12 Найти предел функции

$$\lim_{x\to 0} x\operatorname{ctg} x = (1)$$

$$\lim_{x\to 0} x = (x)_{x=0} = 0$$

$$\lim_{x\to 0} \operatorname{ctg} x = \infty$$

$$(1) = (0\cdot\infty)$$

$$x\operatorname{ctg} x = x\frac{\cos x}{\sin x} = \frac{x}{\sin x}\cos x = \{x\neq 0\} =$$

$$= \cos x\left(\frac{\sin x}{x}\right)^{-1}$$

$$(1) = \lim_{x\to 0}\cos x\left(\frac{\sin x}{x}\right)^{-1} = \frac{\lim\limits_{x\to 0}\cos x}{\lim\limits_{x\to 0}\frac{\sin x}{x}} = (2)$$

$$\lim_{x\to 0}\cos x = (\cos x)_{x=0} = \cos 0 = 1$$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$(2) = \frac{1}{1} = 1$$

Ответ:

$$\boxed{\lim_{x\to 0} x\operatorname{ctg} x = 1}$$