3 Найти: $\int_0^{\ln 2} x e^{-x} dx$

Решение:

$$\int_0^{\ln 2} x e^{-x} dx = \int_0^{\ln 2} x\left(e^{-x} dx\right) = \int_0^{\ln 2} u\,dv = (1)$$

$$u = x \Rightarrow du = dx$$

$$dv = e^{-x} dx = e^{-x}(-1)(-dx) = -\left(e^{-x} d(-x)\right) = -de^{-x} = d\left(-e^{-x}\right) \Rightarrow$$

$$\Rightarrow v = -e^{-x}$$

$$uv = x\left(-e^{-x}\right) = -xe^{-x}$$

$$v\,du = -e^{-x} dx = e^{-x}(-dx) = e^{-x} d(-x) = de^{-x}$$

$$(1) = uv\Big|_0^{\ln 2} - \int_0^{\ln 2} v\,du = -xe^{-x}\Big|_0^{\ln 2} - \int_0^{\ln 2} de^{-x} =$$

$$= -xe^{-x}\Big|_0^{\ln 2} - e^{-x}\Big|_0^{\ln 2} = \left(-xe^{-x} - e^{-x}\right)\Big|_0^{\ln 2} =$$

$$= -e^{-x}(x+1)\Big|_0^{\ln 2} = e^{-x}(x+1)\Big|_{\ln 2}^{0} = (2)$$

$$e^{-x}(x+1)\Big|_{x=0} = e^{-0}(0+1) = 1 \cdot 1 = 1$$

$$e^{-x}(x+1)\Big|_{x=\ln 2} = e^{-\ln 2}(\ln 2 + 1) = \frac{\ln 2 + 1}{e^{\ln 2}} = \frac{\ln 2 + 1}{2}$$

$$(2) = 1 - \frac{\ln 2 + 1}{2} = \frac{2 - (\ln 2 + 1)}{2} = \frac{1 - \ln 2}{2}$$

Решение 2:

$$\int xe^{-x}dx = \int x(e^{-x}dx) = \int u\,dv = (1)$$

$u = x \Rightarrow du = dx$

$dv = e^{-x}dx = e^{-x} - (-dx) = -(e^{-x}d(-x)) = -de^{-x} = d(-e^{-x}) \Rightarrow v = -e^{-x}$

$uv = x(-e^{-x}) = -xe^{-x}$

$v\,du = -e^{-x}dx = e^{-x}(-dx) = e^{-x}d(-x) = de^{-x}$

$$(1) = uv - \int v\,du = -xe^{-x} - \int de^{-x} =$$

$$= -xe^{-x} - (e^{-x} + C) = -xe^{-x} - e^{-x} - C =$$

$$= -e^{-x}(x+1) - C \Rightarrow$$

$$\Rightarrow \int xe^{-x}dx = -e^{-x}(x+1) + C$$

$$\frac{d}{dx}\left(-e^{-x}(x+1) + C\right) = -\frac{d}{dx}\left(e^{-x}(x+1)\right) =$$

$$= -\left(\frac{d}{dx}(e^{-x})(x+1) + e^{-x}\frac{d}{dx}(x+1)\right) =$$

$$= -\left(-e^{-x}(x+1) + e^{-x}\cdot 1\right) = -\left(-xe^{-x} - e^{-x} + e^{-x}\right) = xe^{-x}$$

$$\int xe^{-x}dx = -e^{-x}(x+1) + C \Rightarrow$$

$$\Rightarrow \int_0^{\ln 2} xe^{-x}dx = -e^{-x}(x+1)\Big|_0^{\ln 2} = e^{-x}(x+1)\Big|_{\ln 2}^{0} = (2)$$

$$e^{-x}(x+1)\Big|_{x=0} = e^{-0}(0+1) = 1\cdot 1 = 1$$

$$e^{-x}(x+1)\Big|_{x=\ln 2} = e^{-\ln 2}(\ln 2 + 1) = \frac{\ln 2 + 1}{e^{\ln 2}} = \frac{\ln 2 + 1}{2}$$

$$(2) = 1 - \frac{\ln 2 + 1}{2} = \frac{2 - (\ln 2 + 1)}{2} = \frac{1 - \ln 2}{2}$$

Ответ: $\boxed{\int_0^{\ln 2} xe^{-x}dx = \frac{1 - \ln 2}{2}}$