

Li) Hinweis: $\int \frac{x^2}{x^3+1} dx$

Permutation:

$$x^3+1 = t \Rightarrow$$

$$\Rightarrow d(x^3+1) = dt \Leftrightarrow$$

$$d(x^3+1) = dx^3 = 3x^2 dx$$

$$\Leftrightarrow 3x^2 dx = dt \Leftrightarrow x^2 dx = \frac{dt}{3}$$

$$\frac{x^2}{x^3+1} dx = \frac{x^2 dx}{x^3+1} = \frac{\frac{dt}{3}}{t} = \frac{1}{3} \frac{dt}{t} = \frac{1}{3} d \ln |t| = d \left(\frac{\ln |t|}{3} \right)$$

$$\int \frac{x^2}{x^3+1} dx = \int d \left(\frac{\ln |t|}{3} \right) = \frac{\ln |t|}{3} + C = \frac{1}{3} \ln |x^3+1| + C$$

Permutation 2:

$$\int \frac{x^2}{x^3+1} dx = \int \frac{1}{3} \frac{3x^2 dx}{x^3+1} = \int \frac{1}{3} \frac{dx^3}{x^3+1} =$$

$$= \int \frac{1}{3} d \ln |x^3+1| = \int d \left(\frac{1}{3} \ln |x^3+1| \right) =$$

$$= \frac{1}{3} \ln |x^3+1| + C$$

Überprüfung:

$$\frac{d}{dx} \left(\frac{1}{3} \ln |x^3+1| + C \right) = \frac{d}{dx} \left(\frac{1}{3} \ln |x^3+1| \right) =$$

$$= \frac{1}{3} \frac{d}{dx} \left(\ln |x^3+1| \right) = \frac{1}{3} \cdot \frac{1}{x^3+1} \cdot \frac{d}{dx} (x^3) =$$

$$= \frac{1}{3} \cdot \frac{1}{x^3+1} \cdot 3x^2 = \frac{x^2}{x^3+1}$$

Antwort:

$$\boxed{\int \frac{x^2}{x^3+1} dx = \frac{1}{3} \ln |x^3+1| + C}$$