$$\begin{cases} 2x - 3y + z = 2 \\ 3x - 5y + 5z = 3 \\ 5x - 8y + 6z = 5 \end{cases} ; \begin{cases} x = \\ y = \\ z = \end{cases}$$

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$$\begin{cases} 2x - 3y + z = 2 \\ 3x - 5y + 5z = 3 \\ 5x - 8y + 6z = 5 \end{cases} ; \begin{pmatrix} 2 & -3 & 1 \\ 3 & -5 & 5 \\ 5 & -8 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} ; \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\Delta = \det \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \det \begin{pmatrix} 2 & -3 & 1 \\ 3 & -5 & 5 \\ 5 & -8 & 6 \end{pmatrix} = 2(-5)6 + (-3)5\,5 + 1\,3\cdot(-8) - 1(-5)5 - (-3)3\cdot 6 - 2\cdot 5\cdot(-8) =$$
$$= -60 - 75 - 24 + 25 + 54 + 80 = 159 - 159 = 0$$

$$\begin{cases} 2x - 3y + z = 2 \\ 3x - 5y + 5z = 3 \end{cases} ; \begin{cases} 2x - 3y = -z + 2 \\ 3x - 5y = -5z + 3 \end{cases} ; \begin{pmatrix} 2 & -3 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -z+2 \\ -5z+3 \end{pmatrix} ; \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\Delta = \det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \det \begin{pmatrix} 2 & -3 \\ 3 & -5 \end{pmatrix} = 2(-5) - (-3)3 = -10 + 9 = -1$$

$$\Delta_1 = \det \begin{pmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{pmatrix} = \det \begin{pmatrix} -z+2 & -3 \\ -5z+3 & -5 \end{pmatrix} = (-z+2)(-5) - (-3)(-5z+3) =$$
$$= (5z - 10) + (-15z + 9) = -10z - 1$$

$$\Delta_2 = \det \begin{pmatrix} a_{11} & b_1 \\ a_{12} & b_2 \end{pmatrix} = \det \begin{pmatrix} 2 & -z+2 \\ 3 & -5z+3 \end{pmatrix} = 2(-5z+3) - (-z+2)3 =$$
$$= (-10z + 6) + (3z - 6) = -7z$$

$$\begin{cases} x = x_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{-10z-1}{-1} = 10z + 1 \\ y = x_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{-7z}{-1} = 7z \end{cases} ; \begin{cases} 2x - 3y + z = 2(10z+1) - 3(7z) + z = (20z+2) - 21z + z = 2 \\ 3x - 5y + 5z = 3(10z+1) - 5(7z) + 5z = (30z+3) - 35z + 5z = 3 \\ 5x - 8y + 6z = 5(10z+1) - 8(7z) + 6z = (50z+5) - 56z + 6z = 5 \end{cases}$$

$\triangleleft$

undefined ; $\forall z \quad x = 10z + 1 \land y = 7z$

$$\begin{cases} 4x + 3y + 2z = 1 \\ x + 3y + 5z = 1 \\ 3x + 6y + 9z = 2 \end{cases} ; \begin{cases} x = \\ y = \\ z = \end{cases}$$

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$$\begin{cases} 4x + 3y + 2z = 1 \\ x + 3y + 5z = 1 \\ 3x + 6y + 9z = 2 \end{cases} ; \begin{pmatrix} 4 & 3 & 2 \\ 1 & 3 & 5 \\ 3 & 6 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} ; \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\Delta = \det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \det\begin{pmatrix} 4 & 3 & 2 \\ 1 & 3 & 5 \\ 3 & 6 & 9 \end{pmatrix} = 4\cdot3\cdot9 + 3\cdot5\cdot3 + 2\cdot1\cdot6 - 2\cdot3\cdot3 - 3\cdot1\cdot9 - 4\cdot5\cdot6 =$$
$$= 108 + 45 + 12 - 18 - 27 - 120 =$$
$$= 165 - 165 = 0$$

$$\begin{cases} 4x + 3y + 2z = 1 \\ x + 3y + 5z = 1 \end{cases} ; \begin{cases} 4x + 3y = -2z + 1 \\ x + 3y = -5z + 1 \end{cases} ; \begin{pmatrix} 4 & 3 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2z+1 \\ -5z+1 \end{pmatrix} ; \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\Delta = \det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \det\begin{pmatrix} 4 & 3 \\ 1 & 3 \end{pmatrix} = 4\cdot3 - 3\cdot1 = 12 - 3 = 9$$

$$\Delta_1 = \det\begin{pmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{pmatrix} = \det\begin{pmatrix} -2z+1 & 3 \\ -5z+1 & 3 \end{pmatrix} = (-2z+1)3 - 3(-5z+1) =$$
$$= (-6z+3) + (15z-3) = 9z$$

$$\Delta_2 = \det\begin{pmatrix} a_{11} & b_1 \\ a_{12} & b_2 \end{pmatrix} = \det\begin{pmatrix} 4 & -2z+1 \\ 1 & -5z+1 \end{pmatrix} = 4(-5z+1) - 1(-2z+1) =$$
$$= (-20z+4) + (2z-1) = -18z + 3$$

$$\begin{cases} x = x_1 = \frac{\Delta_1}{\Delta} = \frac{9z}{9} = z \\ y = x_2 = \frac{\Delta_2}{\Delta} = \frac{-18z+3}{9} = -2z + \frac{1}{3} \end{cases} ; \begin{cases} 4x + 3y + 2z = 4z + 3(-2z + \frac{1}{3}) + 2z = 4z + (-6z+1) + 2z = 1 \\ x + 3y + 5z = z + 3(-2z + \frac{1}{3}) + 5z = z + (-6z+1) + 5z = 1 \\ 3x + 6y + 9z = 3z + 6(-2z + \frac{1}{3}) + 9z = 3z + (-12z+2) + 9z = 2 \end{cases}$$

$\triangleleft$

undefined; $\forall z \; x = z \wedge y = -2z + \frac{1}{3}$

-84-

$$\begin{cases} 5x - 6y + z = 4 \\ 3x - 5y - 2z = 3 \\ 2x - y + 3z = 5 \end{cases} ; \quad \begin{cases} x = \\ y = \\ z = \end{cases}$$

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$$\begin{cases} 5x - 6y + z = 4 \\ 3x - 5y - 2z = 3 \\ 2x - y + 3z = 5 \end{cases} ; \quad \begin{pmatrix} 5 & -6 & 1 \\ 3 & -5 & -2 \\ 2 & -1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} ; \quad \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\Delta = \det \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \det \begin{pmatrix} 5 & -6 & 1 \\ 3 & -5 & -2 \\ 2 & -1 & 3 \end{pmatrix} = 5(-5)3 + (-6)(-2)2 + 1\cdot 3(-1) -$$
$$- 1(-5)2 - (-6)3\cdot 3 - 5(-2)(-1) =$$
$$= -75 + 24 - 3 + \cancel{10} + 54 - \cancel{10} =$$
$$= 78 - 78 = 0$$

$$\begin{cases} 5x - 6y + z = 4 \\ 3x - 5y - 2z = 3 \end{cases} ; \quad \begin{cases} 5x - 6y = -z + 4 \\ 3x - 5y = 2z + 3 \end{cases} ; \quad \begin{pmatrix} 5 & -6 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -z+4 \\ 2z+3 \end{pmatrix} ; \quad \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\Delta = \det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \det \begin{pmatrix} 5 & -6 \\ 3 & -5 \end{pmatrix} = 5(-5) - (-6)3 = -25 + 18 = -7$$

$$\Delta_1 = \det \begin{pmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{pmatrix} = \det \begin{pmatrix} -z+4 & -6 \\ 2z+3 & -5 \end{pmatrix} = (-z+4)(-5) - (-6)(2z+3) =$$
$$= (5z - 20) + (12z + 18) = 17z - 2$$

$$\Delta_2 = \det \begin{pmatrix} a_{11} & b_1 \\ a_{12} & b_2 \end{pmatrix} = \det \begin{pmatrix} 5 & -z+4 \\ 3 & 2z+3 \end{pmatrix} = 5(2z+3) - (-z+4)3 =$$
$$= (10z + 15) + (3z - 12) = 13z + 3$$

$$\begin{cases} x = x_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{17z-2}{-7} = \dfrac{-17z+2}{7} \\ y = x_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{13z+3}{-7} = \dfrac{-13z-3}{7} \end{cases}$$

$$\begin{cases} 5x - 6y + z = 5\,\dfrac{-17z+2}{7} - 6\,\dfrac{-13z-3}{7} + z = \dfrac{(-85z+10) + (78z+18) + 7z}{7} = \dfrac{28}{7} = 4 \\ 3x - 5y - 2z = 3\,\dfrac{-17z+2}{7} - 5\,\dfrac{-13z-3}{7} - 2z = \dfrac{(-51z+6) + (75z+15) + 14z}{7} = \dfrac{21}{7} = 3 \\ 2x - y + 3z = 2\,\dfrac{-17z+2}{7} - \dfrac{-13z-3}{7} + 3z = \dfrac{(-34z+4) + (13z+3) + 21z}{7} = \dfrac{7}{7} = 1 \neq 5 \end{cases}$$

Δ inconsistent

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$$\begin{cases} 2x - y + 3z = 4 \\ 3x - 2y + 2z = 3 \\ 5x - 4y = 2 \end{cases} ; \quad \begin{cases} x = \\ y = \\ z = \end{cases}$$

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$$\begin{cases} 2x - y + 3z = 4 \\ 3x - 2y + 2z = 3 \\ 5x - 4y = 2 \end{cases} ; \begin{pmatrix} 2 & -1 & 3 \\ 3 & -2 & 2 \\ 5 & -4 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} ; \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\Delta = \det \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \det \begin{pmatrix} 2 & -1 & 3 \\ 3 & -2 & 2 \\ 5 & -4 & 0 \end{pmatrix} = 2(-2)0 + (-1)2 \cdot 5 + 3 \cdot 3(-4) - $$
$$- 3(-2)5 - (-1)3 \cdot 0 - 2 \cdot 2(-4) =$$
$$= 0 - 10 - 36 + 30 + 0 + 16 = 46 - 46 = 0$$

$$\begin{cases} 2x - y + 3z = 4 \\ 3x - 2y + 2z = 3 \end{cases} ; \begin{cases} 2x - y = -3z + 4 \\ 3x - 2y = -2z + 3 \end{cases} ; \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3z + 4 \\ -2z + 3 \end{pmatrix} ; \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\Delta = \det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \det \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} = 2(-2) - (-1)3 = -4 + 3 = -1$$

$$\Delta_1 = \det \begin{pmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{pmatrix} = \det \begin{pmatrix} -3z + 4 & -1 \\ -2z + 3 & -2 \end{pmatrix} = (-3z+4)(-2) - (-1)(-2z+3) =$$
$$= (6z - 8) + (-2z + 3) = 4z - 5$$

$$\Delta_2 = \det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \det \begin{pmatrix} 2 & -3z + 4 \\ 3 & -2z + 3 \end{pmatrix} = 2(-2z+3) - (-3z+4)3 =$$
$$= (-4z + 6) + (9z - 12) = 5z - 6$$

$$\begin{cases} x = x_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{4z - 5}{-1} = -4z + 5 \\ y = x_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{5z - 6}{-1} = -5z + 6 \end{cases} ;$$

$$2x - y + 3z = 2(-4z+5) - (-5z+6) + 3z = (-8z + 10) + (5z - 6) + 3z = 4$$

$$3x - 2y + 2z = 3(-4z+5) - 2(-5z+6) + 2z = (-12z + 15) + (10z - 12) + 2z = 3$$

$$5x - 4y = 5(-4z+5) - 4(-5z+6) = (-20z + 25) + (20z - 24) = 1 \neq 2$$

Δ inconsistent