AMATH 352: Problem set 1

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Due: Friday January 13, 2017

Instructions:

Complete the following problems. Submit a single Matlab script to Scorelator with your solutions to the problems in the Matlab section. You have five chances to submit your assignment. Your score for the Matlab portion of the assignment will be the highest score from your five submissions. The remaining problems make up the written portion of this assignment. Turn in a write up of these problems digitally (via Canvas) by 5:00pm of the due date.

Column Vectors:

1. Show that for any $\alpha \in \mathbb{R}$ and any $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^m$, $\alpha(\boldsymbol{x} + \boldsymbol{y}) = \alpha\boldsymbol{x} + \alpha\boldsymbol{y}$.
2. Show that for every vector $\boldsymbol{x} \in \mathbb{R}^m$, $1\boldsymbol{x} = \boldsymbol{x}$.

Norms and inner products:

3. Determine whether each of the following functions is a norm. Justify your answer, i.e. if you claim it is a norm, show that it satisfies the five criteria discussed in class, and if not, give a concrete example that shows it doesn't satisfy one of the criteria.
   (a) $\boldsymbol{x} \in \mathbb{R}^3$, $\|\boldsymbol{x}\| := \|\boldsymbol{x}\|_2 - \|\boldsymbol{x}\|_1$
   (b) $\boldsymbol{x} \in \mathbb{R}^3$, $\|\boldsymbol{x}\| := \|\boldsymbol{x}\|_2 + \|\boldsymbol{x}\|_1$
   (c) $\boldsymbol{x} \in \mathbb{R}^3$, $\|\boldsymbol{x}\| :=$ the number of nonzero entries in $\boldsymbol{x}$.
   (d) $\boldsymbol{x} \in \mathbb{R}^3$, $\|\boldsymbol{x}\| := 4|x_1| + |x_1 - x_2 + x_3| + |x_2 + x_3|$

4. Find and sketch the closed unit ball in $\mathbb{R}^2$ for the infinity norm. Justify your drawing (your answer for this problem should be more than just a drawing).

Matlab:

Create and submit to Scorelator a single Matlab script (.m file) which performs the following tasks. Your code should generate the dat files `A1.dat`, `A2.dat`, ..., `A8.dat`.

5. Let $\boldsymbol{x} = \begin{bmatrix} e \\ 5^{1/3} \\ -6\pi \\ 42 \end{bmatrix}, \quad \boldsymbol{y} = \begin{bmatrix} 4 \\ 2 \\ -5 \\ 20 \end{bmatrix}.$

   Compute the following quantities (the `dot`, `norm`, and `acos` commands may be useful here):

   (a) $\|\boldsymbol{x}\|_1,\ \|\boldsymbol{x}\|_2,\ \|\boldsymbol{x}\|_{64},\ \|\boldsymbol{x}\|_\infty$ Store the results in a (4×1 column) vector. Save this vector in `A1.dat`.

   (b) the angle, in radians, between $\boldsymbol{x}$ and $\boldsymbol{y}$. Save the result in `A2.dat`.

6. Compute the following expressions:

   (a) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots - \frac{1}{1003}$. Save the result in `A3.dat`.

   (b) $\frac{1}{1^2\times 3^2} + \frac{1}{3^2\times 5^2} + \frac{1}{5^2\times 7^2} + \ldots$ with 500 terms. Save the result in `A4.dat`.

7. Let $f_k = k\sum_{i=3}^{k} \frac{i+1}{i-1}$. Compute f_{20} and f_{100} and save the results in `A5.dat` and `A6.dat`, respectively.

8. Below is Matlab code which computes the 100th term in the Fibonacci sequence $x_{k+2} = x_{k+1} + x_k$ for $x_1 = 1,\ x_2 = 1$.

```
xk = 1;
xkp1 = 1;
for k = 1:98
    xkp2 = xkp1 + xk;
    xk = xkp1;
    xkp1 = xkp2;
end
% After the above executes, xkp2 contains the 100th term in the sequence
```

   For large enough k, the ratio of successive terms in the Fibonacci sequence should approach the golden ratio $\frac{1+\sqrt{5}}{2} \approx 1.618$. You can test the code given above by comparing the ratio $\frac{x_{100}}{x_{99}}$ against the golden ratio.

Consider the tri-Fibonacci sequence:

$$x_{k+3} = x_{k+2} + x_{k+1} + x_k.$$

Let $x_1 = 0$, $x_2 = 0$, and $x_3 = 1$. Compute the first 100 terms in the tri-Fibonacci sequence and store them in a (100×1 column) vector. Save this vector in `A7.dat`. Compute $\frac{x_{100}}{x_{99}}$ and save the result in `A8.dat`.