

15) Hermites nyrreger pyrrnnyrre:

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\frac{x^2}{2}} = (1)$$

$$\lim_{x \rightarrow 0} 1 - \cos x = (1 - \cos x)_{x=0} = 1 - \cos 0 = 1 - 1 = 0$$

$$\lim_{x \rightarrow 0} \frac{x^2}{2} = \left(\frac{x^2}{2}\right)_{x=0} = \frac{0^2}{2} = \frac{0}{2} = 0$$

$$(1) = \left(\frac{0}{0}\right)$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = \begin{cases} \sin^2 \alpha + \cos^2 \alpha - 1 \\ \cos^2 \alpha - 1 - \sin^2 \alpha \end{cases} = -\begin{cases} 2\sin^2 \alpha \\ 2\cos^2 \alpha \end{cases}$$
$$= (1 - \sin^2 \alpha) - \sin^2 \alpha = 1 - 2\sin^2 \alpha$$

$$1 - \cos x = 1 - \cos\left(2 \cdot \frac{x}{2}\right) = 1 - (1 - 2\sin^2 \frac{x}{2}) = 1 - 1 + 2\sin^2 \frac{x}{2} = 2\sin^2 \frac{x}{2}$$

$$\frac{x^2}{2} = \frac{1}{2} \left(\frac{x}{2}\right)^2 = \frac{1}{2} 4 \left(\frac{x}{2}\right)^2 = 2 \left(\frac{x}{2}\right)^2$$

$$\frac{1 - \cos x}{\frac{x^2}{2}} = \frac{2\sin^2 \frac{x}{2}}{2 \left(\frac{x}{2}\right)^2} = \frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2}\right)^2} = \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2$$

$$(1) = \lim_{x \rightarrow 0} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 = \left(\lim_{x \rightarrow 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 = (2)$$

$$y = \frac{x}{2}$$

$$\lim_{x \rightarrow 0} y = \lim_{x \rightarrow 0} \frac{x}{2} = \left(\frac{x}{2}\right)_{x=0} = \frac{0}{2} = 0$$

$$\lim_{x \rightarrow 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} = \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1$$

$$(2) = 1^2 = 1$$

Omsvar:

$$\boxed{\lim_{x \rightarrow 0} \frac{1 - \cos x}{\frac{x^2}{2}} = 1}$$