$\boxed{11.2}$ $X \in N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad x \in (-\infty, +\infty)$$

$$E[X] = \int_{-\infty}^{+\infty} x f(x)dx = \int_{-\infty}^{+\infty} x \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = (1)$$

$$x \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = (\mu + (x-\mu)) \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx =$$

$$= \mu \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx + (x-\mu) \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx$$

$$(1) = \int_{-\infty}^{+\infty} \mu \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx + \int_{-\infty}^{+\infty} (x-\mu) \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = (2)$$

$$\xi^2 = \frac{(x-\mu)^2}{2\sigma^2} = \left(\frac{x-\mu}{\sqrt{2}\sigma}\right)^2 \Leftrightarrow \xi = \pm \frac{x-\mu}{\sqrt{2}\sigma} \Rightarrow$$

$$\Rightarrow \xi = \frac{x-\mu}{\sqrt{2}\sigma} \Leftrightarrow x - \mu = \sqrt{2}\sigma\xi$$

$$\lim_{x \to -\infty} \xi = \lim_{x \to -\infty} \frac{x-\mu}{\sqrt{2}\sigma} = -\infty$$

$$\lim_{x \to +\infty} \xi = \lim_{x \to +\infty} \frac{x+\mu}{\sqrt{2}\sigma} = +\infty$$

$$d\xi = d\left(\frac{x-\mu}{\sqrt{2}\sigma}\right) = \frac{1}{\sqrt{2}\sigma} d(x-\mu) = \frac{1}{\sqrt{2}\sigma} dx \Leftrightarrow dx = \sqrt{2}\sigma d\xi$$

$$\mu \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = \mu \frac{1}{\sigma\sqrt{2\pi}} e^{-\xi^2} \sqrt{2}\sigma d\xi = \frac{\mu}{\sqrt{\pi}} e^{-\xi^2} d\xi$$

$$(x-\mu) \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = \sqrt{2}\sigma\xi \frac{1}{\sigma\sqrt{2\pi}} e^{-\xi^2} \sqrt{2}\sigma d\xi = \sigma\sqrt{\frac{2}{\pi}} \xi e^{-\xi^2} d\xi$$

$$(2) = \int_{-\infty}^{+\infty} \frac{\mu}{\sqrt{\pi}} e^{-\xi^2} d\xi + \int_{-\infty}^{+\infty} \sigma\sqrt{\frac{2}{\pi}}\, \xi e^{-\xi^2} d\xi =$$

$$= \frac{\mu}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\xi^2} d\xi + \sigma\sqrt{\frac{2}{\pi}} \int_{-\infty}^{+\infty} \xi e^{-\xi^2} d\xi = (3)$$

$$\int_{-\infty}^{+\infty} e^{-\xi^2} d\xi = \int_{-\infty}^{0} e^{-\xi^2} d\xi + \int_{0}^{+\infty} e^{-\xi^2} d\xi = (4)$$

$$\int_{-\infty}^{0} e^{-\xi^2} d\xi = -\int_{0}^{-\infty} e^{-\xi^2} d\xi = \int_{0}^{-\infty} e^{-\xi^2} (-d\xi) = \int_{0}^{-\infty} e^{-(-\xi)^2} d(-\xi) =$$

$$= \int_{0}^{+\infty} e^{-\xi^2} d\xi$$

$$(4) = \int_{0}^{+\infty} e^{-\xi^2} d\xi + \int_{0}^{+\infty} e^{-\xi^2} d\xi = 2\int_{0}^{+\infty} e^{-\xi^2} d\xi = (5)$$

$$\int_{0}^{+\infty} e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2}$$

$$(5) = 2\frac{\sqrt{\pi}}{2} = \sqrt{\pi}$$

$$\int_{-\infty}^{+\infty} \xi e^{-\xi^2} d\xi = \int_{-\infty}^{0} \xi e^{-\xi^2} + \int_{0}^{+\infty} \xi e^{-\xi^2} d\xi = (6)$$

$$\int_{-\infty}^{0} \xi e^{-\xi^2} d\xi = -\int_{0}^{-\infty} \xi e^{-\xi^2} d\xi = -\int_{0}^{-\infty} (-\xi) e^{-\xi^2} (-d\xi) = -\int_{0}^{-\infty} (-\xi) e^{-(-\xi)^2} d(-\xi) =$$

$$= -\int_{0}^{+\infty} \xi e^{-\xi^2} d\xi$$

$$(6) = -\int_{0}^{+\infty} \xi e^{-\xi^2} d\xi + \int_{0}^{+\infty} \xi e^{-\xi^2} d\xi = 0$$

$$(3) = \frac{\mu}{\sqrt{\pi}}\sqrt{\pi} + \sigma\sqrt{\frac{2}{\pi}}\, 0 = \mu$$

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \qquad E[X] = \int_{-\infty}^{+\infty} x f(x)\, dx = \mu}$$

(2)