# A Sample Beamer Presentation

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# Presentation Outline

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# A Counterexample



Consider the function

$$f(x) = \begin{cases} x^2 \sin(1/x), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Let's see what $f'(0)$ is.

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Since $-h \leq h\sin(1/h) \leq h$ and $\lim_{h\to 0}(-h) = \lim_{h\to 0}(h) = 0$, the Squeeze Theorem says $f'(0) = 0$.

## What Really Happens at $x = 0$?



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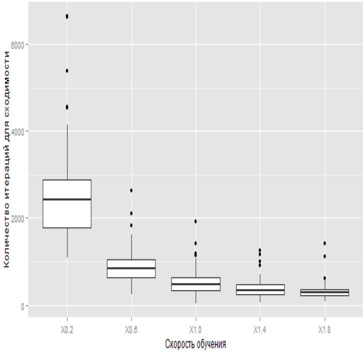


$y = f(x)$, $y = x^2$, $y = -x^2$

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# How to Define "Increasing at a Point"?



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# Our Function with a Slight Twist



Let's modify our function to

$$g(x) = \begin{cases} 0.5x + x^2 \sin(1/x), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Using the definition of derivative as before, we will find that $g'(0) = 0.5$.

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However, $g(x)$ still oscillates enough as $x \to 0$ that there is no open interval containing $x = 0$ that satisfies our definition of $g$ increasing at $x = 0$ even though $g'(0) > 0$.

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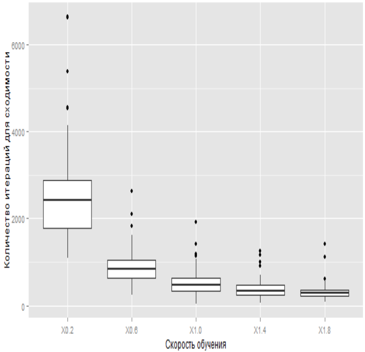


$y = g(x)$, $y = x^2 + 0.5x$, $y = x^2 - 0.5x$

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The function $f(x)$ introduced earlier has other interesting properties, one of which is the fact that while $f'(0)$ exists, $f'(x)$ is discontinuous at $x = 0$.

We leave it to you to work this out for yourself and to explore this interesting function further.

Thank you for your attention today.