[3] $u(x,y,z) = x^2 + y^2 + z^2$

$\bar{c} = (-9, 8, -12)$

$M = (8, -12, 9)$

Найти: $\frac{\partial u}{\partial \bar{c}}(M)$

Решение:

$$\frac{\partial u}{\partial \bar{c}}(M) = \left(\frac{\bar{c}}{|\bar{c}|} \cdot \nabla u\right)(M) = \frac{\bar{c}}{|\bar{c}|} \cdot \nabla u(M) = \frac{\bar{c} \cdot \nabla u(M)}{|\bar{c}|}$$

$$\nabla u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2 + y^2 + z^2) = \frac{d}{dx}(x^2) = 2x$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(x^2 + y^2 + z^2) = \frac{d}{dy}(y^2) = 2y$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}(x^2 + y^2 + z^2) = \frac{d}{dz}(z^2) = 2z$$

$$\nabla u(M) = \left(\frac{\partial u}{\partial x}(M), \frac{\partial u}{\partial y}(M), \frac{\partial u}{\partial z}(M)\right)$$

$$\frac{\partial u}{\partial x}(M) = 2x\Big|_{x=8} = 2 \cdot 8 = 16$$

$$\frac{\partial u}{\partial y}(M) = 2y\Big|_{y=-12} = 2(-12) = -24$$

$$\frac{\partial u}{\partial z}(M) = 2z\Big|_{z=9} = 2 \cdot 9 = 18$$

$$\bar{C}\cdot\nabla u = (C_x, C_y, C_z)\cdot\left(\frac{\partial u}{\partial x}(M), \frac{\partial u}{\partial y}(M), \frac{\partial u}{\partial z}(M)\right) =$$

$$= C_x\frac{\partial u}{\partial x}(M) + C_y\frac{\partial u}{\partial y}(M) + C_z\frac{\partial u}{\partial z}(M) = \Big[$$

$C_x = -9,\ C_y = 8,\ C_z = -12$

$\frac{\partial u}{\partial x}(M) = 16,\ \frac{\partial u}{\partial y}(M) = -24,\ \frac{\partial u}{\partial z}(M) = 18$

$$\Big] = (-9)\cdot 16 + 8\cdot(-24) + (-12)\cdot 18 =$$

$$= -\left(3^2\cdot 2^4 + 2^3\cdot 3\cdot 2^3 + 3\cdot 2^2\cdot 2\cdot 3^2\right) =$$

$$= -\left(2^4 3^2 + 2^6\cdot 3 + 2^3\cdot 3^3\right) =$$

$$= -2^3\cdot 3\left(2\cdot 3 + 2^3 + 3^2\right) =$$

$$= -3\cdot 8\left(6+8+9\right) = -3\cdot 8\cdot 23 = -3\cdot 184 = -552$$

$$|\bar{C}| = \sqrt{\bar{C}^2} = \sqrt{\bar{C}\cdot\bar{C}} = \sqrt{(C_x, C_y, C_z)\cdot(C_x, C_y, C_z)} = \sqrt{C_x^2 + C_y^2 + C_z^2} = \Big[$$

$C_x = -9,\ C_y = 8,\ C_z = -12$

$$\Big] = \sqrt{(-9)^2 + 8^2 + (-12)^2} = \sqrt{81 + 64 + 144} = \sqrt{289} = \sqrt{17^2} = 17$$

$$\frac{\partial u}{\partial \bar{C}}(M) = \frac{\bar{C}\cdot\nabla u(M)}{|\bar{C}|} = \Big[$$

$\bar{C}\cdot\nabla u(M) = -552$

$|\bar{C}| = 17$

$$\Big] = \frac{-552}{17} = -\frac{552}{17}$$

Ответ: $\boxed{\frac{\partial u}{\partial \bar{C}}(M) = -\frac{552}{17}}$