Lecture 8

Trend & Seasonality (finish); Difference Operator; Backshift Operator
Textbook Sections: NA

Difference Operator

The difference operator ∇ is defined as

$$\nabla X_t = X_t - X_{t-1}.$$

The differenced series ∇X_t measures the change between consecutive values X_t.

We obtain a k-order difference $\nabla^k X_t$ by applying difference operator applied iteratively k times. For example, a second-order difference is computed as

$$\nabla^2 X_t = \nabla(X_t - X_{t-1}) = \nabla X_t - \nabla X_{t-1} = (X_t - X_{t-1}) - (X_{t-1} - X_{t-1}) = X_t - 2X_{t-1} + X_{t-2}.$$

Given observations $x_1, \cdots, x_n$, we can compute the sample differences in the same way as $\nabla x_t = x_t - x_{t-1}$ for $t = 2, \cdots, n$. Note that we can only obtain $n-1$ first order differences, $n-2$ second order differences, and so on.

There are several reasons to be aware of the difference operator.

1. The differenced values ∇X_t may be of interest in some problems (i.e., we can analyze consecutive changes, rather than consecutive values).
2. Differencing can be used to remove trend.
3. We will focus on a large class of models called ARIMA models. Differences play a role in some of these models.

Differencing to Remove Trend

If $\{X_t\}$ has a polynomial trend of degree k, then the k^{th} difference series $\{\nabla^k X_t\}$ will not have a trend. That is, if $\{X_t\}$ has the form

$$X_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \cdots + \beta_k t^k + Y_t,$$

where $\beta_0, \cdots, \beta_k$ are constants, and $\{Y_t\}$ is stationary, then

$$\nabla^k X_t = \beta + \nabla^k Y_t,$$

where β is a constant.

Even when the data does not have a perfect polynomial trend, differencing can be effective in removing the trend. It is quick and easy to inspect the differenced data alongside the raw data. If the differenced data looks better (closer to stationary) than the raw data, we can fit a model to the differenced data instead.

Backshift Operator

The backshift operator is defined as

$$B(X_t) = X_{t-1}.$$

This operator simply shifts the time index back by one value. If applied iteratively $k > 0$ times, this operator will shift the index back by k values,

$$B^k(X_t) = X_{t-k}.$$

Note that we can express ∇X_t as $\nabla X_t = X_t - B(X_t)$.

The backshift operator is common to time series notation, and will play an important role when we consider $ARMA(p, q)$ models.

Further Info

More information about differencing and related topics can be found at *https://www.otexts.org/fpp/8/1*. We will touch on some of these topics later in the course, but feel free to peruse the resource now.