

$$(1.4) \quad X \in N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad x \in (-\infty, +\infty)$$

$$F(x) = \int_{-\infty}^x f(y) dy = \int_{-\infty}^x \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right) dy = (1)$$

$$z^2 = \frac{(y-\mu)^2}{2\sigma^2} = \left(\frac{y-\mu}{\sqrt{2}\sigma}\right)^2 \Leftrightarrow z = \pm \frac{y-\mu}{\sqrt{2}\sigma} \Rightarrow$$

$$\Rightarrow z = \frac{y-\mu}{\sqrt{2}\sigma}$$

$$\lim_{y \rightarrow -\infty} z = \lim_{y \rightarrow -\infty} \frac{y-\mu}{\sqrt{2}\sigma} = -\infty$$

$$(z)_{y=x} = \left(\frac{y-\mu}{\sqrt{2}\sigma}\right)_{y=x} = \frac{x-\mu}{\sqrt{2}\sigma}$$

$$dz = d\left(\frac{y-\mu}{\sqrt{2}\sigma}\right) = \frac{1}{\sqrt{2}\sigma} d(y-\mu) = \frac{1}{\sqrt{2}\sigma} dy \Leftrightarrow dy = \sqrt{2}\sigma dz$$

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-z^2} \sqrt{2}\sigma dz = \frac{1}{\sqrt{\pi}} e^{-z^2} dz$$

$$(1) = \int_{-\infty}^{\frac{x-\mu}{\sqrt{2}\sigma}} \frac{1}{\sqrt{\pi}} e^{-z^2} dz = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{x-\mu}{\sqrt{2}\sigma}} e^{-z^2} dz =$$

$$= \frac{1}{\sqrt{\pi}} \left(\int_{-\infty}^0 e^{-z^2} dz + \int_0^{\frac{x-\mu}{\sqrt{2}\sigma}} e^{-z^2} dz \right) = (2)$$

$$\int_{-\infty}^0 e^{-z^2} dz = - \int_0^{-\infty} e^{-z^2} dz = \int_0^{\infty} e^{-z^2} (-dz) = \int_0^{\infty} e^{-(-z)^2} d(-z) =$$

$$= \int_0^{\infty} e^{-z^2} dz = \frac{\sqrt{\pi}}{2}$$

$$\int_0^{\frac{x-\mu}{\sqrt{2}\sigma}} e^{-z^2} dz = \frac{\sqrt{\pi}}{2} \frac{2}{\sqrt{\pi}} \int_0^{\frac{x-\mu}{\sqrt{2}\sigma}} e^{-z^2} dz = (2)$$

$$\text{erf}(x) := \frac{2}{\sqrt{\pi}} \int_0^x e^{-z^2} dz$$

$$(3) = \frac{\sqrt{\pi}}{2} \text{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)$$

$$(2) = \frac{1}{\sqrt{\pi}} \left[\frac{\sqrt{\pi}}{2} + \frac{\sqrt{\pi}}{2} \text{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right) \right] = \frac{1}{2} \left[1 + \text{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right) \right]$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$F(x) = \int_{-\infty}^x f(x) dx = \frac{1}{2} \left[1 + \text{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right) \right]$$