

2.1

Q- no:

$$|\text{cov } xy| \leq \sqrt{D[x] D[y]}$$

Sol:

$$\mu_x = E[x]$$

$$\mu_y = E[y]$$

$$\sigma_x^2 = D[x] = E[(x - E[x])^2] = E[(x - \mu_x)^2] \geq 0$$

$$\sigma_y^2 = D[y] = E[(y - E[y])^2] = E[(y - \mu_y)^2] \geq 0$$

$$\text{cov } xy = E[(x - E[x])(y - E[y])] = E[(x - \mu_x)(y - \mu_y)]$$

$$Z = \sigma_y x - \sigma_x y$$

$$D[Z] = D[\sigma_y x - \sigma_x y] =$$

$$= E[(\sigma_y x - \sigma_x y - E[\sigma_y x - \sigma_x y])^2] =$$

$$= E[(\sigma_y x - \sigma_x y - (\sigma_y E[x] - \sigma_x E[y]))^2] =$$

$$= E[(\sigma_y x - \sigma_x y - (\sigma_y \mu_x - \sigma_x \mu_y))^2] =$$

$$= E[(\sigma_y (x - \mu_x) - \sigma_x (y - \mu_y))^2] =$$

$$= E[\sigma_y^2 (x - \mu_x)^2 - 2\sigma_y \sigma_x (x - \mu_x)(y - \mu_y) + \sigma_x^2 (y - \mu_y)^2] =$$

$$= E[\sigma_y^2 (x - \mu_x)^2 + \sigma_x^2 (y - \mu_y)^2 - 2\sigma_x \sigma_y (x - \mu_x)(y - \mu_y)] =$$

$$= \sigma_y^2 E[(x - \mu_x)^2] + \sigma_x^2 E[(y - \mu_y)^2] - 2\sigma_x \sigma_y E[(x - \mu_x)(y - \mu_y)] =$$

$$= \sigma_y^2 \sigma_x^2 + \sigma_y^2 \sigma_x^2 - 2\sigma_x \sigma_y \text{cov } xy =$$

$$= 2\sigma_x \sigma_y (\sigma_x \sigma_y - \text{cov } xy) \geq 0 \Rightarrow$$

①

$$\Rightarrow \sigma_x \sigma_y - \text{cov}_{xy} \geq 0 \Leftrightarrow$$

$$\Leftrightarrow \text{cov}_{xy} \leq \sigma_x \sigma_y$$

$$z_2 = b_y x + b_x y$$

$$D[z_2] = D[b_y x + b_x y] =$$

$$= E[(b_y x + b_x y) - E(b_y x + b_x y)]^2 =$$

$$= E[(b_y x + b_x y) - (b_y E[x] + b_x E[y])]^2 =$$

$$= E[(b_y x + b_x y) - (b_y \mu_x + b_x \mu_y)]^2 =$$

$$= E[(b_y (x - \mu_x) + b_x (y - \mu_y))]^2 =$$

$$= E[b_y^2 (x - \mu_x)^2 + 2b_y b_x (x - \mu_x)(y - \mu_y) + b_x^2 (y - \mu_y)^2] =$$

$$= E[b_y^2 (x - \mu_x)^2 + b_x^2 (y - \mu_y)^2 + 2b_x b_y (x - \mu_x)(y - \mu_y)] =$$

$$= b_y^2 E[(x - \mu_x)^2] + b_x^2 E[(y - \mu_y)^2] + 2b_x b_y E[(x - \mu_x)(y - \mu_y)] =$$

$$= b_y^2 \sigma_x^2 + b_x^2 \sigma_y^2 + 2b_x b_y \text{cov}_{xy} =$$

$$= 2b_x b_y (b_x \sigma_y + \text{cov}_{xy}) \geq 0 \Rightarrow$$

$$\Rightarrow b_x \sigma_y + \text{cov}_{xy} \geq 0 \Leftrightarrow$$

$$\Leftrightarrow \text{cov}_{xy} \geq -b_x \sigma_y$$

$$\begin{cases} \text{cov}_{xy} \leq b_x \sigma_y \\ \text{cov}_{xy} \geq -b_x \sigma_y \end{cases} \Leftrightarrow -b_x \sigma_y \leq \text{cov}_{xy} \leq b_x \sigma_y \Leftrightarrow$$

$$\Leftrightarrow |\text{cov}_{xy}| \leq b_x \sigma_y \quad \square$$

$$|\text{cov}_{xy}| \leq \sqrt{D[x]D[y]}$$