Original Article

A study of the frequency, amplitude and magnetic field dependence of the dynamic stiffness of magnetorheological elastomers

Dennis J Manjaly[1], Allen Chacko[1], Patel J Kannaihalal[1] ,Umanath R Poojary[2] and Gangadharan KV[3]

Abstract

The purpose of a damper is to isolate mechanisms or payloads from excessive shocks and vibrations that are imparted during the functioning of a machine. This could take the form of car suspensions, building foundations, air-borne isolators and even on satellites where precise positioning of instruments is necessary to acquire data without error. Passive vibration isolators have been used for many of these purposes ever since the advent of the industrial revolution. The first was in the form of leaf-springs in carriages and automobiles. But, passive isolators can function to avoid excessive vibration around, or in a range including one particular natural frequency – the natural frequency of the system. Modern active vibration isolators are capable of changing this natural frequency and can function more flexibly. Active vibration isolators bring this to effect by using special dedicated actuators. However, these actuators demand a lot of power. At this juncture, another class of isolators, namely 'semi-active' vibration isolators emerge. The advent of smart materials made it easier to design semi-active means where, materials dynamically alter their material properties to perform vibration isolation. The paper models the elasticity of a smart material called a magnetorheological elastomer using the non-linear Bouc-Wen model. The paper begins with a brief introduction to MR elastomers and popular modeling techniques employed. It further speaks about the Bouc-Wen model, the linearization used and how the transfer function between the force and displacement was obtained. Further, the description of the experimental setup and the instrumentation used is also specified. The parameters are estimated using a trust region search algorithm using MATLAB Simulink to calculate the parameters of a linearized Bouc-Wen model. A generalized equation relating the dynamic stiffness with the frequency and amplitude of the strain and the applied magnetic field.

Keywords*:* transfer function, smart materials, semi-active, magnetorheological elastomers, trust region search

Introduction

A magnetorheological elastomer (MRE) is a material that is today classified as a 'smart material' as in Cilian et al (2007). This is due to its unique ability to alter its own material properties on the application of a magnetic field. It is composed of an elastomeric matrix with a specific concentration of iron particles and certain binding agents that ensure the cohesiveness of the system (Miao – 2013, Li et al. 2014). The grade of the iron used and its concentration greatly influence the response of the substance to the magnetic field as they directly influence the magnetic permeability of the material. On the application of a magnetic field, the suspended iron inside the matrix will experience a magnetic torque and attempts to align with the magnetic field lines in the form of chains – depending on the manner in which the sample was cured (Wang et al. 2007). Hence, the formation of chains increases the material stiffness of the elastomer in the direction transverse to that of the applied magnetic field. In fact, it will be later shown that the storage moduli – the component of the stiffness that contributes to the storage of energy and the loss moduli – the component which is responsible for the removal of energy from the system continue to increase in the presence of an increasing magnetic field (Brinson and Brinson 2008).

Magnetorheological elastomers find numerous applications as a consequence of their ability to change stiffness when in the presence of a magnetic field (Li and Xang 2013, Choi and Xiong – 2010). A small list would include intelligent suspensions (Deng and Gong 2006), seismic isolation (Li et al 2013, Dyke et al 2010), medical devices etc. A semi-active vibration isolator is one such device that makes use of this unique property of MRE (Bazinenkov and Mikhailov 2015) . It serves to protect fragile payloads from violent oscillations by actively altering the natural frequency of the system in response to the disturbing vibrations. This is necessary because if the forced vibrations go unchecked, the system may approach resonance and will get damaged subsequently as in the case of earthquakes. A suitable control strategy (Sciulli 1997) also has to be engineered to bring this to effect. To prevent this, the frequency of the disturbing vibrations are detected via accelerometers and a control strategy uses this feedback to produce a magnetic field that changes the stiffness of the elastomeric layer on which the system is mounted (Fu et al 2013). Hence for the most efficient functioning of the isolator,

[1]Department of Mechanical Engineering , National Institute of Technology Karnataka,
Mangalore ,India

[2]Department of Mechanical Engineering, National Institute of Technology Karnataka,
Mangalore, India

[3]Department of Mechanical Engineering, National Institute of Technology Karnataka,
Mangalore, India

Corresponding author Dennis Joshy, Department of Mechanical Engineering, National Institute of Technology Karnataka, Surathkal, 575025, Mangalore, India email: dennis.joshy@gmail.com

knowledge of the dynamic response of MR elastomer is of great importance and this is the primary motivation of this paper.

Before beginning to describe the method of characterization employed in this paper, a few basic details about vibration isolation will be delineated in this section using a single degree of freedom system(SDOF):

The stiffness and the damping properties of MRE is represented by the storage and the loss moduli. Hereafter, the G_{real} and G_{img} components will represent the Storage and the Loss factor of the elastomer. Thus, the equivalent complex stiffness is an imaginary quantity, which will later be shown to be amplitude, frequency and magnetic field dependent;

$$G = G_{real} + iG_{img}$$

The natural frequencies of a system are dependent on the system parameters that are responsible for its stiffness and damping. Consequently, the real and imaginary components of the effective stiffness will control the natural frequency and the shape of the frequency response curves. In the frequency response curve of the system, the natural frequency is the point where the maximum magnification factor is reached. This is infinite for undamped systems and a peak at the damped natural frequency(ω_n) value for a damped system. As the stiffness of the system is now controllable, the frequency response curves and natural frequency can be shifted. As the response curve shifts, the magnification factor at the initial loading frequency is reduced and damage is averted. The ability of the MRE to be used as a vibration isolator is derived from the fact that G_{real} and G_{img} can be controlled. Suppose the forced oscillations say from a machine shop is too close to the point of resonance of a glass object mounted on an MR platform, the isolation system applies a magnetic field to alter G_{real} and G_{img} in such a way that frequency response curve re-organizes itself to a new natural frequency and a decreased magnification factor $\frac{F(s)}{X(s)}$. Figure 2 demonstrates this effect.

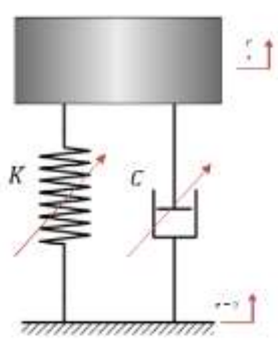

Figure - 1 - viscoelastic models with variable stiffness and damping

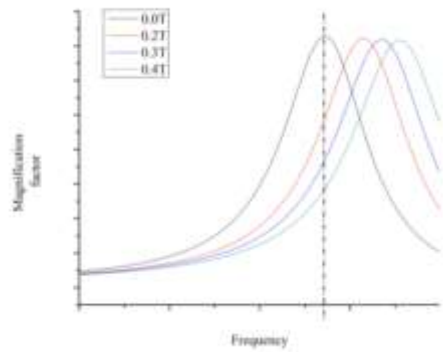

Figure 2 - Shifting Displacement Transmissibility Curves

This is the expected behavior of the MRE isolator; the response curve shifts away from resonance and reduces the magnification factor. The isolation can be considered effective if the curve shifts to an FRF value less than 1 and his occurs for $\frac{\omega}{\omega_n}$ ratio equal to $\sqrt{2}$.

The development of a controller that can accomplish vibration isolation effectively depends largely on the modeling assumptions involved. The present study assumes the properties of magnetorheological elastomers to be analogous to that of a viscoelastic material. Viscoelasticity is the unique property of materials behaving as rubber over a certain range of temperatures and then glass-like after a certain region of transition between them (Ciao and Deng 2009, Christenson 2012). In some examples, the materials tend to act as solids for specific conditions and then as a semi-solid at the other extreme – as the popular example of cornstarch demonstrates. Cornstarch behaves as a solid when the force of impact is very large and as a liquid if it is small. Such transitions are accompanied by drastic changes in material properties. This makes modeling of such materials a difficult task. Particularly, for the problem statement discussed here, the presence of the iron particles inside the elastomeric matrix increases the complexity of the problem. The iron filings induce an amplitude dependence for the response function of the elastomer and a weak frequency dependence as will be demonstrated in the subsequent sections (Lejon 2012, Betton 2008, Drozdov 1997, Patankar 2009).

A multitude of linear and non-linear models have been developed to study visco-elastic behavior. Most elastomers are characterized by visco-elastic models. As MRE uses an elastomeric matrix as the base for the iron particles, the models that are used for studying visco-elastic materials can also be applied for studying MRE with certain modifications (Chen 2007). Another interesting technique is the application of fractional derivative models using the principles of fractional calculus (Gil-Negrete et. al 2008, Pritz 1996, Sasso 2011, Zhu et al 2012). Fractional models are capable of capturing important viscoelastic effects. In comparison to conventional models, fractional models have fewer parameters and at the same time can accurately capture important viscoelastic effects. Bagely and Torvik (1983) gives a very cogent study on the existence of the theoretical basis for the fractional model and how it is superior to the models that were existing at that time. Guo (2014) implements a magnetoviscoelasticity parametric model with an Abel dashpot to describe the fractional order system

Significant study has also been dedicated to the use of nonlinear models. The works of Berg (1996), indicate that even at such an early period, the scientific community had recognized their importance in modeling viscoelastic materials. Berg(1996), makes use of nonlinear spring model to study vehicle dynamics. Gil-Negrete et. al (2008), makes use of a nonlinear model to present a holistic study of amplitude dependence and fractional order viscoelasticity.

A hysteretic system follows linear behavior till a certain region in the stress-strain curve. This is the region within which Hooke`s law is applicable. It is close to the equilibrium point. Beyond this linear region, the system behaves rather unpredictably and the linear models fail to capture this behavior. The so-called ‘non-linearity’ arises due to the friction that arises between the hydrocarbon chains within the matrix material, other material defects and discontinuities at high amplitudes of vibration (Betton 2008). This is where non-linear viscoelastic models play a significant role in simplifying the process of characterization. The hysteretic modeling approach can capture such afore-mentioned nonlinear behavior.

Significant research effort has been dedicated to parameter identification using such hysteresis model. Iskandarani and Karimi (2011) describes the use of hysteresis models for an SAS rotational MR damper. It is a brief treatise to the standard hysteretic models available. The Bingham plastic model, another non-linear model that can model slip is widely used. Dyke et al. (1996), uses a Bingham model to study MR dampers. The Bouc-Wen model is a similar model and has found application in several domains including elastomers, metals and advanced materials. Studies on modeling MR dampers are prevalent as is evident from the works of Yang et al. (2013) and Behrooz et al. (2014). Shih and Sung (2005) uses the Bouc-Wen model to study the hysteretic behavior of rhombic low yield strength steel. Rakotondrabe (2010) made use of this model to compensate for the hysteretic non-linearity in piezoelectric actuators. Ceravolo et al (2008). gives an example where parametric estimation for the model has been completed from seismic response data.

This paper explores the aspect of modelling the elastomer`s response using the Bouc-Wen model. Mathematically, the Bouc-Wen model has been studied rigorously. Perhaps, the most comprehensive treatise on the subject has been provided by Ikhouane (2007) in the book ‘Systems with Hysteresis’, where he details all the mathematical nuances of this system and an entire chapter has been devoted to parameter identification. The linearization provided by Hurtado and Barbat (2000) plays a significant role in this paper. A linearized version of the non-linear Bouc-Wen model is used to characterize the response of the MR elastomer in the amplitude range of 0.1mm to 0.3mm over a frequency of 6 – 20Hz and varying magnetic field.

The hysteresis loops of force-displacement data of four different frequencies, four different magnetic fields and three different amplitudes of vibration (specified in the next few sections) are fitted to the linearized Bouc-Wen model. The estimated parameters are used to evaluate the transfer functions in each case to calculate the real and imaginary components of the effective modulus – respectively the storage and the loss moduli. The storage and the loss moduli are further fitted for amplitude, frequency and magnetic field to develop a generalized expression.

Experimental

Experiments were conducted where a sample of MRE (dimensions) was subjected to shear stresses. The apparatus consisted of a horizontal shaking device mounted on a heavy metallic bed. The sample under study was given a shear sinusoidal excitation along the horizontal axis. Force transducers collected the drive and transfer values on either side of the sample. The differences between drive and transfer stiffness has been explained in Poojary et al. (2016). Two electromagnets were kept in transverse direction to apply a magnetic field up to a magnitude of 0.4T. The data was acquired by the software LabVIEW by National Instruments using the accelerometer and Data Acquisition unit NI9234 at a sampling rate of 25600 samples per second. The shaker is coupled with a function generator and is together capable of applying strains of amplitude up to 0.3mm at a frequency of 20Hz. The setup is the same setup that was used by in Poojary et al. (2016).

Processing of the acquired data was done using another LABVIEW program that served to extract approximately one full wavelength of information that appears first in the sample. This was done for data ranging from a frequency of 6Hz at 0T of magnetic field until 20Hz at 0.4T of magnetic field over amplitudes varying from 0.1mm to 0.3mm. The parameter estimation using this data was done in MATLAB.

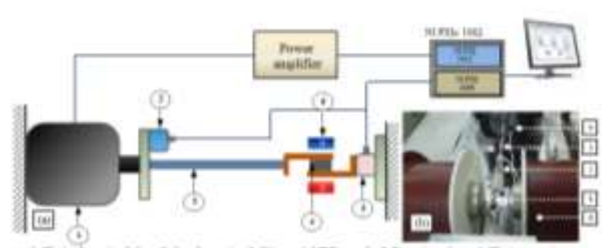

Figure 3 - Diagram of experimental setup

Sample hysteresis loops of some of the data that was used in the parameter estimation.

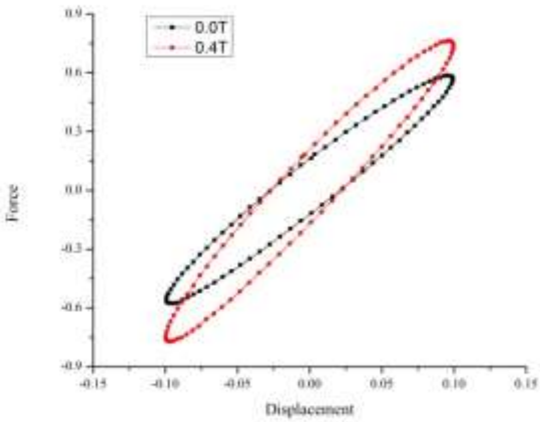

Figure 5 - Hysteresis loops for 6Hz 0.1mm amplitude and 0.0T-0.4T magnetic field

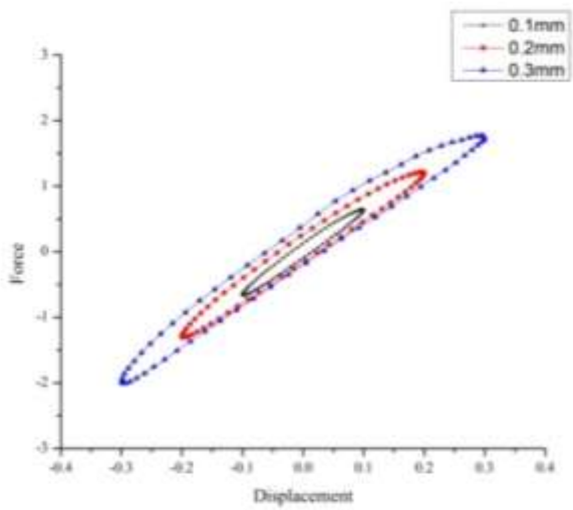

Figure 4 - Hysteresis loops for 16Hz 0T and 0.1-0.3mm amplitude

The shapes and the configurations of the hystersis loops are indicative of the effect of amplitude and magnetic field on the mechanical preperties of the material of the material. For instance, figure 4 depicts that as the magnetic field increases from 0 to 0.4T, the average slope of the hysteresis loop has increased; that is the material has become stiffer. Also, in figure 5, on close observation, the hysteresis loops start to become irregular and is indicative of the system starting to deviate from linear conditions.

Linear and nonlinear system models

Two of the classical models used in this respect are the Kelvin-Voigt model and the Maxwell model (figure 6a and 6b) (Betten 2008). Different models have different advantages. For instance, the Kelvin Voigt model can describe the system response better under the conditions of creep while the Maxwell model is better at describing the system under relaxation conditions. Considering these, a conscious choice is to be made regarding which model to use. Other models include derivatives or combinations of the above mentioned models. These consist of cells of the above units arranged in series or in parallel. The standardized linear model is the generalized representation of this configuration (Wiechert 1889). It is essentially a spring element with n units of Maxwell cells arranged in parallel. It may be thought of as each cell representing each molecular chain of the elastomeric matrix to accurately characterize the dynamics associated with each chain. The spring and damper together attempts to describe the elastic behavior of each elastomeric chain. Such models are exceptionally good at modelling in the linear regions of vibrations.

The simplest approach to model a hysteretic system such as this is to simplify it to a spring-mass-damper system. The output and input data are collected from experiments using shakers and signal acquisition systems. After properly conditioning the signal, a curve fitting algorithm is used to estimate the parameters – the mass, the spring stiffness and the damping co-efficient.

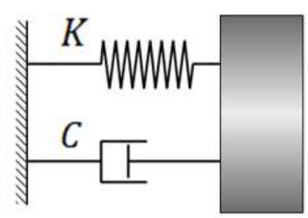

Figure – 6b. The Kelvin Voigt model

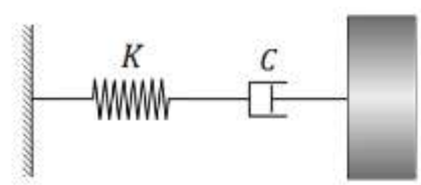

Figure – 6a The Maxwell model

Non-linear systems

Non-linear viscoelastic models can characterize response beyond linear viscoelastic models. Systems like the the Preisach model and the Bouc-Wen model of hysteresis (Ikhoune 2007) are non-linear models. They are capable of describing the nature of vibrations in high amplitudes. This paper considers the Bouc-Wen model of hysteresis to study the response of the MR elastomer as a varying magnetic field is applied. The Bouc Wen model is described as follows.

The Bouc-Wen model is usually represented as:

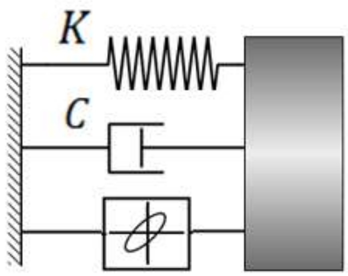

Figure - 7 The Bouc Wen model

Modelling the system as a spring mass damper with a hysteretic element in parallel;

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F \qquad (1)$$

Where z is a non-observable parameter described by a non-linear equation:

$$\frac{dz}{dt}=A\frac{dx}{dt}-\beta\left|\frac{dx}{dt}\right||z|^{n-1}-\gamma\frac{dx}{dt}|z|^{n} \qquad (2)$$

Where A,β,γ and n are parameters that describe the hysteresis loop (Ikhoune 2007). The model is famous for its flexibility to describe several hysteretic systems. The constants k_1 and k_2 together control the restoring spring force. The parameter $k_1$represents the elastic component while the parameter k_2 represents the hysteretic component of the restoring force (Ikhoune 2007). Each parameter of the above governing equations influences the hysteresis loops in different ways. This could be in the form of vertical intercept, amplitude, thickness or area under the graph.

It is difficult to solve the non-linear Bouc-Wen system to obtain closed form solutions for the differential equations. Hence, it is a common practice to linearize the system and use a Laplace transform to obtain a frequency response function of the system. By conducting parameter estimation with advanced optimization algorithms using software like MATLAB, the variation of the parameters across frequency, amplitude and magnetic field can be easily studied.

The Bouc-Wen model can be modified in several ways to simplify the system and to make it computationally faster for parameter estimation. The system can also be normalized to reduce the number of parameters without any compromise in rigorosity. Finally, (Ikhoune 2007). provides a study of how the parameters of the model affect the hysteresis loop. The book provides a comprehensive study of the Bouc-Wen model regarding the hysteresis loops, parameter identification etc.

Linearization of the model

Obtaining the transfer function is key to studying the storage and the loss moduli of the elastomer. The frequency response function is crucial for identifying the effective stiffness value. Although the parameter estimation can be accomplished using the non-linear form of the equation, it is not possible to obtain a transfer function for the differential equation. Hence, the following linearization is assumed:

$$\frac{dz}{dt}=S_e x+C_e\frac{dx}{dt}+k_e z \qquad (3)$$

Hurtado (2000) describes how the linearization is statistically valid and how the parameters of the model are calculable. S_e, C_e and k_e are constants that are used to describe the non-observable parameter z. There are many other linearized versions of the Bouc Wen. As a result of this linearization, the error function resulting from fitting the hysteresis loop using the linearized Bouc Wen model will be more than that of the nonlinear version.

Some of the important aspects, according to Hurtado (2000), about the linearization include that the parameter C_e should be zero. This is further corroborated by the observation that while conducting parameter estimation, the value of the error function at the final iteration was greater when the linear x term was included. So, the parameter estimation was conducted without this term.

There are also time domain solutions for the Bouc-Wen model which involves complicated expressions consisting of gamma functions, additional parameters and series solutions. Although Kozlov (2011) provides a time domain solution of the system, using such a solution is computationally expensive for the present study and was not considered

The transfer function

Applying a Fourier Transform to the equation describing transfer stiffness properties, the frequency domain relationship between the displacement and the restoring force is given by:

$$k_1x(s)+k_2z(s)+iC\omega x(s)=F(s) \qquad (4)$$

Applying a Fourier Transform to the linearized equation Eq. [3]:

$$i\omega z(s)=S_e x(s)+i\omega C_e x(s)+k_e z(s) \qquad (5)$$

$$z(s)=\frac{i\omega C_e}{i\omega-k_e}x(s) \qquad (6)$$

Substituting the expression for $Z(s)$ in Eq. [4],

$$k_1 x(s) + k_2 \frac{i\omega C_e}{i\omega - k_e} x(s) + iC\omega x(s) = F(s) \qquad (7)$$

The transfer function is then:

$$\frac{F(s)}{x(s)} = k_1 + k_2 \frac{i\omega C_e}{i\omega - k_e} + iC\omega \qquad (8)$$

Converting the denominator into a purely real number;

$$G_{eq} = k_1 + k_2 \frac{\omega^2 C_e}{\omega^2 + k_2} + i\omega \left(C - \frac{k_2 k_e \omega C_e}{k_e^2 + \omega^2} \right) \qquad (9)$$

Where,

$$G_{real} = k_1 + k_2 \frac{\omega^2 C_e}{\omega^2 + k_2} \qquad (10)$$

$$G_{img} = \omega \left(C - \frac{k_2 k_e \omega C_e}{k_e^2 + \omega^2} \right) \qquad (11)$$

Parameter Estimation

The parameter estimation tool of Simulink was loaded with the data collected from the experiment. The Levenberg-Marquadat method was applied using a trust-region-reflective algorithm. The data included the following data sets. Each data set was an excel file with both displacement and force data in the time domain.

- 0.0T magnetic field displacements from 0.1mm, 0.15mm, 0.2mm of frequencies for 6Hz, 10Hz, 14Hz, 20Hz
- 0.2T magnetic field displacements from 0.1mm, 0.15mm, 0.2mm of frequencies for 6Hz, 10Hz, 14Hz, 20Hz
- 0.3T magnetic field displacements from 0.1mm, 0.15mm, 0.2mm of frequencies for 6Hz, 10Hz, 14Hz, 20Hz
- 0.4T magnetic field displacements from 0.1mm, 0.15mm, 0.2mm of frequencies for 6Hz, 10Hz, 14Hz
- 0.4T magnetic field displacements from 0.1mm, 0.15mm frequencies for 20Hz

Line search methods and trust region methods are two broad classifications of optimization algorithms. The trust region methods can implement different algorithms like Levenberg-Marquadat techniques. The 'Trust Region Reflective Algorithm' is implemented using a nonlinear least squares method.

Sample hysteresis loops that were fitted:

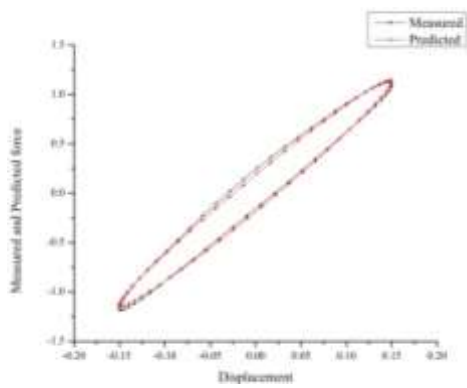

Figure 8 - Fitting hysteresis loops for 10Hz 0T 0.1mm amplitude

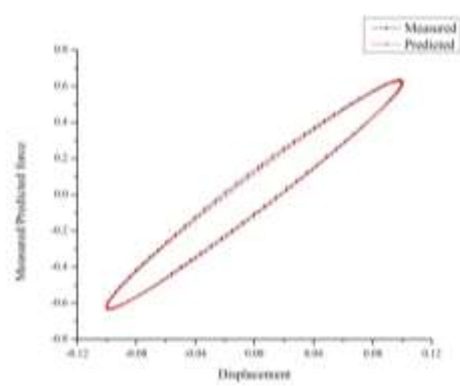

Figure 9 – Fitting hysteresis loops for 14Hz 0.4T and 0.1mm amplitude (Force in Newtons and displacements in mm)

The following parameters where estimated:

- The spring stiffness k_1
- The co-efficient of damping C
- The co-efficient of the unobservable hysteretic parameter k_2
- The co-efficient k_e in the linearized model
- The co-efficient C_e in the linearized model

Although, the non-linear Bouc-Wen model experiences no difficulty in fitting such data, the linearized version of the same model will not so easily fit it. The increased fitting error at higher amplitudes is an evidence of the nonlinear nature of the system.

This data is tabulated as follows:

Table 1 - Estimated Parameter Values

0T	**0.1mm**				**0.15mm**				**0.2mm**			
	6	10	14	20	6	10	14	20	6	10	14	20
C	0.013562	0.018705	0.012189	0.008278	0.014246	0.014209	0.011521	0.008116	0.035091	0.018241	0.011993	0.007766
K_1	5.65481	6.209994	6.440549	6.635052	5.005858	5.798701	6.239434	6.475685	5.638087	6.004017	6.151514	6.409856
K_2	0.155726	0.017983	0.015847	0.005188	0.171131	0.071222	0.022162	0.004199	0.000718	0.010791	0.010951	0.005792
C_e	0.155727	0.017939	0.015916	0.00519	0.171136	0.071219	0.022255	0.00419	0.000719	0.010735	0.01094	0.007112
K_e	1.111394	15.92002	25.53512	42.14152	13.93355	4.475975	7.528152	40.98313	60.06741	23.01163	14.5269	88.98403

0.2T	**0.1mm**				**0.15mm**				**0.2mm**			
	6	10	14	20	6	10	14	20	6	10	*14*	20
C	0.036859	0.015475	0.01322	9.05E-05	0.002336	0.019146	0.008787	0.008527	0.022427	0.019088	0.012567	0.008275
K_1	6.074082	6.273381	6.401657	0.999958	5.615058	5.854026	6.181719	6.374046	5.818496	6.014826	6.151046	6.315567
K_2	0.062854	0.070293	0.008734	9.24E-05	0.195925	0.040824	0.065692	0.008471	0.128437	0.018521	0.007359	0.010337
C_e	0.062836	0.070292	0.008719	9.24E-05	0.195927	0.042833	0.065686	0.008468	0.128438	0.018511	0.007344	0.006554
K_e	5.445951	2.845835	44.5843	17.2041	1.349884	9.784784	14.95101	14.56226	0.435525	8.315492	16.99195	68.42162

0.3T	**0.1mm**				**0.15mm**				**0.2mm**			
	6	10	14	20	6	10	14	20	6	10	*14*	20
C	0.029964	0.022485	0.012189	0.009779	0.001114	0.022268	0.007949	0.009643	0.026253	0.021706	0.014078	0.009418
K_1	7.190542	7.264774	6.440549	7.688615	6.643286	6.788926	7.166823	7.454485	6.905465	6.990707	7.071458	7.331507
K_2	0.135869	0.016031	0.015847	0.010309	0.212093	0.01135	0.085109	0.005703	0.139454	0.01254	0.007532	0.004845
C_e	0.135866	0.014296	0.015916	0.010343	0.212097	0.117494	0.075553	0.005693	0.139451	0.012502	0.011825	0.004836
K_e	0.972323	23.91866	25.53512	28.49274	0.992875	18.65039	7.321847	35.75839	0.306262	17.30877	30.18704	25.33281

0.4T	**0.1mm**				**0.15mm**				**0.2mm**			
	6	10	14	20	6	10	14	20	6	10	*14*	20
C	0.045664	0.049587	0.015197	0.009969	0.000459	0.004174	0.014305	0.009616	0.045756	0.022125	0.014348	0.003497
K_1	7.454235	7.534582	7.692846	7.857303	6.803768	6.991248	7.540305	7.751187	7.133713	7.339365	7.485779	7.323604
K_2	0.070392	0.010817	0.009734	0.004872	0.217612	0.138302	0.001459	0.007213	0.031426	0.008424	0.011128	0.106042
C_e	0.07046	0.010856	0.009722	0.007702	0.217615	0.13934	0.001611	0.007197	0.031136	0.008403	0.011148	0.106029
K_e	6.048122	22.56824	19.74238	52.33587	1.030825	2.246108	5.90279	29.13841	15.12531	33.34187	7.173464	2.076573

Table 2 - Dimensions of the parameters in the linearized equation

Parameter	**Unit**
k_1	N/mm
k_2	N
C_e	1/mm
C	N-s/mm
K_e	1/s

Linear correlations are most popularly used for implementing in control algorithms (Wang 2008), although nonlinear relations are also becoming popula, which are comparatively more involved and time consuming to implement. Considering this, in the present study, a linear model is used to obtain the relationship between stiffness, frequency, amplitude and magnetic field.

Expressions of real and imaginary stiffness

The parameters were used to calculate the transfer functions described in section 3.2 and the values of G_{real} and G_{img} were calculated. The values of G_{real} and G_{img} were fit using the parameters of amplitude, magnetic field and the frequency. So, the approximate functions of stiffness of the magnetorheological elastomer are:

$$G_{real} = 1.4147A + 3.700B + 0.01421f + 5.3376 \quad (10)$$

$$G_{img} = -0.825A + 0.6469B + 0.00109f + 1.017441 \quad (11)$$

Thus, the final expression of effective stiffness will be of the form:

$$G_{eq} = G_{real}(f, A, B) + G_{img}(f, A, B) \quad (12)$$

Results and Discussion

Observing each term of the expressions obtained in the preceding section, multiple details can be gleaned. For instance, the MR elastomer seems to have a residual amount of storage moduli – 5.3376 N/mm^2 and a loss moduli of 1.017441 N/mm^2 in its static state (without any applied vibrations or magnetic field). Further, as expected, there is a positive correlation with the magnetic field. However, the storage moduli has a positive correlation with the amplitude and the frequency while the loss moduli has a negative correlation with the amplitude. Also, the dependence of stiffness (both storage and loss moduli) on the amplitude is almost 80 times stronger than the dependence on the frequency. Verifying and explaining the basis for this would be an interesting area of research.

The data sets that were used for the validation are 8Hz, 12Hz, 16Hz and 18Hz data, 0.1 and 0.2 mm amplitudes and 0 – 0.4T magnetic fields. The procedure involved using a simple Simulink model of sine waves to calculate the amplitude and phase values of the displacement data sets through the same parameter estimation algorithm that was employed earlier. The storage and the loss moduli values of the data sets were calculated using Eq. [13]. The storage and the loss moduli were then expressed in the form of frequency domain transfer functions in the Eulerian notation. The product of the transfer function and the displacement gave the force values for each data set. The force values were used in the calculation of the fitness value using Eq.[14] and this value indicates how well the expression predicts the modulus values for the above test cases.

Table 3 - Calculated Fitness values

8Hz	0.1mm	0.2mm	12Hz	0.1mm	0.2mm
0.0T	91.477	92.432	0.0T	91.125	94.267
0.2T	95.629	90.281	0.2T	94.119	92.092
0.3T	92.729	92.356	0.3T	91.995	95.412
0.4T	92.404	93.945	0.4T	92.613	93.380
16Hz	0.1mm	0.2mm	20Hz	0.1mm	0.2mm
0.0T	90.154	92.905	0.0T	88.815	90.911
0.2T	97.311	92.290	0.2T	91.483	93.453
0.3T	91.148	94.679	0.3T	91.191	93.988
0.4T	91.305	93.157	0.4T	92.142	-

Verification

In order to verify the expression obtained in the preceding section, it is necessary to show how well it is useful for data that lies intermediate to the frequencies that were used for parameter estimation. Hence, for the purpose of validation, the frequencies 8Hz, 12Hz, 14Hz and 18Hz were used – further specifications of the validating data set is given below.The method used to judge how well the linearized Bouc-Wen model fits the hysteresis loops is by calculating a value called the fitness value. Norouzi et al (2015) and uses the fitness values to check how well a 'modified viscoelastic Kelvin-Voigt' model fits the experimental data. Syeung-Hung et al (2012) also uses this definition of fitness value to study how well the model captures the specimen behvior.

The expression for fitness value is given by:

$$Fitness = \left[1 - \frac{norm(F_p - F_m)}{norm(F_m - \overline{F}_m)}\right] \times 100 \qquad (13)$$

Conclusion

Smart materials in application in several areas. In depth study and documentation about their properties ae essential to make proper use of them. With the present study, it is endeavored to build a smart vibration isolator implementing a control algorithm like Proportional-Derivative and Integral control. This may find applications in a suspensions, engine mounts, positioning of telescopes and sensitive measurement equipment on satellites, seismic isolation and even in prosthetics. The use of semi-active means to isolate structures also saves much energy as opposed to the active methods that rely on dedicated actuators.

A specimen of MR elastomer has been studied using a hysteretic model of vibration – the nonlinear Bouc-Wen model. Data was acquired from the experiment in which sinusoidal strain was the input. The resulting response was measured and fitted to a linearized version of the Bouc-Wen model. The parameters from the analysis were used to evaluate the transfer functions and the expressions for storage and loss moduli were fitted with respect to frequency, magnetic field and amplitude. The fitness value obtained from the previous section indicates how well the Bouc-Wen model is capable of describing the hysteretic behavior of systems. The fitness value were calculated using the storage and the loss moduli from Eq. [13]. Fitness values of nearly 97% indicate the high reliability of using the expression and consequently, the linearized Bouc-Wen model over this frequency range, as opposed to the available linear systems.

Acknowledgments

The authors acknowledge the funding support from SOLVE: The Virtual Lab@NITK (Grant:No. F.16-35/2009-DL Ministry of Human Resources Development) (www.solve.nitk.ac.in) and experimental facility provided by Centre for System Design (CSD): A Centre of excellence at NITK-Surathkal.

References:

Banks HT and Pinter GA (2000) Damping: Hysteretic Damping and Model, North Carolina State University, Raleigh, N.C. USA

Bazinenkov AM, Mikhailov VP (2014) Active and Semi Active Vibration Isolation Systems Based on Magnetorheological. In: 2nd International Conference on Dynamics and Vibroacoustics of Machines (DVM2014) September 15 –17, 2014 , pp. 170 – 174, Samara, Russia: Elsevier Publishing

Bagley L et al (1983) A Theoretical Basis for the Application of Fractional Calculus to Viscoelasticity, Journal of Rheology 27, 201

Behrooz M, Xiaojie W and Gordaninejad F (2014), Modeling of a new semi-active/passive magnetorheological elastomer isolator, Smart Materials and Structures 23 045013 (7pp)

Betten J (2008) Creep Mechanics, 3rd Edition, Springer publishing

Berg M (1998) A Non-Linear Rubber Spring Model for Rail Vehicle Dynamics Analysis, Vehicle System Dynamics, International Journal of Vehicle Mechanics and Mobility, 30 pp. 197-212

Brenson HF and Brenson CF (2008) Polymer Engineering Science and Viscoelasticity : An Introduction, Second Edition, Springer US

Ceravolo R, Demarie1 GV, and Erlicher S (2008) Instantaneous identification of Bouc-Wen-type hysteretic systems from seismic response data. In: Key Engineering Materials, 2007, 347, pp.331-338

Christensen R (2010) Theory of viscoelasticity, an introduction, Dover Publishing

Choi WJ, Xiong YP and Shenoi RA (2010) Vibration characteristics of sandwich beams with steel skins and magnetorheological elastomer cores Advances in Structural Engineering ,13, 837-847.

Drozdov A (1997) Viscoelastic Structures, Academic Press

Dyke SJ, Spencer BF Jr, Sain MK (1996) Modeling and control of magnetorheological dampers for seismic response reduction, Smart, Materials and Structures, Volume 5, Number 5

Fu J, Yu M, Dong XM et al (2013) Magnetorheological elastomer and its application on impact buffer. Journal of Physics: Conference Series 412 (2013) 012032, doi:10.1088/1742-6596/412/1/012032

Fei Guo et al. (2014) Viscoelastic Parameter Model of Magnetorheological Elastomers Based on Abel Dashpot. In: Advances in Mechanical Engineering Volume 6, Article ID 629386, 12 pages, doi: 10.1155/2014/629386, Hindawi Publishing Corporation.

Gil-Negrete N, Vinolas J, Leif K (2008) A Nonlinear Rubber Material Model Combining Fractional Order Viscoelasticity and Amplitude Dependent Effects, Journal of Applied Mechanics 76(1) 011009 – doi:10.1115/1.2999454

Guo M (2013), Development of adaptive structures working with magnetorheological elastomers and magnetic force, PhD thesis, University of Wollongong, Australia

Hua-xia D, Xing-long G (2007) Application of magnetorheological elastomer to vibration absorber. Communications in Nonlinear Science and Numerical Simulation 13: 1938–1947

Hurtado JE, Barbat AH (2000) Equivalent linearization of the Bouc–Wen hysteretic model. In: Engineering Structures 22, pp. 1121–1132

Ikhouane F and Rodellar J (2007), Systems with Hysteresis, Analysis, Identification and Control using the Bouc–Wen Model. Wiley-Blackwell publishers

Iskandarani Y and Karimi HR (2011) Hysteresis Model Parameters Identification For The SAS Rotational MR Damper. In: WSEAS Transactions On Systems And Control, Issue 10, Volume 6, October 2011

Jonas L (2012), On the frequency, dynamic strain amplitude, pre-strain, temperature and magnetic field strength dependence of magneto-sensitive elastomers, PhD Thesis, Royal Institute of Technology, Stockholm, Sweden.

Kozlov DV (2011) , Approximate Analytical Solution of the Bouc-Wen Hysteresis Model by the Fourier Transform, : International Siberian Conference on Control and Communications (SIBCON), 10.1109/SIBCON.2011.6072599

Kuankuan C and Yimin D (2009), Design and Analysis of an Automotive Vibration Reduction System Based on Magneto-Rheological Elastomer. Proceedings of the 2009 IEEE International Conference on Mechatronics and Automation, August 9 – 12, 2009, pp. 1645-1649, Changchun, China, doi: 10.1109/ICMA.2009.5246273

Li WH, Zhang XZ, Du H(2013) Magnetorheological elastomers and their applications. In: Visakh, PM, Thomas S, Chandra AK, Mathew AP (eds) Advances in Elastomers I: Blends and Interpenetrating Networks. Germany: Springer, pp.357-374.

Li Y, Li J, Li W et al (2014). A state-of-the-art review on magnetorheological elastomer devices. Smart Materials and Structures, 23 (12), 1-24.

Li J, Li Y, Li W et al (2013). Development of adaptive seismic isolators for ultimate seismic protection of civil structures. Sensors and Smart Structures Technologies for Civil, Mechanical, and Aerospace Systems (pp. 1-12)

Lin Chen et al. (2007) Investigation on magnetorheological elastomers based on natural rubber. In: Journal of Material Science 42:5483–5489

Mayergoyz ID(2003) Mathematical model of hysteresis and their application, second edition, Springer Publishing

Norouzi M et al (2015) A new approach for modeling of magnetorheological elastomers. Journal of Intelligent Material Systems and Structures, 1 -15

Patankar KA (2009), Linear And Nonlinear Viscoelastic Characterization Of Proton Exchange Membranes And Stress Modeling For Fuel Cell Applications, PhD Thesis, Virginia Polytechnic University

Park SW (2001) Analytical modeling of viscoelastic dampers for structural and vibration control, International Journal of Solids and Structures 38 8065-8092

Poojary UR et. al (2016) Experimental investigation on the effect of magnetic field on strain dependent dynamic stiffness of magnetorheological elastomer. In: Rheologica Acta Volume 55, Issue 11, pp. 993–1001. DOI 10.1007/s00397-016-0975-y

Poojary UR (2016) et. al Magnetic field and frequency dependent LVE limit characterization of magnetorheological elastomer. In: jBraz. Soc. Mech. Sci. Eng. DOI 10.1007/s40430-016-0592-9

Poojary UR et. al (2016) Dynamic blocked transfer stiffness method of characterizing the magnetic field and frequency dependent dynamic viscoelastic properties of MRE. In: Korea-Australia Rheology Journal, 28(4), pp. 301-313. DOI: 10.1007/s13367-016-0031-6.

Pritz T (1996) Analysis Of Four-Parameter Fractional Derivative Model Of Real Solid Materials, Journal Of Sound And Vibration 195(1), 103-115

Rakotondrabe M (2010) Bouc-Wen modeling and inverse multiplicative structure to compensate hysteresis nonlinearity in piezoelectric actuators. In: IEEE Transactions on Automation Science and Engineering Volume: 8, Issue: 2, April 2011, pp. 428 - 431

Ruddy C, Ahearne E and Byrne G (2007) A Review Of Magnetorheological Elastomers: Properties And Applications, Advanced Manufacturing Science (AMS) Research Centre, Mechanical Engineering, University College Dublin, Belfield, Dublin 4, Ireland.

Sasso M, Palmieri G and Amodio D (2011) Application of fractional derivative models in linear viscoelastic problems. In: Mechanics of Time-Dependent Materials November 2011, Volume 15, Issue 4, pp 367–387

Scuilli D (1997) Dynamics and Control for Vibration Isolation Design. PhD Thesis, Virginia Polytechnic University, Blacksburg, USA

Seung-Hyun et al. (2012) Modeling Of Magneto-Rheological Elastomers For Harmonic Shear Deformation, IEEE Transactions On Magnetics, Vol. 48, No. 11, pp. 3080 - 3083

Shih M, Sung W (2005) A model for hysteretic behavior of rhombic low yield strength steel added damping and stiffness. In: Computers and Structures 83, 895–908

Wang K, Tang J (2008) Nonlinear High Precision Robust Control With Hysteresis Compensation. In: Adaptive Structural Systems with Piezoelectric Transducer Circuitry, Springer Publishing.

Wiechert, E (1889); "Ueber elastische Nachwirkung", PhD Thesis, Königsberg University, Germany

Xudong Z and Xilin L (2011), Parametric Identification of Bouc-Wen Model and Its Application in Mild Steel Damper Modeling, The Proceedings of the Twelfth East Asia-Pacific Conference on Structural Engineering and Construction – EASEC12, Procedia Engineering 14, 318–324

Yang J, Du H, Li W et al (2013). Experimental study and modeling of a novel magnetorheological elastomer isolator. In: Smart Materials and Structures, 22 (11), 1-14.

Yinling W et al. (2007) - Preparation and Properties of Magnetorheological Elastomers Based on Silicon Rubber/Polystyrene Blend Matrix. Journal of Applied Polymer Science, Epub ahead of print 27 March 2006, DOI 10.1002/app.24598

Zhou H, Zhou Y, Xiao JZ (2007) Active Vibration Control Of Terfenol-D Rod Of Giant Magnetostrictive Actuator With Nonlinear Constitutive Relations. In: Journal Of Theoretical And Applied Mechanics 45, 4, pp. 953-967, Warsaw 2007

Zhu J et. Al (2012) Magnetoviscoelasticity parametric model of an MR elastomer vibration mitigation Device. In: Smart Materials Structures, 21 075034 (12pp), doi:10.1088/0964-1726/21/7/075034