II Найти: $\int \frac{2x+3}{(x-2)(x+5)}\,dx$

Решение:

$(\alpha,\beta)\in[\mathbb{R}^2/(0,0)]$: $2x+3=\alpha(x-2)+\beta(x+5)$

$\alpha(x-2)+\beta(x+5)=(\alpha x-2\alpha)+(\beta x+5\beta)=(\alpha x+\beta x)+(-2\alpha+5\beta)=$

$=(\alpha+\beta)x+(-2\alpha+5\beta)$

$2x+3=\alpha(x-2)+\beta(x+5) \Leftrightarrow (\alpha+\beta)x+(-2\alpha+5\beta)=2x+3 \Leftrightarrow$

$\Leftrightarrow \begin{cases} \alpha+\beta=2 \\ -2\alpha+5\beta=3 \end{cases} \Leftrightarrow \begin{pmatrix} 1 & 1 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}=\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\Delta=\det\begin{pmatrix} 1 & 1 \\ -2 & 5 \end{pmatrix}=1\cdot 5-1\,(-2)=5+2=7$

$\Delta=7\neq 0 \Rightarrow \exists \begin{pmatrix} 1 & 1 \\ -2 & 5 \end{pmatrix}^{-1} \Rightarrow \exists \alpha,\beta\in\mathbb{R}: \begin{pmatrix} \alpha \\ \beta \end{pmatrix}=\begin{pmatrix} 1 & 1 \\ -2 & 5 \end{pmatrix}^{-1}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\Delta_1=\det\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}=2\cdot 5-1\cdot 3=10-3=7$

$\Delta_2=\det\begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}=1\cdot 3-2(-2)=3+4=7$

$\alpha=\frac{\Delta_1}{\Delta}=\frac{7}{7}=1$

$\beta=\frac{\Delta_2}{\Delta}=\frac{7}{7}=1$

$2x+3=\alpha(x-2)+\beta(x+5)=1\cdot(x-2)+1(x+5)=(x-2)+(x+5) \Rightarrow$

$\Rightarrow \frac{2x+3}{(x-2)(x+5)}=\frac{(x-2)+(x+5)}{(x-2)(x+5)}=\frac{x-2}{(x-2)(x+5)}+\frac{x+5}{(x-2)(x+5)}=$

$=\frac{1}{x+5}+\frac{1}{x-2} \Rightarrow$

$$\Rightarrow \frac{2x+3}{(x-2)(x+5)}dx = \left(\frac{1}{x+5} + \frac{1}{x-2}\right)dx =$$

$$= \frac{dx}{x+5} + \frac{dx}{x-2} = \frac{d(x+5)}{x+5} + \frac{d(x-2)}{x-2} =$$

$$= d\ln|x+5| + d\ln|x-2| = d\left(\ln|x+5| + \ln|x-2|\right) =$$

$$= d\ln\left(|x+5||x-2|\right) = d\ln|(x+5)(x-2)| \Rightarrow$$

$$\Rightarrow \int \frac{2x+3}{(x-2)(x+5)}dx = \int d\ln|(x+5)(x-2)| =$$

$$= \ln|(x+5)(x-2)| + C$$

Проверка:

$$\frac{d}{dx}\left(\ln|(x+5)(x-2)| + C\right) = \frac{d}{dx}\ln|(x+5)(x-2)| =$$

$$= \frac{1}{(x+5)(x-2)} \cdot \frac{d}{dx}\left((x+5)(x-2)\right) = \frac{\frac{d}{dx}(x+5)\,(x-2) + (x+5)\,\frac{d}{dx}(x-2)}{(x+5)(x-2)} =$$

$$= \frac{1\cdot(x-2) + (x+5)\cdot 1}{(x+5)(x-2)} = \frac{(x-2)+(x+5)}{(x+5)(x-2)} =$$

$$= \frac{2x+3}{(x-2)(x+5)}$$

Ответ: $\boxed{\int \frac{2x+3}{(x-2)(x+5)}dx = \ln|(x+5)(x-2)| + C}$