

[5]

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x + \sqrt{x + \sqrt{x}}}}{\sqrt{2x+1}} = (1)$$

$$\sqrt{x + \sqrt{x + \sqrt{x}}} = \{x \neq 0\} =$$

$$= \sqrt{x \left(1 + \frac{\sqrt{x + \sqrt{x}}}{x}\right)} = \sqrt{x} \sqrt{1 + \sqrt{\frac{x + \sqrt{x}}{x^2}}} =$$

$$= \sqrt{x} \sqrt{1 + \sqrt{\frac{1}{x} + \frac{\sqrt{x}}{x^2}}}$$

$$\sqrt{2x+1} = \{x \neq 0\} =$$

$$= \sqrt{x \left(2 + \frac{1}{x}\right)} = \sqrt{x} \sqrt{2 + \frac{1}{x}}$$

$$\frac{\sqrt{x + \sqrt{x + \sqrt{x}}}}{\sqrt{2x+1}} = \{x \neq 0\} =$$

$$= \frac{\sqrt{x} \sqrt{1 + \sqrt{\frac{1}{x} + \frac{\sqrt{x}}{x^2}}}}{\sqrt{x} \sqrt{2 + \frac{1}{x}}} = \frac{\sqrt{1 + \sqrt{\frac{1}{x} + \frac{\sqrt{x}}{x^2}}}}{\sqrt{2 + \frac{1}{x}}}$$

$$(1) = \lim_{x \rightarrow \infty} \frac{\sqrt{1 + \sqrt{\frac{1}{x} + \frac{\sqrt{x}}{x^2}}}}{\sqrt{2 + \frac{1}{x}}} = \frac{\sqrt{1 + \left[\lim_{x \rightarrow \infty} \frac{1}{x} + \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{x^2} \right]}}{\sqrt{2 + \lim_{x \rightarrow \infty} \frac{1}{x}}} = \left\{ \right.$$

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0; \quad \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{x^2} = \lim_{x \rightarrow \infty} x^{-\frac{3}{2}} = 0$$

$$\left\{ = \frac{\sqrt{1 + \sqrt{0 + 0}}}{\sqrt{2 + 0}} = \frac{\sqrt{1 + 0}}{\sqrt{2 + 0}} = \frac{1}{\sqrt{2}} \right.$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x + \sqrt{x + \sqrt{x}}}}{\sqrt{2x+1}} = \frac{1}{\sqrt{2}}$$