Discussion 10

1. Find the expression of the spectral densities of the following processes. Use the expression

$$f_X(\omega) = \gamma_X(0) + 2\sum_{h=1}^{\infty} \gamma_X(h)\cos(2\pi\omega h),$$

and properties of $ARMA(p, q)$ processes.

a) $\{X_t\}$ is white noise with $\sigma^2 = 4$.

b) $\{X_t\}$ is an $MA(1)$ process with $\theta = -0.2$ and $\sigma^2 = 4$.

c) $\{X_t\}$ is an $MA(2)$ process with $\theta_1 = 0.6$, $\theta_2 = -0.2$ and $\sigma^2 = 4$.

2. For each of the processes below, explain why the process is a linear filter of another process, find the filter coefficients, the power transform function $|g(\omega)|^2$, and the spectral density fuction of the output process. The following expressions will be useful.

$$Y_t = \sum_{j=-\infty}^{\infty} \beta_j X_t,$$

$$f_Y(\omega) = |g(\omega)|^2 f_X(\omega),$$

$$g(\omega) = \sum_{j=-\infty}^{\infty} \beta_j e^{-2\pi i\omega j}.$$

Compare your answers to the $AR(p)$ and $MA(q)$ spectral densities from lecture.

a) $\{X_t\}$ is an $MA(2)$ process with $\theta_1 = 0.6$, $\theta_2 = -0.2$ and $\sigma^2 = 4$.

b) $\{X_t\}$ is an $AR(2)$ process with $\phi_1 = 0.4$, $\phi_2 = 0.2$ and $\sigma^2 = 4$.

3. Suppose $\{X_t\}$ is an $MA(2)$ process with $\theta_1 = 0.6$, $\theta_2 = -0.2$ and $\sigma^2 = 4$, and $\{Y_t\}$ is a linear filter of $\{X_t\}$. In each case below, find the ACVF of $\{Y_t\}$ for $h = 0, \pm1, \pm2, \cdots$. Use the expression

$$\gamma_Y(h) = \sum_{k=-\infty}^{\infty} \left(\sum_{j=-\infty}^{\infty} \beta_j \beta_{j+k} \right) \gamma_X(h-k).$$

a) $Y_t = (X_{t-1} + X_t + X_{t-1})/3$

b) $Y_t = X_t - X_{t-1}$