$\boxed{2}$

Найти условный экстремум $u(x,y) = 2x^2 + 12xy + 32y^2 + 15$
при условии $x^2 + 16y^2 = 64$

Решение:

$u(x,y) = 2x^2 + 12xy + 32y^2 + 15$; $D(u) = \mathbb{R}^2$

$x^2 + 16y^2 = 64 \Leftrightarrow x^2 + 16y^2 - 64 = 0 \Leftrightarrow$

$\Leftrightarrow f(x,y) = 0$ | $f(x,y) = x^2 + 16y^2 - 64$; $D(f) = \mathbb{R}^2$

$L(x,y,\lambda) = u(x,y) + \lambda f(x,y)$

$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(2x^2 + 12xy + 32y^2 + 15) = 2\frac{d}{dx}(x^2) + 12y\frac{d}{dx}(x) =$

$= 2 \cdot 2x + 12y \cdot 1 = 4x + 12y = 4(x + 3y)$

$\frac{\partial u}{\partial y}(2x^2 + 12xy + 32y^2 + 15) = 12x\frac{d}{dy}(y) + 32\frac{d}{dy}(y^2) =$

$= 12x \cdot 1 + 32 \cdot 2y = 12x + 64y = 4(3x + 16y)$

$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 + 16y^2 - 64) = \frac{d}{dx}(x^2) = 2x$

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 + 16y^2 - 64) = 16\frac{d}{dy}(y^2) = 16 \cdot 2y = 32y$

$\frac{\partial L}{\partial x} = \frac{\partial}{\partial x}(u + \lambda f) = \frac{\partial u}{\partial x} + \lambda\frac{\partial f}{\partial x} = 4x + 12y + \lambda \cdot 2x =$

$= (2\lambda + 4)x + 12y = 2((\lambda + 2)x + 6y)$

$\frac{\partial L}{\partial y} = \frac{\partial}{\partial y}(u + \lambda f) = \frac{\partial u}{\partial y} + \lambda\frac{\partial f}{\partial y} = 12x + 64y + \lambda \cdot 32y =$

$= 12x + (32\lambda + 64)y = 4(3x + 8(\lambda + 2)y)$

$\frac{\partial L}{\partial \lambda} = \frac{\partial}{\partial \lambda}(u + \lambda f) = f\frac{d}{d\lambda}(\lambda) = f \cdot 1 = f = x^2 + 16y^2 - 64$

$$\begin{cases} \dfrac{\partial L}{\partial x} = 0 \\[2mm] \dfrac{\partial L}{\partial y} = 0 \\[2mm] \dfrac{\partial L}{\partial \lambda} = 0 \end{cases} \Leftrightarrow \begin{cases} 2\big((\lambda+2)x+6y\big) = 0 \\[2mm] 4\big(3x+8(\lambda+2)y\big) = 0 \\[2mm] x^2+16y^2-64=0 \end{cases} \Leftrightarrow \begin{cases} (\lambda+2)x+6y = 0 \\[2mm] 3x+8(\lambda+2)y = 0 \\[2mm] x^2+16y^2=64 \end{cases} \overset{(1)}{\Leftrightarrow}$$

$$\begin{cases} (\lambda+2)x+6y=0 \\ 3x+8(\lambda+2)y=0 \end{cases} \Leftrightarrow \begin{pmatrix} (\lambda+2) & 6 \\ 3 & 8(\lambda+2) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Delta = \det \begin{pmatrix} (\lambda+2) & 6 \\ 3 & 8(\lambda+2) \end{pmatrix} = (\lambda+2)\,8(\lambda+2) - 6\cdot 3 =$$

$$= 2^3(\lambda+2)^2 - 2\cdot 3^2 = 2\left(2^2(\lambda+2)^2 - 3^2\right) = 2\left(\big(2(\lambda+2)\big)^2 - 3^2\right) =$$

$$= 2\big(2(\lambda+2)-3\big)\big(2(\lambda+2)+3\big) = 2(2\lambda+1)(2\lambda+7)$$

$$\Delta = 0 \Leftrightarrow 2(2\lambda+1)(2\lambda+7) = 0 \Leftrightarrow$$

$$\Leftrightarrow (2\lambda+1)(2\lambda+7) = 0 \Leftrightarrow (2\lambda+1) = 0 \wedge (2\lambda+7) = 0 \Leftrightarrow$$

$$\Leftrightarrow \lambda = -\frac{1}{2} \wedge \lambda = -\frac{7}{2}$$

$$\begin{pmatrix} \lambda+2 & 6 \\ 3 & 8(\lambda+2) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow \left(\begin{array}{cc|c} (\lambda+2) & 6 & 0 \\ 3 & 8(\lambda+2) & 0 \end{array}\right)$$

$$\lambda = -\frac{1}{2} \Leftrightarrow \lambda + 2 = -\frac{1}{2} + 2 = \frac{3}{2}$$

$$\left(\begin{array}{cc|c} (\lambda+2) & 6 & 0 \\ 3 & 8(\lambda+2) & 0 \end{array}\right) \Leftrightarrow \left(\begin{array}{cc|c} \frac{3}{2} & 6 & 0 \\ 3 & 8\frac{3}{2} & 0 \end{array}\right) \Leftrightarrow$$

$$\Leftrightarrow \left(\begin{array}{cc|c} \frac{3}{2} & 6 & 0 \\ 3 & 3\cdot 4 & 0 \end{array}\right) \Leftrightarrow \left(\begin{array}{cc|c} \frac{2}{3}\cdot\frac{3}{2} & \frac{2}{3}\cdot 6 & \frac{2}{3}\cdot 0 \\ \frac{1}{3}\cdot 3 & \frac{1}{3}\cdot 3\cdot 4 & \frac{1}{3}\cdot 0 \end{array}\right) \Leftrightarrow$$

$$\Leftrightarrow \left(\begin{array}{cc|c} 1 & 2\cdot 2 & 0 \\ 1 & 4 & 0 \end{array}\right) \Leftrightarrow \left(\begin{array}{cc|c} 1 & 4 & 0 \\ 1 & 4 & 0 \end{array}\right) \Leftrightarrow$$

$$\Leftrightarrow x + 4y = 0 \Leftrightarrow y = -\frac{x}{4}$$

$$\lambda = -\frac{7}{2} \Leftrightarrow \lambda + 2 = -\frac{7}{2} + 2 = -\frac{3}{2}$$

$$\left(\begin{array}{cc|c} (\lambda+2) & 6 & 0 \\ 3 & 8(\lambda+2) & 0 \end{array}\right) \Leftrightarrow \left(\begin{array}{cc|c} -\frac{3}{2} & 6 & 0 \\ 3 & 8\left(-\frac{3}{2}\right) & 0 \end{array}\right) \Leftrightarrow$$

$$\Leftrightarrow \left(\begin{array}{cc|c} -\frac{3}{2} & 6 & 0 \\ 3 & -3\cdot 4 & 0 \end{array}\right) \Leftrightarrow \left(\begin{array}{cc|c} \left(-\frac{2}{3}\right)\left(-\frac{3}{2}\right) & \left(-\frac{2}{3}\right)6 & \left(-\frac{2}{3}\right)\cdot 0 \\ \frac{1}{3}\cdot 3 & \frac{1}{3}\cdot(-3\cdot 4) & \frac{1}{3}\cdot 0 \end{array}\right) \Leftrightarrow$$

$$\Leftrightarrow \left(\begin{array}{cc|c} 1 & -2\cdot 2 & 0 \\ 1 & -4 & 0 \end{array}\right) \Leftrightarrow \left(\begin{array}{cc|c} 1 & -4 & 0 \\ 1 & -4 & 0 \end{array}\right) \Leftrightarrow$$

$$\Leftrightarrow x - 4y = 0 \Leftrightarrow y = \frac{x}{4}$$

$$\Delta \neq 0 \Leftrightarrow \neg\left(\lambda = -\tfrac{1}{2} \vee \lambda = -\tfrac{7}{2}\right) \Leftrightarrow$$

$$\Leftrightarrow \neg\left(\lambda = -\tfrac{1}{2}\right) \wedge \neg\left(\lambda = -\tfrac{7}{2}\right) \Leftrightarrow \lambda \neq -\tfrac{1}{2} \wedge \lambda \neq -\tfrac{7}{2}$$

$$\begin{pmatrix} (\lambda+1) & 6 \\ 3 & 8(\lambda+2) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (\lambda+2) & 6 \\ 3 & 8(\lambda+2) \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow x = 0 \wedge y = 0$$

$$\overset{(1)}{\Leftrightarrow} \left[\begin{array}{l} \left[\begin{array}{l} \begin{cases} \lambda = -\frac{1}{2} \\ y = -\frac{x}{4} \end{cases} \\ \begin{cases} \lambda = -\frac{7}{2} \\ y = \frac{x}{4} \end{cases} \\ \begin{cases} \lambda \neq -\frac{1}{2} \wedge \lambda \neq -\frac{7}{2} \\ x = 0 \wedge y = 0 \end{cases} \end{array}\right. \\ x^2 + 16y^2 = 64 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \lambda = -\frac{1}{2} \\ y = -\frac{x}{4} \\ x^2 + 16y^2 = 64 \end{cases} \\ \begin{cases} \lambda = -\frac{7}{2} \\ y = \frac{x}{4} \\ x^2 + 16y^2 = 64 \end{cases} \\ \begin{cases} \lambda \neq -\frac{1}{2} \wedge \lambda \neq -\frac{7}{2} \\ x = 0 \wedge y = 0 \\ x^2 + 16y^2 = 64 \end{cases} \end{array}\right. \overset{(2)}{\Leftrightarrow}$$

$$\begin{cases} y = -\frac{x}{4} \\ x^2 + 16y^2 = 64 \end{cases} \Leftrightarrow$$

$$x^2+16y^2 \Big|_{y=-\frac{x}{4}} = x^2 + 16\left(-\frac{x}{4}\right)^2 = x^2 + 16\frac{x^2}{16} = x^2 + x^2 = 2x^2$$

$$\Leftrightarrow \begin{cases} y = -\frac{x}{4} \\ 2x^2 = 64 \end{cases} \Leftrightarrow$$

$$2x^2 = 64 \Leftrightarrow x^2 = 32 = 2^5 \Leftrightarrow x = 2^{\frac{5}{2}} \vee x = -2^{\frac{5}{2}}$$

$$\Leftrightarrow \begin{cases} y = -\frac{x}{4} \\ \left[\begin{array}{l} x = 2^{\frac{5}{2}} \\ x = -2^{\frac{5}{2}} \end{array}\right. \end{cases} \Leftrightarrow \left[\begin{array}{l} \begin{cases} x = 2^{\frac{5}{2}} = 2^{2\frac{1}{2}} = 2^2\sqrt{2} = 4\sqrt{2} \\ y = -\frac{x}{4} = -\frac{x}{2^2} = -2^{\frac{5}{2}}2^{-2} = -2^{\frac{1}{2}} = -\sqrt{2} \end{cases} \\ \begin{cases} x = -2^{\frac{5}{2}} = -2^{2\frac{1}{2}} = -2^2\sqrt{2} = -4\sqrt{2} \\ y = -\frac{x}{4} = -\frac{x}{2^2} = -\left(-2^{\frac{5}{2}}\right)2^{-2} = 2^{\frac{1}{2}} = \sqrt{2} \end{cases} \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left(x = 4\sqrt{2} \wedge y = -\sqrt{2}\right) \vee \left(x = -4\sqrt{2} \wedge y = \sqrt{2}\right)$$

$$\begin{cases} y = \frac{x}{4} \\ x^2 + 16y^2 = 64 \end{cases} \Leftrightarrow$$

$$x^2+16y^2 \Big|_{y=\frac{x}{4}} = x^2 + 16\left(\frac{x}{4}\right)^2 = x^2 + 16\frac{x^2}{16} = x^2 + x^2 = 2x^2$$

$$\Leftrightarrow \begin{cases} y = \frac{x}{4} \\ 2x^2 = 64 \end{cases} \Leftrightarrow$$

$$2x^2 = 64 \Leftrightarrow x^2 = 32 = 2^5 \Leftrightarrow x = 2^{\frac{5}{2}} \vee x = -2^{\frac{5}{2}}$$

$$\Leftrightarrow \begin{cases} y = \frac{x}{4} \\ \left[\begin{array}{l} x = 2^{\frac{5}{2}} \\ x = -2^{\frac{5}{2}} \end{array}\right. \end{cases} \Leftrightarrow \left[\begin{array}{l} \begin{cases} x = 2^{\frac{5}{2}} = 2^{2\frac{1}{2}} = 2^2\sqrt{2} = 4\sqrt{2} \\ y = \frac{x}{4} = \frac{x}{2^2} = 2^{\frac{5}{2}}\,2^{-2} = 2^{\frac{1}{2}} = \sqrt{2} \end{cases} \\ \begin{cases} x = -2^{\frac{5}{2}} = -2^{2\frac{1}{2}} = -2^2\sqrt{2} = -4\sqrt{2} \\ y = \frac{x}{4} = \frac{x}{2^2} = \left(-2^{\frac{5}{2}}\right)2^{-2} = -2^{\frac{1}{2}} = -\sqrt{2} \end{cases} \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left(x = 4\sqrt{2} \wedge y = \sqrt{2}\right) \vee \left(x = -4\sqrt{2} \wedge y = -\sqrt{2}\right)$$

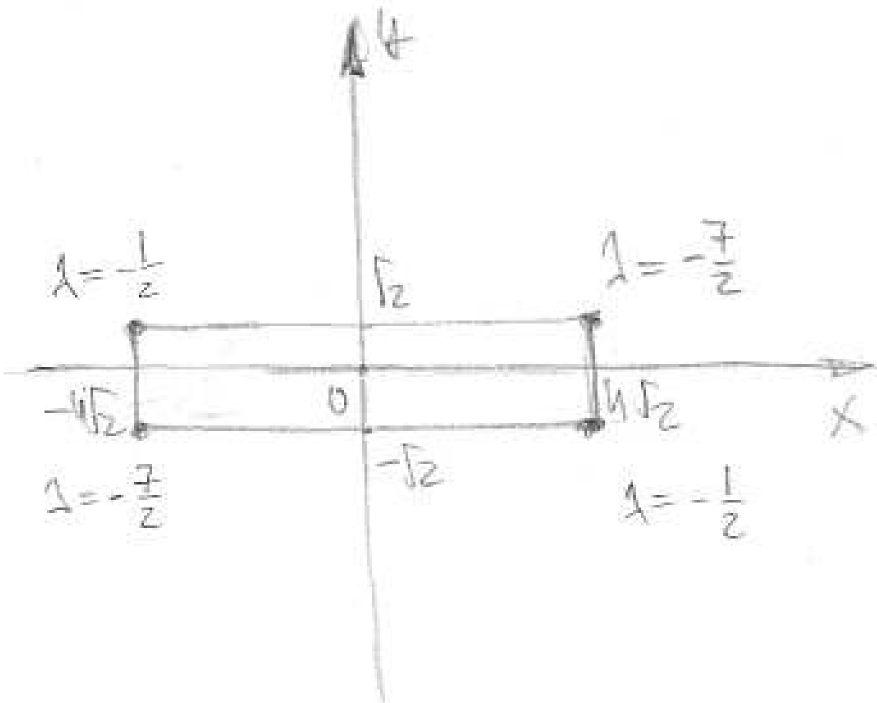
$$\begin{cases} x = 0 \wedge y = 0 \\ x^2 + 16y^2 = 64 \end{cases} \Leftrightarrow$$

$$x^2 + 16y^2 \Big|_{x=0;\, y=0} = 0^2 + 16 \cdot 0^2 = 0 + 0 = 0$$

$$\Leftrightarrow \begin{cases} x = 0 \wedge y = 0 \\ 0 = 64 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow (x, y) \in \emptyset$$

$$\overset{(2)}{\Leftrightarrow} \left[\begin{array}{l} \begin{cases} \lambda = -\frac{1}{2} \\ \left[\begin{array}{l} x = 4\sqrt{2} \wedge y = -\sqrt{2} \\ x = -4\sqrt{2} \wedge y = \sqrt{2} \end{array} \right. \end{cases} \\ \begin{cases} \lambda = -\frac{7}{2} \\ \left[\begin{array}{l} x = 4\sqrt{2} \wedge y = \sqrt{2} \\ x = -4\sqrt{2} \wedge y = -\sqrt{2} \end{array} \right. \end{cases} \end{array} \right.$$



$$\boxed{\begin{array}{l} \left(\lambda = -\frac{1}{2} \wedge \left((x = 4\sqrt{2} \wedge y = -\sqrt{2}) \vee (x = -4\sqrt{2} \wedge y = \sqrt{2}) \right)\right) \vee \\ \vee \left(\lambda = -\frac{7}{2} \wedge \left((x = 4\sqrt{2} \wedge y = \sqrt{2}) \vee (x = -4\sqrt{2} \wedge y = -\sqrt{2}) \right)\right) \\ \frac{\partial L}{\partial x}(x, y, \lambda) = 0 \wedge \frac{\partial L}{\partial y}(x, y, \lambda) = 0 \wedge \frac{\partial L}{\partial \lambda}(x, y, \lambda) = 0 \end{array}}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(4x+12y\right) = 4\frac{d}{dx}(x) = 4\cdot 1 = 4$$

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial y}\left(4x+12y\right) = 12\frac{d}{dy}(y) = 12\cdot 1 = 12$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}\left(12x+64y\right) = 12\frac{d}{dx}(x) = 12\cdot 1 = 12 = \frac{\partial^2 u}{\partial y \partial x}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(12x+64y\right) = 64\frac{d}{dy}(y) = 64\cdot 1 = 64$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(2x\right) = 2\frac{d}{dx}(x) = 2\cdot 1 = 2$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(2x\right) = 0$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(32y\right) = 0 = \frac{\partial^2 f}{\partial y \partial x}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(32y\right) = 32\frac{d}{dy}(y) = 32\cdot 1 = 32$$

$$\frac{\partial^2 L}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial L}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \lambda\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \lambda\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) =$$

$$= \frac{\partial^2 u}{\partial x^2} + \lambda\frac{\partial^2 f}{\partial x^2} = 4 + \lambda 2 = 2\lambda + 4 = 2(\lambda+2)$$

$$\frac{\partial^2 L}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial L}{\partial x}\right) = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \lambda\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) + \lambda\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) =$$

$$= \frac{\partial^2 u}{\partial y \partial x} + \lambda\frac{\partial^2 f}{\partial y \partial x} = 12 + \lambda\cdot 0 = 12$$

$$\frac{\partial^2 L}{\partial \lambda \partial x} = \frac{\partial}{\partial \lambda}\left(\frac{\partial L}{\partial x}\right) = \frac{\partial}{\partial \lambda}\left(\frac{\partial u}{\partial x} + \lambda\frac{\partial f}{\partial x}\right) = \frac{\partial f}{\partial x}\frac{d}{d\lambda}(\lambda) = \frac{\partial f}{\partial x}\cdot 1 = \frac{\partial f}{\partial x}$$

$$= \frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial^2 L}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial L}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} + \lambda\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) + \lambda\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) =$$

$$= \frac{\partial^2 u}{\partial x \partial y} + \lambda\frac{\partial^2 f}{\partial x \partial y} = 12 + \lambda \cdot 0 = 12 = \frac{\partial^2 L}{\partial y \partial x}$$

$$\frac{\partial^2 L}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial L}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} + \lambda\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) + \lambda\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) =$$

$$= \frac{\partial^2 u}{\partial y^2} + \lambda\frac{\partial^2 f}{\partial y^2} = 64 + \lambda \cdot 32 = 32\lambda + 64 = 32(\lambda + 2)$$

$$\frac{\partial^2 L}{\partial \lambda \partial y} = \frac{\partial}{\partial \lambda}\left(\frac{\partial L}{\partial y}\right) = \frac{\partial}{\partial \lambda}\left(\frac{\partial u}{\partial y} + \lambda\frac{\partial f}{\partial y}\right) = \frac{\partial f}{\partial y}\frac{d}{d\lambda}(\lambda) = \frac{\partial f}{\partial y} \cdot 1 =$$

$$= \frac{\partial f}{\partial y} = 32y$$

$$\frac{\partial^2 L}{\partial x \partial \lambda} = \frac{\partial}{\partial x}\left(\frac{\partial L}{\partial \lambda}\right) = \frac{\partial}{\partial x}(f) = \frac{\partial f}{\partial x} = 2x = \frac{\partial^2 f}{\partial \lambda \partial x}$$

$$\frac{\partial^2 L}{\partial y \partial \lambda} = \frac{\partial}{\partial y}\left(\frac{\partial L}{\partial \lambda}\right) = \frac{\partial}{\partial y}(f) = \frac{\partial f}{\partial y} = 32y = \frac{\partial^2 f}{\partial \lambda \partial y}$$

$$\frac{\partial^2 L}{\partial \lambda^2} = \frac{\partial}{\partial \lambda}\left(\frac{\partial L}{\partial \lambda}\right) = \frac{\partial}{\partial \lambda}(f) = 0$$

$$d^2L = \frac{\partial^2 L}{\partial x^2}dx^2 + \frac{\partial^2 L}{\partial y^2}dy^2 + \frac{\partial^2 L}{\partial \lambda^2}d\lambda^2 +$$

$$+ 2\frac{\partial^2 L}{\partial x \partial y}dxdy + 2\frac{\partial^2 L}{\partial y \partial \lambda}dyd\lambda + 2\frac{\partial^2 L}{\partial \lambda \partial x}d\lambda dx =$$

$$= 2(\lambda+2)dx^2 + 32(\lambda+2)dy^2 + 0d\lambda^2 +$$

$$+ 2 \cdot 12 dxdy + 2 \cdot 32y\, dyd\lambda + 2 \cdot 2x\, d\lambda dx =$$

$$= 2\left((\lambda+2)(dx^2 + 32dy^2) + 12dxdy + 2(xdx + 16ydy)d\lambda\right) = (3)$$

$f(x,y)=0 \Rightarrow df=0 \Leftrightarrow$

$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = 2x dx + 32y dy = 2(x dx + 16 y dy)$

$\Leftrightarrow 2(x dx + 16 y dy) = 0 \Leftrightarrow x dx + 16 y dy = 0$

$(3) = 2\left((\lambda+2)(dx^2+32dy^2) + 12 dx dy + 2 \cdot 0 \, d\lambda\right) =$

$= 2\left((\lambda+2)(dx^2+32dy^2) + 12 dx dy\right)$

$d^2L(x,y,-\frac{1}{2}) = 2\left((\lambda+2)(dx^2+32dy^2) + 12 dx dy\right)\Big|_{\lambda=-\frac{1}{2}} =$

$\lambda = -\frac{1}{2} \Leftrightarrow \lambda+2 = -\frac{1}{2}+2 = \frac{3}{2}$

$= 2\left(\frac{3}{2}(dx^2+32dy^2) + 12 dx dy\right) = 2\frac{3}{2}\left(dx^2+32dy^2 + \frac{2}{3}\cdot 12 dx dy\right) =$

$= 3(dx^2 + 32dy^2 + 2\cdot 4 dx dy) = 3\left(dx^2 + 2dx\,4dy + (4dy)^2 + 16dy^2\right) =$

$= 3\left((dx+4dy)^2 + 16dy^2\right)$

$\boxed{\lambda = -\frac{1}{2} \ \forall (x,y) \in \mathbb{R}^2 \ \forall (dx,dy) \in \mathbb{R}^2/\{(0,0)\} \quad d^2L(x,y,\lambda) > 0}$

$d^2L(x,y,-\frac{7}{2}) = 2\left((\lambda+2)(dx^2+32dy^2) + 12 dx dy\right)\Big|_{\lambda=-\frac{3}{2}} =$

$\lambda = -\frac{7}{2} \Leftrightarrow \lambda+2 = -\frac{7}{2}+2 = -\frac{3}{2}$

$= 2\left(-\frac{3}{2}(dx^2+32dy^2) + 12 dx dy\right) = 2\left(-\frac{3}{2}\right)\left(dx^2+32dy^2 + \left(-\frac{2}{3}\right) 12 dx dy\right) =$

$= -3(dx^2 + 32dy^2 - 2\cdot 4 dx dy) = -3\left(dx^2 - 2dx\,4dy + (4dy)^2 + 16dy^2\right) =$

$= -3\left((dx-4dy)^2 + 16dy^2\right)$

$\boxed{\lambda = -\frac{7}{2} \ \forall (x,y) \in \mathbb{R}^2 \ \forall (dx,dy) \in \mathbb{R}^2\setminus\{(0,0)\} \quad d^2L(x,y,-\frac{7}{2}) < 0}$

Ответ:

$(x, y) = (4\sqrt{2}, -\sqrt{2})$

$(x, y) = (-4\sqrt{2}, \sqrt{2})$

точки условного минимума

$(x, y) = (4\sqrt{2}, \sqrt{2})$

$(x, y) = (-4\sqrt{2}, \sqrt{2})$

точки условного максимума