Дано:

$$A = \begin{pmatrix} -1 & -6 \\ 2 & 6 \end{pmatrix}$$

Найти:

$\lambda, X \quad AX = \lambda X$

Решение:

$$\det(A - \lambda E) = \det\begin{pmatrix} -1-\lambda & -6 \\ 2 & 6-\lambda \end{pmatrix} = (-1-\lambda)(6-\lambda) - (-6)2 =$$

$$= (\lambda+1)(\lambda-6) + 12 = (\lambda\lambda + \lambda(-6) + 1\cdot\lambda + 1\cdot(-6)) + 12 =$$

$$= (\lambda^2 - 5\lambda - 6) + 12 = \lambda^2 - 5\lambda + 6 =$$

$$= (\lambda^2 - 2\lambda) + (-3\lambda + 6) = \lambda(\lambda-2) - 3(\lambda-2) = (\lambda-2)(\lambda-3)$$

$$\det(A - \lambda E) = 0 \Leftrightarrow (\lambda-2)(\lambda-3) = 0 \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \lambda - 2 = 0 \\ \lambda - 3 = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \lambda = 2 \\ \lambda = 3 \end{array}\right.$$

$$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$AX = \lambda X \Leftrightarrow (A - \lambda E)X = 0 \Leftrightarrow \begin{pmatrix} -1-\lambda & -6 \\ 2 & 6-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\lambda = 2$

$-1-\lambda = -1-2 = -3$

$6-\lambda = 6-2 = 4$

$$(A - \lambda E)X = 0 \Leftrightarrow \begin{pmatrix} -3 & -6 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} -3x_1 - 6x_2 = 0 \\ 2x_1 + 4x_2 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 + 2x_2 = 0 \\ x_1 + 2x_2 = 0 \end{cases} \Leftrightarrow x_1 + 2x_2 = 0 \Leftrightarrow$$

$$\Leftrightarrow x_2 = -\frac{x_1}{2}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ -\frac{x_1}{2} \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix} \Rightarrow x = \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix}$$

$\lambda = 3$

$-1-\lambda = -1-3 = -4$

$6-\lambda = 6-3 = 3$

$$(A-\lambda E)x = 0 \Leftrightarrow \begin{pmatrix} -4 & -6 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} -4x_1 - 6x_2 = 0 \\ 2x_1 + 3x_2 = 0 \end{cases} \Leftrightarrow \begin{cases} 2x_1 + 3x_2 = 0 \\ 2x_1 + 3x_2 = 0 \end{cases} \Leftrightarrow 2x_1 + 3x_2 = 0 \Leftrightarrow$$

$$\Leftrightarrow x_2 = -\frac{2}{3}x_1$$

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ -\frac{2}{3}x_1 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ -\frac{2}{3} \end{pmatrix} \Rightarrow x = \begin{pmatrix} 1 \\ -\frac{2}{3} \end{pmatrix}$$

Проверка:

$\lambda = 2, \; X = \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix}$

$$AX = \begin{pmatrix} -1 & -6 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} (-1)\cdot 1 + (-6)\cdot(-\frac{1}{2}) \\ 2\cdot 1 + 6\cdot(-\frac{1}{2}) \end{pmatrix} = \begin{pmatrix} (-1) + 3 \\ 2 + (-3) \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} = 2\begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix} = \lambda X$$

$\lambda = 3, \; X = \begin{pmatrix} 1 \\ -\frac{2}{3} \end{pmatrix}$

$$AX = \begin{pmatrix} -1 & -6 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} 1 \\ -\frac{2}{3} \end{pmatrix} = \begin{pmatrix} (-1)\cdot 1 + (-6)(-\frac{2}{3}) \\ 2\cdot 1 + 6\,(-\frac{2}{3}) \end{pmatrix} = \begin{pmatrix} -1 + 4 \\ 2 - 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} = 3\begin{pmatrix} 1 \\ -\frac{2}{3} \end{pmatrix} = \lambda X$$

Ответ:

$$\boxed{\begin{array}{l} \lambda = 2, \; X = \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix} \\ \lambda = 3, \; X = \begin{pmatrix} 1 \\ -\frac{2}{3} \end{pmatrix} \end{array}}$$