# Homework 6

## Due Date: 3:00 PM Wednesday March 15

## General Instructions:

You may work on the assignment in groups of up to 3 students from the class.

Include a title page with the names and student ID numbers of all group members.

Submit your homework by the time it is due in lecture or in my mailbox in 4118 MSB.

Include all used computer code in an appendix at the end of the assignment.

## Assignment:

1. The file *deaths.txt* contains the yearly numbers of deaths from homicide and suicide (per 100,000 people) in Australia for the years 1915-2004.
   a) Plot the raw periodogram of this data. Also plot three smoothed periodograms (modified Daniell) averaging over 3, 7, and 11 frequencies. Which amount of smoothing would you choose here? Explain.
   b) Now use the approach outlined in lecture (using criterion $Q$) to choose the optimal amount of smoothing for the modified Daniell kernel. Consider the values $L = 3, 5, 7, \cdots, 45$. Report the chosen value of $L$, and plot the corresponding smoothed periodogram.
   You may use the provided script *sta137_smoothingPgrm.R*.
   c) Use `auto.arima()` to select the most appropriate ARMA model using the AICc criterion. Write the estimated parameters and their standard errors for the selected model. Use the ACF and PACF plots to investigate whether the residuals from this model can be described as white noise.
   d) Plot the spectral density function of the model selected in part (c) and the smoothed periodogram (with the chosen amount of smoothing in part (b)) side by side. Comment.

   **Note:** Be sure to plot the spectral density and the periodograms on the *original*, not the log scale.

2. Let $\{X_t\}$ be an $MA(1)$ sequence with $\theta = 0.5$ and $\sigma^2 = 4$.
   a) Write down the first difference $\{\nabla X_t\}$ of the sequence $\{X_t\}$ as an $MA(2)$ model by explicitly wring down its the parameters.
   b) Obtain an explicit expression for the spectral density function $f(\omega)$ of $\{\nabla X_t\}$, where $\omega$ is the frequency on $[-0.5, 0.5]$.
   c) Plot the spectral density functions of $\{X_t\}$ and $\{\nabla X_t\}$. Comment on the difference in the shapes of the plots.

3. Let $\{Z_t\}$ be a white noise sequence with $\sigma^2 = 4$. In each case below, obtain the explicit expression of the spectral density function of $\{X_t\}$. Then plot the spectral density functions of $\{Z_t\}$ and $\{X_t\}$, and comment on the differences.
   a) $X_t = 9Z_t$
   b) $X_t = (Z_{t+1} + Z_t + Z_{t-1})/\sqrt{3}$
   c) $X_t = 0.8Z_{t-1} - 0.2Z_{t-2} + Z_t$
   d) $X_t = Z_t - Z_{t-1}$
   e) $X_t = 0.1Z_{t+2} + 0.25Z_{t+1} + 0.3Z_t + 0.25Z_{t-1} + 0.1Z_{t-2}$