ROB 101 - Computational Linear Algebra
HW #7

Profs Grizzle and Ghaffari

Due 9 PM = 21:00 ET on Thurs, November 4, 2021

There are five (5) HW problems plus a *jupyter notebook* to complete and turn in. Problem 5 is worth double points.

1. Read Chapter 10 of our ROB 101 Booklet, `Notes for Computational Linear Algebra`. Based on your reading of the material, summarize in your own words:

   (a) The purpose of the chapter;

   (b) Two things you found the most challenging in the Chapter or the most interesting.

2. The vectors $v_1 = e_1 - e_2$ and $v_2 = e_1 + e_2$ form a basis of $\mathbb{R}^2$. Make a sketch of the corresponding coordinate grid (by hand or with Julia will earn the same points) and also label with a big dot the point (-1,3) corresponding to $-v_1 + 3v_2$.

3. Find a basis for the two-dimensional subspace of $\mathbb{R}^4$ defined by

$$V := \left\{ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4 \mid x_1 + x_3 = 0, \text{ and } x_2 - x_4 = 0 \right\}.$$

   Explain your reasoning.

4. For the 2×2 matrix A, indicate which of the following vectors are eigenvectors, and for those that are eigenvectors, give the corresponding eigenvalue.

$$A = \begin{bmatrix} 8.0 & -7.0 \\ -14.0 & 1.0 \end{bmatrix}$$

   (a) $v_1 = \begin{bmatrix} 1.0 \\ 2.0 \end{bmatrix}$

   (b) $v_2 = \begin{bmatrix} 0.0 \\ 3.0 \end{bmatrix}$

   (c) $v_3 = \begin{bmatrix} -1.0 \\ 1.0 \end{bmatrix}$

   Remark: The problem can be worked using the definition of eigenvectors and eigenvalues. You do not have to compute $\det(\lambda I - A)$ nor find vectors in null$(A - \lambda I)$.

5. Create a Cheat Sheet for the Linear Algebra portion of ROB 101, which means Chapters 2 through 10.

   (a) This problem will count double. Think of it as Problems 5 and 6 combined.

   (b) You can write your solution out and scan it as usual for upload to GradeScope or you can typeset it. It's your call.

   (c) You can use any format you wish. Feel free to make/copy images of formulas from the pdf of the book and paste them into a word document. In Windows, I use `Snip & Sketch` as my tool for doing that.

(d) You are not expected to cover everything. Make judicious choices. While you might think to organize your material around subject headings such as *Facts on Equations*, *Matrix Facts*, *Cool Algorithms*, *etc.*, **you are not required to do so**.

(e) If you are struggling to bring the various pieces of the course material together, then this problem is your opportunity to work on that!

(f) Even if you are sailing through the material, just imagine how useful a good quality cheat sheet may be when you need to use Linear Algebra in a future course!

This is the end of the drill problems. The second part of the HW set is once again a jupyter notebook. **Please go to** the course Canvas Site and complete the assignment titled "juliahw7".

Hints

Hints: Prob. 2 See Figures 10.1 and 10.2 in the book.

Hints: Prob. 3 See Example 10.1 in the book. It's solution shows you how to find candidate basis vectors. Don't forget that you need to show the vectors are linearly independent and their span is all of V.

Hints: Prob. 4 Suppose that we are given $v_4 = \begin{bmatrix} -3.0 \\ 3.0 \end{bmatrix}$. We compute $Av_4 = \begin{bmatrix} -45.0 \\ 45.0 \end{bmatrix}$. We then immediately observe that

$$Av_4 = 15v_4$$

and conclude that v_4 **is an eigenvector and that the corresponding eigenvalue is** $\lambda = 15$.

Suppose instead that we are given $v_5 = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$. We compute $Av_5 = \begin{bmatrix} 1.0 \\ -13.0 \end{bmatrix}$. We then ask ourselves is there a constant λ such that $Av_5 = \lambda v_5$? In other words, is there a solution to

$$Av_5 = \begin{bmatrix} 1.0 \\ -13.0 \end{bmatrix} = \begin{bmatrix} \lambda \\ \lambda \end{bmatrix} = \lambda \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} = \lambda v_5?$$

The answer is clearly **NO** because we cannot have $\lambda = 1.0$ and $\lambda = -13.0$ at the same time! Hence, v_5 **is not an eigenvector**. And, because it is not an eigenvector, there is no eigenvalue to give!