$\boxed{6}$ Найти производную функции:

$y(x) = x\ln(x+\sqrt{x^2+1}) - \sqrt{x^2+1}$

Решение:

$y'(x) = \left(x\ln\left(x+\sqrt{x^2+1}\right) - \sqrt{x^2+1}\right)' =$

$= \left(x\ln\left(x+\sqrt{x^2+1}\right)\right)' - \left(\sqrt{x^2+1}\right)' =$

$= (x)'\ln\left(x+\sqrt{x^2+1}\right) + x\left(\ln\left(x+\sqrt{x^2+1}\right)\right)' - \left(\sqrt{x^2+1}\right)' =$

$= \ln\left(x+\sqrt{x^2+1}\right) + x\left(\ln\left(x+\sqrt{x^2+1}\right)\right)' - \left(\sqrt{x^2+1}\right)' = (1)$

$\left(\sqrt{x^2+1}\right)' = \left((x^2+1)^{\frac{1}{2}}\right)' = \frac{1}{2}(x^2+1)^{\frac{1}{2}-1}(x^2+1)' =$

$= \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(x^2)' = \frac{1}{2}(x^2+1)^{-\frac{1}{2}}2x =$

$= x(x^2+1)^{-\frac{1}{2}} = \frac{x}{\sqrt{x^2+1}}$

$\ln\left(x+\sqrt{x^2+1}\right)' = \frac{\left(x+\sqrt{x^2+1}\right)'}{x+\sqrt{x^2+1}} = \frac{(x)' + \left(\sqrt{x^2+1}\right)'}{x+\sqrt{x^2+1}} =$

$= \frac{1+\left(\sqrt{x^2+1}\right)'}{x+\sqrt{x^2+1}} = \left(1+\frac{x}{\sqrt{x^2+1}}\right)\frac{1}{x+\sqrt{x^2+1}} =$

$= \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\frac{1}{x+\sqrt{x^2+1}} = \frac{x+\sqrt{x^2+1}}{x+\sqrt{x^2+1}}\frac{1}{\sqrt{x^2+1}} =$

$= \frac{1}{\sqrt{x^2+1}}$

$(1) = \ln\left(x+\sqrt{x^2+1}\right) + x\frac{1}{\sqrt{x^2+1}} - \frac{x}{\sqrt{x^2+1}} =$

$= \ln\left(x+\sqrt{x^2+1}\right)$

Ответ: $y'(x) = \ln\left(x+\sqrt{x^2+1}\right)$

варіант 2.

$$y'(x) = \left(x \ln\left(x+\sqrt{x^2+1}\right) - \sqrt{x^2+1}\right)' =$$

$$= \left(x \ln\left(x+\sqrt{x^2+1}\right)\right)' - \left(\sqrt{x^2+1}\right)' =$$

$$= (x)' \ln\left(x+\sqrt{x^2+1}\right) + x\left(\ln\left(x+\sqrt{x^2+1}\right)\right)' - \left(\sqrt{x^2+1}\right)' =$$

$$= \ln\left(x+\sqrt{x^2+1}\right) + x\left(\ln\left(x+\sqrt{x^2+1}\right)\right)' - \left(\sqrt{x^2+1}\right)' = (1)$$

$$\left(\ln\left(x+\sqrt{x^2+1}\right)\right)' = \frac{\left(x+\sqrt{x^2+1}\right)'}{x+\sqrt{x^2+1}} = \frac{(x)' + \left(\sqrt{x^2+1}\right)'}{x+\sqrt{x^2+1}} =$$

$$= \frac{1+\left(\sqrt{x^2+1}\right)'}{x+\sqrt{x^2+1}}$$

$$x\left(\ln\left(x+\sqrt{x^2+1}\right)\right)' - \left(\sqrt{x^2+1}\right)' = x\,\frac{1+\left(\sqrt{x^2+1}\right)'}{x+\sqrt{x^2+1}} - \left(\sqrt{x^2+1}\right)' = (2)$$

$$g(x) := \sqrt{x^2+1}$$

$$(2) = x\,\frac{1+g'(x)}{x+g(x)} - g'(x) = \frac{x(1+g'(x)) - g'(x)(x+g(x))}{x+g(x)} =$$

$$= \frac{x + xg'(x) - xg'(x) - g(x)g'(x)}{x+g(x)} = \frac{x - g(x)g'(x)}{x+g(x)} = (3)$$

$$g(x) = \sqrt{x^2+1}$$

$$g'(x) = \left(\sqrt{x^2+1}\right)' = \left((x^2+1)^{\frac{1}{2}}\right)' = \frac{1}{2}(x^2+1)^{\frac{1}{2}-1}(x^2+1)' =$$

$$= \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(x^2)' = \frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x) =$$

$$= x(x^2+1)^{-\frac{1}{2}} = \frac{x}{\sqrt{x^2+1}} = \frac{x}{g(x)}$$

$$(3) = \frac{x - g(x)\frac{x}{g(x)}}{x+g(x)} = \frac{x-x}{x+g(x)} = \frac{0}{x+g(x)} = 0$$

$$(1) = \ln\left(x+\sqrt{x^2+1}\right) + 0 = \ln\left(x+\sqrt{x^2+1}\right)$$