Дано:

$\det A = 4$

Найти:

$\det(A^2)$

$\det(A^T)$

$\det(2A)$

Решение:

$\boxed{1}$

$A, B \in \mathbb{R}^{n\times n} \quad \det(AB) = \det A \det B$

$k \in \mathbb{N} \quad \det(A^k) = \det(A A^{k-1}) = \det A \det(A^{k-1}) =$

$= \det A \det(A A^{k-2}) = \det A \det A \det A^{k-2} = (\det A)^2 \det A^{k-2} =$

$= \dots = (\det A)^l (\det A)^{k-l} =$

$= \dots = (k - l = 1 \Leftrightarrow l = k-1) = (\det A)^{k-1} \det A = (\det A)^k$

$k \in \mathbb{N} \quad \det(A^k) = (\det A)^k$

$\det(A^2) = (\det A)^2 = (\det A = 4) = 4^2 = 16$

$\boxed{2}$

$A \in \mathbb{R}^{n\times n} \quad \det(A^T) = \det A$

$\det(A^T) = \det A = 4$

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$A \in \mathbb{R}^{n\times n}$ $A = \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} \\ \vdots \\ a_{n\cdot} \end{pmatrix}$

$\lambda \in \mathbb{R}$ $\forall k = \overline{1,n}$ $\det \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} \\ \vdots \\ \lambda a_{k\cdot} \\ \vdots \\ a_{n\cdot} \end{pmatrix} = \lambda \det \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} \\ \vdots \\ a_{k\cdot} \\ \vdots \\ a_{n\cdot} \end{pmatrix} = \lambda \det A$

$\lambda A = \lambda \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} \\ \vdots \\ a_{n\cdot} \end{pmatrix} = \begin{pmatrix} \lambda a_{1\cdot} \\ \lambda a_{2\cdot} \\ \vdots \\ \lambda a_{n\cdot} \end{pmatrix}$

$\det(\lambda A) = \det \begin{pmatrix} \lambda a_{1\cdot} \\ \lambda a_{2\cdot} \\ \lambda a_{3\cdot} \\ \vdots \\ \lambda a_{n\cdot} \end{pmatrix} = \lambda \det \begin{pmatrix} a_{1\cdot} \\ \lambda a_{2\cdot} \\ \lambda a_{3\cdot} \\ \vdots \\ \lambda a_{n\cdot} \end{pmatrix} = \lambda \lambda \det \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} \\ \lambda a_{3\cdot} \\ \vdots \\ \lambda a_{n\cdot} \end{pmatrix} = \lambda^2 \det \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} \\ \lambda a_{3\cdot} \\ \vdots \\ \lambda a_{n\cdot} \end{pmatrix} =$

$= \ldots = \lambda^k \det \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} \\ \vdots \\ a_{k-1\cdot} \\ a_{k\cdot} \\ \lambda a_{k\cdot} \\ \vdots \\ \lambda a_n \end{pmatrix} = \ldots = (k=n) = \lambda^n \det \begin{pmatrix} a_{1\cdot} \\ a_{2\cdot} \\ \vdots \\ a_{n\cdot} \end{pmatrix} = \lambda^n \det A$

$\lambda \in \mathbb{R}$ $A \in \mathbb{R}^{n\times n}$ $\det(\lambda A) = \lambda^n \det A$

$\det(2A) = 2^n \det A = (\det A = 4) = 2^n \cdot 4 = 2^n 2^2 = 2^{n+2}$

Ombem:

$A \in \mathbb{R}^{n\times n}$ $k \in \mathbb{N}$ $\det(A^k) = (\det A)^k$ $\det(A^T) = \det A$ $\lambda \in \mathbb{R}$ $\det(\lambda A) = \lambda^n \det A$	$\det A = 4$	$\det(A^2) = (\det A)^2 = 16$ $\det(A^T) = \det A = 4$ $\det(2A) = 2^n \det A = 2^{n+2}$