

1.5) Найти предел функции:

$$\lim_{x \rightarrow 0} \frac{\sqrt{2} x^2 \sin 4x}{(1 - \cos 2x)^{\frac{3}{2}}} = (1)$$

$$\lim_{x \rightarrow 0} (1 - \cos 2x)^{\frac{3}{2}} = (1 - \cos 2x)^{\frac{3}{2}} \Big|_{x=0} = \\ = (1 - \cos(2 \cdot 0))^{\frac{3}{2}} = (1 - \cos 0)^{\frac{3}{2}} = (1 - 1)^{\frac{3}{2}} = 0^{\frac{3}{2}} = 0$$

$$\lim_{x \rightarrow 0} (\sqrt{2} x^2 \sin 4x) = \sqrt{2} x^2 \sin 4x \Big|_{x=0} = \\ = \sqrt{2} \cdot 0^2 \sin(4 \cdot 0) = \sqrt{2} \cdot 0^2 \cdot \sin 0 = \sqrt{2} \cdot 0 \cdot 0 = 0$$

$$(1 - \cos 2x)^{\frac{3}{2}} = \{ \\ \cos 2x = \cos^2 x - \sin^2 x = (1 - \sin^2 x) - \sin^2 x = 1 - 2 \sin^2 x \\ \} = (1 - (1 - 2 \sin^2 x))^{\frac{3}{2}} = (1 - 1 + 2 \sin^2 x)^{\frac{3}{2}} = (2 \sin^2 x)^{\frac{3}{2}} = \\ = 2^{\frac{3}{2}} (\sin x)^{2 \cdot \frac{3}{2}} = 2^{\frac{3}{2}} \sin^3 x$$

$$\sin 4x = \sin(2 \cdot 2x) = \{ \sin 2x = 2 \sin x \cos x \} = \\ = 2 \sin 2x \cos 2x = 2 (2 \sin x \cos x) \cos 2x = \\ = 4 \cos x \cos 2x \sin x$$

$$\frac{\sqrt{2} x^2 \sin 4x}{(1 - \cos 2x)^{\frac{3}{2}}} = \sqrt{2} x^2 \frac{\sin 4x}{(1 - \cos 2x)^{\frac{3}{2}}} =$$

$$= \sqrt{2} x^2 \frac{4 \cos x \cos 2x \sin x}{2\sqrt{2} \sin^3 x} = \frac{\sqrt{2}}{2\sqrt{2}} x^2 \cos x \cos 2x \frac{\sin x}{\sin^3 x} =$$

$$= 2 x^2 \frac{\cos x \cos 2x}{\sin^2 x} = 2 \cos x \cos 2x \frac{x^2}{\sin^2 x} = \{x \neq 0\} =$$

$$= 2 \cos x \cos 2x \left( \frac{\sin x}{x} \right)^{-2}$$

$$(1) = \lim_{x \rightarrow 0} 2 \cos x \cos 2x \left( \frac{\sin x}{x} \right)^{-2} =$$

$$= 2 \lim_{x \rightarrow 0} (\cos x \cos 2x) \left( \lim_{x \rightarrow 0} \frac{\sin x}{x} \right)^{-2} = \{$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} (\cos x \cos 2x) = \cos x \cos 2x \Big|_{x=0} = \cos 0 \cos(2 \cdot 0) = \cos 0 \cos 0 = 1 \cdot 1 = 1$$

$$\} = 2 \cdot 1 \cdot 1^{-2} = 2$$

Daher:

$$\boxed{\lim_{x \rightarrow 0} \frac{\sqrt{2} x^2 \sin 4x}{(1 - \cos 2x)^{\frac{3}{2}}} = 2}$$