Cardiod Arc-en-Ciel

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Algorithm:

From:

http://mathworld.wolfram.com/Cardioid.html

The cardioid may also be generated as follows. Draw a circle C and fix a point A on it. Now draw a set of circles centered on the circumference of C and passing through A. The envelope of these circles is then a cardioid (Pedoe 1995).

Code:

Define the circle C and an arbitrary point A:

In[1]:=
```
CircleCPoints = Table[
  {Cos[2 Pi i / 39], Sin[2 Pi i / 39]},
  {i, 1, 39}
 ]

PointA = {0, 1}
```

Out[1]=

$$\{\{\text{Cos}[\tfrac{2\pi}{39}], \text{Sin}[\tfrac{2\pi}{39}]\}, \{\text{Cos}[\tfrac{4\pi}{39}], \text{Sin}[\tfrac{4\pi}{39}]\}, \{\text{Cos}[\tfrac{2\pi}{13}], \text{Sin}[\tfrac{2\pi}{13}]\}, \{\text{Cos}[\tfrac{8\pi}{39}], \text{Sin}[\tfrac{8\pi}{39}]\}, \{\text{Sin}[\tfrac{19\pi}{78}], \text{Cos}[\tfrac{19\pi}{78}]\}, \{\text{Sin}[\tfrac{5\pi}{26}], \text{Cos}[\tfrac{5\pi}{26}]\}, \{\text{Sin}[\tfrac{11\pi}{78}], \text{Cos}[\tfrac{11\pi}{78}]\}, \{\text{Sin}[\tfrac{7\pi}{78}], \text{Cos}[\tfrac{7\pi}{78}]\},$$

$$\{\text{Sin}[\tfrac{\pi}{26}], \text{Cos}[\tfrac{\pi}{26}]\}, \{-\text{Sin}[\tfrac{\pi}{78}], \text{Cos}[\tfrac{\pi}{78}]\}, \{-\text{Sin}[\tfrac{5\pi}{78}], \text{Cos}[\tfrac{5\pi}{78}]\}, \{-\text{Sin}[\tfrac{3\pi}{26}], \text{Cos}[\tfrac{3\pi}{26}]\}, \{-\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\}, \{-\text{Sin}[\tfrac{17\pi}{78}], \text{Cos}[\tfrac{17\pi}{78}]\}, \{-\text{Cos}[\tfrac{3\pi}{13}], \text{Sin}[\tfrac{3\pi}{13}]\}, \{-\text{Cos}[\tfrac{7\pi}{39}], \text{Sin}[\tfrac{7\pi}{39}]\},$$

$$\{-\text{Cos}[\tfrac{5\pi}{39}], \text{Sin}[\tfrac{5\pi}{39}]\}, \{-\text{Cos}[\tfrac{\pi}{13}], \text{Sin}[\tfrac{\pi}{13}]\}, \{-\text{Cos}[\tfrac{\pi}{39}], \text{Sin}[\tfrac{\pi}{39}]\}, \{-\text{Cos}[\tfrac{\pi}{39}], -\text{Sin}[\tfrac{\pi}{39}]\}, \{-\text{Cos}[\tfrac{\pi}{13}], -\text{Sin}[\tfrac{\pi}{13}]\}, \{-\text{Cos}[\tfrac{5\pi}{39}], -\text{Sin}[\tfrac{5\pi}{39}]\}, \{-\text{Cos}[\tfrac{7\pi}{39}], -\text{Sin}[\tfrac{7\pi}{39}]\},$$

$$\{-\text{Cos}[\tfrac{3\pi}{13}], -\text{Sin}[\tfrac{3\pi}{13}]\}, \{-\text{Sin}[\tfrac{17\pi}{78}], -\text{Cos}[\tfrac{17\pi}{78}]\}, \{-\tfrac{1}{2}, -\tfrac{\sqrt{3}}{2}\}, \{-\text{Sin}[\tfrac{3\pi}{26}], -\text{Cos}[\tfrac{3\pi}{26}]\}, \{-\text{Sin}[\tfrac{5\pi}{78}], -\text{Cos}[\tfrac{5\pi}{78}]\}, \{-\text{Sin}[\tfrac{\pi}{78}], -\text{Cos}[\tfrac{\pi}{78}]\}, \{\text{Sin}[\tfrac{\pi}{26}], -\text{Cos}[\tfrac{\pi}{26}]\}, \{\text{Sin}[\tfrac{7\pi}{78}], -\text{Cos}[\tfrac{7\pi}{78}]\},$$

$$\{\text{Sin}[\tfrac{11\pi}{78}], -\text{Cos}[\tfrac{11\pi}{78}]\}, \{\text{Sin}[\tfrac{5\pi}{26}], -\text{Cos}[\tfrac{5\pi}{26}]\}, \{\text{Sin}[\tfrac{19\pi}{78}], -\text{Cos}[\tfrac{19\pi}{78}]\}, \{\text{Cos}[\tfrac{8\pi}{39}], -\text{Sin}[\tfrac{8\pi}{39}]\}, \{\text{Cos}[\tfrac{2\pi}{13}], -\text{Sin}[\tfrac{2\pi}{13}]\}, \{\text{Cos}[\tfrac{4\pi}{39}], -\text{Sin}[\tfrac{4\pi}{39}]\}, \{\text{Cos}[\tfrac{2\pi}{39}], -\text{Sin}[\tfrac{2\pi}{39}]\}, \{1, 0\}\}$$

Out[2]= {0, 1}

For each point in CircleCPoints, construct a circle with that point as its centre such that it passes through A. In other words, each such circle must satisfy:

r^2=(x – CircleCPoints[[i, 1]])^2 + (y – CircleCPoints[[i, 2]])^2

in which x and y take on the values of PointA[[1]] and PointA[[2]], respectively, and in which

r

is the radius of the circle in question.

For each point in CircleCPoints, the radius of the required circle is:

In[3]:=
```
ConstructedCircleRadii = Table[
  Sqrt[(PointA[[1]] - CircleCPoints[[i, 1]])^2 + (PointA[[2]] - CircleCPoints[[i, 2]])^2],
  {i, Length[CircleCPoints]}
 ];
```

Give the third result as an example:

In[4]:=
```
ConstructedCircleRadii[[3]]
```

Out[4]= $\sqrt{\mathrm{Cos}\left[\frac{2\pi}{13}\right]^2 + \left(1 - \mathrm{Sin}\left[\frac{2\pi}{13}\right]\right)^2}$

Formulate the equation for the upper hemisphere of each to be constructed circle in the form:

$y = f(x)$

Recalling that each circle satisfies:

$r^2 = (x - \text{CircleCPoints}[[i, 1]])^2 + (y - \text{CircleCPoints}[[i, 2]])^2$, (1)

one rearranges to solve for y:

$r^2 - (x - \text{CircleCPoints}[[i, 1]])^2 = (y - \text{CircleCPoints}[[i, 2]])^2$. (2)

Taking square roots of each side provides:

$\sqrt{r^2 - (x - \text{CircleCPoints}[[i, 1]])^2} = (y - \text{CircleCPoints}[[i, 2]])$, (3)

and solving for y gives:

$y = \text{CircleCPoints}[[i,2]] + \sqrt{r^2 - (x - \text{CircleCPoints}[[i, 1]])^2}$. (4)

Construct the equations for the upper hemisphere of each circle:

In[5]:=
```
UpperHemisphereEquations = Table[
   CircleCPoints[[i, 2]] + Sqrt[
     ConstructedCircleRadii[[i]]^2 -
      (x - CircleCPoints[[i, 1]])^2
    ],
   {i, Length[CircleCPoints]}
  ];
```

Give the third result as an example.

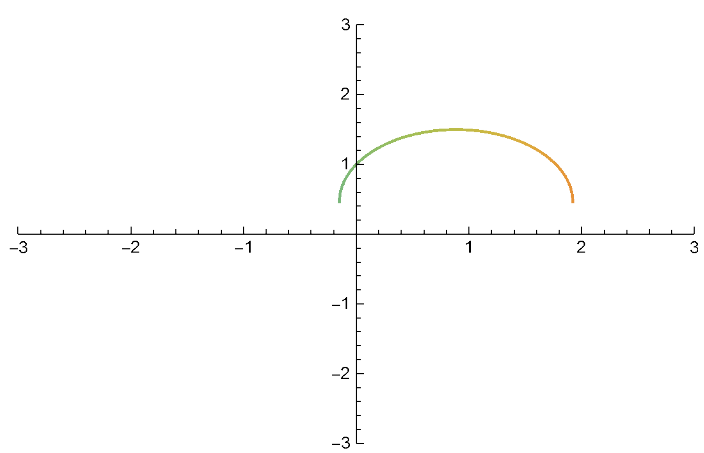
In[6]:=
```
UpperHemisphereEquations[[3]]
```

Out[6]=
$$\sqrt{-\left(x-\mathrm{Cos}\left[\frac{2\pi}{13}\right]\right)^2+\mathrm{Cos}\left[\frac{2\pi}{13}\right]^2+\left(1-\mathrm{Sin}\left[\frac{2\pi}{13}\right]\right)^2}+\mathrm{Sin}\left[\frac{2\pi}{13}\right]$$

Plot each upper hemisphere. Note the color formula in PlotStyle. Show the third in the list as an example.

In[7]:=
```
UpperHemispherePlots = Table[
   Plot[
    UpperHemisphereEquations[[i]],
    {
     x,
     -ConstructedCircleRadii[[i]] + CircleCPoints[[i, 1]],
     ConstructedCircleRadii[[i]] + CircleCPoints[[i, 1]]
    },
    ColorFunction → (ColorData["Rainbow"][Rescale[#1, {-3, 3}]] &),
    ColorFunctionScaling → False,
    PlotRange → {{-3, 3}, {-3, 3}}
   ],
   {i, Length[ConstructedCircleRadii]}
  ];
```

In[8]:=
```
UpperHemispherePlots[[3]]
```

Out[8]=


Construct the equations for the lower hemisphere of each circle:

Recalling that each circle satisfies:

r^2=(x – CircleCPoints[[i, 1]])^2 + (y – CircleCPoints[[i, 2]])^2, (1) one rearranges to solve for y:

r^2 - (x – CircleCPoints[[i, 1]])^2 = (y – CircleCPoints[[i, 2]])^2.   (2)

Taking square roots of each side, and, this time, taking the *negative* root of the right hand side, provides:

$\sqrt{r^2 - (x - \text{CircleCPoints}[[i, 1]])^2}$ = - (y – CircleCPoints[[i, 2]]),   (5)

and solving for y gives:

y = CircleCPoints[[i,2]] - $\sqrt{r^2 - (x - \text{CircleCPoints}[[i, 1]])^2}$.   (6)

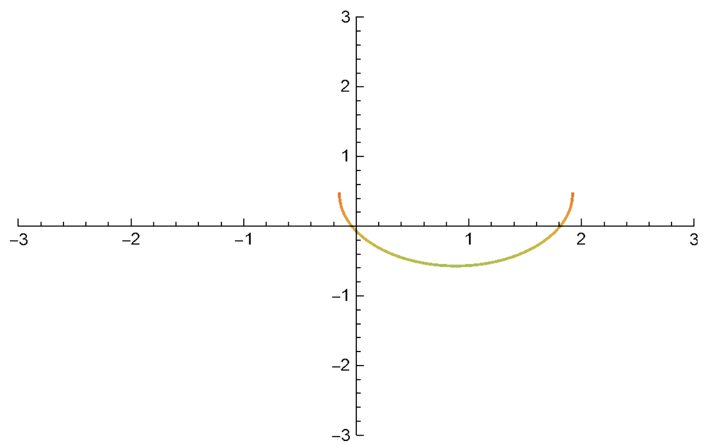
In[9]:=
```
LowerHemisphereEquations = Table[
   CircleCPoints[[i, 2]] -
    Sqrt[
     ConstructedCircleRadii[[i]]^2 -
      (x - CircleCPoints[[i, 1]])^2
    ],
   {i, Length[CircleCPoints]}
  ];
```

Plot each lower hemisphere. Show the third in the list as an example.

In[10]:=
```
LowerHemispherePlots = Table[
   Plot[
    LowerHemisphereEquations[[i]],
    {
     x,
     -ConstructedCircleRadii[[i]] + CircleCPoints[[i, 1]],
     ConstructedCircleRadii[[i]] + CircleCPoints[[i, 1]]
    },
    ColorFunction → (ColorData["Rainbow"][Rescale[#2, {-3, 1}]] &),
    ColorFunctionScaling → False,
    PlotRange → {{-3, 3}, {-3, 3}}
   ],
   {i, Length[ConstructedCircleRadii]}
  ];
```

In[11]:=
```
LowerHemispherePlots[[3]]
```

Out[11]=

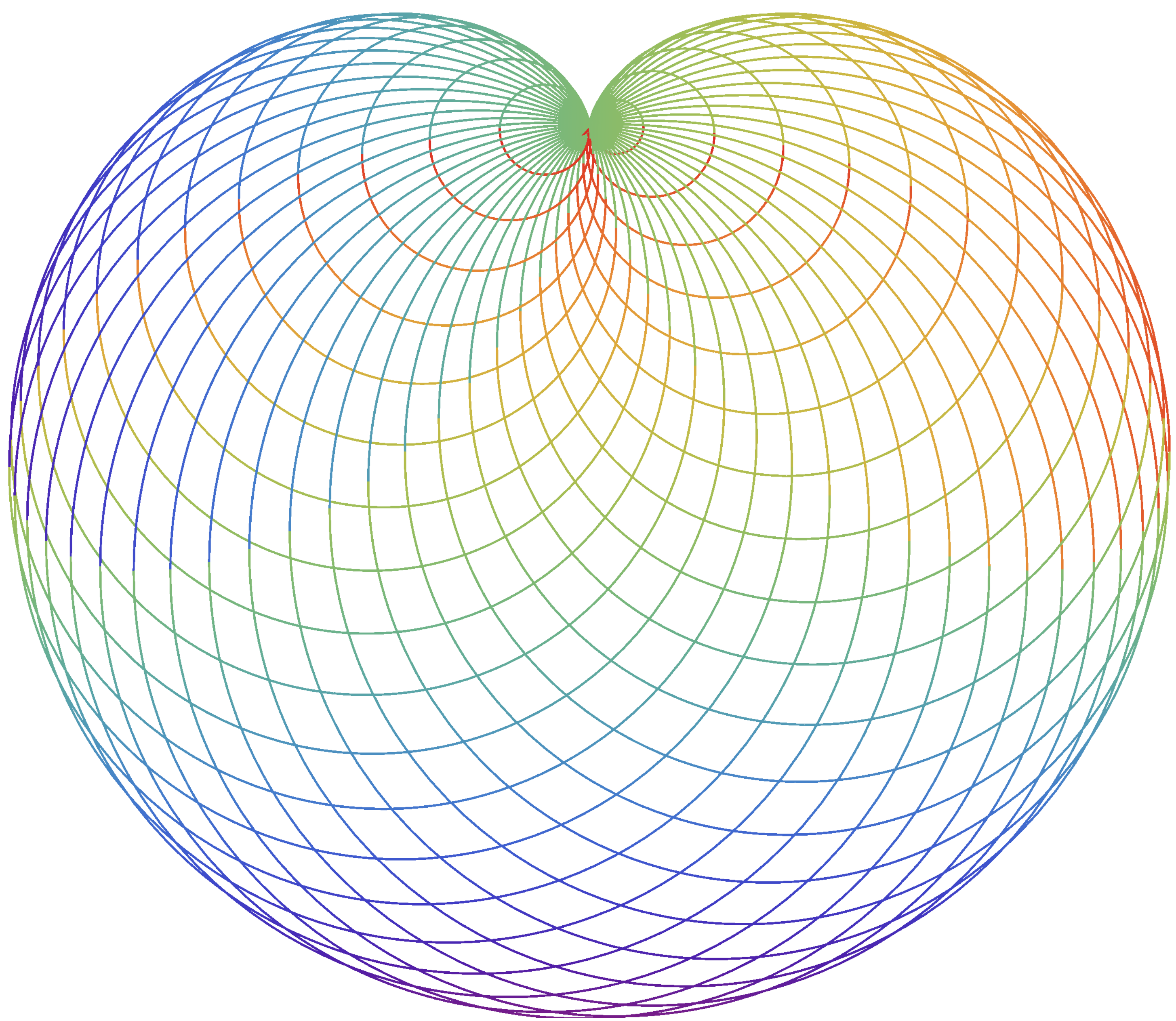


In[12]:=
```
ConstructedCirclePlots =
  Table[
   Show[
    UpperHemispherePlots[[i]],
    LowerHemispherePlots[[i]],
    AspectRatio → 1
   ],
   {i, Length[UpperHemispherePlots]}
  ];

CardiodPlot = Show[
  Table[
   ConstructedCirclePlots[[i]],
   {i, Length[ConstructedCirclePlots]}
  ],
  Axes → False,
  ImageSize → {12 × 72, 12 × 72}
 ]

Export["2LU2D 1UL2R.jpg",
 CardiodPlot]
```

Out[13]=



Out[14]= `2LU2D 1UL2R.jpg`

VisibleSpectrum!!