$\boxed{5}$ Найти производную функции.

$$y(x) = \ln\left(x + \sqrt{x^2+1}\right)$$

Решение:

$$y'(x) = \left(\ln\left(x+\sqrt{x^2+1}\right)\right)' = \frac{\left(x+\sqrt{x^2+1}\right)'}{x+\sqrt{x^2+1}} = (1)$$

$$\left(x+\sqrt{x^2+1}\right)' = (x)' + \left(\sqrt{x^2+1}\right)' = (2)$$

$$(x)' = 1$$

$$\left(\sqrt{x^2+1}\right)' = \left(\left(x^2+1\right)^{\frac{1}{2}}\right)' = \frac{1}{2}\left(x^2+1\right)^{\frac{1}{2}-1}\left(x^2+1\right)' =$$

$$= \frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}\left(x^2\right)' = \frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}} 2x =$$

$$= x\left(x^2+1\right)^{-\frac{1}{2}} = \frac{x}{\sqrt{x^2+1}}$$

$$(2) = 1 + \frac{x}{\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}$$

$$(1) = \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}} \cdot \frac{1}{x+\sqrt{x^2+1}} = \frac{x+\sqrt{x^2+1}}{x+\sqrt{x^2+1}} \cdot \frac{1}{\sqrt{x^2+1}} =$$

$$= \frac{1}{\sqrt{x^2+1}}$$

Ответ:

$$y'(x) = \frac{1}{\sqrt{x^2+1}}$$