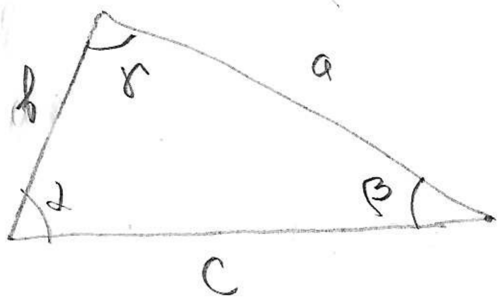
Дано:

$$d(a,b,c,\alpha) := \begin{vmatrix} a^2 & b\sin\alpha & c\sin\alpha \\ b\sin\alpha & 1 & \cos\alpha \\ c\sin\alpha & \cos\alpha & 1 \end{vmatrix}$$



(1)

Д-ть:

$(1) \Rightarrow d(a,b,c,\alpha) = 0$

$(1) \Rightarrow d(b,c,a,\beta) = 0$

$(1) \Rightarrow d(c,a,b,\gamma) = 0$

Д-во:

$$d(a,b,c,\alpha) := \begin{vmatrix} a^2 & b\sin\alpha & c\sin\alpha \\ b\sin\alpha & 1 & \cos\alpha \\ c\sin\alpha & \cos\alpha & 1 \end{vmatrix} =$$

$$= a^2\cdot 1\cdot 1 + (b\sin\alpha)\cos\alpha\,(c\sin\alpha) + (c\sin\alpha)(b\sin\alpha)\cos\alpha -$$

$$- (c\sin\alpha)\cdot 1\cdot(c\sin\alpha) - (b\sin\alpha)\cdot(b\sin\alpha)\cdot 1 - a^2\cdot\cos\alpha\cdot\cos\alpha =$$

$$= a^2 + bc\sin^2\alpha\cos\alpha + bc\sin^2\alpha\cos\alpha -$$

$$- c^2\sin^2\alpha - b^2\sin^2\alpha - a^2\cos^2\alpha =$$

$$= a^2(1-\cos^2\alpha) - b^2\sin^2\alpha - c^2\sin^2\alpha + 2bc\sin^2\alpha\cos\alpha =$$

$$= a^2\sin^2\alpha - b^2\sin^2\alpha - c^2\sin^2\alpha + 2bc\sin^2\alpha\cos\alpha =$$

$$= \left(a^2 - (b^2+c^2) + 2bc\cos\alpha\right)\sin^2\alpha$$

по т. косинусов

$(1) \Rightarrow \quad a^2 = b^2 + c^2 - 2bc\cos\alpha \Leftrightarrow$

$a^2 - (b^2 + c^2) + 2bc\cos\alpha = 0 \Rightarrow$

$d(a,b,c,\alpha) = \left(a^2 - (b^2+c^2) + 2bc\cos\alpha\right)\sin^2\alpha = 0$

$(1) \Rightarrow d(a,b,c,\alpha) = 0$

по т. косинусов

$(1) \Rightarrow \quad b^2 = c^2 + a^2 - 2ca\cos\beta \Leftrightarrow$

$b^2 - (c^2 + a^2) + 2ca\cos\beta = 0 \Rightarrow$

$d(b,c,a,\beta) = \left(b^2 - (c^2+a^2) + 2ca\cos\beta\right)\sin^2\beta = 0$

$(1) \Rightarrow d(b,c,a,\beta) = 0$

по т. косинусов

$(1) \Rightarrow \quad c^2 = a^2 + b^2 - 2ab\cos\gamma \Leftrightarrow$

$c^2 - (a^2 + b^2) + 2ab\cos\gamma = 0 \Rightarrow$

$d(c,a,b,\gamma) = \left(c^2 - (a^2+b^2) + 2ab\cos\gamma\right)\sin^2\gamma = 0$

$(1) \Rightarrow d(c,a,b,\gamma) = 0$

$$\begin{vmatrix} 1 & 0 & 1+i \\ 0 & 1 & i \\ 1-i & -i & 1 \end{vmatrix} =$$

$$= 1\cdot 1\cdot 1 + 0\cdot i\,(1-i) + (1+i)\,0\cdot(-i) -$$

$$- (1+i)\,1\cdot(1-i) - 0\cdot 0\cdot 1 - 1\cdot i\cdot(-i) =$$

$$= 1 - (1+i)(1-i) + i^2 = 1 - |1+i|^2 + i^2 =$$

$$= 1 - 2 - 1 = -2$$

$$\begin{vmatrix} x & a+ib & c+id \\ a-ib & y & e+if \\ c-id & e-if & z \end{vmatrix} =$$

$= xyz + (a+ib)(e+if)(c-id) + (c+id)(a-ib)(e-if) -$

$- (c+id)\,y\,(c-id) - (a+ib)(a-ib)z - x(e+if)(e-if) =$

$= xyz + (a+ib)(c-id)(e+if) + (a-ib)(c+id)(e-if) -$

$- x(e+if)(e-if) - y(c+id)(c-id) - z(a+ib)(a-ib) = \{$

$(a+ib)(c-id)(e+if) = \{$

$(a+ib)(c-id) = ac + a(-id) + (ib)c + (ib)(-id) =$

$= ac - iad + ibc - i^2bd = ac + bd + i(-ad+bc)$

$\} = (ac+bd)e + (ac+bd)(if) + i(-ad+bc)e + i(-ad+bc)(if) =$

$= (ac+bd)e + i(ac+bd)f + i(-ad+bc)e + i^2(-ad+bc)f =$

$= (ac+bd)e - (-ad+bc)f + i\big((ac+bd)f + (-ad+bc)e\big) =$

$= ace + bde + adf - bcf + i(acf + bdf - ade + bce)$

$(a-ib)(c+id)(e-if) = \{$

$(a-ib)(c+id) = ac + a(id) + (-ib)c + (-ib)(id) =$

$= ac + iad - ibc - i^2bd = ac + bd + i(ad-bc)$

$\} = (ac+bd)e + (ac+bd)(-if) + i(ad-bc)e + i(ad-bc)(-if) =$

$$= (ac+bd)e - i(ac+bd)f + i(ad-bc)e - i^2(ad-bc)f =$$

$$= (ac+bd)e + (ad-bc)f + i\big(-(ac+bd)f + (ad-bc)e\big) =$$

$$= ace + bde + adf - bcf - i\big(acf + bdf - ade + bce\big)$$

$$(a+ib)(e+if)(c-id) + (c+id)(a-ib)(e-if) =$$

$$= ace + bde + adf - bcf + \cancel{i(acf + bdf - ade + bce)} +$$

$$+ ace + bde + adf - bcf \cancel{- i(acf + bdf - ade + bce)} =$$

$$= 2(ace + bde + adf - bcf)$$

$$(e+if)(e-if) = ee + e(-if) + (if)e + (if)(-if) =$$

$$= e^2 \cancel{- ief} \cancel{+ ief} - i^2 f^2 = e^2 + f^2$$

$$(c+id)(c-id) = (c+id)\overline{(c+id)} = |c+id|^2 = c^2+d^2$$

$$(a+ib)(a-ib) = (a+ib)\overline{(a+ib)} = |a+ib|^2 = a^2+b^2$$

$$\Big\} = xyz + 2(ace + bde + adf - bcf) -$$

$$- x(e^2+f^2) - y(c^2+d^2) - z(a^2+b^2)$$

Дано: $\varepsilon = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

Найти:

$$d(\varepsilon) = \begin{vmatrix} 1 & \varepsilon & \varepsilon^2 \\ \varepsilon^2 & 1 & \varepsilon \\ \varepsilon & \varepsilon^2 & 1 \end{vmatrix}$$

Решение:

$$d(\varepsilon) = \begin{vmatrix} 1 & \varepsilon & \varepsilon^2 \\ \varepsilon^2 & 1 & \varepsilon \\ \varepsilon & \varepsilon^2 & 1 \end{vmatrix} =$$

$$= 1 \cdot 1 \cdot 1 + \varepsilon \cdot \varepsilon \cdot \varepsilon + \varepsilon^2 \varepsilon^2 \cdot \varepsilon^2 - \varepsilon^2 \cdot 1 \cdot \varepsilon - \varepsilon \cdot \varepsilon^2 \cdot 1 - 1 \cdot \varepsilon \cdot \varepsilon^2 =$$

$$= 1 + \varepsilon^3 + \varepsilon^6 - \varepsilon^3 - \varepsilon^3 - \varepsilon^3 = \varepsilon^6 - 2\varepsilon^3 + 1 = (\varepsilon^3 - 1)^2$$

$$\varepsilon = -\frac{1}{2} + i\frac{\sqrt{3}}{2} =$$

$$= -\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = -\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) =$$

$$= -e^{-i\frac{\pi}{3}} = e^{i\pi} e^{-i\frac{\pi}{3}} = e^{i\left(\pi - \frac{\pi}{3}\right)} = e^{i\frac{2}{3}\pi}$$

$$\varepsilon^3 = \left(e^{i\frac{2}{3}\pi}\right)^3 = e^{i3\frac{2}{3}\pi} = e^{i2\pi} = 1$$

$$\varepsilon^3 - 1 = 0$$

$$d = (\varepsilon^3 - 1)^2 = 0^2 = 0$$

Ответ:

$$\begin{vmatrix} 1 & \varepsilon & \varepsilon^2 \\ \varepsilon^2 & 1 & \varepsilon \\ \varepsilon & \varepsilon^2 & 1 \end{vmatrix} = (\varepsilon^3 - 1) = \left\{ \varepsilon = -\frac{1}{2} + i\frac{\sqrt{3}}{2} \right\} = 0$$

Дано:

$\varepsilon = \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi$

Найти:

$$\begin{vmatrix} 1 & 1 & \varepsilon \\ 1 & 1 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & 1 \end{vmatrix}$$

Решение:

$$\Delta(\varepsilon) = \begin{vmatrix} 1 & 1 & \varepsilon \\ 1 & 1 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & 1 \end{vmatrix} =$$

$$= 1\cdot1\cdot1 + 1\cdot\varepsilon^2\cdot\varepsilon^2 + \varepsilon\cdot1\cdot\varepsilon - \varepsilon\cdot1\cdot\varepsilon^2 - 1\cdot1\cdot1 - 1\cdot\varepsilon^2\cdot\varepsilon =$$

$$= 1 + \varepsilon^4 + \varepsilon^2 - \varepsilon^3 - 1 - \varepsilon^3 = \varepsilon^4 - 2\varepsilon^3 + \varepsilon^2 =$$

$$= \varepsilon^2(\varepsilon^2 - 2\varepsilon + 1) = \varepsilon^2(\varepsilon - 1)^2$$

$$\varepsilon = \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi = e^{i\frac{2}{3}\pi} =$$

$$= e^{i\left(\left(\frac{2}{3}\pi - \pi\right) + \pi\right)} = e^{i\left(-\frac{\pi}{3} + \pi\right)} = e^{-i\frac{\pi}{3}}e^{i\pi} = e^{-i\frac{\pi}{3}}(-1) = -e^{-i\frac{\pi}{3}} =$$

$$= -\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) = -\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right) = -\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\varepsilon^2 = \left(e^{i\frac{2}{3}\pi}\right)^2 = e^{i2\frac{2}{3}\pi} = e^{i\frac{4}{3}\pi} =$$

$$= e^{i\left(\frac{\pi}{3} + \pi\right)} = e^{i\frac{\pi}{3}}e^{i\pi} = e^{i\frac{\pi}{3}}(-1) = -e^{i\frac{\pi}{3}} =$$

$$= -\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = -\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

$$\varepsilon^3 = \left(e^{i\frac{2}{3}\pi}\right)^3 = e^{i3\frac{2}{3}\pi} = e^{i2\pi} = 1$$

$$\varepsilon^4 = \left(e^{i\frac{2}{3}\pi}\right)^4 = e^{i4\frac{2}{3}\pi} = e^{i\frac{8}{3}\pi} =$$

$$= e^{i\left(\frac{2}{3}\pi + 2\pi\right)} = e^{i\frac{2}{3}\pi}e^{i2\pi} = e^{i\frac{2}{3}\pi}\cdot1 = e^{i\frac{2}{3}\pi} = \varepsilon$$

$$\varepsilon = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\varepsilon - 1 = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) - 1 = -\frac{3}{2} + i\frac{\sqrt{3}}{2}$$

$$(\varepsilon-1)^2 = \left(-\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^2 =$$

$$= \left(-\frac{3}{2}\right)^2 + 2\left(-\frac{3}{2}\right)\left(i\frac{\sqrt{3}}{2}\right) + \left(i\frac{\sqrt{3}}{2}\right)^2 = \frac{9}{4} - i\frac{3\sqrt{3}}{2} + i^2\frac{3}{4} =$$

$$= \frac{9}{4} - i\frac{3\sqrt{3}}{2} - \frac{3}{4} = \frac{3}{2} - i\frac{3\sqrt{3}}{2}$$

$$d = \varepsilon^2(\varepsilon-1)^2 = -\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\left(\frac{3}{2} - i\frac{3\sqrt{3}}{2}\right) = \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\left(-\frac{3}{2} + i\frac{3\sqrt{3}}{2}\right) =$$

$$= \frac{1}{2}\left(-\frac{3}{2}\right) + \frac{1}{2}\left(i\frac{3\sqrt{3}}{2}\right) + \left(i\frac{\sqrt{3}}{2}\right)\left(-\frac{3}{2}\right) + \left(i\frac{\sqrt{3}}{2}\right)\left(i\frac{3\sqrt{3}}{2}\right) =$$

$$= -\frac{3}{4} + \cancel{i\frac{3\sqrt{3}}{4}} - \cancel{i\frac{3\sqrt{3}}{4}} + i^2\frac{9}{4} = -\frac{3}{4} - \frac{9}{4} = -3$$

$$d = \varepsilon^4 - 2\varepsilon^3 + \varepsilon^2 = \{\varepsilon^4 = \varepsilon\} = \varepsilon - 2\varepsilon^3 + \varepsilon^2 = \varepsilon + \varepsilon^2 - 2\varepsilon^3 =$$

$$= \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) - 2\cdot 1 =$$

$$= -\frac{1}{2} + \cancel{i\frac{\sqrt{3}}{2}} - \frac{1}{2} - \cancel{i\frac{\sqrt{3}}{2}} - 2 = -3$$

<u>Ответ:</u>

$$\begin{vmatrix} 1 & 1 & \varepsilon \\ 1 & 1 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & 1 \end{vmatrix} = \varepsilon^2(\varepsilon-1)^2 = \left\{\varepsilon = \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi\right\} = -3$$

Дано:

$$\varepsilon = \cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi$$

Найти:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & \varepsilon & \varepsilon^2 \\ 1 & \varepsilon^2 & \varepsilon \end{vmatrix}$$

Решение:

$$d(\varepsilon) := \begin{vmatrix} 1 & 1 & 1 \\ 1 & \varepsilon & \varepsilon^2 \\ 1 & \varepsilon^2 & \varepsilon \end{vmatrix} =$$

$$= 1\cdot\varepsilon\cdot\varepsilon + 1\cdot\varepsilon^2\cdot 1 + 1\cdot 1\cdot\varepsilon^2 - 1\cdot\varepsilon\cdot 1 - 1\cdot 1\cdot\varepsilon - 1\cdot\varepsilon^2\cdot\varepsilon^2 =$$

$$= \varepsilon^2 + \varepsilon^2 + \varepsilon^2 - \varepsilon - \varepsilon - \varepsilon^4 = 3\varepsilon^2 - 2\varepsilon - \varepsilon^4 =$$

$$= -\varepsilon^4 + 3\varepsilon^2 - 2\varepsilon$$

$$\varepsilon = \cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi = e^{i\frac{4}{3}\pi} =$$

$$= e^{i\left(\frac{\pi}{3}+\pi\right)} = e^{i\frac{\pi}{3}}e^{i\pi} = e^{i\frac{\pi}{3}}(-1) = -e^{i\frac{\pi}{3}} =$$

$$= -\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = -\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

$$\varepsilon^2 = \left(e^{i\frac{4}{3}\pi}\right)^2 = e^{i2\frac{4}{3}\pi} = e^{i\frac{8}{3}\pi} =$$

$$= e^{i\left(\frac{2}{3}\pi + 2\pi\right)} = e^{i\frac{2}{3}\pi}e^{i2\pi} = e^{i\frac{2}{3}\pi}\cdot 1 = e^{i\frac{2}{3}\pi} =$$

$$= e^{i\left(\left(\frac{2}{3}\pi - \pi\right)+\pi\right)} = e^{i\left(-\frac{\pi}{3}+\pi\right)} = e^{-i\frac{\pi}{3}}e^{i\pi} = e^{-i\frac{\pi}{3}}(-1) = -e^{-i\frac{\pi}{3}} =$$

$$= -\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) = -\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right) = -\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\varepsilon^4 = \left(e^{i\frac{4}{3}\pi}\right)^4 = e^{i4\frac{4}{3}\pi} = e^{i\frac{16}{3}\pi} =$$

$$= e^{i\left(\frac{4}{3}\pi + 4\pi\right)} = e^{i\frac{4}{3}\pi}e^{i4\pi} = e^{i\frac{4}{3}\pi}\cdot 1 = e^{i\frac{4}{3}\pi} = \varepsilon$$

$$d = -\varepsilon^4 + 3\varepsilon^2 - 2\varepsilon =$$

$$= \left\{ \varepsilon^4 = \varepsilon \right\} = -\varepsilon + 3\varepsilon^2 - 2\varepsilon = 3\varepsilon^2 - 3\varepsilon = 3(\varepsilon^2 - \varepsilon) =$$

$$= 3\left(\left(\cancel{-\frac{1}{2}} + i\frac{\sqrt{3}}{2}\right) + \left(\cancel{\frac{1}{2}} + i\frac{\sqrt{3}}{2}\right)\right) = 3\left(2i\frac{\sqrt{3}}{2}\right) = 3(i\sqrt{3}) = i3\sqrt{3}$$

<u>Ответ:</u>

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & \varepsilon & \varepsilon^2 \\ 1 & \varepsilon^2 & \varepsilon \end{vmatrix} = -\varepsilon^4 + 3\varepsilon^2 - 2\varepsilon = \left\{ \varepsilon = \cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi \right\} = i3\sqrt{3}$$