$\boxed{2.1}$

Д-ть:

$$|\operatorname{cov} xy| \leq \sqrt{D[x]\,D[y]}$$

Д-во:

$$\mu_x := E[x]$$
$$\mu_y := E[y]$$
$$\sigma_x^2 := D[x] := E\left[(x - E[x])^2\right] = E\left[(x-\mu_x)^2\right] \geq 0$$
$$\sigma_y^2 := D[y] := E\left[(y - E[y])^2\right] = E\left[(y-\mu_y)^2\right] \geq 0$$
$$\operatorname{cov}_{xy} := E\left[(x - E[x])(y - E[y])\right] = E\left[(x-\mu_x)(y-\mu_y)\right].$$

$$z = \sigma_y x - \sigma_x y$$
$$D[z] = D[\sigma_y x - \sigma_x y] =$$
$$= E\left[\left((\sigma_y x - \sigma_x y) - E[\sigma_y x - \sigma_x y]\right)^2\right] =$$
$$= E\left[\left((\sigma_y x - \sigma_x y) - (\sigma_y E[x] - \sigma_x E[y])\right)^2\right] =$$
$$= E\left[\left((\sigma_y x - \sigma_x y) - (\sigma_y \mu_x - \sigma_x \mu_y)\right)^2\right] =$$
$$= E\left[\left(\sigma_y(x-\mu_x) - \sigma_x(y-\mu_y)\right)^2\right] =$$
$$= E\left[\sigma_y^2(x-\mu_x)^2 - 2\sigma_y(x-\mu_x)\sigma_x(y-\mu_y) + \sigma_x^2(y-\mu_y)^2\right] =$$
$$= E\left[\sigma_y^2(x-\mu_x)^2 + \sigma_x^2(y-\mu_y)^2 - 2\sigma_x\sigma_y(x-\mu_x)(y-\mu_y)\right] =$$
$$= \sigma_y^2 E\left[(x-\mu_x)^2\right] + \sigma_y^2 E\left[(y-\mu_y)^2\right] - 2\sigma_x\sigma_y E\left[(x-\mu_x)(y-\mu_y)\right] =$$
$$= \sigma_y^2\sigma_x^2 + \sigma_y^2\sigma_x^2 - 2\sigma_x\sigma_y \operatorname{cov}_{xy} =$$
$$= 2\sigma_x\sigma_y\left(\sigma_x\sigma_y - \operatorname{cov}_{xy}\right) \geq 0 \Rightarrow$$

$$\Rightarrow \sigma_x\sigma_y - cov_{xy} \geq 0 \Leftrightarrow$$

$$\Leftrightarrow cov_{xy} \leq \sigma_x \sigma_y$$

$$z_2 = \sigma_y x + \sigma_x y$$

$$D[z_2] = D[\sigma_y x + \sigma_x y] =$$

$$= E\left[\left((\sigma_y x + \sigma_x y) - E(\sigma_y x + \sigma_x y)\right)^2\right] =$$

$$= E\left[\left((\sigma_y x + \sigma_x y) - (\sigma_y E[x] + \sigma_x E[y])\right)^2\right] =$$

$$= E\left[\left((\sigma_y x + \sigma_x y) - (\sigma_y \mu_x + \sigma_x \mu_y)\right)^2\right] =$$

$$= E\left[\left(\sigma_y(x-\mu_x) + \sigma_x(y-\mu_y)\right)^2\right] =$$

$$= E\left[\sigma_y^2(x-\mu_x)^2 + 2\sigma_y(x-\mu_x)\sigma_x(y-\mu_y) + \sigma_x^2(y-\mu_y)^2\right] =$$

$$= E\left[\sigma_y^2(x-\mu_x)^2 + \sigma_x^2(y-\mu_y)^2 + 2\sigma_x\sigma_y(x-\mu_x)(y-\mu_y)\right] =$$

$$= \sigma_y^2 E[(x-\mu_x)^2] + \sigma_x^2 E[(y-\mu_y)^2] + 2\sigma_x\sigma_y E[(x-\mu_x)(y-\mu_y)] =$$

$$= \sigma_y^2\sigma_x^2 + \sigma_x^2\sigma_y^2 + 2\sigma_x\sigma_y\, cov_{xy} =$$

$$= 2\sigma_x\sigma_y(\sigma_x\sigma_y + cov_{xy}) \geq 0 \Rightarrow$$

$$\Rightarrow \sigma_x\sigma_y + cov_{xy} \geq 0 \Leftrightarrow$$

$$\Leftrightarrow cov_{xy} \geq -\sigma_x\sigma_y$$

$$\begin{cases} cov_{xy} \leq \sigma_x\sigma_y \\ cov_{xy} \geq -\sigma_x\sigma_y \end{cases} \Leftrightarrow -\sigma_x\sigma_y \leq cov_{xy} \leq \sigma_x\sigma_y \Leftrightarrow$$

$$\Leftrightarrow |cov_{xy}| \leq \sigma_x\sigma_y \quad \square$$

$$\boxed{|cov_{xy}| \leq \sqrt{D[x]D[y]}}$$