$\boxed{1.1}$ $X \sim N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \qquad x \in (-\infty, +\infty)$$

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = (1)$$

$$\xi^2 = \frac{(x-\mu)^2}{2\sigma^2} = \left(\frac{x-\mu}{\sqrt{2}\sigma}\right)^2 \Leftrightarrow \xi = \pm \frac{x-\mu}{\sqrt{2}\sigma} \Rightarrow$$

$$\Rightarrow \xi = \frac{x-\mu}{\sqrt{2}\sigma}$$

$$\lim_{x \to -\infty} \xi = \lim_{x \to -\infty} \frac{x-\mu}{\sqrt{2}\sigma} = -\infty$$

$$\lim_{\xi \to +\infty} \xi = \lim_{x \to +\infty} \frac{x-\mu}{\sqrt{2}\sigma} = +\infty$$

$$d\xi = d\left(\frac{x-\mu}{\sqrt{2}\sigma}\right) = \frac{1}{\sqrt{2}\sigma} d(x-\mu) = \frac{1}{\sqrt{2}\sigma} dx \Leftrightarrow$$

$$\Leftrightarrow dx = \sqrt{2}\sigma d\xi$$

$$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = \frac{1}{\sigma\sqrt{2\pi}} e^{-\xi^2} \sqrt{2}\sigma d\xi =$$

$$= \frac{1}{\sqrt{\pi}} e^{-\xi^2} d\xi$$

$$(1) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi}} e^{-\xi^2} d\xi = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\xi^2} d\xi = (2)$$

$$\int_{-\infty}^{+\infty} e^{-\xi^2} d\xi = \int_{-\infty}^{0} e^{-\xi^2} d\xi + \int_{0}^{+\infty} e^{-\xi^2} d\xi = (3)$$

$$\eta = -\xi$$

$$\lim_{\xi \to -\infty} \eta = \lim_{\xi \to -\infty} (-\xi) = +\infty$$

$$(\eta)_{\xi=0} = (-\xi)_{\xi=0} = 0$$

$$d\eta = d(-\xi) = -d\xi \Leftrightarrow d\xi = -d\eta$$

$$e^{-\xi^2} d\xi = e^{-(-\eta)^2} (-d\eta) = -e^{-\eta^2} d\eta$$

$$\int_{-\infty}^{0} e^{-\xi^2} d\xi = \int_{+\infty}^{0} (-1) e^{-\eta^2} d\eta = -\int_{+\infty}^{0} e^{-\eta^2} d\eta =$$

$$= -(-1) \int_{0}^{+\infty} e^{-\eta^2} d\eta = \int_{0}^{+\infty} e^{-\eta^2} d\eta = \int_{0}^{+\infty} e^{-\xi^2} d\xi$$

$$(3) = \int_{0}^{+\infty} e^{-\xi^2} d\xi + \int_{0}^{+\infty} e^{-\xi^2} d\xi = 2 \int_{0}^{+\infty} e^{-\xi^2} d\xi = (4)$$

$$\int_{0}^{+\infty} e^{-\xi^2} d\xi = \frac{\sqrt{\pi}}{2}$$

$$(4) = 2 \cdot \frac{\sqrt{\pi}}{2} = \sqrt{\pi}$$

$$(2) = \frac{1}{\sqrt{\pi}} \sqrt{\pi} = 1$$

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \qquad \int_{-\infty}^{+\infty} f(x)\, dx = 1}$$