$\boxed{1.2}$ $u(x,y,z) = 3y^3z^2 + 5x^5\ln z - \sin x\cos y + 11y - 9x + 3z - 20$

Найти: $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}$; du

$\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y\partial x}, \frac{\partial^2 u}{\partial z\partial x}; \frac{\partial^2 u}{\partial x\partial y}, \frac{\partial^2 u}{\partial y^2}, \frac{\partial^2 u}{\partial z\partial y}; \frac{\partial^2 u}{\partial z\partial x}, \frac{\partial^2 u}{\partial z\partial y}, \frac{\partial^2 u}{\partial z^2}; d^2u$

Решение:

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(3y^3z^2 + 5x^5\ln z - \sin x\cos y + 11y - 9x + 3z - 20\right) =$$

$$= 5\ln z\frac{d}{dx}(x^5) - \cos y\frac{d}{dx}(\sin x) - 9\frac{d}{dx}(x) =$$

$$= 5\ln z\, 5x^4 - \cos y\cos x - 9 = 25x^4\ln z - \cos x\cos y - 9$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(3y^3z^2 + 5x^5\ln z - \sin x\cos y + 11y - 9x + 3z - 20\right) =$$

$$= 3z^2\frac{d}{dy}(y^3) - \sin x\frac{d}{dy}(\cos y) + 11\frac{d}{dy}(y) =$$

$$= 3z^2(3y^2) - \sin x(-\sin y) + 11\cdot 1 = 9y^2z^2 + \sin x\sin y + 11$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(3y^3z^2 + 5x^5\ln z - \sin x\cos y + 11y - 9x + 3z - 20\right) =$$

$$= 3y^3\frac{d}{dz}(z^2) + 5x^5\frac{d}{dz}(\ln z) + 3\frac{d}{dz}(z) =$$

$$= 3y^3\,2z + 5x^5\frac{1}{z} + 3 = 6y^3z + 5\frac{x^5}{z} + 3$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz =$$

$$= \left(25x^4\ln z - \cos x\cos y - 9\right)dx +$$

$$+ \left(9y^2z^2 + \sin x\sin y + 11\right)dy +$$

$$+ \left(6y^3z + 5\frac{x^5}{z} + 3\right)dz$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(25x^4 \ln z - \cos x \cos y - 9\right) =$$

$$= 25 \ln z \frac{d}{dx}\left(x^4\right) - \cos y \frac{d}{dx}\left(\cos x\right) =$$

$$= 25 \ln z \, 4x^3 - \cos y\left(-\sin x\right) = 100x^3 \ln z + \sin x \cos y$$

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial y}\left(25x^4 \ln z - \cos x \cos y - 9\right) =$$

$$= -\cos x \frac{d}{dy}\left(\cos y\right) = -\cos x\left(-\sin y\right) = \cos x \sin y$$

$$\frac{\partial^2 u}{\partial z \partial x} = \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial z}\left(25x^4 \ln z - \cos x \cos y - 9\right) =$$

$$= 25x^4 \frac{d}{dz}\left(\ln z\right) = 25x^4 \frac{1}{z} = 25\frac{x^4}{z}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}\left(9y^2z^2 + \sin x \sin y + 11\right) =$$

$$= \sin y \frac{d}{dx}\left(\sin x\right) = \sin y \cos x = \cos x \sin y =$$

$$= \frac{\partial^2 u}{\partial y \partial x}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(9y^2z^2 + \sin x \sin y + 11\right) =$$

$$= 9z^2 \frac{d}{dy}\left(y^2\right) + \sin x \frac{d}{dy}\left(\sin y\right) = 9z^2 \, 2y + \sin x \cos y =$$

$$= 18yz^2 + \sin x \cos y$$

$$\frac{\partial^2 u}{\partial z \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial z}\left(9y^2z^2 + \sin x \sin y + 11\right) =$$

$$= 9y^2 \frac{d}{dz}\left(z^2\right) = 9y^2 \, 2z = 18y^2 z$$

$$\frac{\partial^2 u}{\partial x \partial z} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}\right) = \frac{\partial}{\partial x}\left(6y^3 z + 5\frac{x^5}{z} + 3\right) =$$

$$= \frac{5}{z}\frac{d}{dx}\left(x^5\right) = \frac{5}{z}\,5x^4 = 25\frac{x^4}{z} =$$

$$= \frac{\partial^2 u}{\partial z \partial x}$$

$$\frac{\partial^2 u}{\partial y \partial z} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial z}\right) = \frac{\partial}{\partial y}\left(6y^3 z + 5\frac{x^5}{z} + 3\right) =$$

$$= 6z\frac{d}{dy}\left(y^3\right) = 6z\,3y^2 = 18y^2 z =$$

$$= \frac{\partial^2 u}{\partial z \partial y}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right) = \frac{\partial}{\partial z}\left(6y^3 z + 5\frac{x^5}{z} + 3\right) =$$

$$= 6y^3\frac{d}{dz}\left(z\right) + 5x^5\frac{d}{dz}\left(\frac{1}{z}\right) = 6y^3 \cdot 1 + 5x^5\left(-\frac{1}{z^2}\right) =$$

$$= 6y^3 - 5\frac{x^5}{z^2}$$

$$d^2u = d(du) = d\left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz\right) = d\left(\frac{\partial u}{\partial x}dx\right) + d\left(\frac{\partial u}{\partial y}dy\right) + d\left(\frac{\partial u}{\partial z}dz\right) =$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}dx\right)dx + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}dx\right)dy + \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}dx\right)dz +$$

$$+ \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}dy\right)dx + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}dy\right)dy + \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial y}dy\right)dz +$$

$$+ \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}dz\right)dx + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial z}dz\right)dy + \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}dz\right)dz =$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)dx\,dx + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)dx\,dy + \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}\right)dx\,dz +$$

$$+ \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)dy\,dx + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)dy\,dy + \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial y}\right)dy\,dz +$$

$$+ \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}\right)dz\,dx + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial z}\right)dz\,dy + \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)dz\,dz =$$

$$= \frac{\partial^2 u}{\partial x^2}dx^2 + \frac{\partial^2 u}{\partial y\partial x}dx\,dy + \frac{\partial^2 u}{\partial z\partial x}dx\,dz +$$

$$+ \frac{\partial^2 u}{\partial x\partial y}dy\,dx + \frac{\partial^2 u}{\partial y^2}dy^2 + \frac{\partial^2 u}{\partial z\partial y}dy\,dz +$$

$$+ \frac{\partial^2 u}{\partial x\partial z}dz\,dx + \frac{\partial^2 u}{\partial y\partial z}dz\,dy + \frac{\partial^2 u}{\partial z^2}dz^2 =$$

$$= \frac{\partial^2 u}{\partial x^2}dx^2 + \frac{\partial^2 u}{\partial y^2}dy^2 + \frac{\partial^2 u}{\partial z^2}dz^2 +$$

$$+ \left(\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial^2 u}{\partial y\partial x}\right)dx\,dy + \left(\frac{\partial^2 u}{\partial y\partial z} + \frac{\partial^2 u}{\partial z\partial y}\right)dy\,dz + \left(\frac{\partial^2 u}{\partial x\partial z} + \frac{\partial^2 u}{\partial z\partial x}\right)dz\,dx = \Big[$$

$$\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial^2 u}{\partial y\partial x}\ ;\ \frac{\partial^2 u}{\partial y\partial z} = \frac{\partial^2 u}{\partial z\partial y}\ ;\ \frac{\partial^2 u}{\partial y\partial z} = \frac{\partial^2 u}{\partial z\partial y}$$

$$\Big] = \frac{\partial^2 u}{\partial x^2}dx^2 + \frac{\partial^2 u}{\partial y^2}dy^2 + \frac{\partial^2 u}{\partial z^2}dz^2 +$$

$$+ 2\frac{\partial^2 u}{\partial x\partial y}dx\,dy + 2\frac{\partial^2 u}{\partial y\partial z}dy\,dz + 2\frac{\partial^2 u}{\partial z\partial x}dz\,dx =$$

$$= \left(100x^3\ln z + \sin x\cos y\right)dx^2 +$$

$$+ \left(18yz^2 + \sin x\cos y\right)dy^2 +$$

$$+ \left(6y^3 - 5\frac{x^5}{z^2}\right)dz^2 +$$

$$+ 2\cos x\sin y\, dx dy +$$

$$+ 2\cdot 18y^2 z\, dy dz +$$

$$+ 2\cdot 25\frac{x^4}{z}dz dx$$

Ombem:

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz =$$

$$= \left(25x^4\ln z - \cos x\cos y - 9\right)dx +$$

$$+ \left(9y^2z^2 + \sin x\sin y + 11\right)dy +$$

$$+ \left(6y^3z + 5\frac{x^5}{z} + 3\right)dz$$

$$d^2u = \frac{\partial^2 u}{\partial x^2}dx^2 + \frac{\partial^2 u}{\partial y^2}dy^2 + \frac{\partial^2 u}{\partial z^2}dz^2 +$$

$$+ 2\frac{\partial^2 u}{\partial x\partial y}dx dy + 2\frac{\partial^2 u}{\partial y\partial z}dy dz + 2\frac{\partial^2 u}{\partial z\partial x}dz dx =$$

$$= \left(100x^3\ln z + \sin x\cos y\right)dx^2 +$$

$$+ \left(18yz^2 + \sin x\cos y\right)dy^2 +$$

$$+ \left(6y^3 - 5\frac{x^5}{z^2}\right)dz^2 +$$

$$+ 2\cos x\sin y\, dx dy +$$

$$+ 2\cdot 18y^2 z\, dy dz +$$

$$+ 2\cdot 25\frac{x^4}{z}dz dx$$