7 Найти: $\int x \sin x \, dx$

Решение:

$\int x \sin x \, dx = \int u \, dv =$

$u = x \implies du = dx$

$dv = \sin x \, dx = -(-\sin x \, dx) = -d\cos x = d(-\cos x) \implies$

$\implies v = -\cos x$

$= uv - \int v \, du = x(-\cos x) - \int (-\cos x) \, dx =$

$= -x \cos x + \int \cos x \, dx =$

$\int \cos x \, dx = \int d(\sin x) = \sin x + C$

$= -x \cos x + \sin x + C$

Проверка:

$\frac{d}{dx}(-x\cos x + \sin x + C) = -\frac{d}{dx}(x\cos x) + \frac{d}{dx}(\sin x) =$

$= -\left(\frac{d}{dx}(x)\cos x + x\frac{d}{dx}(\cos x)\right) + \frac{d}{dx}(\sin x) =$

$= -\left(1 \cdot \cos x + x(-\sin x)\right) + \cos x = -(\cos x - x\sin x) + \cos x =$

$= -\cos x + x\sin x + \cos x = x \sin x$

Ответ: $\boxed{\int x \sin x \, dx = -x \cos x + \sin x + C}$