STA13: Elementary Statistics

Lecture 17
Book sections 6.1 - 6.6

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Elements of Hypotheses Tests

These are the main elements involved in hypothesis tests.

It's a good idea to approach problems with this list in mind.

- Significance Level
- Parameter of Interest
- Two Hypotheses
- Test Statistic
- P-value
- Conclusion

Outline of the Process

- Figure out the parameter of interest
- State the hypotheses
- Obtain the test statistic
  - Obtain point estimate of parameter
  - Standardize it according to H_0
  - $\dfrac{\text{(point est.) - (mean of point est. under } H_0\text{)}}{\text{standard error of point est.}}$
- Compute the p-value
- State the conclusion:
  - Reject H_0 if p-value $< \alpha$
  - Fail to reject H_0 if p-value $> \alpha$

Example 1

A sample of size $n = 50$ produces $\bar{x} = 21$ and $s = 1.5$.
Is there reason to believe that the population mean is actually greater than 20?
Test at the $\alpha = 0.05$ significance level.

- Parameter of Interest
- Hypotheses
- Test Statistic
- Rejection Region
- P-value
- Conclusion

Example 1

A sample of size $n = 50$ produces $\bar{x} = 21$ and $s = 1.5$. Is there reason to believe that the population mean is actually greater than 20?
Test at the $\alpha = 0.05$ significance level.

- Parameter of Interest: Mean μ
- Hypotheses $H_0 : \mu = 20$
  $H_A : \mu > 20$
- Test Statistic $z = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{21-20}{1.5/\sqrt{50}} = 4.71$
- Rejection Region Reject H_0 if $z > z_\alpha = 1.645$
- P-value $P(Z > 4.71) \approx 0$
- Conclusion Reject H_0
  - test statistic is in the rejection region
  - p-value $< \alpha$

Example 2

I want to find out if a coin is fair ($p = P(heads) = 0.5$).
I flip the coin 100 times and it comes up heads 42 times.
Is there enough evidence to conclude the coin is biased?
Test at the $\alpha = 0.01$ significance level.

- Parameter of Interest
- Hypotheses
- Test Statistic
- Rejection Region
- P-value
- Conclusion

Example 2

I want to find out if a coin is fair ($p = P(heads) = 0.5$).
I flip the coin 100 times and it comes up heads 42 times.
Is there enough evidence to conclude the coin is biased?
Test at the $\alpha = 0.01$ significance level.

- Parameter of Interest: Proportion p
- Hypotheses $H_0 : p = 0.5$
  $H_A : p \neq 0.5$
- Test Statistic
  $z = \frac{\hat{p}-p_0}{\sqrt{\hat{p}(1-\hat{p})/n}} = \frac{0.42-0.5}{\sqrt{0.42(1-0.42)/100}} = -1.62$
- Rejection Region Reject H_0 if $|z| > z_{\alpha/2} = 2.58$
- P-value $P(Z > |-1.62|) = 2P(Z < -1.62) \approx 0.105$
- Conclusion Fail to reject H_0
  - test statistic is not in the rejection region
  - p-value $> \alpha$

P-value or Rejection Region?

We have two ways of making a conclusion.

- Check if test statistic is in the rejection region
- Check if the p-value is less than α

They are equivalent, and we will focus on the p-value.

Many journals, articles, and reports will list the p-value.

Remember: smaller p-value $=$ more evidence against H_0.

What to Do if α Isn't Given?

- We reject H_0 if the p-value is less than α.
- Sometimes the value of α isn't specified.
- Common values of α: 0.1, 0.05, 0.01.
- You may use $\alpha = 0.05$ by default.

p-value	Conclusion
p-value ≈ 0	Reject for all reasonable α
p-value < 0.01	Reject for most α
$0.01 <$ p-value < 0.1	May reject, depending on α
p-value > 0.1	Fail to reject for all reasonable α

Two Types of Errors

There are two ways we can make the wrong conclusion:

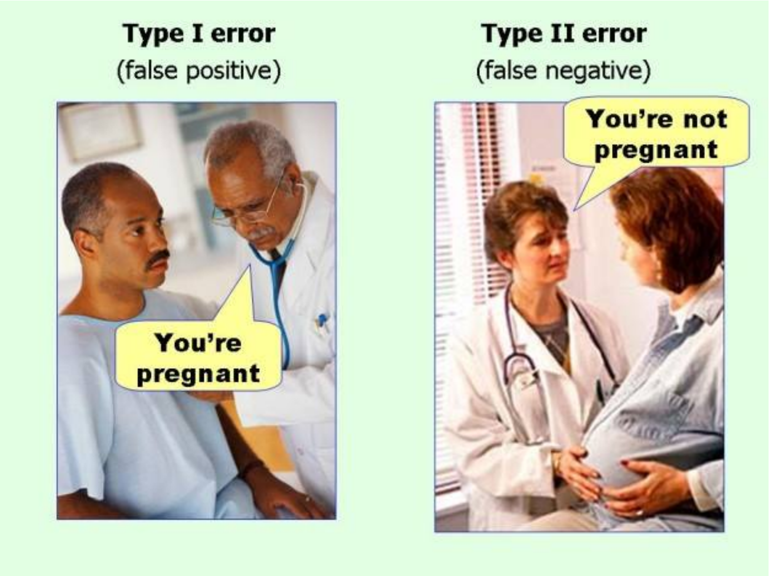
Type I Error: Reject H_0, when H_0 is actually true.

Type II Error: Fail to reject H_0 when H_0 is actually false.

	Reject H_0	Fail to Reject H_0
H_0 is True	Type I Error	Correct!
H_0 is False	Correct!	Type II Error

Two Types of Errors

H_0 : not pregnant



Type I Error

Type I Error: Reject H_0, when H_0 is actually true.

- We reject H_0 if the test-statistic is "too unlikely"
- Particularly, if the observed statistic is in the unlikeliest $(\alpha)100\%$ of possible values.
- There's always a chance that H_0 is true, and we just got an unusual sample.
- Probability of committing Type I error is always α.

  $\text{P(Type I)} = \alpha$

Type II Error

Type II Error: Fail to reject H_0 when H_0 is actually false.

- If H_0 is false, then the parameters are something other than what H_0 suggests.
- The power of a test is the probability of *not* committing Type II error.
  - Depends on the true parameter values
  - Different case by case
  - We won't compute it

Which is Worse?

Ideally we'd like to minimize the probabilities of both errors.

Unfortunately that's not possible.

Sometimes it is more important to avoid one than the other.

Example: A man is on trial for murder.

- H_0: The man is innocent.
- Type I error: convict an innocent man.
- Type II error: let a murderer walk free.

Which is worse???

Connection to Confidence Intervals

Mathematically, confidence intervals (CIs) and hypothesis tests are very similar.

The following are equivalent:

- $H_0 : \mu = 20$ is rejected in a two-tailed test using α.
- The 100(1-α)% CI for μ does not contain 20.

The following are equivalent:

- $H_0 : p = 0.2$ is rejected in a two-tailed test using α.
- The 100(1-α)% CI for p does not contain 0.2.

Keep in mind that α has to be the same in both cases.