-1-

$$\begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} = 53 - 2\cdot 7 = 15 - 14 = 1$$

-2-

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1\cdot 4 - 2\cdot 3 = 4 - 6 = -2$$

-3-

$$\begin{vmatrix} 3 & 2 \\ 8 & 5 \end{vmatrix} = 3\cdot 5 - 2\cdot 8 = 15 - 16 = -1$$

-4-

$$\begin{vmatrix} 6 & 9 \\ 8 & 12 \end{vmatrix} = 6\cdot 12 - 9\cdot 8 = 72 - 72 = 0$$

-5-

$$\begin{vmatrix} a^2 & ab \\ ab & b^2 \end{vmatrix} = a^2 b^2 - (ab)(ab) = (ab)^2 - (ab)^2 = 0$$

-6-

$$\begin{vmatrix} n+1 & n \\ n & n-1 \end{vmatrix} = (n+1)(n-1) - nn = (n^2-1) - n^2 = -1$$

-7-

$$\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix} =$$

$$= (a+b)(a+b) - (a-b)(a-b) = (a+b)^2 - (a-b)^2 =$$

$$= (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2) = 4ab$$

-8-

$$\begin{vmatrix} a^2+ab+b^2 & a^2-ab+b^2 \\ a+b & a-b \end{vmatrix} =$$

$$= (a^2+ab+b^2)(a-b) - (a^2-ab+b^2)(a+b) =$$

$$= \left[(a^2+b^2)(a-b) + ab(a-b)\right] - \left[(a^2+b^2)(a+b) - ab(a+b)\right] =$$

$$= \left[(a^2+b^2)(a-b) - (a^2+b^2)(a+b)\right] + \left[ab(a-b) + ab(a+b)\right] =$$

$$= (a^2+b^2)\left[(a-b)-(a+b)\right] + ab\left[(a-b)+(a+b)\right] =$$

$$= (a^2+b^2)(-2b) + ab(2a) = \cancel{-2a^2b} - 2b^3 + \cancel{2a^2b} =$$

$$= -2b^3$$

-9-

$$\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} = \cos\alpha\cos\alpha - (-\sin\alpha)\sin\alpha = \cos^2\alpha + \sin^2\alpha = 1$$

-10-

$$\begin{vmatrix} \sin\alpha & \cos\alpha \\ \sin\beta & \cos\beta \end{vmatrix} = \sin\alpha\cos\beta - \cos\alpha\sin\beta = \sin(\alpha-\beta)$$

-11-

$$\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\beta & \cos\beta \end{vmatrix} = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \cos(\alpha+\beta)$$

-12-

$$\begin{vmatrix} \sin\alpha + \sin\beta & \cos\beta + \cos\alpha \\ \cos\beta - \cos\alpha & \sin\alpha - \sin\beta \end{vmatrix} =$$

$$= (\sin\alpha + \sin\beta)(\sin\alpha - \sin\beta) - (\cos\beta + \cos\alpha)(\cos\beta - \cos\alpha) =$$

$$= \left[\sin\alpha\sin\alpha + \sin\alpha(-\sin\beta) + \sin\beta\sin\alpha + \sin\beta(-\sin\beta)\right] -$$

$$- \left[\cos\beta\cos\beta + \cos\beta(-\cos\alpha) + \cos\alpha\cos\beta + \cos\alpha(-\cos\alpha)\right] =$$

$$= \left[\sin^2\alpha - \cancel{\sin\alpha\sin\beta} + \cancel{\sin\alpha\sin\beta} - \sin^2\beta\right] -$$

$$- \left[\cos^2\beta - \cancel{\cos\alpha\cos\beta} + \cancel{\cos\alpha\cos\beta} - \cos^2\alpha\right] =$$

$$= (\sin^2\alpha + \cos^2\alpha) - (\sin^2\beta + \cos^2\beta) = 1 - 1 = 0$$

-13-

$$\begin{vmatrix} 2\sin\varphi\cos\varphi & 2\sin^2\varphi - 1 \\ 2\cos^2\varphi - 1 & 2\sin\varphi\cos\varphi \end{vmatrix} =$$

$$= (2\sin\varphi\cos\varphi)(2\sin\varphi\cos\varphi) - (2\sin^2\varphi - 1)(2\cos^2\varphi - 1) =$$

$$= (2\sin\varphi\cos\varphi)^2 - \left[(2\sin^2\varphi)(2\cos^2\varphi) - 2\sin^2\varphi - 2\cos^2\varphi + 1\right] =$$

$$= \cancel{4\sin^2\varphi\cos^2\varphi} - \left[\cancel{4\sin^2\varphi\cos^2\varphi} - 2(\sin^2\varphi + \cos^2\varphi) + 1\right] =$$

$$= 2(\sin^2\varphi + \cos^2\varphi) - 1 =$$

$$= \{\sin^2\varphi + \cos^2\varphi = 1\} = 2 - 1 = 1$$

-14-

$$\begin{vmatrix} \frac{1-t^2}{1+t^2} & \frac{2t}{1+t^2} \\ \frac{-2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{vmatrix} =$$

$$= \frac{1-t^2}{1+t^2}\,\frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2}\,\frac{-2t}{1+t^2} = \frac{(1-t^2)^2+(2t)^2}{(1+t^2)^2} =$$

$$= \frac{(1-2t^2+t^4)+4t^2}{(1+t^2)^2} = \frac{1+2t^2+t^4}{(1+t^2)^2} = \frac{(1+t^2)^2}{(1+t^2)^2} =$$

$$= 1$$

-15-

$$\begin{vmatrix} \frac{(1-t)^2}{1+t^2} & \frac{2t}{1+t^2} \\ \frac{2t}{1+t^2} & -\frac{(1+t)^2}{1+t^2} \end{vmatrix} =$$

$$= \frac{(1-t)^2}{1+t^2}(-1)\frac{(1+t)^2}{1+t^2} - \frac{2t}{1+t^2}\,\frac{2t}{1+t^2} = -\frac{(1-t)^2(1+t)^2+(2t)^2}{(1+t^2)^2} = \Big\{$$

$$(1-t)^2(1+t)^2+(2t)^2 = [(1-t)(1+t)]^2+(2t)^2 = (1-t^2)^2+(2t)^2 =$$

$$= (1-2t^2+t^4)+4t^2 = 1+2t^2+t^4 = (1+t^2)^2$$

$$\Big\} = -\frac{(1+t^2)^2}{(1+t^2)^2} = -1$$

-16-

$$\begin{vmatrix} \frac{1+t^2}{1-t^2} & \frac{2t}{1-t^2} \\ \frac{2t}{1-t^2} & \frac{1+t^2}{1-t^2} \end{vmatrix} =$$

$$= \frac{1+t^2}{1-t^2}\,\frac{1+t^2}{1-t^2} - \frac{2t}{1-t^2}\,\frac{2t}{1-t^2} = \frac{(1+t^2)^2-(2t)^2}{(1-t^2)^2} =$$

$$= \frac{(1+2t^2+t^4)-4t^2}{(1-t^2)^2} = \frac{1-2t^2+t^4}{(1-t^2)^2} = \frac{(1-t^2)^2}{(1-t^2)^2} = 1$$

-17-

$$\begin{vmatrix} 1 & \log_b a \\ \log_a b & 1 \end{vmatrix} = 1 - \log_b a \log_a b = 1 - \frac{\log_b a}{\log_b a} = 1-1 = 0$$

-18-

$$\begin{vmatrix} a & c+di \\ c-di & b \end{vmatrix} =$$

$$= ab - (c+di)(c-di) = ab - \left[c^2-(di)^2\right] = ab - (c^2-d^2i^2) =$$

$$= \{ i^2 = 1 \} = ab - (c^2+d^2)$$

-19-

$$\begin{vmatrix} a+bi & b \\ 2a & a-bi \end{vmatrix} =$$

$$= (a+bi)(a-bi) - b(2a) = \left[a^2-(bi)^2\right] - 2ab = (a^2-b^2i^2) - 2ab =$$

$$= \{ i^2 = 1 \} = (a^2+b^2) - 2ab = a^2 - 2ab + b^2 = (a-b)^2$$

-20-

$$\begin{vmatrix} \cos\alpha + i\sin\alpha & 1 \\ 1 & \cos\alpha - i\sin\alpha \end{vmatrix} =$$

$$= (\cos\alpha + i\sin\alpha)(\cos\alpha - i\sin\alpha) - 1 = \left[(\cos\alpha)^2 - (i\sin\alpha)^2\right] - 1 =$$

$$= (\cos^2\alpha - i^2\sin^2\alpha) - 1 =$$

$$= \{i^2 = 1\} = (\cos^2\alpha + \sin^2\alpha) - 1 =$$

$$= \{\sin^2\alpha + \cos^2\alpha\} = 1 - 1 = 0$$

-21-

$$\begin{vmatrix} a+bi & c+di \\ -c+di & a-bi \end{vmatrix} =$$

$$= (a+bi)(a-bi) - (c+di)(-c+di) = (a+bi)(a-bi) + (c+di)(c-di) =$$

$$= \left[a^2 - (bi)^2\right] + \left[c^2 - (di)^2\right] = (a^2 - b^2i^2) + (c^2 - d^2i^2) =$$

$$= \{i^2 = 1\} = a^2 + b^2 + c^2 + d^2$$