4. $U(x,y,z) = e^{x^2+y^2+z^2}$

$\bar{c} = (4, -13, -16)$

$L = (-16, 4, -13)$

Найти: $\frac{\partial U}{\partial \bar{c}}(L)$

Решение:

$$\frac{\partial U}{\partial \bar{c}} = \frac{\bar{c}}{|\bar{c}|} \cdot \nabla U = \frac{\bar{c} \cdot \nabla U}{|\bar{c}|}$$

$$\nabla U = \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right)$$

$$\frac{\partial U}{\partial x} = \frac{\partial}{\partial x}\left(e^{x^2+y^2+z^2}\right) = e^{x^2+y^2+z^2}\frac{\partial}{\partial x}\left(x^2+y^2+z^2\right) = U\frac{d}{dx}(x^2) = U \cdot 2x = 2xU$$

$$\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\left(e^{x^2+y^2+z^2}\right) = e^{x^2+y^2+z^2}\frac{\partial}{\partial y}\left(x^2+y^2+z^2\right) = U\frac{d}{dy}(y^2) = U \cdot 2y = 2yU$$

$$\frac{\partial U}{\partial z} = \frac{\partial}{\partial z}\left(e^{x^2+y^2+z^2}\right) = e^{x^2+y^2+z^2}\frac{\partial}{\partial z}\left(x^2+y^2+z^2\right) = U\frac{d}{dz}(z^2) = U \cdot 2z = 2zU$$

$$\bar{c} \cdot \nabla U = (c_x, c_y, c_z) \cdot \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right) = c_x\frac{\partial U}{\partial x} + c_y\frac{\partial U}{\partial y} + c_z\frac{\partial U}{\partial z} = \Big[$$

$c_x = 4,\ c_y = -13,\ c_z = -16$

$\frac{\partial U}{\partial x} = 2xU,\ \frac{\partial U}{\partial y} = 2yU,\ \frac{\partial U}{\partial z} = 2zU$

$$\Big] = 4 \cdot 2xU + (-13)\,2yU + (-16)\,2zU =$$

$$= 2(4x - 13y - 16z)U$$

$$|\bar{c}| = \sqrt{\bar{c}^2} = \sqrt{\bar{c}\,\bar{c}} = \sqrt{(c_x, c_y, c_z)\cdot(c_x, c_y, c_z)} = \sqrt{c_x^2 + c_y^2 + c_z^2} = \Big[$$

$c_x = 4,\ c_y = -13,\ c_z = -16$

$$\Big] = \sqrt{4^2 + (-13)^2 + (-16)^2} = \sqrt{16 + 169 + 256} = \sqrt{441} = \sqrt{21^2} = 21$$

$$\frac{\partial u}{\partial \bar{c}} = \frac{\bar{c} \cdot \nabla u}{|\bar{c}|} = \left[ \right.$$

$$\bar{c} \cdot \nabla u = 2(4x - 13y - 16z)u$$

$$|\bar{c}| = 21$$

$$\left. \right] = 2(4x - 13y - 16z)u\,21^{-1} = \frac{2}{21}(4x - 13y - 16z)u =$$

$$= \frac{2}{21}(4x - 13y - 16z)e^{x^2+y^2+z^2}$$

$$\frac{\partial u}{\partial \bar{c}}(L) = \frac{2}{21}(4x - 13y - 16z)\,e^{x^2+y^2+z^2} \;\Big|\; = (1)$$

$$x = -16,\ y = 4,\ z = -13$$

$$x^2 + y^2 + z^2 = (-16)^2 + 4^2 + 13^2 = 256 + 16 + 169 = 441$$

$$4x - 13y - 16z = 4(-16) - 13 \cdot 4 - 16(-13) =$$

$$= -4 \cdot 16 - 4 \cdot 13 + 13 \cdot 16 = 12 \cdot 13 - 4 \cdot 16 = 156 - 64 = 92$$

$$(1) = \frac{2}{21} \cdot 92\, e^{441} = \frac{184}{21} e^{441}$$

Ответ: $\boxed{\frac{\partial u}{\partial \bar{c}}(L) = \frac{184}{21} e^{441}}$