$\boxed{1.1}$ $u(x,y)=5x^2+7y^3\sin x-\ln x\,\mathrm{tg}\,y+10y-13$

Найти: $\frac{\partial u}{\partial x}$; $\frac{\partial u}{\partial y}$; du;

$\frac{\partial^2 u}{\partial x^2}$; $\frac{\partial^2 u}{\partial y \partial x}$; $\frac{\partial^2 u}{\partial x \partial y}$; $\frac{\partial^2 u}{\partial y^2}$; d^2u

Решение:

$$\frac{\partial u}{\partial x}=\frac{\partial}{\partial x}\left(5x^2+7y^3\sin x-\ln x\,\mathrm{tg}\,y+10y-13\right)=$$

$$=5\frac{d}{dx}(x^2)+7y^3\frac{d}{dx}(\sin x)-\mathrm{tg}\,y\frac{d}{dx}(\ln x)=$$

$$=5\cdot 2x+7y^3\cos x-\mathrm{tg}\,y\,\frac{1}{x}=10x+7y^3\cos x-\frac{\mathrm{tg}\,y}{x}$$

$$\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left(5x^2+7y^3\sin x-\ln x\,\mathrm{tg}\,y+10y-13\right)=$$

$$=7\sin x\frac{d}{dy}(y^3)-\ln x\frac{d}{dy}(\mathrm{tg}\,y)+10\frac{d}{dy}(y)=$$

$$=7\sin x\,3y^2-\ln x\,\frac{1}{\cos^2 y}+10\cdot 1=21y^2\sin x-\frac{\ln x}{\cos^2 y}+10$$

$$du=\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy=$$

$$=\left(10x+7y^3\cos x-\frac{\mathrm{tg}\,y}{x}\right)dx+\left(21y^2\sin x-\frac{\ln x}{\cos^2 y}+10\right)dy$$

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)=\frac{\partial}{\partial x}\left(10x+7y^3\cos x-\frac{\mathrm{tg}\,y}{x}\right)=$$

$$=10\frac{d}{dx}(x)+7y^3\frac{d}{dx}(\cos x)-\mathrm{tg}\,y\frac{d}{dx}\left(\frac{1}{x}\right)=$$

$$=10\cdot 1+7y^3(-\sin x)-\mathrm{tg}\,y\left(-\frac{1}{x^2}\right)=10-7y^3\sin x+\frac{\mathrm{tg}\,y}{x^2}$$

$$\frac{\partial^2 u}{\partial y\partial x}=\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)=\frac{\partial}{\partial y}\left(10x+7y^3\cos x-\frac{\mathrm{tg}\,y}{x}\right)=$$

$$=7\cos x\frac{d}{dy}(y^3)-\frac{1}{x}\frac{d}{dy}(\mathrm{tg}\,y)=$$

$$=7\cos x\,3y^2-\frac{1}{x}\frac{1}{\cos^2 y}=21y^2\cos x-\frac{1}{x\cos^2 y}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}\left(21y^2 \sin x - \frac{\ln x}{\cos^2 y} + 10\right) =$$

$$= 21y^2 \frac{d}{dx}(\sin x) - \frac{1}{\cos^2 y}\frac{d}{dx}(\ln x) =$$

$$= 21y^2 \cos x - \frac{1}{\cos^2 y}\frac{1}{x} = 21y^2 \cos x - \frac{1}{x\cos^2 y} =$$

$$= \frac{\partial^2 u}{\partial y \partial x}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(21y^2 \sin x - \frac{\ln x}{\cos^2 y} + 10\right) =$$

$$= 21 \sin x \frac{d}{dy}(y^2) - \ln x \frac{d}{dy}\left(\frac{1}{\cos^2 y}\right) =$$

$$= 21 \sin x (2y) - \ln x \left(\frac{-2}{\cos^3 y}\right)(-\sin y) =$$

$$= 42y \sin x - 2\frac{\ln x \sin y}{\cos^3 y}$$

$$d^2 u = d(du) = d\left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy\right) = d\left(\frac{\partial u}{\partial x}dx\right) + d\left(\frac{\partial u}{\partial y}dy\right) =$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}dx\right)dx + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}dx\right)dy + \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}dy\right)dx + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}dy\right)dy =$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)dx\,dx + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)dx\,dy + \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)dy\,dx + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)dy\,dy =$$

$$= \frac{\partial^2 u}{\partial x^2}dx^2 + \frac{\partial^2 u}{\partial y \partial x}dx\,dy + \frac{\partial^2 u}{\partial x \partial y}dy\,dx + \frac{\partial^2 u}{\partial y^2}dy^2 =$$

$$= \left[\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}\right] = \frac{\partial^2 u}{\partial x^2}dx^2 + 2\frac{\partial^2 u}{\partial x \partial y}dx\,dy + \frac{\partial^2 u}{\partial y^2}dy^2 =$$

$$= \left(10 - 7y^3 \sin x + \frac{\operatorname{tg} y}{x^2}\right)dx^2 +$$

$$+ 2\left(21y^2 \cos x - \frac{1}{x\cos^2 y}\right)dx\,dy +$$

$$+ \left(42y \sin x - 2\frac{\ln x \sin y}{\cos^3 y}\right)dy^2$$

Вариант 2:

$$du = d\left(5x^2 + 7y^3\sin x - \ln x\, \operatorname{tg} y + 10y - 13\right) =$$

$$= 5dx^2 + 7d\left(y^3\sin x\right) - d\left(\ln x\, \operatorname{tg} y\right) + 10\, dy = (1)$$

$$dx^2 = \frac{d}{dx}(x^2)\, dx = 2x\, dx$$

$$d(y^3\sin x) = d(y^3)\sin x + y^3 d(\sin x) = \frac{d}{dy}(y^3)dy \sin x + y^3\frac{d}{dx}(\sin x)\, dx =$$

$$= 3y^2 dy \sin x + y^3\cos x\, dx = y^3\cos x\, dx + 3y^2\sin x\, dy$$

$$d\left(\ln x\, \operatorname{tg} y\right) = d(\ln x)\operatorname{tg} y + \ln x\, d(\operatorname{tg} y) = \frac{d}{dx}(\ln x)\, dx\, \operatorname{tg} y + \ln x\frac{d}{dy}(\operatorname{tg} y)\, dy =$$

$$= \frac{1}{x}dx\, \operatorname{tg} y + \ln x\frac{1}{\cos^2 y}dy = \frac{\operatorname{tg} y}{x}dx + \frac{\ln x}{\cos^2 y}dy$$

$$(1) = 5(2x\, dx) + 7\left(y^3\cos x\, dx + 3y^2\sin x\, dy\right) - \left(\frac{\operatorname{tg} y}{x}dx + \frac{\ln x}{\cos^2 y}dy\right) + 10y =$$

$$= \left(10x + 7y^3\cos x - \frac{\operatorname{tg} y}{x}\right)dx + \left(21y^2\sin x - \frac{\ln x}{\cos^2 y} + 10\right)dy =$$

$$= \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy \Rightarrow$$

$$\frac{\partial u}{\partial x} = 10x + 7y^3\cos x - \frac{\operatorname{tg} y}{x}$$

$$\frac{\partial u}{\partial y} = 21y^2\sin x - \frac{\ln x}{\cos^2 y} + 10$$

$$d^2u = d(du) = d\left(\left(10 + 7y^3\cos x - \frac{\operatorname{tg} y}{x}\right)dx + \left(21y^2\sin x - \frac{\ln x}{\cos^2 y} + 10\right)dy\right) =$$

$$= d\left(10x + 7y^3\cos x - \frac{\operatorname{tg} y}{x}\right)dx + d\left(21y^2\sin x - \frac{\ln x}{\cos^2 y} + 10\right)dy =$$

$$= \left(10\,dx + 7\,d(y^3\cos x) - d\left(\frac{\operatorname{tg} y}{x}\right)\right)dx + \left(21\,d(y^2\sin x) - d\left(\frac{\ln x}{\cos^2 y}\right)\right)dy = (2)$$

$$d(y^3\cos x) = d(y^3)\cos x + y^3 d(\cos x) = \frac{d}{dy}(y^3)\,dy\cos x + y^3\frac{d}{dx}(\cos x)\,dx =$$

$$= 3y^2 dy\cos x + y^3(-\sin x)dx = -y^3\sin x\,dx + 3y^2\cos x\,dy$$

$$d\left(\frac{\operatorname{tg} y}{x}\right) = d(\operatorname{tg} y)\frac{1}{x} + \operatorname{tg} y\, d\left(\frac{1}{x}\right) = \frac{d}{dy}(\operatorname{tg} y)\,dy\frac{1}{x} + \operatorname{tg} y\frac{d}{dx}\left(\frac{1}{x}\right)dx =$$

$$= \frac{1}{\cos^2 y}dy\frac{1}{x} + \operatorname{tg} y\left(-\frac{1}{x^2}\right)dx = -\frac{\operatorname{tg} y}{x^2}dx + \frac{dy}{x\cos^2 y}$$

$$d(y^2\sin x) = d(y^2)\sin x + y^2 d(\sin x) = \frac{d}{dy}(y^2)\,dy\sin x + y^2\frac{d}{dx}(\sin x)\,dx =$$

$$= 2y\,dy\sin x + y^2\cos x\,dx = y^2\cos x\,dx + 2y\sin x\,dy$$

$$d\left(\frac{\ln x}{\cos^2 y}\right) = d(\ln x)\frac{1}{\cos^2 y} + \ln x\, d\left(\frac{1}{\cos^2 y}\right) = \frac{d}{dx}(\ln x)\,dx\frac{1}{\cos^2 y} + \ln x\frac{d}{dy}\left(\frac{1}{\cos^2 y}\right)dy =$$

$$= \frac{1}{x}dx\frac{1}{\cos^2 y} + \ln x\left(\frac{-2}{\cos^3 y}\right)(-\sin y)dy = \frac{dx}{x\cos^2 y} + 2\frac{\ln x\sin y}{\cos^3 y}dy$$

$$(2) = \left(10\,dx + 7(-y^3\sin x\,dx + 3y^2\cos x\,dy) - \left(-\frac{\operatorname{tg} y}{x^2}dx + \frac{dy}{x\cos^2 y}\right)\right)dx +$$

$$+ \left(21(y^2\cos x\,dx + 2y\sin x\,dy) - \left(\frac{dx}{x\cos^2 y} + 2\frac{\ln x\sin y}{\cos^3 y}dy\right)\right)dy =$$

$$= \left(\left(10 - 7y^3\sin x + \frac{\operatorname{tg} y}{x^2}\right)dx + \left(21y^2\cos x - \frac{1}{x\cos^2 y}\right)dy\right)dx +$$

$$+ \left(\left(21y^2\cos x - \frac{1}{x\cos^2 y}\right)dx + \left(42y\sin x - 2\frac{\ln x\sin y}{\cos^3 y}\right)dy\right)dy =$$

$$= \left(10 - 7y^3\sin x + \frac{\operatorname{tg} y}{x^2}\right)dx^2 +$$

$$+ 2\left(21y^2\cos x - \frac{1}{x\cos^2 y}\right)dx\,dy +$$

$$+ \left(42y\sin x - 2\,\frac{\ln x \sin y}{\cos^3 y}\right)dy^2 =$$

$$= \frac{\partial^2 u}{\partial x^2}dx^2 + 2\frac{\partial^2 u}{\partial x \partial y}dx\,dy + \frac{\partial^2 u}{\partial y^2}dy^2$$

$$\frac{\partial^2 u}{\partial x^2} = 10 - 7y^3\sin x + \frac{\operatorname{tg} y}{x^2}$$

$$\frac{\partial^2 u}{\partial x \partial y} = 21y^2\cos x - \frac{1}{x\cos^2 y} = \frac{\partial^2 u}{\partial y \partial x}$$

$$\frac{\partial^2 u}{\partial y^2} = 42y\sin x - 2\,\frac{\ln x \sin y}{\cos^3 y}$$

Ответ:

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy =$$

$$= \left(10x + 7y^3\cos x - \frac{\operatorname{tg} y}{x}\right)dx +$$

$$+ \left(21y^2\sin x - \frac{\ln x}{\cos^2 y} + 10\right)dy$$

$$du^2 = \frac{\partial^2 u}{\partial x^2}dx^2 + 2\frac{\partial^2 u}{\partial x \partial y}dx\,dy + \frac{\partial^2 u}{\partial y^2}dy^2 =$$

$$= \left(10 - 7y^3\sin x + \frac{\operatorname{tg} y}{x^2}\right)dx^2 +$$

$$+ 2\left(21y^2\cos x - \frac{1}{x\cos^2 y}\right)dx\,dy +$$

$$+ \left(42y\sin x - 2\,\frac{\ln x \sin y}{\cos^3 y}\right)dy^2$$