Homework 5

Problem 1 (Sampling distribution of $\bar{x}$)
Random samples of size n are taken from a population with mean μ and standard deviation σ. Describe the distribution of the sample mean in each case. Be as specific as possible.

(a) $n = 35$, $\mu = 18$, $\sigma = 3$

(b) $n = 100$, $\mu = 0.1$, $\sigma = 0.04$

(c) $n = 9$, $\mu = 51$, $\sigma = 13$

Problem 2 (Sampling distribution of $\hat{p}$)
Random samples of size n are taken from a binomial population with p. Describe the distribution of the sample proportion $\hat{p}$ in each case. Be as specific as possible.

(a) $n = 35$, $p = 0.3$

(b) $n = 100$, $p = 0.45$

(c) $n = 9$, $p = 0.78$

Problem 3 (Sampling distribution of $\bar{x}$)
A random sample of size $n = 60$ was selected from a population with mean 106 and s.d. 12.

(a) Describe the distribution of the sample mean $\bar{x}$.

(b) What is the probability that $\bar{x}$ exceeds 110?

(c) What is the probability that $\bar{x}$ deviates from 106 by no more than 4?

Problem 4 (Sampling distribution of the sample sum)
The time I spend waiting for the bus on any given day has a distribution with mean 4 minutes and variance off 0.5 minutes. What is the probability that I spend more than 2 hours and 10 minutes waiting for the bus in one month (30 days)? You may assume that waiting times on different days are independent of each other.
HINT: Is there a sum of random variables somewhere in here?

Problem 5 (Sampling distribution of $\hat{p}$)
Suppose 66% of adults in the US agree that the costs of prescription drugs are unreasonably high. A sample of 1000 US adults is randomly chosen. Let $\hat{p}$ denote the proportion of adults in the sample who say that prescription drugs are too expensive.

(a) Describe the sampling distribution of $\hat{p}$.

(b) What is the probability that $\hat{p}$ exceeds 0.68?

(c) What is the probability that $\hat{p}$ is between 0.64 and 0.68?

Problem 6 (Confidence interval for μ - large sample)
In each of the following scenarios estimate

(i) the standard error of the sample mean $\bar{x}$,

(ii) the width of a 95% confidence interval for the population mean μ.

(a) $n = 100$, $\bar{x} = 12$, $s^2 = 1$

(b) $n = 75$, $\bar{x} = 31$, $s^2 = 5$

Problem 7 (Confidence interval for μ - large sample)
Find a $(1 - \alpha)100\%$ confidence interval for the population mean μ in each case.

(a) $\alpha = 0.10$, $n = 60$, $\bar{x} = 2$, $s^2 = 0.16$

(b) $\alpha = 0.01$, $n = 32$, $\bar{x} = 151$, $s^2 = 13$

(c) $\alpha = 0.05$, $n = 77$, $\bar{x} = 14$, $s^2 = 9$

Problem 8 (Confidence interval for p)
Find a $(1 - \alpha)100\%$ confidence interval for the population proportion p in each case.

(a) $\alpha = 0.10$, $n = 100$, $\hat{p} = 0.43$

(b) $\alpha = 0.01$, $n = 64$, $\hat{p} = 0.19$

(c) $\alpha = 0.05$, $n = 100$, $\hat{p} = 0.88$

Problem 9 (Confidence interval for μ - small sample)
A random sample of $n = 10$ observations from a population produced a sample mean $\bar{x} = 26.8$ and a sample standard deviation $s = 6.5$. Construct a 95% confidence interval for the population mean (μ). Interpret your interval.

Problem 10 (Confidence interval for μ - small sample)
The weights in pounds of weekly garbage for 25 households is:

5.50, 13.2, 15.5, 16.5, 18, 7.5, 4.0, 3.3, 9.0, 17.6, 12.5, 2.5, 7.5, 8.5, 4.9,
10.0, 14.0, 10.0, 16.6, 2.9, 15.3, 13.3, 6.6, 9.7, 13.4

Use this data construct a 99% confidence interval for the population mean of weekly garbage for all households in the city. In your computations use the assumption that the population is approximately normal.

Problem 11 (Confidence interval for p)
A random sample of 300 shoppers at a large supermarket includes 234 who regularly use discount coupons. Construct a 95% confidence interval for the proportion of all shoppers at that supermarket who regularly use coupons. Interpret your interval.

Textbook Problems

Lecture 12: 4.138, 4.146, 4.150, 4.154
Lectures 13, 14: 5.4, 5.6, 5.10, 5.12, 5.28, 5.30, 5.34
Lecture 15: 5.48, 5.52, 5.54