

$$X = 0.(216)$$

$$\begin{aligned} 10^3 X &= 10^3 \cdot 0.(216) = \\ &= 216.(216) = \\ &= 216 + 0.(216) = \\ &= 216 + X \end{aligned}$$

$$10^3 X - X = 216 \quad ; \quad 10^3 X - X = (10^3 - 1)X = 999X$$

$$999X = 216$$

$$X = \frac{216}{999} = (1)$$

$$\begin{aligned} 216 &= 210 + 6 = 21 \cdot 10 + 6 = (3 \cdot 7)(2 \cdot 5) + 6 = \\ &= (2 \cdot 3)(5 \cdot 7) + 6 = 6 \cdot 35 + 6 = 6(35 + 1) = 6 \cdot 36 = \\ &= 6 \cdot (4 \cdot 9) = (6 \cdot 4) \cdot 9 = 9 \cdot 24 \end{aligned}$$

$$999 = 9 \cdot 111$$

$$X = \frac{9 \cdot 24}{9 \cdot 111} = \frac{24}{111}$$

$$X = 0.(216) = \frac{24}{111}$$