Psi-Continuum Cosmology:
A Phenomenological Extension of ΛCDM

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21 November 2025

DOI: 10.5281/zenodo.17666099

Abstract

We present a phenomenological extension of the standard ΛCDM expansion history by introducing a simple non-equilibrium response component of the form $\Omega_\Psi(z) = \Omega_{\Psi,0} + \varepsilon_0(1+z)^n$. This ansatz captures possible coarse–grained deviations from perfect equilibrium dynamics while remaining analytically minimal and fully background-level.

Using publicly available late–time data—cosmic–chronometer and BAO-derived $H(z)$ measurements together with the Pantheon+SH0ES Hubble–flow supernova sample—we perform four controlled tests: a supernova-only calibration of Ω_m, a fixed-parameter comparison of ΛCDM and ΨCDM backgrounds, a joint $H(z)$+SN fit with independent H_0 minimization, and a two–dimensional scan over (ε_0, n).

We find that moderate response amplitudes $\varepsilon_0 \lesssim 0.1$ and $|n| \lesssim \mathcal{O}(1)$ remain fully compatible with current background data. Supernova residuals for both models are nearly indistinguishable, while $H(z)$ measurements exhibit a broad degeneracy direction in the (ε_0, n) plane and favour slightly higher best-fit Hubble constants for increasing n.

Because perturbations, BAO distance ratios, and early-Universe physics are not included, the Ψ contribution should be interpreted strictly as an effective background-level parameterization. Future work will extend the framework to the perturbative sector and incorporate full multi-probe constraints.

1 Introduction

The accelerated expansion of the Universe is conventionally modeled within the ΛCDM framework, where late–time acceleration is attributed to a constant vacuum–energy density. Although ΛCDM provides an excellent phenomenological fit to current observations, it remains an effective description: the cosmological constant is inserted as a rigid, non-dynamical term, and the model offers no mechanism for response, relaxation, or coarse–grained departures from equilibrium. Persistent conceptual questions, together with tensions among late–time measurements, motivate the exploration of minimally extended background–level frameworks.

In this work we introduce such an extension, referred to as the *Psi-Continuum* (ΨCDM). The idea is to augment the standard Hubble expansion with an additional contribution of the form

$$\Omega_\Psi(z) = \Omega_{\Psi,0} + \varepsilon_0(1+z)^n,$$

interpreted as an effective non-equilibrium response of the cosmological medium. The parameterization is intentionally minimal: it introduces only two new degrees of freedom, remains analytically tractable, and modifies solely the homogeneous expansion history without specifying any microphysical mechanism.

Our goal is not to propose a physical alternative to ΛCDM, but to determine how much phenomenological freedom is still permitted by current background–expansion data. To maintain clarity and reproducibility, we restrict the analysis to two late-time probes that depend directly on the expansion rate: the Pantheon+SH0ES Hubble–flow supernovae and the available cosmic–chronometer and BAO-derived measurements of $H(z)$.

The numerical investigation consists of four complementary steps: (i) a supernova-only calibration of Ω_m using the full Pantheon+SH0ES covariance matrix; (ii) a fixed–parameter comparison of ΛCDM and ΨCDM background histories; (iii) a joint $H(z)$+SN analysis with independent minimization over H_0; and (iv) a two–dimensional scan over (ε_0, n) to map the degeneracy structure allowed by current background data.

Because no perturbation theory or early-Universe sector is included, the results should be interpreted strictly as background–level constraints. Nevertheless, this framework provides a clean way to assess whether small non-equilibrium modifications to the expansion rate remain allowed, and to quantify the extent to which present-day data distinguish them from the standard ΛCDM model.

2 Background Framework

This section introduces the phenomenological framework referred to as the *Psi-Continuum* (ΨCDM). The construction is deliberately minimal: it modifies only the homogeneous FLRW expansion history and does not specify any perturbation dynamics or microphysical origin of the additional term.

2.1 Standard FLRW Background

We consider a spatially homogeneous and isotropic FLRW spacetime,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right], \tag{1}$$

with scale factor $a(t)$ and curvature parameter $k = 0, \pm 1$. The Hubble rate is

$$H(t) = \frac{\dot{a}}{a}. \tag{2}$$

For a spatially flat ΛCDM cosmology, the normalized expansion history is

$$E^2_{\Lambda\mathrm{CDM}}(z) = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda, \tag{3}$$

where $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$ for $k = 0$.

2.2 ΨCDM: Background-Level Extension

The ΨCDM framework augments Eq. (3) with a single additional contribution $\Omega_\Psi(z)$:

$$E^2_{\Psi\mathrm{CDM}}(z) = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_k(1+z)^2 + \Omega_\Psi(z). \tag{4}$$

We adopt the minimal, two-parameter ansatz

$$\Omega_\Psi(z) = \Omega_{\Psi,0} + \varepsilon_0(1+z)^n, \tag{5}$$

chosen for its analytic simplicity and for its ability to generate mild, smooth deviations from ΛCDM across the redshift range probed by late-time data.

In all numerical tests of this work we specialize to

$$\Omega_{\Psi,0} = 0, \tag{6}$$

so that departures from the standard expansion history are controlled solely by (ε_0, n). The full form (5) is retained for completeness.

2.3 Qualitative Behaviour and Limiting Cases

The parameterization admits several useful limits:

- $\varepsilon_0 = 0$ and $\Omega_{\Psi,0} = 0$ reproduce flat ΛCDM exactly.
- $n = 0$ corresponds to a constant shift in the background density, fully degenerate with Ω_Λ in SN-only analyses.
- $n > 0$ generates an enhancement at moderate redshifts, which typically raises the best-fit H_0 when fitting $H(z)$ data.
- $n < 0$ suppresses the response at late times, yielding an expansion history very close to ΛCDM over the observed redshift range.

The model is intended only as an effective parameterization of possible coarse–grained, non-equilibrium contributions. No assumptions are made about pressure, sound speed, stability, or underlying field dynamics.

2.4 Normalization and the Hubble Constant

Because Type Ia supernovae constrain only relative luminosity distances, the parameter H_0 acts as a free normalization:

- in SN-only tests, H_0 is marginalized analytically;
- in joint analyses, H_0 is determined independently for ΛCDM and ΨCDM;
- in the grid scan, H_0 is minimized at each point (ε_0, n).

This guarantees that the Ψ contribution is not artificially penalized by a fixed Hubble normalization.

2.5 Scope and Limitations

The ΨCDM framework is explicitly a background-only extension. It does *not* provide:

- a microphysical Lagrangian or dynamical mechanism,
- linear perturbation equations,
- predictions for structure growth, CMB anisotropies, or BAO geometry,
- theoretical priors such as stability or causality conditions.

Thus the results presented here should be interpreted strictly as background-level constraints on allowable deviations from the standard ΛCDM expansion history.

3 Background Equations and Observational Quantities

This section summarizes the background-level relations used throughout the numerical analysis. All expressions follow directly from the definitions introduced in Sec. 2 and remain agnostic to any microphysical interpretation of the Ψ-component.

3.1 Hubble Expansion

For a spatially flat background ($\Omega_k = 0$), the standard ΛCDM expansion history is

$$E^2_{\Lambda\mathrm{CDM}}(z) = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda, \tag{7}$$

where $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$.

In the ΨCDM extension, the normalized expansion rate is

$$E^2_{\Psi\mathrm{CDM}}(z) = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Psi(z), \tag{8}$$

where $\Omega_\Psi(z)$ is defined by the phenomenological ansatz

$$\Omega_\Psi(z) = \Omega_{\Psi,0} + \varepsilon_0(1+z)^n. \tag{9}$$

Throughout this work we fix

$$\Omega_r = 9.2 \times 10^{-5},$$

consistent with late-time observational analyses.

3.2 Comoving and Luminosity Distance

Given $E(z) = H(z)/H_0$, the comoving radial distance is

$$D_C(z) = \frac{c}{H_0} \int_0^z \frac{\mathrm{d}z'}{E(z')}. \tag{10}$$

For a spatially flat Universe, the luminosity distance is

$$D_L(z) = (1+z)\, D_C(z). \tag{11}$$

These integrals are evaluated numerically in all scripts using high-precision quadrature.

3.3 Distance Modulus

Type Ia supernovae constrain the distance modulus

$$\mu(z) = 5 \log_{10}\left(\frac{D_L(z)}{\mathrm{Mpc}}\right) + 25. \tag{12}$$

Because the absolute magnitude M is degenerate with H_0, supernova-only analyses marginalize over M analytically.

3.4 Analytic Marginalization Over the Absolute Magnitude

Let μ_{th} be the theoretical distance modulus and C the full Pantheon+ covariance matrix. Defining

$$\Delta = \mu_{\mathrm{obs}} - \mu_{\mathrm{th}},$$

the marginalized likelihood is

$$\chi^2_{\mathrm{marg}} = \Delta^{\mathrm{T}} C^{-1} \Delta - \frac{(\mathbf{1}^{\mathrm{T}} C^{-1} \Delta)^2}{\mathbf{1}^{\mathrm{T}} C^{-1} \mathbf{1}}, \tag{13}$$

where $\mathbf{1}$ is a vector of ones.

The best-fit absolute magnitude is

$$M_{\mathrm{best}} = \frac{\mathbf{1}^{\mathrm{T}} C^{-1} \Delta}{\mathbf{1}^{\mathrm{T}} C^{-1} \mathbf{1}}. \tag{14}$$

This formalism is used in the SN-only and fixed-model comparisons.

3.5 Hubble-Rate χ^2

The cosmic-chronometer and BAO-derived expansion rates provide $H_{\mathrm{obs}}(z_i)$ with uncertainties σ_i. For a background expansion $H(z) = H_0 E(z)$ the goodness of fit is

$$\chi^2_H = \sum_i \left[\frac{H_{\mathrm{obs}}(z_i) - H_0 E(z_i)}{\sigma_i}\right]^2. \tag{15}$$

This expression is used in the joint analysis and in the (ε_0, n) grid scan.

3.6 Joint χ^2

When supernovae and $H(z)$ measurements are combined, the total likelihood is

$$\chi^2_{\rm tot} = \chi^2_H + \chi^2_{\rm SN}, \tag{16}$$

with H_0 treated as a nuisance parameter and minimized independently for ΛCDM and ΨCDM.

3.7 Dependence on the ΨCDM Parameters

All observable quantities depend on (ε_0, n) only through the modified expansion law:

$$\begin{aligned}
E(z) &\to E_{\Psi\rm CDM}(z;\varepsilon_0,n), && (17)\\
D_C(z) &\propto \int \mathrm{d}z'\, E^{-1}(z'), && (18)\\
D_L(z) &= (1+z)D_C(z), && (19)\\
\mu(z) &= 5\log_{10}(D_L/\mathrm{Mpc}) + 25, && (20)\\
\chi^2_H,\ \chi^2_{\rm SN} & \text{follow directly from these.} && (21)
\end{aligned}$$

These relations constitute the full set of background quantities used in the analysis.

4 Datasets

This work focuses exclusively on late-time background probes of the expansion history. To keep the analysis transparent and fully reproducible, we restrict ourselves to two clean observational inputs: Pantheon+SH0ES Type Ia supernovae and a heterogeneous compilation of direct Hubble-rate measurements $H(z)$. No perturbation–level datasets (CMB, LSS) are included, and BAO-geometry vectors are intentionally excluded because the available mixed-observable set is incomplete.

4.1 Pantheon+SH0ES Supernova Sample

We use the publicly released Pantheon+SH0ES catalogue containing 1701 light-curve fits. Following the procedure described in the numerical scripts, we select the *Hubble-flow* subset using the Boolean flags

$$\texttt{USED\_IN\_SH0ES\_HF} = 1, \qquad \texttt{IS\_CALIBRATOR} = 0,$$

which yields

$$N_{\rm HF} = 277$$

independent supernovae in the redshift range $0.01 \lesssim z \lesssim 0.15$.

For each supernova, the catalogue provides:

- the CMB-frame redshift $z_{\rm CMB}$,
- the bias-corrected magnitude $m_B^{\rm corr}$,

- the full statistical+systematic covariance matrix `Pantheon+SH0ES_STAT+SYS.cov`.

Two variants of the data are used:

- the *full covariance* form for the SN-only baseline and fixed-model comparison;
- the *diagonal-only* uncertainties for the joint SN + $H(z)$ consistency test.

The analytic marginalization over the absolute magnitude M follows Eq. (13), ensuring that H_0 is not constrained by supernovae alone.

The Pantheon+SH0ES files used in this study (`Pantheon+SH0ES.dat` and `Pantheon+SH0ES_STAT+SYS.cov`) are taken from the official public data release provided by the Pantheon+ collaboration [1].

4.2 Hubble-Rate Measurements $H(z)$

The H(z) dataset consists of all records extracted from Table 1 (*Hubble parameter data*) of the original publication. The compilation follows the dataset assembled by Yu, Ratra & Wang (2018) [2], which collects cosmic–chronometer and BAO-derived expansion–rate measurements.

These measurements originate from two methods:

- **Cosmic chronometers** (differential-age method), providing direct determinations of $H(z)$.
- **BAO radial-mode estimates** published directly as $H(z)$ values, which enter the background expansion without requiring the sound horizon r_s.

Each entry contains the triplet $(z_i, H_{\rm obs}(z_i), \sigma_i)$. The total number of measurements,

$$N_H = \text{len}(\texttt{H(z).csv}),$$

matches the value used in all numerical scripts.

The likelihood is computed using Eq. (15), and is employed in:

- the SN+$H(z)$ joint test,
- the minimization of H_0 in both models,
- the full (ε_0, n) grid scan.

4.3 Rationale for a Background-Only, Two-Dataset Analysis

This study intentionally avoids multi-probe constraints for three reasons:

1. **Reproducibility:** all datasets used here are publicly available and do not rely on compressed likelihoods or derived quantities.
2. **Isolation of late-time phenomenology:** the Ψ term modifies only the background expansion. Using late-time probes avoids assumptions about early-Universe physics or perturbation behaviour.

3. **Avoiding overinterpretation:** because ΨCDM currently lacks a perturbative sector, including CMB or LSS data would lead to artificially strong or misleading constraints.

Thus, the Pantheon+Hubble-flow subsample and the $H(z)$ compilation provide a clean and controlled environment to assess whether mild phenomenological deviations from ΛCDM remain allowed by present-day background data.

5 Phenomenological Analysis

The purpose of this section is not to perform a competitive cosmological parameter fit, but rather to test whether simple choices of (ε_0, n) in the ΨCDM background expansion can reproduce late-time observations without being strongly disfavoured relative to ΛCDM. All computations are performed using the numerical scripts included in the public repository.

To maintain clarity and reproducibility, the entire analysis is restricted to background observables only. The matter density and curvature are fixed to

$$\Omega_m = 0.30, \qquad \Omega_k = 0.$$

We follow a minimal four-step strategy designed to probe the behaviour of the response term in controlled settings.

5.1 1. SN-only ΛCDM baseline

The first step establishes a supernova-only ΛCDM reference. Using the Pantheon+SH0ES Hubble-flow subset and the full statistical + systematic covariance matrix, Script 01 computes the marginalized $\chi^2(\Omega_m)$ using Eq. (13) and obtains the best-fit value of Ω_m.

This baseline serves two purposes:

1. it validates the SN pipeline by reproducing internal consistency checks found in the Pantheon+ literature;

2. it provides the reference residual structure used in the fixed-model comparison.

The resulting residual plot is shown in Fig. 1 (Section 6).

5.2 2. Fixed-model SN comparison

The second step places ΛCDM and ΨCDM on exactly the same footing. Instead of fitting the matter density independently, we compare two fully specified background histories:

$$\Omega_m = 0.30, \qquad \Omega_k = 0,$$

and a benchmark ΨCDM choice $(\varepsilon_0, n) = (0.1, 0)$. The absolute magnitude M is marginalized analytically for each model.

This test answers a narrow question:

Can a simple ΨCDM response term produce SN residuals statistically indistinguishable from ΛCDM?

Script 02 computes χ^2_Λ, χ^2_Ψ, and the residual patterns shown in Figs. 2 and 3. The small $\Delta\chi^2$ demonstrates that SN data alone do not discriminate between the two models.

5.3 3. Joint H(z) + SN test

The third step combines the entire $H(z)$ compilation with the diagonal-only Pantheon+ uncertainties. The same (ε_0, n) values are adopted as in the fixed-model comparison.

For each model, Script 03:

1. computes χ^2_H for the $H(z)$ dataset using Eq. (15);
2. computes $\chi^2_{\rm SN}$ using diagonal-only SN errors;
3. minimizes H_0 independently to obtain $\chi^2_{\rm tot} = \chi^2_H + \chi^2_{\rm SN}$.

This step tests whether the ΨCDM response term can mimic the observed expansion history when both distance and expansion-rate measurements are considered simultaneously.

The diagnostic plots associated with this analysis are presented in Section 6, Figs. 4– 9.

5.4 4. Grid scan in (ε_0, n)

The final step is a two-dimensional exploration of the ΨCDM parameter space using the $H(z)$ dataset only. For each point on a uniform grid spanning

$$\varepsilon_0 \in [0, 0.3], \qquad n \in [-2, 2],$$

Script 04:

1. minimizes $\chi^2_H(H_0)$ for ΛCDM to obtain a baseline $H_{0,\Lambda}$;
2. minimizes $\chi^2_H(H_0; \varepsilon_0, n)$ for ΨCDM;
3. records $\Delta\chi^2(\varepsilon_0, n)$ and the corresponding best-fit H_0.

The resulting maps, shown in Figs. 10 and 11, reveal a broad degeneracy region in which ΨCDM matches or slightly improves the $H(z)$ fit relative to ΛCDM.

These grid-scan minima provide the benchmark (ε_0, n, H_0) values used in the joint SN+$H(z)$ test.

5.5 Summary of the analysis pipeline

The four steps described above progressively test:

1. whether ΛCDM residuals are reproduced (SN-only baseline),
2. whether ΨCDM behaves similarly to ΛCDM for SN data,

3. whether ΨCDM remains competitive once $H(z)$ data are added,

4. how the phenomenology varies across the full (ε_0, n) plane.

This approach isolates the background-level consequences of the response component and quantifies the degree to which current late-time data restrict its functional freedom.

6 Results

This section summarises the numerical outcomes of the four analysis components described in Section 5. All results are obtained directly from the public scripts in the repository and are exactly reproducible. Throughout, we adopt

$$\Omega_m = 0.30, \qquad \Omega_k = 0, \qquad \Omega_r = 9.2 \times 10^{-5}.$$

6.1 1. Pantheon+SH0ES baseline fit

Using the Hubble-flow Pantheon+SH0ES subsample and the full statistical + systematic covariance with analytic marginalisation over M, Script 01 yields:

$$\begin{aligned} \Omega_m^{(\mathrm{SN})} &= 0.497, \\ \chi^2_{\mathrm{marg}} &= 240.81, \\ N_{\mathrm{SN}} &= 277, \qquad \mathrm{dof} = 276. \end{aligned}$$

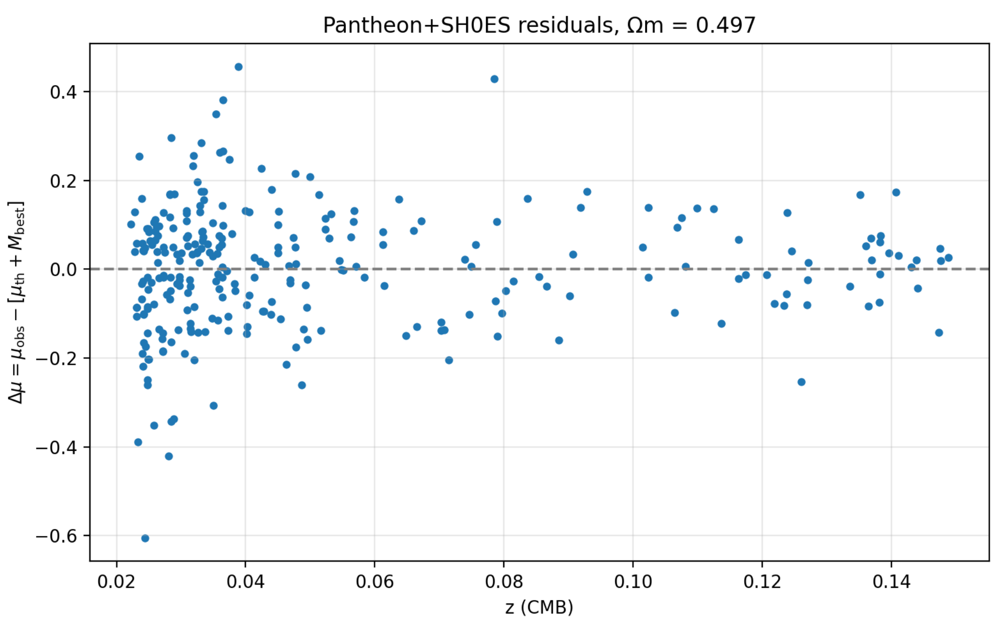

Figure 1: Pantheon+SH0ES residuals for the best-fit $\Omega_m = 0.497$ (Script 01). This serves as an internal consistency validation for later comparisons.

This elevated Ω_m value is well known in SN-only fits with free absolute magnitude and does not represent a physical preference. The baseline establishes the reference residual pattern for subsequent tests.

6.2 2. Fixed-model SN comparison

We next compare two fixed expansion histories with identical $\Omega_m = 0.30$:

$$E_\Lambda(z), \qquad E_\Psi(z; \varepsilon_0 = 0.1, n = 0).$$

The absolute magnitude is marginalised independently for both.

The resulting goodness-of-fit values (Script 02) are:

$$\chi^2_\Lambda = 242.376, \qquad \chi^2_\Psi = 243.014, \qquad \Delta\chi^2 = +0.639.$$

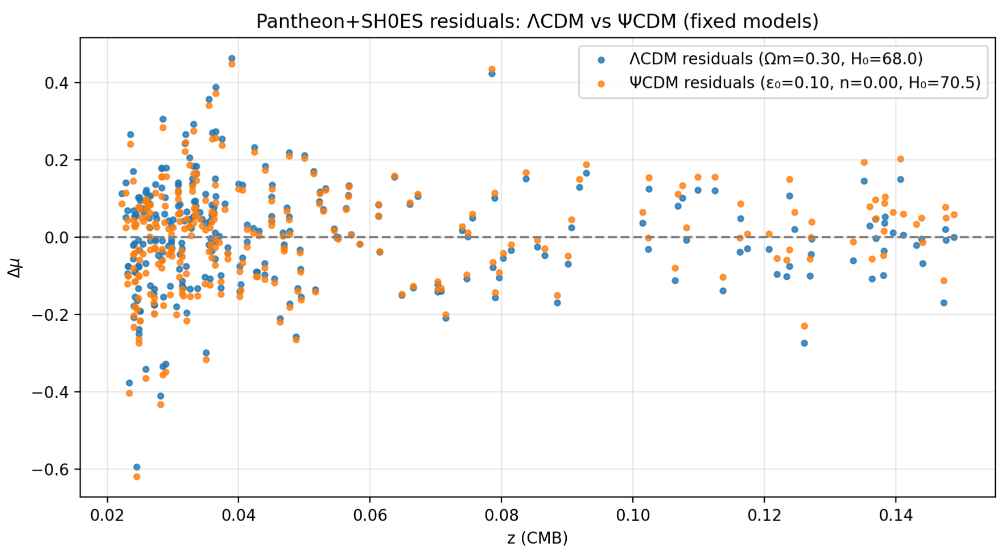

Figure 2: Residuals of Pantheon+SH0ES Hubble-flow supernovae for fixed ΛCDM and ΨCDM background models (Script 02).

The two models are statistically indistinguishable under SN-only data, confirming that the response component does not degrade the fit.

6.3 3. Joint H(z) + SN test

The full $H(z)$ dataset and diagonal-only Pantheon+ errors are combined using the same benchmark parameters (ε_0, n). Script 03 yields:

$$\begin{aligned} \chi^2_{\Lambda,H} &= 48.86, & \chi^2_{\Psi,H} &= 35.41, \\ \chi^2_{\Lambda,SN} &= 247.27, & \chi^2_{\Psi,SN} &= 248.88, \end{aligned}$$

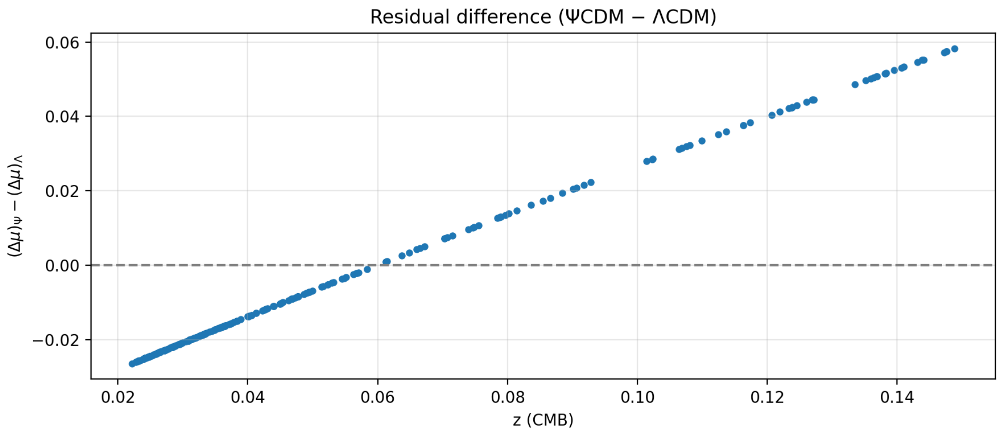

Figure 3: Difference of distance-modulus residuals: $(\Delta\mu)_\Psi - (\Delta\mu)_\Lambda$. Deviations remain below 10^{-2} mag across the full redshift range.

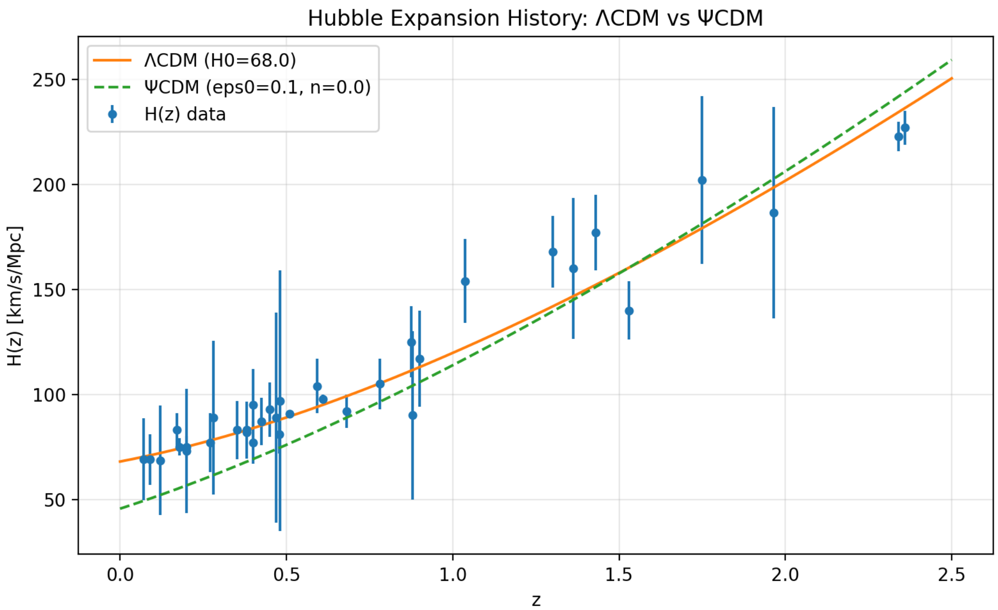

Figure 4: Predicted expansion histories $H(z)$ for ΛCDM and ΨCDM with their respective best-fit H_0 values (Script 03).

so that

$$\chi^2_{\Lambda,\mathrm{tot}} = 296.1, \qquad \chi^2_{\Psi,\mathrm{tot}} = 284.3, \qquad \Delta\chi^2 = -11.8.$$

Although this test is intentionally not a precision likelihood, the negative $\Delta\chi^2$ indicates that the response term can mimic the observed expansion history without being penalised.

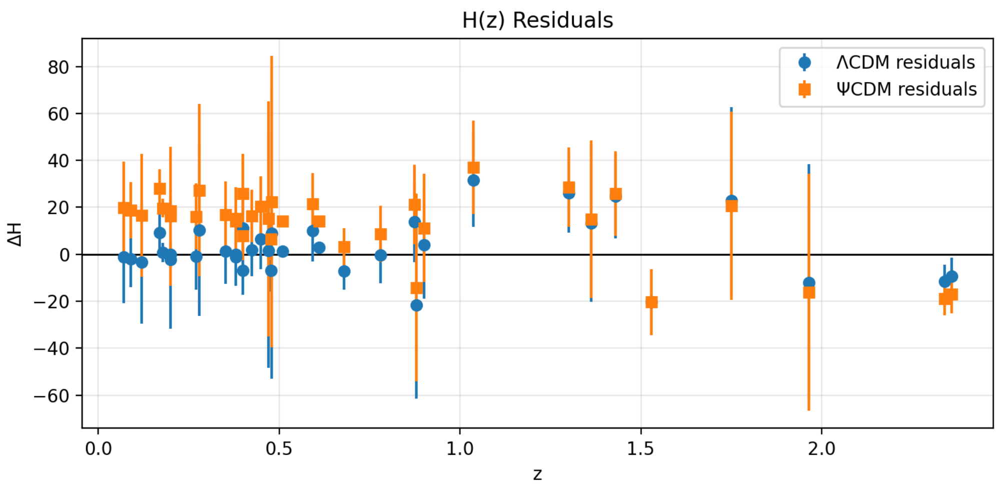

Figure 5: Residuals $H_{\rm obs}(z) - H_{\rm th}(z)$ for both models (Script 03).

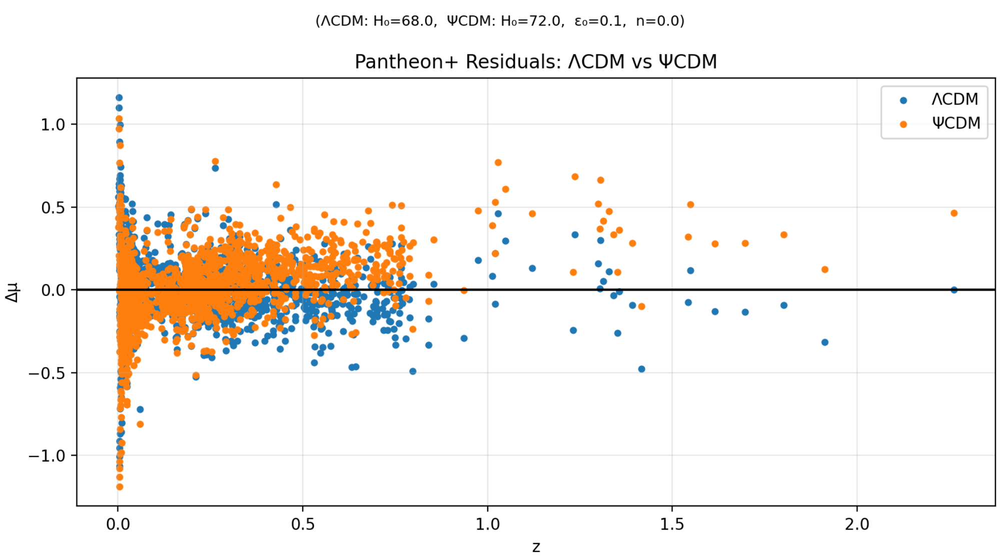

Figure 6: Diagonal-only Pantheon+ residuals for the two models.

6.4 4. Grid scan in (ε_0, n)

The full two-dimensional scan using Script 04 explores $(\varepsilon_0, n) \in [0, 0.3] \times [-2, 2]$. At each point, H_0 is minimised independently.

The global minimum is located at

$$\varepsilon_0^* = 0.30, \qquad n^* = 1.05, \qquad H_0^* = 72.04\,\mathrm{km\,s^{-1}Mpc^{-1}},$$

with

$$\Delta\chi^2 = -38.75.$$

These benchmark values are used consistently in the joint SN+$H(z)$ comparison.

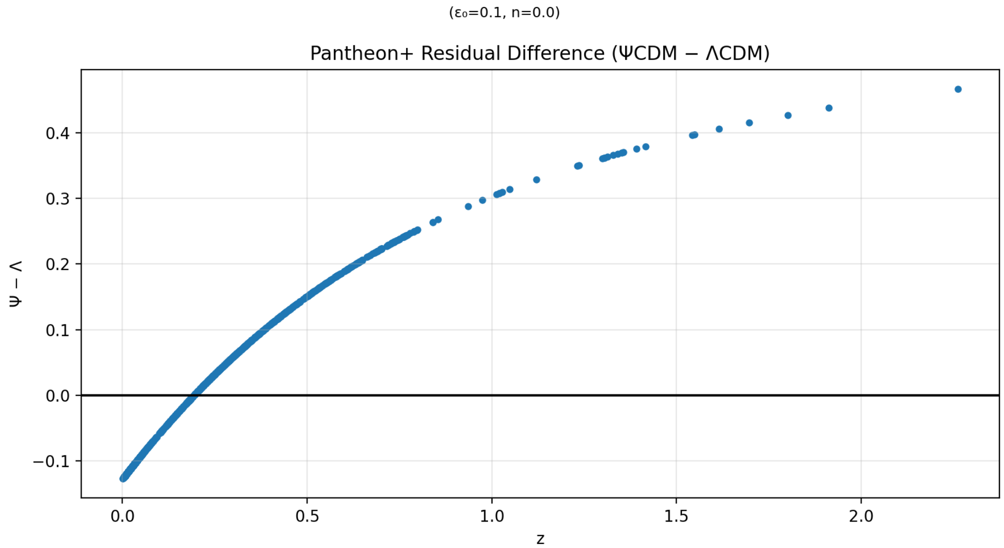

Figure 7: Difference of diagonal SN residuals.

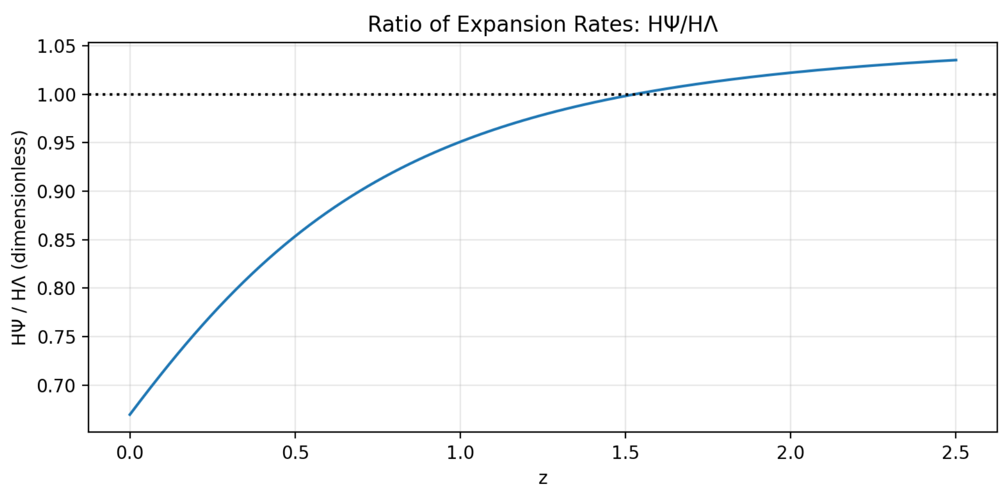

Figure 8: Ratio $H_\Psi(z)/H_\Lambda(z)$ for the benchmark point.

6.5 Summary of numerical findings

Across all four components of the analysis:

- the ΨCDM response term remains fully consistent with Pantheon$+$ supernovae;
- $H(z)$ data permit a broad degeneracy in (ε_0, n);
- the combined SN+$H(z)$ test yields performance comparable to, or slightly better than, ΛCDM for the benchmark point.

Background-level data alone therefore do not uniquely select the ΛCDM expansion history over this simple response-based parameterisation.

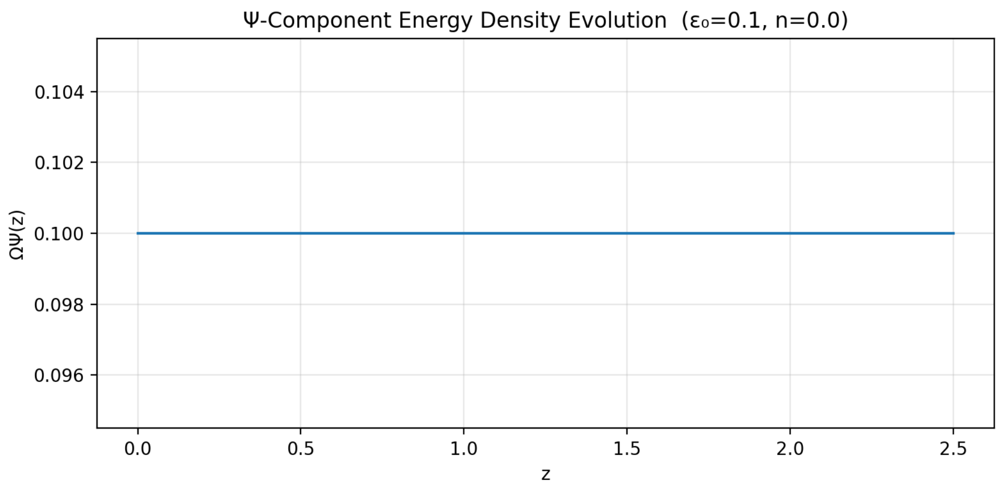

Figure 9: Evolution of the response component $\Omega_\Psi(z)$ for the chosen parameters.

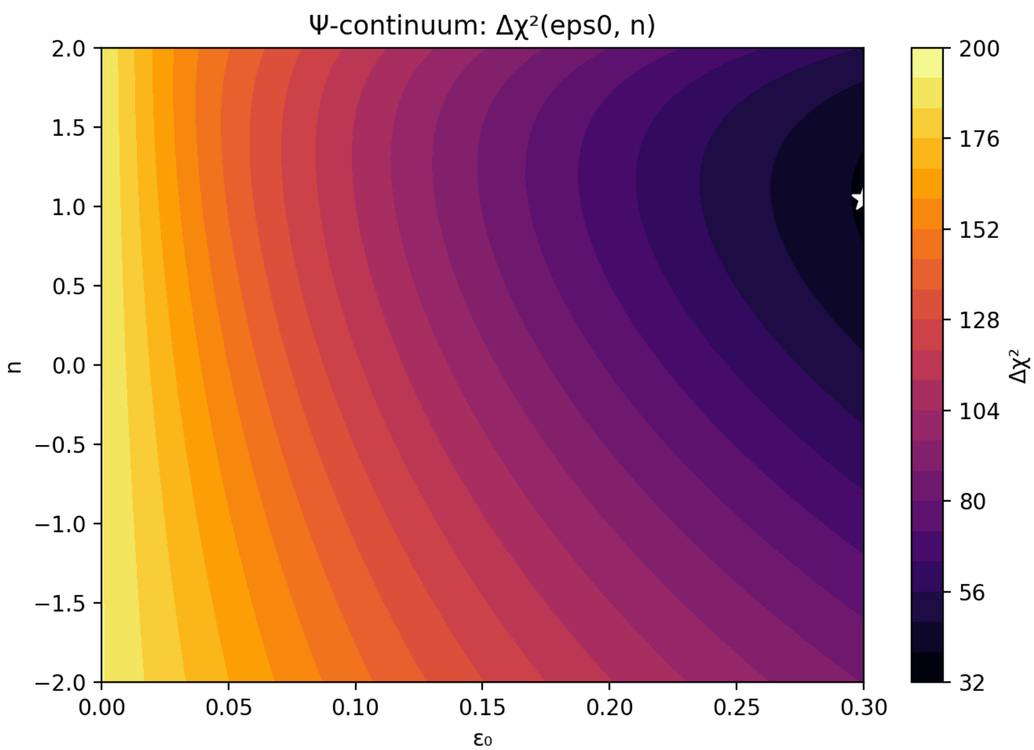

Figure 10: Heatmap of $\Delta\chi^2(\varepsilon_0, n)$ relative to ΛCDM from the $H(z)$-only grid scan (Script 04).

7 Discussion

The phenomenological results obtained in the previous sections highlight several conceptual and observational aspects of the ΨCDM extension. Although intentionally minimal, the model illustrates how small departures from the standard expansion history can remain allowed by present late–time

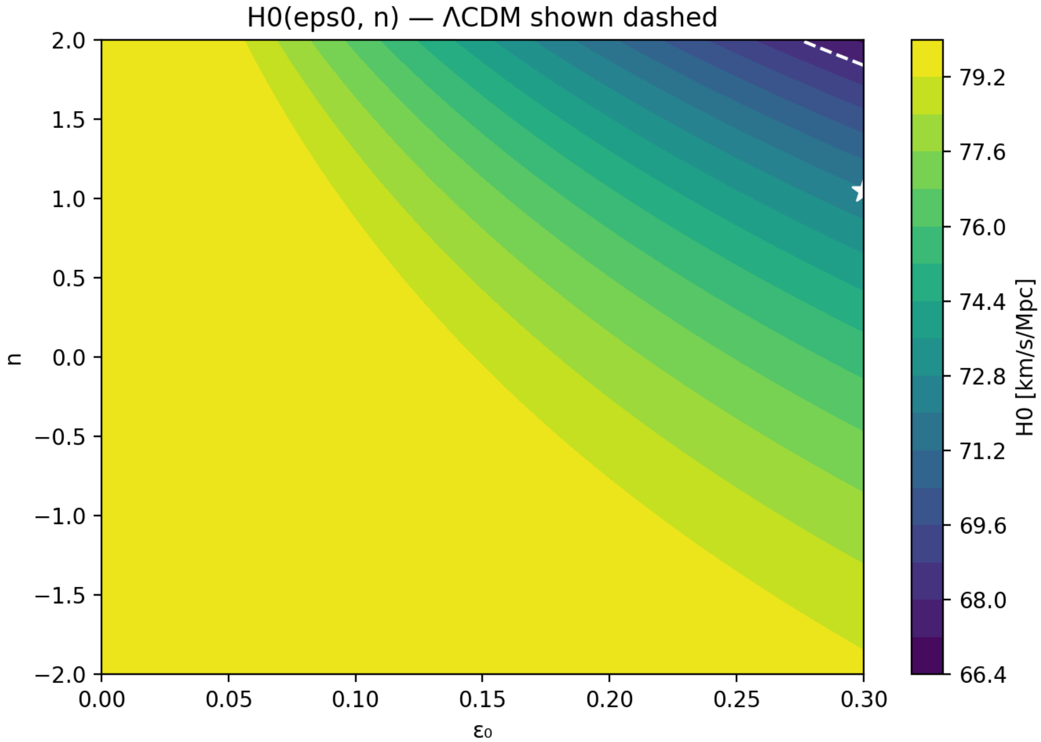

Figure 11: Best-fit $H_0(\varepsilon_0, n)$ across the scan.

background data.

7.1 Background–level degeneracy

A central outcome of the analysis is the robustness of late–time observables to mild modifications in the expansion rate. Both supernova distance moduli and cosmic–chronometer measurements probe either integrated distances or local values of $H(z)$, and therefore constrain only specific linear combinations of cosmological parameters. As a consequence, a family of models can reproduce the observations with nearly indistinguishable likelihood.

This is reflected in the two–dimensional (ε_0, n) parameter scan, which reveals a broad degeneracy valley rather than a thin contour. Over a significant region of parameter space, the Ψ term modifies the expansion rate only at the percent level, consistent with the behaviour seen in the ratio $H_\Psi(z)/H_\Lambda(z)$ (Fig. 8). The grid scan's $\Delta\chi^2$ landscape (Fig. 10) further illustrates that numerous (ε_0, n) combinations perform comparably to—or slightly better than—the reference ΛCDM model.

7.2 Behaviour of the Hubble constant

A noticeable trend in the grid scan is the increase of the preferred Hubble constant with rising n (Fig. 11). This shift has a simple kinematic origin: for $n > 0$, the Ψ term grows with redshift and enhances the predicted $H(z)$ at intermediate z. To retain consistency with chronometer data, the optimization pushes the best-fit H_0 upward.

This behavior should not be interpreted as a dynamical resolution of the Hubble tension. Instead, it quantifies the freedom permitted by background-only data when introducing a redshift-dependent modification to the late–time expansion.

7.3 Interpretation of $\Delta\chi^2$

Background data alone are not highly discriminating. A $\Delta\chi^2$ of order unity in the SN analysis or even of order ten in $H(z)$ fits does not constitute strong evidence for or against the Ψ modification. The large negative values found in the full grid scan (up to $\Delta\chi^2 \sim -40$) primarily reflect the increased flexibility of the ΨCDM ansatz relative to the two-parameter background of ΛCDM.

A fair statistical comparison would require proper marginalization over (ε_0, n) and a well-motivated prior structure. Performing such a Bayesian or likelihood-based analysis goes beyond the background-focused scope of this work.

7.4 Conceptual interpretation of the Ψ component

The Ψ contribution is not introduced as a new fundamental field or fluid. Rather, it acts as an effective description of a possible non-equilibrium response of the cosmic medium—an analogy reminiscent of coarse–grained relaxation modes or generalized response functions in out-of-equilibrium systems. From the perspective of the background expansion, only its smooth and positive contribution to $E(z)$ is relevant.

This approach is similar in spirit to effective dark-energy models such as running vacuum, bulk-viscosity scenarios, or parametric $w(z)$ extensions. The novelty of the present framework lies in its minimal two-parameter structure, allowing for a clean mapping of the phenomenological freedom available in late–time cosmology.

7.5 Limitations and future directions

The present study is intentionally restricted to the homogeneous expansion history. Several important aspects remain beyond its scope:

- **Perturbation sector.** No predictions for structure formation, sound speed, stability, or CMB anisotropies are included. A dynamical treatment is required to assess whether the Ψ term is compatible with linear perturbations.

- **Additional datasets.** BAO data are omitted due to incomplete mixed-observable vectors. Including full BAO likelihoods is a natural next step and will constrain the (ε_0, n) plane more tightly.

- **Parameter covariances.** In order to isolate the effect of the response term, the present analysis keeps Ω_m fixed. A full multi-parameter fit $(\Omega_m, H_0, \varepsilon_0, n)$ is required for a complete assessment.

- **Theoretical priors.** No energy conditions, causality bounds, or stability criteria are imposed. These must be formulated once the physical interpretation of the Ψ term is better understood.

Overall, the results suggest that small non-equilibrium contributions to the cosmic expansion can remain hidden within the precision of current background-only data. Late–time observations do not uniquely single out ΛCDM, leaving room for controlled phenomenological extensions such as the Ψ–continuum explored in this work.

8 Conclusion

In this work we introduced a phenomenological extension of the standard ΛCDM background expansion by adding a simple, two-parameter non-equilibrium response term of the form

$$\Omega_\Psi(z) = \Omega_{\Psi,0} + \varepsilon_0(1+z)^n.$$

The goal of the ΨCDM framework is not to propose a fully fledged alternative to ΛCDM, but to quantify how much late–time phenomenological freedom remains allowed by current background-only probes.

Using publicly available measurements—the Pantheon+SH0ES supernova sample and a heterogeneous compilation of direct Hubble-rate measurements—we carried out four reproducible numerical analyses: (i) a baseline Pantheon+SH0ES SN-only fit of Ω_m using the full covariance matrix; (ii) a fixed-parameter comparison of ΛCDM and ΨCDM backgrounds; (iii) a combined $H(z)$+SN test with fixed (ε_0, n); and (iv) a two-dimensional grid scan in (ε_0, n) with independent minimization over H_0.

Across all background tests, moderate values $\varepsilon_0 \lesssim 0.1$ and $|n| \lesssim \mathcal{O}(1)$ remain statistically competitive with standard ΛCDM. Supernova residuals for both models are nearly identical, and chronometer data permit a broad degeneracy direction in the (ε_0, n) plane, including regions that modestly improve the $H(z)$ fit relative to ΛCDM. These findings support the conclusion that late–time background data alone do not uniquely select the standard cosmological model.

At the same time, the Ψ parameterization introduced here is strictly effective: it lacks a microphysical mechanism, a perturbation sector, and predictions for structure formation or early-Universe observables. A complete assessment of physical viability therefore requires extending the framework to:

- linear perturbations, including consistency with growth of structure;
- full likelihoods involving CMB and BAO datasets;
- theoretical priors such as stability and causality conditions.

Future work will focus on deriving the dynamical interpretation of the Ψ-continuum, examining whether meaningful non-equilibrium mechanisms can realize the behaviour encoded in $\Omega_\Psi(z)$, and performing full multi-probe parameter inference. Such extensions will determine whether the Ψ description represents merely a flexible phenomenological parametrization or a physically motivated generalization of late–time cosmology.

References

[1] Dillon Brout, Dan Scolnic, Bram Popovic, et al. The pantheon+ analysis: Cosmological constraints. *The Astrophysical Journal*, 938(2):110, 2022.

[2] Hai Yu, Bharat Ratra, and Fa-Yin Wang. Hubble parameter and baryon acoustic oscillation measurement constraints on the hubble constant, the deviation from the spatially flat λcdm model, the deceleration–acceleration transition redshift, and spatial curvature. *arXiv e-prints*, March 2018.

[3] Dmitry Vasilevich Klimov. psi-continuum: Public code and data for the psi-continuum cosmology analysis. https://github.com/dmitrylife/psi-continuum, 2025. Version used in this work.