STA13: Elementary Statistics

Lecture 8

Book Sections 4.1-4.2

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Random Variable

A random variable is a mapping from the sample space S to the real numbers.

$$\text{Experiment} \rightarrow \underbrace{\text{Outcome}}_{\text{in } S} \rightarrow \underbrace{\text{Random Variable } X}_{\text{on the number line}}$$

- A number value is assigned to each simple event in S.
- Value of the random variable depends on the outcome.
- Since the outcomes are random, so are the values.

Random Variable

$$\text{Experiment} \rightarrow \underbrace{\text{Outcome}}_{\text{in } S} \rightarrow \underbrace{\text{Random Variable } X}_{\text{on the number line}}$$

Experiment: Toss a fair die once, and record the value.

Outcome:	1	2	3	4	5	6
X:	1	2	3	4	5	6
Y:	10	20	30	40	50	60
Z:	1	0	1	0	1	0

X, Y and Z are all valid random variables.

Types of Random Variables

A discrete random variable has finitely or countably many possible values.

A continuous random variable has uncountably many possible values (a continuous range of values).

Types of Random Variables

Discrete random variables:

- Number of heads in 10 tosses of a coin $(0, 1, 2, \cdots, 10)$.
- Number of bikers I see on the way home $(0, 1, 2, \cdots)$.

Continuous random variables:

- Time (in sec) for a chimpanzee to solve a Rubik's cube.
- Height of a randomly selected person.

Probability Distribution

- An experiment produces different outcomes.
  Some are more likely than others.
- A random variable takes on different values.
  Some are more likely than others.
- It's useful to know what values a random variable takes, and how likely these values are.

Probability Distribution of a Discrete R.V.

The probability distribution of a r.v. is a way of describing what values the r.v. takes, and how likely those values are.

It may be expressed as either a table of values and probabilities, or a function.

Probability Distribution of a Discrete R.V.

If X is a discrete r.v., then we can express the probability of each possible value.

The probability that X equals k is $p(k)$, or $P(X = k)$.

Here $\{X = k\}$ is the event composed of all the simple events in S, for which X takes the value k.

Example

You roll a fair six-sided die. Whatever number x comes up, I will pay you $X = x^2 - 5$ dollars. Find the distribution of X.

Distribution of X:

k	-4	-1	4	11	20	31
$p(k)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Mean and Variance of a Discrete R.V.

Suppose X is a discrete random variable with probability distribution $p(x)$.

- Mean (Expected value): $\mu_X = E[X] = \sum kp(k)$
- Variance: $\sigma_X^2 = Var[X] = \sum (k - \mu)^2 p(k)$
- Standard deviation: $\sigma_X = SD[X] = \sqrt{\sigma_X^2}$

In each case, the summation is over all possible values of X.

Example

You roll a fair six-sided die. Whatever number x comes up, I will pay you $X = x^2 - 5$ dollars.

k	-4	-1	4	11	20	31
$p(k)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Mean:
$(-4)\frac{1}{6} + (-1)\frac{1}{6} + (4)\frac{1}{6} + (11)\frac{1}{6} + (20)\frac{1}{6} + (31)\frac{1}{6} = 10.17$

Variance:
$[(-4 - 10.17)^2 \times \frac{1}{6}] + [(-1 - 10.17)^2 \times \frac{1}{6}] +$
$[(4 - 10.17)^2 \times \frac{1}{6}] + [(11 - 10.17)^2 \times \frac{1}{6}] +$
$[(20 - 10.17)^2 \times \frac{1}{6}] + [(31 - 10.17)^2 \times \frac{1}{6}] = 178.9667$

SD: $\sqrt{178.9667} = 13.378$

Probability Calculation

We'll deal with questions like

- What is $P[X = 3]$?
- What is $P[X < 3]$?
- What is $P[X \geq 3]$?

How can this be done for a discrete random variable?

Example

X has the following distribution:

k	-4	-2	0	1	2
$p(k)$	0.1	0.2	0.3	0.2	0.2

- What is the mean of X?
- What is the variance of X?
- What is $P(X = 4)$?
- What is $P(X \leq 2)$?

Example

X has the following distribution:

k	-4	-2	0	1	2
$p(k)$	0.1	0.2	0.3	0.2	0.2

- What is the mean of X?
  $-4(0.1) + -2(0.2) + 0(0.3) + 1(0.2) + 2(.2) = -0.2$
- What is the variance of X?

k	-4	-2	0	1	2
$(k-(-0.2))^2$	14.44	3.24	0.04	0.64	3.24

  $14.44(0.1)+3.24(0.2)+0.04(0.3)+0.64(0.2)+3.24(0.2)$
- What is $P(X = 0)$? 0.3
- What is $P(X \leq 0)$? $0.1 + 0.2 + 0.3 = 0.6$