Analysis Final Project
Optimal Determination of Global Desalination Plants

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***Abstract*—One of the NAE grand challenges for engineering is to provide access to clean water worldwide. Many countries and regions are land locked or do not have access to potable water due to infrastructure or pollution. To ease the worldwide water shortage a combination of strategic water filtration and improved agricultural methods should be applied. This project will focus on determining the optimal location for desalination plants in coastal areas to maximize the population served while minimizing the economic impact and energy consumption. In doing so, a mathematical model will be created to map the total population that can be reached from a single location while accounting for energy requirements to transport the water. The mathematical model will include down sampling regions with a K-means algorithm to produce a finite distribution for the population of any given continent. With this K-means population, a continuous model proportional to the water horsepower required at any given radial distance will be created. These models can be summed for each population group to produce a continuous cost map. An optimization will then occur to determine the least possible cost for creating desalination systems.**

I. Introduction

To explore a relevant case for water desalination, the western coast of Africa was used as a sample population. The population boundaries were chosen as (20°W, 15°S) to (20°E, 15°N). The actual city location and population data [1] were plotted on this map for analysis as seen in Fig. 1.

The end goal of this project is to determine the optimal coastal location for desalination plants to serve water-deprived populations across country borders.

The analysis for this project includes:

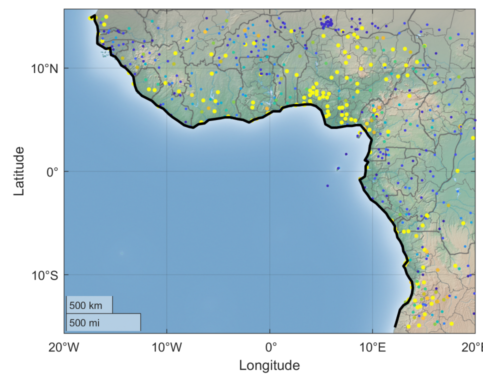

Figure 1. Plot of Population Centers for Western Africa

1) Clustering the city populations
2) Finding the minimum distance to the coast from the cluster centers
3) Generating a polynomial for the coast, local to each cluster
4) Performing a minimization along the coastal polynomial

Each of these steps will be explored individually in the next section.

A. Literature Review

The generation of a cost function was loosely based on [2] and their efforts to optimize the location of desalination plants around Egypt. The key differences are that our algorithm utilize euclidean distance instead of square distance and the search method is constrained by the coastline. Using this constraint allows for the analysis of non-coastal populations while maintaining optimality close to the shoreline to reduce the quantity of salinated water transfer. As it will be seen later, a population cap of

16 million people per desalination plant was utilized to determine the number of plants required. This population cap was determined based on the size of modern desalination plant bids. [3] provides reference to two recent desalination project bids (100,000 m^3/day and 150,000 m^3/day). With a high daily allotment of drinking water per day of 2 gallons, disregarding agricultural requirements, 16 million people yields roughly 120,000 m^3/day of water. Modern ship-based desalination plants such as the ones mentioned by [4] are the primary reason for constraining the optimization to the coastline. This allows for approximate locations either over water or on land.

II. Mathematical Analysis

A. Simple Country

When planning to accomplish this task, our team first envisioned a hypothetical "simple country". Simple country has rectangular, land-locked boarders with the exception of a single coastline defined by an arbitrary Nth order polynomial. The only other relevant features to this simple country are a handful of population centers which represent the locations where people tend to be most clustered. The number of population centers and their locations are also entirely arbitrary in simple country.

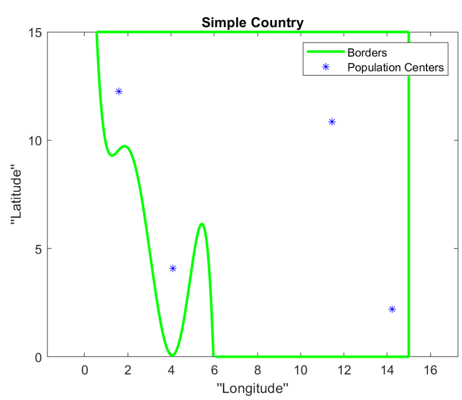

Figure 2. Map of Simple Country

As a first attempt at solving this problem, a point grid was set up on simple country and each point was checked to determine the shortest distance from that point of interest (POI) to the polynomial coastline. This is accomplished by computing the distance from the POI to one hundred different points on the coastline. Then the smallest of these distances is determined to be that POI's minimum distance to the coast. This process could be performed analytically to get a more accurate result, but this discrete, algorithmic method worked as a proof of concept. The resulting cost map is show in Fig. 3

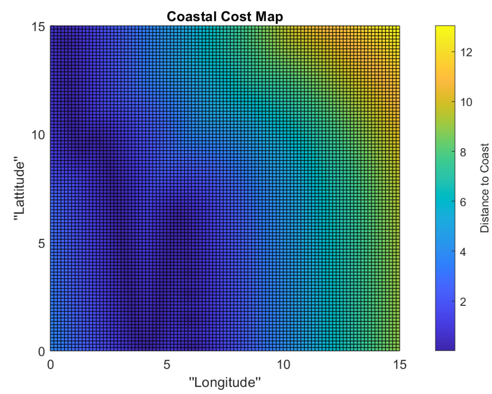

Figure 3. Map of costs associated with reaching the coast

Next, a similar process is performed to determine the distance from each POI to each population cluster. The result is that each point on the grid has a corresponding cost associated with each cluster. This forms four separate cost maps associated with each cluster. Each cost map is then weighted based on the cluster population relative to the total population of simple country. The sum of these cost maps is shown in Fig. 4. With no weighting, the minimum of this cost function would be the centroid of the clusters.

Determining optimal locations using a distance based cost applies to much more than desalination plants. The same proximity based cost function can be applied to retail stores, warehouses, and even community health centers as shown in [5]. In this paper, the authors propose a cost function that can be used to determine the optimal location of a community health center so that it maximizes the amount of people served, while minimizing the distance that those people need to travel to get treatment. Although the cost function in [5] is a bit more inclusive, one of its main elements is a simple proximity based cost just like the one we implement.

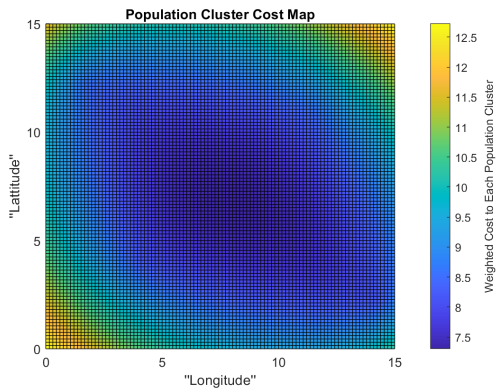

Figure 4. Map of costs associated with reaching each population cluster

To get the final cost map, the coastal cost map and the population cluster cost map are added together as shown in 5

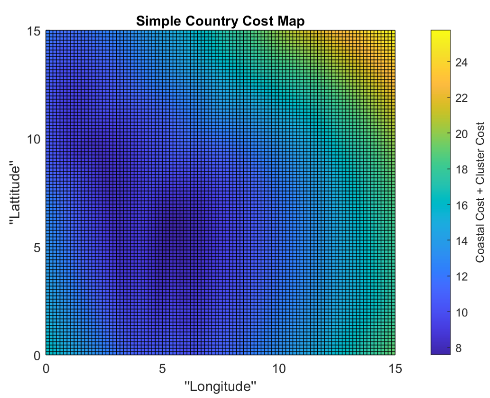

Figure 5. Total cost developed for placing a desalination plant in simple country

B. Population Clustering

Moving into the real population data for Western Africa, a K-means algorithm was utilized in an effort to group the cities into clusters with similar total populations. A maximum population cap for each cluster was chosen at 16 million. The psuedocode for a K-means algorithm can be seen below:

1) Place the cluster centroids randomly
2) Repeat the following steps for N iterations or until convergence
3) For every point, find the nearest centroid and assign the point to that centroid
4) For every cluster, calculate the mean of the assigned points and move the cluster to this mean

This algorithm was fine tuned to our application by using a weighted mean to pull the centroid toward the high population cities. This was found to optimize the clustering and avoid spurious assignments. The second modification was to limit the total population that could be assigned to a single cluster. To do this, the centroids were ranked by their proximity to the current city being evaluated and placement of the city was attempted as long as the centroid population in addition to the current city's population was less than the cap. As centroids 'filled', this pushed points to clusters later in the priority queue producing undesirable assignments. To avoid this issue, and increase convergence speeds, the cities were sorted by descending population prior to cluster assignment. This assignment scheme filled the clusters with high-population cities initially and allowed for more local small-population cities to fill in the gaps. The resulting clusters can be seen in Fig. 6

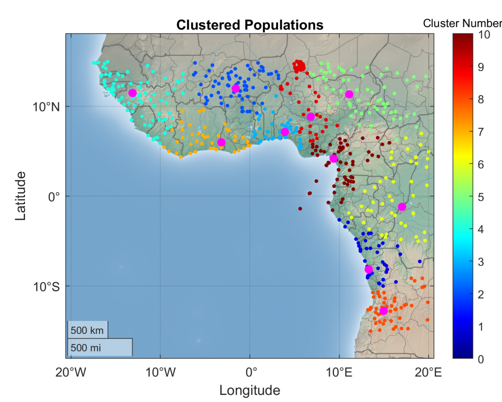

Figure 6. Populations sorted by weighted K-means algorithm

C. Closest Coastal Location

The next step in the path to determining the optimal desalination plant location was to determine the closest coastal location for each cluster. Using the preloaded Matlab coastline data from the Mapping Toolbox, a discrete minimum was found. Using this minimum, the coastal points were filtered to only include coastline within a 7.5 degree circle.

D. Polynomial Approximation

To perform a Lagrange minimization and constrain the solution to the coast, a continuous function for the coastline is required. In our case, a 6th order polynomial fit was used. It should be noticed; however, that when the coast is aligned vertically, it is impossible to provide a best fit in the x-y space due to having multiple y values at each x coordinate. To remedy this problem with relative simplicity, the coastline and cities being observed were rotated to a point where a proper coast function, $C(x)$, can exist. In practice this was accomplished by determining the best fit line of a 1st order polynomial ($y = mx + b$) through the coastline and rotating by $\theta = atan2(m, 1)$. For visualization and to ensure that the results were accurate, the discretized polynomial was rotated back to the original orientation and plotted in Fig. 7

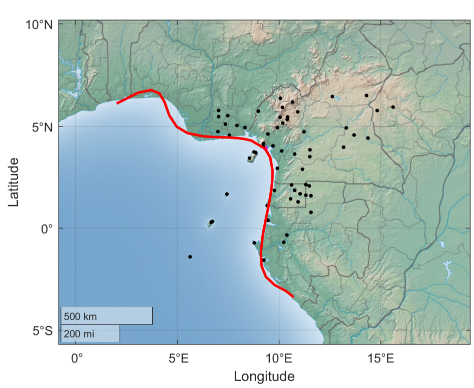

Figure 7. Analytic Coastline Rotated to Original Orientation

E. Desalination Plant Optimization

With the rotated coastline function and transformed city locations, the transformed location of the desalination plan can be optimized. The cost for each city was decided to be the population-weighted radial distance. Although this is a simple cost function, it is fairly indicative of energy using a standard head loss equation. The total cost function in indicial notation can be seen in Eq. 1 where x_i, y_i, and P_i are the longitude, latitude, and population of each city.

$$F(x, y) = P_i\sqrt{(x - x_i)^2 + (y - y_i)^2} \quad (1)$$

1) Analytical Solution: To determine the optimal coast location with an analytic method, the Lagrange minimization method was utilized with the coastline as the boundary condition. This method was chosen primarily because it provides an exact solution and does not rely on an iterative algorithm. The equation that was minimized can be seen in Eq. 2.

$$H(x, y) = F(x, y) + \lambda * (y - C(x)) \quad (2)$$

The minimum cost solution can then be found by solving the system of equations given by:

$$\frac{\partial H}{\partial x} = 0, \frac{\partial H}{\partial y} = 0, \frac{\partial H}{\partial \lambda} = 0 \quad (3)$$

Implementing this method in Matlab is fairly simple when the problem is setup as a set of symbolic equations, but for more than one city the solve() command tended to yield incredibly slow results. Alternatively the vpasolve() command could be used but is heavily reliant on selecting proper initial conditions due to the divergence of the partial derivatives.

Another minimization technique was tested to determine the optimal location of inland desalination plants such that:

$$\begin{aligned} F(x, y) = &P_i\sqrt{(x - x_i)^2 + (y - y_i)^2} \\ &+ P_i\sqrt{(x - x_c)^2 + (y - y_c)^2} \end{aligned} \quad (4)$$

where x_c and y_c are the closest points to the coast from x and y. Using this method required an extra computation to determine the minimum distance to the coast. As it can be imagined, this required taking the first two derivatives of $C(x)$ and solving for the global minimum x value manually after finding all local minima. Because of this extra step, doing a dual optimization was abandoned; however, in a discrete manner this method provided excellent results. Overall, this cost function was abandoned for simplicity but could be used in the future.

2) Discrete Solution: As a faster but less accurate solution to this problem, a completely discrete method can be utilized. Using Eq. 1 an interpolated set of coastline points given by the 6th order polynomial or raw discrete coastline from Matlab can be plugged in to easily determine the cost along the coast. From there, the minimum value can be determined using the min() command, selecting the minimum cost using brute force. Two of these discrete locations can be seen in Fig. 8 and Fig. 9

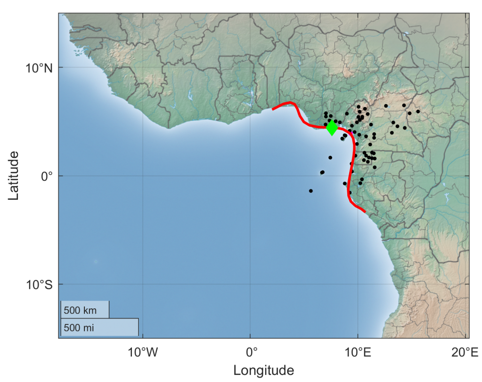

Figure 8. Example 1 - Final Location of Desalination Plant

F. Solution Analysis

Because we utilized the Lagrange method, our solution is exact given the limitations of the problem. There are some quantifiable inaccuracies due to the approximation of the coast line as a 6th order polynomial. These could easily be remedied by selecting the discrete coastal point closest to the optimal solution or by using a higher order polynomial fit. From a more general standpoint, the overall method is only as accurate the the cost function being used. A more accurate cost function could be developed by adding in factors such as water salinity, if the plant should be over land or water, cost and availability of piping, and regional economy. These factors were excluded from the analysis for simplicity.

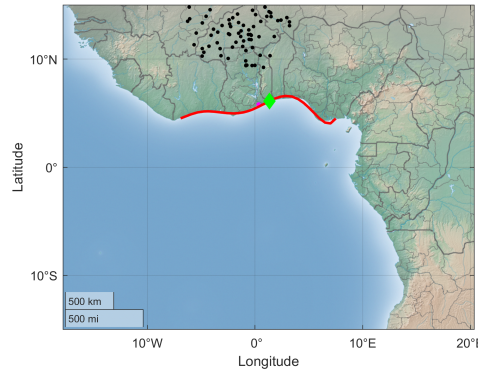

Figure 9. Example 2 - Final Location of Desalination Plant

III. Conclusions

In this project, we analyzed the coast of Western Africa to demonstrate a method for the optimization of coastal desalination plants. In doing so, a K-means population clustering algorithm was developed along with a method for fitting discrete coastal geolocations. These algorithms were paired with both analytical and discrete methods to determine the optimal location to place water filtration and desalination plants to best serve coastal and inland populations.

To further improve this process, different analytic methods could be utilized to bridge the gap between the slow but accurate Lagrange method and the fast but somewhat inaccurate discrete method. One such analytic method would be gradient descent as it is relatively fast and easy to symbolically calculate the two variable gradient of the cost function. Allowing for such methods would be highly beneficial if more complex cost functions are utilized.

REFERENCES

[1] "World cities database," https://simplemaps.com/data/world-cities, accessed: 2022-11-07.

[2] M. Mahmoud, H. Fahmy, and J. Labadie, "Multicriteria siting and sizing of desalination facilities with geographic information system," *Journal of Water Resources Planning and Management-asce - J WATER RESOUR PLAN MAN-ASCE*, vol. 128, 03 2002.

[3] I. Sutzkover, D. Hasson, and R. Semiat, "Report on the israel desalination society conference - large seawater desalination projects: a timely solution to israel's pressing water needs," *Desalination*, vol. 128, 04 2000.

[4] "Uk firm develops nuclear-powered desalination ship concept," https://splash247.com/shipping-shown-new-business-line-drinking-water/, accessed: 2022-11-28.

[5] P. M. Griffin, C. R. Scherrer, and J. L. Swann, "Optimization of community health center locations and service offerings with statistical need estimation," *IIE Transactions*, vol. 40, no. 9, pp. 880–892, 2008. [Online]. Available: https://doi.org/10.1080/07408170802165864

APPENDIX

https://github.com/dmj17b/AnalysisProject