Homework 3

Problem 1 (Events)
In each of the following scenarios, define the experiment and describe the sample space. In each case, all the simple events equally likely?

(a) Two fair coins are tossed, and the value (H or T) of each is recorded.

(b) Two fair coins are tossed, and the number of coins that came up heads is recorded.

(c) Two cards are selected randomly from a shuffled deck and their values are recorded.

Problem 2 (Probability Rules)
A sample space contains 10 simple events, $E_1, E_2, E_3, \cdots, E_{10}$. In each case list the probabilities of the 10 simple events.

(a) The simple events have equal probabilities.

(b) $P(E_1) = P(E_2) = P(E_6) = P(E_8) = 0.1$,
$P(E_3) = P(E_5) = P(E_9) = 0.16$,
$P(E_4) = P(E_7) = P(E_{10}) = ?$

(c) $P(E_1) = 0.45$, $3P(E_2) = 0.45$, and the other eight events have equal probabilities.

Problem 3 (Counting Rules)
Calculate the following:

(a) $9!$

(b) $\frac{15!}{10!}$

(c) $\frac{15!}{10!5!}$

(d) ${}_{10}C_4$

(e) $\binom{10}{7}$

(f) $\binom{10}{10}$

You can check your answers using Google by typing things like "9!" or "10 choose 4."

Problem 4 (Event Relations)
Two fair dice (die A and die B) are thrown one by one. The number facing up on each die is recorded.

(a) Write down all possible outcomes.

(b) What is the probability that die A has 2 facing up?

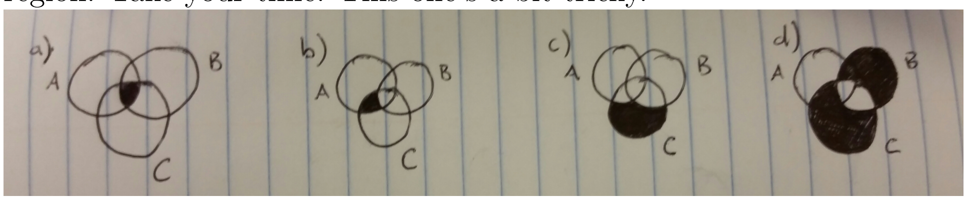
(c) What is the probability that die B has 5 facing up?

(d) Are the events in (b) and (c) mutually exclusive?
If yes, explain why. If no, list the outcomes they have in common.

(e) What is the probability that both dice have even numbers facing up?

Problem 5 (Counting Rules)
You are planning a road trip and want to invite some friends. You have 10 friends with whom you'd like to travel, but there are only 3 empty seats in the car. How many ways are there to select 3 friends to take on the road trip?

Problem 6 (Event Relations)
Use intersections, unions and complements to describe the set corresponding to each shaded region. Take your time. This one's a bit tricky.



Problem 7 (Probability Rules)
You're given this information:

$P(A) = 0.48$
$P(B) = 0.3$
$P(A \cup B) = 0.57$

Compute:

(a) $P(A^C)$

(b) $P(B^C)$

(c) $P(A \cap B)$

(d) $P((A \cup B)^C)$

(e) $P(A^C \cup B)$

(f) $P(A \cap B^C)$

Problem 8 (Probability Rules)
A study of the behavior of a large number of drug offenders after treatment for drug abuse suggests that the likelihood of conviction within a 2-year period after treatment may depend on the offender's education. The proportions of the total number of cases that fall into four education/conviction categories are shown in the table below:

Education	Convicted	Not Convicted	Totals
10 Years or More	0.10	0.30	0.40
9 Years or Less	0.27	0.33	0.60
Totals	0.37	0.63	1.00

Suppose a single offender is selected at random from the treatment program. The events of interest are:
A: The offender has 10 or more years of education.
B: The offender is convicted within 2 years of completing treatment.

Find the probabilities of the following:

(a) A

(b) B

(c) $A \cap B$

(d) $A \cup B$

(e) A^C

(f) $(A \cup B)^C$

(g) $(A \cap B)^C$

Problem 9 (Probability Rules)
A deck of cards is shuffled. Find the probabilities of the following events.
(Note: A standard deck consists of 52 cards, of which 13 are hearts)

(a) The top card is a heart.

(b) The bottom card is not a heart.

(c) Both the top and bottom cards are hearts.

(d) The top card is a heart, and the bottom card is not a heart.

(e) The top card is a heart given that the bottom card is not a heart.

Problem 10 (Conditional Probability) The probability that a certain movie will win an award for acting is 0.15, the probability that it will win an award for direcing is 0.23, and the probability that it will win both is 0.09. Find the probabilities of the following.

(a) The movie wins an award for acting, given that it wins both awards.

(b) The movie wins an award for acting, given that it wins exactly one award.

(c) The movie wins an award for acting, given that it wins at least one award.

Problem 11 (Conditional Probability)
The probability that a famous runner will enter the Boston marathon is 0.60. If he enters, the probability that last year's winner will win again is 0.18. If the famous runner doesn't enter, the probability that last year's winner will win again is 0.66. What is the probability that last year's winner will win again?

Problem 12 (Conditional Probability)
In a state where cars have to be tested for the emission of pollutants, 25% of all cars emit excessive amounts of pollutants. When tested, 99% of all cars that emit excessive amounts of pollutants will fail, but 17% of the cars that do not emit ex- cessive amounts of pollutants will also fail. What is the probability that a car that fails the test actually emits excessive amounts of pollutants?

Problem 13 (Discrete Random Variables)
A random variable X has the following probability distribution:

k	0	1	2	3	4	5	6
$p(k)$	0.1	0.25	0.15	?	0.1	0.15	0.05

(a) Find $p(3)$.

(b) How do you know X is a discrete random variable?

(c) Find μ_X, σ_X^2, and σ_X.

(d) Find the probability that $X \geq 2$.

Problem 14 (Discrete Random Variables)
You toss three fair coins. Whatever the number of heads x, I will pay you $x^2 - 3$ dollars. What are your expected winnings? What is the standard deviation of the winnings?

Problem 15 (Binomial Random Variables)
A fair coin is tossed 10 times. Let X denote the number of heads.

(a) Is this a binomial experiment? Check the conditions.

(b) What is the distribution of X?

(c) Find $E[X]$, $Var[X]$ and $SD[X]$.

(d) What is the probability that there are exactly two heads?

(e) What is the probability that there are more than two heads?

Problem 16 (Binomial Random Variables)
Suppose that 10% of the fields in a given area are infested with whitefly. Twenty fields in the area are randomly selected and checked for whitefly. Let X denote the number of fields (of the 20) found to contain whitefly. Assume the field infestations are independent of each other.

(a) What is the distribution of X?

(b) Find $E[X]$, $Var[X]$ and $SD[X]$.

(c) Find $P[X > 3]$.

(d) Suppose $X = 10$. What might you conclude about the whitefly infestation?

(e) Is it realistic to assume that the field infestations are independent?

Textbook Problems

Lecture 6: 3.36, 3.40, 3.44, 3.46
Lecture 7: 3.66, 3.68, 3.78, 3.80
Lecture 8: 4.2, 4.4, 4.20, 4.22, 4.24
Lecture 9: 4.44, 4.46, 4.47