

2) Aufgabe: $\int x(x^2+1)^{50} dx$

Ansatz:

$$\begin{aligned} x(x^2+1)^{50} dx &= (x^2+1)^{50} \cdot \frac{1}{2} \cdot 2x dx = (x^2+1)^{50} \cdot \frac{1}{2} \cdot \frac{d}{dx}(x^2) dx = (x^2+1)^{50} \cdot \frac{1}{2} dx^2 = \\ &= \frac{1}{2} (x^2+1)^{50} dx^2 = \frac{1}{2} \cdot \frac{1}{51} \cdot 51 (x^2+1)^{50} dx^2 = \frac{1}{2} \cdot \frac{1}{51} \cdot \frac{d}{dx^2} \left((x^2+1)^{51} \right) dx^2 = \\ &= \frac{1}{2} \cdot \frac{1}{51} d(x^2+1)^{51} = \frac{1}{102} d(x^2+1)^{51} = \frac{d}{d(x^2+1)^{51}} \left(\frac{(x^2+1)^{51}}{102} \right) d(x^2+1)^{51} = \\ &= d\left(\frac{(x^2+1)^{51}}{102}\right) \end{aligned}$$

$$\int x(x^2+1)^{50} dx = \int d\left(\frac{(x^2+1)^{51}}{102}\right) = \frac{(x^2+1)^{51}}{102} + C$$

Probe:

$$\begin{aligned} \frac{d}{dx} \left(\frac{(x^2+1)^{51}}{102} + C \right) &= \frac{d}{dx} \left(\frac{(x^2+1)^{51}}{102} \right) = \\ &= \frac{d}{d(x^2+1)^{51}} \left(\frac{(x^2+1)^{51}}{102} \right) \cdot \frac{d(x^2+1)^{51}}{dx} = \frac{1}{102} \cdot \frac{d}{dx} \left((x^2+1)^{51} \right) = \\ &= \frac{1}{102} \cdot \frac{d}{d(x^2+1)} \left((x^2+1)^{51} \right) \cdot \frac{d(x^2+1)}{dx} = \frac{1}{102} \cdot 51 (x^2+1)^{50} \cdot \frac{d}{dx} (x^2+1) = \\ &= \frac{1}{102} \cdot 51 (x^2+1)^{50} \cdot \frac{d}{dx} (x^2) = \frac{1}{102} \cdot 51 (x^2+1)^{50} \cdot 2x = \\ &= \frac{2 \cdot 51}{102} x (x^2+1)^{50} = x(x^2+1)^{50} \end{aligned}$$

Ordnung: $\boxed{\int x(x^2+1)^{50} dx = \frac{(x^2+1)^{51}}{102} + C}$