$\boxed{8}$ Найти: $\int e^x \sin x\,dx$

Решение:

$$\int e^x \sin x\,dx = \int \sin x\, e^x dx = \int u\,dv =$$

$$u = \sin x \Rightarrow du = d\sin x = \cos x\,dx$$

$$dv = e^x dx = de^x \Rightarrow v = e^x$$

$$= uv - \int v\,du = \sin x \cdot e^x - \int e^x \cos x\,dx =$$

$$= e^x \sin x - \int \cos x\, e^x dx = e^x \sin x - \int u\,dv =$$

$$u = \cos x \Rightarrow du = d\cos x = -\sin x\,dx$$

$$dv = e^x dx = de^x \Rightarrow v = e^x$$

$$= e^x \sin x - \left(uv - \int v\,du\right) = e^x \sin x - \left(\cos x\, e^x - \int e^x(-\sin x\,dx)\right) =$$

$$= e^x \sin x - \left(e^x \cos x - (-1)\int e^x \sin x\,dx\right) = e^x \sin x - \left(e^x \cos x + \int e^x \sin x\,dx\right) =$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x\,dx = e^x(\sin x - \cos x) - \int e^x \sin x\,dx$$

$$\int e^x \sin x\,dx = e^x(\sin x - \cos x) - \int e^x \sin x\,dx \Leftrightarrow$$

$$\Leftrightarrow 2\int e^x \sin x\,dx = e^x(\sin x - \cos x) \Leftrightarrow$$

$$\Leftrightarrow \int e^x \sin x\,dx = \frac{e^x(\sin x - \cos x)}{2} \Rightarrow$$

$$\Rightarrow \int e^x \sin x\,dx = \frac{e^x(\sin x - \cos x)}{2} + C$$

Решение 2:

$$\int e^x \sin x\,dx = \int u\,dv =$$

$u = e^x \Rightarrow du = de^x = e^x dx$

$dv = \sin x\,dx = -(-\sin x\,dx) = -d\cos x = d(-\cos x) \Rightarrow v = -\cos x$

$$= uv - \int v\,du = e^x(-\cos x) - \int (-\cos x)e^x dx =$$

$$= -e^x \cos x - (-1)\int e^x \cos x\,dx = -e^x \cos x + \int e^x \cos x\,dx =$$

$$= -e^x \cos x + \int u\,dv =$$

$u = e^x \Rightarrow du = de^x = e^x dx$

$dv = \cos x\,dx = d\sin x \Rightarrow v = \sin x$

$$= -e^x \cos x + \left(uv - \int v\,du\right) = -e^x \cos x + \left(e^x \sin x - \int \sin x\, e^x dx\right) =$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x\,dx = e^x(\sin x - \cos x) - \int e^x \sin x\,dx$$

$$\int e^x \sin x\,dx = e^x(\sin x - \cos x) - \int e^x \sin x\,dx \Leftrightarrow$$

$$\Leftrightarrow 2\int e^x \sin x\,dx = e^x(\sin x - \cos x) \Leftrightarrow$$

$$\Leftrightarrow \int e^x \sin x\,dx = \frac{e^x(\sin x - \cos x)}{2} \Rightarrow$$

$$\Rightarrow \int e^x \sin x\,dx = \frac{e^x(\sin x - \cos x)}{2} + C$$

Проверка:

$$\frac{d}{dx}\left(\frac{e^x(\sin x-\cos x)}{2}+C\right)=\frac{1}{2}\frac{d}{dx}\left(e^x(\sin x-\cos x)\right)=$$

$$=\frac{1}{2}\left(\frac{d}{dx}(e^x)(\sin x-\cos x)+e^x\frac{d}{dx}(\sin x-\cos x)\right)=$$

$$\frac{d}{dx}(e^x)=e^x$$

$$\frac{d}{dx}(\sin x-\cos x)=\frac{d}{dx}(\sin x)-\frac{d}{dx}(\cos x)=\cos x-(-\sin x)=\cos x+\sin x$$

$$=\frac{1}{2}\left(e^x(\sin x-\cos x)+e^x(\cos x+\sin x)\right)=$$

$$=\frac{e^x}{2}(\sin x-\cos x+\cos x+\sin x)=\frac{e^x}{2}2\sin x=e^x\sin x$$

Ответ:

$$\boxed{\int e^x\sin x\,dx=\frac{e^x(\sin x-\cos x)}{2}+C}$$