

2

	n	m	k	s
1	8	5	2	3
2	12	5	4	3

Permutation:

$$p = \sum C_{m_1}^{i_1} C_{n_1-m_1}^{k_1-i_1} C_{m_2}^{i_2} C_{n_2-m_2}^{k_2-i_2} \left( C_{n_1}^{k_1} C_{n_2}^{k_2} \right)^{-1} =$$

$$i_1 + i_2 = s \Leftrightarrow i_2 = s - i_1 \Rightarrow$$

$$\Rightarrow k_2 - i_2 = k_2 - (s - i_1) = k_2 - s + i_1$$

$$p = \sum_i C_{m_1}^i C_{n_1-m_1}^{k_1-i} C_{m_2}^{s-i} C_{n_2-m_2}^{k_2-s+i} \left( C_{n_1}^{k_1} C_{n_2}^{k_2} \right)^{-1}$$

$$\begin{cases} 0 \leq i \leq m_1 \\ 0 \leq k_1 - i \leq n_1 - m_1 \\ 0 \leq s - i \leq m_2 \\ 0 \leq k_2 - s + i \leq n_2 - m_2 \end{cases}$$

$$0 \leq k_1 - i \Leftrightarrow i \leq k_1$$

$$k_1 - i \leq n_1 - m_1 \Leftrightarrow k_1 + m_1 - n_1 \leq i$$

$$0 \leq s - i \Leftrightarrow i \leq s$$

$$s - i \leq m_2 \Leftrightarrow s - m_2 \leq i$$

$$0 \leq k_2 - s + i \Leftrightarrow s - k_2 \leq i$$

$$k_2 - s + i \leq n_2 - m_2 \Leftrightarrow i \leq n_2 + s - m_2 - k_2$$

$$\begin{cases} 0 \leq i \leq m_1 \\ k_1 + m_1 - n_1 \leq i \leq k_1 \\ S - m_2 \leq i \leq S \\ S - k_2 \leq i \leq n_2 + S - m_2 - k_2 \end{cases}$$

$$z_1 = \max(0, k_1 + m_1 - n_1, S - m_2, S - k_2)$$

$$z_2 = \min(m_1, k_1, S, n_2 + S - m_2 - k_2)$$

$$z_1 \leq i \leq z_2$$

$$p = \sum_{i=z_1}^{z_2} C_{m_1}^i C_{n_1-m_1}^{k_1-i} C_{m_2}^{S-i} C_{n_2-m_2}^{k_2-S+i} (C_{n_1}^{k_1} C_{n_2}^{k_2})^{-1}$$

$$z_1 = \max(0, 2+5-8, 3-5, 3-4) = \max(0, -1, -2, -1) = 0$$

$$z_2 = \min(5, 2, 3, 12+3-5-4) = \min(5, 2, 3, 6) = 2$$

$$C_{m_1}^i C_{n_1-m_1}^{k_1-i} C_{m_2}^{S-i} C_{n_2-m_2}^{k_2-S+i} = C_5^i C_{8-5}^{2-i} C_5^{3-i} C_{12-5}^{4-3+i} =$$

$$= C_5^i C_3^{2-i} C_5^{3-i} C_8^{i+1}$$

$$C_{n_1}^{k_1} C_{n_2}^{k_2} = C_8^2 C_{12}^3$$

$$p = \sum_{i=0}^2 C_5^i C_3^{2-i} C_5^{3-i} C_8^{i+1} (C_8^2 C_{12}^3)^{-1}$$

Problem:

$$z_1 = \max(0, k_1 + m_1 - n_1, S - m_2, S - k_2)$$

$$z_2 = \min(m_1, k_1, S, n_2 + S - m_2 - k_2)$$

$$p = \sum_{i=z_1}^{z_2} C_{m_1}^i C_{n_1-m_1}^{k_1-i} C_{m_2}^{S-i} C_{n_2-m_2}^{k_2-S+i} (C_{n_1}^{k_1} C_{n_2}^{k_2})^{-1}$$

$$p = \sum_{i=0}^2 C_5^i C_3^{2-i} C_5^{3-i} C_8^{i+1} (C_8^2 C_{12}^3)^{-1}$$