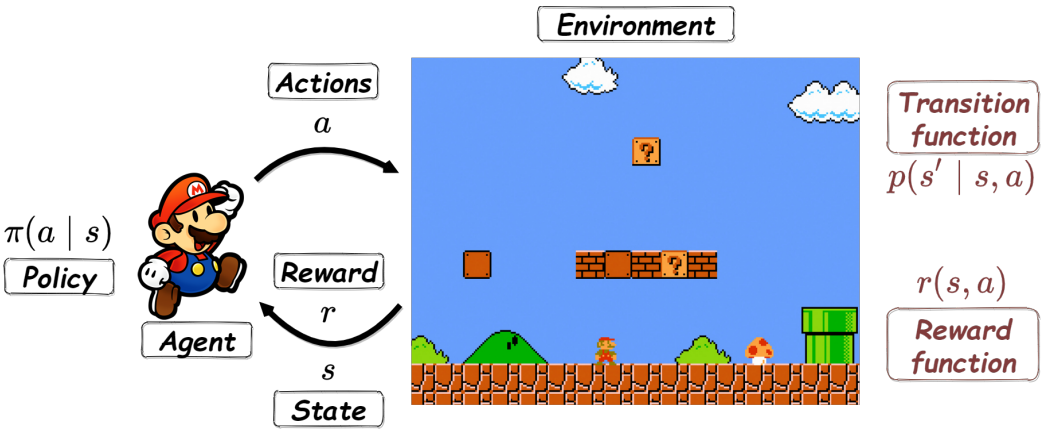
Deep Q-Learning

Reinforcement Learning

D. Sorokin

March 10, 2025

Benchmarks: Video Games



Reminder: Q-learning

Bellman optimality equation for Q^*: $\forall s, a$:

$$Q^*(s,a) := r(s,a) + \gamma \mathbb{E}_{s' \sim p(s'|s,a)} \max_{a'} Q^*(s', a')$$

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Q-learning can learn Q^* from trial and error in *finite* MDPs: $|\mathcal{S}| \ll \infty, |\mathcal{A}| \ll \infty$

Important!

Q-learning is off-policy

Reminder: Q-learning update intuition

For transition (s, a, r, s'):

$$Q^*_{k+1}(s,a) \leftarrow Q^*_k(s,a) + \alpha_k \underbrace{\left(\overbrace{r + \gamma \max_{a'} Q^*_k(s', a')}^{\textbf{Bellman target}} - Q^*_k(s,a) \right)}_{\textbf{temporal difference}}$$

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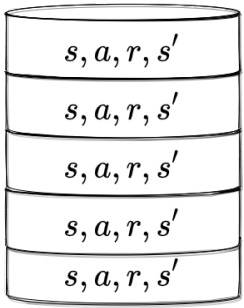
$s' \sim p(s' \mid s, a)$ — **random variable** from interaction experience.

Experience Replay



Collect all experienced transitions in **replay buffer**!

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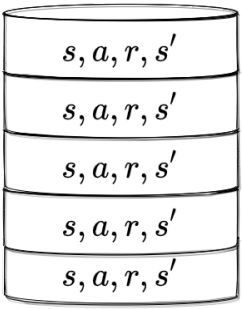

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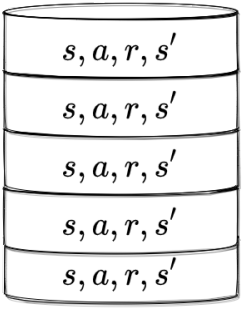

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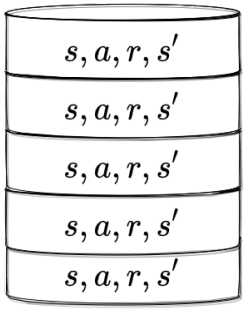

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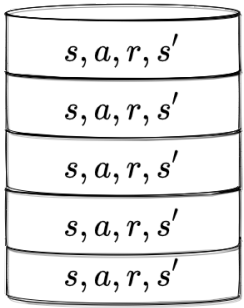

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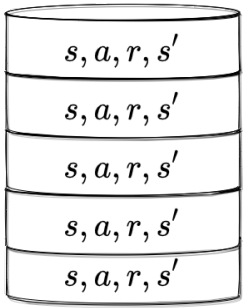

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- ✓ smoothes out with size of experience replay!

Q-learning with experience replay

Q-learning (online)

Initialize $Q^*(s, a)$ arbitrarily;

observe s_0;

for $k = 0, 1, 2 \ldots$

- take action $a_k \sim \varepsilon$-greedy$(Q^*(s_k, a))$;
- observe r_k, s_{k+1};

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- store (s_k, a_k, r_k, s_{k+1}) in $\mathcal{D}$;
- sample (s, a, r, s') from $\mathcal{D}$;
- $y := r + \gamma \max_{a'} Q^*(s', a')$;
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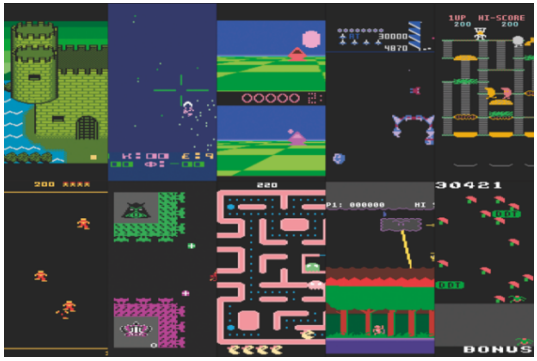
- No game-specific features.
- Finite-state case... not quite finite.
- $|\mathcal{A}| \leqslant 18$



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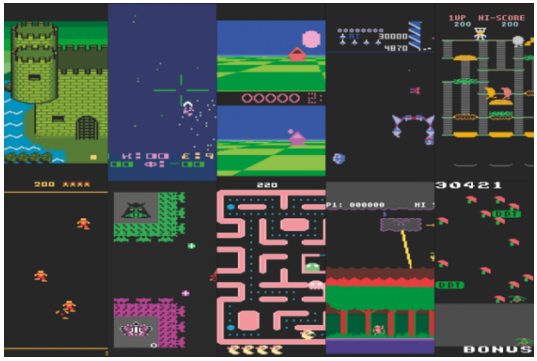
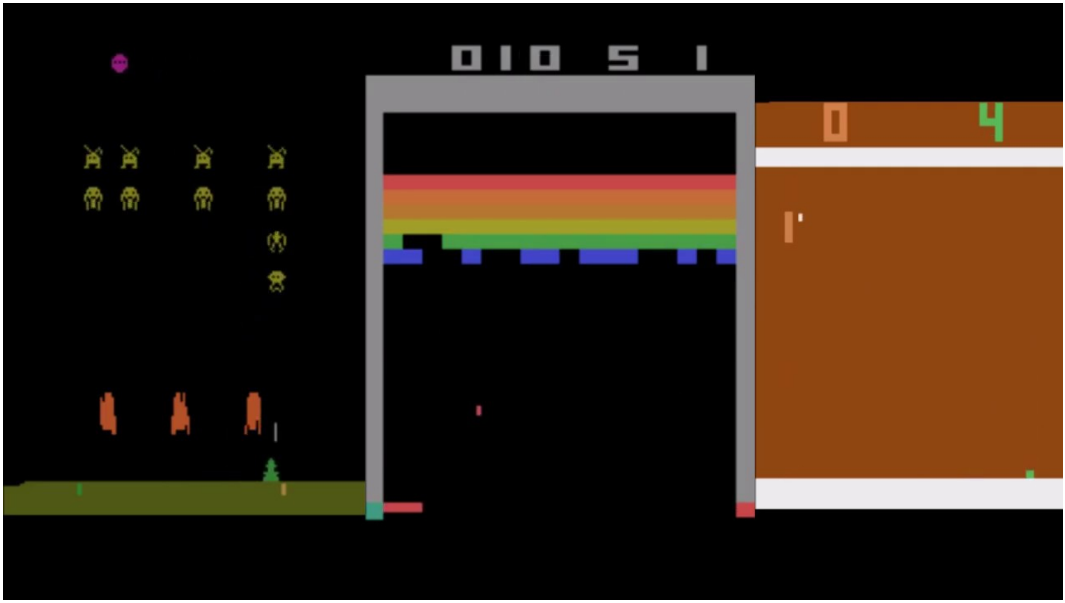


Approximate $Q^*(s, a)$ with neural network!

Is Atari game a MDP?



Practical notes: preprocessing

Action selection frequency:

- Framestack;
- Frameskip;
- MaxAndSkip;
- (sometimes) Sticky actions;

Atari-specific preprocessing:

- EpisodicLife;
- FireReset;

Standard tricks:

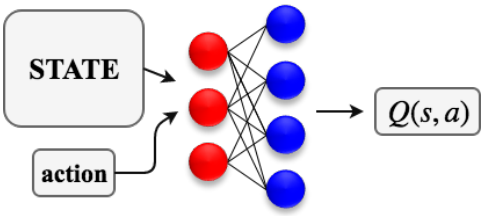
- Crop image;
- Rescale (often to 84x84);
- Grayscale;

Reward preprocessing:

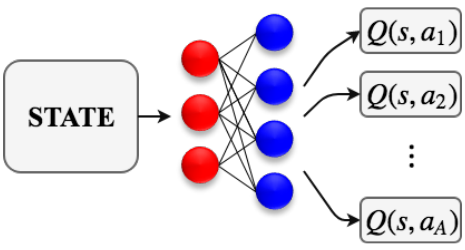
(!) Clip reward to $\{-1, 0, 1\}$;

Deep Q-network

Option 1



Option 2



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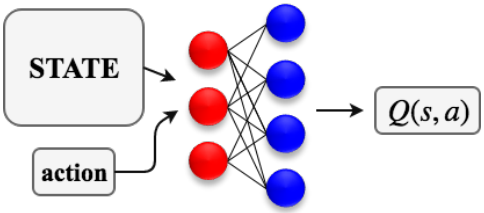

× $\underset{a}{\operatorname{argmax}}\, Q^*(s, a, \theta)$ — expensive!

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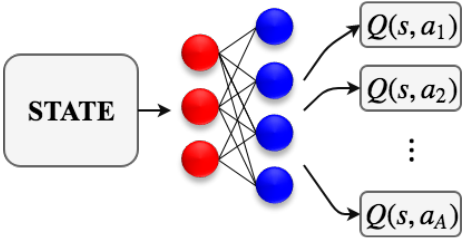
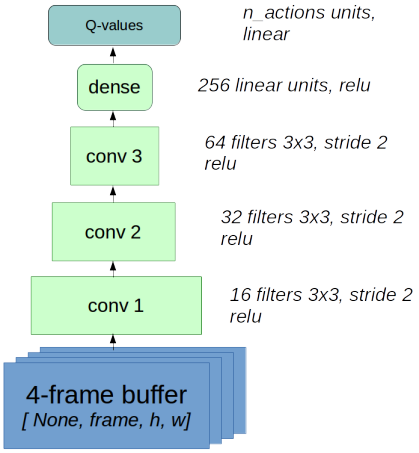


✓ $\underset{a}{\operatorname{argmax}}\, Q^*(s, a, \theta)$ — one forward pass.

Deep Q-network: architecture



- 3-4 convolutional layers followed by 1-2 dense layers;
- stride > 1 for size reduction; linear layers still wide;

Think twice before using:

- × max pooling
- × batch normalization
- × dropout

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Given:

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- loss function $\text{Loss}(y, \hat{y})$;
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- we have to collect all data ourself
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Important

Q-learning is SGD.

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Why stop gradients from target?

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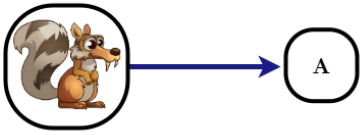

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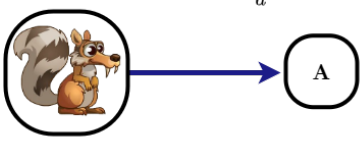

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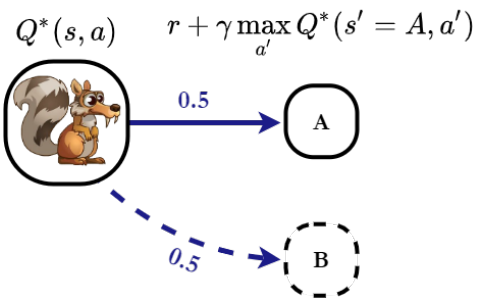

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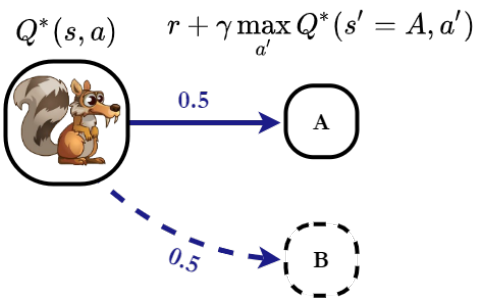
Note that unlike supervised learning:

- target «depends» on our own model;
- it is still *better* that we currently have; («provides direction»)



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- MSE loss function: $\text{Loss}(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$

Consider the following **regression task**:

- s, a is input;
- $y \in \mathbb{R}$ is target:

$$y(s, a) := r + \gamma \max_{a'} Q^*(s', a', \theta_k)$$

  where $r = r(s, a), s' \sim p(s' \mid s, a)$;
- MSE loss function: $\text{Loss}(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$

$$\mathbb{E}_{(s,a,r,s')}(y - Q^*(s, a, \theta_{k+1}))^2 \to \min_{\theta_{k+1}}$$

Solving Bellman equations: deep version

$$\mathbb{E}_{(s,a,r,s')}(y - Q^*(s, a, \theta_{k+1}))^2 \to \min_{\theta_{k+1}}$$

What solution is optimal?

$$Q^*(s, a, \theta^*_{k+1}) =$$

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What solution is optimal?

$$Q^*(s, a, \theta^*_{k+1}) = \mathbb{E}_{r,s'|s,a} y =$$

Solving Bellman equations: deep version

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$$= \underbrace{r(s, a) + \gamma \mathbb{E}_{s' \sim p(s'|s,a)} \max_{a'} Q^*(s', a', \theta_k)}_{\text{value iteration update}}$$

Target network

For how long to solve one regression task?

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- One SGD iteration

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✓ ≈1000 SGD iterations

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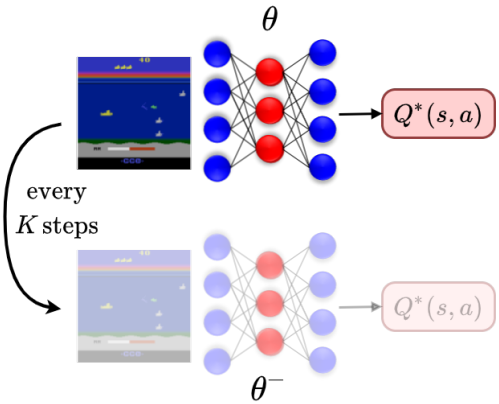

Store a copy of your old Q-network $Q^*(s, a, \theta^-)$

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For how long to solve one regression task?

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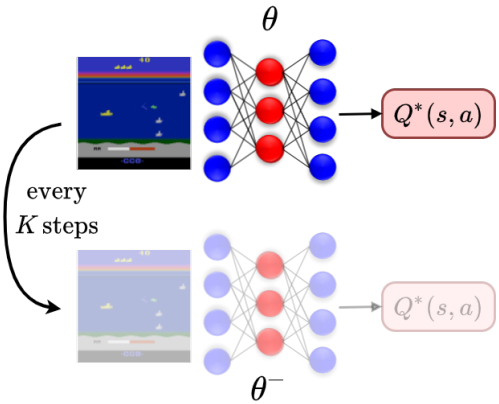
✓ ≈1000 SGD iterations

a) $\theta^- \leftarrow \theta$ every K SGD iterations

b) $\theta^- \leftarrow (1-\beta)\theta^- + \beta\theta$



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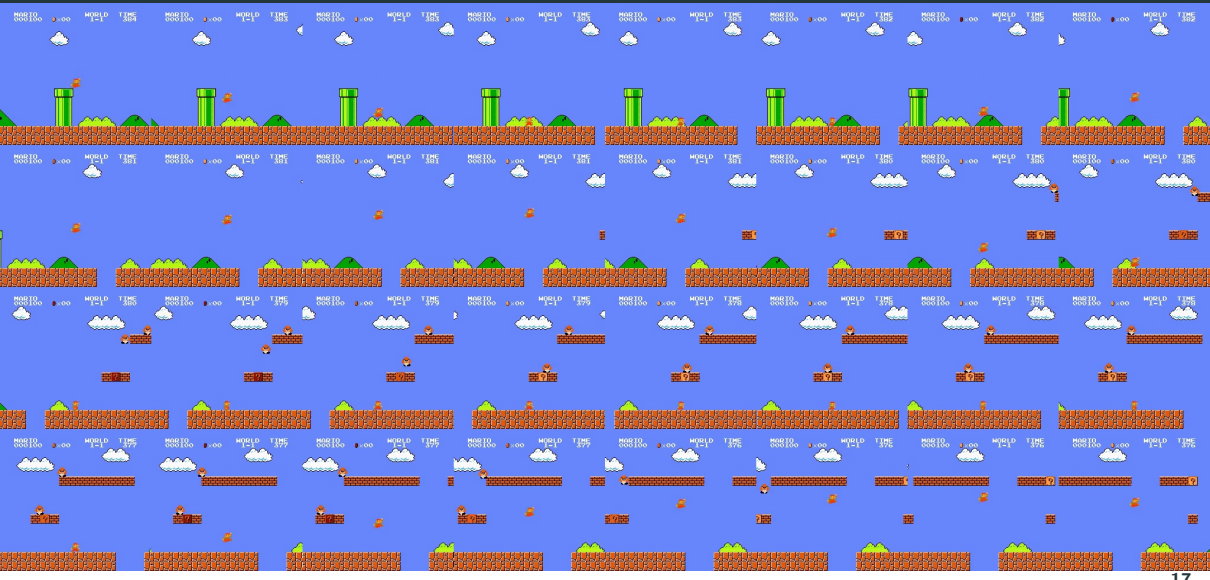
Distributional shift issue



Experience Replay

Do we still need exploration?

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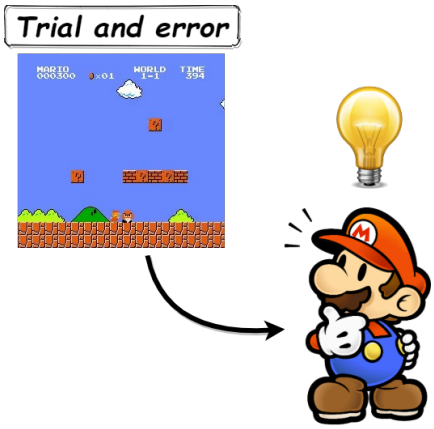

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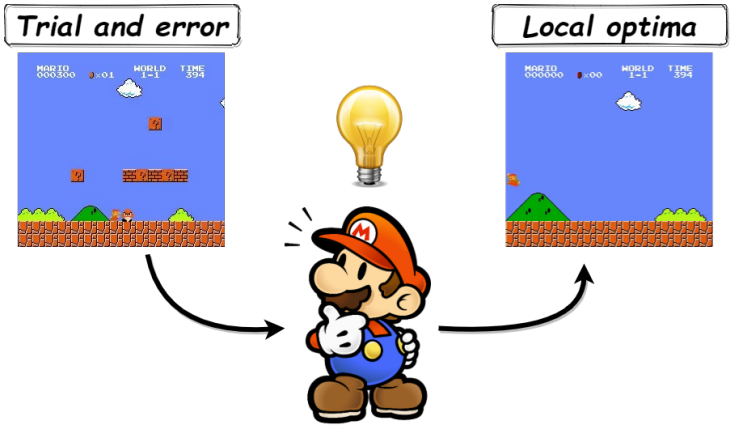

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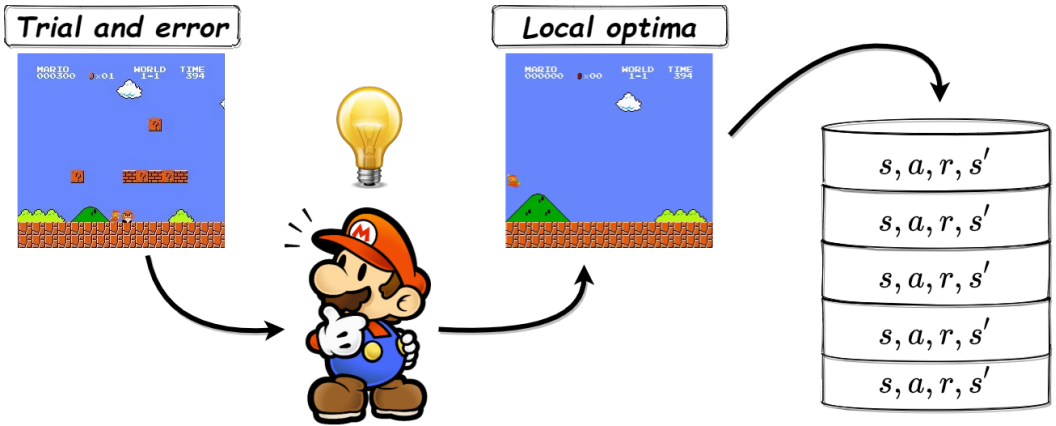

Full alrogithm

Deep Q-learning

Initialize $Q^*(s, a, \theta)$ arbitrarily, $\theta^- := \theta$, $\mathcal{D} = \varnothing$;

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for $k = 0, 1, 2 \ldots$

- take action $a_k \sim \varepsilon$-greedy$(Q^*(s_k, a, \theta))$;
- observe $r_k, s_{k+1}, \mathrm{done}_{k+1}$, store $(s_k, a_k, r_k, s_{k+1}, \mathrm{done}_{k+1})$ in $\mathcal{D}$;

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- perform a step of gradient descent:

$$\theta \leftarrow \theta - \frac{\alpha}{B} \sum_{\mathbb{T}} \nabla_\theta \left(Q^*(s, a, \theta) - y(\mathbb{T}) \right)^2$$

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- update target network: if $k \bmod K = 0$: $\theta^- \leftarrow \theta$

There is a lot to improve

Overestimation Bias and how to fight it

$$\max_{a'}(Q^*(s', a', \theta) + \underbrace{\varepsilon(s', a')}_{\text{approximation error or luck}})$$

Overestimation Bias and how to fight it

$$\mathbb{E}_{\varepsilon} \max_{a'}(Q^*(s', a', \theta) + \underbrace{\varepsilon(s', a')}_{\text{approximation error or luck}}) \geqslant \max_{a'} Q^*(s', a')$$

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Decouple **action selection** and **action evaluation**:

$$\max_{a'} Q^*(s', a') = \overbrace{Q^*(s', \underbrace{\operatorname{argmax}_{a'} Q^*(s', a')}_{\text{action selection}})}^{\text{action evaluation}}$$

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Decouple **action selection** and **action evaluation**:

$$\max_{a'} Q^*(s', a') = \overbrace{Q^*(s', \underbrace{\operatorname{argmax}_{a'} Q^*(s', a')}_{\text{action selection}})}^{\text{action evaluation}} \approx Q_1^*(s', \operatorname*{argmax}_{a'} Q_2^*(s', a'))$$

Action Evaluation: options

Name	Networks	Targets
Double Q-learning	Q_1^* Q_2^*	$y_1 = r + \gamma Q_?^*(s', \underset{a'}{\operatorname{argmax}}\, Q_?^*(s', a'))$

Action Evaluation: options

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Action Evaluation: options

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Double Q-learning	Q_1^* Q_2^*	$y_1 = r + \gamma Q_2^*(s', \underset{a'}{\operatorname{argmax}}\, Q_1^*(s', a'))$ $y_2 = r + \gamma Q_1^*(s', \underset{a'}{\operatorname{argmax}}\, Q_2^*(s', a'))$

Action Evaluation: options

Name	Networks	Targets
~~Double Q-learning~~ Twin DQN	Q_1^* Q_2^*	$y_1 = r + \gamma Q_2^*(s', \underset{a'}{\operatorname{argmax}}\, Q_1^*(s', a'))$ $y_2 = r + \gamma Q_1^*(s', \underset{a'}{\operatorname{argmax}}\, Q_2^*(s', a'))$
Double DQN	Q^* Q_-^* — target network	

Action Evaluation: options

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Double DQN	Q^* Q_-^* — target network	$y = r + \gamma Q_-^*(s', \operatorname{argmax}_{a'} Q^*(s', a'))$

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Double DQN	Q^* Q_-^* — target network	$y = r + \gamma Q_-^*(s', \operatorname{argmax}_{a'} Q^*(s', a'))$
Twin DQN («Clipped Double DQN»)	Q_1^* Q_2^*	

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Double DQN	Q^* Q_-^* — target network	$y = r + \gamma Q_-^*(s', \operatorname{argmax}_{a'} Q^*(s', a'))$
Twin DQN («Clipped Double DQN»)	Q_1^* Q_2^*	$y_1 = r + \gamma \min_{i=1,2} Q_i^*(s', \operatorname{argmax}_{a'} Q_1^*(s', a'))$

Action Evaluation: options

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Twin DQN («Clipped Double DQN»)	Q_1^* Q_2^*	$y_1 = r + \gamma \min_{i=1,2} Q_i^*(s', \operatorname{argmax}_{a'} Q_1^*(s', a'))$ $y_2 = r + \gamma \min_{i=1,2} Q_i^*(s', \operatorname{argmax}_{a'} Q_2^*(s', a'))$

Literature

- Playing Atari with Deep Reinforcement Learning (2013);
- Deep Reinforcement Learning with Double Q-learning (2015);