

$$\boxed{1.1} \quad X \sim N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad x \in (-\infty, +\infty)$$

$$\int_{-\infty}^{+\infty} f(x) dx = \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = (1)$$

$$\frac{x^2}{\sigma^2} = \frac{(x-\mu)^2}{2\sigma^2} = \left(\frac{x-\mu}{\sqrt{2}\sigma}\right)^2 \Leftrightarrow \frac{x-\mu}{\sqrt{2}\sigma} = \pm \frac{x-\mu}{\sqrt{2}\sigma} \Rightarrow$$

$$\Rightarrow \frac{x-\mu}{\sqrt{2}\sigma}$$

$$\lim_{x \rightarrow -\infty} \frac{x-\mu}{\sqrt{2}\sigma} = -\infty$$

$$\lim_{x \rightarrow +\infty} \frac{x-\mu}{\sqrt{2}\sigma} = +\infty$$

$$d\frac{x-\mu}{\sqrt{2}\sigma} = \frac{1}{\sqrt{2}\sigma} d(x-\mu) = \frac{1}{\sqrt{2}\sigma} dx \Leftrightarrow$$

$$\Leftrightarrow dx = \sqrt{2}\sigma d\frac{x-\mu}{\sqrt{2}\sigma}$$

$$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{\sigma^2}} \sqrt{2}\sigma d\frac{x-\mu}{\sqrt{2}\sigma} =$$

$$= \frac{1}{\sqrt{\pi}} e^{-\frac{x^2}{\sigma^2}} d\frac{x-\mu}{\sqrt{2}\sigma}$$

$$(1) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi}} e^{-\frac{x^2}{\sigma^2}} d\frac{x-\mu}{\sqrt{2}\sigma} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\frac{x^2}{\sigma^2}} d\frac{x-\mu}{\sqrt{2}\sigma} = (2)$$

$$\int_{-\infty}^{+\infty} e^{-z^2} dz = \int_{-\infty}^0 e^{-z^2} dz + \int_0^{+\infty} e^{-z^2} dz = (3)$$

$$\eta = -z$$

$$\lim_{z \rightarrow -\infty} \eta = \lim_{z \rightarrow -\infty} (-z) = +\infty$$

$$(\eta)_{z=0} = (-z)_{z=0} = 0$$

$$d\eta = d(-z) = -dz \Leftrightarrow dz = -d\eta$$

$$e^{-z^2} dz = e^{-(-\eta)^2} (-d\eta) = -e^{-\eta^2} d\eta$$

$$\int_{-\infty}^0 e^{-z^2} dz = \int_{+\infty}^0 (-1) e^{-\eta^2} d\eta = - \int_{+\infty}^0 e^{-\eta^2} d\eta =$$

$$= -(-1) \int_0^{+\infty} e^{-\eta^2} d\eta = \int_0^{+\infty} e^{-\eta^2} d\eta = \int_0^{+\infty} e^{-z^2} dz$$

$$(3) = \int_0^{+\infty} e^{-z^2} dz + \int_0^{+\infty} e^{-z^2} dz = 2 \int_0^{+\infty} e^{-z^2} dz = (4)$$

$$\int_0^{+\infty} e^{-z^2} dz = \frac{\sqrt{\pi}}{2}$$

$$(4) = 2 \cdot \frac{\sqrt{\pi}}{2} = \sqrt{\pi}$$

$$(2) = \frac{1}{\sqrt{\pi}} \sqrt{\pi} = 1$$

$$\boxed{f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad \int_{-\infty}^{+\infty} f(x) dx = 1}$$