equations Operator Theory **90** (2018), no. 2, Art. 19, 36 pp.

[18] Ber A., Chilin V., Sukochev F., Zanin D. *Fuglede-Putnam theorem for locally measurable operators.* Proc. Amer. Math. Soc. **146** (2018), no. 4, 1681–1692.

[19] Dykema K., Sukochev F., Zanin D. *An upper triangular decomposition theorem for some unbounded operators affiliated to II_1-factors.* Israel J. Math. **222** (2017), no. 2, 645–709.

[20] Sukochev F., Usachev A., Zanin D. *Singular traces and residues of the $\zeta-$function.* Indiana Univ. Math. J. **66** (2017), no. 4, 1107–1144.

[21] Dykema K., Sukochev F., Zanin D. *Determinants associated to traces on operator bimodules.* J. Oper. Th. **78**:1 (2017), 119–134.

[22] Sukochev F., Zanin D. *Fubini theorem in noncommutative geometry.* J. Funct. Anal. **272** (2017), no. 3, 1230–1264.

[23] Sukochev F., Tulenov K., Zanin D. *Nehari type theorem for non-commutative Hardy spaces.* J. Geom. Anal. **27** (2017), no. 3, 1789–1802.

[24] Jiao Y., Sukochev F., Zanin D., Zhou D. *Noncommutative martingale inequalities in symmetric operator spaces.* J. Funct. Anal. **272** (2017), no. 3, 976–1016.

[25] Carey A., Gesztesy F., Grosse H., Levitina G., Potapov D., Sukochev F., Zanin D. *Trace formulas for a class of non-Fredholm operators: a review.* Reviews in Mathematical Physics, Vol. 28, No. 10 (2016) 1630002.

[26] Dykema K., Noles J., Sukochev F., Zanin D. *On reduction theory and Brown measure for closed unbounded operators.* J. Funct. Anal. **271** (2016), no. 12, 3403–3422.

[27] Dykema K., Sukochev F., Zanin D. *Algebras of Log-Integrable Functions and Operators.* Complex Anal. Oper. Theory. **10** (2016), no. 8, 1775–1787.

[28] Jiao Y., Sukochev F., Xie G., Zanin D. Φ-moment inequalities for independent and freely independent random variables. J. Funct. Anal. 270 (2016), no. 12, 4558–4596.

[29] Carey A., Gesztesy F., Levitina G., Potapov D., Sukochev F., Zanin D. *On index theory for non-Fredholm operators: a* $(1+1)-$*dimensional example.* Math.Nachr. **289** (2016), no. 5-6, 575–609.

[30] Aubrunn G., Sukochev F., Zanin D. *Catalysis in the trace class and weak trace class ideals.* Proc. Amer. Math. Soc. **144** (2016), no. 6, 2461–2471.

[31] Dykema K., Sukochev F., Zanin D. *Holomorphic functional calculus on upper triangular forms in finite von Neumann algebras.* Illinois J. Math. **59** (2015), no. 3, 819–824.

[32] Astashkin S., Sukochev F., Zanin D. *On uniqueness of distribution of a random variable whose independent copies span a subspace in* L_p. Studia Math. **230** (2015), no. 1, 41–57.

[33] Semenov E., Sukochev F., Usachev A., Zanin D. *Banach limits and traces on* $\mathcal{L}_{1,\infty}$. Adv. Math. **285** (2015), 568–628.

[34] Potapov D., Sukochev F., Usachev A., Zanin D. *Singular traces and perturbation formulae of higher order.* J. Funct. Anal. **269** (2015), no. 5, 1441–1481.

[35] Caspers M., Potapov D., Sukochev F., Zanin D. *Weak type estimates for the absolute value mapping.* J. Operator Theory **73** (2015), no. 2, 361–384.

[36] Sukochev F., Usachev A., Zanin D. *Dixmier traces generated by exponentiation invariant generalised limits.* J. Noncommut. Geom. **8** (2014), no. 2, 321–336.

[37] Potapov D., Sukochev F., Tomskova A., Zanin D. *Frechet differentiability of the norm of* L_p-*spaces associated with arbitrary von Neumann algebras.* C. R. Math. Acad. Sci. Paris **352** (2014), no. 11, 923–927.

[38] Astashkin S., Sukochev F., Zanin D. *Disjointification inequalities in symmetric quasi-Banach spaces and their applications.* Pacific J. Math. **270** (2014), no. 2, 257–285.

[39] Potapov D., Sukochev F., Zanin D. *Krein's trace theorem revisited.* J. Spectr. Theory **4** (2014), no. 2, 415–430.

[40] Sukochev F., Zanin D. *Dixmier traces are weak* dense in the set of all fully symmetric traces.* J. Funct. Anal. **266** (2014), no. 10, 6158–6173.

[41] Levitina G., Pietsch A., Sukochev F., Zanin D. *Completeness of quasi-normed operator ideals generated by* $s-$*numbers.* Indag. Math. (N.S.) **25** (2014), no. 1, 49–58.

[42] Sukochev F., Usachev A., Zanin D. *Generalized limits with additional invariance properties and their applications to noncommutative geometry.* Adv. Math. **239** (2013), 164–189.

[43] Sukochev F., Usachev A., Zanin D. *On the distinction between the classes of Dixmier and Connes-Dixmier traces.* Proc. Amer. Math. Soc. 141 (2013), no. 6, 2169–2179.

[44] Sukochev F., Zanin D. *Johnson-Schechtman inequalities in the free probability theory.* J. Funct. Anal. **263** (2012), no. 10, 2921–2948.

[45] Sukochev F., Zanin D. $\zeta-$*function and heat kernel formulae.* J. Funct. Anal. **260** (2011), no. 8, 2451–2482.

[46] Sedaev A., Sukochev F., Zanin D. *Lidskii-type formulae for Dixmier traces.* Integral Equations Operator Theory **68** (2010), no. 4, 551–572.

[47] Kalton N., Sukochev F., Zanin D. *Orbits in symmetric spaces. II.* Studia Math. **197** (2010), no. 3, 257–274.

[48] Astashkin S., Zanin D., Semenov E., Sukochev F. *The Kruglov operator and operators defined by random permutations.* (Russian) Funktsional. Anal. i Prilozhen. **43** (2009), no. 2, 3–18; translation in Funct. Anal. Appl. **43** (2009), no. 2, 83–95.

[49] Sukochev F., Zanin D. *Orbits in symmetric spaces.* J. Funct. Anal. **257** (2009), no. 1, 194–218.

[50] Sukochev F., Zanin D. *Khinchin inequality and Banach-Saks type properties in rearrangement-invariant spaces.* Studia Math. **191** (2009), no. 2, 101–122.

[51] Ganikhodzhaev N., Zanin D. *On a necessary condition for the ergodicity of quadratic operators defined on a two-dimensional simplex.* (Russian) Uspekhi Mat. Nauk **59** (2004), no. 3 (357), 161–162; translation in Russian Math. Surveys **59** (2004), no. 3, 571–572.

Fully refereed conference proceedings

Additional research outputs (including non-traditional research outputs)

D10. Research Opportunity and Performance Evidence (ROPE) - Currently held ARC projects

(This information is auto-populated from the Future Fellowship candidate's RMS profile and will include any active project which has not yet had a Final Report approved and the project file closed by the ARC. You will not be able to submit an application to the ARC that involves a researcher who has an overdue Final Report on an ARC-funded project. If there are any concerns with the information recorded here, contact Your organisation's Research Office.)

Identifier	Investigators	Admin Organisation	Project Title	Funding	End Date	Final Report Due Date	Final Report Status
DE150100030	Dr Dmitriy Zanin	The University of New South Wales	A new concept of independence in noncommutative probability theory.	$300,000	29/06/2018	29/06/2019	Draft

D11. Research Opportunity and Performance Evidence (ROPE) - Detail the number of students the Future Fellowship candidate has supervised over the last five years

(Provide the details of students the Future Fellowship candidate has supervised over the last five years. (No more than 350 characters, approximately 50 words))

PhD students: Edward Mcdonald (completion in 3 months), Dominic Vella (completion in 6 months), Jinghao Huang (completion in 1 year), Nathan Jackson (just started). Master student: Georges Nader (just started). Edward is an outstanding mathematician and is actively involved into this project. Jinghao and Nathan can also be involved.

D12. Eligibility - Relevant Qualification

(Select the qualification which is most relevant to the application.)

Degree/Award Title	Awarding Organisation	Conferral Date
PhD	Flinders University	02/06/2011

D13. Eligibility - Does the Future Fellowship candidate hold a professional equivalent to a PhD as certified by You?

(Where the Future Fellowship candidate does not hold a PhD, evidence must be provided to You, and You must certify that the Future Fellowship candidate holds a professional equivalent to a PhD.)

No

D14. Eligibility - Has the Future Fellowship candidate been granted an extension by You, to the eligibility period due to a significant career interruption as outlined in subsection B3.13 of the grant guidelines?

(If the Future Fellowship candidate's qualification relevant to this application (listed in question D12) was awarded prior to 1 March 2004 and they have had a significant career interruption (as listed in subsection B3.13 of the grant guidelines), the Future Fellowship candidate will need to seek an extension to the eligibility period through their Deputy Vice-Chancellor (Research).)

No

D15. Eligibility - Select the category of career interruption claimed (more than one may be selected)

*(Choose all types of career interruptions which have been claimed and granted by the Future Fellowship candidate's Deputy Vice-Chancellor (Research).
Select a type of interruption and click 'Add'.)*

D16. Eligibility - What is the total period of extension that the Future Fellowship candidate has claimed?

(Select the period of time which most closely equals the total period of extension claimed.)

D17. Eligibility - What is the Future Fellowship candidate's current academic level?

(Select the Future Fellowship candidate's current academic level from the menu below. If the Future Fellowship candidate is not employed at an Australian university, or is an international researcher, select "Other" and upload a letter from the DVCR or equivalent justifying the salary level requested in the Project Cost Part of the application.)

Select the Future Fellowship candidate's current academic level from the drop-down below.

Level B

D18. Eligibility - Academic level justification

(Upload a letter from the DVCR or equivalent justifying the salary level requested in the Project Cost Part of the application. This question is only mandatory if you have selected 'Other' because the Future Fellowship candidate is not employed at an Australian university, is an international researcher OR has chosen a Salary level which does not align with their academic level because they have experienced significant interruptions to their academic career, due to family responsibilities as primary care giver and/or due to working with a relevant industry. (No more than one A4 page))

No PDF file uploaded.

D19. Eligibility - What will the Future Fellowship candidate's time commitment (percentage of their time) be to the Administering Organisation?

(It is a requirement under the grant guidelines that the Future Fellow spend a minimum of 0.2 FTE annually located at Your organisation (the Administering Organisation).)

100

D20. Eligibility - What will the Future Fellowship candidate's time commitment (percentage of time) be to research activities related to this project?

(It is a requirement under the grant guidelines that the Future Fellow spend a minimum of 80 per cent of full time equivalent (0.8 FTE) (or pro-rata part-time equivalent) of their time on research and research capacity-building activities related to the proposed Future Fellowship.)

90

D21. Eligibility - Current Research Fellowship or Award funded by other Australian Government agencies

(Do not list Fellowships and Awards granted by the ARC. Only list Fellowships and Awards from other agencies.)
Does the Future Fellowship candidate hold a current Research Fellowship or Award funded by other Australian Government agencies?

No

D22. Project/Role relinquishment or application withdrawal

(Named participants on successful applications for Australian Laureate Fellowships, Future Fellowships, Centres of

Excellence or Special Research Initiative projects must meet the project limit requirements. If required, this may be achieved by relinquishing existing project(s), or relinquishing role(s) on existing projects, or withdrawing application(s), where allowed, that would exceed the project limits. The Future Fellowship candidate must nominate and adequately justify the proposed relinquishment(s) if these applications were successful. We will determine the outcome of the Future Fellowship candidate's nominated relinquishment(s).
Failure to provide this information will jeopardise the eligibility of their applications. Provide project/application ID(s) and the justification for each separated by a comma.
(No more than 100 characters))

Part E - Project Cost (FT190100442)

E1. What is the proposed budget for the project?

(There are rules around what funds can be requested from the ARC. You must adhere to the scheme specific requirements listed in the grant guidelines. Refer to the Instructions to Applicants for detailed instructions on how to fill out the budget section.)

Remunerated Participants

Dr Dmitriy Zanin	Future Fellowship	Level 1 from year 1 annually for 4 years

Total requested budget: $678,680

Year 1

Description	ARC
	Cash
Total	170,720
Personnel	154,920
Dr Dmitriy Zanin (Future Fellowship)	154,920
Travel	15,800
Travel funds for conferences and research visits	15,800

Year 2

Description	ARC
	Cash
Total	170,820
Personnel	154,920
Dr Dmitriy Zanin (Future Fellowship)	154,920
Travel	15,900
Travel funds for conferences and research visits	15,900

Year 3

Description	ARC
	Cash
Total	167,920
Personnel	154,920
Dr Dmitriy Zanin (Future Fellowship)	154,920
Travel	13,000
Travel funds for conferences and research visits	13,000

Year 4

Description	ARC

	Cash
Total	169,220
Personnel	154,920
Dr Dmitriy Zanin (Future Fellowship)	154,920
Travel	14,300
Travel funds for conferences and research visits	14,300

Part F - Budget Justification (FT190100442)

F1. Justification of Future Fellowship non-salary funding requested from the ARC

(Fully justify, in terms of need and cost, each budget item requested from the ARC. Use the same headings as in the Description column in the Project Cost Part of this application. (Upload a PDF of no more than four A4 pages and within the required format))

Uploaded PDF file follows on next page.

F1. Justification of Future Fellowship non-salary funding requested from the ARC.

Travel

Collaboration is a central part of the development, conduct and impact of Mathematics research. The ideas in Section C1 were developed as a result of conversations with overseas colleagues, the technical expertise of colleagues will accelerate the achievement of results, and it is through joint publication and networks that the impact of the Fellowship is maximised.

Below we describe and justify funds for travel and collaboration requested from the ARC. We indicate where our collaborators are contributing external funding. The UNSW School of Mathematics and Statistics will provide the Fellow with additional travel funds. Section F2 describes the contribution of UNSW funds to collaboration during the Fellowship.

Year 1: 15800 AUD in total. Costs are estimated.

(a) Professor Nigel Higson from Penn State University, USA, is a leading expert in non-commutative geometry and group representations. The Fellow discussed with him Aims 6 and 7 during a conference in Chengdu, China (May 2018) and during his short-term visit to UNSW (Dec 2018). Collaboration with Professor Higson is important for accelerating progress on Aims 6 and 7 of the project and will greatly increase the impact of achieved results. Professor Higson invited the Fellow to his home institution for the duration of up to 1 month. The Fellow and one of the PhD students plan to visit him in January 2020. It is an excellent opportunity for a PhD student in operator algebras and non-commutative analysis; the techniques in group representations he will learn from Professor Higson will allow him to contribute to Aim 6 of the project. Funding requested: return economy airfare Sydney-New York 2x2500 AUD; per diem 30 days x 100 AUD/day + 30 days x 80 AUD/day; 10400 AUD in total. Accommodation for the Fellow and for the PhD student and local transportation will be provided by Professor Higson.

(b) Professor Kordyukov from Ufa University, Russia, is known for his work in the geometry of the manifolds and foliation theory. The Fellow has continuous e-mail contact with him regarding Aims 1, 2 and 3 and his involvement is required for the success of these aims. For example, building a spectral triple from compact manifold is simple, but in the non-compact case this is a substantial result communicated to us by Kordyukov. Professor Kordyukov invited the Fellow to Ufa for the duration up to 1 month. The Fellow plans to visit him in May or June 2020. Funding requested: return economy airfare Sydney-Ufa and local transportation 3000 AUD; accommodation 30 days x 50 AUD/day; per diem 30 days x 30 AUD/day; 5400 AUD in total.

Year 2: 15900 AUD in total. Details are provided below. Costs are estimated.

(a) Reciprocal visit of Professor Higson to UNSW will ensure the success of Aims 6 and 7. The Fellow invited him to visit UNSW in 2021 (precise dates to be determined at a later stage) for the duration up to 1 month. Funding requested: accommodation 30 days x 150 AUD/day; per diem 30 days x 100 AUD/day; 7500 AUD in total. Airfare will be paid by Professor Higson.

(b) Professor Ponge from Seoul National University, South Korea, was a PhD student of Alain Connes and is the top specialist in non-commutative conformal geometry. The computations on the Minakshisundaram-Plejel theorem mentioned in Section C1 will be investigated in collaboration with Professor Ponge. His contribution and expertise are central for Aims 4 and 5 of the project. Professor Ponge invited the Fellow to visit him at Seoul for the duration up to 1 month. The Fellow and one of the PhD students will visit him in 2021 (precise dates to be determined at a later stage). The PhD student will learn Professor Ponge's developments on the relation between heat expansion and conformal geometry and work intesively on calculations for the noncommutative torus and noncommutative plane examples. Funding requested: return economy airfare Sydney-Seoul 2x1500 AUD; per diem 30 days x 100 AUD/day + 30 days x 80 AUD/day; 8400 AUD in total. Accommodation for the Fellow and for the PhD student will be paid by Professor Ponge.

Year 3: 13000 AUD in total. Details are provided below. Costs are estimated.

(a) Reciprocal visit of Professor Ponge to UNSW should accelerate the finalisation of Aims 4 and 5. The Fellow invited him to visit UNSW in 2022 (precise dates to be determined at a later stage) for the duration up to 1 month. Funding requested: accommodation 30 days x 150 AUD/day; per diem 30 days x 100 AUD/day; 7500 AUD in total. Airfare will be paid by Professor Ponge.

(b) Dr Caspers from Delft University of Technology, Netherlands, is a top specialist in both Double Operator Integrals and in quantum groups and also a long term collaborator of the Fellow. His expertise in Double Operator Integrals will be beneficial for Aim 5 as the transference principle invented by Dr Caspers may yield new integral formulae similar to the ones in the Approach to Aim 5. On the other hand, Dr Caspers knowledge of quantum groups is extremely relevant for Aim 6 (his unpublished note shows some progress on spectral triples for certain quantum groups). Dr Caspers invited the Fellow to visit him in Delft for the duration up to 1 month. The Fellow plans to visit him in 2022 (precise dates to be determined at a later stage). Funding requested: return economy airfare Sydney-Amsterdam and local transportation 2500 AUD; per diem 30 days x

100 AUD/day; 5500 AUD in total. Accommodation for the Fellow will be paid by Dr Caspers.

Year 4: 14300 AUD in total. Details are provided below. Costs are estimated.

(a) Professor Dykema from Texas A&M University, USA, is one of the creators of free Probability Theory and a world leading expert in Operator Theory. He is a long-term collaborator of the Fellow and conversations with him were important for the foundation of this project. Collaboration with Professor Dykema is invaluable in the later stages of the project to maximise impact and look for wider applications. He invited the Fellow to visit his home institution for the duration up to 1 month. The Fellow will visit him in 2023 (precise dates to be determined at a later stage). Funding requested: return economy Sydney-College Station 2500 AUD; per diem 30 days x 100 AUD/day; 5500 AUD in total. Accommodation for the Fellow will be paid by Professor Dykema.

(b) Reciprocal visit of Dr Caspers to UNSW serves the purpose of developing future applications (mostly, to representations of quantum groups) and strategy. Dr Caspers will visit the Fellow in 2023 (precise dates to be determined at a later stage). Funding requested: 50% of the return economy airfare Amsterdam-Sydney 1300 AUD; accommodation 30 days x 150 AUD/day; per diem 30 days x 100 AUD/day; 8800 AUD in total. Airfare will be partially paid by Dr Caspers.

F2. Details of Administering Organisation contributions

(Provide an explanation of how Your organisation's contributions will support the proposed project. Use the same headings as in the Description column in the Project Cost Part of this application. (Upload a PDF of no more than one A4 page and within the required format))

Uploaded PDF file follows on next page.

F2. Details of Administering Organisation contributions.

Travel

UNSW School of Mathematics and Statistics will provide the Fellow with travel funds (10000 AUD annually). Using these funds, the Fellow and PhD students will attend international and domestic conferences which reduces funds requested from the ARC and increases collaboration and impact. The travel plan below does not interfere with the research visits specified in Section F1. All costs are estimated.

Year 1: International. 6000 AUD in total.

(a) Conference "Interactions of non-commutative analysis and quantum information theory" at Harbin Institute of Technology, China, June 2020. The Fellow will deliver preliminary results on Aims 1 and 2. Funding utilised: return economy airfare Sydney-Harbin 1500 AUD; accommodation 5 days x 80 AUD/day; per diem 5 days x 80 AUD/day; 2300 AUD in total.

(b) International Workshop on Operator Theory and Applications at Lancaster University, UK, July 2020. The Fellow will deliver preliminary results on Aim 3. Funding utilised: return economy airfare Sydney-London and local transportation 2500 AUD; accommodation 5 days x 120 AUD/day; per diem 5 days x 120 AUD/day; 3700 AUD in total.

Year 2: International. 10000 AUD in total.

(a) Great Plains Operator Theory Symposium (USA, location to be determined). Attendees: Fellow and one of the PhD students. The Fellow will deliver results on Aims 1, 2 and 3. Funding utilised: return economy airfare 2x2500 AUD; accommodation 2 x 5 days x 80 AUD/day; per diem 5 days x 100 AUD/day + 5 days x 80 AUD/day; 6700 AUD in total.

(b) Conference "Non-commutative harmonic analysis and non-commutative probability theory" at Bedlewo, Poland. Attendees: Fellow. The Fellow will deliver preliminary results on Aim 6. Funding utilised: return economy airfare Sydney-Warsaw and local transportation 2500 AUD; conference fee 300 AUD; per diem 7 days x 70 AUD/day; 3290 AUD in total.

Year 3: International. 10000 AUD in total.

(a) Canadian Operator Symposium (Canada, location to be determined). Attendees: Fellow and one of the PhD students. The Fellow will deliver results on Aims 4 and 5. Funding utilised: return economy airfare and local transportation 2x2500 AUD; accommodation 2 x 5 days x 100 AUD/day; per diem 5 days x 100 AUD/day + 5 days x 80 AUD/day; 6900 AUD in total.

(b) Conference "Quantum groups in non-commutative geometry" in Oberwolfach. Attendees: Fellow. The Fellow will deliver preliminary results on Aim 7. Funding utilised: return economy airfare Sydney-Frankfurt 2500 AUD; per diem 6 days x 100 AUD/day; 3100 AUD in total.

Year 4: International. 10000 AUD in total.

(a) Conference "Operator Theory 33" (Romania, location to be determined). Attendees: Fellow and both PhD students. The Fellow will deliver the final results on Aims 6 and 7. Funding utilised: return economy airfare Sydney-Bucharest and local transportation 3x2500 AUD; accommodation 3 x 5 days x 100 AUD/day; per diem 5 days x 80 AUD/day + 2 x 5 days x 60 AUD/day; 10000 AUD in total.

(b) Conference travel at this stage of the project should target international conferences that would maximise impact of the results. This may result from invitations or conferences not yet scheduled four years in advance. Administering Organisation funds will be utilised for these opportunities and may adjust conference plans as above.

Years 1-4: Domestic. 4000 AUD in total.

The outcomes of this project will be of interest in Australia, and broaden existing research strengths in new directions with the development of asymptotics for noncommutative Laplacians as mentioned under "Benefit and Collaboration" in Section C1. UNSW Funding to an estimated amount of 4000 AUD will be utilised as appropriate during the Fellowship to allow the Fellow and PhD students to communicate the progress and results of the Fellowship at Australian meetings and interact with other research groups within Australia (e.g. ANU, Adelaide, Sydney, Wollongong).

F3. Does this application request funding for research activities, infrastructure or a project previously funded, or currently being funded, with Australian Government funding (from the ARC or elsewhere)?

(This is a ‘Yes’ or ’No’ question. If 'Yes' provide the project ID and outline the similarities of the research and explain how it will be managed.)

No

Funded Project ID

Outline the similarities and explain how these similarities will be managed if this application is funded. (No more than 2000 characters, approximately 285 words)

F4. Does this application request funding for research activities or infrastructure which are the subject of an application already submitted to the ARC?

(This is a ‘Yes’ or ’No’ question. If 'Yes' provide the application ID and outline the similarities of the research.)

No

Provide the application ID

Outline the similarities and explain why more than one application has been submitted for the same research. (No more than 2000 characters, approximately 285 words)

Part G - Research Support and Statements on Progress (FT190100442)

G1. Research support

(For the Future Fellowship candidate on this application, provide details of:
i) current submitted ARC applications (i.e. for which the outcome has not yet been announced);
ii) any newly funded ARC projects which are not yet showing in the Future Fellowship candidate's question (Currently held ARC projects); and
iii) research funding from non-ARC sources (in Australia and overseas). For research funding from non-ARC sources, list all projects/proposals/awards/fellowships awarded or requests submitted involving the Future Fellowship candidate for funding for the years 2018 to 2023 inclusive.)

Uploaded PDF file follows on next page.

G1: Research Support

Current submitted ARC Proposals

Description (All named investigators on any proposal or grant/ project/ fellowship in which a participant is involved, project title, scheme and round)	**Same Research Area**	**Support status** (Requested, Current, Past)	**Project ID**	**2018** $'000	**2019** $'000	**2020** $'000	**2021** $'000	**2022** $'000	**2023** $'000
Dr Dmitriy Zanin, Non-commutative Laplacians, quantum symmetries and the Chern character, ARC, Future Fellowship 2019.	yes	R	FT190100442	0	0	171	171	168	170

G2. Statements on Progress for ARC-funded projects

(A progress statement must be provided for any currently funded ARC project that involves the Future Fellowship candidate named on this application. This requirement applies to all ARC funding with the exception of ARC Centres of Excellence, Supporting Responses to Commonwealth Science Council Priorities, Learned Academies Special Projects and Special Research Initiatives schemes. Refer to the Instructions to Applicants for further information. (Upload a PDF of no more than one A4 page for each project))

Certification

Certification by the Deputy/Pro Vice-Chancellor (Research) or their delegate or equivalent in the Administering Organisation

I certify that—

- I have read, understood and complied with the *Grant Guidelines for the Discovery Program (2018)*, (the grant guidelines) and, to the best of my knowledge all details provided in this application form and in any supporting documentation are true and complete in accordance with the grant guidelines.
- Proper enquiries have been made and I am satisfied that the Future Fellowship candidate meets the requirements specified in the grant guidelines, including having been awarded a PhD between 1 March 2004 and 1 March 2014. Where the Future Fellowship candidate has allowable career interruptions, sufficient evidence has been provided to the Administering Organisation and based on this evidence, I certify that the candidate's PhD award date together with allowable career interruptions (as listed under subsection B2.6 of the grant guidelines) would be commensurate with being awarded a PhD on or between 1 March 2004 and 1 March 2014.
- Where the Future Fellowship candidate holds a research higher degree, which is not a PhD, sufficient evidence has been provided to the Administering Organisation and based on this evidence, I certify that the candidate's qualification meets the level 10 criteria of the *Australian Qualifications Framework Second Edition January 2013*, or is a professional equivalent to a PhD.
- Upon request from the ARC, this organisation will provide evidence to support a career interruption justification in relation to the PhD Award date.
- The ARC reserves the right to audit any evidence on which an application is based.
- I will notify the ARC if there are any changes to the Future Fellowship candidate after the submission of this application.
- The Future Fellowship candidate is responsible for the authorship and intellectual content of this application, and has appropriately cited sources and acknowledged significant contributions to this application.
- To the best of my knowledge, all Conflicts of Interest relating to parties involved in or associated with this application have been disclosed to the Administering Organisation, and, if the application is successful, I agree to manage all Conflicts of Interest relating to this application in accordance with the *Australian Code for the Responsible Conduct of Research* (2018), the *ARC Conflict of Interest and Confidentiality Policy* and any relevant successor documents.
- I have obtained the agreement, attested to by written evidence, of all the relevant persons and organisations necessary to allow the project to proceed. This written evidence has been retained and will be provided to the ARC if requested.
- This application complies with the eligible research requirements set out in the *ARC Medical Research Policy*, located on the ARC website.
- This application does not request funding for the same research activities, infrastructure or project previously funded or currently being funded through any other Commonwealth funding.
- If this application is successful, I am prepared to have the project carried out as set out in this application and agree to abide by the terms and conditions of the grant guidelines and the *Grant Agreement for the Discovery Program (2018)*.
- The project can be accommodated within the general facilities of this organisation and if applicable, within the facilities of other relevant organisations specified in this application, and sufficient working and office space is available for any proposed additional staff.
- This organisation supports this application and, if successful, will provide the Future Fellowship candidate with an appropriate appointment for the duration of the Fellowship.
- All funds for this project will only be spent for the purpose for which they are provided.
- The project will not be permitted to commence until appropriate ethical clearance(s) has/have been obtained and all statutory requirements have been met.
- I consent, on behalf of all the parties, to this application being referred to third parties, including to overseas

parties, who will remain anonymous, for assessment purposes.

- I consent, on behalf of all the parties, to the ARC copying, modifying and otherwise dealing with information contained in this application.
- To the best of my knowledge, the Privacy Notice appearing at the top of this application has been drawn to the attention of the Future Fellowship candidate whose personal details have been provided in the Personnel section.