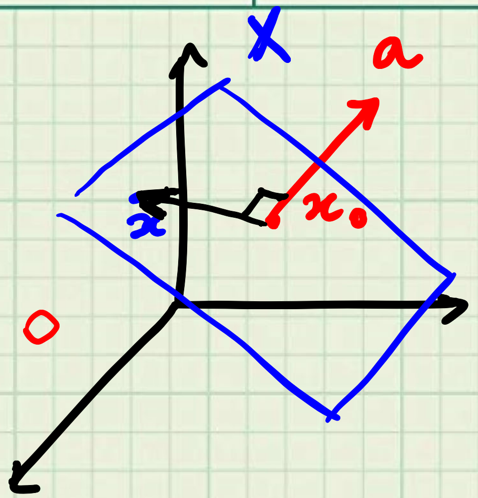


$a \cdot (x - x_0) = 0$

$a \cdot x = a x_0 =: d$

$a^T x = d \quad , d \in \mathbb{R}$

$a, x \in \mathbb{R}^3$

* normal vector

$\rightarrow a^T x = a_1 x_1 + a_2 x_2 + a_3 x_3 = d \rightarrow$ distance from the origin

Any point such as $x \in \mathbb{R}^3$ that satisfies the plane equation lies on the plane.

- Example: Given $a = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $x_0 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, form a plane in $\mathbb{R}^3$.

$a \cdot (x - x_0) = 0$

$d = a \cdot x_0 = 1(0) + (-2)(1) + 3(-1) = -5$

$$\boxed{1x_1 - 2x_2 + 3x_3 = -5}$$

- Hyperplane: generalization to $\mathbb{R}^n$

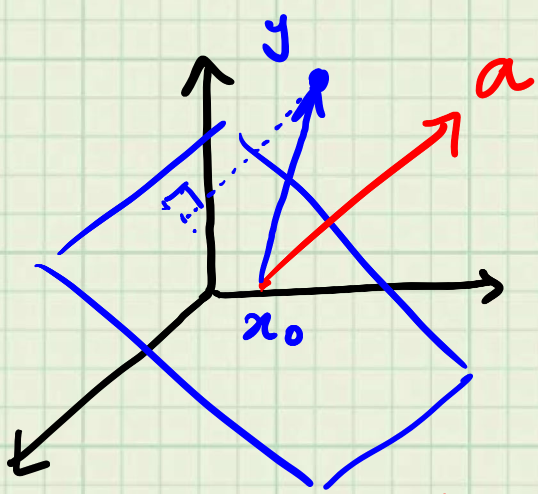
A hyperplane is a set of points in $\mathbb{R}^n$ such that $a^T(x - x_0) = 0$.

$x, x_0, a \in \mathbb{R}^n$.

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = a^T x_0 =: d$$

$d \in \mathbb{R}$

- Distance of a point to the hyperplane



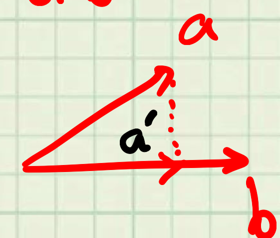
any point on the plane

$$a^T(y - x_0) = a^T y - \underbrace{a^T x_0}_{=d}$$

is not zero because y is not on the plane

$$D = \frac{|a^T y - d|}{\|a\|}, \quad \|a\| \neq 0$$

$a.b$



$e_b = \frac{b}{\|b\|}$

unit vector along b

$a' = (a.b)\, e_b$

- Quadratic programming

A Quadratic program (QP) is like this:

Useful Fact about QPs

We consider the QP

$$
\begin{aligned}
x^* = \quad & \underset{x \in \mathbb{R}^m}{\arg\min} \quad \frac{1}{2} x^\top Q x + q x \\
& A_{in} x \preceq b_{in} \\
& A_{eq} x = b_{eq} \\
& lb \preceq x \preceq ub
\end{aligned}
\tag{11.55}
$$

and assume that Q is symmetric ($Q^\top = Q$) and **positive definite**[a] ($x \neq 0 \implies x^\top Q x > 0$), and that the subset of $\mathbb{R}^m$ defined by the constraints is non empty, that is

$$
C := \{x \in \mathbb{R}^m \mid A_{in} x \preceq b_{in},\ A_{eq} x = b_{eq},\ lb \preceq x \preceq ub\} \neq \emptyset. \tag{11.56}
$$

Then x^* exists and is unique.

[a] Positive definite matrices are treated in Chapter A.3.

* A minimizer of (11.55) is feasible if it satisfies all constraints.

Example 1:

$J(x_1, x_2) =$
$(x_1 - 2)^2 + (x_2 - 1)^2$

$x_1 + 2x_2 \leq 12$

$3x_1 + 3x_2 \leq 25$

$x_1 \leq 7$

$x_2 \leq 5$

$x_1 \geq 0$

$x_2 \geq 0$