$\boxed{23}$ $u(x,y,z) = xy^2 + z^3 - xyz$

$\bar{b} = (12, -8, 9)$

$P = (1,1,2)$

Найти: $\frac{\partial u}{\partial b}(P)$

Решение:

$$\frac{\partial u}{\partial b}(P) = \left(\frac{\bar{b}}{|\bar{b}|} \cdot \nabla u\right)(P) = \frac{\bar{b}}{|\bar{b}|} \cdot \nabla u(P) = \frac{\bar{b} \cdot \nabla u(P)}{|\bar{b}|}$$

$$\nabla u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(xy^2 + z^3 - xyz\right) = y^2 \frac{d}{dx}(x) - yz\frac{d}{dx}(x) =$$

$$= y^2 \cdot 1 - yz \cdot 1 = y^2 - yz$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(xy^2 + z^3 - xyz\right) = x\frac{d}{dy}(y^2) - xz\frac{d}{dy}(y) =$$

$$= x \cdot 2y - xz \cdot 1 = 2xy - xz$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(xy^2 + z^3 - xyz\right) = \frac{d}{dz}(z^3) - xy\frac{d}{dz}(z) =$$

$$= 3z^2 - xy \cdot 1 = 3z^2 - xy$$

$$\nabla u(P) = \left(\frac{\partial u}{\partial x}(P), \frac{\partial u}{\partial y}(P), \frac{\partial u}{\partial z}(P)\right)$$

$$\frac{\partial u}{\partial x}(P) = y^2 - yz\Big|_{x=1, y=1, z=2} = 1^2 - 1 \cdot 2 = 1 - 2 = -1$$

$$\frac{\partial u}{\partial y}(P) = 2xy - xz\Big|_{x=1, y=1, z=2} = 2 \cdot 1 \cdot 1 - 1 \cdot 2 = 2 - 2 = 0$$

$$\frac{\partial u}{\partial z}(P) = 3z^2 - xy\Big|_{x=1, y=1, z=2} = 3 \cdot 2^2 - 1 \cdot 1 = 3 \cdot 4 - 1 = 12 - 1 = 11$$

$$\vec{b}\cdot\nabla u(P) = (b_x, b_y, b_z)\cdot\left(\frac{\partial u}{\partial x}(P), \frac{\partial u}{\partial y}(P), \frac{\partial u}{\partial z}(P)\right) =$$

$$= b_x\frac{\partial u}{\partial x}(P) + b_y\frac{\partial u}{\partial y}(P) + b_z\frac{\partial u}{\partial z}(P) \Bigg|_{\substack{b_x=12,\ b_y=-8,\ b_z=9 \\ \frac{\partial u}{\partial x}(P)=-1,\ \frac{\partial u}{\partial y}(P)=0,\ \frac{\partial u}{\partial z}(P)=11}} =$$

$$= 12(-1) + (-8)\cdot 0 + 9\cdot 11 = -12 + 0 + 99 = 99 - 12 = 87$$

$$|\vec{b}| = \sqrt{\vec{b}^2} = \sqrt{\vec{b}\cdot\vec{b}} = \sqrt{(b_x, b_y, b_z)\cdot(b_x, b_y, b_z)} = \sqrt{b_x^2 + b_y^2 + b_z^2} \Bigg|_{b_x=12,\ b_y=-8,\ b_z=9} =$$

$$= \sqrt{12^2 + (-8)^2 + 9^2} = \sqrt{144 + 64 + 81} = \sqrt{289} = \sqrt{17^2} = 17$$

$$\frac{\partial u}{\partial \vec{b}}(P) = \frac{\vec{b}\cdot\nabla u(P)}{|\vec{b}|} \Bigg|_{\substack{\vec{b}\cdot\nabla u(P)=87 \\ |\vec{b}|=17}} = \frac{87}{17}$$

Ответ: $\frac{\partial u}{\partial \vec{b}}(P) = \frac{87}{17}$

Решение 2:

$$\frac{\partial u}{\partial b}(P) = \left(\frac{\bar{b}}{|\bar{b}|}\cdot\nabla u\right)(P) = \left(\frac{\bar{b}\cdot\nabla u}{|\bar{b}|}\right)(P)$$

$$\nabla u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(xy^2+z^3-xyz\right) = y^2\frac{d}{dx}(x) - yz\frac{d}{dx}(x) =$$

$$= y^2\cdot 1 - yz\cdot 1 = y^2 - yz$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(xy^2+z^3-xyz\right) = x\frac{d}{dy}(y^2) - xz\frac{d}{dy}(y) =$$

$$= x\cdot 2y - xz\cdot 1 = 2xy - xz$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(xy^2+z^3-xyz\right) = \frac{d}{dz}(z^3) - xy\frac{d}{dz}(z) =$$

$$= 3z^2 - xy\cdot 1 = 3z^2 - xy$$

$$\bar{b}\cdot\nabla u = (b_x, b_y, b_z)\cdot\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right) = b_x\frac{\partial u}{\partial x} + b_y\frac{\partial u}{\partial y} + b_z\frac{\partial u}{\partial z} =$$

$$= b_x(y^2-yz) + b_y(2xy-xz) + b_z(3z^2-xy) =$$

$$= b_x y^2 + 3b_z z^2 + (2b_y - b_z)xy - b_x yz - b_y zx \Big| = \quad b_x=12,\ b_y=-8,\ b_z=9$$

$$= 12y^2 + 3\cdot 9z^2 + (2(-8)-9)xy - 12yz - (-8)zx =$$

$$= 12y^2 + 27z^2 - 25xy - 12yz + 8zx$$

$$|\bar{b}| = \sqrt{\bar{b}^2} = \sqrt{\bar{b}\cdot\bar{b}} = \sqrt{(b_x, b_y, b_z)\cdot(b_x, b_y, b_z)} = \sqrt{b_x^2+b_y^2+b_z^2} \Big| = \quad b_x=12,\ b_y=-8,\ b_z=9$$

$$= \sqrt{12^2+(-8)^2+9^2} = \sqrt{144+64+81} = \sqrt{289} = \sqrt{17^2} = 17$$

$$\frac{\partial u}{\partial b} = \frac{\bar{b}\cdot\nabla u}{|\bar{b}|} = \left(12y^2+27z^2-25xy-12yz+8zx\right)\cdot 17^{-1}$$

$$\frac{\partial u}{\partial \bar{b}}(P) = \left(\frac{\bar{b} \cdot \nabla u}{|\bar{b}|}\right)(P) =$$

$$= \left(12y^2 + 27z^2 - 25xy - 12yz + 8zx\right) \cdot 17^{-1} \Big|_{x=1,\,y=1,\,z=2} =$$

$$= \left(12 \cdot 1^2 + 27 \cdot 2^2 - 25 \cdot 1 \cdot 1 - 12 \cdot 1 \cdot 2 + 8 \cdot 2 \cdot 1\right) 17^{-1} =$$

$$= \left(12 + 4 \cdot 27 - 25 - 24 + 16\right) 17^{-1} =$$

$$= \left(12 + 108 + 16 - (25 + 24)\right) 17^{-1} = (136 - 49)\, 17^{-1} = (100 - 13) \cdot 17^{-1} =$$

$$= \frac{87}{17}$$

Ответ: $\frac{\partial u}{\partial \bar{b}}(P) = \frac{87}{17}$