4 Дано:

$X \sim N(\mu, \sigma^2)$

$\mu = 174$ см

$\sigma = 8$ см

Найти:

1. $p(X > 182 \text{ см})$
2. $p(X > 190 \text{ см})$
3. $p(166 \text{ см} < X \leq 190 \text{ см})$
4. $p(166 \text{ см} \leq X \leq 182 \text{ см})$
5. $p(158 \text{ см} \leq X \leq 190 \text{ см})$
6. $p(X \leq 150 \text{ см} \vee X \geq 190 \text{ см})$
7. $p(X \leq 150 \text{ см} \vee X \geq 198 \text{ см})$
8. $p(X < 166 \text{ см})$

Решение:

$X \sim N(\mu, \sigma^2)$

$f(x; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)}{2\sigma^2}\right)$

$F(x; \mu, \sigma^2) = \int_{-\infty}^{x} f(x; \mu, \sigma^2) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\right] = \Phi\left(\frac{x-\mu}{\sigma}\right)$

$\Phi(x) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right]$

$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\xi^2} d\xi$

$\operatorname{erf}(-x) = \frac{2}{\sqrt{\pi}} \int_0^{-x} e^{-\xi^2} d\xi =$

$= -\frac{2}{\sqrt{\pi}} (-1) \int_0^{-x} e^{-\xi^2} d\xi = -\frac{2}{\sqrt{\pi}} \int_{\xi=0}^{\xi=-x} e^{-\xi^2} (-d\xi) =$

$= -\frac{2}{\sqrt{\pi}} \int_{-\xi=0}^{-\xi=x} e^{-(-\xi)^2} d(-\xi) = -\frac{2}{\sqrt{\pi}} \int_0^x e^{-\xi^2} d\xi =$

$= -\operatorname{erf}(x)$

$\Phi(-x) = \frac{1}{2}\left[1 + \operatorname{erf}\left(-\frac{x}{\sqrt{2}}\right)\right] =$

$= \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right] = \frac{1}{2}\left[2 - \left(1 + \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right)\right] =$

$= 1 - \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right] = 1 - \Phi(x)$

$\Phi(x) - \Phi(-x) = \Phi(x) - (1 - \Phi(x)) = 2\Phi(x) - 1 \iff$

$\iff \Phi(x) = \frac{1}{2}\left[1 + \Phi(x) - \Phi(-x)\right]$

1. $P(X > 182\text{ cm}) =$

$$P(X > X_0) = \int_{X_0}^{+\infty} f(x)\,dx = \int_{-\infty}^{X_0} f(x)\,dx + \int_{X_0}^{+\infty} f(x)\,dx - \int_{-\infty}^{X_0} f(x)\,dx =$$

$$= \int_{-\infty}^{+\infty} f(x)\,dx - \int_{-\infty}^{X_0} f(x)\,dx = 1 - \int_{-\infty}^{X_0} f(x)\,dx = 1 - F(X_0) = (1)$$

$$z_0 = \frac{X_0 - \mu}{\sigma} = \frac{182\text{ cm} - 174\text{ cm}}{8\text{ cm}} = \frac{8\text{ cm}}{8\text{ cm}} = \frac{8}{8} = 1$$

$$F(X_0) = \Phi(z_0) = \Phi(1)$$

$$(1) = 1 - \Phi(1) =$$

$$= 1 - \left[\frac{\Phi(1) - \Phi(-1)}{2} + \frac{1}{2}\right] = \frac{1}{2} - \frac{\Phi(1) - \Phi(-1)}{2} \approx$$

$$\approx \frac{0.32}{2} = 0.16$$

2. $P(X > 190\text{ cm}) =$

$$= P(X > X_0) = 1 - F(X_0) =$$

$$z_0 = \frac{X_0 - \mu}{\sigma} = \frac{190\text{ cm} - 174\text{ cm}}{8\text{ cm}} = \frac{16\text{ cm}}{8\text{ cm}} = \frac{16}{8} = 2$$

$$F(X_0) = \Phi(z_0) = \Phi(2)$$

$$(2) = 1 - \Phi(2) =$$

$$= 1 - \left[\frac{\Phi(2) - \Phi(-2)}{2} + \frac{1}{2}\right] = \frac{1}{2} - \frac{\Phi(2) - \Phi(-2)}{2} \approx$$

$$\approx \frac{0.046}{2} = 0.023$$

3. $p(166\,cm \le X \le 190\,cm) =$

$$= p(x_1 \le X \le x_2) = \int_{x_1}^{x_2} f(x)\,dx = \int_{-\infty}^{x_1} f(x)\,dx + \int_{x_1}^{x_2} f(x)\,dx - \int_{-\infty}^{x_1} f(x)\,dx =$$

$$= \int_{-\infty}^{x_2} f(x)\,dx - \int_{-\infty}^{x_1} f(x)\,dx = F(x_2) - F(x_1) = (3)$$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{166\,cm - 174\,cm}{8\,cm} = \frac{-8\,cm}{8\,cm} = -\frac{8}{8} = -1$$

$$F(x_1) = \Phi(z_1) = \Phi(-1)$$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{190\,cm - 174\,cm}{8\,cm} = \frac{16\,cm}{8\,cm} = \frac{16}{8} = 2$$

$$F(x_2) = \Phi(z_2) = \Phi(2)$$

$$(3) = \Phi(2) - \Phi(-1) =$$

$$= \left[\frac{\Phi(2) - \Phi(-2)}{2} + \frac{1}{2}\right] - \left[\frac{\Phi(-1) - \Phi(1)}{2} + \frac{1}{2}\right] =$$

$$= \frac{\Phi(2) - \Phi(-2)}{2} + \frac{\Phi(1) - \Phi(-1)}{2} \approx$$

$$\approx \frac{0.68 + 0.96}{2} = \frac{1.64}{2} = 0.82$$

Ответ:

1. $p(X > 182\text{см}) = 1 - \Phi(1) =$

$= \frac{1}{2}\left[1 - (\Phi(1) - \Phi(-1))\right] \approx 0.16$

2. $p(X > 190\text{см}) = 1 - \Phi(2) =$

$= \frac{1}{2}\left[1 - (\Phi(2) - \Phi(-2))\right] \approx 0.023$

3. $p(166\text{см} \leq X \leq 190\text{см}) = \Phi(2) - \Phi(1) =$

$= \frac{\Phi(2) - \Phi(-2)}{2} + \frac{\Phi(1) - \Phi(-1)}{2} \approx 0.82$