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See also: regression, derivation of regression formula

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Regression

Confidence Interval

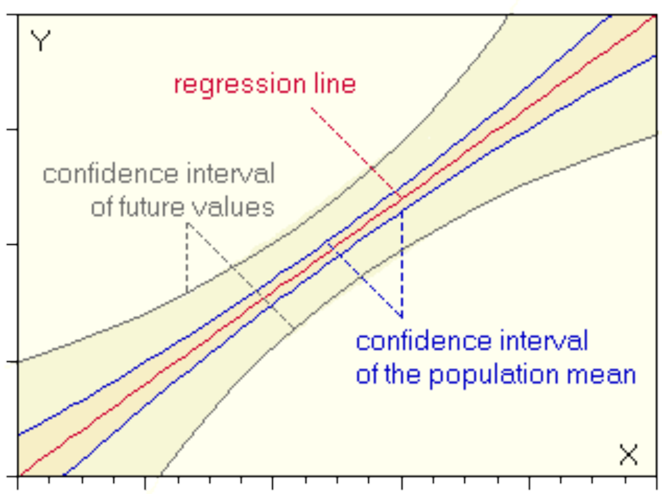
When calculating a regression line, one estimates the mean of the population of Y at any value of X. Thus the regression line represents the mean $\hat{Y}_i$ at any value of the independent variable X. This estimated mean is normally distributed, and one may ask the question about the confidence interval of the estimated Y. It can be shown that the ratio ($\hat{Y}$ - m_y)/s_2 follows a t-distribution with n-2 degrees of freedom. From this, one can calculate the confidence interval of the estimated Y by the following equation

$$\hat{Y} \pm s_2\, t_{n-2}$$

with

$$s_2^2 = s_{y,x}^2 \left[\frac{1}{n} + \frac{(x - \bar{x})^2}{[x^2]} \right] \qquad s_{y,x}^2 = \sum \frac{(Y_i - \hat{Y}_i)^2}{n-2} \qquad [x^2] = \sum_i (x_i - \bar{x})^2$$

If you are plotting the confidence interval for the population regression line, you see that these lines are hyperbolic. This means that the confidence interval depends on the value of X. The farther the value of X departs from $\bar{x}$, the larger is the confidence interval (inner, blue curve of figure below). The band formed by the confidence interval for all X values is also called **Working-Hotelling confidence band**.



In practical situations the confidence interval of the population mean is not so frequently required, while most of the inferences are based upon the estimation of a distinct, future $\hat{Y}_i$, which is not known at the time when the regression is calculated and which is independent from any previous values. The confidence interval for future observations is given by the following equation:

$$\hat{Y} \pm s_3 \, t_{n-2}$$

with

$$s_3^2 = s_{y,x}^2 \left[1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{[x^2]} \right] \qquad s_{y,x}^2 = \sum \frac{(Y_i - \hat{Y}_i)^2}{n-2} \qquad [x^2] = \sum_i (x_i - \bar{x})^2$$

The confidence interval for actual (future) values of Y is wider than the confidence interval for the population mean (outer, gray curve in figure above). This demonstrates the fact that the estimate of the actual value of Y is less precise than the estimate of the mean of Y.

Last Update: 2005-Jul-16