

[21] Найми предел функции.

$$\lim_{x \rightarrow 0} \frac{\cos x \sin^2 3x}{4x \operatorname{tg} \frac{x}{3}} = (1)$$

$$\lim_{x \rightarrow 0} \cos x \sin^2 3x = (\cos x \sin^2 3x)_{x=0} = \\ = \cos 0 \sin^2 3 \cdot 0 = \cos 0 \sin^2 0 = 1 \cdot 0^2 = 1 \cdot 0 = 0$$

$$\lim_{x \rightarrow 0} 4x \operatorname{tg} \frac{x}{3} = (4x \operatorname{tg} \frac{x}{3})_{x=0} = \\ = 4 \cdot 0 \cdot \operatorname{tg} \frac{0}{3} = 4 \cdot 0 \cdot \operatorname{tg} 0 = 4 \cdot 0 \cdot 0 = 0$$

$$\frac{\cos x \sin^2 3x}{4x \operatorname{tg} \frac{x}{3}} = \frac{\cos x \sin^2 3x}{4x \frac{\sin \frac{x}{3}}{\cos \frac{x}{3}}} = \frac{\cos x \cos \frac{x}{3}}{4x} \frac{\sin^2 3x}{\sin \frac{x}{3}} =$$

$$= \frac{\cos x \cos \frac{x}{3}}{4x} (3x)^2 \frac{\sin^2 3x}{(3x)^2} \frac{3}{x} \frac{\frac{x}{3}}{\sin \frac{x}{3}} =$$

$$= \frac{(3x)^2 \cdot 3}{4x \cdot x} \cos x \cos \frac{x}{3} \left( \frac{\sin 3x}{3x} \right)^2 \left( \frac{\sin \frac{x}{3}}{\frac{x}{3}} \right)^{-1} =$$

$$= \frac{3^3 x^2}{4 x^2} \cos x \cos \frac{x}{3} \left( \frac{\sin 3x}{3x} \right)^2 \left( \frac{\sin \frac{x}{3}}{\frac{x}{3}} \right)^{-1} =$$

$$= \frac{27}{4} \cos x \cos \frac{x}{3} \left( \frac{\sin 3x}{3x} \right)^2 \left( \frac{\sin \frac{x}{3}}{\frac{x}{3}} \right)^{-1}$$

$$(1) = \lim_{x \rightarrow 0} \frac{27}{4} \cos x \cos \frac{x}{3} \left( \frac{\sin 3x}{3x} \right)^2 \left( \frac{\sin \frac{x}{3}}{\frac{x}{3}} \right)^{-1} =$$

$$= \frac{27}{4} \lim_{x \rightarrow 0} \cos x \lim_{x \rightarrow 0} \cos \frac{x}{3} \left( \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \right)^2 \left( \lim_{x \rightarrow 0} \frac{\sin \frac{x}{3}}{\frac{x}{3}} \right)^{-1} = (2)$$

$$\lim_{x \rightarrow 0} \cos x = \{x > 0\} = (\cos x)_{x=0} =$$

$$= \cos(x:0) = \cos 0 = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \{$$

$$x > 0; y = x; \lim_{x \rightarrow 0} y = \lim_{x \rightarrow 0} x = (x)_{x=0} = x:0 = 0$$

$$\} = \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1$$

$$(2) = \frac{27}{4} \cdot 1 \cdot 1 \cdot 1^2 \cdot 1^{-1} = \frac{27}{4}$$

Problem:

$$\lim_{x \rightarrow 0} \frac{\cos x \sin^2 3x}{4x \lg \frac{x}{3}} = \frac{27}{4}$$