1. $u(x,y,z) = x^3 + 3xy^2 + z^2 - 39x - 36y + 2z + 26$

Найти: $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}$;

$\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y \partial x}, \frac{\partial^2 u}{\partial z \partial x}$; $\frac{\partial^2 u}{\partial x \partial y}, \frac{\partial^2 u}{\partial y^2}, \frac{\partial^2 u}{\partial z \partial y}$; $\frac{\partial^2 u}{\partial x \partial z}, \frac{\partial^2 u}{\partial y \partial z}, \frac{\partial^2 u}{\partial z^2}$;

Решение:

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(x^3 + 3xy^2 + z^2 - 39x - 36y + 2z + 26\right) =$$

$$= \frac{d}{dx}\left(x^3\right) + 3y^2 \frac{d}{dx}(x) - 39\frac{d}{dx}(x) =$$

$$= 3x^2 + 3y^2 \cdot 1 - 39 \cdot 1 = 3x^2 + 3y^2 - 39 = 3\left(x^2 + y^2 - 13\right)$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(x^3 + 3xy^2 + z^2 - 39x - 36y + 2z + 26\right) =$$

$$= 3x\frac{d}{dy}\left(y^2\right) - 36\frac{d}{dy}(y) =$$

$$= 3x \cdot 2y - 36 \cdot 1 = 6xy - 36 = 6(xy - 6)$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(x^3 + 3xy^2 + z^2 - 39x - 36y + 2z + 26\right) =$$

$$= \frac{d}{dz}\left(z^2\right) + 2\frac{d}{dz}(z) =$$

$$= 2z + 2 \cdot 1 = 2z + 2 = 2(z+1)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(3x^2 + 3y^2 - 39\right) =$$

$$= 3\frac{d}{dx}\left(x^2\right) = 3 \cdot 2x = 6x$$

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial y}\left(3x^2 + 3y^2 - 39\right) =$$

$$= 3\frac{d}{dy}\left(y^2\right) = 3 \cdot 2y = 6y$$

$$\frac{\partial^2 u}{\partial z \partial x} = \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial z}\left(3x^2 + 3y^2 - 39\right) = 0$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}\left(6xy - 3z\right) =$$

$$= 6y \frac{d}{dx}(x) = 6y \cdot 1 = 6y =$$

$$= \frac{\partial^2 u}{\partial y \partial x}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(6xy - 3z\right) =$$

$$= 6x \frac{d}{dy}(y) = 6x \cdot 1 = 6x$$

$$\frac{\partial^2 u}{\partial z \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial z}\left(6xy - 3z\right) = 0$$

$$\frac{\partial^2 u}{\partial x \partial z} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}\right) = \frac{\partial}{\partial x}\left(2z + 2\right) = 0 =$$

$$= \frac{\partial^2 u}{\partial z \partial x}$$

$$\frac{\partial^2 u}{\partial y \partial z} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial z}\right) = \frac{\partial}{\partial y}\left(2z + 2\right) = 0 =$$

$$= \frac{\partial^2 u}{\partial z \partial y}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right) = \frac{\partial}{\partial z}\left(2z + 2\right) =$$

$$= 2 \frac{d}{dz}(z) = 2 \cdot 1 = 2$$

Вариант 2:

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$du = d\left(x^3 + 3xy^2 + z^2 - 39x - 36y + 2z + 26\right) =$$

$$= d(x^3) + 3\,d(xy^2) + d(z^2) - 39\,d(x) - 36\,d(y) + 2\,d(z) = (1)$$

$$d(x^3) = \frac{d}{dx}(x^3)\,dx = 3x^2dx$$

$$d(xy^2) = d(x)\,y^2 + x\,d(y^2) = y^2dx + x\,\frac{d}{dy}(y^2)dy =$$

$$= y^2dx + x\,2y\,dy = y^2dx + 2xy\,dy$$

$$d(z^2) = \frac{d}{dz}(z^2)dz = 2z\,dz$$

$$(1) = 3x^2dx + 3(y^2dx + 2xy\,dy) + 2z\,dz - 39dx - 36dy + 2dz =$$

$$= (3x^2 + 3y^2 - 39)\,dx + (6xy - 36)\,dy + (2z + 2)dz =$$

$$= 3(x^2 + y^2 - 13)\,dx + 6(xy - 6)\,dy + 2(z + 1)dz$$

$$\frac{\partial u}{\partial x} = 3(x^2 + y^2 - 13)$$

$$\frac{\partial u}{\partial y} = 6(xy - 6)$$

$$\frac{\partial u}{\partial z} = 2(z + 1)$$

$$d^2U = d(dU) = d\left(\frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz\right) = d\left(\frac{\partial U}{\partial x}dx\right) + d\left(\frac{\partial U}{\partial y}dy\right) + d\left(\frac{\partial U}{\partial z}dz\right) =$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial x}dx\right)dx + \frac{\partial}{\partial y}\left(\frac{\partial U}{\partial x}dx\right)dy + \frac{\partial}{\partial z}\left(\frac{\partial U}{\partial x}dx\right)dz +$$

$$+ \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial y}dy\right)dx + \frac{\partial}{\partial y}\left(\frac{\partial U}{\partial y}dy\right)dy + \frac{\partial}{\partial z}\left(\frac{\partial U}{\partial y}dy\right)dz +$$

$$+ \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial z}dz\right)dx + \frac{\partial}{\partial y}\left(\frac{\partial U}{\partial z}dz\right)dy + \frac{\partial}{\partial z}\left(\frac{\partial U}{\partial z}dz\right)dz =$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial x}\right)dx\,dx + \frac{\partial}{\partial y}\left(\frac{\partial U}{\partial x}\right)dx\,dy + \frac{\partial}{\partial z}\left(\frac{\partial U}{\partial x}\right)dx\,dz +$$

$$+ \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial y}\right)dy\,dx + \frac{\partial}{\partial y}\left(\frac{\partial U}{\partial y}\right)dy\,dy + \frac{\partial}{\partial z}\left(\frac{\partial U}{\partial y}\right)dy\,dz +$$

$$+ \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial z}\right)dz\,dx + \frac{\partial}{\partial y}\left(\frac{\partial U}{\partial z}\right)dz\,dy + \frac{\partial}{\partial z}\left(\frac{\partial U}{\partial z}\right)dz\,dz =$$

$$= \frac{\partial^2 U}{\partial x^2}dx^2 + \frac{\partial^2 U}{\partial y\partial x}dy\,dx + \frac{\partial^2 U}{\partial z\partial x}dz\,dx +$$

$$+ \frac{\partial^2 U}{\partial x\partial y}dx\,dy + \frac{\partial^2 U}{\partial y^2}dy^2 + \frac{\partial^2 U}{\partial z\partial y}dz\,dy +$$

$$+ \frac{\partial^2 U}{\partial x\partial z}dx\,dz + \frac{\partial^2 U}{\partial y\partial z}dy\,dz + \frac{\partial^2 U}{\partial z^2}dz^2 =$$

$$= \frac{\partial^2 U}{\partial x^2}dx^2 + \frac{\partial^2 U}{\partial y^2}dy^2 + \frac{\partial^2 U}{\partial z^2}dz^2 +$$

$$+ \left(\frac{\partial^2 U}{\partial x\partial y} + \frac{\partial^2 U}{\partial y\partial x}\right)dx\,dy + \left(\frac{\partial^2 U}{\partial z\partial y} + \frac{\partial^2 U}{\partial y\partial z}\right)dy\,dz + \left(\frac{\partial^2 U}{\partial x\partial z} + \frac{\partial^2 U}{\partial z\partial x}\right)dx\,dz =$$

$$\frac{\partial^2 U}{\partial x\partial y} = \frac{\partial^2 U}{\partial y\partial x} \;;\; \frac{\partial^2 U}{\partial y\partial z} = \frac{\partial^2 U}{\partial z\partial y} \;;\; \frac{\partial^2 U}{\partial x\partial z} = \frac{\partial^2 U}{\partial z\partial x}$$

$$= \frac{\partial^2 U}{\partial x^2}dx^2 + \frac{\partial^2 U}{\partial y^2}dy^2 + \frac{\partial^2 U}{\partial z^2}dz^2 +$$

$$+ 2\frac{\partial^2 U}{\partial x\partial y}dx\,dy + 2\frac{\partial^2 U}{\partial y\partial z}dy\,dz + 2\frac{\partial^2 U}{\partial x\partial z}dx\,dz$$

$$d^2u = d(du) = d\left((3x^2+3y^2-3z)dx + (6xy-3y)dy + (2z+2)dz\right) =$$

$$= d\left((3x^2+3y^2-3z)dx\right) + d\left((6xy-3y)dy\right) + d\left((2z+2)dz\right) =$$

$$= d(3x^2+3y^2-3z)dx + d(6xy-3y)dy + d(2z+2)dz =$$

$$= \left(3\,d(x^2)+3d(y^2)\right)dx + 6\,d(xy)dy + 2d(z)dz = (2)$$

$$d(x^2) = \frac{d}{dx}(x^2)dx = 2x\,dx$$

$$d(y^2) = \frac{d}{dy}(y^2)dy = 2y\,dy$$

$$d(xy) = d(x)y + x\,d(y) = y\,dx + x\,dy$$

$$(2) = \left(3\cdot 2x\,dx + 3\cdot 2y\,dy\right)dx + 6\left(y\,dx + x\,dy\right)dy + 2\,dz\,dz =$$

$$= 6x\,dx\,dx + 6y\,dy\,dx + 6y\,dx\,dy + 6x\,dy\,dy + 2\,dz\,dz =$$

$$= 6x\,dx^2 + 2\cdot 6y\,dx\,dy + 6x\,dy^2 + 2\,dz^2 =$$

$$= 6x\,dx^2 + 6x\,dy^2 + 2\,dz^2 + 2\cdot 6y\,dx\,dy$$

$$\frac{\partial^2 u}{\partial x^2} = 6x \;;\; \frac{\partial^2 u}{\partial y^2} = 6x \;;\; \frac{\partial^2 u}{\partial z^2} = 2$$

$$\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial^2 u}{\partial y\partial x} = 6y \;;\; \frac{\partial^2 u}{\partial z\partial y} = \frac{\partial^2 u}{\partial y\partial z} = 0 \;;\; \frac{\partial^2 u}{\partial z\partial x} = \frac{\partial^2 u}{\partial x\partial z} = 0$$

Ответ:

$$\frac{\partial u}{\partial x} = 3(x^2+y^2-1z) \;;\; \frac{\partial u}{\partial y} = 6(xy-6) \;;\; \frac{\partial u}{\partial z} = 2(z+1)$$

$$\frac{\partial^2 u}{\partial x^2} = 6x \;;\; \frac{\partial^2 u}{\partial y^2} = 6x \;;\; \frac{\partial^2 u}{\partial z^2} = 2$$

$$\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial^2 u}{\partial y\partial x} = 6y \;;\; \frac{\partial^2 u}{\partial z\partial y} = \frac{\partial^2 u}{\partial y\partial z} = 0 \;;\; \frac{\partial^2 u}{\partial z\partial x} - \frac{\partial^2 u}{\partial x\partial z} = 0$$