$2 \quad A \in \mathbb{R}^{3\times3} \Rightarrow |A| = |A^T|$

$\triangleleft \; A = (a_{ij}) \in \mathbb{R}^{3\times3}$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$\begin{array}{lccccccccccccc}
 & = & a_{11} & a_{22} & a_{33} & + & a_{12} & a_{23} & a_{31} & + & a_{13} & a_{21} & a_{32} & - \\
i & & 1 & 2 & 3 & & 1 & 2 & 3 & & 1 & 2 & 3 & \\
j & & 1 & & & & 1 & & & & 1 & & &
\end{array}$$

$$\begin{array}{lccccccccccccc}
 & - & a_{13} & a_{22} & a_{31} & - & a_{12} & a_{21} & a_{33} & - & a_{11} & a_{23} & a_{32} & = \\
i & & 1 & 2 & 3 & & 1 & 2 & 3 & & 1 & 2 & 3 & \\
j & & 3 & & & & 2 & & & & 1 & & &
\end{array}$$

$$\begin{array}{lccccccccccccc}
 & = & a_{11} & a_{22} & a_{33} & + & a_{31} & a_{12} & a_{23} & + & a_{21} & a_{32} & a_{13} & - \\
i & & 1 & & & & 3 & & & & 2 & & & \\
j & & 1 & 2 & 3 & & 1 & 2 & 3 & & 1 & 2 & 3 &
\end{array}$$

$$\begin{array}{lccccccccccccc}
 & - & a_{31} & a_{22} & a_{13} & - & a_{21} & a_{12} & a_{33} & - & a_{11} & a_{32} & a_{23} & = \\
i & & 3 & & & & 2 & & & & 1 & & & \\
j & & 1 & 2 & 3 & & 1 & 2 & 3 & & 1 & 2 & 3 &
\end{array}$$

$$\begin{array}{lccccccccccccc}
 & = & a_{11} & a_{22} & a_{33} & + & a_{21} & a_{32} & a_{13} & + & a_{31} & a_{12} & a_{23} & - \\
i & & 1 & & & & 2 & & & & 3 & & & \\
j & & 1 & 2 & 3 & & 1 & 2 & 3 & & 1 & 2 & 3 &
\end{array}$$

$$\begin{array}{lccccccccccccc}
 & - & a_{31} & a_{22} & a_{13} & - & a_{21} & a_{12} & a_{33} & - & a_{11} & a_{32} & a_{23} & = \\
i & & 3 & & & & 2 & & & & 1 & & & \\
j & & 1 & 2 & 3 & & 1 & 2 & 3 & & 1 & 2 & 3 &
\end{array}$$

1

$$= \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}^t & a_{12}^t & a_{13}^t \\ a_{21}^t & a_{22}^t & a_{23}^t \\ a_{31}^t & a_{32}^t & a_{33}^t \end{vmatrix} = |A^t| \triangleright$$

?

$A=(a_{ij})\in\mathbb{R}^{3\times3} \wedge (i\in\{1,2,3\}\ \forall j\in\{1,2,3\}\ a_{ij}=0) \Rightarrow$

$|A|=0$

$\triangle A=(a_{ij})\in\mathbb{R}^{3\times3}$

$$|A|=\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}=$$

$$\begin{array}{ccccccccccccc} = & a_{11} & a_{22} & a_{33} & + & a_{12} & a_{23} & a_{31} & + & a_{13} & a_{21} & a_{32} & -\\ i & 1 & 2 & 3 & & 1 & 2 & 3 & & 1 & 2 & 3 & \\ - & a_{13} & a_{22} & a_{31} & - & a_{12} & a_{21} & a_{33} & - & a_{11} & a_{23} & a_{32} & =\\ i & 1 & 2 & 3 & & 1 & 2 & 3 & & 1 & 2 & 3 & \end{array}$$

$i\in\{1,2,3\}\ \forall j\in\{1,2,3\}\ a_{ij}=0$

$= 0+0+0-0-0-0=0$ ▷

?

$A = (a_{ij}) \in \mathbb{R}^{3\times3} \wedge \left(\forall i \in \{1,2,3\}\ j \in \{1,2,3\}\ a_{ij} = 0 \right) \Rightarrow$

$|A| = 0$

$\triangleleft\ A = (a_{ij}) \in \mathbb{R}^{3\times3}$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$\begin{array}{cccccccccccccc}
= & a_{11} & a_{22} & a_{33} & + & a_{12} & a_{23} & a_{31} & + & a_{13} & a_{21} & a_{32} & - & \\
i & 1 & 2 & 3 & & 1 & 2 & 3 & & 1 & 2 & 3 & & \\
- & a_{13} & a_{22} & a_{31} & - & a_{12} & a_{21} & a_{33} & - & a_{11} & a_{23} & a_{32} & = & \\
i & 1 & 2 & 3 & & 1 & 2 & 3 & & 1 & 2 & 3 & &
\end{array}$$

$$\begin{array}{cccccccccccccc}
= & a_{11} & a_{22} & a_{33} & + & a_{31} & a_{12} & a_{23} & + & a_{21} & a_{32} & a_{13} & - & \\
j & 1 & 2 & 3 & & 1 & 2 & 3 & & 1 & 2 & 3 & & \\
- & a_{31} & a_{22} & a_{13} & - & a_{21} & a_{12} & a_{33} & - & a_{11} & a_{32} & a_{23} & = & \Big\{ \\
j & 1 & 2 & 3 & & 1 & 2 & 3 & & 1 & 2 & 3 & &
\end{array}$$

$\forall i \in \{1,2,3\}\ j \in \{1,2,3\}\ a_{ij} = 0$

$\Big\} = 0 + 0 + 0 - 0 - 0 - 0 = 0\ \triangleright$