

2.5

$$\Phi(z) := \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right] ; \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\xi^2} d\xi$$

$$\begin{aligned} 1. \operatorname{erf}(-x) &= \frac{2}{\sqrt{\pi}} \int_0^{-x} e^{-\xi^2} d\xi = - \frac{2}{\sqrt{\pi}} \int_0^x e^{-\xi^2} (-d\xi) = - \frac{2}{\sqrt{\pi}} \int_0^x e^{-(-\xi)^2} d(-\xi) = \\ &= - \frac{2}{\sqrt{\pi}} \int_0^x e^{-\xi^2} d\xi = -\operatorname{erf}(x) \end{aligned}$$

2.

$$\begin{aligned} \Phi(-z) &= \frac{1}{2} \left[1 + \operatorname{erf}\left(-\frac{z}{\sqrt{2}}\right) \right] = \frac{1}{2} \left[1 + \operatorname{erf}\left(-\frac{z}{\sqrt{2}}\right) \right] = \frac{1}{2} \left[1 - \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right] = \\ &= \frac{1}{2} \left[2 - \left(1 + \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right) \right] = 1 - \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right] = 1 - \Phi(z) \end{aligned}$$

3.

$$\begin{aligned} \Phi(z) - \Phi(-z) &= \Phi(z) - [1 - \Phi(z)] \quad \Phi(z) - 1 + \Phi(z) = \\ &= 2\Phi(z) - 1 \Leftrightarrow \end{aligned}$$

$$\Leftrightarrow 2\Phi(z) = \Phi(z) - \Phi(-z) + 1 \Leftrightarrow$$

$$\Leftrightarrow \Phi(z) = \frac{\Phi(z) - \Phi(-z)}{2} + \frac{1}{2}$$

$\Phi(z) := \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right]$ $\Phi(-z) = 1 - \Phi(z)$	$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\xi^2} d\xi$ $\operatorname{erf}(-x) = -\operatorname{erf}(x)$
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$$\Phi(z) - \Phi(-z) = 2\Phi(z) - 1 \Leftrightarrow \Phi(z) = \frac{\Phi(z) - \Phi(-z)}{2} + \frac{1}{2}$$