$\boxed{1.6}$ Найти предел последовательности

$$\lim_{n\to\infty}\left(\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\ldots+\frac{1}{(n-1)n}\right)=(1)$$

$$\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\ldots+\frac{1}{(n-1)n}=(2)$$

$$\left(\frac{1}{(k-1)k}\right)_{k=2}=\frac{1}{(2-1)2}=\frac{1}{1\cdot2}$$

$$\left(\frac{1}{(k-1)k}\right)_{k=3}=\frac{1}{(3-1)3}=\frac{1}{2\cdot3}$$

$$\left(\frac{1}{(k-1)k}\right)_{k=4}=\frac{1}{(4-1)4}=\frac{1}{3\cdot4}$$

$$(2)=\sum_{k=2}^{n}\frac{1}{(k-1)k}=(3)$$

$k\in\mathbb{N}\wedge k\geq2$

$p,q\in\mathbb{Z}$

$$\frac{p}{k-1}+\frac{q}{k}=\frac{pk+q(k-1)}{(k-1)k}=\frac{pk+qk-q}{(k-1)k}=\frac{(p+q)k-q}{(k-1)k}=$$

$$=\{p+q=0;\ p=-q\}=\frac{0k-q}{(k-1)k}=\frac{-q}{(k-1)k}=\Big\{$$

$-q=1;\ q=-1$

$p=-q=-(-1)=1$

$$\Big\}=\frac{-(-1)}{(k-1)k}=\frac{1}{(k-1)k}$$

$$\frac{1}{(k-1)k}=\frac{p}{k-1}+\frac{q}{k}=\frac{1}{k-1}+\frac{(-1)}{k}=\frac{1}{k-1}-\frac{1}{k}$$

$$(3) = \sum_{k=2}^{n}\left(\frac{1}{k-1} - \frac{1}{k}\right) = \sum_{k=2}^{n}\frac{1}{k-1} - \sum_{k=2}^{n}\frac{1}{k} = \Big\{$$

$$\sum_{k=2}^{n}\frac{1}{k-1} = \sum_{k=2}^{k=n}\frac{1}{k-1} = \sum_{k-1=2-1}^{k-1=n-1}\frac{1}{k-1} = \sum_{k-1=1}^{k-1=n-1}\frac{1}{k-1} = \sum_{k=1}^{k=n-1}\frac{1}{k} = \sum_{k=1}^{n-1}\frac{1}{k}$$

$$\Big\} = \sum_{k=1}^{n-1}\frac{1}{k} - \sum_{k=2}^{n}\frac{1}{k} = \Big\{$$

$$\sum_{k=1}^{n-1}\frac{1}{k} = \left(\frac{1}{k}\right)_{k=1} + \sum_{k=2}^{n-1}\frac{1}{k} = 1 + \sum_{k=2}^{n-1}\frac{1}{k}$$

$$\sum_{k=2}^{n}\frac{1}{k} = \sum_{k=2}^{n-1}\frac{1}{k} + \left(\frac{1}{k}\right)_{k=n} = \sum_{k=2}^{n-1}\frac{1}{k} + \frac{1}{n}$$

$$\Big\} = \left(1 + \sum_{k=2}^{n-1}\frac{1}{k}\right) - \left(\sum_{k=2}^{n-1}\frac{1}{k} + \frac{1}{n}\right) =$$

$$= 1 + \sum_{k=2}^{n-1}\frac{1}{k} - \sum_{k=2}^{n-1}\frac{1}{k} - \frac{1}{n} = 1 - \frac{1}{n}$$

$$(1) = \lim_{n\to\infty}\left(1 - \frac{1}{n}\right) = 1 - \lim_{n\to\infty}\frac{1}{n} = 1 - 0 = 1$$

Svar:

$$\boxed{\lim_{n\to\infty}\left(\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \dots \frac{1}{(n-1)n}\right) = 1}$$