8 Найти предел функции:

$$\lim_{x \to 4} \frac{\sqrt{1+2x}-3}{\sqrt{x}-2} = (1)$$

$$\lim_{x \to 4} \sqrt{1+2x}-3 = \left(\sqrt{1+2x}-3\right)_{x=4} =$$

$$= \sqrt{1+2\cdot4}-3 = \sqrt{1+8}-3 = \sqrt{9}-3 = 3-3 = 0$$

$$\lim_{x \to 4} \sqrt{x}-2 = \left(\sqrt{x}-2\right)_{x=4} = \sqrt{4}-2 = 2-2 = 0$$

$$(1) = \left(\frac{0}{0}\right)$$

$$\frac{\sqrt{1+2x}-3}{\sqrt{x}-2} = \frac{\left(\sqrt{1+2x}-3\right)\left(\sqrt{1+2x}+3\right)}{\left(\sqrt{x}-2\right)\left(\sqrt{x}+2\right)} \cdot \frac{\left(\sqrt{x}+2\right)}{\left(\sqrt{1+2x}+3\right)} = (2)$$

$$\left(\sqrt{1+2x}-3\right)\left(\sqrt{1+2x}+3\right) = \left(\sqrt{1+2x}\right)^2 - 3^2 =$$

$$= 1+2x-9 = 2x-8 = 2(x-4)$$

$$\left(\sqrt{x}-2\right)\left(\sqrt{x}+2\right) = \left(\sqrt{x}\right)^2 - 2^2 = x-4$$

$$(2) = \frac{2(x-4)}{(x-4)} \cdot \frac{\sqrt{x}+2}{\sqrt{1+2x}+3} = \{x-4 \neq 0; x \neq 4\} = 2\frac{\sqrt{x}+2}{\sqrt{1+2x}+3}$$

$$(1) = \lim_{x \to 4} 2\frac{\sqrt{x}+2}{\sqrt{1+2x}+3} = \left(2\frac{\sqrt{x}+2}{\sqrt{1+2x}+3}\right)_{x=4} =$$

$$= 2\frac{\sqrt{4}+2}{\sqrt{1+2\cdot4}+3} = 2\frac{\sqrt{4}+2}{\sqrt{9}+3} = 2\cdot\frac{2+2}{3+3} = 2\cdot\frac{2\cdot2}{2\cdot3} = 2\cdot\frac{2}{3} = \frac{4}{3}$$

Ответ:

$$\lim_{x \to 4} \frac{\sqrt{1+2x}-3}{\sqrt{x}-2} = \frac{4}{3}$$