

$$[2.2] \quad n = 5 \cdot 10^3; p = 4 \cdot 10^{-4};$$

$$k = 2$$

Решение 1 (Биномиальное распределение):

$$n \in \mathbb{Z} \quad n \geq 0 \quad \Omega = \{ (x_1, x_2, \dots, x_n) \mid \forall i=1, n \quad x_i \in \{1, 0\} \}$$

$$p \in \mathbb{R} \quad 0 < p \leq 1 \quad \forall i=1, n \quad p(x_i=1) = p$$

$$\Xi_n = \sum_{i=1}^n x_i$$

$$\Xi_n \sim \text{Bin}(n, p)$$

$$q = 1 - p$$

$$k \in \mathbb{Z} \quad 0 \leq k \leq n \quad p(\Xi_n = k) = f(k, n, p) = C_n^k p^k q^{n-k}$$

$$C_n^k = C_{5000}^2 = \frac{5000 \cdot 4999}{2!} = \frac{5000 \cdot 4999}{2} = 2500 \cdot 4999 =$$

$$= 2.5 \cdot 10^3 \cdot 4.999 \cdot 10^3 = 25 \cdot 4.999 \cdot 10^6$$

$$p^k = (4 \cdot 10^{-4})^2 = 4^2 \cdot (10^{-4})^2 = 16 \cdot 10^{-8} = 1.6 \cdot 10 \cdot 10^{-9} = 1.6 \cdot 10^{-7}$$

$$q = 1 - p = 1 - 0.0004 = 0.9996$$

$$n - k = 5000 - 2 = 4998$$

$$q^{n-k} = 0.9996^{4998}$$

$$C_n^k p^k = (25 \cdot 4.999 \cdot 10^6) \cdot (1.6 \cdot 10^{-7}) = (1.6 \cdot 2.5) (10^6 \cdot 10^{-7}) \cdot 4.999 =$$

$$= (1.6 \cdot 2.5) 10^{-1} \cdot 4.999 = (1)$$

$$1.6 \cdot 2.5 = (16 \cdot 10^{-1}) (25 \cdot 10^{-1}) = (16 \cdot 25) \cdot (10^{-1} \cdot 10^{-1}) = 4^2 \cdot 5^2 \cdot 10^{-2} = (4 \cdot 5)^2 \cdot 10^{-2} =$$

$$= (20)^2 \cdot 10^{-2} = (2 \cdot 10)^2 \cdot 10^{-2} = 2^2 \cdot 10^2 \cdot 10^{-2} = 2^2 = 4$$

$$(1) = 4 \cdot 10^{-1} \cdot 4.999 = 0.4 \cdot 4.999$$

$$p(\Xi_n = 2) = f(2, 5 \cdot 10^3, 4 \cdot 10^{-4}) = 0.4 \cdot 4.999 \cdot 0.9996^{4998}$$

Пример 2 (распределение Пуассона):

$$\xi_n \sim \text{Pois}(\lambda)$$

$$\lambda = np$$

$$p(\xi_n = k) = f(k; \lambda) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$$\lambda = np = 5 \cdot 10^3 \cdot 4 \cdot 10^{-4} = (5 \cdot 4) (10^3 \cdot 10^{-4}) = 20 \cdot 10^{-1} = 2$$

$$p(\xi_n = k) = f(k; 2) = \frac{2^k}{k!} e^{-2}$$

$$p(\xi_n = 2) = f(2; 2) = \frac{2^2}{2!} e^{-2} = \frac{4}{2} e^{-2} = 2e^{-2}$$

Ответ:

$$\xi_n \sim \text{Bin}(n, p) \quad p(\xi_n = 2) = 0.4 \cdot 4.999 \cdot 0.9996^{4998}$$

$$\xi_n \sim \text{Pois}(\lambda) \quad p(\xi_n = 2) = 2e^{-2}$$