$\boxed{1.1}$ Найти предел последовательности:

$$\lim_{n\to\infty}\frac{(23-2n^2)(3n^2+17)^2}{4n^6+n-1}=(1)$$

$$(23-2n^2)(3n^2+17)^2=n^2\left(\frac{23}{n^2}-2\right)\left(n^2\left(3+\frac{17}{n^2}\right)\right)^2=$$

$$=n^2\left(-2+\frac{23}{n^2}\right)n^4\left(3+\frac{17}{n^2}\right)^2=n^6\left(-2+\frac{23}{n^2}\right)\left(3+\frac{17}{n^2}\right)^2$$

$$4n^6+n-1=n^6\left(4+\frac{n}{n^6}-\frac{1}{n^6}\right)=n^6\left(4+\frac{1}{n}-\frac{1}{n^6}\right)$$

$$\frac{(23-2n^2)(3n^2+17)^2}{4n^6+n-1}=\frac{n^6\left(-2+\frac{23}{n^2}\right)\left(3+\frac{17}{n^2}\right)^2}{n^6\left(4+\frac{1}{n}-\frac{1}{n^6}\right)}=$$

$$=\frac{\left(-2+\frac{23}{n^2}\right)\left(3+\frac{17}{n^2}\right)^2}{4+\frac{1}{n}-\frac{1}{n^6}}$$

$$(1)=\lim_{n\to\infty}\frac{\left(-2+\frac{23}{n^2}\right)\left(3+\frac{17}{n^2}\right)^2}{4+\frac{1}{n}-\frac{1}{n^6}}=$$

$$=\frac{\left(-2+23\lim\limits_{n\to\infty}\frac{1}{n^2}\right)\left(3+17\lim\limits_{n\to\infty}\frac{1}{n^2}\right)^2}{4+\lim\limits_{n\to\infty}\frac{1}{n}-\lim\frac{1}{n^6}}=$$

$$=\frac{(-2+23\cdot 0)(3+17\cdot 0)^2}{4+0-0}=\frac{(-2)(3)^2}{4}=-\frac{9}{2}$$

Ответ: $\lim\limits_{n\to\infty}\dfrac{(23-2n^2)(3n^2+17)^2}{4n^6+n-1}=-\dfrac{9}{2}$

1.2 Найти предел последовательности

$$\lim_{n\to\infty} \frac{(97-2n)^3}{2n(3n^2+15)+8n} = (1)$$

$$(97-2n)^3 = n^3\left(\frac{97}{n}-2\right)^3 = n^3\left(-2+\frac{97}{n}\right)^3$$

$$2n(3n^2+15)+8n = 2n\,n^2\left(3+\frac{15}{n^2}\right)+8n =$$

$$= 2n^3\left(3+\frac{15}{n^2}\right)+8n = n^3\left(2\left(3+\frac{15}{n^2}\right)+\frac{8n}{n^3}\right) =$$

$$= n^3\left(2\left(3+\frac{15}{n^2}\right)+\frac{8}{n^2}\right)$$

$$\frac{(97-2n)^3}{2n(3n^2+15)+8n} = \frac{n^3\left(-2+\frac{97}{n}\right)^3}{n^3\left(2\left(3+\frac{15}{n^2}\right)+\frac{8}{n^2}\right)} =$$

$$= \frac{\left(-2+\frac{97}{n}\right)^3}{2\left(3+\frac{15}{n^2}\right)+\frac{8}{n^2}}$$

$$(1) = \lim_{n\to\infty} \frac{\left(-2+\frac{97}{n}\right)^3}{2\left(3+\frac{15}{n^2}\right)+\frac{8}{n^2}} =$$

$$= \frac{\left(-2+97\lim\limits_{n\to\infty}\frac{1}{n}\right)^3}{2\left(3+15\lim\limits_{n\to\infty}\frac{1}{n^2}\right)+8\lim\limits_{n\to\infty}\frac{1}{n^2}} =$$

$$= \frac{(-2+97\cdot 0)^3}{2(3+15\cdot 0)+8\cdot 0} = \frac{(-2+0)^3}{2\cdot(3+0)+0} = \frac{(-2)^3}{2\cdot 3} = -\frac{4}{3}$$

Ответ: $\lim\limits_{n\to\infty} \frac{(97-2n)^3}{2n(3n^2+15)+8n} = -\frac{4}{3}$

1.3 Найти предел последовательности:

$$\lim_{n\to\infty} \frac{2n^3+13n(n+18)}{(27-n)(2n+19)^2} = (1)$$

$$2n^3+13n(n+18) = n^3\left(2+\frac{13n(n+18)}{n^3}\right) =$$

$$= n^3\left(2+\frac{13(n+18)}{n^2}\right) = n^3\left(2+13\left(\frac{n}{n^2}+\frac{18}{n^2}\right)\right) =$$

$$= n^3\left(2+13\left(\frac{1}{n}+\frac{18}{n^2}\right)\right)$$

$$(27-n)(2n+19)^2 = n\left(\frac{27}{n}-1\right)n^2\left(2+\frac{19}{n}\right)^2 =$$

$$= n^3\left(-1+\frac{27}{n}\right)\left(2+\frac{19}{n}\right)^2$$

$$\frac{2n^3+13n(n+18)}{(27-n)(2n+19)^2} = \frac{n^3\left(2+13\left(\frac{1}{n}+\frac{18}{n^2}\right)\right)}{n^3\left(-1+\frac{27}{n}\right)\left(2+\frac{19}{n}\right)^2} =$$

$$= \frac{2+13\left(\frac{1}{n}+\frac{18}{n^2}\right)}{\left(-1+\frac{27}{n}\right)\left(2+\frac{19}{n}\right)^2}$$

$$(1) = \lim_{n\to\infty} \frac{2+13\left(\frac{1}{n}+\frac{18}{n^2}\right)}{\left(-1+\frac{27}{n}\right)\left(2+\frac{19}{n}\right)^2} =$$

$$= \frac{2+13\left(\lim\limits_{n\to\infty}\frac{1}{n}+18\lim\limits_{n\to\infty}\frac{1}{n^2}\right)}{\left(-1+27\lim\limits_{n\to\infty}\frac{1}{n}\right)\left(2+19\lim\limits_{n\to\infty}\frac{1}{n}\right)^2} =$$

$$= \frac{2+13(0+18\cdot 0)}{(-1+27\cdot 0)(2+19\cdot 0)^2} = \frac{2+0}{(-1+0)(2+0)^2} = \frac{2}{(-1)2^2} = -\frac{1}{2}$$

Ответ: $\lim\limits_{n\to\infty} \frac{2n^3+13n(n+18)}{(27-n)(2n+19)^2} = -\frac{1}{2}$

$\boxed{1.4}$ Найти предел последовательности

$$\lim_{n\to\infty} \sqrt{n^2+1} - n = (1)$$

$$\sqrt{n^2+1} - n = \frac{\left(\sqrt{n^2+1} - n\right)\left(\sqrt{n^2+1} + n\right)}{\sqrt{n^2+1} + n} =$$

$$= \frac{\left(\sqrt{n^2+1}\right)^2 - n^2}{\sqrt{n^2+1} + n} = \frac{(n^2+1) - n^2}{\sqrt{n^2+1} + n} = \frac{1}{\sqrt{n^2+1} + n} =$$

$$= \frac{1}{\sqrt{n^2\left(1+\frac{1}{n^2}\right)} + n} = \frac{1}{n\sqrt{1+\frac{1}{n^2}} + n} = \frac{1}{n\left(\sqrt{1+\frac{1}{n^2}} + 1\right)}$$

$$= \frac{1}{n\left(1+\sqrt{1+\frac{1}{n^2}}\right)}$$

$$(1) = \lim_{n\to\infty} \frac{1}{n\left(1+\sqrt{1+\frac{1}{n^2}}\right)} = \lim_{n\to\infty} \frac{1}{n} \lim_{n\to\infty} \frac{1}{1+\sqrt{1+\frac{1}{n^2}}} = (2)$$

$$\lim_{n\to\infty} \frac{1}{n} = 0$$

$$\lim_{n\to\infty} \frac{1}{1+\sqrt{1+\frac{1}{n^2}}} = \frac{1}{1+\sqrt{1+\lim\limits_{n\to\infty}\frac{1}{n^2}}} =$$

$$= \frac{1}{1+\sqrt{1+0}} = \frac{1}{1+\sqrt{1}} = \frac{1}{2}$$

$$(2) = \frac{0}{2} = 0$$

Ответ: $\lim\limits_{n\to\infty} \sqrt{n^2+1} - n = 0$

1.5 Найти предел последовательности

$$\lim_{n\to\infty} \frac{(-4)^n + 5\cdot 7^n}{(-4)^{n-1} + 7^{n+2}} = (1)$$

$$(-4)^n + 5\cdot 7^n = 7^n\left(\frac{(-4)^n}{7^n} + 5\right) = 7^n\left(\left(-\frac{4}{7}\right)^n + 5\right) =$$

$$= 7^n\left(5 + \left(-\frac{4}{7}\right)^n\right)$$

$$(-4)^{n-1} + 7^{n+2} = (-4)^{n-1} + 7^3 7^{n-1} = 7^{n-1}\left(\frac{(-4)^{n-1}}{7^{n-1}} + 7^3\right) =$$

$$= 7^{n-1}\left(\left(-\frac{4}{7}\right)^{n-1} + 7^3\right) = 7^{n-1}\left(7^3 + \left(-\frac{4}{7}\right)^{n-1}\right)$$

$$\frac{(-4)^n + 5\cdot 7^n}{(-4)^{n-1} + 7^{n+2}} = \frac{7^n\left(5 + \left(-\frac{4}{7}\right)^n\right)}{7^{n-1}\left(7^3 + \left(-\frac{4}{7}\right)^{n-1}\right)} = \frac{7\left(5 + \left(-\frac{4}{7}\right)^n\right)}{7^3 + \left(-\frac{4}{7}\right)^{n-1}}$$

$$(1) = \lim_{n\to\infty} \frac{7\left(5 + \left(-\frac{4}{7}\right)^n\right)}{7^3 + \left(-\frac{4}{7}\right)^{n-1}} =$$

$$= \frac{7\left(5 + \lim\limits_{n\to\infty}\left(-\frac{4}{7}\right)^n\right)}{7^3 + \lim\limits_{n\to\infty}\left(-\frac{4}{7}\right)^{n-1}} = (2)$$

$$\lim_{n\to\infty}\left(-\frac{4}{7}\right)^{n-1} = \lim_{(n-1)+1\to\infty}\left(-\frac{4}{7}\right)^{n-1} = \lim_{n-1\to\infty}\left(-\frac{4}{7}\right)^{n-1} = \lim_{n\to\infty}\left(-\frac{4}{7}\right)^n$$

$$(2) = \frac{7\left(5 + \lim\limits_{n\to\infty}\left(-\frac{4}{7}\right)^n\right)}{7^3 + \lim\limits_{n\to\infty}\left(-\frac{4}{7}\right)^n} = \frac{7(5+0)}{7^3+0} = \frac{7\cdot 5}{7^3} = \frac{5}{7^2} = \frac{5}{49}$$

Ответ: $\lim\limits_{n\to\infty} \frac{(-4)^n + 5\cdot 7^n}{(-4)^{n-1} + 7^{n+2}} = \frac{5}{49}$