ROB 101 - Computational Linear Algebra
HW #9

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Due 9 PM = 21:00 ET on Thurs, Dec. 2, 2021

There are six (6) HW problems plus a *jupyter notebook* to complete and turn in. The drill problems this week go against the spirit of ROB 101 in that you are doing by hand things that belong on a computer. However, humans learn better by working out simple examples by hand!

1. Read Chapter 12 of our ROB 101 Booklet, `Notes for Computational Linear Algebra`. Based on your reading of the Chapter, summarize in your own words:

   (a) The purpose of the Chapter;

   (b) Two things you found the most challenging or the most interesting.

2. Your objective is to compute

$$x^* = \arg\min_{x \in \mathbb{R}} f(x)$$

   for $f(x) = (x+2)(x-2) = x^2 - 4$. To make the calculations a snap, we'll do the Gradient Descent Algorithm by hand with the a priori knowledge from Calculus that the analytical derivative is $\frac{df(x)}{dx} = 2x$. This way, you have no approximate derivatives to compute.

   (a) Starting with the initial condition $x_0 = 3$ and using a step size of $s = 1.0$, perform Gradient Descent and fill in Table 1.

   (b) Starting with the initial condition $x_0 = 3$ and using a step size of $s = 0.25$, perform Gradient Descent and fill in Table 2.

Write down the algorithm and then fill in the tables. You do not have to show any intermediate computations.

Table 1: Gradient Descent Results.

k	x_k	$-\frac{df(x_k)}{dx}$
0	3	
1		
2		
3		
4		Leave Blank

Table 2: Gradient Descent Results.

k	x_k	$-\frac{df(x_k)}{dx}$
0	3	
1		
2		
3		
4		Leave Blank

3. Your objective is to compute

$$x^* = \arg\min_{x \in \mathbb{R}} f(x)$$

for the functions given below by applying Newton's Method to the fist derivative. To make the calculations a snap, we give you analytical expressions for all of the required derivatives.

(a) Using the formulas in the book for a step size of $s = 1.0$, complete Table 3 for the function $f(x) = (x+2)(x-2) = x^2 - 4$ by applying Newton's Method to the first derivative. To minimize the number of calculations, we give you the derivatives

- $\frac{df(x)}{dx} = 2x$
- $\frac{d^2 f(x)}{dx^2} = 2$

(b) Using the formulas in the book for a step size of $s = 1.0$, complete Table 4 for the function $f(x) = x^4 - x^3 + x^2 - 4$ by applying Newton's Method to the first derivative. To minimize the number of calculations, we give you the derivatives

- $\frac{df(x)}{dx} = 4x^3 - 3x^2 + 2x$
- $\frac{d^2 f(x)}{dx^2} = 12x^2 - 6x + 2$

(c) Did you notice anything special about part (a)?

(d) In part (b), did you converge to a local min or a local max?

Write down the algorithm and then fill in the tables. You do not have to show any intermediate computations.

Table 3: Second-order Optimization.

k	x_k	$-\frac{df(x_k)}{dx}$	$\frac{d^2 f(x_k)}{dx^2}$
0	3		
1			
2			
3			
4		Skip	Skip

Table 4: Second-order Optimization.

k	x_k	$-\frac{df(x_k)}{dx}$	$\frac{d^2 f(x_k)}{dx^2}$
0	3		
1			
2			
3			
4		Skip	Skip

4. Compute a numerical approximation of the gradient, using symmetric differences, for the function

$$f(x_1, x_2) = x_2 \sin(x_1) + x_1 x_2$$

at the following points:

(a) $x_0 = \begin{bmatrix} -1.0 \\ 1.0 \end{bmatrix}$

(b) $x_0 = \begin{bmatrix} 0.0 \\ -1.0 \end{bmatrix}$

You are free to chose $h > 0$ as you wish. Do report your value for h. Also, when you form the gradient, ask yourself, "Is it a row vector or a column vector? ".

5. Compute a numerical approximation of the Hessian using symmetric differences for the function

$$f(x_1, x_2) = x_2 \sin(x_1) + x_1 x_2$$

at the following point: $x_0 = \begin{bmatrix} \pi/2 \\ -1.0 \end{bmatrix}$. Use $\delta = h = 0.1$ for the symmetric differences. Use your favorite formula from Chapter 12. Search on the word "Hessian" in the PDF of the book!

[Remark: Don't forget the term $4h\delta$ in the denominator, or you will estimate that the Hessian is the zero matrix!

6. This is your last drill problem of the term. You challenge is to make up a problem based on material in Chapter 12 and solve it. Don't go crazy! We do not want to work very hard to check your answer. Clear enough? The problem should be appropriate for hand computations.

This is the end of the drill problems. The second part of the HW set is once again a jupyter notebook. **Please go to** the course Canvas Site and complete the assignment titled "juliahw9". It is delightfully short! We kind of over did it on juliahw8, so we are making it up to you!

Hints

There are no hints!