14 Найти предел функции:

$$\lim_{x\to 0} \frac{\operatorname{arctg} x}{x} = (1)$$

$$\lim_{x\to 0} \operatorname{arctg} x = (\operatorname{arctg} x)_{x=0} = \operatorname{arctg} 0 = 0$$

$$\lim_{x\to 0} x = (x)_{x=0} = 0$$

$$(1) = \left(\frac{0}{0}\right)$$

$$y = \operatorname{arctg} x \Leftrightarrow x = \operatorname{tg} y$$

$$\lim_{x\to 0} y = \lim_{x\to 0} \operatorname{arctg} x = 0$$

$$(1) = \lim_{y\to 0} \frac{y}{\operatorname{tg} y} = (2)$$

$$\frac{y}{\operatorname{tg} y} = y\left(\frac{\sin y}{\cos y}\right)^{-1} = \{y \neq 0\} = \cos y \left(\frac{\sin y}{y}\right)^{-1}$$

$$(2) = \lim_{y\to 0} \cos y \left(\frac{\sin y}{y}\right)^{-1} = \lim_{y\to 0} \cos y \left(\lim_{y\to 0} \frac{\sin y}{y}\right)^{-1} = (3)$$

$$\lim_{y\to 0} \cos y = (\cos y)_{y=0} = \cos 0 = 1$$

$$\lim_{y\to 0} \frac{\sin y}{y} = 1$$

$$(3) = \frac{1}{1} = 1$$

Ответ:

$$\lim_{x\to 0} \frac{\operatorname{arctg} x}{x} = 1$$