$n \in \mathbb{N} \wedge n \geq 2 \quad D = \{(x_i, y_i) \mid i = \overline{1,n}\}$

$\alpha \in R \quad y = f(x|\alpha) = \alpha x$

$i = \overline{1,n} \quad \varepsilon_i = y_i - f(x_i|\alpha) = y_i - \alpha x_i$

$$u(\alpha) = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} [y_i - f(x_i|\alpha)]^2 = \sum_{i=1}^{n} (y_i - \alpha x_i)^2$$

$$u(\alpha) \to \min_{\alpha \in R}$$

$\boxed{1}$

$$du(\alpha) = \frac{du}{d\alpha} d\alpha$$

$$\frac{du}{d\alpha} = \frac{d}{d\alpha} \sum_{i=1}^{n} (y_i - \alpha x_i)^2 = \sum_{i=1}^{n} \frac{d}{d\alpha} (y_i - \alpha x_i)^2 = (1.1)$$

$$\frac{d}{d\alpha}(y_i - \alpha x_i)^2 = 2(y_i - \alpha x_i) \frac{d}{d\alpha}(y_i - \alpha x_i) = 2(y_i - \alpha x_i)(-x_i) =$$

$$= 2x_i(\alpha x_i - y_i) = 2(\alpha x_i^2 - x_i y_i)$$

$$(1.1) = \sum_{i=1}^{n} 2(\alpha x_i^2 - x_i y_i) = 2\left(\alpha \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i y_i\right)$$

$$\frac{du}{d\alpha} = 0 \Rightarrow du = \frac{du}{d\alpha} d\alpha = 0\, d\alpha = 0$$

$$\frac{du}{d\alpha} = 2\left(\alpha \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i y_i\right) = 0 \Leftrightarrow$$

$$\Leftrightarrow \alpha \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i y_i = 0 \Leftrightarrow \alpha \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i \Rightarrow$$

$$\Rightarrow \alpha = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} \quad \Big| \quad \sum_{i=1}^{n} x_i^2 \neq 0$$

$\boxed{2}$

$$d^2u(\alpha) = \frac{d}{d\alpha}(du) = \frac{d}{d\alpha}\left(\frac{du}{d\alpha}d\alpha\right) = \frac{d}{d\alpha}\left(\frac{du}{d\alpha}\right)d\alpha = \frac{d^2u}{d\alpha^2}d\alpha$$

$$\frac{d^2u}{d\alpha^2} = \frac{d}{d\alpha}\left(\frac{du}{d\alpha}\right) = \frac{d}{d\alpha}\ 2\left(\alpha\sum_{i=1}^{n}x_i^2 - \sum_{i=1}^{n}x_iy_i\right) = 2\sum_{i=1}^{n}x_i^2 \Rightarrow$$

$$\Rightarrow \frac{d^2u}{d\alpha^2} > 0 \ \Big|\ \sum_{i=1}^{n}x_i^2 \neq 0 \Rightarrow$$

$$\Rightarrow \begin{cases} d^2u > 0 \mid d\alpha > 0 \\ d^2u < 0 \mid d\alpha < 0 \end{cases}$$

$\boxed{3}$

$$\begin{cases} \frac{du}{d\alpha} = 0 \Rightarrow du(\alpha) = \frac{du}{d\alpha}(\alpha)\,d\alpha = 0 \\ \frac{d^2u}{d\alpha^2} > 0 \Rightarrow \begin{cases} d^2u > 0 \mid d\alpha > 0 \\ d^2u < 0 \mid d\alpha < 0 \end{cases} \end{cases} \Leftrightarrow \alpha = \arg\min_{\alpha \in \mathbb{R}}(u)$$

$\boxed{4}$

$$\sum_{i=1}^{n}x_i^2 = n\,\frac{1}{n}\sum_{i=1}^{n}x_i^2 = n\,\overline{x^2}$$

$$\sum_{i=1}^{n}x_i^2 \neq 0 \mid n \in \mathbb{N} \Rightarrow \overline{x^2} \neq 0$$

$$\alpha = \frac{\sum_{i=1}^{n}x_iy_i}{\sum_{i=1}^{n}x_i^2} = \frac{\frac{1}{n}\sum_{i=1}^{n}x_iy_i}{\frac{1}{n}\sum_{i=1}^{n}x_i^2} = \frac{\overline{xy}}{\overline{x^2}}$$

$n \in \mathbb{N} \wedge n \geq 2 \quad D = \{(x_i; y_i) \mid i = \overline{1,n}\}$; выборка

$\alpha \in \mathbb{R} \quad y = f(x \mid \alpha) = \alpha x$; регрессионная модель

$i = \overline{1,n} \quad \varepsilon_i = y_i - f(x_i \mid \alpha) = y_i - \alpha x_i$

$$u(\alpha) = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} [y_i - f(x \mid \alpha)]^2 = \sum_{i=1}^{n} (y_i - \alpha x_i)^2 ;$$

функция ошибок

$u(\alpha) \to \min\limits_{\alpha}$; критерий

$\overline{x^2} \neq 0$

$$\alpha = \underset{\alpha \in \mathbb{R}}{\operatorname{argmin}}(u)$$

$$\alpha = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} = \frac{\overline{xy}}{\overline{x^2}}$$