Project 2

Design Optimization of Brake Disc Geometry

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MAE 598 Fall 2021

Nov. 17 2021

Problem Statement

The thickness, outer diameter, and inner diameter of a brake rotor is optimized to minimize volume, maximum stress, and maximum temperature while maximizing the first natural frequency to avoid failure by resonance. The optimization is performed using ANSYS's Design of Experiments and Optimization tools.

Analysis Setup

The input parameters are the thickness, outer diameter, and inner diameter of the brake rotor. The output parameters are the volume of the disc, maximum stress, maximum temperature, and first natural frequency which were obtained through static structural, thermal, and modal analyses. The details of the setup can be found in the ANSYS DOE and Design Optimization Tutorial (Yi and Vipradas).

Design of Experiments

The design space is explored by varying the input parameters and observing the impact on the output parameters. The input parameters have a continuous range of values within a set of constraints. The constraints for the input parameters are shown in Table 1.

Table 1. Constraints of Input Parameters

Input Parameters	Constraints
Thickness	15 in – 25 in
Outer Diameter	124 in – 160 in
Inner Diameter	66 in – 90 in

Changes to the input parameters will affect each of the objectives differently in such a way that there will be trade-offs. For example, increasing the thickness will increase the volume but also reduce the maximum stress. The effects on the objectives of each of the input parameters can be further analyzed by sensitivity analyses.

Response Surface

A response surface is fitted to the design points found in the design of experiments. The response surface can be used to predict the output parameter values given the input parameters. There are multiple algorithms that can be used to fit a surface to the design points. The default on ANSYS is to fit a 2^{nd} order polynomial surface which may not capture the non-linear and high-order data. Additionally, the changes in the objectives are not expected to be smooth. There are other methods such as a non-parametric regression or Kriging method which may yield better results. However, ANSYS developed the Genetic Aggregation method which can combine multiple response surfaces with cross-validation testing to yield a desirable result for most cases.

After the response surface is generated, the goodness of fit curve is observed to see how closely the model fits to the training data points. In this case, the coefficient of determination is nearly one for all objectives which indicates that the model has been closely fit to the data. However, untested verification points are also generated to test the predictive capabilities of the response surface. If the verification points are not captured by the model, the verification points are added as refinement points to improve the response surface. After four to five refinement iterations, the verification points were acceptably close to the response surface. It should also be noted that the verification points do not need to be too closely fitted to the model as the aim is to optimize which can be done without an exceptionally accurate predictive model. In Figure 2, a few response surface quality metrics are shown. From the verification points, the response surface is adequate to perform an optimization.

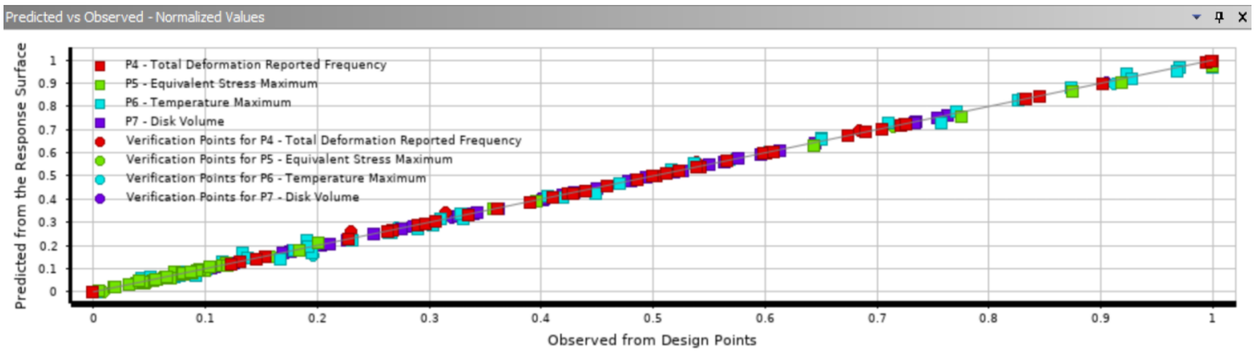

Figure 1. Goodness of Fit after Refinement

	P4 - Total Deformation Reported Frequency	P5 - Equivalent Stress Maximum	P6 - Temperature Maximum	P7 - Disk Volume
Coefficient of Determination (Best Value...				
Learning Points	1	0.99924	0.99739	1
Cross-Validation on Learning Points	0.99755	0.96293	0.99298	1
Root Mean Square Error (Best Value =...				
Learning Points	0.00014184	1.3507E+05	1.2454	1.3423E-09
Verification Points	9.5746	1.808E+05	1.3123	1.6076E-07
Cross-Validation on Learning Points	9.582	9.4491E+05	2.0446	1.721E-07
Relative Maximum Absolute Error (Best Value = ...				
Learning Points	0	8.3884	11.952	0
Verification Points	11.963	7.5279	11.19	0.22216
Cross-Validation on Learning Points	9.4284	70.101	21.391	0.26686
Relative Average Absolute Error (Best Value =...				
Learning Points	0	1.9416	4.2042	0
Verification Points	2.2109	2.5263	4.0191	0.01974
Cross-Validation on Learning Points	4.1783	12.596	6.4102	0.034612

Figure 2. Response Surface Quality Metrics

Sensitivity Analysis

The sensitivity analyses show how the objectives change with each of the input parameters. The objectives' responses to changes in each of the isolated input parameters are shown in Figures 3-11. The presence of monotonicity can be observed from these figures. Figure 3 shows that the frequency always increases with increases in rotor thicknesses while Figure 5 shows that the max temperature decreases. In Figure 6, the frequency consistently decreases with increases in rotor outer diameter.

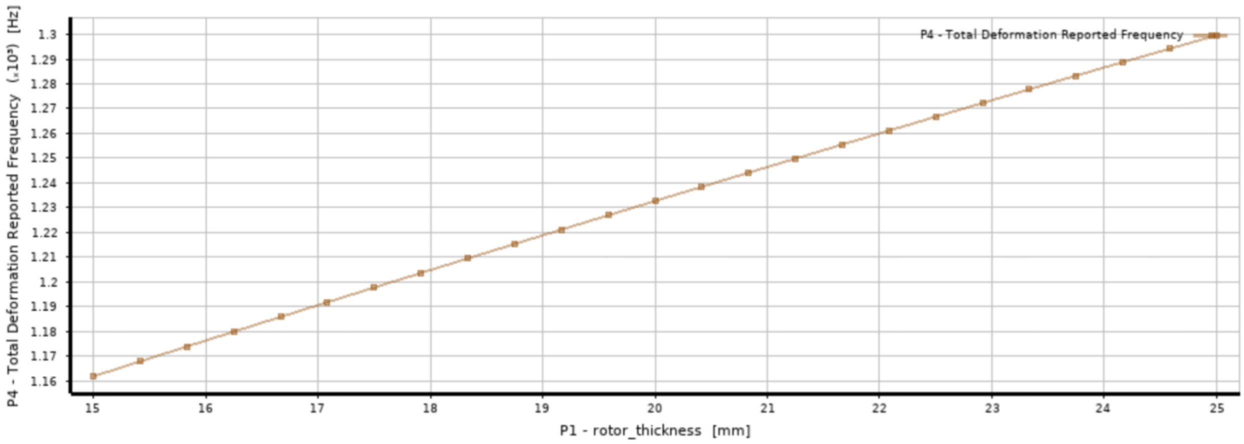

Figure 3. Frequency Response to Changes in Rotor Thickness

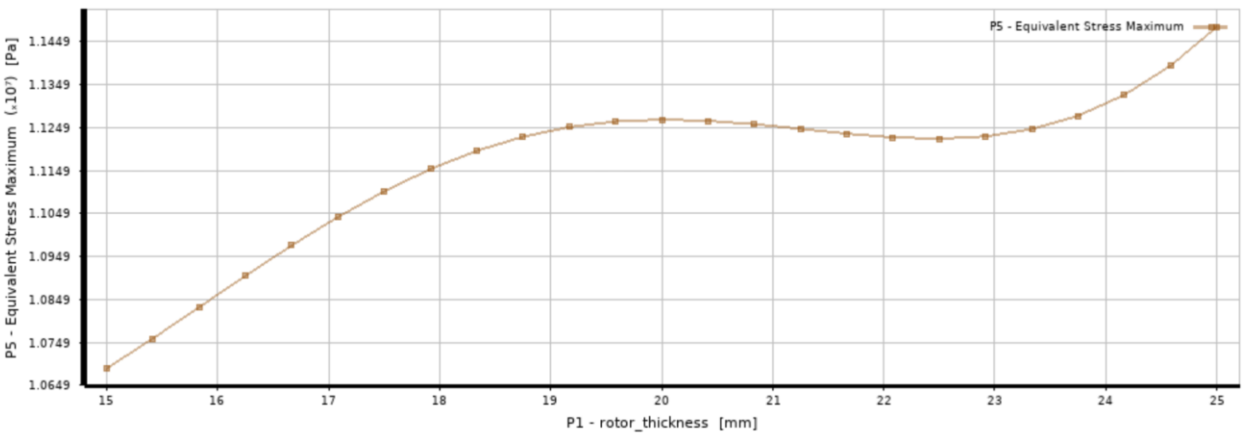

Figure 4. Max Equivalent Stress Response to Changes in Rotor Thickness

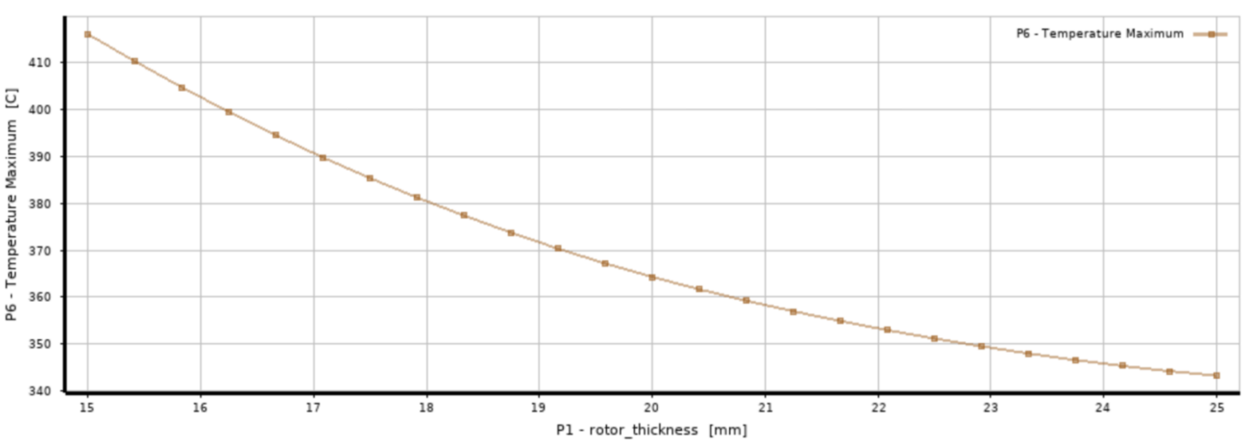

Figure 5. Max Temperature Response to Changes in Rotor Thickness

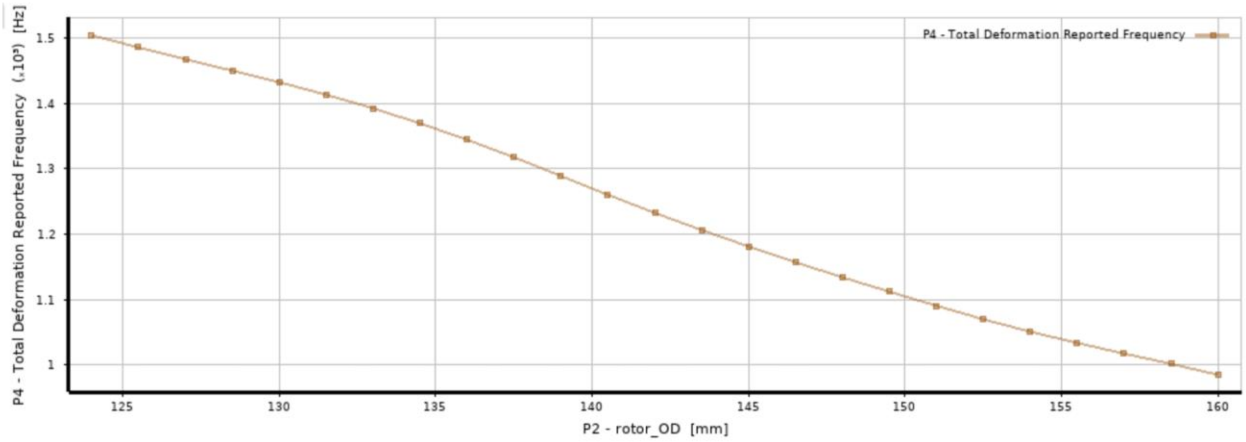

Figure 6. Frequency Response to Changes in Rotor Outer Diameter

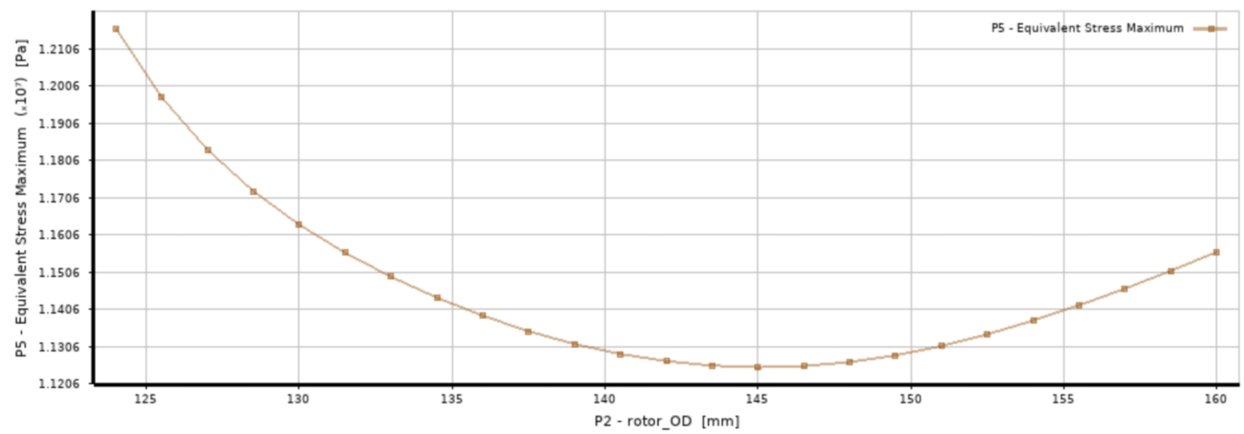

Figure 7. Max Equivalent Stress Response to Changes in Rotor Outer Diameter

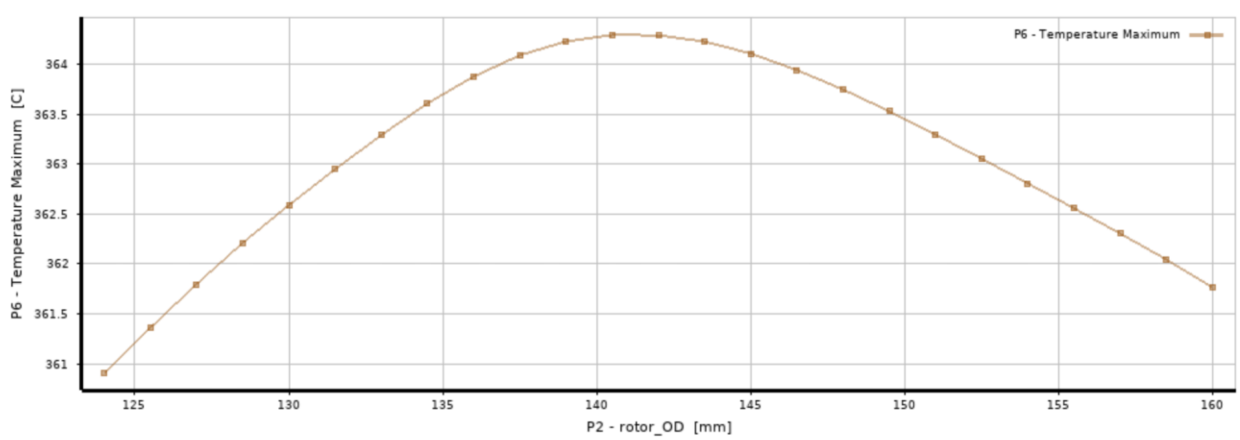

Figure 8. Max Temperature Response to Changes in Rotor Outer Diameter

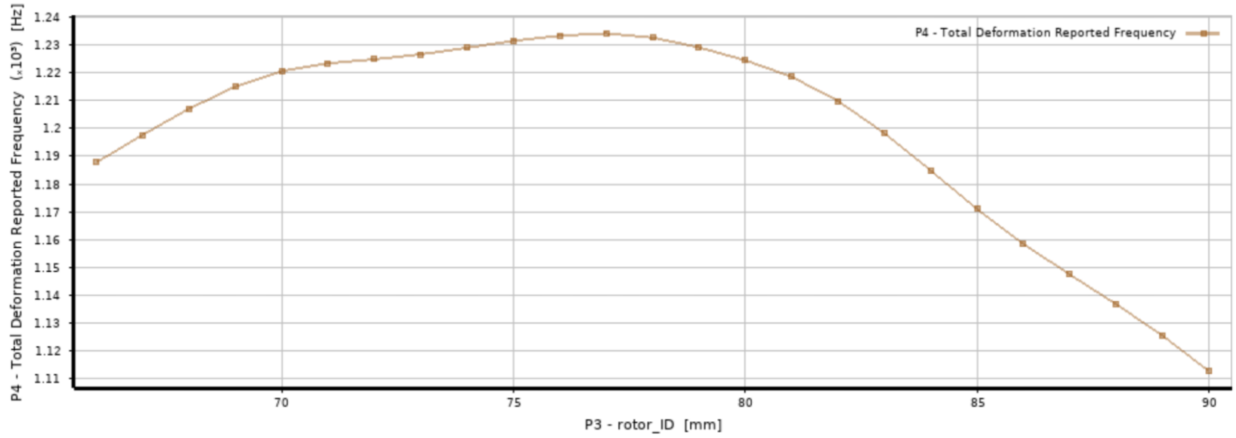

Figure 9. Frequency Response to Changes in Rotor Inner Diameter

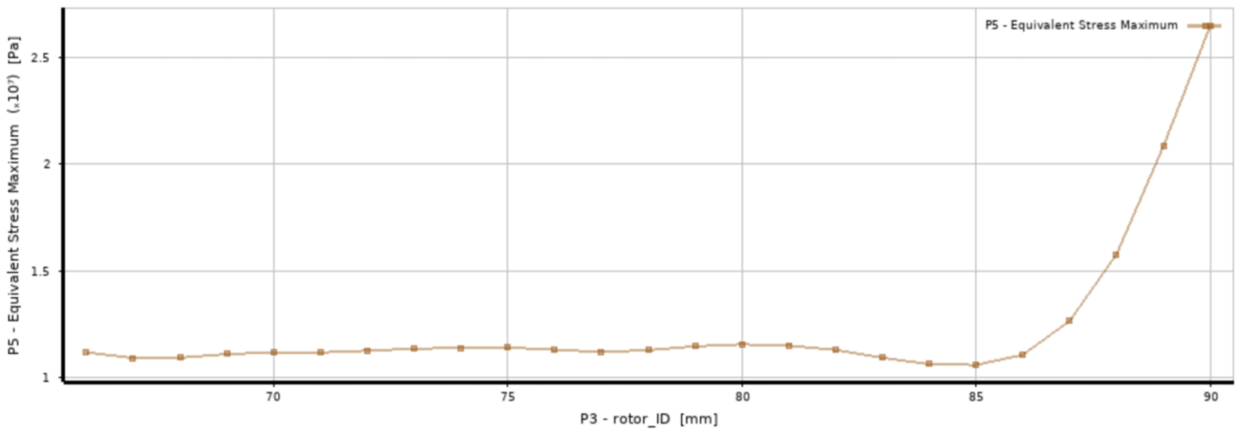

Figure 10. Max Equivalent Stress Response to Changes in Rotor Inner Diameter

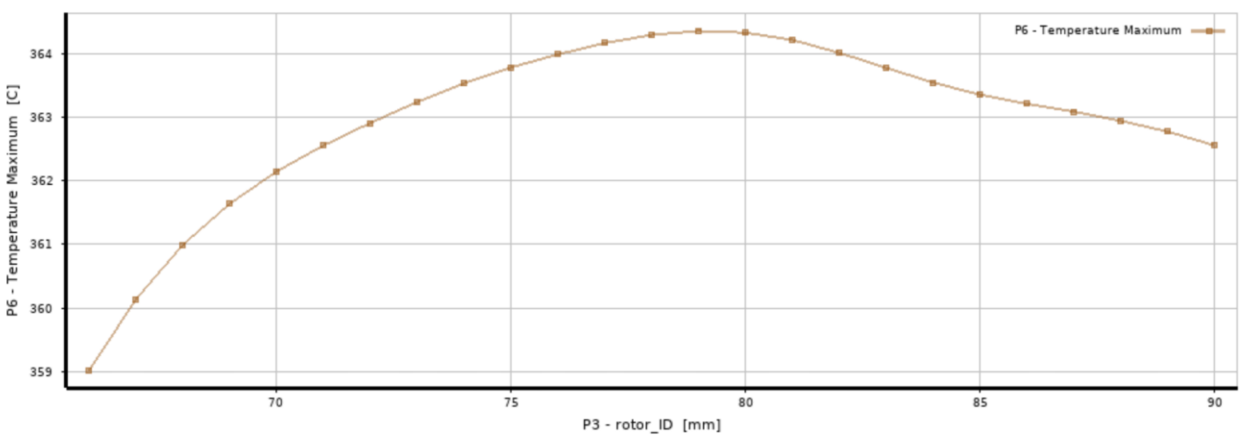

Figure 11. Max Temperature Response to Changes in Rotor Inner Diameter

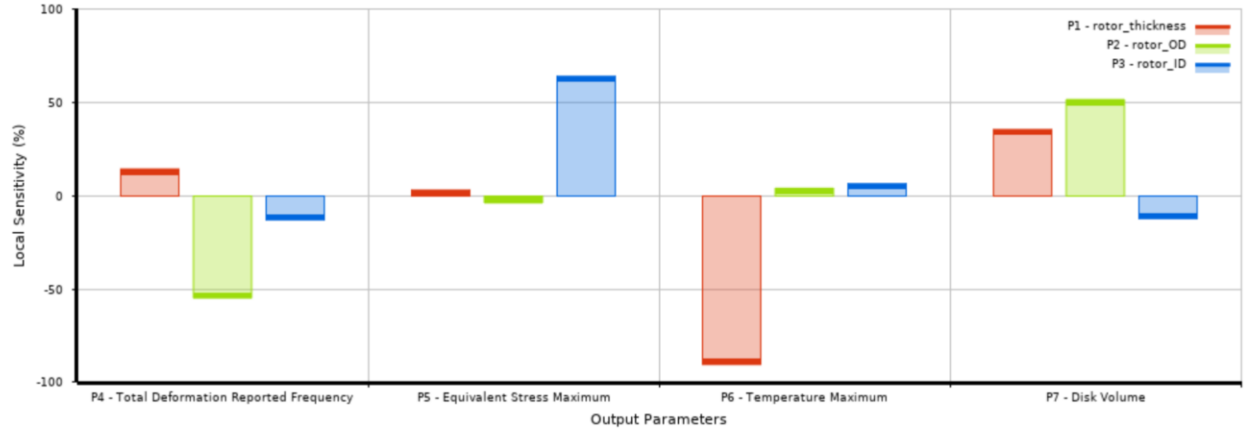

Figure 12. Local Sensitivity at a Design Point

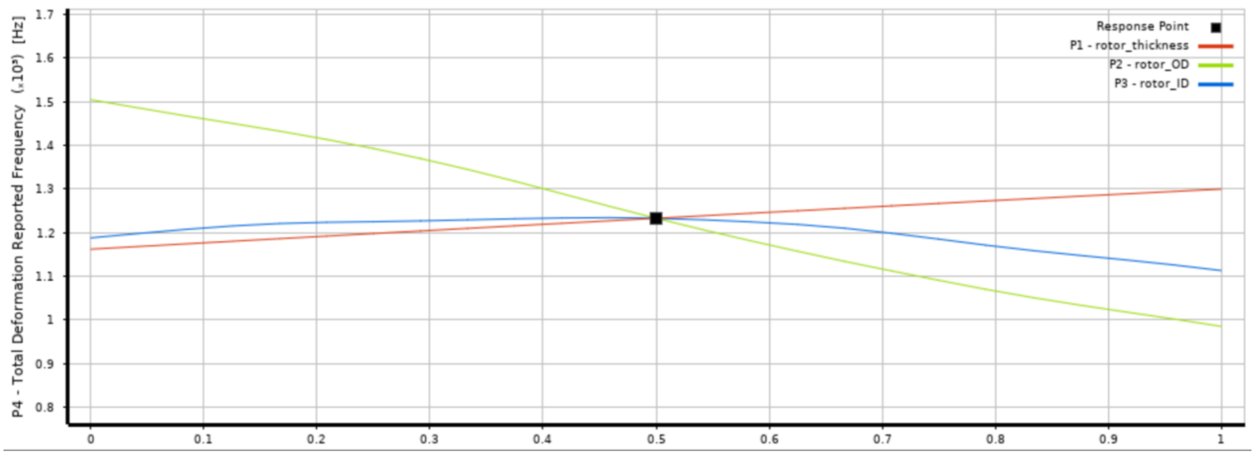

Figure 13. Local Sensitivity Curve for Frequency

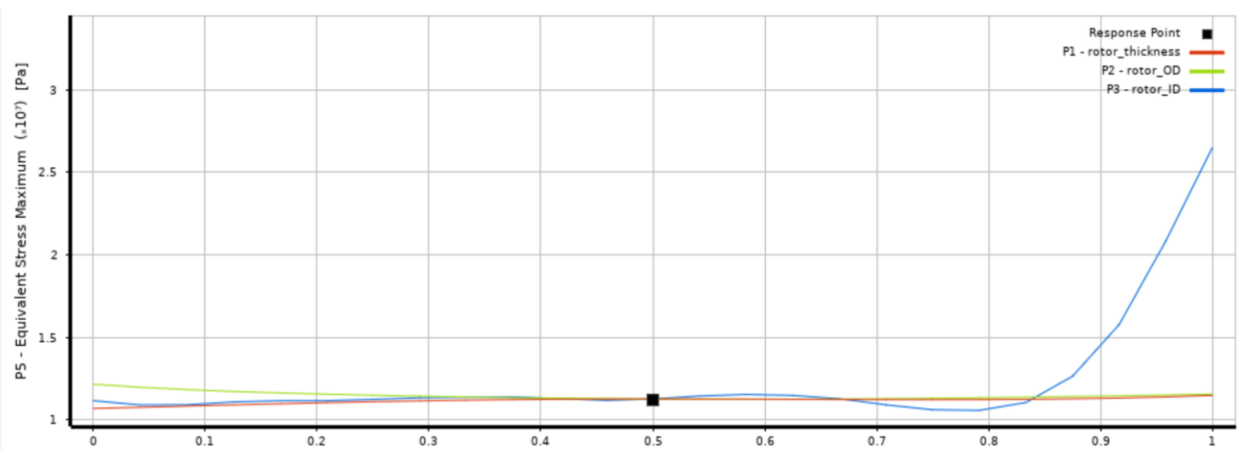

Figure 14. Local Sensitivity Curve for Max Equivalent Stress

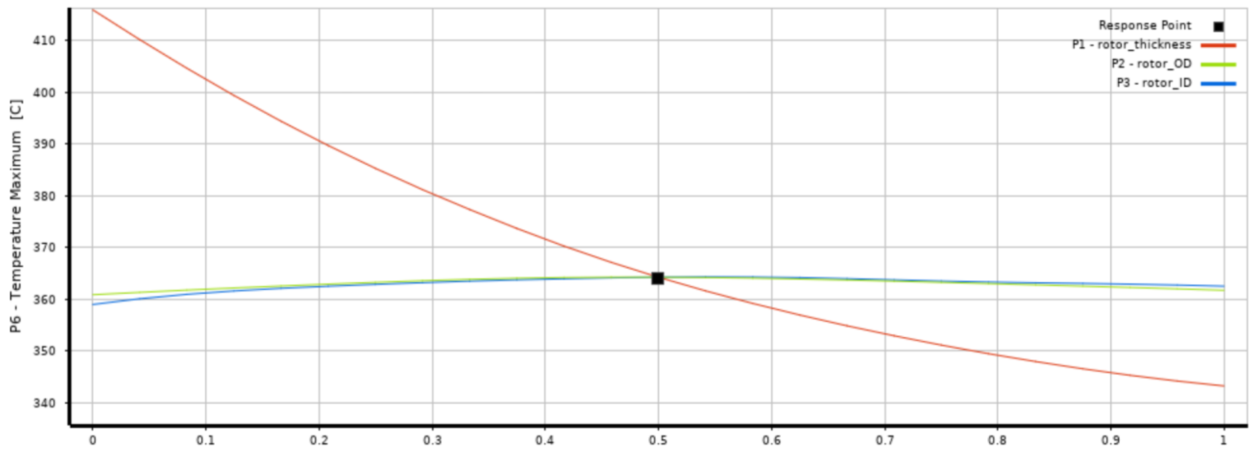

Figure 15. Local Sensitivity Curve for Max Temperature

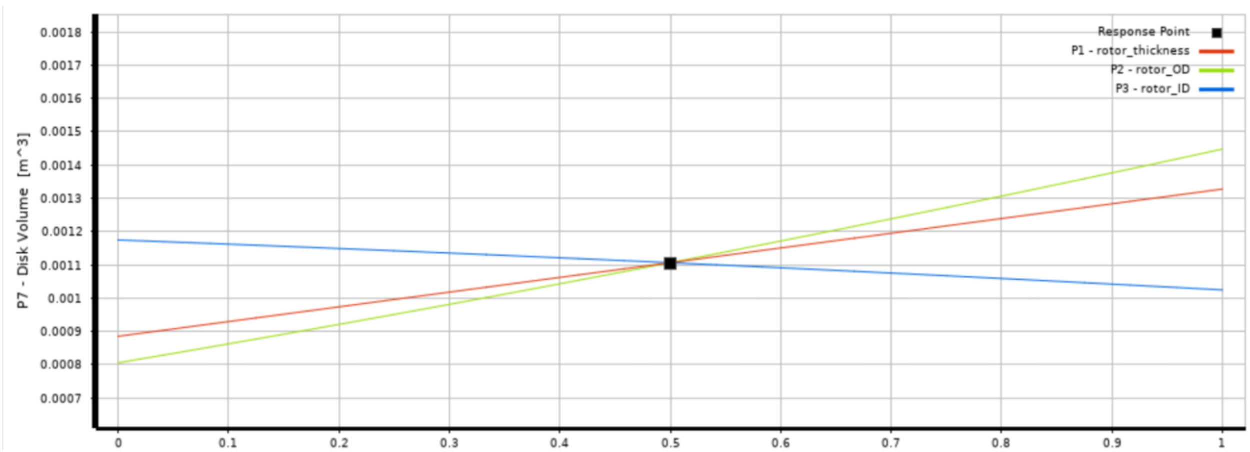

Figure 16. Local Sensitivity Curve for Disc Volume

Optimization

Multi-Objective Genetic Algorithm (MOGA) is used to optimize all the objectives as opposed to selecting one objective to optimize and setting the other objectives as constraints. The settings for the optimization are shown in Table 2. The objectives, target values, and constraints are tabulated in Figure 17.

Table 2. Optimization Properties

Property	Value
Method Name	MOGA
Estimated Number of Evaluations	2000
Number of Initial Samples	100
Number of Samples Per Iteration	100
Maximum Allowable Pareto Percentage	70
Maximum Number of Iterations	20
Maximum Number of Candidates	3

Name	Parameter	Objective			Constraint			
		Type	Target	Tolerance	Type	Lower Bound	Upper Bound	Tolerance
Maximize P4; P4 >= 1200 Hz	P4 - Total Deformation Reported Frequency	Maximize	1200		Values >= Lower Bound	1200		0.001
Minimize P5; P5 <= 1.4E+07 Pa	P5 - Equivalent Stress Maximum	Minimize	1.4E+07		Values <= Upper Bound		1.4E+07	0.001
Minimize P6; P6 <= 400 C	P6 - Temperature Maximum	Minimize	400		Values <= Upper Bound		400	0.001
Minimize P7	P7 - Disk Volume	Minimize	0		No Constraint			

Figure 17. Optimization Objectives and Constraints

The optimization yielded three candidate points which are verified by running the design points. The objectives are compared with a reference point which is the initial design of the brake rotor. As seen in Figure 18, all the candidates meet the requirements, but each has trade-offs that the designer might consider depending on the relative importance of each objective. The local sensitivity in Figure 19 can be used to visualize how the input parameters change the output parameters within the candidate design space.

Name	P1 - rotor_thickness (mm)	P2 - rotor_OD (mm)	P3 - rotor_ID (mm)	P4 - Total Deformation Reported Frequency (Hz)		P5 - Equivalent Stress Maximum (Pa)		P6 - Temperature Maximum (C)		P7 - Disk Volume (m^3)	
				Parameter Value	Variation from Reference	Parameter Value	Variation from Reference	Parameter Value	Variation from Reference	Parameter Value	Variation from Reference
Candidate Point 1	16.947	124.13	85.144	1315.8	-18.89%	1.1614E+07	-2.32%	388.25	14.17%	0.00068239	-31.53%
Candidate Point 1 (verified)				1305.9	-19.50%	1.1216E+07	-5.67%	383.54	12.79%	0.0006823	-31.54%
Candidate Point 2	16.947	124.12	84.548	1332.1	-17.88%	1.1323E+07	-4.77%	388.24	14.17%	0.00068549	-31.22%
Candidate Point 2 (verified)				1323.7	-18.40%	1.1087E+07	-6.76%	385.77	13.45%	0.0006854	-31.23%
Candidate Point 3	18.113	124.15	85.799	1311.2	-19.17%	1.1024E+07	-7.29%	376.55	10.74%	0.00070868	-28.89%
Candidate Point 3 (verified)				1297	-20.05%	1.1389E+07	-4.22%	373.9	9.95%	0.0007086	-28.90%
Initial Design	25	125	75	1622.2	0.00%	1.189E+07	0.00%	340.05	0.00%	0.00099663	0.00%

Figure 18. Candidate Points with Respect to Reference/Initial Design

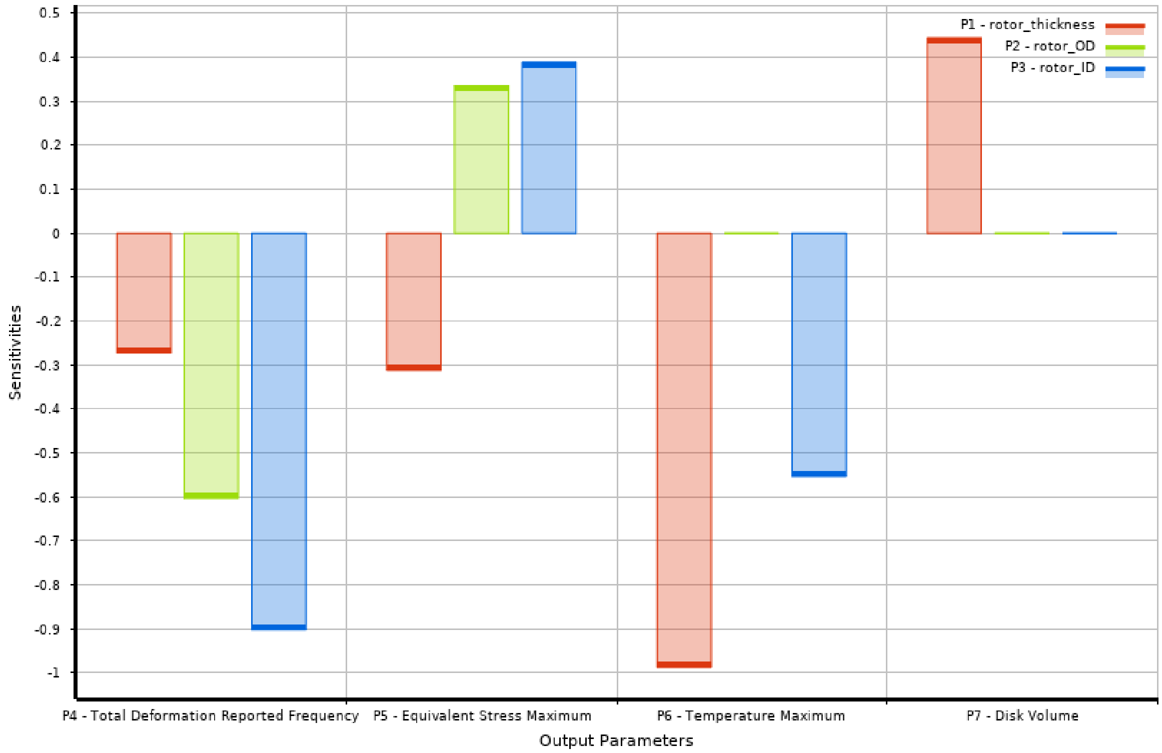

Figure 19. Local Sensitivity with Candidate Points

To compare the results of the optimization, Candidate Point 2 from Figure 18 will be compared with the initial design. The initial design began with a thickness of 25 mm, outer diameter of 125 mm, and inner diameter of 75 mm. The optimized design reduced the thickness to 16.95 mm, reduced the outer diameter by 1 mm to 124.12 mm, and increased the inner diameter from 75 to 86 mm. Through these geometric changes to the input parameters, the volume decreased 31.23%, the max equivalent stress decreased by 6.76%, but decreased the frequency by 18.4% and increased the maximum temperature by 13.45%. However, all the objectives meet the engineering requirements with a significant decrease in volume and a smaller reduction in stress. The optimized design is reasonable and comparable to existing brake rotors in terms of size.