

Dано:

$$A = \begin{pmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 4 & 3 \\ 2 & 3 & 5 & 6 \end{pmatrix}$$

Найти:

rank A

Решение:

$$\text{rank } A = \text{rank} \begin{pmatrix} a_{1.} \\ a_{2.} \\ a_{3.} \\ a_{4.} \end{pmatrix} = \text{rank} \begin{pmatrix} a_{1.} \\ a_{2.} - a_{1.} \\ a_{3.} - 2a_{1.} \\ a_{4.} \end{pmatrix} = (1)$$

$$a_{2.} - a_{1.} = (0 \ 0 \ 2 \ 2) - (0 \ 0 \ 2 \ 1) = (0-0 \ 0-0 \ 2-2 \ 2-1) = \\ = (0 \ 0 \ 0 \ 1)$$

$$a_{3.} - a_{2.} = (0 \ 0 \ 4 \ 3) - 2(0 \ 0 \ 2 \ 1) = (0-0 \ 0-0 \ 4-4 \ 3-2) = \\ = (0 \ 0 \ 0 \ 1) = a_{2.} - a_{1.}$$

$$(1) = \text{rank} \begin{pmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 3 & 5 & 6 \end{pmatrix} = \text{rank} \begin{pmatrix} a_{1.} \\ a_{2.} \\ a_{3.} \\ a_{4.} \end{pmatrix} =$$

$$= (a_{2.} = a_{3.}) = \text{rank} \begin{pmatrix} a_{1.} \\ a_{2.} \\ a_{4.} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 3 & 5 & 6 \end{pmatrix} = \text{rank} \begin{pmatrix} a_{1.} \\ a_{2.} \\ a_{3.} \end{pmatrix} =$$

$$= \text{rank} \begin{pmatrix} a_{3.} \\ a_{1.} \\ a_{2.} \end{pmatrix} = \text{rank} \begin{pmatrix} 2 & 3 & 5 & 6 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = 3$$

Итого:

$$\text{rank } A = 3$$