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$$\alpha = (1, 9, 6, 3, 2, 5, 4, 7, 8)$$

$$s(\alpha) =$$

▷

$$k = |\alpha| = 9$$

$$\forall i \in (1, 2, \dots, k-1) \quad A_i = \{ \alpha_j \mid i < j \wedge \alpha_i > \alpha_j \}, \quad s_i = |A_i|$$

$$s = \sum_{i=1}^{k-1} s_i$$

$i$	$\alpha_i$	$s_i$	$A_i$
1	1	0	$\emptyset$
2	9	7	$\{6, 3, 2, 5, 4, 7, 8\}$
3	6	4	$\{3, 2, 5, 4\}$
4	3	1	$\{2\}$
5	2	0	$\emptyset$
6	5	1	$\{4\}$
7	4	0	$\emptyset$
8	7	0	$\emptyset$
$s$		13	

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$$s(\alpha) = 13$$

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$$A = (7, 5, 6, 4, 1, 3, 2)$$

$$Z(A) =$$

D

$$K = |A| = 7$$

$$\forall i = (1, 2, \dots, k-1) \quad A_i = \{A_j \mid i < j \text{ and } A_j > A_i\} \quad \delta_i = |A_i|$$

$$Z = \sum_{i=1}^{k-1} \delta_i$$

$i$	$A_i$	$\delta_i$	$A_i$
1	7	6	$\{5, 6, 4, 1, 3, 2\}$
2	5	4	$\{4, 1, 3, 2\}$
3	6	4	$\{4, 1, 3, 2\}$
4	4	3	$\{1, 3, 2\}$
5	1	0	$\emptyset$
6	3	1	$\{2\}$
$\Sigma$		18	

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$$Z(A) = 18$$

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$$d = (1, 3, 5, 7, \dots, 2n-1, 2, 4, 6, 8, \dots, 2n)$$

$$z(d) =$$

▷

$$k = |d| = 2n$$

$$\forall i \in (1, 2, \dots, k-1) \quad A_i = \{d_j \mid i < j \wedge d_i > d_j\} \quad z_i = |A_i|$$

$$z = \sum_{i=1}^{k-1} z_i$$

i	$d_i$	$z_i$	$A_i$
1	1	0	$\emptyset$
2	3	1	$\{2\}$
3	5	2	$\{2, 4\}$
4	7	3	$\{2, 4, 6\}$

$$i \quad 2i-1 \quad i-1 \quad \{2, 4, 6, \dots, 2i-2\} \longleftrightarrow \left\{ \frac{2}{2}, \frac{4}{2}, \frac{6}{2}, \dots, \frac{2i-2}{2} \right\} = \{1, 2, 3, \dots, i-1\}$$

$$n \quad 2n-1 \quad n-1 \quad \{2, 4, 6, \dots, 2n-2\}$$

$$n+1 \quad 2 \quad 0 \quad \emptyset$$

$$n+2 \quad 4 \quad 0 \quad \emptyset$$

$$n+3 \quad 6 \quad 0 \quad \emptyset$$

$$n+4 \quad 8 \quad 0 \quad \emptyset$$

$$n+j \quad 2j \quad 0 \quad \emptyset$$

$$2n-1 \quad 2n-2 \quad 0 \quad \emptyset$$

$$z = \sum_{i=1}^{k-1} z_i = \sum_{i=1}^{2n-1} z_i = \sum_{i=1}^n z_i + \sum_{i=n+1}^{2n-1} z_i = \left\{ \sum_{i=1}^{2n-1} z_i = \sum_{i=n+1}^{2n-1} 0 = 0 \right\} =$$

$$= \sum_{i=1}^n z_i = \{ \forall i \in (2, 3, \dots, n) \quad z_i = z_{i-1} + d \quad d=1 \} =$$

$$= \frac{a_1 + a_n}{2} \cdot n = \{ a_1 = 0; a_n = a_1 + (n-1)d = n-1 \} = \frac{0 + (n-1)}{2} \cdot n = \frac{n(n-1)}{2}$$

$$\triangle z(d) = \frac{n(n-1)}{2}$$

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$$d = (2, 4, 6, \dots, 2n, 1, 3, 5, \dots, 2n-1)$$

$$\delta(d) =$$

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$$k = |d| = 2n$$

$$\forall i \in (1, 2, \dots, k-1) \quad A_i = \{a_i \mid i < j \text{ and } d_j > d_i\} \quad \delta_i = |A_i|$$

$$\delta = \sum_{i=1}^{k-1} \delta_i$$

i	$d_i$	$\delta_i$	$A_i$
1	2	1	$\{1\}$
2	4	2	$\{1, 3\}$
3	6	3	$\{1, 3, 5\}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	$2i$	i	$\{1, 3, 5, \dots, 2i-1\} \Leftrightarrow$ $\Leftrightarrow \{1+1, 3+1, 5+1, \dots, (2i-1)+1\} = \{2, 4, 6, \dots, 2i\} \Leftrightarrow$ $\Leftrightarrow \{\frac{2}{2}, \frac{4}{2}, \frac{6}{2}, \dots, \frac{2i}{2}\} = \{1, 2, 3, \dots, i\}$
n	2n	n	$\{1, 3, 5, \dots, 2n-1\}$
n+1	1	0	$\emptyset$
n+2	3	0	$\emptyset$
n+3	5	0	$\emptyset$
n+j	$2j-1$	0	$\emptyset$
2n-1	2n-3	0	$\emptyset$

$$\delta = \sum_{i=1}^{k-1} \delta_i = \sum_{i=1}^{2n-1} \delta_i = \sum_{i=1}^n \delta_i + \sum_{i=n+1}^{2n-1} \delta_i = \left\{ \sum_{i=1}^{2n-1} \delta_i = \sum_{i=n+1}^{2n-1} 0 = 0 \right\} =$$

$$= \sum_{i=1}^n \delta_i = \{ \forall i \in (2, 3, \dots, n) \quad \delta_i = \delta_{i-1} + d \quad d=1 \} =$$

$$= \frac{\delta_1 + \delta_n}{2} n = \{ \delta_1 = 1; \delta_n = \delta_1 + (n-1)d = 1 + (n-1) = n \} =$$

$$= \frac{1+n}{2} n = \frac{n(n+1)}{2}$$

▷

$$\delta(d) = \frac{n(n+1)}{2}$$

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