$\boxed{6}$ Найти: $\int \operatorname{arctg} x \, dx$

Решение:

$$\int \operatorname{arctg} x \, dx = \int u \, dv =$$

$$u = \operatorname{arctg} x \Rightarrow du = d \operatorname{arctg} x = \frac{dx}{x^2+1}$$

$$dv = dx \Rightarrow v = x$$

$$= uv - \int v \, du = \operatorname{arctg} x \cdot x - \int x \frac{dx}{x^2+1} = x \operatorname{arctg} x - \int \frac{x}{x^2+1} dx =$$

$$\frac{x}{x^2+1} dx = \frac{1}{2} \frac{2x \, dx}{x^2+1} = \frac{1}{2} \frac{dx^2}{x^2+1} = \frac{1}{2} d\left(\ln |x^2+1|\right) = \frac{1}{2} d \ln (x^2+1) =$$

$$= d\left(\frac{1}{2} \ln (x^2+1)\right) \Rightarrow$$

$$\Rightarrow \int x \frac{dx}{x^2+1} = \int d\left(\frac{1}{2} \ln (x^2+1)\right) = \frac{1}{2} \ln (x^2+1) + C$$

$$= x \operatorname{arctg} x - \frac{1}{2} \ln (x^2+1) + C$$

Проверка:

$$\frac{d}{dx}\left(x \operatorname{arctg} x - \frac{1}{2} \ln (x^2+1) + C\right) = \frac{d}{dx}(x \operatorname{arctg} x) - \frac{1}{2} \frac{d}{dx} \ln (x^2+1) =$$

$$= \frac{d}{dx}(x) \operatorname{arctg} x + x \frac{d}{dx}(\operatorname{arctg} x) - \frac{1}{2} \frac{1}{x^2+1} \frac{d}{dx}(x^2+1) =$$

$$= 1 \cdot \operatorname{arctg} x + x \cdot \frac{1}{x^2+1} - \frac{1}{2} \frac{1}{x^2+1} \cdot 2x =$$

$$= \operatorname{arctg} x + \frac{x}{x^2+1} - \frac{2x}{2(x^2+1)} = \operatorname{arctg} x + \frac{x}{x^2+1} - \frac{x}{x^2+1} =$$

$$= \operatorname{arctg} x$$

Ответ: $\boxed{\int \operatorname{arctg} x = x \operatorname{arctg} x - \frac{1}{2} \ln (x^2+1) + C}$