

4.2 Permutation:

$$P = \left[\begin{pmatrix} C_7^0 C_{10-7}^{2-0} \\ C_7^1 C_{10-7}^{2-1} \\ C_7^2 C_{10-7}^{2-2} \end{pmatrix} \begin{pmatrix} C_9^2 C_{11-9}^{2-2} \\ C_9^1 C_{11-9}^{2-1} \\ C_9^0 C_{11-9}^{2-0} \end{pmatrix} + \right. \\ \left. + \begin{pmatrix} C_7^1 C_{10-7}^{2-1} \\ C_7^2 C_{10-7}^{2-2} \end{pmatrix} \begin{pmatrix} C_9^0 C_{11-9}^{2-0} \end{pmatrix} \right] \left[C_{10}^2 C_{11}^2 \right]^{-1} =$$

$$= \left[\begin{pmatrix} C_4^0 C_3^2 \\ C_4^1 C_3^1 \\ C_4^2 C_3^0 \end{pmatrix} \begin{pmatrix} C_5^2 C_2^0 \\ C_5^1 C_2^1 \\ C_5^0 C_2^2 \end{pmatrix} + \begin{pmatrix} C_4^1 C_3^1 \\ C_4^2 C_3^0 \end{pmatrix} \begin{pmatrix} C_5^0 C_2^2 \end{pmatrix} \right] \left[C_{10}^2 C_{11}^2 \right]^{-1} =$$

$$= \left(C_3^2 C_9^2 + C_4^1 C_3^1 C_5^1 C_2^1 + C_4^2 C_2^2 \right) \left(C_{10}^2 C_{11}^2 \right)^{-1} =$$

$$= \frac{C_3^2 C_9^2}{C_{10}^2 C_{11}^2} + \frac{C_4^1 C_3^1 C_5^1 C_2^1}{C_{10}^2 C_{11}^2} + \frac{C_4^2 C_2^2}{C_{10}^2 C_{11}^2} = (1)$$

$$\frac{C_3^2 C_9^2}{C_{10}^2 C_{11}^2} = \frac{C_3^2}{C_{10}^2} \cdot \frac{C_9^2}{C_{11}^2} = \frac{A_3^2}{2!} \left(\frac{A_{10}^2}{2!} \right)^{-1} \frac{A_9^2}{2!} \left(\frac{A_{11}^2}{2!} \right)^{-1} = \frac{A_3^2}{A_{10}^2} \frac{A_9^2}{A_{11}^2} =$$

$$= \frac{3 \cdot 2}{10 \cdot 9} \cdot \frac{9 \cdot 8}{11 \cdot 10} = \left(\frac{3}{10} \cdot \frac{2}{9} \right) \cdot \left(\frac{9}{11} \cdot \frac{8}{10} \right) =$$

$$= \frac{2 \cdot 3 \cdot 8 \cdot 9}{\cancel{8 \cdot 10 \cdot 10 \cdot 11}} = \frac{2 \cdot 3}{11 \cdot 25} = \frac{3 \cdot 4}{11 \cdot 25} = \frac{12}{11 \cdot 25}$$

$$\frac{C_7^2 C_2^2}{C_{10}^2 C_{11}^2} = \frac{C_7^2}{C_{10}^2} \cdot \frac{C_2^2}{C_{11}^2} = \frac{A_7^2}{A_{10}^2} \frac{A_2^2}{A_{11}^2} = \frac{7 \cdot 6}{10 \cdot 9} \cdot \frac{2 \cdot 1}{11 \cdot 10} = \left(\frac{7}{10} \cdot \frac{6}{9} \right) \cdot \left(\frac{2}{11} \cdot \frac{1}{10} \right) =$$

$$= \frac{2 \cdot 6 \cdot 7}{\cancel{9 \cdot 10 \cdot 10 \cdot 11}} = \frac{7}{3 \cdot 5 \cdot 5 \cdot 11} = \frac{7}{3 \cdot 11 \cdot 25}$$

$$\frac{C_7^1 C_3^1 C_9^1 C_2^1}{C_{10}^2 C_{11}^2} = \frac{C_4^1 C_3^1}{C_{10}^2} \cdot \frac{C_8^1 C_2^1}{C_{11}^2} =$$

$$= 7 \cdot 3 \left(\frac{10 \cdot 9}{2!} \right)^{-1} \cdot 8 \cdot 2 \left(\frac{11 \cdot 10}{2!} \right)^{-1} =$$

$$= 2 \frac{7 \cdot 3}{10 \cdot 9} \cdot 2 \frac{8 \cdot 2}{11 \cdot 10} = \left(\frac{7}{10} \cdot \frac{3}{9} + \frac{3}{10} \cdot \frac{7}{9} \right) \left(\frac{8}{11} \cdot \frac{2}{10} + \frac{2}{11} \cdot \frac{8}{10} \right) =$$

$$= \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 3 \cdot 7 \cdot \cancel{8}}{\cancel{9} \cdot \cancel{10} \cdot \cancel{10} \cdot 11} = \frac{2 \cdot 3 \cdot 7}{5 \cdot 5 \cdot 11} = \frac{6 \cdot 7}{11 \cdot 25} = \frac{42}{11 \cdot 25}$$

$$(i) = \frac{12}{11 \cdot 25} + \frac{7}{3 \cdot 11 \cdot 25} + \frac{42}{11 \cdot 25} = \frac{3 \cdot 12 + 7 + 3 \cdot 42}{3 \cdot 11 \cdot 25} =$$

$$= \frac{36 + 7 + 126}{33 \cdot 25} = \frac{169}{825}$$

Answer:

$$P = (C_3^2 C_9^2 + C_4^1 C_3^1 C_9^1 C_2^1 + C_4^2 C_2^2) (C_{10}^2 C_{11}^2)^{-1} = \frac{169}{825}$$