$\boxed{5}$ $u(x,y,z) = \log_{21}(x^2+y^2+z^2)$

$F = (-19, 8, -4)$

Найти: $\dfrac{\partial u}{\partial \nabla u}(F)$

Решение:

$$\frac{\partial u}{\partial \nabla u} = \frac{\nabla u}{|\nabla u|} \cdot \nabla u = \frac{\nabla u^2}{|\nabla u|} = |\nabla u| =$$

$$= \left|\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)\right| = \left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right)^{\frac{1}{2}}$$

$$u = \log_{21}(x^2+y^2+z^2) \Leftrightarrow u = \log_a Z^2 \quad \Big| \quad Z = (x^2+y^2+z^2)^{\frac{1}{2}}, \ a = 21$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\log_a Z^2\right) = \frac{1}{Z^2 \ln a} \frac{\partial Z^2}{\partial x} = \Big[$$

$$\frac{\partial Z^2}{\partial x} = \frac{\partial}{\partial x}\left(x^2+y^2+z^2\right) = \frac{d}{dx}\left(x^2\right) = 2x$$

$$\Big] = \frac{1}{Z^2 \ln a} 2x = \frac{2}{\ln a} \frac{x}{Z^2}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\log_a Z^2\right) = \frac{1}{Z^2 \ln a} \frac{\partial Z^2}{\partial y} = \frac{1}{Z^2 \ln a} 2y = \frac{2}{\ln a} \frac{y}{Z^2}$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(\log_a Z^2\right) = \frac{1}{Z^2 \ln a} \frac{\partial Z^2}{\partial z} = \frac{1}{Z^2 \ln a} 2z = \frac{2}{\ln a} \frac{z}{Z^2}$$

$$|\nabla u| = \left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right)^{\frac{1}{2}} =$$

$$= \left(\left(\frac{2}{\ln a} \frac{x}{Z^2}\right)^2 + \left(\frac{2}{\ln a} \frac{y}{Z^2}\right)^2 + \left(\frac{2}{\ln a} \frac{z}{Z^2}\right)^2\right)^{\frac{1}{2}} =$$

$$= \left(\left(\frac{2}{\ln a}\right)^2 \frac{x^2}{Z^4} + \left(\frac{2}{\ln a}\right)^2 \frac{y^2}{Z^4} + \left(\frac{2}{\ln a}\right)^2 \frac{z^2}{Z^4}\right)^{\frac{1}{2}} =$$

$$= \left(\left(\frac{2}{\ln a}\right)^2 \frac{x^2+y^2+z^2}{z^4} \right)^{\frac{1}{2}} = \left(\left(\frac{2}{\ln a}\right)^2 \frac{z^2}{z^4} \right)^{\frac{1}{2}} = \left(\left(\frac{2}{\ln a}\right)^2 \frac{1}{z^2} \right)^{\frac{1}{2}} =$$

$$= \left(\left(\frac{2}{z\ln a}\right)^2 \right)^{\frac{1}{2}} = \left| \frac{2}{z\ln a} \right| = (1)$$

$u(x,y,z) = \log_a z^2(x,y,z) \mid z(x,y,z) = (x^2+y^2+z^2)^{\frac{1}{2}},\ a = 21$

$D(u) = \{ (x,y,z) \in \mathbb{R} \mid z^2(x,y,z) > 0 \}$

$z^2 = x^2+y^2+z^2 > 0 \Leftrightarrow \neg(x^2+y^2+z^2 = 0) \Leftrightarrow \neg(x=0 \wedge y=0 \wedge z=0) \Leftrightarrow$

$\Leftrightarrow \neg(x=0) \vee \neg(y=0) \vee \neg(z=0) \Leftrightarrow x \neq 0 \vee y \neq 0 \vee z \neq 0$

$D(u) = \{ (x,y,z) \mid x \neq 0 \vee y \neq 0 \vee z \neq 0 \} \Rightarrow$

$\Rightarrow z = (x^2+y^2+z^2)^{\frac{1}{2}} > 0$

$\ln a = \ln 21 > 0$

$\frac{2}{z\ln a} > 0$

$(1) = \frac{2}{z\ln a}$

$$\frac{\partial u}{\partial \nabla u}(F) = |\nabla u|(F) = \left(\frac{2}{z\ln a}\right)(F) = \frac{2}{z(F)\ln a} = (2)$$

$$z(F) = (x^2+y^2+z^2)^{\frac{1}{2}} \Big|_{x=-19,\ y=8,\ z=-4} =$$

$$= ((-19)^2 + 8^2 + (-4)^2)^{\frac{1}{2}} = (361+64+16)^{\frac{1}{2}} = (441)^{\frac{1}{2}} = (21^2)^{\frac{1}{2}} = 21$$

$$(2) = \frac{2}{21\ln 21}$$

Ответ: $\boxed{\frac{\partial u}{\partial \nabla u}(F) = \frac{2}{21\ln 21}}$