1.4 Найти предел последовательности

$$\lim_{n\to\infty} \sqrt{n^2+1} - n = (1)$$

$$\sqrt{n^2+1} - n = \frac{\left(\sqrt{n^2+1} - n\right)\left(\sqrt{n^2+1} + n\right)}{\sqrt{n^2+1} + n} =$$

$$= \frac{\left(\sqrt{n^2+1}\right)^2 - n^2}{\sqrt{n^2+1} + n} = \frac{(n^2+1) - n^2}{\sqrt{n^2+1} + n} = \frac{1}{\sqrt{n^2+1} + n} =$$

$$= \frac{1}{\sqrt{n^2\left(1+\frac{1}{n^2}\right)} + n} = \frac{1}{n\sqrt{1+\frac{1}{n^2}} + n} = \frac{1}{n\left(\sqrt{1+\frac{1}{n^2}} + 1\right)} =$$

$$= \frac{1}{n\left(1+\sqrt{1+\frac{1}{n^2}}\right)}$$

$$(1) = \lim_{n\to\infty} \frac{1}{n\left(1+\sqrt{1+\frac{1}{n^2}}\right)} = \lim_{n\to\infty} \frac{1}{n} \lim_{n\to\infty} \frac{1}{1+\sqrt{1+\frac{1}{n^2}}} = (2)$$

$$\lim_{n\to\infty} \frac{1}{n} = 0$$

$$\lim_{n\to\infty} \frac{1}{1+\sqrt{1+\frac{1}{n^2}}} = \frac{1}{1+\sqrt{1+\lim\limits_{n\to\infty}\frac{1}{n^2}}} =$$

$$= \frac{1}{1+\sqrt{1+0}} = \frac{1}{1+\sqrt{1}} = \frac{1}{2}$$

$$(2) = \frac{0}{2} = 0$$

Ответ: $\boxed{\lim_{n\to\infty} \sqrt{n^2+1} - n = 0}$