ROB 101 - Computational Linear Algebra
HW #3

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Due 9 PM = 21:00 ET on Thurs, Sept 23, 2021

There are six (6) HW problems plus a *jupyter notebook* to complete and turn in. The drill problems this week go against the spirit of ROB 101 in that you are doing by hand things that belong on a computer. However, humans learn better by working out simple examples by hand!

1. Read Chapters 4 and 5 of our ROB 101 Booklet, `Notes for Computational Linear Algebra`. Based on your reading of the Chapters, summarize in your own words:

   (a) Choose a chapter and summarize its purpose;

   (b) Two things you found the most challenging or the most interesting.

2. If the sizes for the indicated vectors or matrices are compatible for multiplication, then perform the multiplications using "standard multiplication." Otherwise, state why the multiplication is not defined.

   (a) $M_a = \begin{bmatrix} -2 & 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

   (b) $M_b = \begin{bmatrix} 1 & -2 \\ 4 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

   (c) $M_c = \begin{bmatrix} 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 4 & -3 \end{bmatrix}$

   (d) $M_d = \begin{bmatrix} -2 & 2 & 7 \\ -2 & 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 4 & -3 \\ 5 & 2 \end{bmatrix}$

   (e) $M_e = \begin{bmatrix} 1 & -2 \\ 4 & -3 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & 2 & 7 \\ -2 & 2 & 7 \\ 1 & -5 & 0 \end{bmatrix}$

3. Let $A = \begin{bmatrix} 1 & -2 & 0 & 4 \\ 4 & -3 & 2 & -1 \\ 5 & 2 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} -2 & 2 \\ 2 & 7 \\ 1 & 4 \\ -2 & -3 \end{bmatrix}$

   (a) Compute a_{12}, the element of A on the first row and second column. Show clearly the rows and columns you are multiplying to form a_{12}.

   (b) Compute a_{32}. Show clearly your work.

   (c) Does A have a 23-element? Explain.

4. Re-do four of the matrix multiplications from Prob. 2, but this time, doing the matrix multiplication as the "sum of columns times rows." In each case, show the intermediate matrices that you form before adding them up to get the final answer.

   (a) Compute M_a

(b) Compute M_b

(c) Compute M_c

(d) Compute M_d

5. Let $A = \begin{bmatrix} 2 & -2 & 1 \\ 4 & 0 & 0 \\ 0 & -1 & 3 \end{bmatrix}$

(a) Let C be the first column of A and let R be the first row of A. Compute $\text{Temp} := A - C \cdot R$.

(b) Let C and R be as above, and then rename C after normalizing by its first element: $C = C/C[1]$. Compute $\text{Temp} := A - C \cdot R$.

(c) Why did we scale C so that its first entry became 1.0?

(d) Now suppose $\text{Temp} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & -1 & 3 \end{bmatrix}$. Define C and R so that the only non-zero element of $\text{Temp} - C \cdot R$ is in the bottom right-hand corner of the matrix. To be extra clear, Temp should have the form

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{bmatrix},$$

where $*$ is some number.

6. LU factorization and solutions of linear equations.

(a) You are given the set of linear equations

$$\begin{aligned} 3x_1 - 4x_2 &= -2 \\ -2x_1 + 10x_2 &= 16. \end{aligned}$$

Write them in the form $Ax = b$. Then use the algorithm from the class notes to do the LU factorization of $A = L \cdot U$. Finally, solve the equation $L \cdot Ux = b$ using a combination of forward and back substitution.

(b) Consider the system of linear equations

$$\underbrace{\begin{bmatrix} 1 & -2 & 1 \\ 2 & 0 & 0 \\ 3 & -3 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}}_{b}, \tag{1}$$

and you are given the LU Factorization of $A = L \cdot U$ is

$$L = \begin{bmatrix} 1.0 & 0.0 & -0.0 \\ 2.0 & 1.0 & -0.0 \\ 3.0 & 0.75 & 1.0 \end{bmatrix} \quad U = \begin{bmatrix} 1.0 & -2.0 & 1.0 \\ 0.0 & 4.0 & -2.0 \\ 0.0 & 0.0 & -0.5 \end{bmatrix}$$

Solve for x.

This is the end of the drill problems. The second part of the HW set is once again a jupyter notebook. **Please go to** the course Canvas Site and complete the assignment titled "juliahw3".

Hints

Hints: Prob. 2 Write down the sizes of each given vector or matrix and then check how the columns and rows match up.

Hints: Prob. 3 Suppose $A = B \cdot C$ and the number of columns of B is equal to the number of rows of C. Then $a_{ij} = b_i^{\text{row}} \cdot c_j^{\text{col}}$, where b_i^{row} is the i-th row of B and c_j^{col} is the j-th column of C.

Hints: Prob. 4 Suppose $A = B \cdot C$, where B is $n \times k$ and C is $k \times m$. Then

$$A = \sum_{i=1}^{k} b_i^{\text{col}} \cdot c_i^{\text{row}}$$

Hints: Prob. 6-(a) If $Ax = b$ and $A = L \cdot U$ is an LU Factorization of A, then you solve for x by

- Solve first $L \cdot y = b$ to obtain y
- Next solve $U \cdot x = y$.

Then, $Ax = (L \cdot U)\, x = L \cdot (Ux) = L \cdot y = b$