2) $u(x,y,z) = \frac{256}{x} + \frac{x^2}{y} + \frac{y^2}{z} + z^2$

Найти:

$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}$

$\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y \partial x}, \frac{\partial^2 u}{\partial z \partial x}; \frac{\partial^2 u}{\partial x \partial y}, \frac{\partial^2 u}{\partial y^2}, \frac{\partial^2 u}{\partial z \partial y}; \frac{\partial^2 u}{\partial x \partial z}, \frac{\partial^2 u}{\partial y \partial z}, \frac{\partial^2 u}{\partial z^2}$

Решение:

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\frac{256}{x} + \frac{x^2}{y} + \frac{y^2}{z} + z^2\right) = 256 \frac{d}{dx}\left(\frac{1}{x}\right) + \frac{1}{y}\frac{d}{dx}\left(x^2\right) =$$

$$= 256\left(-\frac{1}{x^2}\right) + \frac{1}{y} 2x = -\frac{256}{x^2} + 2\frac{x}{y} = -\left(\frac{16}{x}\right)^2 + 2\frac{x}{y}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{256}{x} + \frac{x^2}{y} + \frac{y^2}{z} + z^2\right) = x^2 \frac{d}{dy}\left(\frac{1}{y}\right) + \frac{1}{z}\frac{d}{dy}\left(y^2\right) =$$

$$= x^2\left(-\frac{1}{y^2}\right) + \frac{1}{z} 2y = -\frac{x^2}{y^2} + 2\frac{y}{z} = -\left(\frac{x}{y}\right)^2 + 2\frac{y}{z}$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(\frac{256}{x} + \frac{x^2}{y} + \frac{y^2}{z} + z^2\right) = y^2 \frac{d}{dz}\left(\frac{1}{z}\right) + \frac{d}{dz}\left(z^2\right) =$$

$$= y^2\left(-\frac{1}{z^2}\right) + 2z = -\frac{y^2}{z^2} + 2z = -\left(\frac{y}{z}\right)^2 + 2z$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(-\frac{256}{x^2} + 2\frac{x}{y}\right) = -256\frac{d}{dx}\left(\frac{1}{x^2}\right) + \frac{2}{y}\frac{d}{dx}(x) =$$

$$= -256\left(-\frac{2}{x^3}\right) + \frac{2}{y} \cdot 1 = \frac{2 \cdot 256}{x^3} + \frac{2}{y} = 2\left(\frac{16^2}{x^3} + \frac{1}{y}\right)$$

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial y}\left(-\frac{256}{x^2} + 2\frac{x}{y}\right) = 2x\frac{d}{dy}\left(\frac{1}{y}\right) =$$

$$= 2x\left(-\frac{1}{y^2}\right) = -2\frac{x}{y^2}$$

$$\frac{\partial^2 u}{\partial z \partial x} = \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial z}\left(-\frac{256}{x^2} + 2\frac{x}{y}\right) = 0$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}\left(-\frac{x^2}{y^2} + 2\frac{y}{z}\right) = -\frac{1}{y^2}\frac{d}{dx}\left(x^2\right) =$$

$$= -\frac{1}{y^2}2x = -2\frac{x}{y^2} = \frac{\partial^2 u}{\partial y \partial x}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(-\frac{x^2}{y^2} + 2\frac{y}{z}\right) = -x^2\frac{d}{dy}\left(\frac{1}{y^2}\right) + \frac{2}{z}\frac{d}{dy}(y) =$$

$$= -x^2\left(-\frac{2}{y^3}\right) + \frac{2}{z}\cdot 1 = 2\frac{x^2}{y^3} + \frac{2}{z} = 2\left(\frac{x^2}{y^3} + \frac{1}{z}\right)$$

$$\frac{\partial^2 u}{\partial z \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial z}\left(-\frac{x^2}{y^2} + 2\frac{y}{z}\right) = 2y\frac{d}{dz}\left(\frac{1}{z}\right) =$$

$$= 2y\left(-\frac{1}{z^2}\right) = -2\frac{y}{z^2}$$

$$\frac{\partial^2 u}{\partial x \partial z} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}\right) = \frac{\partial}{\partial x}\left(-\frac{y^2}{z^2} + 2z\right) = 0 = \frac{\partial^2 u}{\partial z \partial x}$$

$$\frac{\partial^2 u}{\partial y \partial z} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial z}\right) = \frac{\partial}{\partial y}\left(-\frac{y^2}{z^2} + 2z\right) = -\frac{1}{z^2}\frac{d}{dy}\left(y^2\right) =$$

$$= -\frac{1}{z^2}2y = -2\frac{y}{z^2} = \frac{\partial^2 u}{\partial z \partial y}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right) = \frac{\partial}{\partial z}\left(-\frac{y^2}{z^2} + 2z\right) = -y^2\frac{d}{dz}\left(\frac{1}{z^2}\right) + 2\frac{d}{dz}(z) =$$

$$= -y^2\left(-\frac{2}{z^3}\right) + 2\cdot 1 = 2\frac{y^2}{z^3} + 2 = 2\left(\frac{y^2}{z^3} + 1\right)$$

вариант 2:

$$du = d\left(\frac{256}{x} + \frac{x^2}{y} + \frac{y^2}{z} + z^2\right) = 256\, d\left(\frac{1}{x}\right) + d\left(\frac{x^2}{y}\right) + d\left(\frac{y^2}{z}\right) + d(z^2) = (1)$$

$$d\left(\frac{1}{x}\right) = \frac{d}{dx}\left(\frac{1}{x}\right)dx = -\frac{1}{x^2}dx = -\frac{dx}{x^2}$$

$$d\left(\frac{x^2}{y}\right) = d(x^2)\frac{1}{y} + x^2 d\left(\frac{1}{y}\right) = \frac{d}{dx}(x^2)dx\,\frac{1}{y} + x^2\frac{d}{dy}\left(\frac{1}{y}\right)dy =$$

$$= 2x\,dx\,\frac{1}{y} + x^2\left(-\frac{1}{y^2}\right)dy = 2\frac{x}{y}dx - \frac{x^2}{y^2}dy = 2\frac{x}{y}dx - \left(\frac{x}{y}\right)^2 dy$$

$$d\left(\frac{y^2}{z}\right) = d(y^2)\frac{1}{z} + y^2 d\left(\frac{1}{z}\right) = \frac{d}{dy}(y^2)\,dy\,\frac{1}{z} + y^2\frac{d}{dz}\left(\frac{1}{z}\right)dz =$$

$$= 2y\,dy\,\frac{1}{z} + y^2\left(-\frac{1}{z^2}\right)dz = 2\frac{y}{z}dy - \frac{y^2}{z^2}dz = 2\frac{y}{z}dy - \left(\frac{y}{z}\right)^2 dz$$

$$d(z^2) = \frac{d}{dz}(z^2)dz = 2z\,dz$$

$$(1) = 256\left(-\frac{dx}{x^2}\right) + \left(2\frac{x}{y}dx - \left(\frac{x}{y}\right)^2 dy\right) + \left(2\frac{y}{z}dy - \left(\frac{y}{z}\right)^2 dz\right) + 2z\,dz =$$

$$= \left(-\frac{256}{x^2} + 2\frac{x}{y}\right)dx + \left(-\left(\frac{x}{y}\right)^2 + 2\frac{y}{z}\right)dy + \left(-\left(\frac{y}{z}\right)^2 + 2z\right)dz =$$

$$= \left(-\left(\frac{16}{x}\right)^2 + 2\frac{x}{y}\right)dx + \left(-\left(\frac{x}{y}\right)^2 + 2\frac{y}{z}\right)dy + \left(-\left(\frac{y}{z}\right)^2 + 2z\right)dz$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$\frac{\partial u}{\partial x} = -\left(\frac{16}{x}\right)^2 + 2\frac{x}{y}$$

$$\frac{\partial u}{\partial y} = -\left(\frac{x}{y}\right)^2 + 2\frac{y}{z}$$

$$\frac{\partial u}{\partial z} = -\left(\frac{y}{z}\right)^2 + 2z$$

$$d^2u = d(du) = d\left(\left(-\left(\frac{16}{x}\right)^2 + 2\frac{x}{y}\right)dx + \left(-\left(\frac{x}{y}\right)^2 + 2\frac{y}{z}\right)dy + \left(-\left(\frac{y}{z}\right)^2 + 2z\right)dz\right) =$$

$$= d\left(\left(-\left(\frac{16}{x}\right)^2+2\frac{x}{y}\right)dx\right) + d\left(\left(-\left(\frac{x}{y}\right)^2+2\frac{y}{z}\right)dy\right) + d\left(\left(-\left(\frac{y}{z}\right)^2+2z\right)dz\right) =$$

$$= d\left(-\left(\frac{16}{x}\right)^2+2\frac{x}{y}\right)dx + d\left(-\left(\frac{x}{y}\right)^2+2\frac{y}{z}\right)dy + d\left(-\left(\frac{y}{z}\right)^2+2z\right)dz = (2)$$

$$d\left(-\left(\frac{16}{x}\right)^2+2\frac{x}{y}\right) = -d\left(\frac{16}{x}\right)^2 + 2d\left(\frac{x}{y}\right) = -2\left(\frac{16}{x}\right)d\left(\frac{16}{x}\right) + 2d\left(\frac{x}{y}\right) =$$

$$= -2\frac{16}{x}\,16d\left(\frac{1}{x}\right) + 2\left(d(x)\frac{1}{y} + x\,d\left(\frac{1}{y}\right)\right) =$$

$$= -2\frac{16}{x}\,16\frac{d}{dx}\left(\frac{1}{x}\right)dx + 2\left(dx\frac{1}{y} + x\frac{d}{dy}\left(\frac{1}{y}\right)dy\right) =$$

$$= -2\frac{16}{x}\,16\left(-\frac{1}{x^2}\right)dx + 2\left(dx\frac{1}{y} + x\left(-\frac{1}{y^2}\right)dy\right) =$$

$$= 2\left(\frac{16^2}{x^3} + \frac{1}{y}\right)dx - 2\frac{x}{y^2}dy$$

$$d\left(-\left(\frac{x}{y}\right)^2+2\frac{y}{z}\right) = -d\left(\frac{x}{y}\right)^2 + 2d\left(\frac{y}{z}\right) = -2\left(\frac{x}{y}\right)d\left(\frac{x}{y}\right) + 2d\left(\frac{y}{z}\right) =$$

$$= -2\frac{x}{y}\left(d(x)\frac{1}{y} + x\,d\left(\frac{1}{y}\right)\right) + 2\left(d(y)\frac{1}{z} + y\,d\left(\frac{1}{z}\right)\right) =$$

$$= -2\frac{x}{y}\left(dx\frac{1}{y} + x\frac{d}{dy}\left(\frac{1}{y}\right)dy\right) + 2\left(dy\frac{1}{z} + y\frac{d}{dz}\left(\frac{1}{z}\right)dz\right) =$$

$$= -2\frac{x}{y}\left(dx\frac{1}{y} + x\left(-\frac{1}{y^2}\right)dy\right) + 2\left(dy\frac{1}{z} + y\left(-\frac{1}{z^2}\right)dz\right) =$$

$$= -2\frac{x}{y^2}dx + 2\left(\frac{x^2}{y^3}+\frac{1}{z}\right)dy - 2\frac{y}{z^2}dz$$

$$d\left(-\left(\frac{y}{z}\right)^2+2z\right) = -d\left(\frac{y}{z}\right)^2 + 2d(z) = -2\left(\frac{y}{z}\right)d\left(\frac{y}{z}\right) + 2d(z) =$$

$$= -2\left(\frac{y}{z}\right)\left(d(y)\frac{1}{z} + y\,d\left(\frac{1}{z}\right)\right) + 2d(z) =$$

$$= -2\frac{y}{z}\left(dy\frac{1}{z} + y\frac{d}{dz}\left(\frac{1}{z}\right)dz\right) + 2dz =$$

$$= -2\frac{y}{z}\left(dy\frac{1}{z} + y\left(-\frac{1}{z^2}\right)dz\right) + 2dz =$$

$$= -2\frac{y}{z^2}dy + 2\left(\frac{y^2}{z^3}+1\right)dz$$

$$(2) = \left(2\left(\frac{16^2}{x^3}+\frac{1}{y}\right)dx - 2\frac{x}{y^2}dy\right)dx +$$

$$+\left(-2\frac{x}{y^2}dx + 2\left(\frac{x^2}{y^3}+\frac{1}{z}\right)dy - 2\frac{y}{z^2}dz\right)dy +$$

$$+\left(-2\frac{y}{z^2}dy + 2\left(\frac{y^2}{z^3}+1\right)dz\right)dz =$$

$$= 2\left(\frac{16^2}{x^3}+\frac{1}{y}\right)dx\,dx - 2\frac{x}{y^2}dy\,dx -$$

$$- \quad 2\frac{x}{y^2}dx\,dy + 2\left(\frac{x^2}{y^3}+\frac{1}{z}\right)dy\,dy - 2\frac{y}{z^2}dz\,dy -$$

$$- \quad 2\frac{y}{z^2}dy\,dz + 2\left(\frac{y^2}{z^3}+1\right)dz\,dz =$$

$$= 2\left(\frac{16^2}{x^3}+\frac{1}{y}\right)dx^2 + 2\left(\frac{x^2}{y^3}+\frac{1}{z}\right)dy^2 + 2\left(\frac{y^2}{z^3}+1\right)dz^2 +$$

$$- \quad 2\cdot 2\frac{x}{y^2}dx\,dy - 2\cdot 2\frac{y}{z^2}dy\,dz$$

$$d^2u = \frac{\partial^2 u}{\partial x^2}dx^2 + \frac{\partial^2 u}{\partial y^2}dy^2 + \frac{\partial^2 u}{\partial z^2}dz^2 +$$

$$+ 2\frac{\partial^2 u}{\partial x\partial y}dx\,dy + 2\frac{\partial^2 u}{\partial y\partial z}dy\,dz + 2\frac{\partial^2 u}{\partial z\partial x}dz\,dx$$

$$\frac{\partial^2 u}{\partial x^2} = 2\left(\frac{16^2}{x^3}+\frac{1}{y}\right) \; ; \; \frac{\partial^2 u}{\partial y^2} = 2\left(\frac{x^2}{y^3}+\frac{1}{z}\right) \; ; \; \frac{\partial^2 u}{\partial z^2} = 2\left(\frac{y^2}{z^3}+1\right)$$

$$\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial^2 u}{\partial y\partial x} = -2\frac{x}{y^2} \; ; \; \frac{\partial^2 u}{\partial y\partial z} = \frac{\partial^2 u}{\partial z\partial y} = -2\frac{y}{z^2} \; ; \; \frac{\partial^2 u}{\partial z\partial x} = \frac{\partial^2 u}{\partial x\partial z} = 0$$

Onbeen:

$$\frac{\partial u}{\partial x} = -\left(\frac{16}{x}\right)^2 + 2\frac{x}{y}$$

$$\frac{\partial u}{\partial y} = -\left(\frac{x}{y}\right)^2 + 2\frac{y}{z}$$

$$\frac{\partial u}{\partial z} = -\left(\frac{y}{z}\right)^2 + 2z$$

$$\frac{\partial^2 u}{\partial x^2} = 2\left(\frac{16^2}{x^3} + \frac{1}{y}\right)$$

$$\frac{\partial^2 u}{\partial y^2} = 2\left(\frac{x^2}{y^3} + \frac{1}{z}\right)$$

$$\frac{\partial^2 u}{\partial z^2} = 2\left(\frac{y^2}{z^3} + 1\right)$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x} = -2\frac{x}{y^2}$$

$$\frac{\partial^2 u}{\partial y \partial z} = \frac{\partial^2 u}{\partial z \partial y} = -2\frac{y}{z^2}$$

$$\frac{\partial^2 u}{\partial z \partial x} = \frac{\partial^2 u}{\partial x \partial z} = 0$$