STA13: Elementary Statistics

Lecture 9
Book Sections 4.3

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Binomial Setting

A binomial experiment meets the following conditions:

- n identical trials
- the trials are independent
- each trial has two outcomes (*success* and *failure*)
- the probability of success (p) is the same in each trial

Binomial Random Variable

If X is the number of successes in a binomial experiment, then we say X...

- is a binomial random variable with parameters n and p
- has the Binomial(n, p) distribution

Binomial Distribution

X is a random variable with the Binomial(n, p) distribution.

- X takes on values $0, 1, 2, \cdots, n$
- For each value $k = 0, 1, 2, \cdots, n$:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

- $P(X = k) = 0$ for all other k.

Recall: $\binom{n}{k} = {}_nC_k = \frac{n!}{(n-k)!k!}$

Binomial Mean and Variance

Suppose X is a random variable with the Binomial(n, p) distribution.

- Mean (Expected value): np
- Variance: $np(1-p)$
- Standard deviation: $\sqrt{np(1-p)}$

Example - Coins

Let X be the number of heads in 10 tosses of a fair coin.

- What is the distribution of X?
- What is the mean of X?
- What is the standard deviation of X?
- What is $P(X = 4)$?
- What is $P(X \leq 2)$?

Example - Coins

Let X be the number of heads in 10 tosses of a fair coin.

- What is the distribution of X? Binomial(10,0.5)
- What is the mean of X? $np = 10(0.5) = 5$
- What is the s.d. of X? $\sqrt{np(1-p)} = \sqrt{2.5}$
- What is $P(X = 4)$?

  $\binom{n}{k} p^k (1-p)^{n-k} = \binom{10}{4} 0.5^4 0.5^6 \approx 0.205$
- What is $P(X \leq 2)$?

  $P(X = 0) + P(X = 1) + P(X = 2)$

Example - Smokers

Suppose in some city 15% of the adult population are tobacco smokers.

Choose 20 adults from that city at random.

X is the number of smokers in the group of 20.

The distribution of X is Binomial(20, 0.15).

Poisson Distribution

The Poisson Distribution is used to model the number of occurrences of a rare event in a fixed period of time or space.

For example:

- Number of typos in one page of a book.
- Number of earthquakes in CA in one year.

The Poisson distribution has one parameter $\lambda > 0$ that controls how rare the event is.

Poisson Distribution

A Poisson(λ) random variable X takes on values $0, 1, 2, \cdots$ with probabilities:

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Poisson Mean and Variance

Suppose X is a Poisson(λ) random variable.

- Mean (Expected value): λ
- Variance: λ
- Standard deviation: $\sqrt{\lambda}$

Note: the mean and variance are equal.

Example - Earthquakes

Suppose that the average number of major earthquakes that happen on an island in one year is 0.3.
Find the probability that...

- two major earthquakes happen in a year
- no major earthquakes happen in a year
- no major earthquakes happen in 4 years

Example - Earthquakes

Suppose that the average number of major earthquakes that happen on an island in one year is 0.3.

X has the Poisson(0.3) distribution

Find the probability that...

- two major earthquakes happen in a year

  $\frac{\lambda^k e^{-\lambda}}{k!} = \frac{0.3^2 e^{-0.3}}{2!} = 0.033$
- no major earthquakes happen in a year

  $\frac{\lambda^k e^{-\lambda}}{k!} = \frac{0.3^0 e^{-0.3}}{0!} = 0.741$
- no major earthquakes happen in 4 years

  $(0.741)^4 = 0.301$