

Beweis:

$$\det A = 4$$

Hilfsmittel:

$$\det(A^2)$$

$$\det(A^T)$$

$$\det(2A)$$

Rekurrenz:

II

$$A, B \in \mathbb{R}^{n \times n} \quad \det(A+B) = \det A \det B$$

$$k \in \mathbb{N} \quad \det(A^k) = \det(A A^{k-1}) = \det A \det(A^{k-1}) =$$

$$= \det A \det(A A^{k-2}) = \det A \det A \det A^{k-2} = (\det A)^2 \det A^{k-2} =$$

$$= \dots = (\det A)^l (\det A)^{k-l} =$$

$$= \dots = (k-l=1 \Leftrightarrow l=k-1) = (\det A)^{k-1} \det A = (\det A)^k$$

$$k \in \mathbb{N} \quad \det(A^k) = (\det A)^k$$

$$\det(A^2) = (\det A)^2 = (\det A = 4) = 4^2 = 16$$

2

$$A \in \mathbb{R}^{n \times n} \quad \det(A^T) = \det A$$

$$\det(A^T) = \det A = 4$$

3

$$A \in \mathbb{R}^{n \times n} \quad A = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{pmatrix}$$

$$A \in \mathbb{R} \quad \forall k=1, n \quad \det \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ 2a_{k1} \\ \vdots \\ a_{n1} \end{pmatrix} = 2 \det \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{k1} \\ \vdots \\ a_{n1} \end{pmatrix} = 2 \det A$$

$$2A = 2 \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{pmatrix} = \begin{pmatrix} 2a_{11} \\ 2a_{21} \\ \vdots \\ 2a_{n1} \end{pmatrix}$$

$$\det(2A) = \det \begin{pmatrix} 2a_{11} \\ 2a_{21} \\ 2a_{31} \\ \vdots \\ 2a_{n1} \end{pmatrix} = 2 \det \begin{pmatrix} a_{11} \\ 2a_{21} \\ 2a_{31} \\ \vdots \\ 2a_{n1} \end{pmatrix} = 2 \cdot 2 \det \begin{pmatrix} a_{11} \\ a_{21} \\ 2a_{31} \\ \vdots \\ 2a_{n1} \end{pmatrix} = 2^2 \begin{pmatrix} a_{11} \\ a_{21} \\ 2a_{31} \\ \vdots \\ 2a_{n1} \end{pmatrix} =$$

$$= 2^n = 2^k \det \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{k1} \\ a_{k+1} \\ \vdots \\ a_{n1} \end{pmatrix} = 2^{n-k} = (k-n) = 2^{n-k} \det \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{pmatrix} = 2^n \det A$$

$$A \in \mathbb{R} \quad A \in \mathbb{R}^{n \times n} \quad \det(2A) = 2^n \det A$$

$$\det(2A) = 2^n \det A = (\det A = 4) = 2^n \cdot 4 = 2^n 2^2 = 2^{n+2}$$

Umformen:

$$A \in \mathbb{R}^{n \times n}$$

$$k \in \mathbb{N} \quad \det(Ak) = (\det A)k$$

$$\det(A^T) = \det A$$

$$A \in \mathbb{R} \quad \det(2A) = 2^n \det A$$

$$\det A = 4 \quad \det(A^2) = (\det A)^2 = 16$$

$$\det(A^T) = \det A = 4$$

$$\det(2A) = 2^n \det A = 2^{n+2}$$

②