5 Найти предел функции:

$$\lim_{x\to\infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{2x+1}} = (1)$$

$$\sqrt{x+\sqrt{x+\sqrt{x}}} = \{x\neq 0\} =$$

$$= \sqrt{x\left(1+\frac{\sqrt{x+\sqrt{x}}}{x}\right)} = \sqrt{x}\sqrt{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}} =$$

$$= \sqrt{x}\sqrt{1+\sqrt{\frac{1}{x}+\frac{\sqrt{x}}{x^2}}}$$

$$\sqrt{2x+1} = \{x\neq 0\} =$$

$$= \sqrt{x\left(2+\frac{1}{x}\right)} = \sqrt{x}\sqrt{2+\frac{1}{x}}$$

$$\frac{x+\sqrt{x+\sqrt{x}}}{\sqrt{2x+1}} = \{x\neq 0\} =$$

$$= \frac{\sqrt{x}\sqrt{1+\sqrt{\frac{1}{x}+\frac{\sqrt{x}}{x^2}}}}{\sqrt{x}\sqrt{2+\frac{1}{x}}} = \frac{\sqrt{1+\sqrt{\frac{1}{x}+\frac{\sqrt{x}}{x^2}}}}{\sqrt{2+\frac{1}{x}}}$$

$$(1) = \lim_{x\to\infty} \frac{\sqrt{1+\sqrt{\frac{1}{x}+\frac{\sqrt{x}}{x^2}}}}{\sqrt{2+\frac{1}{x}}} = \frac{\sqrt{1+\sqrt{\lim\limits_{x\to\infty}\frac{1}{x}+\lim\limits_{x\to\infty}\frac{\sqrt{x}}{x^2}}}}{\sqrt{2+\lim\limits_{x\to\infty}\frac{1}{x}}} = \{$$

$$\lim_{x\to\infty}\frac{1}{x} = 0;\ \lim_{x\to\infty}\frac{\sqrt{x}}{x^2} = \lim_{x\to\infty} x^{-\frac{3}{2}} = 0$$

$$\} = \frac{\sqrt{1+\sqrt{0+0}}}{\sqrt{2+0}} = \frac{\sqrt{1+0}}{\sqrt{2+0}} = \frac{1}{\sqrt{2}}$$

Ответ:

$$\boxed{\lim_{x\to\infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{2x+1}} = \frac{1}{\sqrt{2}}}$$