[1] Найти: $\int \sin(2x)\,dx$

Решение:

$$\sin(2x)dx = \sin(2x)\,\frac{1}{2}\,2dx = \sin(2x)\,\frac{1}{2}\,\frac{d}{dx}(2x)dx = \sin(2x)\frac{1}{2}d(2x) =$$

$$= \frac{1}{2}\sin(2x)d(2x) = \frac{1}{2}(-1)\left(-\sin(2x)\right)d(2x) = \frac{1}{2}(-1)\frac{d}{d(2x)}\left(\cos(2x)\right)d(2x) =$$

$$= \frac{1}{2}(-1)\,d\cos(2x) = -\frac{1}{2}\,d\cos(2x) = \frac{d}{d\cos(2x)}\left(-\frac{1}{2}\cos(2x)\right)d\cos(2x) =$$

$$= d\left(-\frac{1}{2}\cos(2x)\right)$$

$$\int \sin(2x)dx = \int d\left(-\frac{1}{2}\cos(2x)\right) = -\frac{1}{2}\cos(2x) + C$$

Проверка:

$$\frac{d}{dx}\left(-\frac{1}{2}\cos(2x) + C\right) = \frac{d}{dx}\left(-\frac{1}{2}\cos(2x)\right) =$$

$$= \frac{d}{d\cos(2x)}\left(-\frac{1}{2}\cos(2x)\right)\cdot\frac{d\cos(2x)}{dx} = -\frac{1}{2}\,\frac{d}{dx}(\cos 2x) =$$

$$= -\frac{1}{2}\,\frac{d}{d(2x)}\left(\cos(2x)\right)\frac{d(2x)}{dx} = -\frac{1}{2}\left(-\sin(2x)\right)\frac{d}{dx}(2x) =$$

$$= \left(-\frac{1}{2}\right)(-1)\sin(2x)\cdot 2 = \left(-\frac{1}{2}\right)(-1)\cdot 2\sin(2x) = \sin(2x)$$

Ответ: $\boxed{\int \sin(2x)dx = -\frac{1}{2}\cos(2x) + C}$