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$$\begin{cases}2x+5y=1\\3x+7y=1\end{cases};\quad\begin{cases}x=\\y=\end{cases}$$

$$\triangleright\ \begin{cases}2x+5y=1\\3x+7y=1\end{cases};\quad\begin{pmatrix}2&5\\3&7\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix};\quad\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}b_1\\b_2\end{pmatrix}$$

$$\Delta=\det\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}=\det\begin{pmatrix}2&5\\3&7\end{pmatrix}=2\cdot7-5\cdot3=14-15=-1$$

$$\Delta_1=\det\begin{pmatrix}b_1&a_{12}\\b_2&a_{22}\end{pmatrix}=\det\begin{pmatrix}1&5\\1&7\end{pmatrix}=1\cdot7-5\cdot1=7-5=2$$

$$\Delta_2=\det\begin{pmatrix}a_{11}&b_1\\a_{21}&b_2\end{pmatrix}=\det\begin{pmatrix}2&1\\3&1\end{pmatrix}=2\cdot1-1\cdot3=2-3=-1$$

$$x=x_1=\frac{\Delta_1}{\Delta}=\frac{2}{(-1)}=-2$$

$$y=x_2=\frac{\Delta_2}{\Delta}=\frac{(-1)}{(-1)}=1$$

$\triangleleft\triangleright$

$$\begin{cases}2x+5y=1\\3x+7y=1\end{cases};\quad\left(\begin{array}{cc|c}2&5&1\\3&7&1\end{array}\right)\rightarrow\begin{cases}(1\cdot)=7(1\cdot)-5(2\cdot)\\(2\cdot)=3(1\cdot)-2(2\cdot)\end{cases}\rightarrow$$

$$\rightarrow\left(\begin{array}{cc|c}7\cdot2-5\cdot3&7\cdot5-5\cdot7&7\cdot1-5\cdot1\\3\cdot2-2\cdot3&3\cdot5-2\cdot7&3\cdot1-2\cdot1\end{array}\right)=\left(\begin{array}{cc|c}14-15&35-35&7-5\\6-6&15-14&3-2\end{array}\right)=$$

$$=\left(\begin{array}{cc|c}-1&0&2\\0&1&1\end{array}\right)\rightarrow\{(1\cdot)=(-1)(1\cdot)\}\rightarrow\left(\begin{array}{cc|c}(-1)(-1)&(-1)0&(-1)2\\0&1&1\end{array}\right)=$$

$$=\left(\begin{array}{cc|c}1&0&-2\\0&1&1\end{array}\right);\quad\begin{cases}x=-2\\y=1\end{cases}$$

$\triangleleft$ $x=-2\wedge y=1$

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$$\begin{cases} 2x - 3y = 4 \\ 4x - 5y = 10 \end{cases} ; \quad \begin{cases} x = \\ y = \end{cases}$$

▷

$$\begin{cases} 2x - 3y = 4 \\ 4x - 5y = 10 \end{cases} ; \quad \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 10 \end{pmatrix} ; \quad \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\Delta = \det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \det\begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix} = 2(-5) - (-3)4 = -10 + 12 = 2$$

$$\Delta_1 = \det\begin{pmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{pmatrix} = \det\begin{pmatrix} 4 & -3 \\ 10 & -5 \end{pmatrix} = 4(-5) - (-3)10 = -20 + 30 = 10$$

$$\Delta_2 = \det\begin{pmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{pmatrix} = \det\begin{pmatrix} 2 & 4 \\ 4 & 10 \end{pmatrix} = 2 \cdot 10 - 4 \cdot 4 = 20 - 16 = 4$$

$$x = x_1 = \frac{\Delta_1}{\Delta} = \frac{10}{2} = 5$$

$$y = x_2 = \frac{\Delta_2}{\Delta} = \frac{4}{2} = 2$$

◁▷

$$\begin{cases} 2x - 3y = 4 \\ 4x - 5y = 10 \end{cases} ; \quad \left(\begin{array}{cc|c} 2 & -3 & 4 \\ 4 & -5 & 10 \end{array}\right) \rightarrow \left\{\begin{array}{l} (1\cdot) = 5(1\cdot) - 3(2\cdot) \\ (2\cdot) = 2(1\cdot) - (2\cdot) \end{array}\right\} \rightarrow$$

$$\rightarrow \left(\begin{array}{cc|c} 5\cdot 2 - 3\cdot 4 & 5(-3) - 3(-5) & 5\cdot 4 - 3\cdot 10 \\ 2\cdot 2 - 4 & 2(-3) - (-5) & 2\cdot 4 - 10 \end{array}\right) = \left(\begin{array}{cc|c} 10 - 12 & -15 + 15 & 20 - 30 \\ 4 - 4 & -6 + 5 & 8 - 10 \end{array}\right) =$$

$$= \left(\begin{array}{cc|c} -2 & 0 & -10 \\ 0 & -1 & -2 \end{array}\right) \rightarrow \left\{\begin{array}{l} (1\cdot) = \frac{(1\cdot)}{-2} \\ (2\cdot) = (-1)(2\cdot) \end{array}\right\} \rightarrow \left(\begin{array}{cc|c} \frac{-2}{-2} & \frac{0}{-2} & \frac{-10}{-2} \\ (-1)0 & (-1)(-1) & (-1)(-2) \end{array}\right) =$$

$$= \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 2 \end{array}\right) ; \quad \begin{cases} x = 5 \\ y = 2 \end{cases}$$

◁ $x = 5 \wedge y = 2$

$$\begin{cases} 5x + 7y = 1 \\ x - 2y = 0 \end{cases} ; \quad \begin{cases} x = \\ y = \end{cases}$$

$\triangleright$ $\begin{cases} 5x + 7y = 1 \\ x - 2y = 0 \end{cases} ; \quad \begin{pmatrix} 5 & 7 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} ; \quad \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$$\Delta = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 1 & -2 \end{pmatrix} = 5(-2) - 7 \cdot 1 = -10 - 7 = -17$$

$$\Delta_1 = \begin{pmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{pmatrix} = \begin{pmatrix} 1 & 7 \\ 0 & -2 \end{pmatrix} = 1 \cdot (-2) - 7 \cdot 0 = -2 + 0 = -2$$

$$\Delta_2 = \begin{pmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{pmatrix} = \begin{pmatrix} 5 & 1 \\ 1 & 0 \end{pmatrix} = 5 \cdot 0 - 1 \cdot 1 = 0 - 1 = -1$$

$$x = x_1 = \frac{\Delta_1}{\Delta} = \frac{-2}{-17} = \frac{2}{17}$$

$$y = x_2 = \frac{\Delta_2}{\Delta} = \frac{-1}{-17} = \frac{1}{17}$$

$\triangleleft\triangleright$ $\begin{cases} 5x + 7y = 1 \\ x - 2y = 0 \end{cases} ; \quad \left(\begin{array}{cc|c} 5 & 7 & 1 \\ 1 & -2 & 0 \end{array}\right) \to \left\{ \begin{array}{l} (1.) = 2(1.) + 7(2.) \\ (2.) = (1.) - 5(2.) \end{array} \right\} \to$

$$\to \left(\begin{array}{cc|c} 2 \cdot 5 + 7 \cdot 1 & 2 \cdot 7 + 7(-2) & 2 \cdot 1 + 7 \cdot 0 \\ 5 - 5 \cdot 1 & 7 - 5(-2) & 1 - 5 \cdot 0 \end{array}\right) = \left(\begin{array}{cc|c} 10 + 7 & 14 - 14 & 2 + 0 \\ 5 - 5 & 7 + 10 & 1 - 0 \end{array}\right) =$$

$$= \left(\begin{array}{cc|c} 17 & 0 & 2 \\ 0 & 17 & 1 \end{array}\right) \to \left\{ \begin{array}{l} (1.) = \frac{(1.)}{17} \\ (2.) = \frac{(2.)}{17} \end{array} \right\} \to \left(\begin{array}{cc|c} \frac{17}{17} & \frac{0}{17} & \frac{2}{17} \\ \frac{0}{17} & \frac{17}{17} & \frac{1}{17} \end{array}\right) =$$

$$= \left(\begin{array}{cc|c} 1 & 0 & \frac{2}{17} \\ 0 & 1 & \frac{1}{17} \end{array}\right) ; \quad \begin{cases} x = \frac{2}{17} \\ y = \frac{1}{17} \end{cases}$$

$\triangleleft$ $x = \frac{2}{17} \wedge y = \frac{1}{17}$

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$$\begin{cases} 4x+7y+13=0 \\ 5x+8y+14=0 \end{cases} ; \quad \begin{cases} x= \\ y= \end{cases}$$

I▷ $\begin{cases} 4x+7y+13=0 \\ 5x+8y+14=0 \end{cases} ; \begin{cases} 4x+7y=-13 \\ 5x+8y=-14 \end{cases} ; \begin{pmatrix} 4 & 7 \\ 5 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} -13 \\ -14 \end{pmatrix} ; \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$$\Delta = \det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \det\begin{pmatrix} 4 & 7 \\ 5 & 8 \end{pmatrix} = 4\cdot 8 - 7\cdot 5 = 32-35=-3$$

$$\Delta_1 = \det\begin{pmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{pmatrix} = \det\begin{pmatrix} -13 & 7 \\ -14 & 8 \end{pmatrix} = (-13)\cdot 8 - 7(-14) = -8\cdot 13 + 7\cdot 14 =$$

$$= 7\cdot 14 - 8\cdot 13 = 7\cdot 14 - (7+1)13 = 7\cdot 14 - (7\cdot 13 + 13) = (7\cdot 14 - 7\cdot 13) - 13 = 7(14-13)-13 = 7-13=-6$$

$$\Delta_2 = \det\begin{pmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{pmatrix} = \det\begin{pmatrix} 4 & -13 \\ 5 & -14 \end{pmatrix} = 4(-14) - (-13)5 = -4\cdot 14 + 5\cdot 13 =$$

$$= 5\cdot 13 - 4\cdot 14 = (4+1)\cdot 13 - 4\cdot 14 = (4\cdot 13 + 13) - 4\cdot 14 = (4\cdot 13 - 4\cdot 14) + 13 = 4(13-14)+13 = -4+13 = 9$$

$$x = x_1 = \frac{\Delta_1}{\Delta} = \frac{-6}{-3} = 2$$

$$y = x_2 = \frac{\Delta_2}{\Delta} = \frac{9}{-3} = -3$$

◁II▷ $\begin{cases} 4x+7y+13=0 \\ 5x+8y+14=0 \end{cases} ; \begin{cases} 4x+7y=-13 \\ 5x+8y=-14 \end{cases} ; \left(\begin{array}{cc|c} 4 & 7 & -13 \\ 5 & 8 & -14 \end{array}\right) \to \left\{\begin{array}{l} (1.) = 8(1.) - 7(2.) \\ (2.) = 5(1.) - 4(2.) \end{array}\right\} \to$

$$\to \left(\begin{array}{cc|c} 8\cdot 4 - 7\cdot 5 & 8\cdot 7 - 7\cdot 8 & 8(-13)-7(-14) \\ 5\cdot 4 - 4\cdot 5 & 5\cdot 7 - 4\cdot 8 & 5(-13) - 4(-14) \end{array}\right) = \left(\begin{array}{cc|c} 32-35 & 7\cdot 8 - 7\cdot 8 & 7\cdot 14 - 8\cdot 13 \\ 4\cdot 5 - 4\cdot 5 & 35-32 & 4\cdot 14 - 5\cdot 13 \end{array}\right) = \{$$

$$7\cdot 14 - 8\cdot 13 = 7(13+1) - 8\cdot 13 = (7\cdot 13 + 7) - 8\cdot 13 = (7\cdot 13 - 8\cdot 13) + 7 = (7-8)13 + 7 = -13+7 = -6$$

$$4\cdot 14 - 5\cdot 13 = 4(13+1) - 5\cdot 13 = (4\cdot 13 + 4) - 5\cdot 13 = (4\cdot 13 - 5\cdot 13) + 4 = (4-5)13 + 4 = -13+4 = -9$$

$$\} = \left(\begin{array}{cc|c} -3 & 0 & -6 \\ 0 & 3 & -9 \end{array}\right) \to \left\{\begin{array}{l} (1.) = \frac{(1.)}{(-3)} \\ (2.) = \frac{(2.)}{3} \end{array}\right\} \to \left(\begin{array}{cc|c} \frac{-3}{-3} & \frac{0}{-3} & \frac{-6}{-3} \\ \frac{0}{3} & \frac{3}{3} & \frac{-9}{3} \end{array}\right) =$$

$$= \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \end{array}\right) ; \begin{cases} x=2 \\ y=-3 \end{cases}$$

◁ $x=2 \wedge y=-3$

$$\begin{cases} x\cos\alpha - y\sin\alpha = \cos\beta \\ x\sin\alpha + y\cos\alpha = \sin\beta \end{cases} ; \begin{cases} x = \\ y = \end{cases}$$

D) $$\begin{cases} x\cos\alpha - y\sin\alpha = \cos\beta \\ x\sin\alpha + y\cos\alpha = \sin\beta \end{cases} ; \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix} ; \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\Delta = \det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \det\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \cos\alpha\cos\alpha - (-\sin\alpha)\sin\alpha =$$

$$= \sin^2\alpha + \cos^2\alpha = 1$$

$$\Delta_1 = \det\begin{pmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{pmatrix} = \det\begin{pmatrix} \cos\beta & -\sin\alpha \\ \sin\beta & \cos\alpha \end{pmatrix} = \cos\beta\cos\alpha - (-\sin\alpha)\sin\beta =$$

$$= \cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta)$$

$$\Delta_2 = \det\begin{pmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{pmatrix} = \det\begin{pmatrix} \cos\alpha & \cos\beta \\ \sin\alpha & \sin\beta \end{pmatrix} = \cos\alpha\sin\beta - \cos\beta\sin\alpha =$$

$$= -(\sin\alpha\cos\beta - \cos\alpha\sin\beta) = -\sin(\alpha-\beta)$$

$$x = x_1 = \frac{\Delta_1}{\Delta} = \frac{\cos(\alpha-\beta)}{1} = \cos(\alpha-\beta)$$

$$y = x_2 = \frac{\Delta_2}{\Delta} = \frac{-\sin(\alpha-\beta)}{1} = -\sin(\alpha-\beta)$$

4D) $$\begin{cases} x\cos\alpha - y\sin\alpha = \cos\beta \\ x\sin\alpha + y\cos\alpha = \sin\beta \end{cases} ; \left(\begin{array}{cc|c} \cos\alpha & -\sin\alpha & \cos\beta \\ \sin\alpha & \cos\alpha & \sin\beta \end{array}\right) \to \begin{Bmatrix} (1.) = \cos\alpha(1.) + \sin\alpha(2.) \\ (2.) = \sin\alpha(1.) - \cos\alpha(2.) \end{Bmatrix} \to$$

$$\to \left(\begin{array}{cc|c} \cos\alpha\cos\alpha + \sin\alpha\sin\alpha & \cos\alpha(-\sin\alpha) + \sin\alpha\cos\alpha & \cos\alpha\cos\beta + \sin\alpha\sin\beta \\ \sin\alpha\cos\alpha - \cos\alpha\sin\alpha & \sin\alpha(-\sin\alpha) - \cos\alpha\cos\alpha & \sin\alpha\cos\beta - \cos\alpha\sin\beta \end{array}\right) =$$

$$= \left(\begin{array}{cc|c} \sin^2\alpha + \cos^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \cos\alpha\cos\beta + \sin\alpha\sin\beta \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & -(\sin^2\alpha + \cos^2\alpha) & \sin\alpha\cos\beta - \cos\alpha\sin\beta \end{array}\right) = \Big\{$$

$\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta)$
$\sin\alpha\cos\beta - \cos\alpha\sin\beta = \sin(\alpha-\beta)$

$$\Big\} = \left(\begin{array}{cc|c} 1 & 0 & \cos(\alpha-\beta) \\ 0 & -1 & \sin(\alpha-\beta) \end{array}\right) = \Big\{ (2.) = (-1)(2.) \Big\} = \left(\begin{array}{cc|c} 1 & 0 & \cos(\alpha-\beta) \\ (-1)0 & (-1)(-1) & (-1)\sin(\alpha-\beta) \end{array}\right) =$$

$$= \left(\begin{array}{cc|c} 1 & 0 & \cos(\alpha-\beta) \\ 0 & 1 & -\sin(\alpha-\beta) \end{array}\right) ; \begin{cases} x = \cos(\alpha-\beta) \\ y = -\sin(\alpha-\beta) \end{cases} \Delta$$

5

$$\begin{cases} \alpha \neq \frac{\pi}{2} + 2\pi k,\ k \in \mathbb{Z} \\ x\,tg\alpha + y = \sin(\alpha+\beta) \\ x - y\,tg\alpha = \cos(\alpha+\beta) \end{cases} ; \quad \begin{cases} x = \\ y = \end{cases}$$

$$D \quad \begin{cases} x\,tg\alpha + y = \sin(\alpha+\beta) \\ x - y\,tg\alpha = \cos(\alpha+\beta) \end{cases} ; \quad \begin{pmatrix} tg\alpha & 1 \\ 1 & -tg\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \sin(\alpha+\beta) \\ \cos(\alpha+\beta) \end{pmatrix} ; \quad \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\Delta = \det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{12} \end{pmatrix} = \det\begin{pmatrix} tg\alpha & 1 \\ 1 & -tg\alpha \end{pmatrix} = tg\alpha(-tg\alpha) - 1\cdot 1 = -tg^2\alpha - 1 = -(tg^2\alpha + 1) =$$

$$= \left\{ tg^2\alpha + 1 = \left(\frac{\sin\alpha}{\cos\alpha}\right)^2 + 1 = \frac{\sin^2\alpha}{\cos^2\alpha} + 1 = \frac{\sin^2\alpha + \cos^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha} \right\} = -\frac{1}{\cos^2\alpha}$$

$$\Delta_1 = \det\begin{pmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{pmatrix} = \det\begin{pmatrix} \sin(\alpha+\beta) & 1 \\ \cos(\alpha+\beta) & -tg\alpha \end{pmatrix} = \sin(\alpha+\beta)(-tg\alpha) - 1\cdot\cos(\alpha+\beta) =$$

$$= -\left(\sin(\alpha+\beta)\,tg\alpha + \cos(\alpha+\beta)\right) = \Big\{$$

$$\sin(\alpha+\beta)\,tg\alpha + \cos(\alpha+\beta) = \sin(\alpha+\beta)\frac{\sin\alpha}{\cos\alpha} + \cos(\alpha+\beta) = \frac{\sin(\alpha+\beta)\sin\alpha + \cos(\alpha+\beta)\cos\alpha}{\cos\alpha} =$$

$$= \frac{1}{\cos\alpha}\Big(\sin\alpha(\sin\alpha\cos\beta + \cos\alpha\sin\beta) + \cos\alpha(\cos\alpha\cos\beta - \sin\alpha\sin\beta)\Big) =$$

$$= \frac{1}{\cos\alpha}\left(\sin^2\alpha\cos\beta + \cancel{\sin\alpha\cos\alpha\cos\beta} + \cos^2\alpha\cos\beta - \cancel{\sin\alpha\cos\alpha}\sin\beta\right) =$$

$$= \frac{(\sin^2\alpha + \cos^2\alpha)\cos\beta}{\cos\alpha} = \frac{\cos\beta}{\cos\alpha}$$

$$\Big\} = -\frac{\cos\beta}{\cos\alpha}$$

$$\Delta_2 = \det\begin{pmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{pmatrix} = \det\begin{pmatrix} \operatorname{tg}\alpha & \sin(\alpha+\beta) \\ 1 & \cos(\alpha+\beta) \end{pmatrix} = \operatorname{tg}\alpha\cos(\alpha+\beta) - \sin(\alpha+\beta)\cdot 1 =$$

$$= \cos(\alpha+\beta)\operatorname{tg}\alpha - \sin(\alpha+\beta) = \cos(\alpha+\beta)\frac{\sin\alpha}{\cos\alpha} - \sin(\alpha+\beta) = \frac{\cos(\alpha+\beta)\sin\alpha - \sin(\alpha+\beta)\cos\alpha}{\cos\alpha} =$$

$$= \frac{1}{\cos\alpha}\Big(\sin\alpha(\cos\alpha\cos\beta - \sin\alpha\sin\beta) - \cos\alpha(\sin\alpha\cos\beta + \cos\alpha\sin\beta)\Big) =$$

$$= \frac{1}{\cos\alpha}\left(\cancel{\sin\alpha\cos\alpha\cos\beta} - \sin^2\alpha\sin\beta - \cancel{\sin\alpha\cos\alpha\cos\beta} - \cos^2\alpha\sin\beta\right) =$$

$$= \frac{-(\sin^2\alpha + \cos^2\alpha)\sin\beta}{\cos\alpha} = -\frac{\sin\beta}{\cos\alpha}$$

$$x = x_1 = \frac{\Delta_1}{\Delta} = \left(-\frac{\cos\beta}{\cos\alpha}\right)\left(-\frac{1}{\cos^2\alpha}\right)^{-1} = \frac{\cos\beta}{\cos\alpha}\cos^2\alpha = \cos\alpha\cos\beta$$

$$y = x_2 = \frac{\Delta_2}{\Delta} = \left(-\frac{\sin\beta}{\cos\alpha}\right)\left(-\frac{1}{\cos^2\alpha}\right)^{-1} = \frac{\sin\beta}{\cos\alpha}\cos^2\alpha = \cos\alpha\sin\beta$$

$\triangleleft\triangleright$

$$x\operatorname{tg}\alpha + y = \cos\alpha\cos\beta\frac{\sin\alpha}{\cos\alpha} + \cos\alpha\sin\beta = \sin\alpha\cos\beta + \cos\alpha\sin\beta =$$

$$= \sin(\alpha+\beta)$$

$$x - y\operatorname{tg}\alpha = \cos\alpha\cos\beta - \cos\alpha\sin\beta\frac{\sin\alpha}{\cos\alpha} = \cos\alpha\cos\beta - \sin\alpha\sin\beta =$$

$$= \cos(\alpha+\beta)$$

$\triangleleft$

$$x = \cos\alpha\cos\beta \;\wedge\; y = \cos\alpha\sin\beta$$