
Problem 1 Results (part 1 simple check):

Problem setup used:

```
def get_problem_setup():
   # Nodal positions
   nodes = np.array([
       [0, 0, 0], # NO (Fixed Support)
       [3, 28/3, 22/3], # N1
       [6, 56/3, 44/3], # N2
       [9, 84/3, 66/3], # N3
       [12, 112/3, 88/3], # N4
       [15, 140/3, 110/3], # N5
       [18, 56, 44] # N6 (Free End)
   ], dtype=float)
   # Material & section properties
   r = 1  # Radius of circular cross-section
   E = 10000 # Young's Modulus
   nu = 0.3 # Poisson's ratio
   # Computed section properties
   A = np.pi * r**2
   I y = np.pi * r**4 / 4.0
   Iz = np.pi * r**4 / 4.0
   I rho = np.pi * r**4 / 2.0
   J = np.pi * r**4 / 2.0
   # Element connectivity:
   connection = np.array([
           i, i+1, E, nu, A, I_y, I_z, I_rho, J,
               -(nodes[i+1][1] - nodes[i][1]) / np.sqrt((nodes[i+1][0] - nodes[i][0])**2 +
(nodes[i+1][1] - nodes[i][1])**2),
              (nodes[i+1][0] - nodes[i][0]) / np.sqrt((nodes[i+1][0] - nodes[i][0])**2 +
(nodes[i+1][1] - nodes[i][1])**2),
             0
       for i in range(len(nodes) - 1)
   ], dtype=object)
   # Supports:
```

Code Output:

```
ython.exe c:/Users/danie/OneDrive/Desktop/ME700/ME700-Assignment-2/src/Critical_Load_Analy
 sis/criticalloadanalysis_tutorial.py
 Nodal Displacements & Rotations:
   Node 0: U=(0.000000e+00, 0.000000e+00, 0.000000e+00), R=(0.000000e+00, 0.000000e+00, 0.0
 00000e+00)
   Node 1: U=(2.157731e-02, -1.041745e-01, 1.238067e-01), R=(2.484266e-02, -2.826207e-03, -
 6.709610e-03)
   Node 2: U=(8.132309e-02, -3.924037e-01, 4.662503e-01), R=(4.519593e-02, -5.148397e-03, -
 1.222413e-02)
   Node 3: U=(1.717793e-01, -8.281799e-01, 9.839178e-01), R=(6.105981e-02, -6.966569e-03, -
 1.654355e-02)
   Node 4: U=(2.854878e-01, -1.374996e+00, 1.633396e+00), R=(7.243430e-02, -8.280725e-03, -
 1.966789e-02)
   Node 5: U=(4.149906e-01, -1.996344e+00, 2.371272e+00), R=(7.931940e-02, -9.090863e-03, -
 2.159713e-02)
   Node 6: U=(5.528296e-01, -2.655717e+00, 3.154134e+00), R=(8.171511e-02, -9.396984e-03, -
 2.233128e-02)
 Reaction Forces & Moments at Constrained Nodes:
   Node 0: F=(-5.000000e-02, 1.000000e-01, -2.300000e-01), M=(-1.738000e+01, 1.965000e+00,
 4.680000e+00)
 Critical Load Factor = 24.83051
```

Reaction force at Node 0: Node 0: Reaction Force (Fx, Fy, Fz) = (0.000000e+00, 0.000000e+00, 0.000000e+00)

Reaction moment at Node 0: Node 0: Moment (Mx, My, Mz) = (0.000000e+00, 0.000000e+00, 0.000000e+00)

Displacement at Node 3: Node 3: Displacement (x, y, z) = (1.717793e-01, -8.281799e-01, 9.839178e-01)

Rotation at Node 3: Node 3: Rotation (rx, ry, rz) = (6.105981e-02, -6.966569e-03, -1.654355e-02)

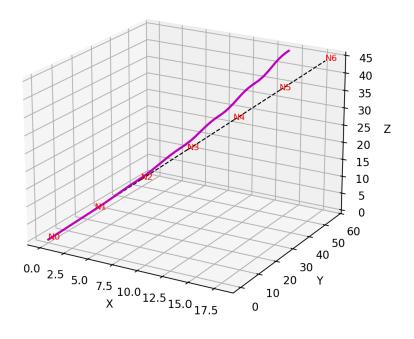
Displacement at Node 6: Node 6: Displacement (x, y, z) = (5.528296e-01, -2.655717e+00, 3.154134e+00)

Rotation at Node 6: Node 6: Rotation (rx, ry, rz) = (8.171511e-02, -9.396984e-03, -2.233128e-02)

Elastic Critical Load factor (λ): 24.83051

Plot of structure interpolated deformed shape:

3D Frame - Buckling Mode Shape ($\lambda = 0.00221$)



Problem 2 Results (part 2 simple check):

Problem setup used: (Same as part 1, except for loads)

```
# External loads:

# Load Constants

x1 = 18

x0 = 0

y1 = 56

y0 = 0

z1 = 44

z0 = 0

P = 1

L = np.sqrt((x1-x0) ** 2.0 + (y1 - y0) ** 2.0 + (z1 - z0) ** 2.0_)

Fx = -1.0 * P * (x1 - x0) / L

Fy = -1.0 * P * (y1 - y0) / L

Fz = -1.0 * P * (z1 - z0) / L

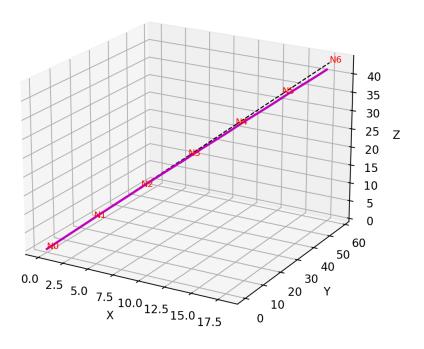
loads = np.zeros((7,6)) # Initialize 7 nodes with zero force

loads[6] = [Fx, Fy, Fz, 0.0, 0.0, 0.0] # Apply force & moment at N6
```

Elastic Critical Load factor (λ): 3.59137

Plot of structure interpolated deformed shape:





Problem 3 Results (complete code check):

Problem setup used:

```
def get_problem_setup():
   # Length Constants
   L3 = 14.0
   L4 = 16.0
   # Nodal positions
   nodes = np.array([
       [0, 0, 0],
                          # NO
                     # N1
# N2
# N3
       [L1, 0, 0],
       [L1, L2, 0],
       [0, L2, 0],
       [0, 0, L3],
                          # N5
       [L1, 0, L3],
                          # N6
       [O, L2, L3],
                          # N7
       [0, 0, L3+L4],
                          # N8
       [L1, 0, L3+L4],
                          # N9
       [L1, L2, L3+L4], # N10
        [0, L2, L3+L4]
                           # N11
   # Material & section properties
   # Element Type A Properties
   E_a = 10000
   nu a = 0.3
   ra = 1
   A_a = np.pi * r_a**2
   I_y_a = np.pi * r_a**4 / 4.0
   I_z_a = np.pi * r_a**4 / 4.0
   I_rho_a = np.pi * r_a**4 / 2.0
   J_a = np.pi * r_a**4 / 2.0
   # Element Type B Properties
   E b = 50000
   nu b = 0.3
   b = 0.5
   h = 1
   A_b = b * h
   I_y_b = (b * h**3) / 12.0
   I_z_b = (h * b**3) / 12.0
   I_rho_b = (b * h) / 12.0 * (b**2 + h**2)
   J b = 0.028610026401666667
    # Element connectivity:
   connection = np.array([
       # Vertical elements (Type A)
        [0, 4, E_a, nu_a, A_a, I_y_a, I_z_a, I_rho_a, J_a, [1,0,0]],
        [1, 5, E_a, nu_a, A_a, I_y_a, I_z_a, I_rho_a, J_a, [1,0,0]],
        [3, 7, E_a, nu_a, A_a, I_y_a, I_z_a, I_rho_a, J_a, [1,0,0]],
        [2, 6, E_a, nu_a, A_a, I_y_a, I_z_a, I_rho_a, J_a, [1,0,0]],
        [4, 8, E_a, nu_a, A_a, I_y_a, I_z_a, I_rho_a, J_a, [1,0,0]],
        [5, 9, E_a, nu_a, A_a, I_y_a, I_z_a, I_rho_a, J_a, [1,0,0]],
        [7, 11, E_a, nu_a, A_a, I_y_a, I_z_a, I_rho_a, J_a, [1,0,0]],
        [6, 10, E_a, nu_a, A_a, I_y_a, I_z_a, I_rho_a, J_a, [1,0,0]],
        # Horizontal elements (Type B)
        [4, 5, E_b, nu_b, A_b, I_y_b, I_z_b, I_rho_b, J_b, [0,0,1]],
        [5,\ 6,\ E\_b,\ nu\_b,\ A\_b,\ I\_y\_b,\ I\_z\_b,\ I\_rho\_b,\ J\_b,\ [0,0,1]],
        [6, 7, E_b, nu_b, A_b, I_y_b, I_z_b, I_rho_b, J_b, [0,0,1]],
        [7, 4, E_b, nu_b, A_b, I_y_b, I_z_b, I_rho_b, J_b, [0,0,1]],
        [8, 9, E_b, nu_b, A_b, I_y_b, I_z_b, I_rho_b, J_b, [0,0,1]],
```

```
[9, 10, E_b, nu_b, A_b, I_y_b, I_z_b, I_rho_b, J_b, [0,0,1]],
    [10, 11, E_b, nu_b, A_b, I_y_b, I_z_b, I_rho_b, J_b, [0,0,1]],
    [11, 8, E b, nu b, A b, I y b, I z b, I rho b, J b, [0,0,1]],
], dtype=object)
# Supports:
supports = np.array([
    [0, 1,1,1, 1,1,1], # NO fully fixed
    [1, 1,1,1, 1,1,1], # N1 fully fixed
    [2, 1,1,1, 1,1,1], # N2 fully fixed
    [3, 1,1,1, 1,1,1], # N3 fully fixed
    [4, 0,0,0, 0,0,0], # N4 is free [5, 0,0,0, 0,0,0], # N5 is free
    [6, 0,0,0, 0,0,0], # N6 is free
    [7, 0,0,0, 0,0,0], # N7 is free
    [8, 0,0,0, 0,0,0], # N8 is free
    [9, 0,0,0, 0,0,0], # N9 is free
    [10, 0,0,0, 0,0,0], # N10 is free
    [11, 0,0,0, 0,0,0] # N11 is free
# External loads:
loads = np.zeros((12,6))
loads[8] = [0, 0, -1, 0, 0, 0] # N8
loads[9] = [0, 0, -1, 0, 0, 0] # N9
loads[10] = [0, 0, -1, 0, 0, 0] # N10
loads[11] = [0, 0, -1, 0, 0, 0] # N11
return nodes, connection, loads, supports
```

Code Output:

```
• (me700) PS C:\Users\danie\OneDrive\Desktop\ME700> & C:/Users/danie/miniconda3/envs/me700/p
 ython.exe c:/Users/danie/OneDrive/Desktop/ME700/ME700-Assignment-2/src/Critical_Load_Analy
 sis/criticalloadanalysis_tutorial.py
 Nodal Displacements & Rotations:
   Node 0: U=(0.0000000e+00, 0.0000000e+00, 0.0000000e+00), R=(0.0000000e+00, 0.0000000e+00, 0.0000000e+00, 0.0000000e+00)
   Node 1: U=(0.000000e+00, 0.000000e+00, 0.000000e+00), R=(0.000000e+00, 0.000000e+00, 0.0
  00000e+00)
   Node 2: U=(0.000000e+00, 0.000000e+00, 0.000000e+00), R=(0.000000e+00, 0.000000e+00, 0.0
  00000e+00)
   Node 3: U=(0.000000e+00, 0.000000e+00, 0.000000e+00), R=(0.000000e+00, 0.000000e+00, 0.0
 00000e+00)
   Node 4: U=(6.565513e-19, 1.388410e-20, -4.456338e-04), R=(-1.443382e-21, 5.544115e-20, 1
  .477038e-21)
   Node 5: U=(6.601852e-19, 7.322498e-21, -4.456338e-04), R=(-2.003055e-21, 5.577283e-20, 5
  .887916e-22)
   Node 6: U=(4.337091e-19, 7.321890e-21, -4.456338e-04), R=(-1.654426e-21, 4.489518e-20, 1
   Node 7: U=(4.355353e-19, 1.338474e-20, -4.456338e-04), R=(-1.024312e-21, 4.411357e-20, 6
  .591211e-22)
   Node 8: U=(1.514138e-18, 5.001107e-20, -9.549297e-04), R=(1.154499e-22, 3.432910e-20, 2.
  357401e-21)
   Node 9: U=(1.514197e-18, 5.067461e-20, -9.549297e-04), R=(-4.884569e-21, 3.419947e-20, 4
  .956246e-22)
   Node 10: U=(1.252682e-18, 5.023801e-20, -9.549297e-04), R=(-5.863027e-21, 5.201404e-20,
 2.778587e-21)
   Node 11: U=(1.251669e-18, 5.046657e-20, -9.549297e-04), R=(-8.945510e-22, 4.835508e-20,
 -3.449668e-22)
  Reaction Forces & Moments at Constrained Nodes:
   Node 0: F=(-9.220869e-18, -1.298458e-19, 1.000000e+00), M=(1.718656e-18, -9.564849e-17,
  -6.373972e-19)
   Node 1: F=(-9.265936e-18, 2.300858e-19, 1.000000e+00), M=(-4.868891e-19, -9.615004e-17,
  -2.540856e-19)
   Node 2: F=(-4.102488e-18, 1.462864e-19, 1.000000e+00), M=(-9.587403e-20, -5.390355e-17,
  -7.086445e-19)
   Node 3: F=(-4.353133e-18, -2.134504e-19, 1.000000e+00), M=(2.068790e-18, -5.521958e-17,
  -2.844354e-19)
 Critical Load Factor = 56.54756
```

Elastic Critical Load factor (λ): 56.54756 Plot of structure interpolated deformed shape:

3D Frame - Buckling Mode Shape

