CS 434: Implementation Assignment 1

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Linear Regression

1. The learned weight vector of the test data is:

$$\begin{bmatrix} 3.95843212e+01 \\ -1.01137046e-01 \\ 4.58935299e-02 \\ -2.73038670e-03 \\ 3.07201340e+00 \\ -1.72254072e+01 \\ 3.71125235e+00 \\ 7.15862492e-03 \\ -1.59900210e+00 \\ 3.73623375e-01 \\ -1.57564197e-02 \\ -1.02417703e+00 \\ 9.69321451e-03 \\ -5.85969273e-01 \end{bmatrix}$$

Here is the learned weight vector next to the features that each weight describes:

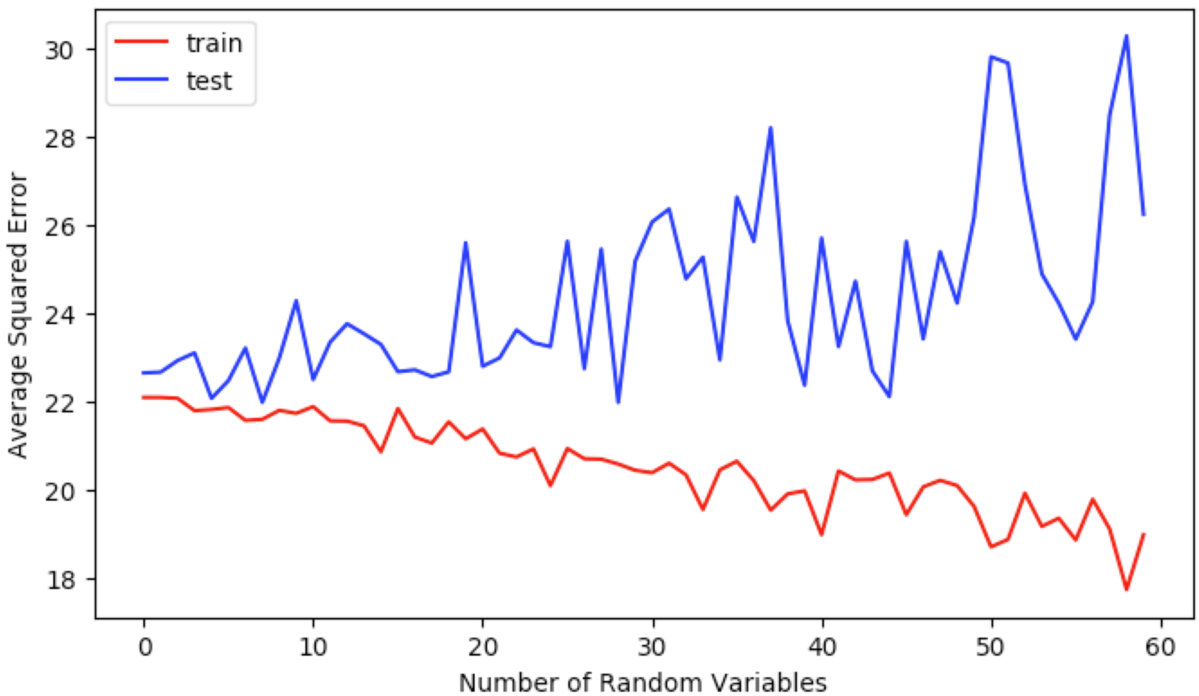
Weight	**Feature**
3.95843212e+01	Dummy
-1.01137046e-01	per capita crime rate by town
4.58935299e-02	proportion of residential land zoned for lots over 25,000sq.ft.
-2.73038670e-03	proportion of non-retail business acres per town
3.07201340e+00	Charles River dummy variable
-1.72254072e+01	nitric oxides concentration (parts per 10 million)
3.71125235e+00	average number of rooms per dwelling
7.15862492e-03	proportion of owner-occupied units built prior to 1940
-1.59900210e+00	weighted distances to five Boston employment centres
3.73623375e-01	index of accessibility to radial highways
-1.57564197e-02	full-value property-tax rate per $10,000
-1.02417703e+00	pupil-teacher ratio by town
9.69321451e-03	1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town
-5.85969273e-01	% lower status of the population

2. Training Dataset ASE: 22.081273187
   Test Dataset ASE: 22.6382562966

3. Training Dataset ASE (Without Dummy): 24.4758827846
   Test Dataset ASE (Without Dummy): 24.2922381757

   The reason the dummy variable improves the accuracy of the optimal weight vector is because it provides the y intercept (or b) for the regression.

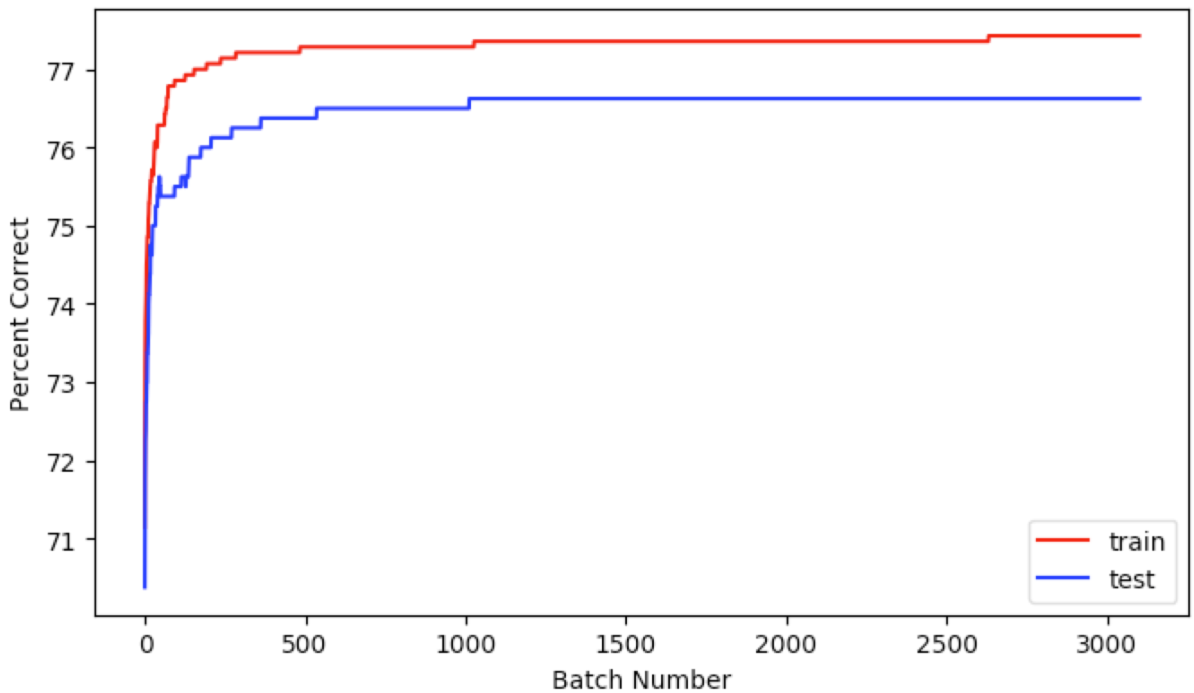
4. The addition of random features increases the accuracy of the regression when applying it to the training data, but also decreases the accuracy of the regression when applied to other data. Adding more features leads to worse performance when it comes to applying the learned weight vector to other datasets. This is because the addition of features allows the weight vector to be trained more precisely, but it comes at the expense of overfitting the regression to the specific case of the training data.



Logistic Regression

1. With $\eta = 10^{-7}$, the gradient descent algorithm's accuracy follows a logarithmic curve. The accuracy of the algoriothm is as follows.

η	**Training Accuracy**	**Testing Accuracy**	**Δ norm**	**Number of Batches**
$\eta = 10^{-7}$	78.0000	77.2500	399	1375



2. Adding regularization will allow the algorithm to train with more sensitivity to the size of the weights. By adding a function of the weights, the regression will be less likely to overfit using extreme weights. This will prevent a misled learning of data, and help make the final weights more accurate. Our L2 algorithm is similar to the regression algorithm, but with the added weight calculation:

$$\sum_{i=1}^{n} l(g(w^T x^i), y^i) + \frac{1}{2}\lambda |w|^2$$

This algorithm translates into code very similar to our initial code for the logistic regression. Below is how this algorithm would function, with the `batch_train` function being called iteratively until the delta norm is lower than a set bound.

```
batch_train(input_matrix, output_matrix, w):
  lambda, eta = 10^-3, 10^-7
  delta = [weight matrix initialized with zeros]
  for each element x,y in input_matrix,output_matrix:
      y_hat = 1 / (1 + exponential(-transposed w * x))
      loss = ((y_hat - y) * x)
      regularization = (|w|^2) / 2
      current_delta = loss + lambda * regularization
      batch_delta = batch_delta + current_delta
  w = w - (eta * batch_delta)
```

3. Using our adapted logistic regression we were able to achieve a maximum training accuracy of 78.4286%, with a testing accuracy of 77.5%. The pseudocode was adopted into python, simply by altering our initial logistic regression function. Listed below are the λ values and their corresponding training and testing accuracy, the Δ normal and the number of batches until convergence. This data was taken with an $\epsilon = 400$

λ	**Training Accuracy**	**Testing Accuracy**	**Δ norm*	* **Number of Batches**
10^{-3}	78.0000	77.2500	399	1375
10^{-2}	78.0000	77.2500	399	1375
10^{-1}	78.0000	77.2500	399	1375
10^{1}	77.5714	76.8750	398	348
10^{2}	76.8571	75.7500	390	103
10^{3}	75.7857	74.3750	343	21

This data shows an expected trend. As we increase our λ value, which is the strength of the regularization, both training and testing accuracies increase. As the algorithm becomes more sensitive to weight size, the training accuracy decreases. This is expected because λ should lessen any overfitting, which would make the training data fit less accurately. It is also interesting that the training data seems to have such discrete maximums. This may be attributed to the smaller data size, or possibly a inherent plateau, due to ambiguous training data points. Our testing accuracy also seems to plateau at 77.25% accuracy, which may be attributed to similar issues.

These charts display the batch accuracy over time for $\lambda = 10^3$, $\lambda = 10^2$, $\lambda = 10^1$, and $\lambda = 10^{-1}$. With a value of 10^3 and 10^2 you can see that it seems to fit well, and then minimizes the size of the weights. This overcorrection causes a falling accuracy, which can be seen in both training and testing data. With $\lambda = 10^1$, the accuracy is closer to what we want. However, by further minimizing the influence of the regularization, we can get similar accuracy for both training and testing sets. Using a value of $\lambda = 10^{-1}$ the chart closely resembles the chart from the plan logistic regression, because as λ decreases the influence of the regularization decreases. Having a slight influence though, allows our accuracy to be higher than the accuracy seen by just logistic regression.

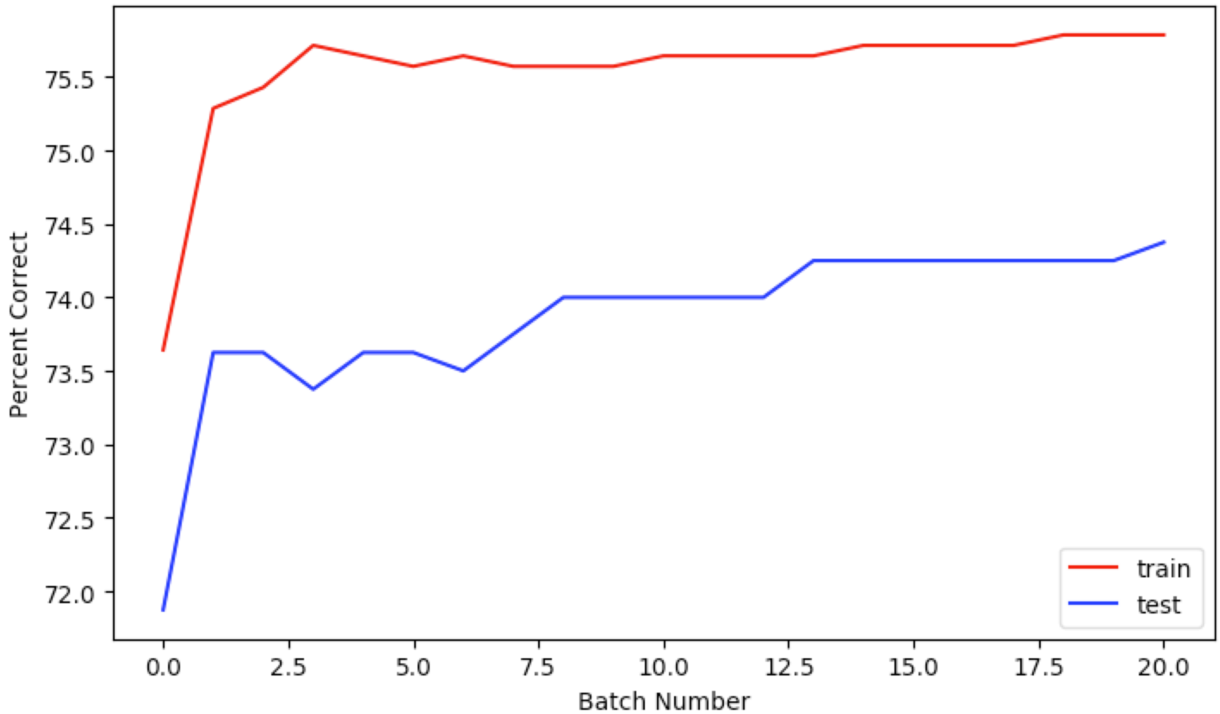


Figure 1: $\lambda = 10^3$

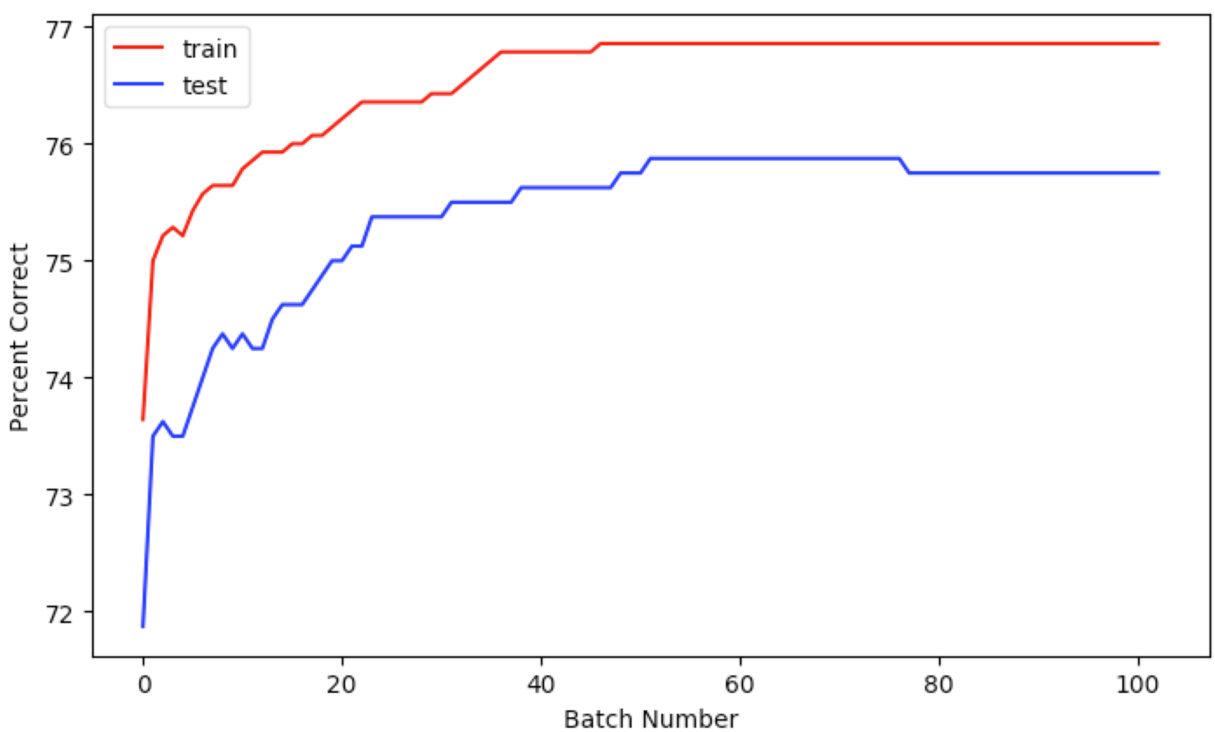


Figure 2: $\lambda = 10^2$

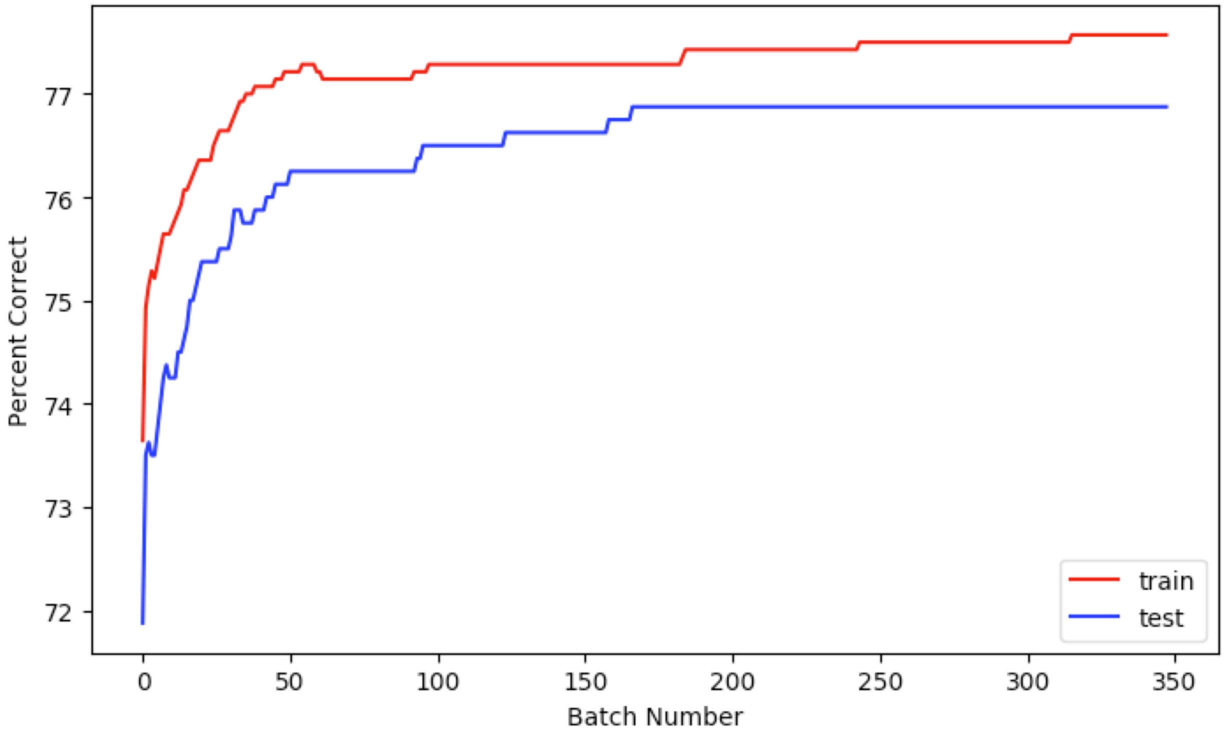


Figure 3: $\lambda = 10^1$

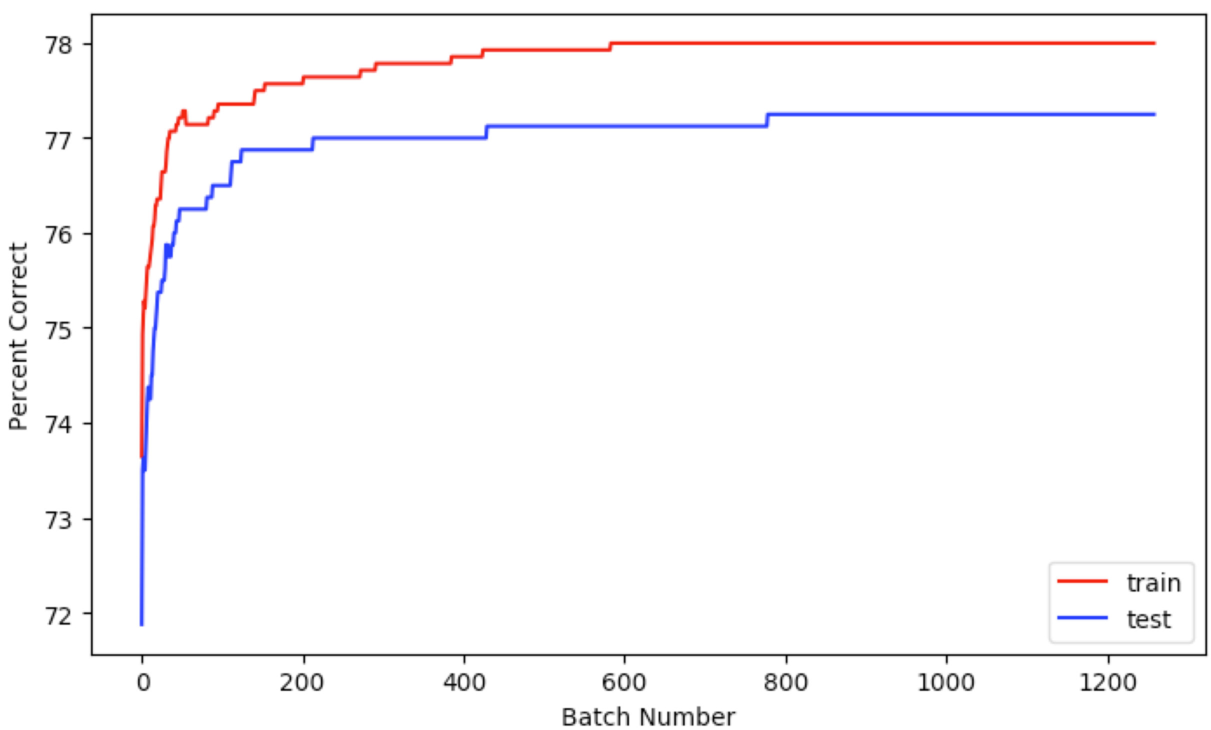


Figure 4: $\lambda = 10^{-1}$