3 Непрерывное равномерное распределение

$X \sim U(a,b) \quad a<b$

1. плотность вероятности

$$f(x|a,b) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & x<a \vee b<x \end{cases}$$

2. нормировка

$$\int_{-\infty}^{\infty} f(x|a,b)\,dx = \int_{-\infty}^{a} f(x|a,b)\,dx + \int_{a}^{b} f(x|a,b)\,dx + \int_{b}^{+\infty} f(x|a,b)\,dx =$$

$$= \int_{-\infty}^{a} 0\,dx + \int_{a}^{b} \frac{1}{b-a}\,dx + \int_{b}^{+\infty} 0\,dx = \int_{a}^{b} \frac{1}{b-a}\,dx =$$

$$= \frac{1}{b-a}\int_{a}^{b} dx = \frac{1}{b-a}\, x\Big|_{a}^{b} = \frac{1}{b-a}(b-a) = 1$$

3. математическое ожидание

$$E[X] = \int_{-\infty}^{\infty} x f(x|a,b)\,dx = \int_{-\infty}^{a} x f(x|a,b)\,dx + \int_{a}^{b} x f(x|a,b)\,dx + \int_{b}^{+\infty} x f(x|a,b)\,dx =$$

$$= \int_{-\infty}^{a} x\cdot 0\,dx + \int_{a}^{b} x \frac{1}{b-a}\,dx + \int_{b}^{+\infty} x\cdot 0\,dx = \int_{a}^{b} x\frac{1}{b-a}\,dx =$$

$$= \frac{1}{b-a}\int_{a}^{b} x\,dx = \frac{1}{b-a}\,\frac{x^2}{2}\Big|_{a}^{b} = \frac{1}{2(b-a)}\, x^2\Big|_{a}^{b} = \frac{1}{2(b-a)}(b^2-a^2) =$$

$$= \frac{(b-a)(b+a)}{2(b-a)} = \frac{b+a}{2}$$

4. дисперсия

$$D[X] = \int_{-\infty}^{\infty} (x - E[X])^2 f(x|a,b)\,dx =$$

$$= \int_{-\infty}^{a} (x - E[X])^2 f(x|a,b)\,dx + \int_{a}^{b} (x - E[X])^2 f(x|a,b)\,dx + \int_{b}^{+\infty} (x - E[X])^2 f(x|a,b)\,dx =$$

$$= \int_{-\infty}^{a} (x - E[X])^2 \cdot 0\,dx + \int_{a}^{b} \left(x - \frac{b+a}{2}\right)^2 \frac{1}{b-a}\,dx + \int_{b}^{+\infty} (x - E[X])^2\, 0\,dx =$$

$$= \int_{a}^{b} \left(x - \frac{b+a}{2}\right)^2 \frac{1}{b-a}\,dx = \frac{1}{b-a} \int_{a}^{b} \left(x - \frac{b+a}{2}\right)^2 dx =$$

$$= \frac{1}{b-a} \frac{1}{3} \left(x - \frac{b+a}{2}\right)^3 \Bigg|_a^b = \frac{1}{3(b-a)} \left(x - \frac{b+a}{2}\right)^3 \Bigg|_a^b =$$

$$= \frac{1}{3(b-a)} \left[\left(b - \frac{b+a}{2}\right)^3 - \left(a - \frac{b+a}{2}\right)^3\right] = \frac{1}{3(b-a)} \left[\frac{(2b-b-a)^3}{2^3} - \frac{(2a-b-a)^3}{2^3}\right] =$$

$$= \frac{1}{3 \cdot 2^3 (b-a)} \left[(b-a)^3 - (a-b)^3\right] = \frac{1}{3 \cdot 2^3 (b-a)} \left[(b-a)^3 - (-1)^3 (b-a)^3\right] =$$

$$= \frac{2(b-a)^3}{3 \cdot 2^3 (b-a)} = \frac{(b-a)^2}{3 \cdot 2^2} = \frac{(b-a)^2}{12}$$

5. функция распределения

$$P(X \le x) = F(x|a,b) = \int_{-\infty}^{x} f(\xi|a,b)\,d\xi =$$

$$= \begin{cases} \int_{-\infty}^{x} f(\xi|a,b)\,d\xi, & x < a \\ \int_{-\infty}^{x} f(\xi|a,b)\,d\xi, & a \le x \le b \\ \int_{-\infty}^{x} f(\xi|a,b)\,d\xi, & b < x \end{cases} = (5)$$

$x < a$

$$\int_{-\infty}^{x} f(\xi|a,b)\,d\xi = \int_{-\infty}^{x} 0\,d\xi = 0$$

$a \le x \le b$

$$\int_{-\infty}^{x} f(\xi|a,b)\,d\xi = \int_{-\infty}^{a} f(\xi|a,b)\,d\xi + \int_{a}^{x} f(\xi|a,b)\,d\xi =$$

$$= \int_{-\infty}^{a} 0\,d\xi + \int_{a}^{x} \frac{1}{b-a}\,d\xi = \int_{a}^{x} \frac{1}{b-a}\,d\xi =$$

$$= \frac{1}{b-a}\int_{a}^{x} d\xi = \frac{1}{b-a}\,\xi\Big|_{a}^{x} = \frac{1}{b-a}(x-a) = \frac{x-a}{b-a}$$

$b < x$

$$\int_{-\infty}^{x} f(\xi|a,b)\,d\xi = \int_{-\infty}^{a} f(\xi|a,b)\,d\xi + \int_{a}^{b} f(\xi|a,b)\,d\xi + \int_{b}^{x} f(\xi|a,b)\,d\xi =$$

$$= \int_{-\infty}^{a} 0\,d\xi + \int_{a}^{b} \frac{1}{b-a}\,d\xi + \int_{b}^{x} 0\,d\xi = \int_{a}^{b} \frac{1}{b-a}\,d\xi =$$

$$= \frac{1}{b-a}\int_{a}^{b} d\xi = \frac{1}{b-a}\,\xi\Big|_{a}^{b} = \frac{1}{b-a}(b-a) = 1$$

$$(5.1) = \begin{cases} 0 & , X < a \\ \frac{x-a}{b-a} & , a \le X \le b \\ 1 & , b < X \end{cases} = (5.2)$$

$$\left.\frac{x-a}{b-a}\right|_{x=a} = \frac{a-a}{b-a} = \frac{0}{b-a} = 0$$

$$\left.\frac{x-a}{b-a}\right|_{x=b} = \frac{b-a}{b-a} = 1$$

$$(5.2) = \begin{cases} 0 & , x \le a \\ \frac{x-a}{b-a} & , a < x < b \\ 1 & , b \le X \end{cases}$$

$X \sim U(a,b) \quad a < b$

$$f(x|a,b) = \begin{cases} \frac{1}{b-a} & , a \le X \le b \\ 0 & , X < a \vee b < X \end{cases}$$

$$\int_{-\infty}^{\infty} f(x|a,b)\,dx = 1$$

$$E[X] = \int_{-\infty}^{\infty} x\, f(x|a,b)\,dx = \frac{a+b}{2}$$

$$D[X] = \int_{-\infty}^{+\infty} (x - E[X])^2 f(x|a,b)\,dx = \frac{(b-a)^2}{12}$$

$$p(X \le x) = F(x|a,b) = \begin{cases} 0 & , x < a \\ \frac{x-a}{b-a} & , a \le x \le b \\ 1 & , b < x \end{cases} = \begin{cases} 0 & , x \le a \\ \frac{x-a}{b-a} & , a < x < b \\ 1 & , b \le x \end{cases}$$