

Algorithm 3 | Input $\varepsilon, x_0, R: V(x_*, x_0) \leq R^2$
 $L_0 > 0$

1. $L_{k+1} := L_k/2$;

2. $x_{k+1} := \operatorname{argmin}_{x \in Q} \{ \langle \nabla f(x_k), x \rangle + L_{k+1} V(x, x_k) \}$;

3. If $0 \leq \langle \nabla f(x_k), x_{k+1} - x_k \rangle + L_{k+1} V(x_{k+1}, x_k) \leq \frac{\varepsilon}{2}$

Then $k := k+1$ and goto 1

Else $L_{k+1} := 2 \times L_{k+1}$, and goto 2

4. Stopping rule $\frac{R^2}{S_N} \leq \frac{\varepsilon}{2} \quad (S_N \geq \frac{2R^2}{\varepsilon})$

5. Output $\hat{x} = \frac{1}{S_N} \sum_{k=0}^{N-1} x_k$ $\operatorname{argmin}_{k \geq 0} f(x_k)$, $S_N = \sum_{k=0}^{N-1} L_{k+1}^{-1}$

Algorithm 4 | Input $x_0, R: V(x_*, x_0) \leq R^2$

1. $L_{k+1} := L_k/2$, $\delta_{k+1} := \delta_k/2$; $L_0 > 0, \delta_0 > 0$

2. $x_{k+1} := \operatorname{argmin}_{x \in Q} \{ \langle \nabla f(x_k), x \rangle + L_{k+1} V(x, x_k) \}$;

3. If $0 \leq \langle \nabla f(x_k), x_{k+1} - x_k \rangle + L_{k+1} V(x_{k+1}, x_k) + \delta_{k+1}$

Then $k := k+1$ and goto 1

Else $L_{k+1} := 2 \times L_{k+1}$, $\delta_{k+1} := 2 \times \delta_{k+1}$ and goto 2

4. Output $\hat{x} = \frac{1}{S_N} \sum_{k=0}^{N-1} x_k$ $\operatorname{argmin}_{k \geq 0} f(x_k)$

$$f(\hat{x}) - f(x_*) \leq \frac{R^2}{c} + \frac{1}{c} \sum_{k=0}^{N-1} \delta_{k+1}.$$