$\boxed{1.1}$

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$

$$e^{-i\alpha} = \cos(-\alpha) + i\sin(-\alpha) = \cos\alpha + i(-\sin\alpha) = \cos\alpha - i\sin\alpha$$

$\boxed{1.2}$

$$\frac{e^{i\alpha} - e^{-i\alpha}}{2i} = \frac{1}{2i}\left[(\cos\alpha + i\sin\alpha) - (\cos\alpha - i\sin\alpha)\right] =$$

$$= \frac{2i\sin\alpha}{2i} = \sin\alpha$$

$$\frac{e^{i\alpha} + e^{-i\alpha}}{2} = \frac{1}{2}\left[(\cos\alpha + i\sin\alpha) + (\cos\alpha - i\sin\alpha)\right] =$$

$$= \frac{2\cos\alpha}{2} = \cos\alpha$$

$\boxed{2}$

$$\sin^2\alpha + \cos^2\alpha =$$

$$\sin^2\alpha = \left(\frac{e^{i\alpha} - e^{-i\alpha}}{2i}\right)^2 = \frac{(e^{i\alpha} - e^{-i\alpha})^2}{(2i)^2} = \frac{1}{4i^2}\left[(e^{i\alpha})^2 - 2e^{i\alpha}e^{-i\alpha} + (e^{-i\alpha})^2\right]$$

$$= -\frac{1}{4}\left(e^{i2\alpha} - 2 + e^{-i2\alpha}\right) = -\frac{1}{4}\left(e^{i2\alpha} + e^{-i2\alpha}\right) + \frac{1}{2}$$

$$\cos^2\alpha = \left(\frac{e^{i\alpha} + e^{-i\alpha}}{2}\right)^2 = \frac{(e^{i\alpha} + e^{-i\alpha})^2}{2^2} = \frac{1}{4}\left[(e^{i\alpha})^2 + 2e^{i\alpha}e^{-i\alpha} + (e^{-i\alpha})^2\right] =$$

$$= \frac{1}{4}\left(e^{i2\alpha} + 2 + e^{-i2\alpha}\right) = \frac{1}{4}\left(e^{i2\alpha} + e^{-i2\alpha}\right) + \frac{1}{2}$$

$$\left(= \left[-\frac{1}{4}\left(e^{i2\alpha} + e^{-i2\alpha}\right) + \frac{1}{2}\right] + \left[\frac{1}{4}\left(e^{i2\alpha} + e^{-i2\alpha}\right) + \frac{1}{2}\right] =\right.$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$\boxed{3.1}$

$$\sin\alpha \cos\beta = \frac{e^{i\alpha}-e^{-i\alpha}}{2i}\,\frac{e^{i\beta}+e^{-i\beta}}{2} =$$

$$= \frac{1}{4i}\left(e^{i\alpha}e^{i\beta}+e^{i\alpha}e^{-i\beta}-e^{-i\alpha}e^{i\beta}-e^{-i\alpha}e^{-i\beta}\right) =$$

$$= \frac{1}{4i}\left[e^{i(\alpha+\beta)}+e^{i(\alpha-\beta)}-e^{-i(\alpha-\beta)}-e^{-i(\alpha+\beta)}\right] =$$

$$= \frac{1}{2}\left[\frac{e^{i(\alpha+\beta)}-e^{-i(\alpha+\beta)}}{2i}+\frac{e^{i(\alpha-\beta)}-e^{-i(\alpha-\beta)}}{2i}\right] =$$

$$= \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

$\boxed{3.2}$

$\beta=\alpha$

$$\sin\alpha\cos\alpha = \frac{1}{2}\left[\sin(\alpha+\alpha)+\sin(\alpha-\alpha)\right] =$$

$$= \frac{1}{2}\left(\sin 2\alpha+\sin 0\right) =$$

$$= \frac{\sin 2\alpha}{2}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$\boxed{3.3}$

$$\begin{cases}\alpha+\beta=\alpha^*\\ \alpha-\beta=\beta^*\end{cases};\quad \begin{cases}(\alpha+\beta)+(\alpha-\beta)=\alpha^*+\beta^*\\ (\alpha+\beta)-(\alpha-\beta)=\alpha^*-\beta^*\end{cases};\quad \begin{cases}2\alpha=\alpha^*+\beta^*\\ 2\beta=\alpha^*-\beta^*\end{cases}$$

$$\begin{cases}\alpha=\dfrac{\alpha^*+\beta^*}{2}\\ \beta=\dfrac{\alpha^*-\beta^*}{2}\end{cases}$$

$$\sin\left(\frac{\alpha^*+\beta^*}{2}\right)\cos\left(\frac{\alpha^*-\beta^*}{2}\right) = \frac{1}{2}\left(\sin\alpha^*+\sin\beta^*\right)$$

$$\sin\alpha^*+\sin\beta^* = 2\sin\left(\frac{\alpha^*+\beta^*}{2}\right)\cos\left(\frac{\alpha^*-\beta^*}{2}\right)$$

$\alpha^*\to\alpha;\ \beta^*\to\beta$

$$\sin\alpha+\sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$\boxed{4.1}$

$$\sin\alpha \sin\beta = \frac{e^{i\alpha}-e^{-i\alpha}}{2i}\,\frac{e^{i\beta}-e^{-i\beta}}{2i} =$$

$$= \frac{1}{(2i)^2}\left(e^{i\alpha}e^{i\beta}-e^{i\alpha}e^{-i\beta}-e^{-i\alpha}e^{i\beta}+e^{-i\alpha}e^{-i\beta}\right) =$$

$$= \frac{1}{4i^2}\left[e^{i(\alpha+\beta)}-e^{i(\alpha-\beta)}-e^{-i(\alpha-\beta)}+e^{-i(\alpha+\beta)}\right] =$$

$$= -\frac{1}{2}\left[\frac{e^{i(\alpha+\beta)}+e^{-i(\alpha+\beta)}}{2}-\frac{e^{i(\alpha-\beta)}-e^{-i(\alpha-\beta)}}{2}\right] =$$

$$= -\frac{1}{2}\left[\cos(\alpha+\beta)-\cos(\alpha-\beta)\right] = \frac{1}{2}\left[-\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

$\boxed{4.2}$

$\beta=\alpha$

$$\sin\alpha\sin\alpha = \frac{1}{2}\left[-\cos(\alpha+\alpha)+\cos(\alpha-\alpha)\right] =$$

$$\sin^2\alpha = \frac{1}{2}\left(-\cos 2\alpha+\cos 0\right) =$$

$$= \frac{1}{2}\left(-\cos 2\alpha+1\right)$$

$$\cos 2\alpha = 1-2\sin^2\alpha = (1-\sin^2\alpha)-\sin^2\alpha = \cos^2\alpha-\sin^2\alpha$$

$\boxed{4.3}$

$$\begin{cases}\alpha+\beta=\alpha^* \\ \alpha-\beta=\beta^*\end{cases} ; \quad \begin{cases}\alpha=\dfrac{\alpha^*+\beta^*}{2} \\ \beta=\dfrac{\alpha^*-\beta^*}{2}\end{cases}$$

$$\sin\left(\frac{\alpha^*+\beta^*}{2}\right)\sin\left(\frac{\alpha^*-\beta^*}{2}\right) = \frac{1}{2}\left(-\cos\alpha^*+\cos\beta^*\right)$$

$$= -\frac{1}{2}\left(\cos\alpha^*-\cos\beta^*\right)$$

$$\cos\alpha^*-\cos\beta^* = -2\sin\left(\frac{\alpha^*+\beta^*}{2}\right)\sin\left(\frac{\alpha^*-\beta^*}{2}\right)$$

$\alpha^*\to\alpha$; $\beta^*\to\beta$

$$\cos\alpha-\cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

5.1

$$\cos\alpha\cos\beta = \frac{e^{i\alpha}+e^{-i\alpha}}{2}\,\frac{e^{i\beta}+e^{-i\beta}}{2} =$$

$$= \frac{1}{4}\left(e^{i\alpha}e^{i\beta}+e^{i\alpha}e^{-i\beta}+e^{-i\alpha}e^{i\beta}+e^{-i\alpha}e^{-i\beta}\right) =$$

$$= \frac{1}{4}\left[e^{i(\alpha+\beta)}+e^{i(\alpha-\beta)}+e^{-i(\alpha-\beta)}+e^{-i(\alpha+\beta)}\right] =$$

$$= \frac{1}{2}\left[\frac{e^{i(\alpha+\beta)}+e^{-i(\alpha+\beta)}}{2}+\frac{e^{i(\alpha-\beta)}+e^{-i(\alpha-\beta)}}{2}\right] =$$

$$= \frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

5.2

$\beta = \alpha$

$$\cos\alpha\cos\alpha = \frac{1}{2}\left[\cos(\alpha+\alpha)+\cos(\alpha-\alpha)\right]$$

$$\cos^2\alpha = \frac{1}{2}\left(\cos 2\alpha+\cos 0\right) =$$

$$= \frac{1}{2}\left(\cos 2\alpha + 1\right)$$

$$\cos 2\alpha = 2\cos^2\alpha - 1 = \cos^2\alpha - \left(1-\cos^2\alpha\right) = \cos^2\alpha - \sin^2\alpha$$

5.3

$$\begin{cases}\alpha+\beta=\alpha^* \\ \alpha-\beta=\beta^*\end{cases};\quad \begin{cases}\alpha = \dfrac{\alpha^*+\beta^*}{2} \\ \beta = \dfrac{\alpha^*-\beta^*}{2}\end{cases}$$

$$\cos\left(\frac{\alpha^*+\beta^*}{2}\right)\cos\left(\frac{\alpha^*-\beta^*}{2}\right) = \frac{1}{2}\left(\cos\alpha^*+\cos\beta^*\right)$$

$$\cos\alpha^*+\cos\beta^* = 2\cos\left(\frac{\alpha^*+\beta^*}{2}\right)\cos\left(\frac{\alpha^*-\beta^*}{2}\right)$$

$\alpha^*\to\alpha;\ \beta^*\to\beta$

$$\cos\alpha+\cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

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$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

$$\sin\beta\cos\alpha = \frac{1}{2}\left[\sin(\beta+\alpha)+\sin(\beta-\alpha)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(-(\alpha-\beta))\right] =$$

$$= \frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$$

$$\begin{cases}\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]\\ \cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]\end{cases}$$

$$\begin{cases}\sin\alpha\cos\beta+\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\cancel{\sin(\alpha-\beta)}\right]+\\ \qquad +\frac{1}{2}\left[\sin(\alpha+\beta)-\cancel{\sin(\alpha-\beta)}\right]\\ \sin\alpha\cos\beta-\cos\alpha\sin\beta = \frac{1}{2}\left[\cancel{\sin(\alpha+\beta)}+\sin(\alpha-\beta)\right]-\\ \qquad -\frac{1}{2}\left[\cancel{\sin(\alpha+\beta)}-\sin(\alpha-\beta)\right]\end{cases}$$

$$\begin{cases}\sin\alpha\cos\beta+\cos\alpha\sin\beta = 2\,\frac{1}{2}\sin(\alpha+\beta)\\ \sin\alpha\cos\beta-\cos\alpha\sin\beta = 2\,\frac{1}{2}\sin(\alpha-\beta)\end{cases}$$

$$\begin{cases}\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta\\ \sin(\alpha-\beta) = \sin\alpha\cos\beta-\cos\alpha\sin\beta\end{cases}$$

$\boxed{7}$

$$\begin{cases} \sin\alpha \sin\beta = \frac{1}{2}\left[-\cos(\alpha+\beta)+\cos(\alpha-\beta)\right] \\ \cos\alpha \cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right] \end{cases}$$

$$\begin{cases} \sin\alpha \sin\beta + \cos\alpha\cos\beta = \frac{1}{2}\left[-\cancel{\cos(\alpha+\beta)}+\cos(\alpha-\beta)\right] + \\ \qquad + \frac{1}{2}\left[\cancel{\cos(\alpha+\beta)}+\cos(\alpha-\beta)\right] \\ \sin\alpha\sin\beta - \cos\alpha\cos\beta = \frac{1}{2}\left[-\cos(\alpha+\beta)+\cancel{\cos(\alpha-\beta)}\right] - \\ \qquad - \frac{1}{2}\left[\cos(\alpha+\beta)+\cancel{\cos(\alpha-\beta)}\right] \end{cases}$$

$$\begin{cases} \sin\alpha\sin\beta + \cos\alpha\cos\beta = 2\,\frac{1}{2}\cos(\alpha-\beta) \\ \sin\alpha\sin\beta - \cos\alpha\cos\beta = -2\,\frac{1}{2}\cos(\alpha+\beta) \end{cases}$$

$$\begin{cases} \cos(\alpha+\beta) = -\sin\alpha\sin\beta + \cos\alpha\cos\beta \\ \cos(\alpha-\beta) = \sin\alpha\sin\beta + \cos\alpha\cos\beta \end{cases}$$

$\boxed{8.1}$

$$\sin\alpha + \sin\beta =$$

$$\sin\alpha = \sin\left(\frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}\right) = \sin\left(\frac{\alpha+\beta}{2}\right)\cos\frac{\alpha-\beta}{2} + \cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\beta = \sin\left(\frac{\beta+\alpha}{2} + \frac{\beta-\alpha}{2}\right) = \sin\left(\frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}\right) =$$

$$= \sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) - \cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$= \left[\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + \cancel{\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}\right] +$$

$$+ \left[\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) - \cancel{\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}\right] =$$

$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$\boxed{8.2}$

$$\sin\alpha - \sin\beta =$$

$$= \left[\cancel{\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)} + \cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right] -$$

$$- \left[\cancel{\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)} - \cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right] =$$

$$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

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$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\sin\alpha\sin\beta = \frac{1}{2}\left[-\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$= 2\cos^2\alpha - 1 =$$

$$= \cos^2\alpha - \sin^2\alpha$$

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = -\sin\alpha\sin\beta + \cos\alpha\cos\beta$$

$$\cos(\alpha-\beta) = \sin\alpha\sin\beta + \cos\alpha\cos\beta$$