Homework 5

Due Date: 3:00 PM Wednesday March 1

General Instructions:

You may work on the assignment in groups of up to 3 students from the class.

Include a title page with the names and student ID numbers of all group members.

Submit your homework by the time it is due in lecture or in my mailbox in 4118 MSB.

Include all used computer code in an appendix at the end of the assignment.

Assignment:

1. Consider the following $ARMA(1,1)$ model

$$X_t - \mu = \phi(X_{t-1} - \mu) + \varepsilon_t + \theta\varepsilon_{t-1}.$$

The series have been observed $\{X_1, .., X_n\}$, $n = 75$. The observed value of X_n is $x_n = 2.1$.

a) Assume that $\mu = 0$, $\phi = 0.5$, and $\theta = 0.7$. From the data it has been calculated that an estimate of ε_n is $\hat{\varepsilon}_n = 0.3$. Obtain the predicted values $\hat{X}_{n+1}$, $\hat{X}_{n+2}$ and $\hat{X}_{n+3}$.

b) For this part assume that $\mu = 10$, $\phi = 0.5$, $\theta = 0.7$, and$\hat{\varepsilon}_n = 0.3$. Obtain the predicted values $\hat{X}_{n+1}$, $\hat{X}_{n+2}$ and $\hat{X}_{n+3}$.

c) For this part, assume that $\{X_t\}$ is the first difference of an observed series $\{Y_t\}$, $X_t = Y_t - Y_{t-1}$, with observed values $y_{n-1} = 5.0$ and $y_n = 7.1$. Assuming that the mean of $\{X_t\}$ is 0.01 and $\hat{\varepsilon}_n = 0.3$, Obtain the predicted values of Y_{n+1}, Y_{n+2} and Y_{n+3}.

Note: Here $\{Y_t\}$ has an $ARIMA(1,1,1)$ expression with $\phi = 0.5$, $\theta = 0.7$.

2. For each of the following processes, use the R function `ARMAacf()` to obtain the ACF and PACF values for lags $h = 1, 2, \cdots, 20$. Plot the values.
   a) $AR(2)$ with $\phi_1 = 0.4$, $\phi_2 = 0.5$
   b) $AR(2)$ with $\phi_1 = -0.4$, $\phi_2 = 0.5$
   c) $MA(2)$ with $\theta_1 = 1.5, \theta_2 = -0.8$
   d) $MA(2)$ with $\theta_1 = -1.5, \theta_2 = -0.8$
   e) $ARMA(2, 2)$ with $\phi_1 = -1.5, \phi_2 = 0.8, \theta_1 = 1.2, \theta_2 = -0.3$.
   f) Look at the plots for parts a) and b). What happens when ϕ_1 changes sign?
   Look at the plots for parts c) and d). What happens when θ_1 changes sign?

3. For each of the following processes, use the R function `ARMAtoMA()` to obtain the coefficients ψ_j with $j = 1, 2, \cdots, 30$ of the causal expression. Plot the values (make a horizontal line at 0, and plot the ψ_j as points, or make line segments as in an ACF plot). Comment on the shape. What happens as j increases?
   a) $AR(2)$ with $\phi_1 = -0.4$, $\phi_2 = 0.5$
   b) $MA(2)$ with $\theta_1 = -0.4$, $\theta_2 = 0.5$
   c) $ARMA(2, 2)$ with $\phi_1 = -1.5, \phi_2 = 0.8, \theta_1 = 1.2, \theta_2 = -0.3$

4. An $AR(2)$ model with mean was deemed appropriate for some data set of size $n = 100$. The estimated model parameters are $\hat{\mu} = 7.0009$, $\hat{\phi}_1 = -1.5378$, $\hat{\phi}_2 = -0.7726$, and $\hat{\sigma}^2 = 0.8383$. The observed values of X_{99} and X_{100} are $x_{99} = 6.8250$ and $x_{100} = 7.1126$.
   a) Obtain the point forecasts of X_{n+h} for $h = 1, ..., 5$.
   b) Obtain estimates of the variances of forecast errors, $\sigma^2(h)$ for $h = 1, ..., 5$. To do this, you will need to compute $\{\hat{\psi}_j\}$, the estimates of the causal representation coefficients. You can do this using the function `ARMAtoMA()` with the estimated model parameters.
   c) Use the previous parts to obtain approximate 95% prediction intervals for X_{n+h} for $h = 1, .., 5$.
   d) For this series we also know the observed values of X_{n+h} ,$h = 1, ..., 5$, are 6.8340, 6.7434, 6.4144, 7.7644, 3.2322. Plot the observed and the forecasted values against time $t = 101, ..., 105$ along with the prediction bands on the same graph. Comment of the results.

5. The file *globTemp.txt* contains measurements of the annual global temperature for the years 1850 - 2012. Take out the last five observations. Use only the first 158 observations for parts a) through c).
   a) Fit an $ARIMA(1, 1, 1)$ model to the first 158 observations. Provide the estimated parameters $\hat{\mu}$, $\hat{\phi}, \hat{\theta}$, and $\hat{\sigma}^2$.
   b) Plot the observed and fitted data on the same graph. Comment.
   c) Obtain the residuals. Use sample ACF and PACF plots, the Box-Ljung test, a histogram, and a Normal QQ-plot to carry out the usual diagnostics. Do the residuals appear to be i.i.d.? Do they appear to be Gaussian?

d) Use your model to obtain the forecasts of the five observations you originally took out. Plot the observed values (that weren't included in fitting the model) and the forecast values against time on the same figure. Comment.