Дано:

$$A = \begin{pmatrix} -2 & 7 & -3 \\ 4 & -14 & 6 \\ -3 & 7 & 13 \end{pmatrix}$$

Д-ть: $\det A = 0$

Д-во:

$\boxed{1}$

$$A \in \mathbb{R}^{n \times n} \quad \det A = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} M_{ij}$$

$$\det A = \sum_{j=1}^{3} (-1)^{1+j} a_{1j} M_{1j} =$$

$$= (-1)^{1+1}(-2) \det\begin{pmatrix} -14 & 6 \\ 7 & 13 \end{pmatrix} + (-1)^{1+2} 7 \det\begin{pmatrix} 4 & 6 \\ -3 & 13 \end{pmatrix} + (-1)^{1+3}(-3) \det\begin{pmatrix} 4 & -14 \\ -3 & 7 \end{pmatrix} =$$

$$= (-2) \det\begin{pmatrix} -14 & 6 \\ 7 & 13 \end{pmatrix} + (-7) \det\begin{pmatrix} 4 & 6 \\ -3 & 13 \end{pmatrix} + (-3) \det\begin{pmatrix} 4 & -14 \\ -3 & 7 \end{pmatrix} =$$

$$= (-2)\left((-14)\cdot 13 - 6\cdot 7\right) + (-7)\left(4\cdot 13 - 6\cdot(-3)\right) + (-3)\left(4\cdot 7 - (-14)(-3)\right) =$$

$$= 2\cdot 13\cdot 14 + 2\cdot 6\cdot 7 - 4\cdot 7\cdot 13 - 3\cdot 6\cdot 7 - 3\cdot 4\cdot 7 + 3\cdot 3\cdot 14 =$$

$$= (2\cdot 13\cdot 14 - 4\cdot 7\cdot 13) + (2\cdot 6\cdot 7 - 3\cdot 4\cdot 7) + (3\cdot 3\cdot 14 - 3\cdot 6\cdot 7) =$$

$$= 0 + 0 + 0 = 0$$

$\boxed{2}$

$$A \in \mathbb{R}^{n \times n} \quad \det A = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} M_{ij}$$

$$\det A = \sum_{i=1}^{3} (-1)^{i+1} a_{i1} M_{i1} =$$

$$= (-1)^{1+1}(-2) \det\begin{pmatrix} -14 & 6 \\ 7 & 13 \end{pmatrix} + (-1)^{2+1} 4 \det\begin{pmatrix} 7 & -3 \\ 7 & 13 \end{pmatrix} + (-1)^{3+1}(-3) \det\begin{pmatrix} 7 & -3 \\ -14 & 6 \end{pmatrix} =$$

$$= (-2)\det\begin{pmatrix} -14 & 6 \\ 7 & 13 \end{pmatrix} + (-4)\det\begin{pmatrix} 7 & -3 \\ 7 & 13 \end{pmatrix} + (-3)\det\begin{pmatrix} 7 & -3 \\ -14 & 6 \end{pmatrix} =$$

$$= (-2)\left((-14)13 - 6\cdot 7\right) + (-4)\left(7\cdot 13 - (-3)7\right) + (-3)\left(7\cdot 6 - (-3)(-14)\right) =$$

$$= 2\cdot 13\cdot 14 + 2\cdot 6\cdot 7 - 4\cdot 7\cdot 13 - 3\cdot 4\cdot 7 - 3\cdot 6\cdot 7 + 3\cdot 3\cdot 14 =$$

$$= (2\cdot 13\cdot 14 - 4\cdot 7\cdot 13) + (2\cdot 6\cdot 7 - 3\cdot 4\cdot 7) + (3\cdot 3\cdot 14 - 3\cdot 6\cdot 7) =$$

$$= 0 + 0 + 0 = 0$$

$\boxed{3}$

$$A \in \mathbb{R}^{n\times n} \quad \det A = \sum_{\{k_1,k_2,\dots,k_n\}} (-1)^{P\{k_1,k_2,\dots,k_n\}} a_{1k_1}a_{2k_2}\dots a_{nk_n}$$

$$\det A = \sum_{\{1,2,3\}} (-1)^{P\{k_1,k_2,k_3\}} a_{1k_1}a_{2k_2}a_{3k_3} =$$

$$= (-1)^{P\{1,2,3\}}(-2)(-14)13 + (-1)^{P\{2,3,1\}}7\cdot 6\cdot(-3) + (-1)^{P\{3,1,2\}}(-3)\cdot 4\cdot 7 +$$
$$+ (-1)^{P\{3,2,1\}}(-3)(-14)(-3) + (-1)^{P\{2,1,3\}}7\cdot 4\cdot 13 + (-1)^{P\{1,3,2\}}(-2)\cdot 6\cdot 7 = (3)$$

$P\{1,2,3\} = 0$; $P\{2,3,1\} = 2$; $P\{3,1,2\} = 2$

$P\{3,2,1\} = 3$; $P\{2,1,3\} = 1$; $P\{1,3,2\} = 1$

$$(3) = (-2)(-14)\cdot 13 + 7\cdot 6\cdot(-3) + (-3)\cdot 4\cdot 7 +$$
$$+ (-1)(-3)(-14)(-3) + (-1)\cdot 7\cdot 4\cdot 13 + (-1)(-2)\cdot 6\cdot 7 =$$

$$= 2\cdot 13\cdot 14 - 3\cdot 6\cdot 7 - 3\cdot 4\cdot 7 + 3\cdot 3\cdot 14 - 4\cdot 7\cdot 13 + 2\cdot 6\cdot 7 =$$

$$= (2\cdot 13\cdot 14 - 4\cdot 7\cdot 13) + (3\cdot 3\cdot 14 - 3\cdot 6\cdot 7) + (2\cdot 6\cdot 7 - 3\cdot 4\cdot 7) =$$

$$= 0 + 0 + 0 = 0$$

$\boxed{4}$

$$A = \begin{pmatrix} -2 & 7 & -3 \\ 4 & -14 & 6 \\ -3 & 7 & 13 \end{pmatrix} = \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} \\ a_{3\cdot} \end{pmatrix}$$

$$a_{1\cdot} = \begin{pmatrix} -2 & 7 & -3 \end{pmatrix}$$

$$a_{2\cdot} = \begin{pmatrix} 4 & -14 & 6 \end{pmatrix} = \begin{pmatrix} 2\cdot 2 & -2\cdot 7 & 2\cdot 3 \end{pmatrix} =$$

$$= \begin{pmatrix} (-2)(-2) & (-2)7 & (-2)(-3) \end{pmatrix} = (-2)\begin{pmatrix} -2 & 7 & -3 \end{pmatrix} = (-2)\, a_{1\cdot} \iff$$

$$\iff 2a_{1\cdot} + a_{2\cdot} = 0 \iff$$

$\iff a_{1\cdot}, a_{2\cdot}$ – линеарно зависни $\Rightarrow$

$$\Rightarrow \det \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} \\ a_{3\cdot} \end{pmatrix} = \det A = 0$$

$\boxed{5}$

$$\det A = \det \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} \\ a_{3\cdot} \end{pmatrix} =$$

$$= \left(a_{2\cdot} = (-2) a_{1\cdot} \right) = \det \begin{pmatrix} a_{1\cdot} \\ (-2) a_{1\cdot} \\ a_{3\cdot} \end{pmatrix} = (-2) \det \begin{pmatrix} a_{1\cdot} \\ a_{1\cdot} \\ a_{3\cdot} \end{pmatrix} = (-2)\cdot 0 = 0$$

$\boxed{6}$

$$\det A = \det \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} \\ a_{3\cdot} \end{pmatrix} = \det \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} + 2a_{1\cdot} \\ a_{3\cdot} \end{pmatrix} =$$

$$= \left(a_{2\cdot} = (-2) a_{1\cdot} \right) = \det \begin{pmatrix} a_{1\cdot} \\ (-2) a_{1\cdot} + 2 a_{1\cdot} \\ a_{3\cdot} \end{pmatrix} = \det \begin{pmatrix} a_{1\cdot} \\ 0 \\ a_{3\cdot} \end{pmatrix} = 0$$