11/8 - 2.1 + 2.2
11/9 - 4.1 + 4.2
11/10 - 4.3 + 1.6
11/11 - 1.1 - 1.3
11/12 - 1.4 + 1.5

Practice Problems

11/8 1. If $A \subset B$, can A and B be independent?

11/8 2. Suppose that you continually collect coupons and that there are m different types. Suppose also that each time a new coupon is obtained it is a type i coupon with probability p_i, $i = 1, \ldots, m$. Suppose that you have just collected your nth coupon. What is the probability that it is a new type?

11/8 3. If X is uniformly distributed over $(-1, 1)$, find the pdf of the random variable $|X|$.

11/8 4. This problem introduces a simple meteorological model, more complicated versions of which have been proposed in literature. Consider a sequence of days, and let R_i denote the event that it rains on day i. Suppose that $P(R_i|R_{i-1}) = \alpha$ and $P(R_i^c|R_{i-1}^c) = \beta$. Suppose further that only today's weather is relevant to predicting tomorrow's, that is, expressions like $P(R_i|R_{i-1} \cap R_{i-2} \cap \ldots \cap R_0)$ are equivalent to $P(R_i|R_{i-1})$.

   a. If the probability of rain today is p, what is the probability of rain tomorrow?

   b. If the probability of rain today is p, what is the probability of rain the day after tomorrow?

11/9 5. If A is independent of B and B is independent of C, then A is independent of C. Prove this statement, or give a counter example if it is false.

11/9 6. Suppose you have a neighbor that occasionally drinks too much beer. Suppose, in fact, that he gets drunk 10 percent of the time when he drinks. When he comes home drunk, he forgets to turn off the porch light 70 percent of the time. When he comes home sober, he remembers to turn off the light 90 percent of the time. You notice that the porch light is on. What is the probability that he came home drunk?

11/9 7. Explain in detail how you would generate a sample of 10 observations from $f(x) = 3x^2$ for $0 < x < 1$ and 0 otherwise.

11/9 8. Let X_1 and X_2 have the joint pdf $f(x_1, x_2) = 2$ for $0 < x_1 < x_2 < 1$. Derive the pdf of $Y = X_2 - X_1$.

11/10 9. Suppose we select a point at random in the interior of a circle of radius 1. Let X be the distance of the selected point from the center of the circle. For $0 < x < 1$, the event $\{X \leq x\}$ is equivalent to the point lying in a circle of radius x. Assume that the probability of the selected point lying in a circle of radius x is equal to the ratio of the area of this circle to the area of the full circle. Following this line of thought, first find the cdf and pdf of X. Then find the cdf and pdf of $Y = X^3$.

11/10 10. Let X have the uniform distribution in $[0, 1]$. Derive the c.d.f. of $Y = \{\log(X)\}^2/2$, where $\log(\cdot)$ represents the natural logarithm.

11/10 11. A class in probability theory consists of 6 men and 4 women. An exam is given and students are ranked ordered according to their performance. Assume that no two students obtain the same score. If all rankings are considered equally likely what is the probability that women receive the top 4 scores?

11/10 12. Divide a line segment into two parts by selecting a point at random. Find the probability that the larger segment is at least three times the shorter. Assume a uniform distribution.

11/11 13. Consider the joint density function $f(x, y)$ of X and Y which is constant over the region defined by the following three statements: $0 \leq x \leq 2$, $0 \leq y \leq 2$ when $0 \leq x \leq 1$ and $0 \leq y \leq 1$ when $1 \leq x \leq 2$. Note that the region is what is left over once you subtract the square region $\{1 \leq x \leq 2; 1 \leq y \leq 2\}$ from the larger square $\{0 \leq x \leq 2; 0 \leq y \leq 2\}$.

a. Draw the support of the joint density and find out what the joint density is equal to in this region.

b. Find the distribution of X given $Y = y$.

c. Compute $E(X|Y = 3/2)$ and $Var(X|Y = 3/2)$.

11/11 14. An actuary is studying the prevalence of three health risk factors, denoted by A, B, and C, within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B, is 1/3. What is the probability that a woman has none of the three risk factors, given that she does not have risk factor A?

11/11 15. The probability that a randomly chosen male has a circulation problem is 0.25. Males who have a circulation problem are twice as likely to be smokers as those who do not have a circulation problem. What is the probability that a male has a circulation problem, given that he is a smoker?

11/11 16. Suppose (X_1, X_2) have the joint pdf, $f_{X_1,X_2}(x_1, x_2) = 1$ for $0 < x_1 < 1$, $0 < x_2 < 1$, and zero elsewhere.

a. Find the joint distribution of $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$. Give a sketch of the support of this distribution.

b. Find the marginal distribution of Y_1.

11/12 17. Let X be a continuous random variable with pdf given by

$$f(x) = |x|/4, \quad 0 \le |x| \le 2.$$

Find $P(1 \le |X| \le 2)$.

11/12 18. Let X and Y be discrete random variables and let g and h be functions such that the following identity holds:

$$P(X = x, Y = y) = g(x)h(y).$$

Assume that the support of the joint pmf of X and Y is rectangular, $S_X \times S_Y$, where S_x and S_Y denote the supports of X and Y, respectively.

a. Express $P(X = x)$ and $P(Y = y)$ in terms of g and h.

b. Show that X and Y are independent.

Hint: First show that $(\sum_{x\in S_X} g(x))(\sum_{y\in S_y} h(y)) = 1$.