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$$\lim_{x \rightarrow 4} \frac{\sqrt{1+2x} - 3}{\sqrt{x} - 2} = (1)$$

$$\lim_{x \rightarrow 4} \sqrt{1+2x} - 3 = (\sqrt{1+2x} - 3)_{x=4} =$$

$$= \sqrt{1+2 \cdot 4} - 3 = \sqrt{1+8} - 3 = \sqrt{9} - 3 = 3 - 3 = 0$$

$$\lim_{x \rightarrow 4} \sqrt{x} - 2 = (\sqrt{x} - 2)_{x=4} = \sqrt{4} - 2 = 2 - 2 = 0$$

$$(1) = \left(\frac{0}{0} \right)$$

$$\frac{\sqrt{1+2x} - 3}{\sqrt{x} - 2} = \frac{(\sqrt{1+2x} - 3)(\sqrt{1+2x} + 3)(\sqrt{x} + 2)}{(\sqrt{x} - 2)(\sqrt{x} + 2)(\sqrt{1+2x} + 3)} = (2)$$

$$(\sqrt{1+2x} - 3)(\sqrt{1+2x} + 3) = (\sqrt{1+2x})^2 - 3^2 =$$

$$= 1+2x - 9 = 2x - 8 = 2(x-4)$$

$$(\sqrt{x} - 2)(\sqrt{x} + 2) = (\sqrt{x})^2 - 2^2 = x - 4$$

$$(2) = \frac{2(x-4)}{(x-4)} \cdot \frac{\sqrt{x} + 2}{\sqrt{1+2x} + 3} = \{x-4 \neq 0, x \neq 4\} = 2 \frac{\sqrt{x} + 2}{\sqrt{1+2x} + 3}$$

$$(1) = \lim_{x \rightarrow 4} 2 \frac{\sqrt{x} + 2}{\sqrt{1+2x} + 3} = \left(2 \frac{\sqrt{x} + 2}{\sqrt{1+2x} + 3} \right)_{x=4} =$$

$$= 2 \frac{\sqrt{4} + 2}{\sqrt{1+2 \cdot 4} + 3} = 2 \frac{\sqrt{4} + 2}{\sqrt{9} + 3} = 2 \cdot \frac{2+2}{3+3} = 2 \cdot \frac{2 \cdot 2}{2 \cdot 3} = 2 \cdot \frac{2}{3} = \frac{4}{3}$$

$$\lim_{x \rightarrow 4} \frac{\sqrt{1+2x} - 3}{\sqrt{x} - 2} = \frac{4}{3}$$