$\boxed{13}$ $X \sim N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad x \in (-\infty, +\infty)$$

$$D[X] = \int_{-\infty}^{+\infty} (x - E[X])^2 f(x)dx = \int_{-\infty}^{+\infty} (x-\mu)^2 \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = (1)$$

$$\xi^2 = \frac{(x-\mu)^2}{2\sigma^2} = \left(\frac{x-\mu}{\sqrt{2}\sigma}\right)^2 \Leftrightarrow \xi = \pm \frac{x-\mu}{\sqrt{2}\sigma} \Rightarrow$$

$$\Rightarrow \xi = \frac{x-\mu}{\sqrt{2}\sigma}$$

$$\xi^2 = \frac{(x-\mu)^2}{2\sigma^2} \Leftrightarrow (x-\mu)^2 = 2\sigma^2\xi^2$$

$$\lim_{x\to-\infty} \xi = \lim_{x\to-\infty} \frac{x-\mu}{\sqrt{2}\sigma} = -\infty$$

$$\lim_{x\to+\infty} \xi = \lim_{x\to+\infty} \frac{x-\mu}{\sqrt{2}\sigma} = +\infty$$

$$d\xi = d\left(\frac{x-\mu}{\sqrt{2}\sigma}\right) = \frac{1}{\sqrt{2}\sigma} d(x-\mu) = \frac{1}{\sqrt{2}\sigma} dx \Leftrightarrow dx = \sqrt{2}\sigma d\xi$$

$$(x-\mu)^2 \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = 2\sigma^2\xi^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\xi^2} \sqrt{2}\sigma d\xi =$$

$$= \frac{2}{\sqrt{\pi}} \sigma^2 \xi^2 e^{-\xi^2} d\xi$$

$$(1) = \int_{-\infty}^{+\infty} \frac{2}{\sqrt{\pi}} \sigma^2 \xi^2 e^{-\xi^2} d\xi = \frac{2}{\sqrt{\pi}} \sigma^2 \int_{-\infty}^{+\infty} \xi^2 e^{-\xi^2} d\xi = (2)$$

$$\int_{-\infty}^{+\infty} \xi^2 e^{-\xi^2} d\xi = \int_{-\infty}^{+\infty} u\,dv = (3)$$

$$\xi^2 e^{-\xi^2} d\xi = \xi\left(\xi e^{-\xi^2} d\xi\right) = u\,dv$$

$$u = \xi \Rightarrow du = d\xi$$

$$dv = \xi e^{-\xi^2} d\xi = \frac{1}{2} e^{-\xi^2}(2\xi\,d\xi) = \frac{1}{2} e^{-\xi^2} d\xi^2 = -\frac{1}{2} e^{-\xi^2}\left(-d\xi^2\right) =$$

$$= -\frac{1}{2} e^{-\xi^2} d(-\xi^2) = -\frac{1}{2}\left(e^{-\xi^2} d(-\xi^2)\right) = -\frac{1}{2} de^{-\xi^2} = d\left(-\frac{1}{2} e^{-\xi^2}\right) \Rightarrow$$

$$\Rightarrow v = -\frac{1}{2} e^{-\xi^2}$$

$$uv = \xi\left(-\frac{1}{2} e^{-\xi^2}\right) = -\frac{1}{2}\xi e^{-\xi^2}$$

$$v\,du = -\frac{1}{2} e^{-\xi^2} d\xi$$

$$uv\Big|_{-\eta}^{\eta} = -\frac{1}{2}\xi e^{-\xi^2}\Big|_{-\eta}^{\eta} = \left(-\frac{1}{2}\xi e^{-\xi^2}\right)_{\xi=\eta} - \left(-\frac{1}{2}\xi e^{-\xi^2}\right)_{\xi=-\eta} =$$

$$= \left(-\frac{1}{2}\eta e^{-\eta^2}\right) - \left(-\frac{1}{2}(-\eta) e^{-(-\eta)^2}\right) = -\frac{1}{2}\eta e^{-\eta^2} - \frac{1}{2}\eta e^{-\eta^2} =$$

$$= 2\left(-\frac{1}{2}\eta e^{-\eta^2}\right) = -\eta e^{-\eta^2}$$

$$uv\Big|_{-\infty}^{+\infty} = \lim_{\eta\to+\infty} uv\Big|_{-\eta}^{\eta} = \lim_{\eta\to+\infty}\left(-\eta e^{-\eta^2}\right) = -\lim_{\eta\to+\infty}\frac{\eta}{e^{\eta^2}} =$$

$$= -\lim_{\eta\to+\infty}\frac{\frac{d}{d\eta}(\eta)}{\frac{d}{d\eta}\left(e^{\eta^2}\right)} = -\lim_{\eta\to+\infty}\frac{1}{2\eta e^{\eta^2}} = -\frac{1}{2}\lim_{\eta\to+\infty}\frac{1}{\eta e^{\eta^2}} = 0 -$$

$$(3) = uv\Big|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} v\,du = -\int_{-\infty}^{+\infty} v\,du =$$

$$= -\int_{-\infty}^{+\infty} -\frac{1}{2} e^{-\xi^2} d\xi = -\left(-\frac{1}{2}\right)\int_{-\infty}^{+\infty} e^{-\xi^2} d\xi = \frac{1}{2}\int_{-\infty}^{+\infty} e^{-\xi^2} d\xi = (4)$$

$$\int_{-\infty}^{+\infty} e^{-\xi^2} d\xi = \int_{-\infty}^{0} e^{-\xi^2} d\xi + \int_{0}^{+\infty} e^{-\xi^2} d\xi = (5)$$

$$\int_{-\infty}^{0} e^{-\xi^2} d\xi = -\int_{0}^{-\infty} e^{-\xi^2} d\xi = \int_{0}^{-\infty} e^{-\xi^2} (-d\xi) = \int_{0}^{-\infty} e^{-(-\xi)^2} d(-\xi) =$$

$$= \int_{0}^{+\infty} e^{-\xi^2} d\xi$$

$$(5) = \int_{0}^{+\infty} e^{-\xi^2} d\xi + \int_{0}^{+\infty} e^{-\xi^2} d\xi = 2\int_{0}^{+\infty} e^{-\xi^2} d\xi = (6)$$

$$\int_{0}^{+\infty} e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2}$$

$$(6) = 2\frac{\sqrt{\pi}}{2} = \sqrt{\pi}$$

$$(4) = \frac{1}{2}\sqrt{\pi} = \frac{\sqrt{\pi}}{2}$$

$$(2) = \frac{2}{\sqrt{\pi}}\sigma^2 \frac{\sqrt{\pi}}{2} = \sigma^2$$

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad D[X] = \int_{-\infty}^{+\infty} (x - E(x))^2 f(x)\, dx = \sigma^2}$$

$$\int_{0}^{+\infty} e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2} \qquad \int_{-\infty}^{+\infty} e^{-\xi^2} d\xi = \sqrt{\pi}$$

$$\int_{-\infty}^{+\infty} \xi e^{-\xi^2} d\xi = 0$$

$$\int_{-\infty}^{+\infty} \xi^2 e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2}$$