$\boxed{20}$ Найти предел функции:

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = (1)$$

$$\lim_{x\to 0} 1-\cos x = (1-\cos x)_{x=0} = 1-\cos 0 = 1-1 = 0$$

$$\lim_{x\to 0} x^2 = (x^2)_{x=0} = 0^2 = 0$$

$$(1) = \left(\frac{0}{0}\right)$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \{\sin^2\alpha + \cos^2\alpha = 1;\ \cos^2\alpha = 1-\sin^2\alpha\} =$$
$$= (1-\sin^2\alpha) - \sin^2\alpha = 1 - 2\sin^2\alpha$$

$$1-\cos x = 1-\cos\left(2\frac{x}{2}\right) = 1-\left(1-2\sin^2\frac{x}{2}\right) = 1-1+2\sin^2\frac{x}{2} =$$
$$= 2\sin^2\frac{x}{2}$$

$$x^2 = \left(2\frac{x}{2}\right)^2 = 2^2\left(\frac{x}{2}\right)^2 = 4\left(\frac{x}{2}\right)^2$$

$$\frac{1-\cos x}{x^2} = \frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2} = \frac{2}{4}\ \frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2} = \frac{1}{2}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2$$

$$(1) = \lim_{x\to 0} \frac{1}{2}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 = \frac{1}{2}\left(\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 = (2)$$

$$y = \frac{x}{2}$$

$$\lim_{x\to 0} y = \lim_{x\to 0}\frac{x}{2} = \frac{0}{2} = 0$$

$$\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}} = \lim_{y\to 0}\frac{\sin y}{y} = 1$$

$$(2) = \frac{1}{2}\cdot 1^2 = \frac{1}{2}$$

Ответ:

$$\boxed{\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}}$$