AMATH 352: Problem Set 2

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January 13, 2017

Matlab Part Due: Friday January 20, 2017
Written Part Due: Monday January 23, 2017

Instructions:

Complete the following problems. Submit a single Matlab script to Scorelator with your solutions to the problems in the Matlab section. You have five chances to submit your assignment. Your score for the Matlab portion of the assignment will be the highest score from your five submissions. The remaining problems (1-5) make up the written portion of this assignment. Turn in a write up of these problems digitally (via Canvas) by 5:00pm of the due date.

Norms:

1. Find and sketch the closed unit ball in $\mathbb{R}^2$ for the infinity norm. Justify your drawing (your answer for this problem should be more than just a drawing).

Real Linear Spaces:

2. Verify that $\mathbb{C}^2$ (the set of column vectors with two entries which are both in $\mathbb{C}$) is a real linear space, i.e. show that it satisfies the 10 defining properties of a real linear space.

3. Which of the following sets W are subspaces of V (and hence are linear spaces themselves)? Justify your answers with arguments showing they are closed under addition and scalar multiplication or counterexamples showing they are not.

(a) $V = \mathbb{R}^4$ and $W = \left\{ \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4 : x_1 + 2x_2 = 0 \text{ and } x_3 - x_4 = 0 \right\}$

(b) $V = C^0(\mathbb{R}, \mathbb{R})$ and $W = \{f \in V : f(3) = 2\}$

(c) $V = \mathcal{F}(\mathbb{R}, \mathbb{R})$ and W is the set of all periodic functions of period 1, i.e. the set of all functions f such that $f(x+1) = f(x)$ for all $x \in \mathbb{R}$.

4. Explain why the following set is not a real linear space:

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : x_1 + x_2 x_3 = 1 \right\}.$$

Span:

5. Answer the following (justify your answers):

(a) Is $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ in span $\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right)$?

(b) Is $\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$ in span $\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right)$?

(c) Is $f(x) = 1$ (the constant function that is 1 for any input x) in span$(2x^2 - 2, x + 3)$?

Matlab (defining functions):

Create and submit to Scorelator a single Matlab script (.m file) which performs the following tasks along with function files `cumulative_geometric_mean.m` and `approximate_pi.m` containing the implementations of your cumulative geometric mean and π approximating functions, respectively. Your code should generate the dat files `A1.dat`, `A2.dat`, `A3.dat`, and `A4.dat`.

6. Watch the video at `https://www.youtube.com/watch?v=qo3AtBoyBdM` on creating your own functions in Matlab. Create a function called `cumulative_geometric_mean` (inside a new m file named `cumulative_geometric_mean.m`) which takes as its input a vector `v` and a positive integer `n` and returns (gives as output) the geometric mean of the first `n` entries of `v`. Recall that the geometric mean of a set of m numbers $x_1,\ x_2, \ldots, x_m$ is

$$(x_1 \cdot x_2 \cdots x_m)^{\frac{1}{m}} = \left(\prod_{i=1}^{m} x_i \right)^{\frac{1}{m}}.$$

Below is some skeleton code to get you started:

```
function output = cumulative_geometric_mean(v,n)

% Your code goes here

end
```

In Matlab, define the (column) vector `v` as

$$v = \begin{bmatrix} 1 \\ 4 \\ 9 \\ 16 \\ \vdots \\ 100^2 \end{bmatrix}.$$

Store `v` in the file `A1.dat`. Next compute the cumulative geometric means of the first $1, 2, 3, \ldots, 30$ entries of `v`. Store the results in a 30×1 column vector and output this vector as `A2.dat`.

7. Below is an identity involving π.

$$\frac{\pi^2 - 8}{16} = \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2(2k+1)^2}.$$

One can generate an approximation for π by truncating the series on the right-hand side after summing a large number of terms, i.e.

$$\frac{\pi^2 - 8}{16} \approx \sum_{k=1}^{N} \frac{1}{(2k-1)^2(2k+1)^2}$$

if N is large enough. Create a funtion called `approximate_pi` which takes as an input a positive integer `n` and returns an approximation of π by using `n` terms in the above series.

Use your function to approximate π using $N = 5, 10, 20, 50$, and 100 terms in the series. Store the results in a 5×1 column vector then save that column vector as `A3.dat`.

How many terms are needed to obtain an approximation that is within `1e-12`$= 10^{-12}$ of the actual value of π? Save your answer to `A4.dat` (note that your answer for this part should be a positive integer, not the actual approximation that is within `1e-12`).

When you submit your homework on Scorelator you will need to upload **both** your main homework script (that generates the dat files) **and** `cumulative_geometric_mean.m` and `approximate_pi.m`. Before clicking submit you must **highlight your main homework script** by clicking on it. If you highlight either of your function files Scorelator will give you a 0.