2.1 $u(x,y) = 4\sin x + 3\cos y + 2x^4y^3 + 4$

Найти: $\nabla u(0,0)$

Решение:

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(4\sin x + 3\cos y + 2x^4y^3 + 4\right) = 4\frac{d}{dx}(\sin x) + 2y^3\frac{d}{dx}(x^4) =$$

$$= 4\cos x + 2y^3 4x^3 = 4\cos x + 2\cdot 4x^3y^3 = 4\left(\cos x + 2x^3y^3\right)$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(4\sin x + 2\cos y + 2x^4y^3 + 4\right) = 3\frac{d}{dy}(\cos y) + 2x^4\frac{d}{dy}(y^3) =$$

$$= 3(-\sin y) + 2x^4 3y^2 = -3\sin y + 2\cdot 3x^4y^2 = 3\left(-\sin y + 2x^4y^2\right)$$

$$\nabla u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right) = \left(4\left(\cos x + 2x^3y^3\right), 3\left(-\sin y + 2x^4y^2\right)\right)$$

$$\nabla u(0,0) = \left(\frac{\partial u}{\partial x}(0,0), \frac{\partial u}{\partial y}(0,0)\right) = [$$

$$\frac{\partial u}{\partial x}(0,0) = 4\left(\cos x + 2x^3y^3\right)\Big|_{\substack{x=0\\y=0}} = 4\left(\cos 0 + 2\cdot 0^3 0^3\right) = 4(1+2\cdot 0) = 4$$

$$\frac{\partial u}{\partial y}(0,0) = 3\left(-\sin y + 2x^4y^2\right)\Big|_{\substack{x=0\\y=0}} = 3\left(-\sin 0 + 2\cdot 0^4 0^2\right) = 3(-0+2\cdot 0) = 0$$

$$] = (4,0)$$

Ответ:

$$\nabla u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right) = \left(4\left(\cos x + 2x^3y^3\right), 3\left(-\sin y + 2x^4y^2\right)\right)$$

$$\nabla u(0,0) = (4,0)$$