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WAVEGUIDE FILTER SYNTHESIS USING MODE-MATCHING METHOD †

Dmitriy A. Dyomin*, [1], Nikolay P. Chubinskiy[1], Ivan V. Filatov[1] and Vasiliy V. Filatov[2]

Abstract—In this paper, a mode matching method is discussed in application of waveguide filter synthesis task. It is customized for analysis of particular waveguide filter with thick symmetric irises. Equivalent circuit model of thick iris discontinuities is also proposed. Parameters of the equivalent circuit are evaluated using mode-matching method. Using this model, an example filter for Ka-band has been designed. The performance of the filter has then been validated by the proposed mode-matching method and finite-elements method.

1. INTRODUCTION

Filters are essential parts of modern high-frequency circuits. Various filter types are used for different frequency bands, including lumped-element networks for sub-GHz applications, while higher frequencies require use of distributed elements such as microstrip or waveguide resonators. Commonly, bandpass filters consist of resonators coupled with each other. Several connections patterns are known, including direct and indirect neighbours coupling. Indirect-coupled filters may be realized as folded structures with couplings between non-adjacent resonators or by using multi-mode resonators. They allow more control over transfer function for the cost of increased complexity and cost of fabrication. In this article, an narrowband direct-coupled waveguide filter for Ka-band is considered. Coupling is performed by irises symmetric around yz-plane (Figure 1).

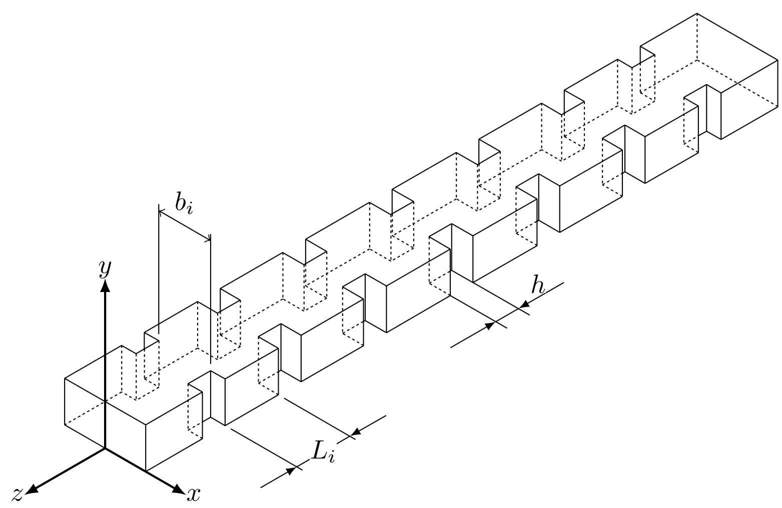

Figure 1. Direct-coupled waveguide filter with symmetric irises

There is a well-known theory of generic direct-coupled filters. A complete set of equations for filter synthesis can be found in [1]. In this paper, a waveguide iris is represented by an equivalent shunt inductance. It can be evaluated from geometric parameters of the iris using relations from [2].

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* Corresponding author: Dmitriy Dyomin (demin.da@mipt.ru).
[1] Moscow Institute of Physics and Technology.
[2] The second affiliation.

However, this approximation works well only for the case of thin irises. A practical Ka-band (wavelength $\lambda \sim 8$ mm) filter fabricated by milling contains "thick" irises ($h \sim 2$ mm, so that h/λ is not negligible anymore). So, we aim at building equivalent circuit model for this type of discontinuity. But at first let us consider direct task of filter analysis.

2. WAVEGUIDE FILTER ANALYSIS USING MODE-MATCHING METHOD

Conventional waveguide filter is a highly resonant structure with large Q factor, so general purpose methods such as FEM, MoM or FDTD converge slowly. Mode-matching method (MMM) has relaxed demands for memory and computation time and is a preferred one for waveguide-domain calculations. Of course, it is not so universal as mesh methods as it requires determination of complete mode set. This imposes no problem for the case of rectangular waveguide of interest as its modes are known in analytical form.

A discussed waveguide filter is depicted on Figure 1. It consists of rectangular waveguide sections connected with each other in a symmetrically. Note that irises are also described as waveguide sections. The only difference is the absence of propagating modes in them. Filter is excited by a fundamental mode TE_{01}, so several mode types are not excited throughout the structure:

- all TM waves
- $TE_{nm}, n \neq 0$ waves
- $TE_{0,2m}$ waves (anti-symmetric around yz-plane while excitation TE_{01} is symmetric).

Remaining are TE_{0m} (m – odd number) waves only. Their normalized E_y components are represented by the following relation:

$$F_n^a(x) = \sqrt{\frac{2}{a}} \cos \frac{\pi x n}{a}, \tag{1}$$

where a is the waveguide width.

Overlapping matrix used to calculate reflection between adjacent sections is (for the case $a \leq b$):

$$V_{mn} = \int_{-a/2}^{a/2} F_m^a(x) F_n^b(x) dx = \begin{cases} \frac{4}{\pi} \frac{a^{-3/2} b^{-1/2} m (-1)^{\frac{m+1}{2}}}{(m/a)^2 - (n/b)^2} \cos\left(\frac{\pi}{2} \frac{a}{b} n\right) & \text{if } \frac{m}{a} \neq \pm \frac{n}{b} \\ \sqrt{\frac{a}{b}} \quad \text{otherwise} \end{cases} \tag{2}$$

Let $\mathbf{Q} = V_{mn}$, $\mathbf{P} = \gamma_n^a / \gamma_m^b V_{nm}$, where γ_n^a are the propagation coefficients for mode $TE_{0,2n+1}$ in a waveguide width a, then generalized S-parameters matrix of the waveguide junction is (according to [3]):

$$\begin{aligned} \mathbf{S}_{11} &= (\mathbf{I} + \mathbf{QP})^{-1} (\mathbf{I} + \mathbf{QP}) \\ \mathbf{S}_{12} &= 2 (\mathbf{I} + \mathbf{QP})^{-1} \mathbf{Q} \\ \mathbf{S}_{21} &= 2 (\mathbf{I} + \mathbf{PQ})^{-1} \mathbf{P} \\ \mathbf{S}_{22} &= (\mathbf{I} + \mathbf{PQ})^{-1} (\mathbf{I} - \mathbf{PQ}) \end{aligned} \tag{3}$$

An example solution for the case of connected waveguides of widths $a = \sqrt{2}$ and $b = 1$ is shown on Figure 2. One can notice the quasi-diagonal structure of the matrix, i. e. energy is efficiently transferred between modes which numbers follows the simple equation $m/n = a/b$. This results from a specific form of overlapping matrix in Equation. 2. Its values are close to $\sqrt{a/b}$ on quasi-diagonal indices and decrease rapidly on off-diagonal elements. If V_{mn} matrix axial ratio unbalances (i. e. other than a/b), then modes that do not have quasi-diagonal counterparts undergo non-physical reflection. That's why increasing number of accounted modes in one section only does not really improve calculation precision. So, calculations described herein have been carried out using matrices with balanced axial ratio.

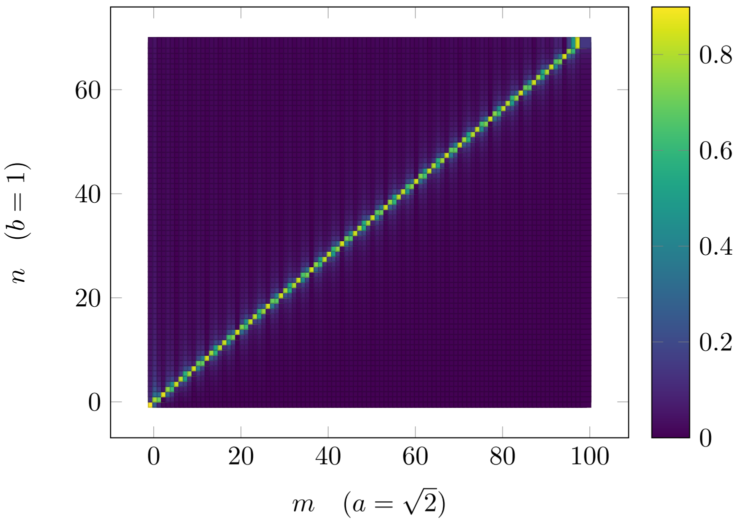

Figure 2. Transmission coefficient matrix S_{12} for a waveguide junction. Waveguide widths are $a = \sqrt{2}$, $b = 1$

Generalized S-matrix of a waveguide is as simple as

$$\mathbf{S}_{11} = \mathbf{S}_{22} = 0, \quad \mathbf{S}_{12} = \mathbf{S}_{21} = \begin{bmatrix} e^{-\gamma_1} & 0 & \dots & 0 \\ 0 & e^{-\gamma_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{-\gamma_N} \end{bmatrix} \tag{4}$$

where γ_n is still a propagation coefficient (real positive for evanescent waves).

By knowing numeric values for generalized S-matrices for all the elements of the waveguide filter, one can combine them to produce generalized S-matrix of the whole filter. If $\mathbf{A}, \mathbf{B}$ are the generalized S matrices, then the generalized S-matrix of the cascaded network $\mathbf{A} \circ \mathbf{B}$ is

$$\begin{aligned} \mathbf{S}_{11} &= \mathbf{A}_{11} + \mathbf{A}_{12} \left(\mathbf{I} - \mathbf{B}_{11}\mathbf{A}_{22}\right)^{-1} \mathbf{B}_{11}\mathbf{A}_{21} \\ \mathbf{S}_{12} &= \mathbf{A}_{12} \left(\mathbf{I} - \mathbf{B}_{11}\mathbf{A}_{22}\right)^{-1} \mathbf{B}_{12} \\ \mathbf{S}_{21} &= \mathbf{B}_{21} \left(\mathbf{I} - \mathbf{A}_{22}\mathbf{B}_{11}\right)^{-1} \mathbf{A}_{21} \\ \mathbf{S}_{22} &= \mathbf{B}_{22} + \mathbf{B}_{21} \left(\mathbf{I} - \mathbf{A}_{22}\mathbf{B}_{11}\right)^{-1} \mathbf{B}_{12} \end{aligned} \tag{5}$$

Using equation (5) is not the only way to find cascaded network parameters. One could convert scattering parameters S to transmission parameters T as described in [4]. Cascading is much simpler in terms of T-matrices. Resultant T-matrix is a matrix multiple of T-matrices of the cascaded network elements. However, conversion between S-matrix and T-matrix forms is not that straightforward:

$$\begin{aligned} \mathbf{T}_{11} &= \mathbf{S}_{12} - \mathbf{S}_{11}\mathbf{S}_{21}^{-1}\mathbf{S}_{22} \\ \mathbf{T}_{12} &= \mathbf{S}_{11}\mathbf{S}_{21}^{-1} \\ \mathbf{T}_{21} &= -\mathbf{S}_{21}^{-1}\mathbf{S}_{22} \\ \mathbf{T}_{22} &= \mathbf{S}_{21}^{-1} \end{aligned} \tag{6}$$

As pointed out in [4], $\mathbf{S}_{21}$ block of generalized $\mathbf{S}$-matrix is rectangular, so it is inverse $\mathbf{S}_{21}^{-1}$ is ambiguous and conversion between S- and T-matrices results in loss of information. By contrast, equation (5) contains inverse values of square matrices only.

Equations eqs. (2) to (5) fully define generalized S-matrix for the whole waveguide filter thus accomplishing filter analysis task.

3. FILTER SYNTHESIS USING EQUIVALENT CIRCUIT

Iris discontinuities in the discussed filter are modeled by shunt inductances shown on Figure 3a. Shunt inductance may be can be converted to impedance inverter using equations from [5] thus yielding more general equivalent circuit shown on Figure 3b. However, thin iris model does not work well for Ka band (wavelength $\lambda \approx 7\ mm$) with typical iris thickness $h \approx 2\ mm$.

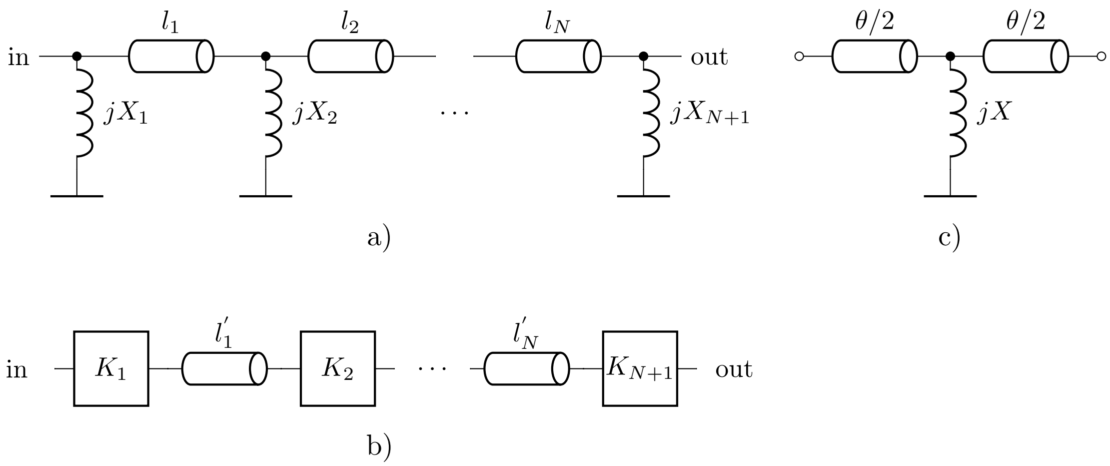

Figure 3. Waveguide filter equivalent circuit. a) – N-section direct-coupled filter equivalent circuit, b) – N-section filter equivalent circuit using impedance inverters, c) – single thick iris equivalent circuit (also used as single inverter equivalent circuit).

Non-zero iris thickness results in additional phase shift which can be modeled by attaching pieces of transmission lines to shunt inductance (Figure 3c). This is a symmetric T-network built of a shunt inductor of impedance X and two transmission lines (electrical lengths $\theta/2$, wave impedance Z_0). Transmission matrix is then

$$\begin{aligned}\begin{bmatrix} a & b \\ c & d \end{bmatrix} &= \begin{bmatrix} \cos\theta/2 & jZ_0\sin\theta/2 \\ j/Z_0\sin\theta/2 & \cos\theta/2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/X & 1 \end{bmatrix} \begin{bmatrix} \cos\theta/2 & jZ_0\sin\theta/2 \\ j/Z_0\sin\theta/2 & \cos\theta/2 \end{bmatrix} \\ &= \begin{bmatrix} \dfrac{Z_0}{2X}\sin\theta + \cos\theta & jZ_0\left(\sin\theta + \dfrac{Z_0}{2X}(1-\cos\theta)\right) \\ \dfrac{j}{Z_0}\left(\sin\theta - \dfrac{Z_0}{2X}(1+\cos\theta)\right) & \dfrac{Z_0}{2X}\sin\theta + \cos\theta \end{bmatrix}\end{aligned} \tag{7}$$

Converting ABCD-parameters to S-matrix, one determines:

$$\begin{aligned} S_{11} = S_{22} &= -\frac{e^{-j\theta}}{1 + 2jX/Z_0} \\ S_{12} = S_{21} &= \frac{2jX}{Z_0}\frac{e^{-j\theta}}{1 + 2jX/Z_0} \end{aligned} \tag{8}$$

So, equivalent circuit parameters may be calculated using S-parameters:

$$\frac{X}{Z_0} = \frac{1}{2}\sqrt{\frac{1}{|S_{11}|^2} - 1} \quad \theta = -\arccos|S_{11}| - \arg\left(-S_{11}\right) \tag{9}$$

S-parameters, in turn, depend on geometry of the particular iris and are evaluated numerically using mode-matching method described in the previous section. Results of these calculations are shown on Figure 4 for iris thickness $h = 2\ mm$.

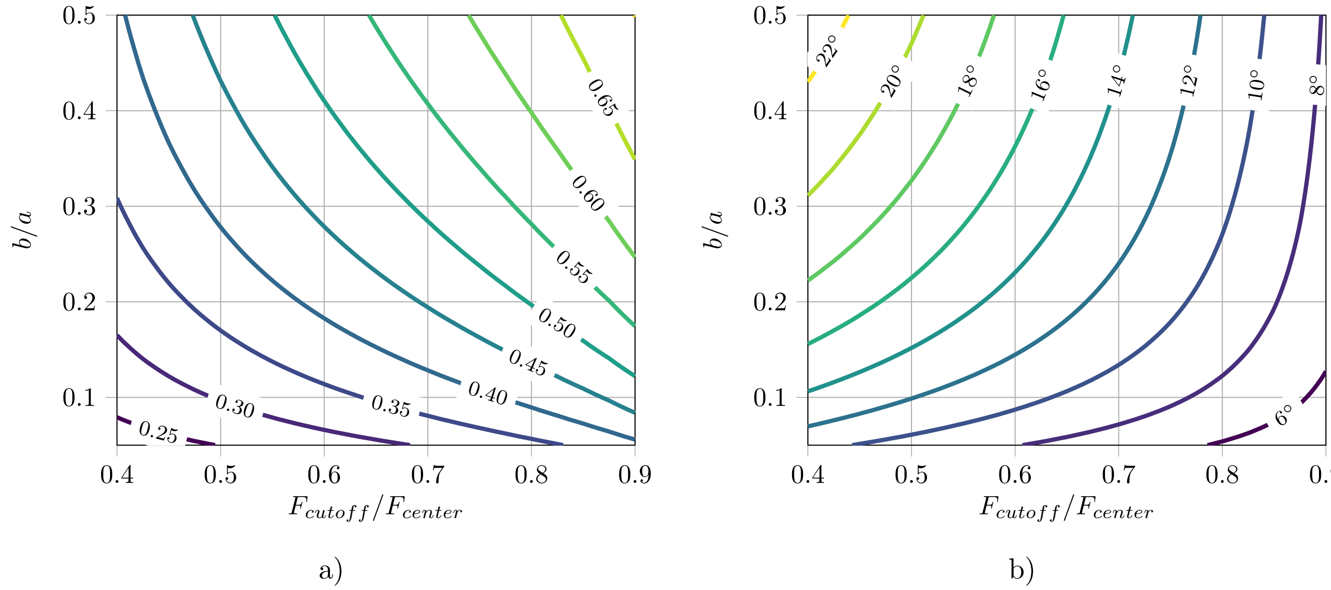

Figure 4. Equivalent circuit parameters for iris thickness $h/a = 0.1$. a) – equivalent impedance X/Z_0, b) – equivalent t-line length θ (cf. Figure 3).

Impedance inverter's equivalent circuit is similar to that of thick iris and depicted on Figure 3c. Conversion between these representations if performed by equations from [5]:

$$\frac{X_i}{Z_0} = \frac{K_i/Z_0}{(K_i/Z_0)^2 - 1}, \quad \theta_i = -2\tan^{-1}\left(\frac{K_i}{Z_0}\right) \tag{10}$$

Note that $\theta_i < 0$, i. e. it is subtracted from adjacent half-wave resonators.

Filter network Figure 3b is similar to a well-known problem of a capacitively-coupled microstrip filter. The design procedure described in [5] is as follows:

- choose low-pass filter prototype (maximally flat, equal ripple, etc.)
- determine selected filter's prototype values $g_0, g_1, \ldots, g_{N+1}$ (cases for maximally-flat and equal-ripple filters are considered in [2])
- determine transformation coefficients:

$$\begin{aligned} \frac{K_1}{Z_0} &= \sqrt{\frac{2}{\pi}\frac{g_0 g_1}{\Delta}}, \\ &\ldots \\ \frac{K_i}{Z_0} &= \frac{2}{\pi\Delta}\sqrt{g_i g_{i+1}} \\ &\ldots \\ \frac{K_N}{Z_0} &= \sqrt{\frac{2}{\pi}\frac{g_N g_{N+1}}{\Delta}} \end{aligned} \tag{11}$$

  where $\Delta = (F_{max} - F_{min})/F_{center}$ – fractional bandwidth of the filter
- determine line lengths $l'_i = \Lambda_g/2$, where $\Lambda_g = \lambda/\sqrt{1 - (F_{cutoff}/F_{center})^2}$ is the wavelength in waveguide at center frequency.

i	X_i/Z_0, mm	l_i, deg
1	0.424	152.9
2	0.125	167.7
3	0.093	169.6
4	0.090	169.6
5	0.093	167.7
6	0.125	152.9
7	0.424	

a)

i	b_i, mm	L_i, mm
1	4.56	4.68
2	3.59	5.50
3	3.39	5.63
4	3.36	5.63
5	3.39	5.50
6	3.59	4.68
7	4.56	

b)

Table 1. Proposed filter parameters: a) – equivalent circuit parameters, b) – filter dimensions.

Given K_i and l'_i, one can determine X_i/Z_0 and l_i using equation (10). A final step is to use mode-matching model together with equation (9) and Figure 4 to evaluate irises' widths b_i and waveguide section lengths L_i.

The procedure described herein has been used to synthesize Ka-band filter for a standard WR28 flange (width $a = 7.112\ mm$) for the following set of parameters:

- frequency band $F_{center} = 30.1\ GHz \pm 5.1\%$
- Tchebyshev filter, order $n = 6$, ripple 0.1 dB
- cutoff frequency $F_{cutoff} = 0.7F_{center}$

Equivalent circuit parameters for this filter is show in Table 1a. Using these data, filter dimensions are evaluated and show in Table 1b. S-parameters of the designed filter has been calculated using 3 different methods: equivalent circuit model, mode matching mode and FEM with the help of commercial software (HFSS). Results are presented on Figure 5.

FEM and mode matching yield almost the same result with slight differences in the pass-band that can be accounted for by finite precision of the numerical methods. However, computation time for mode matching is reduced drastically compared to the FEM (TODO: how much?). Pass-band return loss is less than $-15\ dB$. As for equivalent circuit model, it is the fastest yet the least accurate. Besides, this model shows systematic discrepancies in selectivity: it is higher for $F < F_{center}$ and lower for $F > F_{center}$ than calculated using MMM of FEM.

4. CONCLUSION

In this paper, an application of mode-matching method for waveguide filter analysis and synthesis is discussed. This method has lower demands on CPU time and memory compared to conventional mesh-based techniques such as FEM.

The proposed thick iris model allowed applying conventional synthesis procedure for direct-coupled waveguide filter with tick irises. A band-pass filter has been designed using this procedure. Its center frequency is 30.1 GHz, bandwidth is 5%. The designed filter is quite compact (overall length = ???) and can be used in high-speed satellite transmission lines for separating up- and down-link signals for duplex operation. Simulations using various computational methods are consistent with each other.

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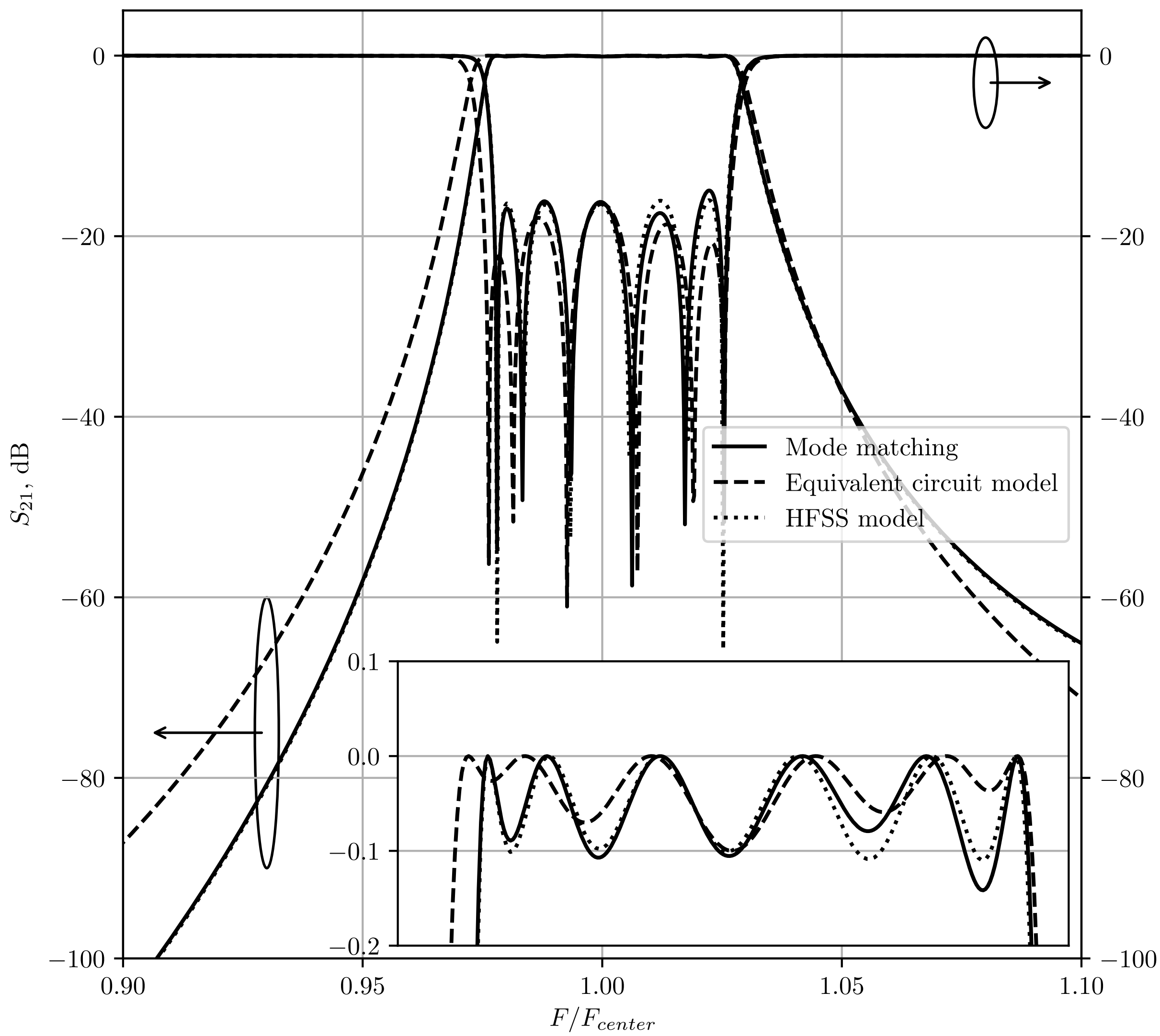

Figure 5. Filter transfer function calculated using various methods.

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