Backpropagation Through Time

Герард Костин

Анализ градиентов в RNN

$$\begin{aligned}
h_t &= f(x_t, h_{t-1}, w_h),\\
o_t &= g(h_t, w_o),
\end{aligned} \tag{9.7.1}$$

$$L(x_1, \ldots, x_T, y_1, \ldots, y_T, w_h, w_o) = \frac{1}{T}\sum_{t=1}^{T} l(y_t, o_t). \tag{9.7.2}$$

$$\begin{aligned}
\frac{\partial L}{\partial w_h} &= \frac{1}{T}\sum_{t=1}^{T} \frac{\partial l(y_t, o_t)}{\partial w_h}\\
&= \frac{1}{T}\sum_{t=1}^{T} \frac{\partial l(y_t, o_t)}{\partial o_t} \frac{\partial g(h_t, w_o)}{\partial h_t} \frac{\partial h_t}{\partial w_h}.
\end{aligned} \tag{9.7.3}$$

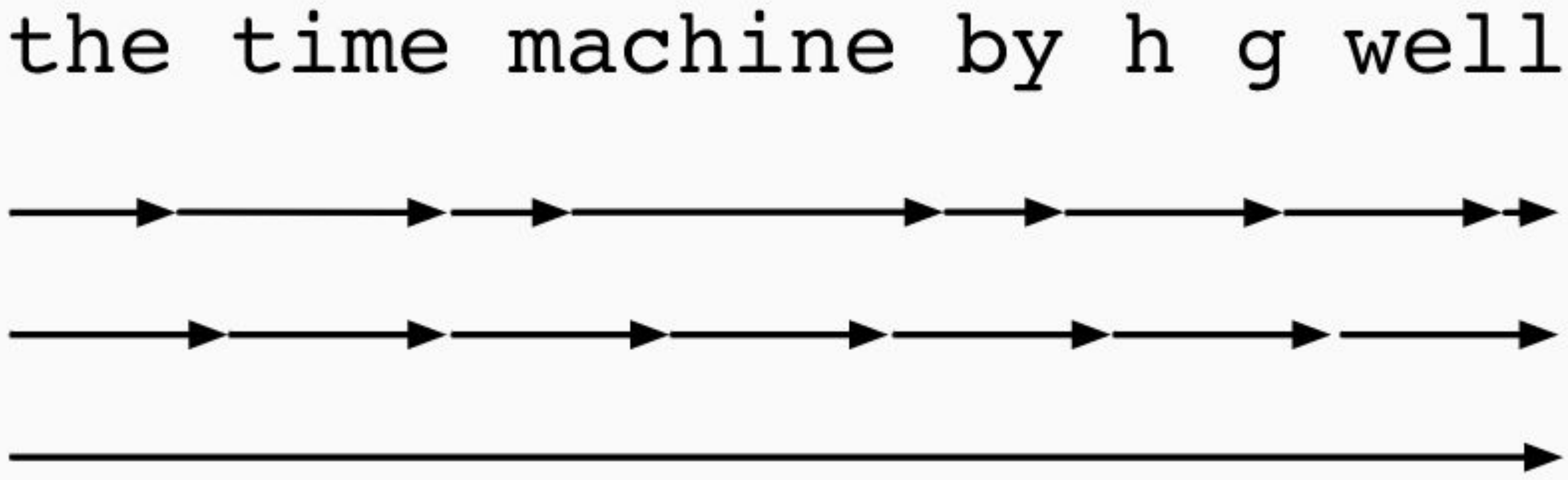
Анализ градиентов в RNN

$$\frac{\partial h_t}{\partial w_h} = \frac{\partial f(x_t, h_{t-1}, w_h)}{\partial w_h} + \frac{\partial f(x_t, h_{t-1}, w_h)}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial w_h}. \tag{9.7.4}$$

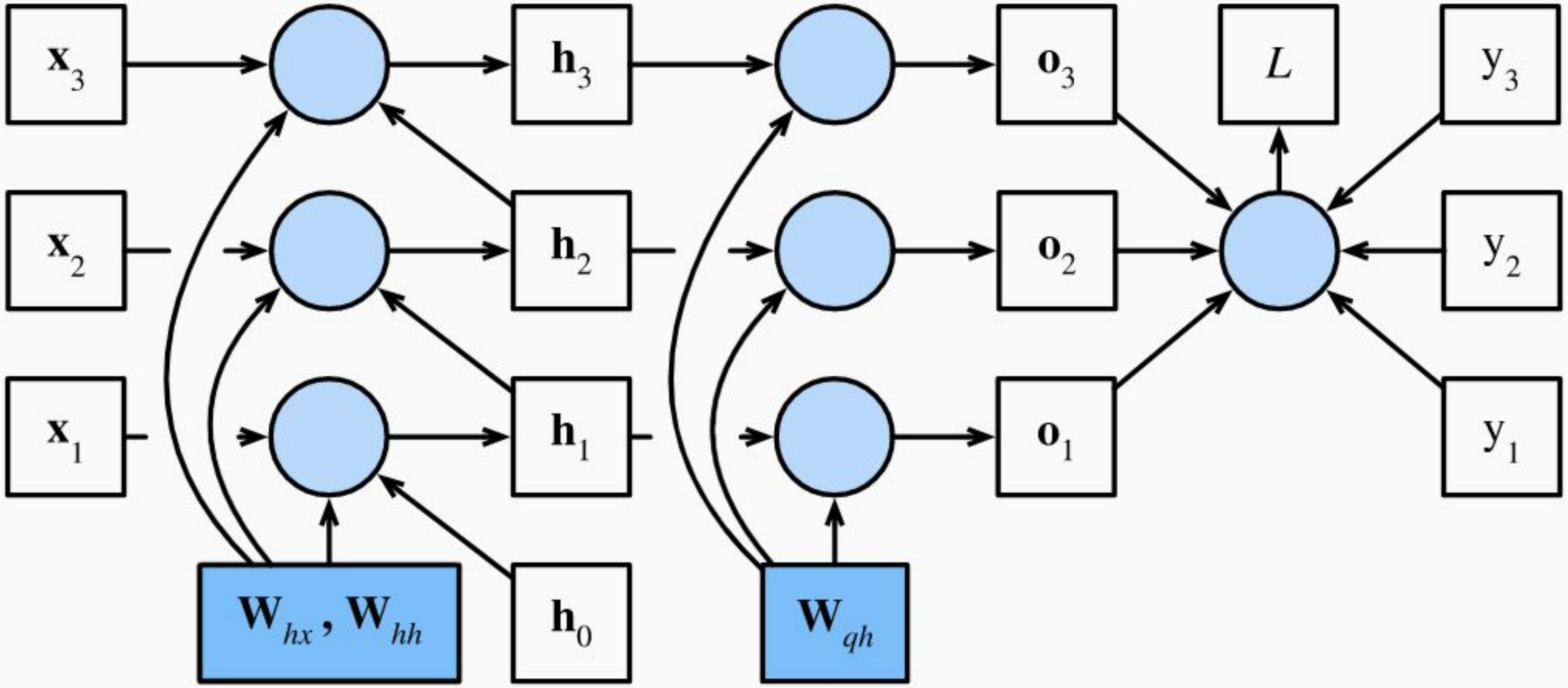
$$\begin{aligned} a_t &= \frac{\partial h_t}{\partial w_h}, \\ b_t &= \frac{\partial f(x_t, h_{t-1}, w_h)}{\partial w_h}, \\ c_t &= \frac{\partial f(x_t, h_{t-1}, w_h)}{\partial h_{t-1}}, \end{aligned} \qquad a_t = b_t + \sum_{i=1}^{t-1} \left(\prod_{j=i+1}^{t} c_j \right) b_i. \tag{9.7.5}$$

$$\frac{\partial h_t}{\partial w_h} = \frac{\partial f(x_t, h_{t-1}, w_h)}{\partial w_h} + \sum_{i=1}^{t-1} \left(\prod_{j=i+1}^{t} \frac{\partial f(x_j, h_{j-1}, w_h)}{\partial h_{j-1}} \right) \frac{\partial f(x_i, h_{i-1}, w_h)}{\partial w_h}. \tag{9.7.7}$$

Как обрезать расчеты в RNN



BPTT по шагам



$$\mathbf{h}_t = \mathbf{W}_{hx}\mathbf{x}_t + \mathbf{W}_{hh}\mathbf{h}_{t-1},$$
$$\mathbf{o}_t = \mathbf{W}_{qh}\mathbf{h}_t,$$

$$L = \frac{1}{T}\sum_{t=1}^{T} l(\mathbf{o}_t, y_t).$$

BPTT по шагам

$$\frac{\partial L}{\partial \mathbf{o}_t} = \frac{\partial l(\mathbf{o}_t, y_t)}{T \cdot \partial \mathbf{o}_t} \in \mathbb{R}^q. \tag{9.7.11}$$

$$\frac{\partial L}{\partial \mathbf{W}_{qh}} = \sum_{t=1}^{T} \text{prod}\left(\frac{\partial L}{\partial \mathbf{o}_t}, \frac{\partial \mathbf{o}_t}{\partial \mathbf{W}_{qh}}\right) = \sum_{t=1}^{T} \frac{\partial L}{\partial \mathbf{o}_t} \mathbf{h}_t^\top, \tag{9.7.12}$$

$$\frac{\partial L}{\partial \mathbf{h}_T} = \text{prod}\left(\frac{\partial L}{\partial \mathbf{o}_T}, \frac{\partial \mathbf{o}_T}{\partial \mathbf{h}_T}\right) = \mathbf{W}_{qh}^\top \frac{\partial L}{\partial \mathbf{o}_T}. \tag{9.7.13}$$

$$\frac{\partial L}{\partial \mathbf{h}_t} = \text{prod}\left(\frac{\partial L}{\partial \mathbf{h}_{t+1}}, \frac{\partial \mathbf{h}_{t+1}}{\partial \mathbf{h}_t}\right) + \text{prod}\left(\frac{\partial L}{\partial \mathbf{o}_t}, \frac{\partial \mathbf{o}_t}{\partial \mathbf{h}_t}\right) = \mathbf{W}_{hh}^\top \frac{\partial L}{\partial \mathbf{h}_{t+1}} + \mathbf{W}_{qh}^\top \frac{\partial L}{\partial \mathbf{o}_t}. \tag{9.7.14}$$

BPTT по шагам

$$\frac{\partial L}{\partial \mathbf{h}_t} = \sum_{i=t}^{T} \left(\mathbf{W}_{hh}^\top\right)^{T-i} \mathbf{W}_{qh}^\top \frac{\partial L}{\partial \mathbf{o}_{T+t-i}}. \tag{9.7.15}$$

$$\begin{aligned}
\frac{\partial L}{\partial \mathbf{W}_{hx}} &= \sum_{t=1}^{T} \text{prod}\left(\frac{\partial L}{\partial \mathbf{h}_t}, \frac{\partial \mathbf{h}_t}{\partial \mathbf{W}_{hx}}\right) = \sum_{t=1}^{T} \frac{\partial L}{\partial \mathbf{h}_t} \mathbf{x}_t^\top,\\
\frac{\partial L}{\partial \mathbf{W}_{hh}} &= \sum_{t=1}^{T} \text{prod}\left(\frac{\partial L}{\partial \mathbf{h}_t}, \frac{\partial \mathbf{h}_t}{\partial \mathbf{W}_{hh}}\right) = \sum_{t=1}^{T} \frac{\partial L}{\partial \mathbf{h}_t} \mathbf{h}_{t-1}^\top,
\end{aligned} \tag{9.7.16}$$