—113—

Д-ть:

$$\begin{vmatrix} a_1+ib_1 & b_1+ia_1 & c_1 \\ a_2+ib_2 & b_2+ia_2 & c_2 \\ a_3+ib_3 & b_3+ia_3 & c_3 \end{vmatrix} = 2\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Д-во:

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} = \begin{pmatrix} a_{\cdot 1} & a_{\cdot 2} & a_{\cdot 3} \end{pmatrix}$$

$$B = \begin{pmatrix} a_1+ib_1 & b_1+ia_1 & c_1 \\ a_2+ib_2 & b_2+ia_2 & c_2 \\ a_3+ib_3 & b_3+ia_3 & c_3 \end{pmatrix} = \begin{pmatrix} b_{\cdot 1} & b_{\cdot 2} & b_{\cdot 3} \end{pmatrix}$$

$$b_{\cdot 1} = \begin{pmatrix} a_1+ib_1 \\ a_2+ib_2 \\ a_3+ib_3 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} ib_1 \\ ib_2 \\ ib_3 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + i\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_{\cdot 1} + i a_{\cdot 2}$$

$$b_{\cdot 2} = \begin{pmatrix} b_1+ia_1 \\ b_2+ia_2 \\ b_3+ia_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \begin{pmatrix} ia_1 \\ ia_2 \\ ia_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + i\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = a_{\cdot 2} + i a_{\cdot 1}$$

$$\det B = \det\begin{pmatrix} b_{\cdot 1} & b_{\cdot 2} & b_{\cdot 3} \end{pmatrix} = \det\begin{pmatrix} a_{\cdot 1}+ia_{\cdot 2} & a_{\cdot 2}+ia_{\cdot 1} & a_{\cdot 3} \end{pmatrix} =$$

$$= \det\begin{pmatrix} a_{\cdot 1} & a_{\cdot 2}+ia_{\cdot 1} & a_{\cdot 3} \end{pmatrix} + \det\begin{pmatrix} ia_{\cdot 2} & a_{\cdot 2}+ia_{\cdot 1} & a_{\cdot 3} \end{pmatrix} =$$

$$= \det\begin{pmatrix} a_{\cdot 1} & a_{\cdot 2} & a_{\cdot 3} \end{pmatrix} + \det\begin{pmatrix} a_{\cdot 1} & ia_{\cdot 1} & a_{\cdot 3} \end{pmatrix} +$$

$$+ \det\begin{pmatrix} ia_{\cdot 2} & a_{\cdot 2} & a_{\cdot 3} \end{pmatrix} + \det\begin{pmatrix} ia_{\cdot 2} & ia_{\cdot 1} & a_{\cdot 3} \end{pmatrix} =$$

$$= \det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3}) + i \det(a_{\cdot 1}\ a_{\cdot 1}\ a_{\cdot 3}) +$$

$$+ i \det(a_{\cdot 2}\ a_{\cdot 2}\ a_{\cdot 3}) + i \cdot i \det(a_{\cdot 2}\ a_{\cdot 1}\ a_{\cdot 3}) = \{$$

$$\det(a_{\cdot 1}\ a_{\cdot 1}\ a_{\cdot 3}) = 0$$

$$\det(a_{\cdot 2}\ a_{\cdot 2}\ a_{\cdot 3}) = 0$$

$$\} = \det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3}) + i^2 \det(a_{\cdot 2}\ a_{\cdot 1}\ a_{\cdot 3}) = \{$$

$$\det(a_{\cdot 2}\ a_{\cdot 1}\ a_{\cdot 3}) = (-1)\det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3})$$

$$\} = \det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3}) + i^2(-1)\det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3}) =$$

$$= (1 - i^2)\det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3}) = \{i^2 = -1\} =$$

$$= 2\det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3}) = 2 \det A$$

$$\det B = 2 \det A$$

—113— ещё раз

$$\begin{vmatrix} a_1+ib_1 & b_1+ia_1 & c_1 \\ a_2+ib_2 & b_2+ia_2 & c_2 \\ a_3+ib_3 & b_3+ia_3 & c_3 \end{vmatrix} =$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & ia_1 & c_1 \\ a_2 & ia_2 & c_2 \\ a_3 & ia_3 & c_3 \end{vmatrix} +$$

$$+ \begin{vmatrix} ib_1 & b_1 & c_1 \\ ib_2 & b_2 & c_2 \\ ib_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} ib_1 & ia_1 & c_1 \\ ib_2 & ia_2 & c_2 \\ ib_3 & ia_3 & c_3 \end{vmatrix} =$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + i \begin{vmatrix} a_1 & a_1 & c_1 \\ a_2 & a_2 & c_2 \\ a_3 & a_3 & c_3 \end{vmatrix} +$$

$(\cdot 1) = (\cdot 2)$

$$+ i \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & b_2 & c_2 \\ b_3 & b_3 & c_3 \end{vmatrix} + ii \begin{vmatrix} b_1 & a_1 & c_1 \\ b_2 & a_2 & c_2 \\ b_3 & a_3 & c_3 \end{vmatrix} =$$

$(\cdot 1) = (\cdot 2)$ $\qquad$ $(\cdot 1) \Leftrightarrow (\cdot 2)$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + i^2(-1) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} =$$

$$= (1-i^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 2 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Д-ть:

$$\begin{vmatrix} a_1 + xb_1 & b_1 + xa_1 & c_1 \\ a_2 + xb_2 & b_2 + xa_2 & c_2 \\ a_3 + xb_3 & b_3 + xa_3 & c_3 \end{vmatrix} = (1-x^2)\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Д-во:

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} = \begin{pmatrix} a_{\cdot 1} & a_{\cdot 2} & a_{\cdot 3} \end{pmatrix}$$

$$B = \begin{pmatrix} a_1 + xb_1 & b_1 + xa_1 & c_1 \\ a_2 + xb_2 & b_2 + xa_2 & c_2 \\ a_3 + xb_3 & b_3 + xa_3 & c_3 \end{pmatrix} = \begin{pmatrix} b_{\cdot 1} & b_{\cdot 2} & b_{\cdot 3} \end{pmatrix}$$

$$b_{\cdot 1} = \begin{pmatrix} a_1 + xb_1 \\ a_2 + xb_2 \\ a_3 + xb_3 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} xb_1 \\ xb_2 \\ xb_3 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + x\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_{\cdot 1} + x a_{\cdot 2}$$

$$b_{\cdot 2} = \begin{pmatrix} b_1 + xa_1 \\ b_2 + xa_2 \\ b_3 + xa_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \begin{pmatrix} xa_1 \\ xa_2 \\ xa_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + x\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = a_{\cdot 2} + x a_{\cdot 1}$$

$$\det B = \det\begin{pmatrix} b_{\cdot 1} & b_{\cdot 2} & b_{\cdot 3} \end{pmatrix} = \det\begin{pmatrix} a_{\cdot 1} + x a_{\cdot 2} & a_{\cdot 2} + x a_{\cdot 1} & a_{\cdot 3} \end{pmatrix} =$$

$$= \det\begin{pmatrix} a_{\cdot 1} & a_{\cdot 2} + x a_{\cdot 1} & a_{\cdot 3} \end{pmatrix} + \det\begin{pmatrix} x a_{\cdot 2} & a_{\cdot 2} + x a_{\cdot 1} & a_{\cdot 3} \end{pmatrix} =$$

$$= \det\begin{pmatrix} a_{\cdot 1} & a_{\cdot 2} & a_{\cdot 3} \end{pmatrix} + \det\begin{pmatrix} a_{\cdot 1} & x a_{\cdot 1} & a_{\cdot 3} \end{pmatrix} +$$

$$+ \det\begin{pmatrix} x a_{\cdot 2} & a_{\cdot 2} & a_{\cdot 3} \end{pmatrix} + \det\begin{pmatrix} x a_{\cdot 2} & x a_{\cdot 1} & a_{\cdot 3} \end{pmatrix} =$$

$= \det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3}) + x\det(a_{\cdot 1}\ a_{\cdot 1}\ a_{\cdot 3}) +$

$+ x\det(a_{\cdot 2}\ a_{\cdot 2}\ a_{\cdot 3}) + x\,x\det(a_{\cdot 2}\ a_{\cdot 1}\ a_{\cdot 3}) = \{$

$\det(a_{\cdot 1}\ a_{\cdot 1}\ a_{\cdot 3}) = 0$

$\det(a_{\cdot 2}\ a_{\cdot 2}\ a_{\cdot 3}) = 0$

$\} = \det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3}) + x\cdot 0 + x\cdot 0 + x^2\det(a_{\cdot 2}\ a_{\cdot 1}\ a_{\cdot 3}) = \{$

$\det(a_{\cdot 2}\ a_{\cdot 1}\ a_{\cdot 3}) = (-1)\det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3})$

$\} = \det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3}) + x^2(-1)\det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3}) =$

$= (1-x^2)\det(a_{\cdot 1}\ a_{\cdot 2}\ a_{\cdot 3}) = (1-x^2)\det A$

$\det B = (1-x^2)\det A$

Примечание:

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$x = i \Rightarrow 1-x^2 = 1-i^2 = 2$

$\det B = (1-x^2)\det A\,\big|_{x=i} \Rightarrow \det B = 2\det A$

-114 – ещё раз

$$\begin{vmatrix} a_1+xb_1 & b_1+xa_1 & c_1 \\ a_2+xb_2 & b_2+xa_2 & c_2 \\ a_3+xb_3 & b_3+xa_3 & c_3 \end{vmatrix} =$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & xa_1 & c_1 \\ a_2 & xa_2 & c_2 \\ a_3 & xa_3 & c_3 \end{vmatrix} +$$

$$+ \begin{vmatrix} xb_1 & b_1 & c_1 \\ xb_2 & b_2 & c_2 \\ xb_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} xb_1 & xa_1 & c_1 \\ xb_2 & xa_2 & c_2 \\ xb_3 & xa_3 & c_3 \end{vmatrix} =$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x \underbrace{\begin{vmatrix} a_1 & a_1 & c_1 \\ a_2 & a_2 & c_2 \\ a_3 & a_3 & c_3 \end{vmatrix}}_{(\cdot 1) = (\cdot 2)} +$$

$$+ x \underbrace{\begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & b_2 & c_2 \\ b_3 & b_3 & c_3 \end{vmatrix}}_{(\cdot 1) = (\cdot 2)} + x x \underbrace{\begin{vmatrix} b_1 & a_1 & c_1 \\ b_2 & a_2 & c_2 \\ b_3 & a_3 & c_3 \end{vmatrix}}_{(\cdot 1) \longleftrightarrow (\cdot 2)} =$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x \cdot 0 + x \cdot 0 + x^2(-1) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} =$$

$$= (1 - x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

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Д-ть:

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (b-a)(c-a)(c-b)$$

Д-во:

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \left\{ (2\cdot) -= (1\cdot);\ (3\cdot) -= (1\cdot);\ |\cdot\cdot| = |\cdot\cdot| \right\} =$$

$$= \begin{vmatrix} 1 & a & bc \\ 1-1 & b-a & ca-bc \\ 1-1 & c-a & ab-bc \end{vmatrix} = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & (-c)(b-a) \\ 0 & c-a & (-b)(c-a) \end{vmatrix} = \Big\{$$

$$(2\cdot) = (b-a)\,(\hat{2\cdot});\ (3\cdot) = (c-a)\,(\hat{3\cdot});\ |\cdot\cdot| = (b-a)(c-a)\,\hat{|\cdot\cdot|}$$

$$\Big\} = (b-a)(c-a)\begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 1 & -b \end{vmatrix} = \left\{ (3\cdot) -= (2\cdot);\ |\cdot\cdot| = |\cdot\cdot| \right\} =$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0-0 & 1-1 & -b-(-c) \end{vmatrix} = (b-a)(c-a)\begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 0 & c-b \end{vmatrix} = \Big\{$$

$$\begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 0 & c-b \end{vmatrix} = \begin{vmatrix} \lambda_1 & * & * \\ 0 & \lambda_2 & * \\ 0 & 0 & \lambda_3 \end{vmatrix} = \lambda_1 \lambda_2 \lambda_3 = 1 \cdot 1 \cdot (c-b) = c-b$$

$$\Big\} = (b-a)(c-a)(c-b)$$

Д-то:

$$\begin{vmatrix}1&a&a^2\\1&b&b^2\\1&c&c^2\end{vmatrix} = (b-a)(c-a)(c-b)$$

Д-во:

$$\begin{vmatrix}1&a&a^2\\1&b&b^2\\1&c&c^2\end{vmatrix} = \left\{ (2\cdot) -= (\cdot 1);\ (3\cdot) -= (\cdot 1);\ |\,''\,| = |\,''\,| \right\} =$$

$$= \begin{vmatrix}1&a&a^2\\1-1&b-a&b^2-a^2\\1-1&c-a&c^2-a^2\end{vmatrix} = \begin{vmatrix}1&a&a^2\\0&b-a&(b-a)(b+a)\\0&c-a&(c-a)(c+a)\end{vmatrix} = \Big\{$$

$$(2\cdot) = (b-a)(\widehat{2\cdot});\ (3\cdot) = (c-a)(\widehat{3\cdot});\ |\,''\,| = (b-a)(c-a)\widehat{|\,''\,|}$$

$$\Big\} = (b-a)(c-a)\begin{vmatrix}1&a&a^2\\0&1&b+a\\0&1&c+a\end{vmatrix} = \left\{ (2\cdot) -= (3\cdot);\ |\,''\,| = |\,''\,| \right\} =$$

$$= (b-a)(c-a)\begin{vmatrix}1&a&a^2\\0&1&b+a\\0-0&1-1&(c+a)-(b+a)\end{vmatrix} = (b-a)(c-a)\begin{vmatrix}1&a&a^2\\0&1&b+a\\0&0&c-b\end{vmatrix} = \Big\{$$

$$\begin{vmatrix}1&a&a^2\\0&1&b+a\\0&0&c-b\end{vmatrix} = \begin{vmatrix}\lambda_1&*&*\\0&\lambda_2&*\\0&0&\lambda_3\end{vmatrix} = \lambda_1\lambda_2\lambda_3 = 1\cdot 1\cdot(c-b) = c-b$$

$$\Big\} = (b-a)(c-a)(c-b)$$

Д-ть:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a+b+c)(b-a)(c-a)(c-b)$$

Д-во:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = \left\{ (\cdot 2) - = (\cdot 1);\ (\cdot 3) = -(\cdot 1);\ |\,|| = |\,|| \right\} =$$

$$= \begin{vmatrix} 1 & 1-1 & 1-1 \\ a & b-a & c-a \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix} = \{$$

$$b^3-a^3 = (b-a)(b^2+ba+a^2)$$
$$c^3-a^3 = (c-a)(c^2+ca+a^2)$$

$$\} = \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & (b-a)(b^2+ba+a^2) & (c-a)(c^2+ca+a^2) \end{vmatrix} = \left\{ \begin{matrix} (\cdot 2) = (b-a)\widehat{(\cdot 2)} \\ (\cdot 3) = (c-a)\widehat{(\cdot 3)} \\ |\,|| = (b-a)(c-a)\,|\,\widehat{||}\,| \end{matrix} \right\} =$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^3 & b^2+ba+a^2 & c^2+ca+a^2 \end{vmatrix} = \left\{ \begin{matrix} (\cdot 3) - = (\cdot 2) \\ |\,|| = |\,|| \end{matrix} \right\} =$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1-1 \\ a^3 & b^2+ba+a^2 & (c^2+ca+a^2)-(b^2+ba+a^2) \end{vmatrix} = \{$$

$$(\cancel{c^2}+ca+\cancel{a^2})-(\cancel{b^2}+ba+\cancel{a^2}) = (c^2-b^2)+(ca-ba) =$$

$$= (c-b)(c+b)+a(c-b) = (c-b)(c+b+a)$$

$$\} = (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a^3 & b^2+ba+a^2 & (c-b)(c+b+a) \end{vmatrix} = \{$$

$$\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a^3 & b^2+ba+a^2 & (c-b)(c+b+a) \end{vmatrix} = \begin{vmatrix} \lambda_1 & 0 & 0 \\ * & \lambda_2 & 0 \\ * & * & \lambda_3 \end{vmatrix} = \lambda_1 \lambda_2 \lambda_3 =$$

$$= 1 \cdot 1 \cdot (c-b)(c+b+a) = (c-b)(c+b+a)$$

$$\} = (b-a)(c-a)(c-b)(c+b+a) =$$

$$= (a+b+c)(b-a)(c-a)(c-b)$$