$\boxed{3}$ Дано:

$$X \sim N(\mu, \sigma^2)$$

$$f(x; \mu, \sigma^2) = \frac{1}{4\sqrt{2\pi}} \exp\left(-\frac{(x+2)^2}{32}\right)$$

<u>Найти:</u> $E[X], D[X], S[X]$

<u>Решение:</u>

<u>1 общее</u>

$$X \sim N(\mu, \sigma^2)$$

$$\left.\begin{aligned} f(x; \mu, \sigma^2) &= \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \\ f(x; \mu, \sigma^2) &= \frac{1}{4\sqrt{2\pi}} \exp\left(-\frac{(x+2)^2}{32}\right) \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \begin{cases} \dfrac{1}{\sigma\sqrt{2\pi}} = \dfrac{1}{4\sqrt{2\pi}} \\ -\dfrac{(x-\mu)^2}{2\sigma^2} = -\dfrac{(x+2)^2}{32} \end{cases} \overset{(1)}{\Leftrightarrow}$$

$$\frac{1}{\sigma\sqrt{2\pi}} = \frac{1}{4\sqrt{2\pi}} \Leftrightarrow \frac{1}{\sigma} = \frac{1}{4} \Leftrightarrow \sigma = 4$$

$$-\frac{(x-\mu)^2}{2\sigma^2} = -\frac{(x+2)^2}{32} \Leftrightarrow \frac{(x-\mu)^2}{\sigma^2} = \frac{(x+2)^2}{16} \Leftrightarrow \left(\frac{x-\mu}{\sigma}\right)^2 = \left(\frac{x+2}{4}\right)^2 \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{aligned} \frac{x-\mu}{\sigma} &= \frac{x+2}{4} \\ \frac{x-\mu}{\sigma} &= -\frac{x+2}{4} \end{aligned}\right. \Leftrightarrow \left[\begin{aligned} x-\mu &= \frac{\sigma}{4}(x+2) \\ x-\mu &= -\frac{\sigma}{4}(x+2) \end{aligned}\right. \Leftrightarrow \left[\begin{aligned} \mu &= x - \frac{\sigma}{4}(x+2) \\ \mu &= x + \frac{\sigma}{4}(x+2) \end{aligned}\right.$$

$$\overset{(1)}{\Leftrightarrow} \begin{cases} \sigma = 4 \\ \left[\begin{aligned} \mu &= x - \frac{\sigma}{4}(x+2) = x - \frac{4}{4}(x+2) = x - (x+2) = -2 \\ \mu &= x + \frac{\sigma}{4}(x+2) = x + \frac{4}{4}(x+2) = x + (x+2) = 2x+2 \end{aligned}\right. \end{cases}$$

$$\begin{cases} \sigma = 4 \\ \left[\begin{array}{l} \mu = -2 \\ \mu = 2x+2 \end{array}\right. \\ \dfrac{\partial \mu}{\partial x} = \dfrac{\partial \sigma}{\partial x} = 0 \end{cases} \Rightarrow \begin{cases} \sigma = 4 \\ \mu = -2 \end{cases}$$

2. частное

$$\left.\begin{array}{l} f(x; \mu, \sigma^2) = \dfrac{1}{\sigma\sqrt{2\pi}} \exp\left(-\dfrac{(x-\mu)^2}{2\sigma^2}\right) \\ f(x, \mu, \sigma^2) = \dfrac{1}{4\sqrt{2\pi}} \exp\left(-\dfrac{(x+2)^2}{32}\right) = \dfrac{1}{4\sqrt{2\pi}} \exp\left(-\dfrac{(x-(-2))^2}{2\cdot 4^2}\right) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \begin{cases} \sigma = 4 \\ \mu = -2 \end{cases}$$

3. подстановка

$$E[X] := \int_{-\infty}^{+\infty} x f(x; \mu, \sigma^2)\, dx = \mu = -2$$

$$D[X] := \int_{-\infty}^{+\infty} (x - E[X])^2 f(x; \mu, \sigma^2)\, dx = \sigma^2 = 4^2 = 16$$

$$S[X] := \sqrt{D[X]} = \sigma = 4$$

Ответ:

$$\boxed{\begin{array}{l} E[X] = -2 \\ D[X] = 16 \\ S[X] = 4 \end{array}}$$