1.4 Найти предел функции:

$$\lim_{x\to 0}\frac{3x\,tg4x}{1-\cos 4x}=(1)$$

$$\lim_{x\to 0}(1-\cos 4x)=(1-\cos 4x)_{x=0}=1-\cos(4\cdot 0)=1-\cos 0=1-1=0$$

$$\lim_{x\to 0}(3x\,tg4x)=(3x\,tg4x)_{x=0}=3\cdot 0\cdot tg4\cdot 0=3\cdot 0\,tg0=3\cdot 0\cdot 0=0$$

$$(1)=\left(\frac{0}{0}\right)$$

$$1-\cos 4x=1-\cos(2\cdot 2x)=\{$$

$$\cos 2t=\cos^2 t-\sin^2 t=(1-\sin^2 t)-\sin^2 t=1-2\sin^2 t$$

$$\}=1-(1-2\sin^2 2x)=1-1+2\sin^2 2x=2\sin^2 2x$$

$$tg4x=\frac{\sin 4x}{\cos 4x}=\frac{\sin(2\cdot 2x)}{\cos 4x}=\{\sin 2t=2\sin t\cos t\}=$$

$$=\frac{2\sin 2x\cos 2x}{\cos 4x}=2\frac{\cos 2x}{\cos 4x}\sin 2x$$

$$\frac{3x\,tg4x}{1-\cos 4x}=3x\,\frac{tg4x}{1-\cos 4x}=$$

$$=3x\,\frac{2\frac{\cos 2x}{\cos 4x}\sin 2x}{2\sin^2 2x}=3\,\frac{\cos 2x}{\cos 4x}\,\frac{x}{\sin 2x}=$$

$$=\frac{3}{2}\,\frac{\cos 2x}{\cos 4x}\,\frac{2x}{\sin 2x}=\{x\neq 0\}=$$

$$=\frac{3}{2}\,\frac{\cos 2x}{\cos 4x}\left(\frac{\sin 2x}{2x}\right)^{-1}$$

$$(1) = \lim_{x\to 0} \frac{3}{2}\,\frac{\cos 2x}{\cos 4x}\left(\frac{\sin 2x}{2x}\right)^{-1} =$$

$$= \frac{3}{2}\lim_{x\to 0}\frac{\cos 2x}{\cos 4x}\left(\lim_{x\to 0}\frac{\sin 2x}{2x}\right)^{-1} = \Big\{$$

$$\lim_{x\to 0}\frac{\sin 2x}{2x} = \Big|\, 3 = 2x;\ \lim_{x\to 0} 3 = \lim_{x\to 0} 2x = (2x)_{x=0} = 2\cdot 0 = 0 \,\Big\} = \lim_{3\to 0}\frac{\sin 3}{3} = 1$$

$$\lim_{x\to 0}\frac{\cos 2x}{\cos 4x} = \left(\frac{\cos 2x}{\cos 4x}\right)_{x=0} = \frac{\cos(2\cdot 0)}{\cos(4\cdot 0)} = \frac{\cos 0}{\cos 0} = \frac{1}{1} = 1$$

$$\Big\} = \frac{3}{2}\cdot 1\cdot 1^{-1} = \frac{3}{2}$$

Onde em:

$$\boxed{\lim_{x\to 0}\frac{3x\,\mathrm{tg}\,4x}{1-\cos 4x} = \frac{3}{2}}$$