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Dano:

$$P(A|B_1) = 0.8; P(A|B_2) = 0.7; P(A|B_3) = 0.9$$

$$P(B_1) = P(B_2) \wedge P(B_3) = P(B_1) + P(B_2)$$

Wymagane:

$$P(B_i|A) \quad i = \overline{1,3}$$

Rozwiazanie:

$$i = \overline{1,3} \quad P(B_i|A) = \frac{P(B_i) P(A|B_i)}{P(A)}$$

$$\begin{cases} P(B_1) = P(B_2) \\ P(B_3) = P(B_1) + P(B_2) \end{cases} \Leftrightarrow \begin{cases} P(B_1) = P(B_2) \\ P(B_3) = P(B_1) + P(B_1) = 2P(B_1) \end{cases}$$

$$\sum_{i=1}^3 P(B_i) = 1$$

$$\sum_{i=1}^3 P(B_i) = P(B_1) + P(B_2) + P(B_3) =$$

$$= P(B_1) + P(B_1) + 2P(B_1) = 4P(B_1)$$

$$4P(B_1) = 1 \Leftrightarrow P(B_1) = \frac{1}{4}$$

$$\begin{cases} P(B_2) = P(B_1) = \frac{1}{4} \\ P(B_3) = 2P(B_1) = 2 \cdot \frac{1}{4} = \frac{1}{2} \end{cases}$$

$$P(A) = \sum_{i=1}^3 P(B_i) P(A|B_i) =$$

$$= P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3) =$$

$$= \frac{1}{4} \cdot \frac{8}{10} + \frac{1}{4} \cdot \frac{7}{10} + \frac{1}{2} \cdot \frac{9}{10} = \frac{8+7+2 \cdot 9}{4 \cdot 10} = \frac{33}{40}$$

$$P(B_1|A) = \frac{P(B_1) P(A|B_1)}{P(A)} =$$

$$= \frac{1}{4} \frac{8}{10} \left(\frac{33}{40}\right)^{-1} = \frac{8}{40} \cdot \left(\frac{33}{40}\right)^{-1} = \frac{8}{33}$$

$$P(B_2|A) = \frac{P(B_2) P(A|B_2)}{P(A)} =$$

$$= \frac{1}{4} \frac{7}{10} \left(\frac{33}{40}\right)^{-1} = \frac{7}{40} \left(\frac{33}{40}\right)^{-1} = \frac{7}{33}$$

$$P(B_3|A) = \frac{P(B_3) P(A|B_3)}{P(A)} =$$

$$= \frac{1}{2} \frac{9}{10} \left(\frac{33}{40}\right)^{-1} = \frac{9}{20} \left(\frac{33}{40}\right)^{-1} = \frac{18}{40} \left(\frac{33}{40}\right)^{-1} = \frac{18}{33}$$

Проверка:

$$1 = \sum_{i=1}^3 P(B_i|A) = \frac{P(B_i) P(A|B_i)}{P(A)} = \frac{P(B_i) P(A|B_i)}{\sum_{i=1}^3 P(B_i) P(A|B_i)}$$

$$\sum_{i=1}^3 P(B_i|A) = \sum_{i=1}^3 \frac{P(B_i) P(A|B_i)}{\sum_{i=1}^3 P(B_i) P(A|B_i)} = \frac{\sum_{i=1}^3 P(B_i) P(A|B_i)}{\sum_{i=1}^3 P(B_i) P(A|B_i)} = 1$$

$$\sum_{i=1}^3 P(B_i|A) = P(B_1|A) + P(B_2|A) + P(B_3|A) =$$

$$= \frac{8}{33} + \frac{7}{33} + \frac{18}{33} = \frac{8+7+18}{33} = \frac{33}{33} = 1$$

Ответ:

$$P(B_1|A) = \frac{8}{33}$$

$$P(B_2|A) = \frac{7}{33}$$

$$P(B_3|A) = \frac{18}{33}$$

Примечание:

A - сразу сессию студентами;

B_i - студент узнан на i -м фактуме;

$A|B_i$ - сразу сессию студентами узнанная на i -м фактуме;

$B_i|A$ - данный сессию студент узнан на i -м фактуме; ②