1. Найти производную $\frac{dy}{dx}$ функции $y(x)$, заданной неявно $F(x,y)=0$, где $F(x,y) = y\sin x + x^2y^3 - 5x - 10y + \cos y$

Решение:

$$F(x,y(x)) = 0 \Rightarrow \frac{dF}{dx}(x,y(x)) = 0$$

$$\frac{dF}{dx}(x,y(x)) = \frac{d}{dx}(y\sin x + x^2y^3 - 5x - 10y + \cos y) =$$

$$= \frac{d}{dx}(y\sin x) + \frac{d}{dx}(x^2y^3) - 5\frac{d}{dx}(x) - 10\frac{d}{dx}(y) + \frac{d}{dx}(\cos y) = (1)$$

$$\frac{d}{dx}(y\sin x) = \frac{d}{dx}(y)\sin x + y\frac{d}{dx}(\sin x) = \frac{dy}{dx}\sin x + y\cos x$$

$$\frac{d}{dx}(x^2y^3) = \frac{d}{dx}(x^2)y^3 + x^2\frac{d}{dx}(y^3) = 2x\,y^3 + x^2\,3y^2\frac{dy}{dx} =$$

$$= 2xy^3 + 3x^2y^2\frac{dy}{dx}$$

$$\frac{d}{dx}(\cos y) = -\sin y\frac{dy}{dx}$$

$$(1) = \left(\frac{dy}{dx}\sin x + y\cos x\right) + \left(2xy^3 + 3x^2y^2\frac{dy}{dx}\right) - 5\cdot 1 - 10\frac{dy}{dx} - \sin y\frac{dy}{dx} =$$

$$= \left(\sin x + 3x^2y^2 - 10 - \sin y\right)\frac{dy}{dx} + \left(y\cos x + 2xy^3 - 5\right) = 0$$

$$\sin x + 3x^2y^2 - 10 - \sin y \neq 0$$

$$\frac{dy}{dx} = -\frac{y\cos x + 2xy^3 - 5}{\sin x + 3x^2y^2 - 10 - \sin y}$$

Ответ:

$$\frac{dy}{dx} = -\frac{y\cos x + 2xy^3 - 5}{\sin x + 3x^2y^2 - 10 - \sin y} \quad \Big| \quad \sin x + 3x^2y^2 - 10 - \sin y \neq 0$$

Вариант 2:

$F(x, y(x)) = 0 \Rightarrow \frac{dF}{dx}(x, y(x)) = 0 \Leftrightarrow$

$\Leftrightarrow \frac{dF}{dx} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{dy}{dx} = 0 \Leftrightarrow \frac{\partial F}{\partial y}\frac{dy}{dx} = -\frac{\partial F}{\partial x} \Rightarrow$

$\Rightarrow \frac{dy}{dx} = -\frac{\partial F}{\partial x}\left(\frac{\partial F}{\partial y}\right)^{-1} \Big| \frac{\partial F}{\partial y} \neq 0$

$F(x, y) = y\sin x + x^2y^3 - 5x - 10y + \cos y$

$\frac{\partial F}{\partial x} = \frac{\partial}{\partial x}\left(y\sin x + x^2y^3 - 5x - 10y + \cos y\right) =$

$= y\frac{d}{dx}(\sin x) + y^3\frac{d}{dx}(x^2) - 5\frac{d}{dx}(x) = (1)$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(x^2) = 2x$

$(1) = y\cos x + y^3 2x - 5\cdot 1 = y\cos x + 2xy^3 - 5$

$\frac{\partial F}{\partial y} = \frac{\partial}{\partial y}\left(y\sin x + x^2y^3 - 5x - 10y + \cos y\right) =$

$= \sin x\frac{d}{dy}(y) + x^2\frac{d}{dy}(y^3) - 10\frac{d}{dy}(y) + \frac{d}{dy}(\cos y) = (2)$

$\frac{d}{dy}(y^3) = 3y^2$

$\frac{d}{dy}(\cos y) = -\sin y$

$(2) = \sin x\cdot 1 + x^2 3y^2 - 10\cdot 1 + (-\sin y) = \sin x + 3x^2y^2 - 10 - \sin y$

$\frac{\partial F}{\partial y} \neq 0 \Leftrightarrow \sin x + 3x^2y^2 - 10 - \sin y \neq 0$

$\frac{dy}{dx} = -\frac{\partial F}{\partial x}\left(\frac{\partial F}{\partial y}\right)^{-1} = -\frac{y\cos x + 2xy^3 - 5}{\sin x + 3x^2y^2 - 10 - \sin y}$