

Daten:

$$A = \begin{pmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 4 & 3 \\ 2 & 3 & 5 & 6 \end{pmatrix}$$

Frage:

rank A

Rechnung:

$$\text{rank } A = \text{rank} \begin{pmatrix} a_{1*} \\ a_{2*} \\ a_{3*} \\ a_{4*} \end{pmatrix} = \text{rank} \begin{pmatrix} a_{1*} \\ a_{2*} - a_{1*} \\ a_{3*} - 2a_{1*} \\ a_{4*} \end{pmatrix} = (i)$$

$$a_{2*} - a_{1*} = (0 \ 0 \ 2 \ 2) - (0 \ 0 \ 2 \ 1) = (0 - 0 \ 0 - 0 \ 2 - 2 \ 2 - 1) = \\ = (0 \ 0 \ 0 \ 1)$$

$$a_{3*} - a_{2*} = (0 \ 0 \ 4 \ 3) - 2(0 \ 0 \ 2 \ 1) = (0 - 0 \ 0 - 0 \ 4 - 4 \ 3 - 2) = \\ = (0 \ 0 \ 0 \ 1) = a_{2*} - a_{1*}$$

$$(i) = \text{rank} \begin{pmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 3 & 5 & 6 \end{pmatrix} = \text{rank} \begin{pmatrix} a_{1*} \\ a_{2*} \\ a_{3*} \\ a_{4*} \end{pmatrix} =$$

$$= (a_{2*} = a_{3*}) = \text{rank} \begin{pmatrix} a_{1*} \\ a_{2*} \\ a_{4*} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 3 & 5 & 6 \end{pmatrix} = \text{rank} \begin{pmatrix} a_{1*} \\ a_{2*} \\ a_{3*} \end{pmatrix} =$$

$$= \text{rank} \begin{pmatrix} a_{3*} \\ a_{1*} \\ a_{2*} \end{pmatrix} = \text{rank} \begin{pmatrix} 2 & 3 & 5 & 6 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = 3$$

Antwort:

rank A = 3