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$$\lim_{x \rightarrow \infty} \frac{x^2-1}{2x^2-x-1} = (1)$$

$$\lim_{x \rightarrow \infty} x^2-1 = \infty$$

$$\lim_{x \rightarrow \infty} 2x^2-x-1 = \infty$$

$$(1) = \left(\frac{\infty}{\infty} \right)$$

$$x^2-1 = \{x \neq 0\} = x^2 \left(1 - \frac{1}{x^2}\right)$$

$$2x^2-x-1 = \{x \neq 0\} = x^2 \left(2 - \frac{x}{x^2} - \frac{1}{x^2}\right) = x^2 \left(2 - \frac{1}{x} - \frac{1}{x^2}\right)$$

$$\frac{x^2-1}{2x^2-x-1} = \{x \neq 0\} = \frac{x^2 \left(1 - \frac{1}{x^2}\right)}{x^2 \left(2 - \frac{1}{x} - \frac{1}{x^2}\right)} = \frac{1 - \frac{1}{x^2}}{2 - \frac{1}{x} - \frac{1}{x^2}}$$

$$(1) = \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x^2}}{2 - \frac{1}{x} - \frac{1}{x^2}} = \frac{1 - \lim_{x \rightarrow \infty} \frac{1}{x^2}}{2 - \lim_{x \rightarrow \infty} \frac{1}{x} - \lim_{x \rightarrow \infty} \frac{1}{x^2}} =$$

$$= \frac{1-0}{2-0-0} = \frac{1+0}{2+0} = \frac{1}{2}$$

$$\boxed{\lim_{x \rightarrow \infty} \frac{x^2-1}{2x^2-x-1} = \frac{1}{2}}$$