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Waveguide filter synthesis using mode-matching method

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Abstract—In this paper, a mode matching method is discussed in application to waveguide filter synthesis task. It is customized for analysis of particular waveguide filter with thick symmetric irises. Equivalent circuit model of thick iris discontinuities is also proposed. Parameters of the equivalent circuit are evaluated using mode-matching method. Using this model, an example filter for Ka-band has been designed. The performance of the filter has then been validated by the proposed mode-matching method and finite-elements method.

1. INTRODUCTION

Filters are essential parts of modern high-frequency circuits. Various filter types are used for different frequency bands, including lumped-element networks for sub-GHz applications, while higher frequencies require use of distributed elements such as microstrip or waveguide resonators. Commonly, bandpass filters consist of resonators coupled with each other. Several connections patterns are known, including direct and indirect neighbours coupling. Indirect-coupled filters may be realized as folded structures with couplings between non-adjacent resonators or by using multi-mode resonators. They allow more control over transfer function for the cost of increased complexity and cost of fabrication. In this article, a narrowband direct-coupled waveguide filter for Ka-band is considered. Coupling is performed by irises symmetric around yz-plane (Figure 1).

There is a well-known theory of generic direct-coupled filters. A complete set of equations for filter synthesis can be found in [1]. In this paper, a waveguide iris is represented by an equivalent shunt inductance. It can be evaluated from geometric parameters of the iris using relations from [2].

However, this approximation works well only for the case of thin irises. A practical Ka-band (wavelength $\lambda \approx 8$ mm) filter fabricated by milling contains "thick" irises ($h \approx 2$ mm, so that h/λ is not negligible anymore). So, we aim at building equivalent circuit model for this type of discontinuity. But at first let us consider direct task of filter analysis.

2. WAVEGUIDE FILTER ANALYSIS USING MODE-MATCHING METHOD

Conventional waveguide filter is a highly resonant structure with large Q factor, so general purpose methods such as FEM, MoM or FDTD converge slowly. Mode-matching method (MMM) has relaxed demands for memory and computation time and is a preferred one for waveguide-domain calculations. Of course, it is not so universal as mesh methods as it requires determination of complete mode set. This imposes no problem for the case of rectangular waveguide of interest as its modes are known in analytical form.

A discussed waveguide filter is depicted on Figure 1. It consists of rectangular waveguide sections connected with each other symmetrically around zy-plane. Note that irises are also described as

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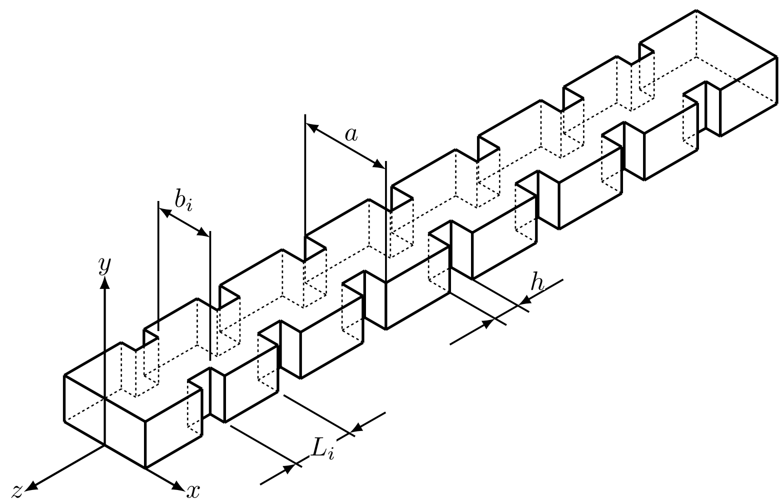

Figure 1. Direct-coupled waveguide filter with symmetric irises

waveguide sections. The only difference is the absence of propagating modes in them. Filter is excited by a fundamental mode TE_{01}, so several mode types are not excited throughout the structure:

- all TM waves
- $TE_{n,m}, n \neq 0$ waves
- $TE_{0,2m}$ waves (anti-symmetric around yz-plane while excitation TE_{01} is symmetric).

Remaining are $TE_{0,2m}$ waves only. Their normalized E_y components are represented by the following relation:

$$E_y[TE_{0,2m}] = F^a_{2m}(x) = \sqrt{\frac{2}{a}} \cos \frac{2\pi x m}{a}, \tag{1}$$

where a is the waveguide width.

Overlapping matrix used to calculate reflection between adjacent sections of widths a and b is for the case $a \leq b$:

$$V_{mn} = \int\limits_{-a/2}^{a/2} F^a_m(x) F^b_n(x) dx = \begin{cases} \dfrac{4}{\pi} \dfrac{a^{-3/2} b^{-1/2} m (-1)^{\dfrac{m+1}{2}}}{(m/a)^2 - (n/b)^2} \cos\left(\dfrac{\pi}{2} \dfrac{a}{b} n\right) & \text{if } \dfrac{m}{a} \neq \pm \dfrac{n}{b} \\ \sqrt{\dfrac{a}{b}} \quad \text{otherwise} \end{cases} \tag{2}$$

Let $\mathbf{Q} = V_{mn}$, $\mathbf{P} = \gamma^a_n / \gamma^b_m V_{nm}$, where γ^a_n are the propagation coefficients for mode $TE_{0,2n+1}$ in a waveguide of width a, then generalized S-parameters matrix of the waveguide junction is (according to [3]):

$$\begin{aligned} \mathbf{S}_{11} &= (\mathbf{I} + \mathbf{QP})^{-1} (\mathbf{I} + \mathbf{QP}) \\ \mathbf{S}_{12} &= 2 (\mathbf{I} + \mathbf{QP})^{-1} \mathbf{Q} \\ \mathbf{S}_{21} &= 2 (\mathbf{I} + \mathbf{PQ})^{-1} \mathbf{P} \\ \mathbf{S}_{22} &= (\mathbf{I} + \mathbf{PQ})^{-1} (\mathbf{I} - \mathbf{PQ}) \end{aligned} \tag{3}$$

Here $\mathbf{I}$ are identity matrices of matching dimensions.

An example solution for the case of connected waveguides of widths $a = 1$ and $b = \sqrt{2}$ is shown on Figure 2. One can notice the quasi-diagonal structure of the matrix, i. e. energy is efficiently transferred

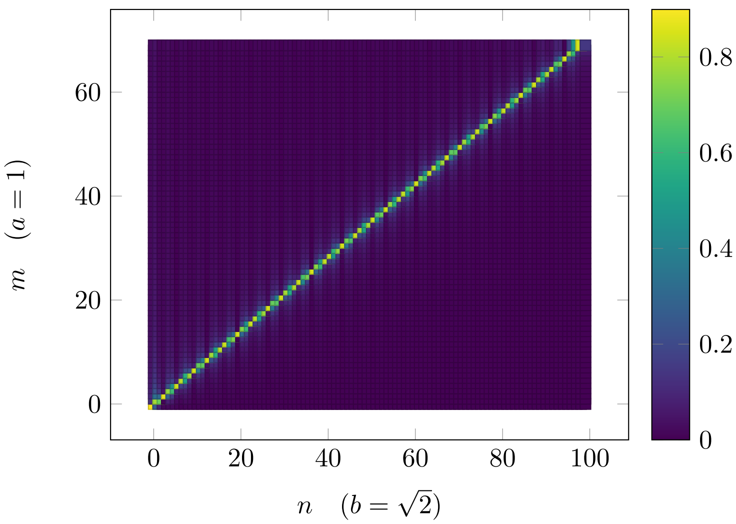

Figure 2. Transmission coefficient matrix S_{12} for a waveguide junction. Waveguide widths are $a = 1$, $b = \sqrt{2}$ (in arbitrary units). Color represents the magnitude of S_{12} matrix elements.

between modes numbers of which follow the simple equation $m/n = a/b$. This results from a specific form of overlapping matrix in Equation. (2). Its values are close to $\sqrt{a/b}$ on quasi-diagonal indices and decrease rapidly on off-diagonal elements. If V_{mn} matrix axial ratio is unbalanced (i. e. other than a/b), then modes that do not have quasi-diagonal counterparts undergo non-physical reflection. That's why increasing number of accounted modes in one section only does not really improve calculation precision. So, calculations described herein have been carried out using matrices with balanced axial ratio.

Generalized S-matrix of a waveguide is as simple as

$$\mathbf{S}_{11} = \mathbf{S}_{22} = 0, \quad \mathbf{S}_{12} = \mathbf{S}_{21} = \begin{bmatrix} e^{-\gamma_1} & 0 & \dots & 0 \\ 0 & e^{-\gamma_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{-\gamma_N} \end{bmatrix} \tag{4}$$

where γ_n is still a propagation coefficient (real positive for evanescent waves).

By knowing numeric values for generalized S-matrices for all the elements of the waveguide filter, one can combine them to produce generalized S-matrix of the whole filter. If $\mathbf{A}, \mathbf{B}$ are the generalized S matrices, then the generalized S-matrix of the cascaded network $\mathbf{A} \circ \mathbf{B}$ is

$$\begin{aligned} \mathbf{S}_{11} &= \mathbf{A}_{11} + \mathbf{A}_{12} \left(\mathbf{I} - \mathbf{B}_{11}\mathbf{A}_{22}\right)^{-1} \mathbf{B}_{11}\mathbf{A}_{21} \\ \mathbf{S}_{12} &= \mathbf{A}_{12} \left(\mathbf{I} - \mathbf{B}_{11}\mathbf{A}_{22}\right)^{-1} \mathbf{B}_{12} \\ \mathbf{S}_{21} &= \mathbf{B}_{21} \left(\mathbf{I} - \mathbf{A}_{22}\mathbf{B}_{11}\right)^{-1} \mathbf{A}_{21} \\ \mathbf{S}_{22} &= \mathbf{B}_{22} + \mathbf{B}_{21} \left(\mathbf{I} - \mathbf{A}_{22}\mathbf{B}_{11}\right)^{-1} \mathbf{B}_{12} \end{aligned} \tag{5}$$

Using equation (5) is not the only way to find cascaded network parameters. One could convert scattering parameters S to transmission parameters T as described in [4]. Cascading is much simpler in terms of T-matrices. Resultant T-matrix is a matrix product of T-matrices of the cascaded network elements. However, conversion between S-matrix and T-matrix forms is not that straightforward:

$$\begin{aligned} \mathbf{T}_{11} &= \mathbf{S}_{12} - \mathbf{S}_{11}\mathbf{S}_{21}^{-1}\mathbf{S}_{22} \\ \mathbf{T}_{12} &= \mathbf{S}_{11}\mathbf{S}_{21}^{-1} \\ \mathbf{T}_{21} &= -\mathbf{S}_{21}^{-1}\mathbf{S}_{22} \\ \mathbf{T}_{22} &= \mathbf{S}_{21}^{-1} \end{aligned} \tag{6}$$

As pointed out in [4], $\mathbf{S}_{21}$ block of generalized $\mathbf{S}$-matrix is rectangular, so its inverse $\mathbf{S}_{21}^{-1}$ is ambiguous and conversion between S- and T-matrices results in loss of information. By contrast, equation (5) contains inverse values of square matrices only.

Equations (2) to (5) fully define generalized S-matrix for the whole waveguide filter thus accomplishing filter analysis task.

3. FILTER SYNTHESIS USING EQUIVALENT CIRCUIT

Iris discontinuities in the discussed filter are modeled by shunt inductances shown on Figure 3a. Shunt inductance can be converted to impedance inverter using equations from [5] thus yielding more general equivalent circuit shown on Figure 3b. However, thin iris model does not work well for Ka band (wavelength $\lambda \approx 8\ mm$) with typical iris thickness $h \approx 2\ mm$.

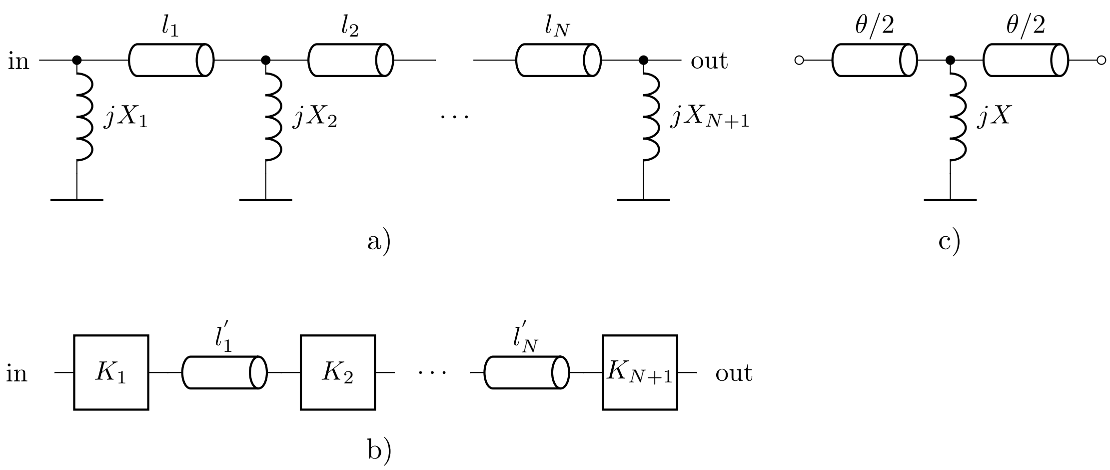

Figure 3. Waveguide filter equivalent circuit. a) N-section direct-coupled filter equivalent circuit, b) N-section filter equivalent circuit using impedance inverters, c) single thick iris equivalent circuit (also used as single inverter equivalent circuit).

Non-zero iris thickness results in additional phase shift which can be modeled by attaching pieces of transmission lines to shunt inductance (Figure 3c). This is a symmetric T-network built of a shunt inductor of impedance X and two transmission lines (electrical lengths $\theta/2$, wave impedance Z_0). Transmission matrix is then

$$\begin{aligned} \begin{bmatrix} A & B \\ C & D \end{bmatrix} &= \begin{bmatrix} \cos\theta/2 & jZ_0\sin\theta/2 \\ j/Z_0\sin\theta/2 & \cos\theta/2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/X & 1 \end{bmatrix} \begin{bmatrix} \cos\theta/2 & jZ_0\sin\theta/2 \\ j/Z_0\sin\theta/2 & \cos\theta/2 \end{bmatrix} \\ &= \begin{bmatrix} \dfrac{Z_0}{2X}\sin\theta + \cos\theta & jZ_0\left(\sin\theta + \dfrac{Z_0}{2X}(1-\cos\theta)\right) \\ \dfrac{j}{Z_0}\left(\sin\theta - \dfrac{Z_0}{2X}(1+\cos\theta)\right) & \dfrac{Z_0}{2X}\sin\theta + \cos\theta \end{bmatrix} \end{aligned} \tag{7}$$

Converting ABCD-parameters to S-matrix, one determines:

$$
\begin{aligned}
S_{11} = S_{22} &= -\frac{e^{-j\theta}}{1+2jX/Z_0} \\
S_{12} = S_{21} &= \frac{2jX}{Z_0}\frac{e^{-j\theta}}{1+2jX/Z_0}
\end{aligned}
\tag{8}
$$

So, equivalent circuit parameters may be calculated using S-parameters:

$$
\frac{X}{Z_0} = \frac{1}{2}\sqrt{\frac{1}{|S_{11}|^2} - 1}, \qquad \theta = -\arccos|S_{11}| - \arg(-S_{11}) \tag{9}
$$

S-parameters, in turn, depend on geometry of the particular iris and are evaluated numerically using mode-matching method described in the previous section. Results of these calculations are shown on Figure 4 for iris thickness $h = 2\ mm$.

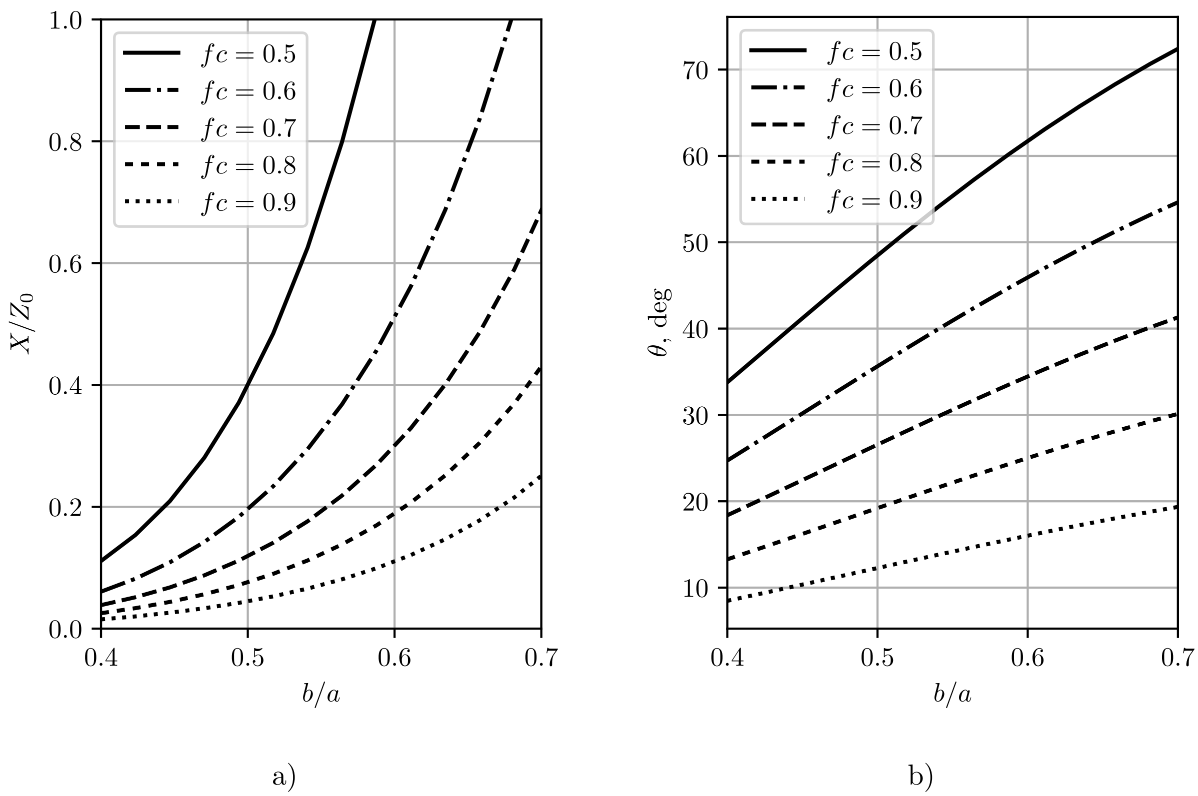

Figure 4. Equivalent circuit parameters for iris thickness $h = 2\ mm$ (waveguide width $a = 7.112\ mm$). Parameter $fc = F_{cutoff}/F_{center}$ is the normalized cutoff frequency. a) equivalent impedance X/Z_0, b) equivalent transmission line length θ (cf. Figure 3).

Impedance inverter's equivalent circuit is similar to that of thick iris and depicted on Figure 3c. Conversion between these representations is performed by equations from [5]:

$$
\frac{X_i}{Z_0} = \frac{K_i/Z_0}{(K_i/Z_0)^2 - 1}, \quad \theta_i = -2\tan^{-1}\left(\frac{K_i}{Z_0}\right) \tag{10}
$$

Note that $\theta_i < 0$, i. e. it is subtracted from adjacent half-wave resonators.

Filter network shown on Figure 3b is similar to a well-known problem of a capacitively-coupled microstrip filter. The design procedure described in [5] is as follows:

i	X_i/Z_0, mm	l_i, deg
1	0.424	152.9
2	0.125	167.7
3	0.093	169.6
4	0.090	169.6
5	0.093	167.7
6	0.125	152.9
7	0.424	

a)

i	b_i, mm	L_i, mm
1	4.56	4.68
2	3.59	5.50
3	3.39	5.63
4	3.36	5.63
5	3.39	5.50
6	3.59	4.68
7	4.56	

b)

Table 1. Proposed filter parameters: a) – equivalent circuit parameters, b) – filter dimensions.

- choose low-pass filter prototype (maximally flat, equal ripple, etc.)
- determine selected filter's prototype values $g_0, g_1, \ldots, g_{N+1}$ (cases for maximally-flat and equal-ripple filters are considered in [2])
- determine transformation coefficients:

$$\begin{aligned} \frac{K_1}{Z_0} &= \sqrt{\frac{2}{\pi}\frac{g_0 g_1}{\Delta}}, \\ &\ldots \\ \frac{K_i}{Z_0} &= \frac{2}{\pi\Delta}\sqrt{g_i g_{i+1}} \\ &\ldots \\ \frac{K_N}{Z_0} &= \sqrt{\frac{2}{\pi}\frac{g_N g_{N+1}}{\Delta}} \end{aligned} \tag{11}$$

where $\Delta = (F_{max} - F_{min})/F_{center}$ – fractional bandwidth of the filter

- determine line lengths $l'_i = \Lambda_g/2$, where $\Lambda_g = \lambda/\sqrt{1-(F_{cutoff}/F_{center})^2}$ is the wavelength in waveguide at center frequency.

Given K_i and l'_i, one can determine X_i/Z_0 and l_i using equation (10). A final step is to use mode-matching model together with equation (9) and Figure 4 to evaluate irises' widths b_i and waveguide section lengths L_i.

The procedure described herein has been used to synthesize Ka-band filter for a standard WR28 flange (width $a = 7.112\ mm$) for the following set of parameters:

- frequency band $F_{center} = 30.1\ GHz \pm 5.1\%$
- Tchebyshev filter, order $n = 6$, ripple 0.1 dB
- cutoff frequency for fundamental mode of WR28 waveguide is $F_{cutoff} = 0.7 F_{center}$

Equivalent circuit parameters for this filter are shown in Table 1a. Using these data, filter dimensions have been evaluated and are shown in Table 1b. S-parameters of the designed filter has been calculated using 3 different methods: equivalent circuit model, mode matching method and FEM with the help of commercial software (HFSS). Results are presented on Figure 5.

FEM and MMM yield almost the same result with slight differences in the pass-band that can be accounted for by finite precision of the numerical methods. However, computation time for mode matching is reduced drastically compared to the FEM: HFSS v15 calculations took 1 hour 36 minutes on desktop with Intel Core i7-2600, while the same results have been obtained using Python (Numpy) code implementing MMM in 49 seconds on the same hardware. Pass-band return loss is less than

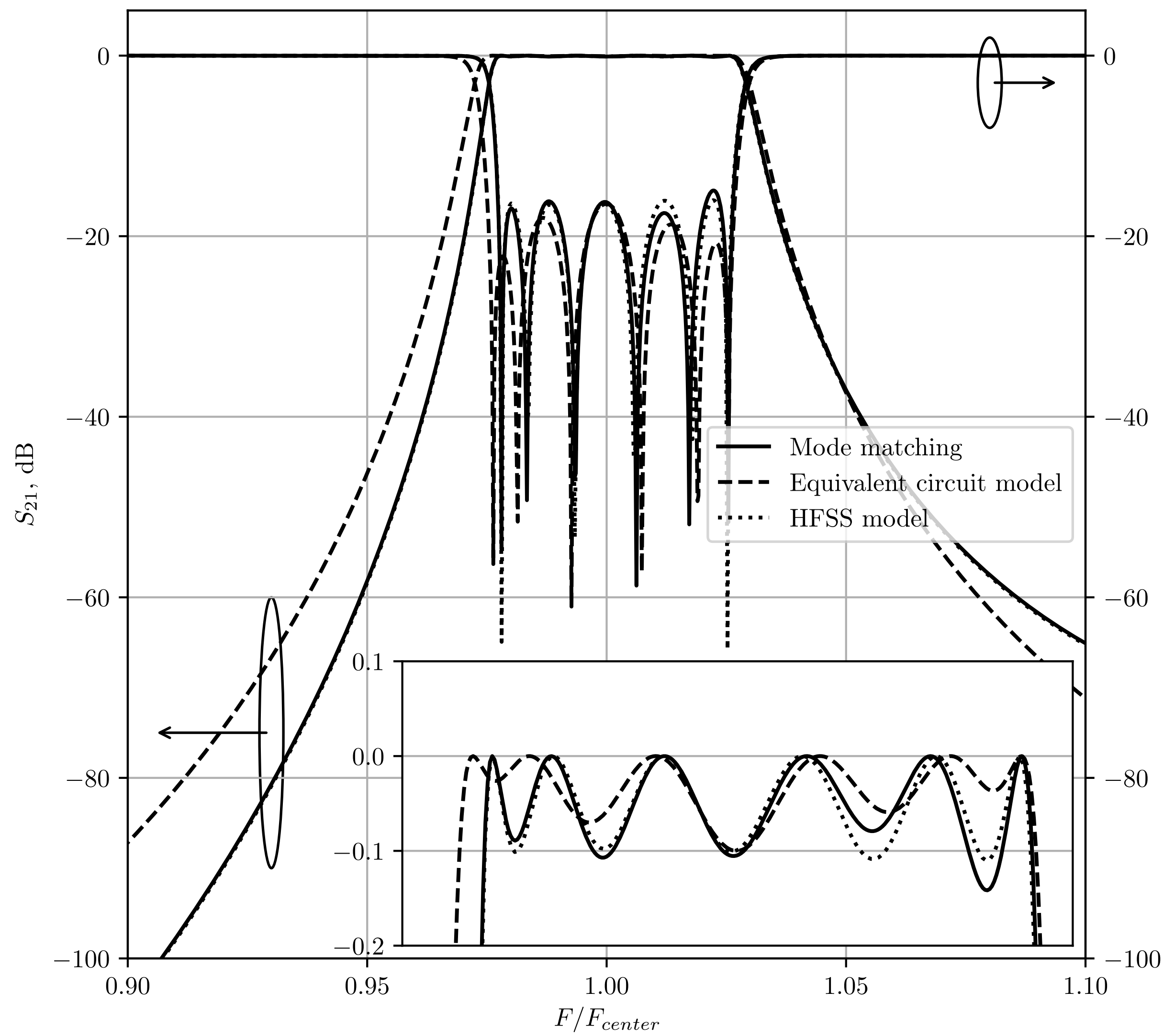

Figure 5. Filter transfer function and reflection coefficient calculated using various methods: mode matching method (solid line), equivalent circuit method (dashed line) and finite elements method (dotted line).

$-15\ dB$. As for equivalent circuit model, it is the fastest yet the least accurate. Besides, this model shows systematic discrepancies in selectivity: it is higher for $F < F_{center}$ and lower for $F > F_{center}$ than calculated using MMM of FEM.

4. CONCLUSION

In this paper, an application of mode-matching method for waveguide filter analysis and synthesis is discussed. This method has lower demands on CPU time and memory compared to conventional mesh-based techniques such as FEM.

The proposed thick iris model allowed applying conventional synthesis procedure for direct-coupled waveguide filter with thick irises. A band-pass filter has been designed using this procedure. Its center frequency is 30.1 GHz, bandwidth is 5%. The designed filter is quite compact (overall length $L = 45.62\ mm$, from Table 1b) and can be used in high-speed satellite transmission lines for separating up- and down-link signals for duplex operation. Simulations using various computational methods are consistent with each other.

HFSS models and developed Python scripts are available on `https://github.com/dmitrodem/pier_filter`.

REFERENCES

1. Cohn, S., "Direct-coupled-resonator filters," *Proceedings of the IRE*, Vol. 45, No. 2, 187–196, 1957.
2. Matthaei, G., Jones, E., and Young, L., *Microwave Filters, Impedance-Matching Networks, and Coupling Structures (Artech Microwave Library).* Artech House, 1980.
3. Conciauro, G., Guglielmi, M., and Sorrentino, R., *Advanced Modal Analysis.* Wiley, 2000.
4. Frei, J., Xiao-Ding Cai, and Muller, S., "Multiport S-Parameter and T-Parameter Conversion With Symmetry Extension," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 56, No. 11, 2493–2504, nov 2008.
5. Pozar, D., *Microwave engineering.* Hoboken, NJ: Wiley, 2012.