Intensity-invariant nonlinear filtering for detection in camouflage

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We introduce a method based on an orthonormal vector space basis representation to detect camouflaged targets in natural environments. The method is intensity invariant so that camouflaged targets are detected independently of the illumination conditions. The detection technique does not require one to know the exact camouflage pattern, but only the class of patterns (e.g., foliage, netting, woods). We use nonlinear filtering and the calculation of several correlations. The nonlinearity of the filtering process also allows high discrimination against false targets. Several experiments confirm the target detectability where strong camouflage might delude even human viewers.

OCIS codes: 100.5010, 070.5010.

1. Introduction

In most pattern recognition problems, objects must be recognized under different illuminations, and discrimination between different objects must be maintained. When the illumination conditions of a scene cannot be controlled, the outputs for similar targets with different illuminations can be quite variable. Many pattern recognition techniques use the correlation operation to detect targets unsegmented from the scene. However, if a given target has a different illumination, the correlation peak height changes. One illumination model that is widely used has a target that is multiplied by an unknown constant factor so that the correlation peak will change by the same amount; in such cases dark targets can be missed. We consider two types of intensity transformation: multiplicative and additive. The additive transformation occurs when the light source intensity changes, thus affecting in the same way the light reflected from the object. Additive-intensity transformations may occur, for example, when different camera settings are used or when scattered light enters the camera. That may introduce a constant difference between the intensity distributions for two identical objects. All the intensity transformations considered here are uniform, which means that every part of an object is affected by the same intensity transformation.

Another way to understand the correlation operation is to consider it as an inner product between two functions, the object and the reference functions. These functions may be considered as vectors in a Hilbert space. From the point of view of vector spaces, intensity-invariant pattern recognition consists of recognizing vectors independently of their length, which can be viewed as an angle measurement between vectors in vector spaces. This angle provides a measure of the similarity between the object and the reference functions. Dickey and Romero[1] defined a normalized correlation as a normalization of the inner product represented by the correlation integral. This method was introduced using the Cauchy–Schwarz inequality and yields correlation peak values that are invariant under a multiplicative factor. Kotynski and Chalasinska-Macukov[2] generalized this algorithm by using the Hölder inequality and ended up with a whole new family of solutions of which the multiplicative is a special case. Arsenault and Belisle[3] showed that it is possible to achieve additive-intensity invariance using only one correlation. They used a composite[4] or synthetic discriminant filter[5] constructed to recognize the target but also to discriminate against the target support (a binary image equal to unity, where the target is present and equal to zero everywhere else). Arsenault and Belisle[3] developed a method invariant to changes of orientation and multiplicative-intensity changes

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Received 24 January 2005; revised manuscript received 18 April 2005; accepted 19 April 2005.
0003-6935/05/265483-08$15.00/0

by combining multiple correlation planes resulting from correlations with different circular harmonic components.[6] Garcia-Martinez *et al.*[7] proposed a method to achieve intensity-invariant recognition using the sliced orthogonal nonlinear generalized correlation.[8,9] Arsenault and Lefebvre[10] used a homomorphic transformation to change a multiplicative-intensity problem into an additive-intensity problem that can be addressed with the synthetic discriminant filter mentioned above. Zhang and Karim[11] have pointed out that the morphological correlation[12] is intensity invariant when the object is brighter than the reference. Chalasinska-Macukow *et al.*[13] used pure phase correlation to achieve linear-intensity invariance, but this works well only for segmented images.

Lefebvre *et al.*[14] defined a nonlinear filtering method known as the locally adaptive contrast-invariant filter (LACIF), which is invariant under any linear-intensity transformation. This LACIF operation uses three correlation operations involving local statistics and nonlinearities. It was applied directly to scenes containing unsegmented targets. One of the advantages of the LACIF method is that no *a priori* information about the constant values involved in the linear illumination model is assumed. The LACIF method can be combined with synthetic discrimination filters to achieve both illumination invariance and out-of-plane rotation invariance.[15] Recently, in Ref. 16 the authors generalized the LACIF for situations in which a linear-intensity gradient across an object is present. It is interesting to consider the LACIF technique in the context of a vector space interpretation. In this paper we use that interpretation to define an orthonormal basis that includes the intensity transformations and we will show that the LACIF from Ref. 14 can be obtained using this basis. The original mathematical derivation of the LACIF[14] was based on the combination of a synthetic discriminant function filter, a normalization idea, and a good intuition in vector space interpretation. However, there was no direct mathematical derivation. In this paper we show that, when we apply the Gram–Schmidt orthonormalization to the same vector space, we can directly derive the LACIF expression.

In addition to the definition of an orthonormal vector space basis used to detect objects independently of the illumination, we have studied the recognition of camouflaged targets. To identify targets that are partially obscured by man-made or natural obscurants, such as camouflage netting, foliage, smoke, or fog, is another particularly challenging issue in real pattern recognition. Recent advances in laser- and computer-related systems have improved the capability to identify targets behind such obscuration.[17] However, those methods deal with imaging more than pattern recognition. Other references for evaluating camouflage effectiveness can be found.[18] Other experimental frameworks for evaluating metrics for the search and discrimination of camouflage patterns are given in Ref. 19. The authors asked human observers to view image stimuli consisting of various target patterns embedded within various background patterns. These psychophysical experiments provided a quantitative basis for the comparison of human judgments to the computed values of target distinctness metrics. However, in all those methods a segmentation of the target from the background is required and the influence of illumination is not considered.

In this paper we show how the vector space representation allows detection through camouflage while maintaining intensity invariance. The method is based on nonlinear filtering in which we use certain correlations after decomposing the target and the camouflage pattern into a specific orthonormal basis. Before describing the filtering process, we review in Section 2 the importance of using orthogonal bases in pattern recognition. As we noted above, an object can be viewed as a vector in a given vector space, and the metric used to measure the similarities between different vectors is simply a correlation operation. The measure of similarity becomes interesting when the object to detect has varied its intensity or when it is camouflaged by another pattern. In Section 3 we show that the LACIF can be obtained when an orthonormal basis is used. The application of the method to camouflaged targets is given in Section 4. In Section 5 we show the results for the camouflaged targets.

2. Basis Functions for Intensity Transformations

As pointed out in Section 1, images can be considered as vectors in a Hilbert space. Moreover, any specific vector can be expressed in terms of an orthonormal basis. For simplicity, we express all functions in one dimension, and the generalization to two-dimensional images is trivial (two-dimensional images may be expressed as one-dimensional vectors by concatenating all the rows of the image into a single row). We begin by expanding $f(x)$, which is the reference image, into a set of orthonormal basis functions or basic images $\{\phi_j(x)\}$:

$$f(x) = \sum_j a_j \phi_j(x), \tag{1}$$

where

$$\langle \phi_j \mid \phi_i \rangle = (\phi_j {}^* \phi_i)(0,\ 0) = \int \phi_j(x)\phi_i(x)\mathrm{d}x = \delta_{ji}. \tag{2}$$

Note that Eq. (2) implies the orthonormalization of the basis and the integral represents the correlation (*) or inner product at the origin between the two elements of the basis.

Let Eq. (1) represent a linear intensity transformation, where the target is multiplied by constant factors a_j. Let us assume also that $s(x)$ is the input scene, so to obtain an intensity-invariant pattern recognition, the invariant operation that we are looking for will give the maximum value (if normalized, this will be equal to unity) when the intensity of the target is changed according to Eq. (1) and less than one if it is not. This means that if

$$s(x) = \sum_j b_j \phi_j(x),$$

then the recognition operation that we define, for instance, with the Ξ symbol, will be $s(x)\ \Xi\ f(x) = 1$. On the contrary, if $s(x)$ contains components outside this basis $\{\phi_j\}$, then $s(x)\ \Xi\ f(x) < 1$.

The last comments can be also expressed as follows: We stated in Section 1 that intensity-invariant correlation methods can be viewed as an angle measurement between two vectors. In the presence of a multiplicative factor $[af(x)]$, where $f(x)$ is the target and a is a constant, the normalized correlation[1] yields the value of the cosine of the angle between the input target and the reference. Because a change of intensity implies a change in the length of the vector, two equal targets with different illuminations will be parallel, so the normalized correlation and the cosine will be equal to one. On the other hand, the normalized correlation between a target different from the reference will give a cosine value smaller than one. Lefebvre *et al.*[14] proposed an extension of the previous multiplicative-intensity transformation to include an additive term. Indeed, if the intensity transformation of the targets is $af(x) + b$, where a and b are unknown parameters that are constant over a single target, the LACIF method is an angular measurement between two vectors that are projected onto the same plane. This plane is orthogonal to the region of support of the reference object. So for the LACIF, as we will show in Section 3, the orthogonal basis is expressed by using two vectors. One is the region of support of the reference object, and the other must be a function that is orthonormal to this region of support.

Summarizing this section and to explain the motivation, we emphasize that intensity invariance must be achieved even when the coefficients that multiply the basis images are unknown so that the method will work for any values that are constant over an area the size of the reference target. In other words, the intensity invariance problem is solved by determining whether an object is a linear combination of that specific basis, that is, whether the unknown object belongs to the subspace spanned by those basis images. If we are able to obtain the operation Ξ, we have defined an intensity-invariant recognition method. In Section 3 we will show the mathematical expression for this Ξ operation.

3. Intensity-Invariant Recognition by Use of an Orthonormal Basis in Vector Space

A linear transformation of intensity over a target can be expressed as

$$f'(x) = af(x) + b \diamond(x), \tag{3}$$

where $\diamond(x)$ is the binary object support that is equal to unity over the support of the target $f(x)$ and equal to zero everywhere else, and a and b are unknown constants. An orthogonal basis for the subspace is selected. The first component is the silhouette $\diamond(x)$. To find another orthogonal vector to $\diamond(x)$, we define $f_0(x)$ as

$$f_0(x) = f(x) - \mu_f \diamond(x), \tag{4}$$

where μ_f is the mean of $f(x)$. Note that $f_0(x)$ is a zero-mean target. Therefore the target can be defined as a linear combination of two orthogonal images (a silhouette and a zero-mean target) as

$$f'(x) = a' f_0(x) + b' \diamond(x). \tag{5}$$

The basis defined by $\{f_0(x), \diamond(x)\}$ is not orthonormal, but we can normalize it to unit length as

$$\left\{ \phi_1 = \hat{\diamond}(x) = \frac{\diamond(x)}{\|\diamond(x)\|};\ \phi_2 = \hat{f}_0(x) = \frac{f_0(x)}{\|f_0(x)\|} \right\},$$

where $\|f_0(x)\| = \sqrt{f_0(x) * f_0(x)}$, and $*$ denotes the correlation, which is the inner product. Taking into account this orthonormal basis $\{\phi_1, \phi_2\}$, the target can now be defined as

$$f'(x) = a'' \hat{f}_0(x) + b'' \hat{\diamond}(x). \tag{6}$$

To find the filtering operation Ξ that we are looking for, consider first the correlation between the scene $s(x)$ and $\hat{f}_0(x)$. If the scene is a linear combination of the basis, then the correlation peak will be

$$[s(x) * \hat{f}_0(x)]^2 = a''^2. \tag{7}$$

We want a filtering operation that not only gives a high value when the target is in the scene but at the same time will yield a result that will be normalized to a value of 1. To normalize the final correlation we need to divide by a''^2. This value can be obtained by calculating the following correlations:

$$[s^2(x) * \hat{\diamond}(x)] = \frac{1}{\sqrt{N}}(a''^2 + b''^2);\ [s(x) * \hat{\diamond}(x)]^2 = b''^2, \tag{8}$$

where N is the number of pixels in the support of the object $f(x)$. So the final filtering operation at the output will be

$$C_{\mathrm{LACIF}}(x) = \frac{[s(x) * \hat{f}_0(x)]^2}{\sqrt{N}[s^2(x) * \hat{\diamond}(x)] - [s(x) * \hat{\diamond}(x)]^2}. \tag{9}$$

Then, if $s(x)$ is a linear combination of the orthonormal basis, the correlation peak will be equal to one and will be smaller than one if it is not. Note that Eq. (9) is the same expression that Lefebvre *et al.* obtained in Ref. 14. In terms of a vector interpretation, the LACIF technique is equivalent to projecting the

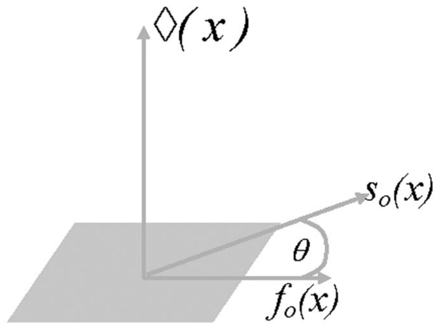


Fig. 1. Vector space interpretation. The LACIF is equivalent to projecting $s(x)$ onto the subspace orthogonal to $\diamond(x)$ and then to calculating the angle between this projected vector and $f_0(x)$.

scene onto the subspace orthogonal to $\diamond(x)$, that is, to project the zero-mean scene in this region of support and then to calculate the cosine of the angle between the scene and the reference.[14] Equation (9) is the operation defined as Ξ mentioned in Section 2. We illustrate the vector interpretation of the LACIF in Fig. 1, from which we conclude that

$$C_{\mathrm{LACIF}}(x) = \cos^2 \theta. \tag{10}$$

To show the LACIF performance, we have chosen two slightly different vehicle images. In Fig. 2(a) we present four replicas of the reference (true target, top) and four replicas of the false target (bottom). Each replica has undergone an intensity transformation with different values of a and b from Eq. (3). These results coincide with those already presented in Ref. 14.

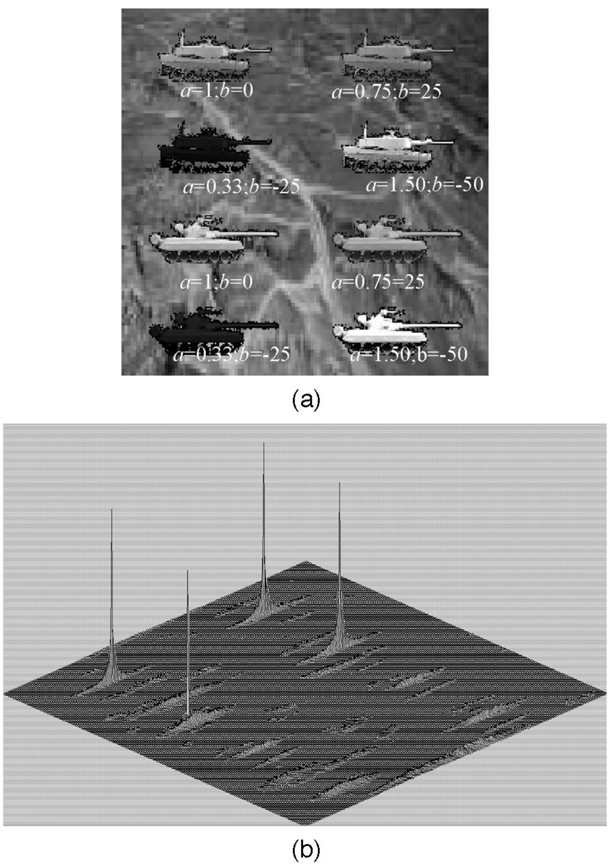


Fig. 2. (a) Scene containing true (top) and false (bottom) targets. (b) Correlation results with disjoint correlated noise.

Figure 2(b) shows the results of our using Eq. (9) for Fig. 2(a). The result is obtained by calculating three correlations and then computing the ratio indicated in the expression. All four upper correlation peaks are very sharp, and the value for all of them is equal to one no matter which parameters a and b have been used to modify the targets. The rest of the correlation peaks for the false targets are lower, which demonstrates good discrimination against correlated noise. This good performance can be understood from the vector space interpretation: All images are intensity images, and therefore every pixel has a positive value. In a multidimensional space this means that all vectors are in the first quadrant. Therefore the angles between images are fairly small, and the squared cosine values are all close to one. But when the vectors are projected onto the subspace orthogonal to $\diamond(x)$, those vectors will be spread out in the vector space, thus increasing the angles between them. The vectors that are defined according to Eq. (6) [see the four upper tanks of Fig. 2(a)] will be parallel to the reference, so the correlation value for those vectors will be equal to one. On the contrary, false objects like those represented in the lower part of Fig. 2(a) cannot be expressed as a linear combination of the basis $\{f_0(x), \diamond(x)\}$, so they will not be parallel to the reference; thus the cosine between the target and the false objects will be lower (a bigger angle implies a smaller cosine), and the correlation values for those vectors will be smaller than one.

4. Intensity-Invariant Recognition in Camouflage Scenes

An important issue in pattern recognition is the identification of targets through obscuration such as foliage, camouflage netting, or other patterns. Recent advances in laser- and computer-related systems improved the capability to identify targets behind such obscuration.[17] However, those methods deal with imaging more than pattern recognition. Other references can be found to evaluate the effectiveness of camouflage.[19] But segmentation of the target from the background is required, and the influence of changes of illumination is not considered.

We now introduce a technique based on finding an orthonormal basis to detect targets that are camouflaged and that also suffer from variations in intensity. We applied the same idea as in Section 3. Invariant-intensity pattern recognition methods can be viewed as the consideration of images as vectors in a vector space and to measuring angles between those vectors. In Section 3 we recognize true targets independently of changes of intensity because we defined the intensity transformation as a linear combination of two orthogonal vectors, the silhouette and the zero-mean reference [see Eq. (3)]. Let us now

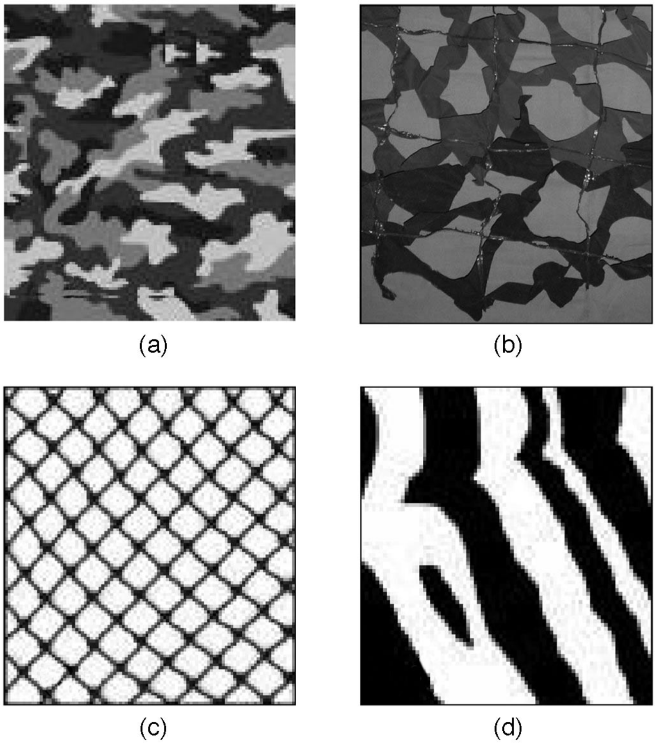


Fig. 3. Different camouflage patterns.

assume that the target not only has changes of intensity but is also hidden by a camouflage pattern such as

$$f'(x) = af(x) + b \Diamond(x) + cg(x), \tag{11}$$

where $g(x)$ is the camouflage pattern and c is a constant. Figure 3 shows some camouflage patterns. The type of camouflage considered in this paper is of the widely used types that would be applied by painting, as opposed to camouflage that hides or obscures parts of the target, such as, for example, netting with foliage. The former does not change the shape of the target, whereas a replacement type would, for example, by obscuring target edges. Our method deals with both additive or multiplicative changes of intensity, but not with replacement.

Equation (11) represents the decomposition of the target into a basis defined by vectors $\{f(x), \Diamond(x), g(x)\}$; however, those vectors are not orthonormal or orthogonal. The intensity invariance problem that we are solving is to determine whether an object is a linear combination of an orthonormal image basis and whether an unknown object belongs to the subspace spanned by those three vectors. Two orthogonal vectors used in Section 3 are $f_0(x)$ and $\Diamond(x)$, but we need a third basis vector, which in this case will be represented by the camouflage pattern. One way to find such a vector is to project $g(x)$ into a plane perpendicular to $\Diamond(x)$, so we calculate the zero mean of $g(x)$ between the region of support of $\Diamond(x)$ as

$$g_0(x) = g(x) - \mu_g \Diamond(x). \tag{12}$$

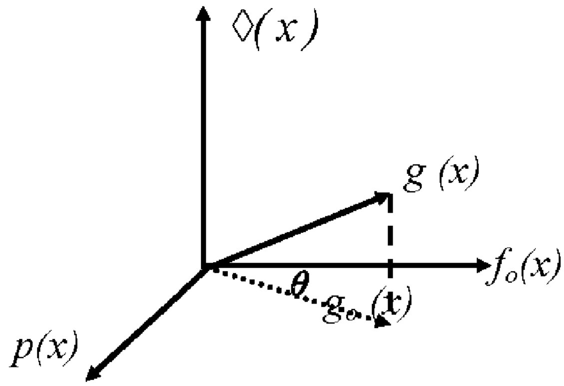


Fig. 4. Three-dimensional representation of the orthogonal vector space basis (in fact the number of dimensions is equal to the number of pixels, so this basis does not span the full vector space of the image).

We find that the inner product between $[g_0(x)^* \Diamond(x)]_{x=0} = 0$, so $g_0(x)$ and $f_0(x)$ are already perpendicular to the silhouette but they are not perpendicular to each other. To find a third vector $p(x)$ that is perpendicular to both $\Diamond(x)$ and $f_0(x)$, we apply the Gram—Schmidt orthogonalization method to $g_0(x)$. Then the third orthogonal vector is

$$p(x) = [g_0(x)^* f_0(x)] \frac{f_0(x)}{\|f_0(x)\|^2} - g_0(x). \tag{13}$$

The resulting orthogonal basis is shown in Fig. 4.

The basis spanned by images $\{\Diamond(x), f_0(x), p(x)\}$ is orthogonal and these vectors can be normalized to unit length $\{\phi_1(x) = \hat{\Diamond}(x), \phi_2(x) = \hat{f}_0(x), \phi_3(x) = \hat{p}(x)\}$. So the target can be written as

$$f'(x) = a''\hat{f}_0(x) + b''\hat{\Diamond}(x) + c''\hat{p}(x). \tag{14}$$

The nonlinear filtering expression is calculated by the same expression as in Section 3, but adapted to a three-vector subspace instead of two. Analogously to Eq. (9), the correlation plane will be

$$C_{\text{LACIF CAM}}(x) = \frac{[s(x)^* \hat{f}_0(x)]^2}{\sqrt{N}\,[s^2(x)^* \hat{\Diamond}(x)] - [s(x)^* \hat{\Diamond}(x)]^2 - [s(x)^* \hat{p}(x)]^2}. \tag{15}$$

To distinguish Eq. (9) from Eq. (15), we call the application of the LACIF to camouflaged images as LACIF CAM. If part of the scene $s(x)$ is a linear combination of the reference target and of the camouflage $g(x)$ so it can be defined by Eq. (14), the corresponding correlation peak will be equal to one, and otherwise will be less than one. In Section 5 we show different experiments to test the LACIF CAM operation.

5. Results of Detection in Camouflage

We have carried out three sets of experiments. The first experiment deals with the detection of a target

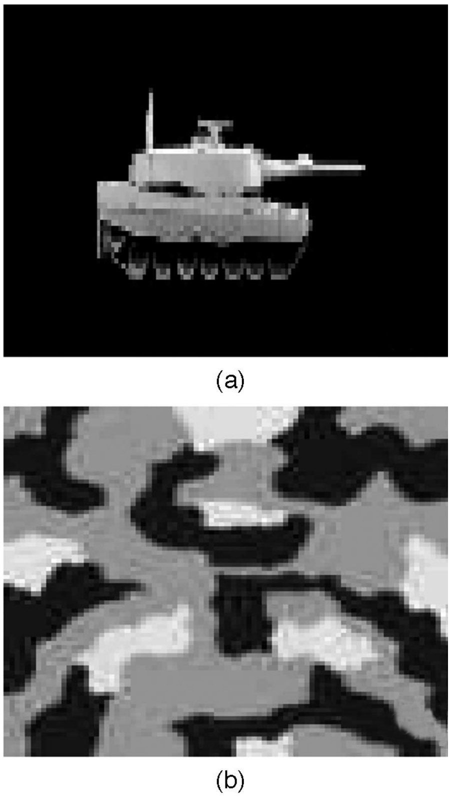


Fig. 5. (a) Target. (b) Camouflage pattern discussed in Subsection 5.A.

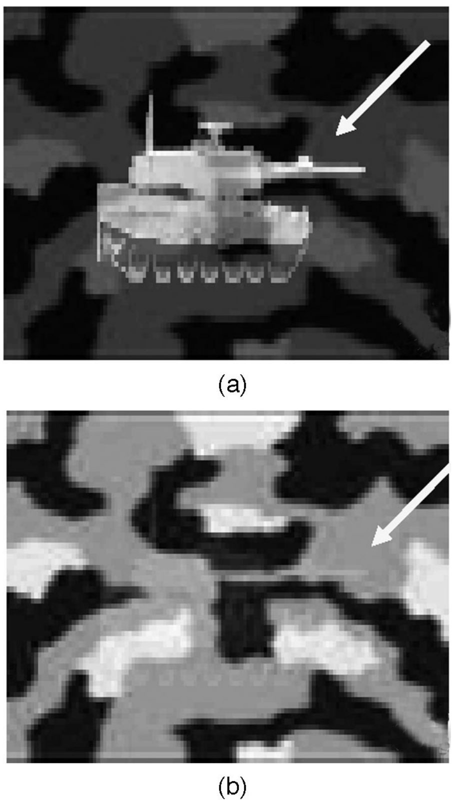


Fig. 6. Weakly camouflaged target. (b) Strongly camouflaged target discussed in Subsection 5.A.

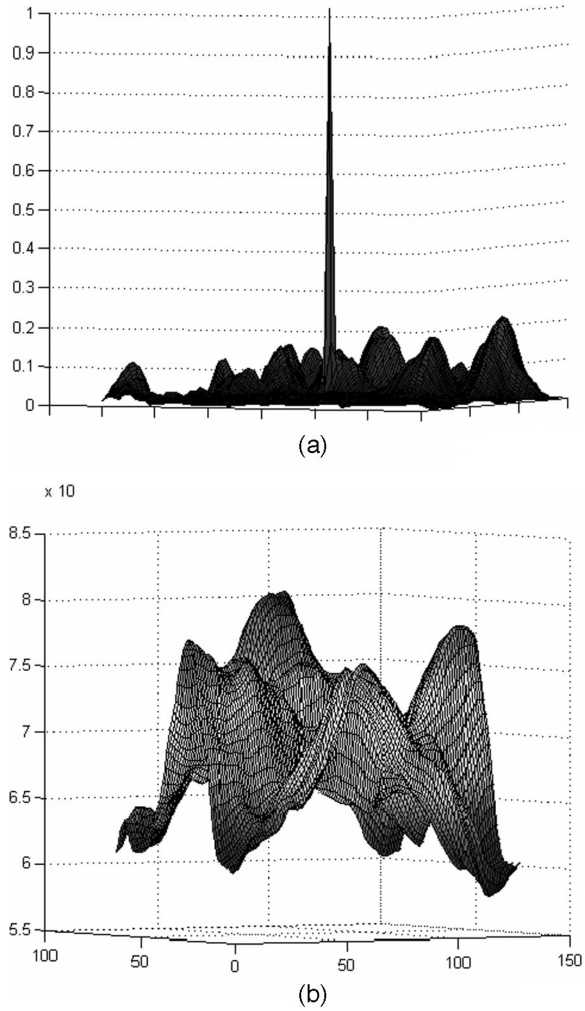


Fig. 7. (a) LACIF CAM for Fig. 6(b). (b) Common matched filtering for Fig. 6(b).

that has been camouflaged in a specific background. In the second experiment we will show the influence of the illumination conditions by changing the contrast of the camouflage pattern and of the target. At the same time we will add other false objects in the scene to verify the discrimination capability of the method. Finally, assuming that there is a certain catalog for typical cases of camouflage, the third experiment will show that if only the class of camouflage is known (e.g., foliage, netting) then we can still detect the target.

A. Experiment 1: Detecting Camouflaged Targets

The reference target is shown in Fig. 5(a), and the camouflage pattern used for this experiment is shown in Fig. 5(b). For this case $f(x)$ is the object to detect, and $g(x)$ is the camouflage pattern. Now we camouflage the image using Eq. (11). Note that we can change the constant values a, b, and c to mask the target with different levels of camouflage, as shown in Figs. 6(a) and 6(b). An arrow is used to mark the position of the target. Note that in Fig. 6(b) the target is difficult to distinguish even by eye. We show in Fig.

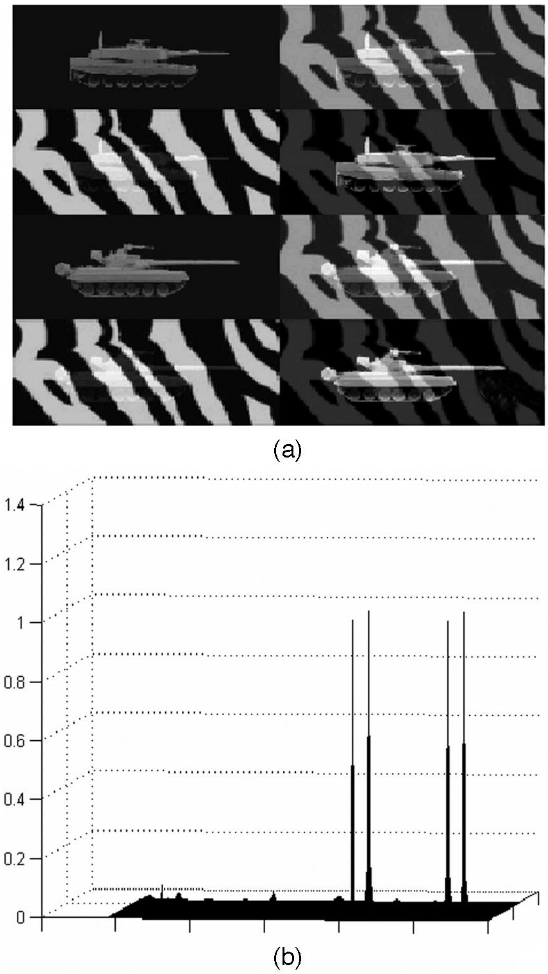


Fig. 8. (a) Input scene with different values of camouflage contrast. The four upper objects are targets and the other four shown below are false objects. (b) The LACIF CAM result.

7(a) the result for the LACIF CAM corresponding to Fig. 6(b), and in Fig. 7(b) the detection operation is the well-known common linear matched filtering. Note from Fig. 7(b) that the correlation peak height for LACIF CAM is equal to one, and the value of constants a, b, and c are considered to be unknown. To show the intensity invariance of the method, we carried out a second experiment.

B. Experiment 2: Invariance to Camouflage Contrast

In this experiment we show the performance of the LACIF CAM when different illumination conditions of the camouflage affect the target. Moreover, we show the discrimination capability by introducing false camouflaged objects in the same scene. We take an input scene with the input target and at the same time the false camouflage object with different values of constants a, b, and c. The input image is shown in Fig. 8(a), and Fig. 8(b) is a plot of the three-dimensional profile of the correlation peaks for the LACIF CAM. Again all four true targets are detected with correlation peaks equal to one. On the other hand, although the camouflage pattern is the same for the two objects, the false targets are not detected. The nonlinearity of the LACIF is the reason for the discrimination, as discussed in Section 3.

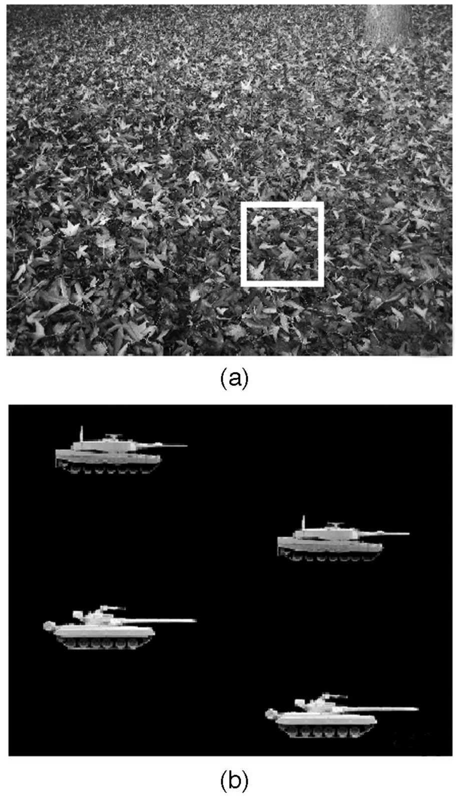
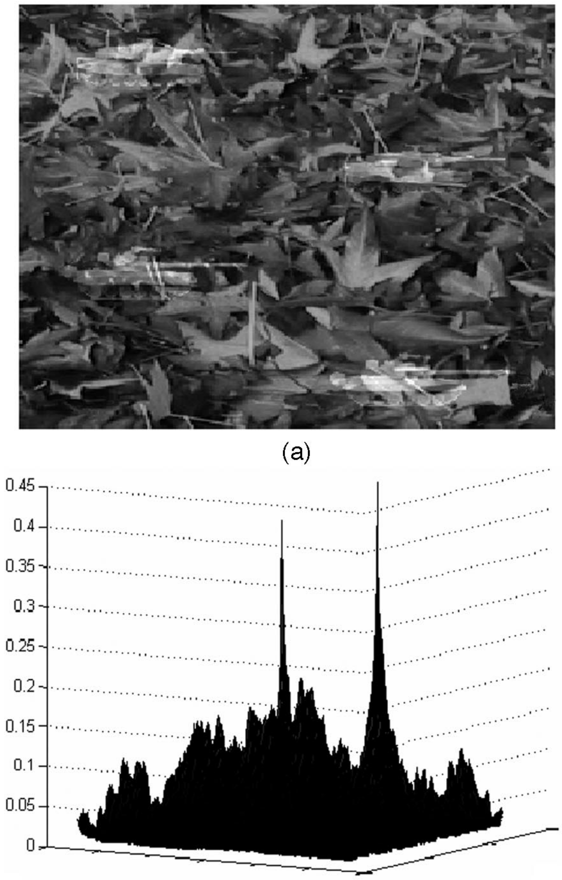


Fig. 9. (a) Background leaf camouflage. (b) Input scene; the two true targets are located in the upper part of the image.

C. Experiment 3: Detection When Only the Class of Camouflage is Known

For camouflage pattern recognition, one might argue that in practice we never know the exact place where the target is hidden because the target can move through the scene. In the previous experiments we showed that, if the camouflage pattern is known *a priori*, we will find an orthonormal basis to perform the filtering. But to test for a more realistic application we chose a typical class of camouflage, for instance, leaves, and we will change the camouflage pattern of the targets to prove the performance of the LACIF CAM. Figure 9(a) shows a background of leaves. We chose an arbitrary part of it to calculate the filter and another different arbitrary part to camouflage the image. Thus Fig. 9(a) is the input scene before the camouflage is applied. Then we took a specific part of Fig. 9(a) to be $g(x)$ and another part to camouflage the input scene [Fig. 9(b)]. Figure 10(a) shows the camouflaged input image. Note that it is difficult to see the camouflaged targets. In Fig. 10(b) we plot the three-dimensional correlation plane for the LACIF CAM. Here the correlation peak value is not equal to one because the vector $g(x)$ is not exactly the same as the leaf pattern used to camouflage. De-



(a)

(b)

Fig. 10. (a) Camouflaged tanks. (b) LACIF CAM correlation results.

spite this we still discriminate the two true targets from the background.

6. Conclusion

Intensity-invariant pattern recognition techniques can be understood in terms of vector spaces. We obtained the previous LACIF expression by means of the orthonormalization of basis functions. We have extended the LACIF pattern recognition method to camouflaged targets. Various experiments were carried out to validate the intensity invariance recognition for the target and for the camouflage. We successfully tested the method when other false targets were in the scene. For real applications, the method detects through the camouflage pattern only when the class of camouflage is known (foliage, fog), but not the specific camouflage pattern. These methods were applied to unsegmented natural scenes. It should be noted that, because the illumination parameters need to be constant only over a small area, the illumination can vary over the scene, which is why we call the method locally adaptive.

This work was supported by a grant from the Natural Sciences and Engineering Research Council of Canada and by FEDER funds and the Spanish Ministerio de Educación y Ciencia under the project FIS2004-06947-C02-01, the D.G. Investigació i Transferència Tecnològica project IIARC0/2004/217, and the Agencia Valenciana de Ciencia y Tecnología project GRUPOS03/117.

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