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Дано:

$X \sim N(\mu, \sigma^2)$

$f(x|\mu,\sigma^2) = \frac{1}{4\sqrt{2\pi}} \exp\left(-\frac{(x+2)^2}{32}\right)$

Найти:

$E[X]; D[X]; S[X]$

Решение:

1.

$X \sim N(\mu, \sigma^2)$

$f(x|\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

$f(x|\mu,\sigma^2) = \frac{1}{4\sqrt{2\pi}} \exp\left(-\frac{(x+2)^2}{32}\right)$

$$\begin{cases} \frac{1}{\sigma\sqrt{2\pi}} = \frac{1}{4\sqrt{2\pi}} \\ -\frac{(x-\mu)^2}{2\sigma^2} = -\frac{(x+2)^2}{32} \end{cases} \overset{(1)}{\Leftrightarrow}$$

$$\frac{1}{\sigma\sqrt{2\pi}} = \frac{1}{4\sqrt{2\pi}} \Leftrightarrow \frac{1}{\sigma} = \frac{1}{4} \Leftrightarrow \sigma = 4$$

$$-\frac{(x-\mu)^2}{2\sigma^2} = -\frac{(x+2)^2}{32} \Leftrightarrow \frac{(x-\mu)^2}{\sigma^2} = \frac{(x+2)^2}{16} \Leftrightarrow \left(\frac{x-\mu}{\sigma}\right)^2 = \left(\frac{x+2}{4}\right)^2 \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \frac{x-\mu}{\sigma} = \frac{x+2}{4} \\ \frac{x-\mu}{\sigma} = -\frac{x+2}{4} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x-\mu = \frac{\sigma}{4}(x+2) \\ x-\mu = -\frac{\sigma}{4}(x+2) \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \mu = x-\frac{\sigma}{4}(x+2) \\ \mu = x+\frac{\sigma}{4}(x+2) \end{array}\right.$$

$$\overset{(1)}{\Leftrightarrow} \begin{cases} \sigma = 4 \\ \left[\begin{array}{l} \mu = x-\frac{\sigma}{4}(x+2) = x-\frac{4}{4}(x+2) = x-(x+2) = -2 \\ \mu = x+\frac{\sigma}{4}(x+2) = x+\frac{4}{4}(x+2) = x+(x+2) = 2x+2 = 2(x+1) \end{array}\right. \end{cases}$$

$$\frac{\partial \mu}{\partial x} = \frac{\partial \sigma}{\partial x} = 0$$

$$\mu = -2 \wedge \sigma = 4$$

2.

$$E[X] = \int_{-\infty}^{+\infty} x f(x \mid \mu, \sigma^2)\, dx = \mu = -2$$

$$D[X] = \int_{-\infty}^{+\infty} (x - E[X])^2 f(x \mid \mu, \sigma^2) = \sigma^2 = 4^2 = 16$$

$$S[X] = \sqrt{D[X]} = \sigma = 4$$

Ответ:

$$\boxed{\begin{aligned} E[X] &= -2 \\ D[X] &= 16 \\ S[X] &= 4 \end{aligned}}$$