Discussion 2

1. Let X and Y be random variables with

$$\mathrm{E}(X) = 2, \quad \mathrm{E}(Y) = 5, \quad \mathrm{Var}(X) = 4, \quad \mathrm{Var}(Y) = 1.$$

   Define $Z = 2X - Y$.

   a) Find $\mathrm{E}(Z)$.

   b) Find $\mathrm{Var}(Z)$ and $\mathrm{Cov}(X, Z)$ if X and Y are independent.

   c) Find $\mathrm{Var}(Z)$ and $\mathrm{Cov}(X, Z)$ if $\mathrm{Cov}(X, Y) = 2$.

2. Let X, Y, and Z be random variables with

$$\mathrm{E}(X) = \mathrm{E}(Y) = \mathrm{E}(Z) = 0, \quad \mathrm{Var}(X) = \mathrm{Var}(Y) = \mathrm{Var}(Z) = 4,$$

$$\mathrm{Cov}(X, Y) = 1, \quad \mathrm{Cov}(X, Z) = 2, \quad \mathrm{Cov}(Y, Z) = 0.$$

   a) Find $\mathrm{Var}(X + Y)$.

   b) Find $\mathrm{Var}(Z + Y)$.

   c) Find $\mathrm{Cov}(X + Y, X + Z)$.

   d) Find $\mathrm{Cov}(X + 3Y, 2Y - Z)$.

3. Consider the process $X_t = Z_t + \theta Z_{t-1}$, where $\{Z_t\}$ are i.i.d. random variables with mean 0 and variance σ^2. In terms of θ and σ^2 express the following:

   a) The variance $\mathrm{Var}(X_t)$.

   b) The autocovariance $\gamma(h) = \mathrm{Cov}(X_t, X_{t+h})$ for lags $h = 1, 2, 3, \cdots$.

   c) The autocorrelation $\rho(h) = \mathrm{Corr}(X_t, X_{t+h})$ for lags $h = 1, 2, 3, \cdots$.

4. Consider the process $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}$, where $\{Z_t\}$ are i.i.d. random variables with mean 0 and variance σ^2. In terms of θ_1, θ_2 and σ^2 express the following:

   a) The variance $\mathrm{Var}(X_t)$.

   b) The autocovariance $\gamma(h) = \mathrm{Cov}(X_t, X_{t+h})$ for lags $h = 1, 2, 3, \cdots$.

c) The autocorrelation $\rho(h) = \mathrm{Corr}(X_t, X_{t+h})$ for lags $h = 1, 2, 3, \cdots$.

5. We say a process $\{X_t\}$ is (weakly) stationary if
   - $\mathrm{E}(X_t)$ is constant for all t,
   - $\mathrm{Var}(X_t)$ is constant for all t,
   - $\mathrm{Cov}(X_t, X_{t+h})$ does not depend on t for all $h = 0, \pm 1, \pm 2, \cdots$.

   Suppose X_t and Y_t are stationary. Show that $X_t + Y_t$ is also stationary.