$\boxed{2.2}$ $u(x,y) = x^2 + y^3 - 3x^3y^2 + 5$

$\bar{a} = (1,1)$

$A = (2,1)$

Найти: $\frac{\partial u}{\partial \bar{a}}(A)$; $\frac{\partial u}{\partial \nabla u}(A)$

Решение:

$$\frac{\partial u}{\partial \bar{a}}(A) = \left(\frac{\bar{a}}{|\bar{a}|} \cdot \nabla u\right)(A) = \frac{\bar{a}}{|\bar{a}|} \cdot \nabla u(A) = \frac{\bar{a} \cdot \nabla u(A)}{|\bar{a}|}$$

$$\nabla u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right)$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(x^2 + y^3 - 3x^3y^2 + 5\right) = \frac{d}{dx}\left(x^2\right) - 3y^2\frac{d}{dx}\left(x^3\right) =$$

$$= 2x - 3y^2\,3x^2 = 2x - 9x^2y^2$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(x^2 + y^3 - 3x^3y^2 + 5\right) = \frac{d}{dy}\left(y^3\right) - 3x^3\frac{d}{dy}\left(y^2\right) =$$

$$= 3y^2 - 3x^2\,2y = 3y^2 - 6x^3y$$

$$\nabla u(A) = \left(\frac{\partial u}{\partial x}(A), \frac{\partial u}{\partial y}(A)\right)$$

$$\frac{\partial u}{\partial x}(A) = 2x - 9x^2y^2\Big|_{x=2,\,y=1} =$$

$$= 2\cdot 2 - 9\cdot 2^2\cdot 1^2 = 4 - 4\cdot 9 = -4\cdot 8 = -32$$

$$\frac{\partial u}{\partial y}(A) = 3y^2 - 6x^3y\Big|_{x=2,\,y=1} =$$

$$= 3\cdot 1^2 - 6\cdot 2^3\cdot 1 = 3 - 6\cdot 8 = 3 - 3\cdot 16 = -3\cdot 15 = -45$$

$$\bar{a}\cdot\nabla u(A) = (a_x, a_y)\cdot\left(\frac{\partial u}{\partial x}(A), \frac{\partial u}{\partial y}(A)\right) = a_x\frac{\partial u}{\partial x}(A) + a_y\frac{\partial u}{\partial y}(A) =$$

$$= \left[a_x = 1, a_y = 1;\ \frac{\partial u}{\partial x}(A) = -32, \frac{\partial u}{\partial y}(A) = -21\right] =$$

$$= 1(-32) + 1\cdot(-45) = -32 - 45 = -77$$

$$|\bar{a}| = \sqrt{\bar{a}^2} = \sqrt{\bar{a}\cdot\bar{a}} = \sqrt{(a_x, a_y)\cdot(a_x, a_y)} = \sqrt{a_x^2 + a_y^2} =$$

$$= \left[a_x = 1,\ a_y = 1\right] =$$

$$= \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

$$\frac{\partial u}{\partial \bar{a}}(A) = \frac{\bar{a}\cdot\nabla u(A)}{|\bar{a}|} =$$

$$= \left[\bar{a}\cdot\nabla u(A) = -77,\ |\bar{a}| = \sqrt{2}\right] =$$

$$= \frac{-77}{\sqrt{2}} = -\frac{77\sqrt{2}}{2}$$

$$\frac{\partial u}{\partial \nabla u} = \frac{\nabla u}{|\nabla u|}\cdot\nabla u = \frac{(\nabla u)^2}{|\nabla u|} = |\nabla u| =$$

$$= \sqrt{\nabla u^2} = \sqrt{\nabla u\cdot\nabla u} = \sqrt{\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right)\cdot\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right)} = \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2}$$

$$\frac{\partial u}{\partial \nabla u}(A) = |\nabla u|(A) = |\nabla u(A)| =$$

$$= \left|\left(\frac{\partial u}{\partial x}(A), \frac{\partial u}{\partial y}(A)\right)\right| = \sqrt{\left(\frac{\partial u}{\partial x}(A)\right)^2 + \left(\frac{\partial u}{\partial y}(A)\right)^2} =$$

$$= \left[\frac{\partial u}{\partial x}(A) = -32,\ \frac{\partial u}{\partial y}(A) = -45\right] =$$

$$= \sqrt{(-32)^2 + (-45)^2} = \sqrt{32^2 + 45^2} = \sqrt{3049}$$

Ответ:

$$\frac{\partial u}{\partial \bar{a}} = -\frac{77\sqrt{2}}{2}$$

$$\frac{\partial u}{\partial \nabla u}(A) = \sqrt{3049}$$