

Demo.

Mitteinholo uporabombo $L \subset C(\mathbb{R})$ krog novem \mathbb{R}

$$f_1(x) = e^x, f_2(x) = 1, f_3(x) = x+1, f_4(x) = x - e^x \in L$$

Nekateri:

Mitteinholo (kl.) zahracemo $f_1(x), f_2(x), f_3(x), f_4(x)$

Povezave:

$$\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in \mathbb{R}$$

$$\alpha_1 f_1(x) + \alpha_2 f_2(x) + \alpha_3 f_3(x) + \alpha_4 f_4(x) =$$

$$= \alpha_1 e^x + \alpha_2 1 + \alpha_3 (x+1) + \alpha_4 (x - e^x) =$$

$$= (\alpha_1 - \alpha_4) e^x + (\alpha_3 + \alpha_4) x + (\alpha_2 + \alpha_3) =$$

$$= (\alpha_1 - \alpha_4) e^x + (\alpha_2 + \alpha_3) x + (\alpha_2 + \alpha_3) \cdot 1 = 0$$

$e^x, x, 1 \in L$ mitteinholo teljazracemo

$$(\alpha_1 - \alpha_4) e^x + (\alpha_2 + \alpha_3) x + (\alpha_2 + \alpha_3) \cdot 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow \alpha_1 - \alpha_4 = 0 \wedge \alpha_2 + \alpha_3 = 0 \wedge \alpha_2 + \alpha_3 = 0$$

$$\begin{cases} \alpha_1 - \alpha_4 = 0 \\ \alpha_2 + \alpha_3 = 0 \\ \alpha_2 + \alpha_3 = 0 \end{cases} \Leftrightarrow \begin{cases} \alpha_4 = \alpha_1 \\ \alpha_3 = -\alpha_4 = -\alpha_1 \\ \alpha_2 = -\alpha_3 = -(-\alpha_1) = \alpha_1 \end{cases} \Leftrightarrow \begin{cases} \alpha_2 = \alpha_1 \\ \alpha_3 = -\alpha_1 \\ \alpha_4 = \alpha_1 \end{cases}$$

$$\exists \alpha_i \in \mathbb{R} \setminus \{0\} \quad \alpha_2 = \alpha_1, \alpha_3 = -\alpha_1, \alpha_4 = \alpha_1 \quad \alpha_1 f_1(x) + \alpha_2 f_2(x) + \alpha_3 f_3(x) + \alpha_4 f_4(x) = 0 \Leftrightarrow$$

$f_1(x), f_2(x), f_3(x), f_4(x)$ mitteinholo zahracemo

Dosevstvo:

$$f_4(x) = x - e^x = -e^x - 1 + (x+1) = -f_1(x) - f_2(x) + f_3(x) \Rightarrow$$

$f_1(x), f_2(x), f_3(x), f_4(x)$ mitteinholo zahracemo