1.5 Найти предел последовательности

$$\lim_{n\to\infty} \frac{(-4)^n + 5\cdot 7^n}{(-4)^{n-1} + 7^{n+2}} = (1)$$

$$(-4)^n + 5\cdot 7^n = 7^n\left(\frac{(-4)^n}{7^n} + 5\right) = 7^n\left(\left(-\frac{4}{7}\right)^n + 5\right) =$$

$$= 7^n\left(5 + \left(-\frac{4}{7}\right)^n\right)$$

$$(-4)^{n-1} + 7^{n+2} = (-4)^{n-1} + 7^3 7^{n-1} = 7^{n-1}\left(\frac{(-4)^{n-1}}{7^{n-1}} + 7^3\right) =$$

$$= 7^{n-1}\left(\left(\frac{-4}{7}\right)^{n-1} + 7^3\right) = 7^{n-1}\left(7^3 + \left(-\frac{4}{7}\right)^{n-1}\right)$$

$$\frac{(-4)^n + 5\cdot 7^n}{(-4)^{n-1} + 7^{n+2}} = \frac{7^n\left(5 + \left(-\frac{4}{7}\right)^n\right)}{7^{n-1}\left(7^3 + \left(-\frac{4}{7}\right)^{n-1}\right)} = \frac{7\left(5 + \left(-\frac{4}{7}\right)^n\right)}{7^3 + \left(-\frac{4}{7}\right)^{n-1}}$$

$$(1) = \lim_{n\to\infty} \frac{7\left(5 + \left(-\frac{4}{7}\right)^n\right)}{7^3 + \left(-\frac{4}{7}\right)^{n-1}} =$$

$$= \frac{7\left(5 + \lim\limits_{n\to\infty}\left(-\frac{4}{7}\right)^n\right)}{7^3 + \lim\limits_{n\to\infty}\left(-\frac{4}{7}\right)^{n-1}} = (2)$$

$$\lim_{n\to\infty}\left(-\frac{4}{7}\right)^{n-1} = \lim_{(n-1)+1\to\infty}\left(-\frac{4}{7}\right)^{n-1} = \lim_{n-1\to\infty}\left(-\frac{4}{7}\right)^{n-1} = \lim_{n\to\infty}\left(-\frac{4}{7}\right)^n$$

$$(2) = \frac{7\left(5 + \lim\limits_{n\to\infty}\left(-\frac{4}{7}\right)^n\right)}{7^3 + \lim\limits_{n\to\infty}\left(-\frac{4}{7}\right)^n} = \frac{7(5+0)}{7^3+0} = \frac{7\,5}{7^3} = \frac{5}{7^2} = \frac{5}{49}$$

Ответ: $\boxed{\lim_{n\to\infty} \frac{(-4)^n + 5\cdot 7^n}{(-4)^{n-1} + 7^{n+2}} = \frac{5}{49}}$