15 Найти предел функции:

$$\lim_{x\to0}\frac{\sqrt{2}x^2\sin 4x}{(1-\cos 2x)^{\frac{3}{2}}}=(1)$$

$$\lim_{x\to0}(1-\cos 2x)^{\frac{3}{2}}=(1-\cos 2x)^{\frac{3}{2}}\Big|_{x=0}=$$

$$=(1-\cos 2\cdot0)^{\frac{3}{2}}=(1-\cos 0)^{\frac{3}{2}}=(1-1)^{\frac{3}{2}}=0^{\frac{3}{2}}=0$$

$$\lim_{x\to0}\sqrt{2}x^2\sin x=\sqrt{2}x^2\sin x\Big|_{x=0}=\sqrt{2}\,0^2\sin 0=\sqrt{2}\cdot0\cdot0=0$$

$$(1)=\left(\frac{0}{0}\right)$$

$$(1-\cos 2x)^{\frac{3}{2}}=\{$$

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha=(1-\sin^2\alpha)-\sin^2\alpha=1-2\sin^2\alpha$$

$$\}=(1-(1-2\sin^2x))^{\frac{3}{2}}=(1-1+2\sin^2x)^{\frac{3}{2}}=(2\sin^2x)^{\frac{3}{2}}=2^{\frac{3}{2}}(\sin^2x)^{\frac{3}{2}}=\{$$

$$(a^2)^{\frac{1}{2}}=|a|\;;\;(a^2)^{\frac{3}{2}}=(a^2)^{\frac{1}{2}\cdot3}=\left((a^2)^{\frac{1}{2}}\right)^3=|a|^3$$

$$\}=2^{\frac{3}{2}}|\sin x|^3$$

$$\sin 4x=\sin(2\cdot2x)=\{$$

$$\sin 2\alpha=2\sin\alpha\cos\alpha$$

$$\}=2\sin 2x\cos 2x=2(2\sin x\cos x)\cos 2x=4\sin x\cos x\cos 2x$$

$$\frac{\sqrt{2}\,x^2 \sin 4x}{(1-\cos 2x)^{\frac{3}{2}}} = \frac{2^{\frac{1}{2}} x^2\, 4 \sin x \cos x \cos 2x}{2^{\frac{3}{2}} |\sin x|^3} =$$

$$= \frac{2^2 2^{\frac{1}{2}}}{2^{\frac{3}{2}}} x^2 \cos x \cos 2x \;\frac{\sin x}{|\sin x|^3} = \Big\}$$

$$\frac{2^2 \cdot 2^{\frac{1}{2}}}{2^{\frac{3}{2}}} = 2^{2+\frac{1}{2}-\frac{3}{2}} = 2^{\frac{5}{2}-\frac{3}{2}} = 2^{\frac{2}{2}} = 2^1 = 2$$

$$\Big\} = 2x^2 \cos x \cos 2x \;\frac{\sin x}{|\sin x|^3} = (2)$$

$$\frac{\sin x}{|\sin x|^3} = \Big\}$$

$$a \neq 0 \quad \frac{a}{|a|^3} = \frac{a}{|a||a|^2} = \frac{a}{|a|a^2} = \frac{a}{|a|}\frac{1}{a^2} = \frac{|a|}{a}\frac{1}{a^2} = \frac{\operatorname{sign} a}{a^2}$$

$$\Big\} = \frac{\operatorname{sign}(\sin x)}{\sin^2 x}$$

$$(2) = 2 \cdot x^2 \cos x \cos 2x \;\frac{\operatorname{sign}(\sin x)}{\sin^2 x} =$$

$$= 2 \operatorname{sign}(\sin x) \cos x \cos 2x \;\frac{x^2}{\sin^2 x} = \{x \neq 0\} =$$

$$= 2 \operatorname{sign}(\sin x) \cos x \cos 2x \left(\frac{\sin x}{x}\right)^{-2}$$

$$\lim_{x\to 0\pm} \frac{\sqrt{2}\,x^2 \sin 4x}{(1-\cos 2x)^{\frac{3}{2}}} =$$

$$= \lim_{x\to 0\pm} 2\,\mathrm{sign}(\sin x) \cos x \cos 2x \left(\frac{\sin x}{x}\right)^{-2} =$$

$$= \lim_{x\to 0\pm} 2(\pm 1) \cos x \cos 2x \left(\frac{\sin x}{x}\right)^{-2} =$$

$$= \pm 2 \lim_{x\to 0\pm} (\cos x \cos 2x) \left(\lim_{x\to 0\pm} \frac{\sin x}{x}\right)^{-2} = (4)$$

$$\lim_{x\to 0\pm} (\cos x \cos 2x) = \lim_{x\to 0} (\cos x \cos 2x) = \cos x \cos 2x \Big|_{x=0} =$$

$$= \cos 0 \cos(2\cdot 0) = \cos 0 \cos 0 = 1\cdot 1 = 1$$

$$\lim_{x\to 0\pm} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$(4) = \pm 2 \cdot 1 \cdot 1^{-2} = \pm 2$$

Ответ:

$$\boxed{\lim_{x\to 0\pm} \frac{\sqrt{2}\,x^2 \sin 4x}{(1-\cos 2x)^{\frac{3}{2}}} = \pm 2}$$