ECO 2302 - Networks in Trade and Macroeconomics

Lecture 2 - Aggregation and Propagation of Shocks in Networks

Motivation

- A key question for the networks literature: why do networks matter?
- Two potential answers:
  - networks matter for the **aggregation** of idiosyncratic shocks
  - networks matter for the **propagation** of shocks across economic entities
- Intuitive idea:
  - network linkages represent economic interactions between nodes
  - hence linkages should transmit effects of shocks across nodes
  - network structure seems like it should also be key for aggregate effects
- But is this really true?

Outline

- Hulten's theorem
  - Hulten (1978, ReStud)
  - a first look at production networks between sectors...
  - and how these matter for shock aggregation
  - an important benchmark result
- Beyond Hulten's theorem
  - Baqaee and Farhi (2019, Ecma)
  - more general characterization of the role of networks in shock aggregation
- The network origins of aggregate fluctuations
  - Acemoglu et al (2012, Ecma)
  - can idiosyncratic shocks to nodes generate aggregate fluctuations?
  - how does the network matter?
- Shock propagation through buyer-seller linkages
  - Barrot and Sauvagnat (2016, QJE)
  - how to identify role of linkages in shock propagation across firms?
  - what characteristics of buyer-seller relationships matter for propagation?

A Simple Benchmark Model

- Hulten (1978, ReStud), "Growth Accounting with Intermediate Inputs"
- Consider an economy with N sectors
- Households:
  - consume final goods from all sectors
  - supply labor inelastically
- Firms in each sector:
  - produce using labor and intermediates from other sectors
  - operate under perfect competition
- The **production network** is the set of input-output linkages between sectors
- We want to study the aggregate effects of sector-level shocks
  - and in particular how this depends on structure of the production network

Demand

- Households choose sector consumption to maximize aggregate consumption:

$$Y = Y\left(C_1, \cdots, C_N\right)$$

  - C_i: consumption of goods from sector i
  - Y: homogeneous of degree one (constant returns)
- Households supply L units of labor inelastically
- Budget constraint:

$$\sum_{i=1}^{N} P_i C_i = L$$

  - P_i: price of goods from sector i
  - wage is taken as numeraire

Demand

- Household utility maximization:

$$\max_{C_i} Y\left(C_1, \cdots, C_N\right)$$

$$\text{s.t. } \sum_{i=1}^{N} P_i C_i = L$$

- First-order conditions:

$$\partial C_i : \qquad \frac{\partial Y}{\partial C_i} = \lambda P_i$$

  - λ: Lagrange multiplier on the budget constraint

Demand

- Since Y is homogeneous of degree one, then Euler's theorem implies:

$$Y = \sum_{i=1}^{N} \frac{\partial Y}{\partial C_i} C_i$$

- Substituting the FOC into the budget constraint, we then have:

$$\lambda = Y/L$$

i.e. the Lagrange multiplier is equal to consumption per capita

Production

- Production technology for firms in sector i:

$$X_i = T_i F_i \left[L_i, \{X_{ij}\}_{j=1}^{N} \right]$$

  - T_i : sector-level TFP
  - L_i: quantity of labor hired
  - X_{ij}: quantity of inputs purchased from sector j
  - F_i: homogeneous of degree one (constant returns)
- Market structure is perfect competition
  - hence all firms earn zero profits

Production

- Firm profit maximization:

$$\max_{L_i, X_{ij}} \left\{ P_i T_i F_i \left(L_i, \{X_{ij}\}_{j=1}^{N} \right) - L_i - \sum_{j=1}^{N} P_j X_{ij} \right\}$$

- First-order conditions:

$$\partial L_i : \qquad P_i T_i \frac{\partial F_i}{\partial L_i} = 1$$

$$\partial X_{ij} : \qquad P_i T_i \frac{\partial F_i}{\partial X_{ij}} = P_j$$

- How do we know that profits are zero?
  - again, application of Euler's theorem to F_i

Market Clearing

- Labor market clearing:

$$\sum_{i=1}^{N} L_i = L$$

- Goods market clearing:

$$C_i + \sum_{j=1}^{N} X_{ji} = X_i$$

Aggregation of Sectoral Shocks

- Now we want to see how shocks to sector TFP (T_i) affect aggregate output (Y)
- We will focus on a *first-order approximation*:

$$\frac{\partial \log Y}{\partial \log T_i} = \frac{T_i}{Y}\frac{\partial Y}{\partial T_i}$$

- In other words, the *elasticity* of aggregate output with respect to TFP in sector i
- **Hulten's Theorem** tells us that:

$$\frac{\partial \log Y}{\partial \log T_i} = \frac{R_i}{GDP} \equiv D_i$$

  - R_i: total sales in sector i
  - GDP: gross domestic product (which here is equal to L)
  - D_i: sales share ("Domar weight") of sector i
- Note that this is true regardless of:
  - the specific form of the utility function Y
  - the specific form of the production functions F_i

Proof of the Theorem

- To prove the theorem, we will rely on the **first welfare theorem:**
  - every competitive equilibrium is Pareto efficient
- In other words:
  - the equilibrium that results from utility/profit maximization...
  - is identical to the solution to the *social planner's problem*

$$\max_{C_i, L_i, X_{ij}} Y\left(C_1, \cdots, C_N\right)$$

$$\text{s.t.} \sum_{i=1}^{N} L_i = L$$

$$C_i + \sum_{j=1}^{N} X_{ji} = T_i F_i \left[L_i, \left\{X_{ij}\right\}_{j=1}^{N}\right]$$

- The planner therefore:
  - chooses the allocations of labor and output...
  - so as to maximize household utility...
  - subject to the labor and output market clearing constraints

Proof of the Theorem

- Lagrangian for the planner's problem:

$$\mathscr{L} = Y + \lambda \underbrace{\left(L - \sum_{i=1}^{N} L_i \right)}_{\text{labor constraint}} + \sum_{i=1}^{N} \mu_i \underbrace{\left[T_i F_i \left[L_i, \{X_{ij}\}_{j=1}^{N} \right] - C_i - \sum_{j=1}^{N} X_{ji} \right]}_{\text{output constraint}}$$

  - λ: Lagrange multiplier on labor market clearing constraint
  - μ_i: Lagrange multiplier on output market clearing constraint for sector i

- First-order conditions:

$$\partial C_i : \qquad \frac{\partial Y}{\partial C_i} = \mu_i$$

$$\partial L_i : \qquad \mu_i T_i \frac{\partial F_i}{\partial L_i} = \lambda$$

$$\partial X_{ij} : \qquad \mu_i T_i \frac{\partial F_i}{\partial X_{ij}} = \mu_j$$

Proof of the Theorem

	planner	market
∂C_i:	$\frac{\partial Y}{\partial C_i} = \mu_i$	$\frac{\partial Y}{\partial C_i} = \lambda P_i$
∂L_i:	$\mu_i T_i \frac{\partial F_i}{\partial L_i} = \lambda$	$P_i T_i \frac{\partial F_i}{\partial L_i} = 1$
∂X_{ij}:	$\mu_i T_i \frac{\partial F_i}{\partial X_{ij}} = \mu_j$	$P_i T_i \frac{\partial F_i}{\partial X_{ij}} = P_j$

- Comparing the planner's solution with the market equilibrium, we see that:

$$\mu_i = \lambda P_i = P_i Y / L$$

- Now, application of the envelope theorem to the planner's problem implies:

$$\frac{\partial Y}{\partial T_i} = \mu_i F_i$$

which we can rewrite as:

$$\frac{T_i}{Y}\frac{\partial Y}{\partial T_i} = \frac{P_i X_i}{L}$$

- This completes the proof

Implications

- What does Hulten's Theorem tell us about the role of the production network?
  - the network is irrelevant for aggregation of sector shocks!
- As long as we can observe sales of each sector...
  - we can compute Domar weights
  - hence we will know aggregate effects of sector shocks to a first-order
  - regardless of the production network underlying the economy
- A very old result in macro...
  - but becoming salient again in the network literature

Beyond Hulten's Theorem

- Baqaee and Farhi (2018), "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem"
- Main insight: Hulten's Theorem is only a first-order result
- Suppose we also care about second-order effects:

$$\frac{\partial^2 \log Y}{\partial \log T_i \partial \log T_j}$$

- Then the production network (amongst other things) matters
- The paper is very technical:
  - derives exact expressions for $\frac{\partial^2 \log Y}{\partial \log T_i \partial \log T_j}$ under very general assumptions
- To develop the main intuition, we will consider an illustrative example

An Illustrative Example

- Suppose that there is only one sector
  - but output from the sector is also used as an input in production
  - i.e. there is *roundabout production*
- Firms are perfectly competitive
- The production function is of the *constant elasticity of substitution* (CES) form:

$$X = T\left[\omega^{\frac{1}{\sigma}} L^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} M^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

  - X: sector output
  - T: sector TFP
  - L: quantity of labor hired
  - M: quantity of output used as inputs
  - ω: weight on labor
  - σ: elasticity of substitution between labor and intermediates
- Note that:
  - as $\sigma \to 1$, production function becomes...Cobb-Douglas
  - as $\sigma \to 0$, production function becomes...Leontief
  - as $\sigma \to \infty$, labor and intermediates become perfect substitutes

An Illustrative Example

- Representative household:
  - supplies 1 unit of labor inelastically
  - consumes Y units of output
- Taking the wage as numeraire, final consumption is hence:

$$Y = 1/P$$

  where P is the price of output
- Market clearing:

$$X = Y + M$$

Competitive Equilibrium

- Cost minimization problem for producers:

$$\min_{L,M} \{L + PM\}$$
$$\text{s.t. } T\left[\omega^{\frac{1}{\sigma}} L^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} M^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = X$$

  - P: price of output
  - wage is taken as numeraire
  - let μ denote Lagrange multiplier on production constraint

- First order conditions can be written as:

$$\partial L: \quad \mu X^{\frac{1}{\sigma}} T^{\frac{\sigma-1}{\sigma}} \omega^{\frac{1}{\sigma}} L^{\frac{\sigma-1}{\sigma}} = L$$
$$\partial M: \quad \mu X^{\frac{1}{\sigma}} T^{\frac{\sigma-1}{\sigma}} (1-\omega)^{\frac{1}{\sigma}} M^{\frac{\sigma-1}{\sigma}} = PM$$

Competitive Equilibrium

- Adding FOCs together implies:

$$\mu X = L + PM$$

- Therefore μ must be equal to the marginal cost of production
  - which is equal to the price of output under perfect competition

$$\mu = P$$

Competitive Equilibrium

- Substituting FOCs into production constraint:

$$P = \frac{1}{T}\left[\omega + (1-\omega)\, P^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

- We can then solve for the output price as:

$$P = \left[\frac{\omega T^{\sigma-1}}{1-(1-\omega)\, T^{\sigma-1}}\right]^{\frac{1}{1-\sigma}}$$

- Hence final consumption is:

$$\begin{aligned} Y &= 1/P \\ &= \left[\frac{\omega T^{\sigma-1}}{1-(1-\omega)\, T^{\sigma-1}}\right]^{\frac{1}{\sigma-1}} \end{aligned}$$

- Now we have an explicit solution for Y and can see how it varies with TFP T

Higher-order Effects of TFP Shocks

- First-order effect of TFP shocks:

$$\frac{\partial \log Y}{\partial \log T} = \xi \equiv \frac{1}{1-(1-\omega)\,T^{\sigma-1}}$$

  which is equal to the Domar weight PX/L (Hulten)

- Second-order effect of TFP shocks:

$$\frac{\partial^2 \log Y}{\partial \log T^2} = \xi\,(\xi-1)\,(\sigma-1)$$

- Note that the Hulten approximation is exact ($\frac{\partial^2 \log Y}{\partial \log T^2} = 0$) if either...
  - there are no intermediates ($\omega = 1$)
  - the production function is Cobb-Douglas ($\sigma = 1$)
- Hence, when there is a non-trivial production network between sectors...
  - higher-order effects generally matter for aggregate effects of sector shocks

Higher-order Effects of TFP Shocks

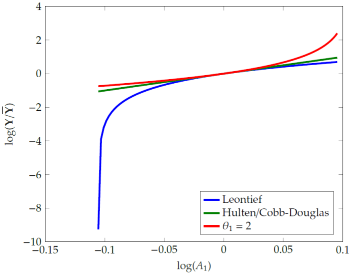


Figure 2: Output as a function of productivity shocks $\log(A_1)$ with variable input-output multiplier effect with steady-state input-output multiplier $\xi = 10$.

Source: Baqaee and Farhi (2018).

- Note that $\log Y$ is linear in $\log T$ only when production is Cobb-Douglas

Network Origins of Aggregate Fluctuations

- Acemoglu et al (2012, Ecma), "The Network Origins of Aggregate Fluctuations"
- Consider an economy with N sectors
- Households:
  - supply one unit of labor inelastically
  - consume final goods from each sector
- Firms:
  - produce using labor and intermediates from other sectors
  - operate under perfect competition
- Key questions:
  - how do fluctuations in sector TFP generate fluctuations in total output?
  - in particular, what happens as $N \to \infty$?
  - do idiosyncratic shocks wash out in the aggregate?
  - how does the network between sectors matter?

Demand

- Households have Cobb-Douglas utility over consumption from each sector:

$$Y = \prod_{i=1}^{N} \left(\frac{C_i}{\alpha_i} \right)^{\alpha_i}$$

  - C_i: consumption from sector i
  - α_i: share of household expenditure on sector i (with $\sum_{i=1}^{N} \alpha_i = 1$)

- Budget constraint:

$$\sum_{i=1}^{N} P_i C_i = 1$$

  - P_i: price of sector i goods
  - wage is taken as numeraire

Demand

- Household utility maximization:

$$\max_{C_i} \prod_{i=1}^{N} \left(\frac{C_i}{\alpha_i}\right)^{\alpha_i}$$

$$\text{s.t. } \sum_{i=1}^{N} P_i C_i = 1$$

- First-order conditions:

$$\partial C_i : \qquad \alpha_i Y = \lambda P_i C_i$$

  - λ: Lagrange multiplier on the budget constraint
- Substituting the FOC into the budget constraint:

$$\lambda = Y$$

- Hence the solution to the household's problem is:

$$P_i C_i = \alpha_i$$

i.e. a fraction α_i of total income ($= 1$) is spent on sector i

Demand

- Aggregate consumption is then:

$$Y = \prod_{i=1}^{N} (P_i)^{-\alpha_i}$$

- Let lower case variables denote logs (i.e. $x \equiv \log X$)
- Then we can rewrite this as:

$$y = -\sum_{i=1}^{N} \alpha_i p_i$$

Production

- Firms in sector i have access to a Cobb-Douglas production function:

$$X_i = T_i \left(\frac{L_i}{1-\gamma}\right)^{1-\gamma} \left(\frac{M_i}{\gamma}\right)^{\gamma}$$

  - T_i: TFP in sector i
  - L_i : quantity of labor hired
  - M_i: aggregate quantity of intermediate inputs used
  - γ: intermediate input share

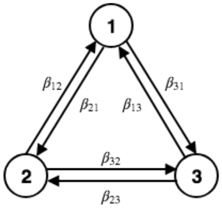
- Intermediate inputs are produced by combining inputs from all other sectors:

$$M_i = \prod_{j=1}^{N} \left(\frac{X_{ij}}{\beta_{ij}}\right)^{\beta_{ij}}$$

  - X_{ij}: quantity of inputs purchased by sector i from sector j
  - β_{ij}: share of sector j in sector i's intermediate input purchases

Production

- Note that the **network** of input-output linkages between sectors is specified by...
  - the matrix of Cobb-Douglas weights, $\{\beta_{ij}\}$



- The network is (potentially) complete along the extensive margin
  - but the weights of the edges along the intensive margin are heterogeneous

Production

- Profit maximization problem for firms in sector i:

$$\max_{L_i, X_{ij}} \left\{ P_i X_i - L_i - \sum_{j=1}^{N} P_j X_{ij} \right\}$$

$$\text{s.t. } X_i = T_i \left(\frac{L_i}{1-\gamma} \right)^{1-\gamma} \left(\frac{M_i}{\gamma} \right)^{\gamma}$$

$$M_i = \prod_{j=1}^{N} \left(\frac{X_{ij}}{\beta_{ij}} \right)^{\beta_{ij}}$$

- First-order conditions:

$$\begin{aligned} \partial L_i : \quad & L_i = (1-\gamma) P_i X_i \\ \partial X_{ij} : \quad & P_j X_{ij} = \gamma \beta_{ij} P_i X_i \end{aligned}$$

Production

- Substituting FOCs into the production function gives:

$$P_i = \frac{1}{T_i} \prod_{j=1}^{N} P_j^{\gamma \beta_{ij}}$$

- In log terms:

$$p_i = -\epsilon_i + \gamma \sum_{j=1}^{N} \beta_{ij} p_j$$

where $\epsilon_i \equiv \log T_i$

Market Clearing

- Labor market clearing:

$$\sum_{i=1}^{N} L_i = L$$

- Goods market clearing:

$$X_i = C_i + \sum_{j=1}^{N} X_{ji}$$

- Substituting the household and firm first-order conditions:

$$R_i = \alpha_i + \gamma \sum_{j=1}^{N} \beta_{ji} R_j$$

where $R_i = P_i X_i$ is total sales of sector i

Upstream and Downstream Networks

- Note that the equilibrium conditions include two "network systems"
- The price (upstream) network:

$$p_i = -\epsilon_i + \gamma \sum_{j=1}^{N} \beta_{ij} p_j$$

  - sector i has low price if TFP is high...
  - or if it uses intensively goods from sectors with low prices
- The sales (downstream) network:

$$R_i = \alpha_i + \gamma \sum_{j=1}^{N} \beta_{ji} R_j$$

  - sector i has high sales if it has high final sales...
  - or if it sells intensively to sectors that have high sales

Upstream and Downstream Networks

- Since these systems are linear, we can easily solve for p and R (and hence X):

$$p = -\left(I - \gamma\beta\right)^{-1}\epsilon$$
$$R = \left(I - \gamma\beta'\right)^{-1}\alpha$$

- How do we know that $I - \gamma\beta$ is invertible?
- Need to show that eigenvalues of $\gamma\beta$ are less than 1 in absolute value
- Note that β is a right stochastic matrix:
  - i.e. a matrix for which all rows sum up to 1
- Theorem: the largest absolute eigenvalue of any stochastic matrix is equal to 1
  - proof - application of the Gershgorin circle theorem
- Therefore largest absolute eigenvalue of $\gamma\beta$ is equal to $\gamma < 1$

Aggregate Fluctuations

- How do fluctuations in $\{\epsilon_i\}_{i=1}^N$ translate into aggregate fluctuations in y?
- As a benchmark, suppose first that:
  - there are no input-output linkages between sectors ($\gamma = 0$)
  - final demand shares are equal ($\alpha_i = 1/N$ for all i)
- Then from the household utility maximization solution:

$$y = -\frac{1}{N}\sum_{i=1}^{N} p_i$$

and from the firm profit maximization solution:

$$p_i = -\epsilon_i$$

- Hence, log final output is:

$$y = \frac{1}{N}\sum_{i=1}^{N} \epsilon_i$$

i.e. an *average* of sectoral log TFPs

Aggregate Fluctuations

- Now suppose that ϵ_i is drawn from some probability distribution, with:
  - zero mean, $\mathbb{E}[\epsilon_i] = 0$
  - finite variance, $\text{var}(\epsilon_i) = \sigma^2$
  - independent draws across sectors
- Then the central limit theorem implies that:

$$\text{var}(y)^{1/2} = \Theta\left(\frac{1}{\sqrt{N}}\right)$$

  i.e. the standard deviation of log GDP scales with $\frac{1}{\sqrt{N}}$

- Hence, $\text{var}(y)^{1/2} \to 0$ as $N \to \infty$
  - i.e. idiosyncratic sector TFP shocks wash out in the aggregate

Aggregate Fluctuations

- Now consider the general case with a production network ($\gamma \in (0,1)$)
- From household's utility maximization problem:

$$y = -\sum_{i=1}^{N} \alpha_i p_i$$

or in matrix form:

$$y = -\alpha' p$$

- From firm's profit maximization problem:

$$p_i = -\epsilon_i + \gamma \sum_{i=1}^{N} \beta_{ij} p_j$$

or in matrix form:

$$p = -\left[I - \gamma\beta\right]^{-1} \epsilon$$

Aggregate Fluctuations

- Combining the two matrix expressions, we get:

$$y = v'\epsilon$$

  where v is the **influence vector** of the economy:

$$v \equiv \left[\boldsymbol{I} - \gamma\beta'\right]^{-1} \alpha$$

- To isolate role of the production network, suppose that $\alpha_i = 1/N$ for all i:

$$v = \frac{1}{N}\left[\boldsymbol{I} - \gamma\beta'\right]^{-1} \mathbf{1}$$

- Then the influence vector depends on the following Leontief inverse:

$$\begin{aligned} V &= \left[\boldsymbol{I} - \gamma\beta'\right]^{-1} \\ &= \boldsymbol{I} + \gamma\left(\beta'\right) + \gamma^2\left(\beta'\right)^2 + \cdots \end{aligned}$$

Aggregate Fluctuations

- The $(i,j)^{th}$-element of V is the effect of shocks to sector i on firm j's cost (i.e. the influence of i on j) through connections of **all degrees**

$$V_{ij} = \underbrace{\mathbf{1}_{[i=j]}}_{\text{own-sector effect}}$$

ⓘ

Aggregate Fluctuations

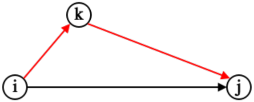
- The $(i,j)^{th}$-element of V is the effect of shocks to sector i on firm j's cost (i.e. the influence of i on j) through connections of **all degrees**

$$V_{ij} = \underbrace{\mathbf{1}_{[i=j]}}_{\text{own-sector effect}} + \underbrace{\gamma\beta_{ji}}_{\text{direct effect}}$$

Aggregate Fluctuations

- The $(i,j)^{th}$-element of V is the effect of shocks to sector i on firm j's cost (i.e. the influence of i on j) through connections of **all degrees**

$$V_{ij} = \underbrace{\mathbf{1}_{[i=j]}}_{\text{own-sector effect}} + \underbrace{\gamma\beta_{ji}}_{\text{direct effect}} + \underbrace{\gamma^2\Sigma_{k=1}^{N}\beta_{jk}\beta_{ki}}_{\text{effect via one sector}}$$



Aggregate Fluctuations

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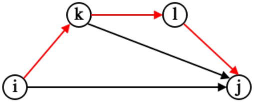

Aggregate Fluctuations

- The $(i,j)^{th}$-element of V is the effect of shocks to sector i on firm j's cost (i.e. the influence of i on j) through connections of **all degrees**

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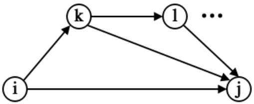


- Multiplying V by unit vector gives total influence of each sector on all sectors
- Multiplying by shock vector then gives aggregate effect of shocks

Aggregate Fluctuations

- How does this connect to Hulten's theorem?
- Output as a function of TFP shocks:

$$y = v^T \epsilon$$

- Note that the influence vector is actually equal to the sales vector:

$$v = R = \left[I - \gamma\beta'\right]^{-1} \alpha$$

- Hence we have:

$$\frac{\partial y}{\partial \epsilon_i} = R_i$$

- Since *GDP* here is equal to 1, R_i is the Domar weight of sector i

The Role of the Production Network

- Now let X_N denote the value of variable X in an economy with N sectors
- Suppose that ϵ_{iN} is drawn from some probability distribution, with:
  - zero mean, $\mathbb{E}\left[\epsilon_{iN}\right] = 0$
  - finite variance, $\text{var}\left(\epsilon_{iN}\right) = \sigma^2$
  - independent draws across sectors
- Since $y = v'\epsilon$, then aggregate volatility is given by:

$$\text{var}\left(y_N\right)^{1/2} = \sigma \sqrt{\sum_{i=1}^{N} v_{iN}^2} = \sigma \left\|v_N\right\|$$

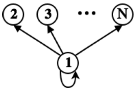
- Aggregate volatility hence scales with Euclidean norm of the influence vector:

$$\text{var}\left(y_N\right)^{1/2} = \Theta\left(\left\|v_N\right\|\right)$$

whereas recall that $\text{var}\left(y_N\right)^{1/2} = \Theta\left(\frac{1}{\sqrt{N}}\right)$ without the network

The Role of the Production Network

- If $v_N = \begin{bmatrix} \frac{1}{N} & \cdots & \frac{1}{N} \end{bmatrix}'$ (empty or symmetric network), then $\|v_N\| = \frac{1}{\sqrt{N}}$
  - idiosyncratic fluctuations wash out as $N \to \infty$
- However, for general network structures:
  - aggregate volatility can decay at rates slower than $\frac{1}{\sqrt{N}}$
  - and need not even approach zero as $N \to \infty$
- For example, consider the following star network:



$$\beta = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix}$$

- One can show that the influence vector is $v_{Ni} = \begin{cases} \frac{1}{N} + \frac{\gamma}{1-\gamma} & i = 1 \\ \frac{1}{N} & i \neq 1 \end{cases}$
- Hence, $\lim_{N\to\infty} \|v_N\| = \frac{\gamma}{1-\gamma}$
  - idiosyncratic shocks generate aggregate volatility

Empirical Application

- How important is the I-O network for aggregate fluctuations in the US economy?
- Study input-output data from the Bureau of Economic Analysis, 1972-2002
  - measures spending on sector i by sector j
- Rough sketch of empirical analysis:
  - calibrate intermediate input share γ and I-O matrix β
  - compute influence vector v_N
  - compute Euclidean norm of influence vector $\|v_N\|$ and compare this to $\frac{1}{\sqrt{N}}$

Empirical Application

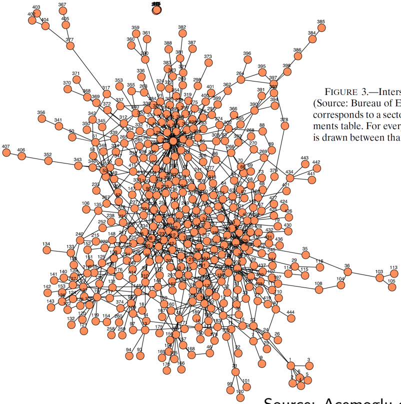


FIGURE 3.—Intersectoral network corresponding to the U.S. input–output matrix in 1997. (Source: Bureau of Economic Analysis. See Section 4 for more details on the data.) Each vertex corresponds to a sector in the 1997 benchmark detailed commodity-by-commodity direct requirements table. For every input transaction above 5% of the total input purchases of a sector, a link is drawn between that sector and the input supplier.

Source: Acemoglu et al (2012).

Empirical Application

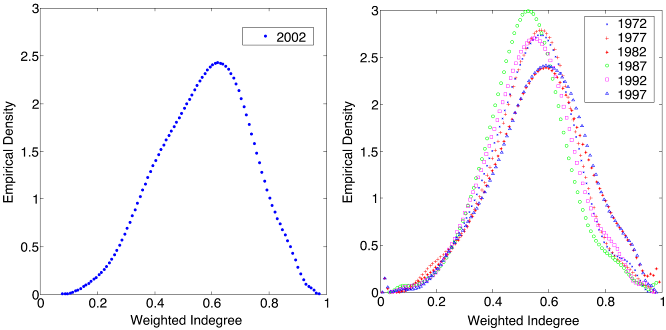


FIGURE 6.—Empirical densities of intermediate input shares (indegrees).
Source: Acemoglu et al (2012).

- Mean intermediate input share, $\gamma = 0.55$

The Role of the Input-Output Network

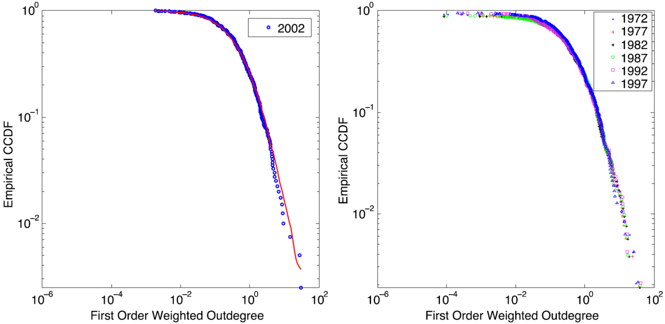


FIGURE 8.—Empirical counter-cumulative distribution function of first-order degrees.
Source: Acemoglu et al (2012).

- Weighted outdegrees are much more heterogeneous than indegrees
- Some sectors supply inputs to many sectors (i.e. general purpose inputs)

The Role of the Input-Output Network

ESTIMATES FOR $\|v_n\|_2$[a]

	1972	1977	1982	1987	1992	1997	2002
$\|v_{n_d}\|_2$	0.098 ($n_d = 483$)	0.091 ($n_d = 524$)	0.088 ($n_d = 529$)	0.088 ($n_d = 510$)	0.093 ($n_d = 476$)	0.090 ($n_d = 474$)	0.094 ($n_d = 417$)
$\|v_{n_s}\|_2$	0.139 ($n_s = 84$)	0.137 ($n_s = 84$)	0.149 ($n_s = 80$)	0.133 ($n_s = 89$)	0.137 ($n_s = 89$)	0.115 ($n_s = 127$)	0.119 ($n_s = 128$)
$\frac{\|v_{n_d}\|_2}{\|v_{n_s}\|_2}$	0.705	0.664	0.591	0.662	0.679	0.783	0.790
$\frac{1/\sqrt{n_d}}{1/\sqrt{n_s}}$	0.417	0.400	0.399	0.418	0.432	0.518	0.554

[a] $\|v_{n_d}\|_2$ denotes estimates obtained from the detailed level input–output BEA data. $\|v_{n_s}\|_2$ denotes estimates obtained from the summary input–output BEA data. The numbers in parentheses denote the total number of sectors implied by each level of disaggregation.

Source: Acemoglu et al (2012).

- Measures of $\|v_N\|$ are approximately twice as large as $\frac{1}{\sqrt{N}}$
  - network amplifies sectoral shocks by a factor of ≈ 2
- Measures of $\|v_N\|$ are smaller when using more disaggregated data (larger N)
- Since $\|v_N\|$ falls more slowly than $\frac{1}{\sqrt{N}}$ as N increases...
  - network effects are more important at higher levels of disaggregation

Identification and Microfoundations of Shock Propagation

- Suppose we want to study how shocks to firms spill over to other firms
- Key challenge: **identification** of network effects
- As a motivating example, consider two firms $i \in \{1, 2\}$ in a supply chain
  - firm 1 supplies inputs for production of firm 2's output

$$1 \longrightarrow 2$$

  - suppose that firm productivities ϕ_i are related by:

$$\phi_1 = \epsilon_1$$
$$\phi_2 = \rho\phi_1 + \epsilon_2$$

    where $\{\epsilon_1, \epsilon_2\}$ are firm-specific shocks
- Now suppose that we want to identify ρ
  - i.e. the strength of spillover effects from firm 1 to firm 2

Identification and Microfoundations of Shock Propagation

- Combining the two equations, we would want to estimate:

$$\phi_2 = \rho\epsilon_1 + \epsilon_2$$

  where $\{\epsilon_1, \phi_2\}$ are observed
- However, note that as long as firm-specific shocks $\{\epsilon_1, \epsilon_2\}$ are correlated...
  - exogeneity assumption is violated and OLS estimates are biased
- Hence, need something else for identification of ρ
- Even if we can cleanly identify ρ...
  - we would also like to develop **microfoundations** for ρ
  - e.g. what explains why ρ might vary across different relationships

Outline

- Barrot and Sauvagnat (2016, QJE), "Input Specificity and the Propagation of Idiosyncratic Shocks in Production Networks"
- Study events where firms are hit by natural disasters (exogenous)
- Use disaster treatment to estimate:
  - *direct effects* on affected firms' sales growth, equity value, etc.
  - *downstream propagation effects* on affected firm's customers
  - *horizontal propagation effects* on other suppliers of the same customers
- Focus on whether inputs provided by suppliers are "specific" to relationship
  - shocks should propagate strongly if buyers cannot easily adjust

Firm-level Data

- Compustat North America Fundamentals Quarterly database:
  - firm-level financial data
  - firm headquarter locations (county level)
  - restrict sample to nonfinancial firms with HQs in the US
- Infogroup:
  - firm headquarter locations (county level)
  - employment and establishment information
- Center for Research in Security Prices:
  - daily stock prices

Buyer-seller Linkages

- SEC regulation SFAS No. 131 requires public firms to disclose:
  - industry segments that account for more than 10% of sales
  - customers (firms) that account for more than 10% of sales
- Identity of major customers reported as name of each customer
- Hence need to match set of reported customers with set of reporting suppliers
  - use phonetic string-matching algorithm to match firm names
  - customers with no match are excluded from sample
  - matched customers represent 75% of total sales in Compustat
- Unique feature: panel data
  - allows tracking of buyer-seller relationships over time
- Main limitations of data:
  - truncation at 10% of sales
  - restricted to public firms
  - no information on products traded

Buyer-seller Linkages

Company Name	Ticker Symbol	Customer Name	Customer Sales
ADC TELECOMMUNICATIONS INC	ADCT	VERIZON COMMUNICATIONS	146
ANR PIPELINE CO	4267A	Wisconsin Gas Co	56
SERVIDYNE INC	SERV.1	KMART HOLDING CORP-PRE AMEND	2.008
ACTIVISION INC	ATVI.1	Wal-Mart Stores	323.347
ADVANCED MICRO DEVICES	AMD	FUJITSU LTD -ADR	875
AEROSONIC CORP	AIM	Lockheed Martin Corp	4.915
AEROSONIC CORP	AIM	Boeing Co	3.072
IDNA INC	IDAI	R&D Strategic Solutions	1.475
IDNA INC	IDAI	PFIZER INC	4.083
AIR T INC	AIRT	Federal Express Corp	41.312
ATRION CORP	ATRI	NOVARTIS CORP.	7.8
ALEXANDER'S INC	ALX	Bloomberg L.P.	63.609
SKYWORKS SOLUTIONS INC	SWKS	MOTOROLA INC	166.398
SKYWORKS SOLUTIONS INC	SWKS	SONY CORP -ADR	79.237
SKYWORKS SOLUTIONS INC	SWKS	Samsung Electronics Co -GDR	55.466
ALPINE GROUP INC	APNI	HOME DEPOT INC	89.563
ALCOA INC	AA.3	North America	15956.99
AMERICAN GREETINGS -CL A	AM.1	WAL-MART STORES	247.355
AMERICAN LOCKER GROUP INC	ALGI	United States Postal Service	6.913
AMERICAN PACIFIC CORP	APFC	Alliant Techsystems Inc	34.172
AMERICAN VANGUARD CORP	AVD	Helena Chemical Company	20.878
AMERICAN VANGUARD CORP	AVD	Agriliance	24.673
AMERICAN VANGUARD CORP	AVD	United Agri Products	28.469
AMGEN INC	AMGN	AmerisourceBergen Corp	4760
AMGEN INC	AMGN	Cardinal Health Inc	2370
AMGEN INC	AMGN	McKesson Corp	2140

Source: Compustat.

Natural Disasters

- SHELDUS (Spatial Hazard and Loss Database for the United States) database
  - data on all major disasters occurring in the US after 1978
  - start date, end date, and identifier code of all affected counties
- Restrict attention to disasters:
  - lasting less than 30 days
  - with total estimated damages above $1 billion 2013 constant dollars

Natural Disasters

List of Major Disasters

Disaster	Date	# Counties	U.S. Employment Affected (%)	Location
Mount St. Helens eruption	May 1980	2	0.03	WA
Hurricane Alicia	August 1983	139	4.72	TX
Hurricane Elena	August 1985	32	0.54	AL, FL, LA, MS
Hurricane Juan	October 1985	66	3.58	AL, FL, LA, MS, TX
Hurricane Hugo	September 1989	71	1.43	NC, SC, VA
Loma earthquake	October 1989	8	2.56	CA
Hurricane Bob	August 1991	54	7.06	MA, ME, NC, NH, NY, RI
Oakland Hills firestorm	October 1991	1	0.54	CA
Hurricane Andrew	August 1992	51	2.67	AL, FL, LA, MS
Hurricane Iniki	September 1992	1	0.02	HI
Blizzard	March 1993	221	11.15	AL, CT, FL, GA, MA, MD, NJ, OH, SC, VA, VT
Northridge earthquake	January 1994	1	3.69	CA
Hurricane Alberto	July 1994	41	0.66	AL, FL, GA
Hurricane Opal	October 1995	186	6.43	AL, FL, GA, LA, MS, NC, SC
Blizzard	January 1996	319	14.57	CT, DE, IN, KY, MA, MD, NC, NJ, NY, PA, VA, WV
Hurricane Fran	September 1996	100	2.02	NC, SC, VA, WV
Ice storm	January 1998	43	1.09	ME, NH, NY, VT
Hurricane Bonnie	August 1998	43	1.26	NC, VA
Hurricane Georges	September 1998	78	3.68	AL, FL, LA, MS
Hurricane Floyd	September 1999	226	15.68	CT, DC, DE, FL, MD, ME, NC, NH, NJ, NY, PA, SC, VA, VT
Hurricane Allison	June 2001	77	4.56	AL, FL, GA, LA, MS, PA, TX
Hurricane Isabel	September 2003	89	4.99	DE, MD, NC, NJ, NY, PA, RI, VA, VT, WV

Source: Barrot and Sauvagnat (2016).

- 41 major disasters of all kinds, e.g. blizzards, earthquakes, floods, hurricanes
- Disasters are generally very localized:
  - each disaster affects at most 22% of U.S. employment

Natural Disasters

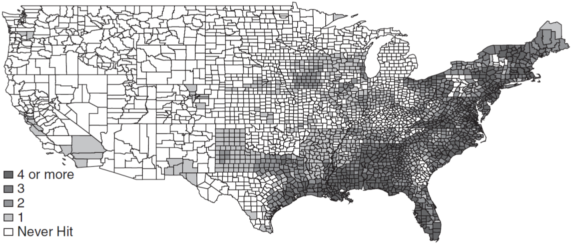


FIGURE II

Major Natural Disaster Frequency by U.S. Counties

Source: Barrot and Sauvagnat (2016).

- Some counties are hit more frequently than others
  - especially counties along southeast coast

Natural Disasters

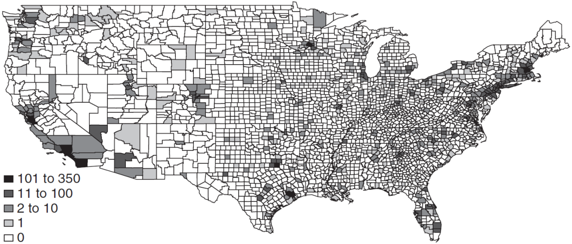


FIGURE III

Location of Sample Suppliers' Headquarters

Source: Barrot and Sauvagnat (2016).

- Location of suppliers in Compustat spans entire U.S. mainland
  - including counties that are never hit and often hit by natural disasters

Input Specificity

- Three different measures of input specificity
- Rauch (1999) classification of industries as:
  - sold on an exchange (e.g. silk)
  - reference priced (e.g. tobacco)
  - differentiated (e.g. household electronics)

Shares of commodity categories in value of total trade (percent)

		1970	1980	1990
Conservative Aggregation	Organized exchange	19.5	27.2	12.6
	Reference priced	24.0	21.3	20.3
	Differentiated	56.5	51.5	67.1
Liberal Aggregation	Organized exchange	24.7	31.7	16.0
	Reference priced	21.8	19.5	19.5
	Differentiated	53.6	48.9	64.6

Source: Rauch (1999).

- Using this classification:
  - compute fraction of differentiated products in each industry
  - supplier is specific if it operates in industry that lies above sample median

Input Specificity

- Investments in R&D
  - compute ratio of R&D to sales at the firm level
  - supplier is specific if this ratio lies above the sample median
  - lag measure by two years
- Patents
  - patent information from Google patents assembled by Kogan et al (2012)
  - count number of patents issued by each firm in previous three years
  - supplier is specific if patent count is above sample median

Summary Statistics

Panel A: Customer sample

Sales growth $(t-4,t)$	80,574	0.102	0.375	−0.606	0.040	1.927
Cogs growth $(t-4,t)$	79358	0.106	0.411	−0.651	0.038	2.193
Disaster hits firm (t)	80,574	0.016	0.126	0.000	0.000	1.000
Disaster hits one supplier (t)	80,574	0.014	0.118	0.000	0.000	1.000
Number of suppliers	80,574	1.383	4.162	0.000	0.000	19.000

Source: Barrot and Sauvagnat (2016).

- Mean sales growth $= 10.2\%$
- Mean probability that firm is hit directly by disaster $= 1.6\%$
- Mean probability that at least one supplier is hit by disaster $= 1.4\%$
- Mean number of suppliers (in-degree) $= 1.38$

Summary Statistics

Panel B: Supplier sample

	Obs.	Mean	Std. dev.	p1	p50	p99
Sales growth $(t-4,t)$	139,976	0.188	0.814	-0.876	0.045	4.568
Disaster hits firm (t)	139,976	0.017	0.127	0.000	0.000	1.000
Disaster hits a customer (t)	139,976	0.008	0.088	0.000	0.000	0.000
Disaster hits a customer's supplier (t)	139,976	0.042	0.200	0.000	0.000	1.000
Number of customers	139,976	0.711	0.964	0.000	0.000	4.000
% Employees at HQs county	102,279	0.597	0.365	0.000	0.667	1.000

Source: Barrot and Sauvagnat (2016).

- Mean sales growth = 18.8%
- Mean probability that firm is hit directly by disaster = 1.7%
- Mean probability that at least one customer is hit by disaster = 0.4%
- Mean number of customers (out-degree) = 0.71

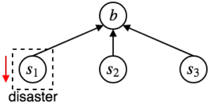
Summary Statistics

	Diff.		R&D		Patent	
	S	NS	S	NS	S	NS
Av. duration of relationships	7.125	6.692	6.373	8.335	7.821	6.618
Av. supplier-customer HQs distance	1,332	1,210	1,502	1,214	1,388	1,219
Av. suppliers' input share	0.022	0.025	0.017	0.023	0.025	0.022

Source: Barrot and Sauvagnat (2016).

- Mean duration of relationships $\approx$ 7 years
- Supplier inputs account for on average only 2.5% of cost of goods sold

Direct Effects



- First, estimate direct effect of natural disasters on firms that get hit
- Baseline regression specification at the firm (i) and quarter (t) level:

$$\Delta R_{i,t} = \alpha + \sum_{\tau=0}^{5} \beta_\tau \cdot DF_{i,t-\tau} + \eta_i + \pi_t + \epsilon_{i,t}$$

  - $\Delta R_{i,t}$: annual sales growth
  - $DF_{i,t-\tau}$: equals 1 if firm is directly hit by disaster τ quarters ago
  - η_i: firm fixed-effects
  - π_t: year-quarter fixed effects
- Coefficients of interest: β_τ

Direct Effects

Natural Disaster Disruptions—Supplier Sales Growth

	Sales Growth $(t-4,t)$			
Disaster hits firm (t)	−0.006	−0.004	−0.001	−0.011
	(0.018)	(0.018)	(0.018)	(0.018)
Disaster hits firm $(t-1)$	−0.045***	−0.045***	−0.032*	−0.039**
	(0.016)	(0.016)	(0.017)	(0.018)
Disaster hits firm $(t-2)$	−0.033*	−0.032*	−0.024	−0.026
	(0.018)	(0.018)	(0.021)	(0.021)
Disaster hits firm $(t-3)$	−0.042**	−0.040**	−0.032	−0.029
	(0.019)	(0.019)	(0.022)	(0.023)
Disaster hits firm $(t-4)$	−0.031	−0.028	−0.029	−0.024
	(0.020)	(0.020)	(0.022)	(0.023)
Disaster hits firm $(t-5)$	−0.007	−0.005	−0.022	−0.019
	(0.020)	(0.020)	(0.023)	(0.023)
Firm FE	Yes	Yes	Yes	Yes
Year-quarter FE	Yes	Yes	Yes	Yes
Size, age, ROA × year-quarter FE	No	Yes	Yes	Yes
State-year FE	No	No	Yes	Yes
Industry-year FE	No	No	No	Yes
Observations	139,976	139,976	139,976	139,976
R^2	0.177	0.192	0.212	0.233

Source: Barrot and Sauvagnat (2016).

- Natural disasters lead to 3-5 percentage point decline in sales growth
  - negative effects last for around 4 quarters
- Effects are not driven by type of firms hit, state, or industry

Direct Effects

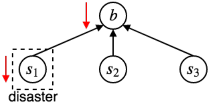
NATURAL DISASTERS DISRUPTIONS—SPECIFIC VERSUS NONSPECIFIC SUPPLIERS

	Sales Growth $(t-4,t)$					
Supplier specificity:	Diff.		R&D		Patent	
Disaster hits firm $(t-4,t-1)$	−0.050***	−0.044***	−0.048***	−0.048***	−0.046***	−0.041***
	(0.017)	(0.016)	(0.012)	(0.012)	(0.016)	(0.015)
Disaster hits specific firm $(t-4,t-1)$	0.023	0.013	0.038	0.044	0.020	0.011
	(0.026)	(0.026)	(0.040)	(0.039)	(0.028)	(0.028)
Specific firm			0.099***	0.090***	−0.060***	−0.030**
			(0.021)	(0.021)	(0.014)	(0.013)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year-quarter FE	Yes	Yes	Yes	Yes	Yes	Yes
Size, age, ROA × year-quarter FE	No	Yes	No	Yes	No	Yes
Observations	139,976	139,976	139,976	139,976	139,976	139,976
R^2	0.177	0.192	0.177	0.192	0.177	0.192

Source: Barrot and Sauvagnat (2016).

- To test whether disasters are different for specific vs. non-specific suppliers
  - add interaction term between disaster dummy and specific supplier dummy
- Negative effects are not larger for specific vs. nonspecific suppliers

Downstream Propagation



- Next, estimate downstream propagation of disaster shock to firms' customers
- Main regression specification at the firm (i) and quarter (t) level:

$$\Delta R_{i,t} = \alpha_0 + \alpha_1 \cdot DS_{i,t-4} + \alpha_2 \cdot DF_{i,t-4} + \eta_i + \pi_t + \epsilon_{i,t}$$

  - $DS_{i,t-4}$: equals 1 if at least one supplier hit by disaster in previous year
- Coefficient of interest: α_1

Downstream Propagation

Downstream Propagation—Baseline

Panel A	Sales Growth $(t-4,t)$			
Disaster hits one supplier $(t-4)$	−0.031***	−0.027***	−0.029***	−0.019**
	(0.009)	(0.008)	(0.008)	(0.008)
Disaster hits firm $(t-4)$	−0.031***	−0.029***	−0.005	−0.003
	(0.011)	(0.011)	(0.009)	(0.009)
Number of suppliers	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Year-quarter FE	Yes	Yes	Yes	Yes
Size, age, ROA × year-quarter FE	No	Yes	Yes	Yes
State-year FE	No	No	Yes	Yes
Industry-year FE	No	No	No	Yes
Observations	80,574	80,574	80,574	80,574
R^2	0.234	0.262	0.300	0.342

Source: Barrot and Sauvagnat (2016).

- Shock to at least one supplier leads to 3.1 percentage point fall in sales growth
- Results survive controlling for firm characteristics, state-year, industry-year

Downstream Propagation

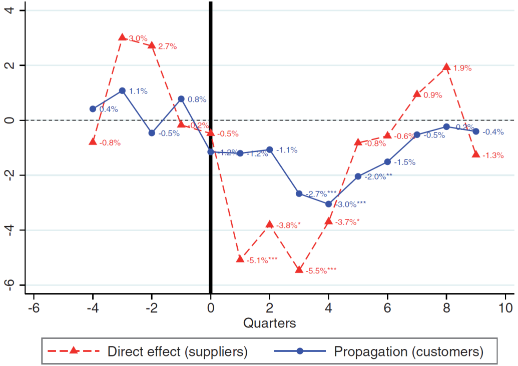
- To study the timing of effects in more detail, estimate the following
- Direct effect on firms hit by disasters - β_τ in the regression:

$$\Delta R_{i,t} = \alpha + \sum_{\tau=-4}^{9} \beta_\tau \cdot DF_{i,t-\tau} + \eta_i + \pi_t + \epsilon_{i,t}$$

- Downstream propagation effect on customers - γ_τ in the regression:

$$\Delta R_{i,t} = \alpha + \sum_{\tau=-4}^{9} \beta_\tau \cdot DF_{i,t-\tau} + \sum_{\tau=-4}^{9} \gamma_\tau \cdot DS_{i,t-\tau} + \eta_i + \pi_t + \epsilon_{i,t}$$

Downstream Propagation



Source: Barrot and Sauvagnat (2016).

- Both direct and downstream effects of disasters persist for several quarters
  - downstream effects last a bit longer than direct effects

Input Specificity

- To study what determines strength of shock propagation...
  - examine the role of **input specificity**
- Key idea: downstream propagation effects should be large if...
  - customers face large costs of switching to alternative suppliers
- Suppliers are more likely to produce specific inputs if they:
  - operate in industries producing differentiated goods
  - have a high level of R&D
  - hold many patents
- Now estimate the following regression:

$$\Delta R_{i,t} = \alpha_0 + \alpha_1^{ns} \cdot DS_{i,t-4}^{ns} + \alpha_1^{s} \cdot DS_{i,t-4}^{s} + \alpha_2 \cdot DF_{i,t-4} + \eta_i + \pi_t + \epsilon_{i,t}$$

  - $DS_{i,t-4}^{ns}$: equals 1 if at least one non-specific supplier hit in previous year
  - $DS_{i,t-4}^{s}$: equals 1 if at least one specific supplier hit in previous year
- Coefficients of interest: α_1^{ns} and α_1^{s}

Input Specificity

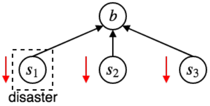
Downstream Propagation—Input Specificity

	Sales Growth $(t-4,t)$					
Supplier Specificity	Diff.		R&D		Patent	
Disaster hits one nonspecific supplier $(t-4)$	−0.002 (0.012)	−0.002 (0.011)	−0.018 (0.011)	−0.011 (0.011)	−0.020* (0.011)	−0.016 (0.010)
Disaster hits one specific supplier $(t-4)$	−0.050*** (0.010)	−0.043*** (0.010)	−0.039*** (0.014)	−0.032** (0.014)	−0.039*** (0.011)	−0.034*** (0.012)
Disaster hits firm $(t-4)$	−0.031*** (0.011)	−0.029*** (0.011)	−0.031*** (0.011)	−0.029*** (0.011)	−0.031*** (0.011)	−0.029*** (0.011)
Number of suppliers	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year-quarter FE	Yes	Yes	Yes	Yes	Yes	Yes
Size, age, ROA × year-quarter FE	No	Yes	No	Yes	No	Yes
Observations	80,574	80,574	80,574	80,574	80,574	80,574
R^2	0.234	0.262	0.234	0.261	0.234	0.262

Source: Barrot and Sauvagnat (2016).

- Downstream effects are significant only when specific suppliers are hit
    - effects are also larger than baseline effects
- Suggests that input specificity is a key driver of strength of shock propagation

Horizontal Propagation



- Finally, estimate *horizontal* propagation of disaster shock:
  - to other suppliers of affected firms' customers
- In theory, direction of effect should depend on...
  - whether suppliers' inputs are substitutes or complements
- Main regression specification at the firm (i) and quarter (t) level:

$$\Delta R_{i,t} = \alpha_0 + \alpha_1 \cdot DC_{i,t-4} + \alpha_2 \cdot DCS_{i,t-4} + \alpha_3 \cdot DF_{i,t-4} + \eta_i + \pi_t + \epsilon_{i,t}$$

  - $DC_{i,t-4}$: equals 1 if at least one customer hit in previous year
  - $DCS_{i,t-4}$: equals 1 if at least one other supplier of firm's customer(s) hit in previous year
- Coefficient of interest: α_2

Horizontal Propagation

HORIZONTAL PROPAGATION—RELATED SUPPLIERS' SALES GROWTH

	Sales Growth $(t-4,t)$			
Supplier Specificity		Diff.	R&D	Patent
Disaster hits firm $(t-4,t-1)$	−0.040***	−0.040***	−0.041***	−0.040***
	(0.013)	(0.013)	(0.013)	(0.013)
Disaster hits one customer $(t-4,t-1)$	0.002	0.001	0.001	0.002
	(0.021)	(0.021)	(0.021)	(0.021)
Disaster hits one customer's supplier $(t-4,t-1)$	−0.038***			
	(0.010)			
Disaster hits one customer's specific supplier $(t-4,t-1)$		−0.047***	−0.048***	−0.040***
		(0.013)	(0.014)	(0.013)
Disaster hits one customer's non-specific supplier $(t-4,t-1)$		−0.011	−0.013	−0.015
		(0.013)	(0.013)	(0.013)
Number of customers' Suppliers	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Year-quarter FE	Yes	Yes	Yes	Yes
Size, age, ROA × year-quarter FE	Yes	Yes	Yes	Yes
Observations	139,976	139,976	139,976	139,976
R^2	0.192	0.192	0.192	0.192

Source: Barrot and Sauvagnat (2016).

- Shock to other suppliers leads to 3.8 percentage point fall in sales growth
  - again, effect is significant only when specific suppliers are hit
- Upstream propagation effects appear to be minimal

Robustness

- The paper is also very careful in testing robustness of the results
- Buyers might be located near affected suppliers
  - control for direct effects of disasters
  - exclude relationships where both HQs are within 300 miles of each other
  - control for customer plants being located near affected suppliers
- Buyers' customer bases might be located near affected suppliers
  - control for any past supplier hit by disaster
- Large natural disasters might affect both buyers and sellers simultaneously
  - restrict to disasters affecting below median number of firms
  - look at effects on exporters vs. non-exporters
- Shocks might be sector- rather than firm-specific if industries are clustered
  - control for whether large share of industry sales are hit

Outline

- Heise (2018), "Firm-to-firm Relationships and the Pass-Through of Shocks"
- Study *pass-through* of shocks from foreign suppliers to US importers
  - e.g. suppose supplier cost increases by 1%
  - what is the resulting change in price charged to the US importer?
- Identification strategy: **exchange rate shocks**
- Key observation: pass-through is increasing in the **age of the relationship**
- Not immediately obvious:
  - e.g. older relationships are more likely to use contracts with fixed prices
- To rationalize this, develop a model of buyer-seller relationship dynamics, with:
  - *relationship-specific capital* that evolves over time
  - *limited commitment* where parties cannot guarantee to stay in relationship
- Simulate model to study effects of break-up of short-term relationships in 2008-2009 great recession on aggregate pass-through of shocks

Data

- Main data source: Longitudinal Firm Trade Transactions Database (LFTTD)
  - entire universe of import transactions by US Firms, 1992-2011
  - based on customs declarations forms, managed by US Census Bureau
- For each transaction, the database records:
  - HS-10 code of product traded
  - value and quantity shipped
  - date of shipment
  - ID code of US importer and foreign exporter
  - flag for related-party trade
- With both value and quantity data, can compute *prices* as unit values
  - i.e. shipment value divided by quantity shipped
- Focus on arms-length relationships only and exclude related-party transactions
- US importer ID can also be linked to Longitudinal Business Database (LBD)
  - annual information at establishment-level about payroll, number of employees, NAICS code of establishment, etc.

Summary Statistics

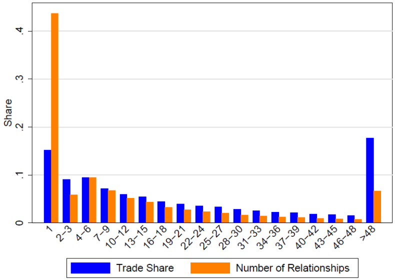
		All relationships	>12 months[1]
		(1)	(2)
(1)	Arms' length trade	38%	32%
(2)	Arms' length trade (always unrelated)	27%	21%
	Arms' length trade		
(3)	Exporters per importer-HS10, per year	2.7	2.2
(4)	Importers per exporter-HS10, per year	1.2	1.2
(5)	HS per importer-exporter, per year	1.9	3.0
(6)	Average gap time between transactions (months)	0.6	0.6
(7)	Average maximum gap time (months)	10.0	—
(8)	Average relationship length (months)	5.7	30.0
(9)	... in Manufacturing	5.9	30.6
(10)	... in Wholesale / Transportation	5.7	30.6
(11)	... in Retail	5.9	28.7

[1] Statistics consider only those relationships that last in total for more than 12 months.

Source: Heise (2018).

Relationship Age and Trade Volumes

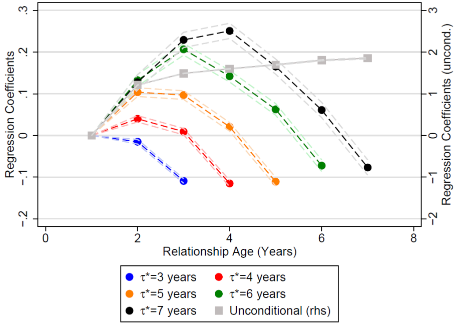
(a) International Trade Relationships (in Months)



Source: Heise (2018).

- 44% of all arms-length relationships in average quarter are less than 1 month old
  - but account for only 15% of value traded
- Relationships older than 12 months account for 53% of value traded
- Relationships older than 4 years account for 18% of value traded

Relationship Age and Trade Volumes



Source: Heise (2018).

- Variation of value traded in a relationship with relationship age is hump-shaped
  - increases in first few years of relationship, then declines toward end
- Longer-lasting relationships (higher τ^*) trade more, both initially and in total

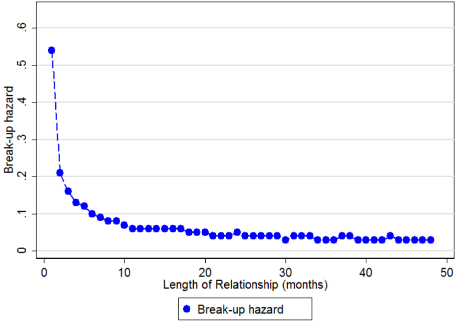
Relationship Age and Trade Prices

	$\ln(\tilde{p}_{mxch})$	$\ln(\tilde{q}_{mxch})$	$\ln(\tilde{p}_{mxch})$	$\ln(\tilde{p}_{mxch})$
	(1)	(2)	(3)	(4)
d_6	$-.0036$	$.0131$	$-.0006$	$-.0018$
	$(.0002)$	$(.0005)$	$(.0002)$	$(.0002)$
d_{11}	$-.0050$	$.0243$	$.0003$	$-.0019$
	$(.0003)$	$(.0006)$	$(.0003)$	$(.0003)$
d_{16}	$-.0069$	$.0332$	$.0002$	$-.0027$
	$(.0004)$	$(.0007)$	$(.0003)$	$(.0004)$
d_{21}	$-.0096$	$.0434$	$-.0006$	$-.0043$
	$(.0003)$	$(.0006)$	$(.0003)$	$(.0004)$
d_{41}	$-.0131$	$.0554$	$-.0019$	$-.0066$
	$(.0004)$	$(.0008)$	$(.0004)$	$(.0005)$
Length_{mx}	$-.0003$	$-.0002$	$-.0007$	$-.0006$
	$(.0000)$	$(.0000)$	$(.0000)$	$(.0000)$
Cont_{mx}	$-.0193$	$.0035$	$-.0314$	$-.0264$
	$(.0003)$	$(.0006)$	$(.0003)$	$(.0004)$
$\ln(q_{mxch})$			$-.2160$	$-.1272$
			$(.0000)$	$(.0000)$
Instruments	No	No	No	Yes
Fixed effects	mxh	mxh	mxh	mxh
Observations	$67,868,000$	$67,868,000$	$67,868,000$	$67,868,000$

Source: Heise (2018).

- Price of relationship ($\tilde{p}$) falls with number of transactions (d)
- This is true even after instrumenting for increase in demand as relationship ages

Relationship Age and Hazard Rates



Source: Heise (2018).

- Older relationships are more likely to survive

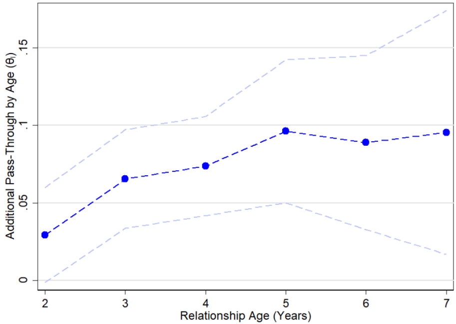
Pass-through of Exchange Rate Shocks

- How does shock propagation vary with relationship age?
- Use exchange rate shocks as source of exogenous variation
  - obtained from OECD Monetary and Financial Statistics database
- Main regression specification at importer (m), exporter (x), exporter country (c), product (h), and quarter (t) level:

$$\Delta p_{mxcht} = \beta_1 \cdot \Delta e_{ct} + \sum_{l=2}^{7} \beta_l \cdot d^l_{mxt} + \sum_{l=2}^{7} \theta_l \cdot d^l_{mxt} \Delta \log e_{ct} + \omega_t + \gamma_{mxh} + \epsilon_{mxcht}$$

  - Δp_{mxcht}: change in log price since last transaction
  - Δe_{ct}: change in log exchange rate since last transaction
  - d^l_{mxt}: equals 1 if relationship is of age l (in years)
  - ω_t: quarter fixed effect
  - γ_{mxh}: importer-exporter-product fixed effect
- Coefficients of interest: β_1 and θ_l

Pass-through of Exchange Rate Shocks

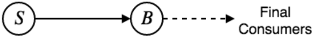


Source: Heise (2018).

- Baseline pass-through, $\beta_1 = .1534$
- Pass-through is increasing with age:
  - e.g. in year 5, $\beta_1 + \theta_5 = .25$

Benchmark Model

- To study buyer-seller relationship dynamics and shock propagation...
  - develop a simple model with one seller (S) and one buyer (B)



- Demand for final output sold by firm B:

$$y_t = \Lambda \left(p_t^f\right)^{-\theta}$$

  - p_t^f : price charged by firm B
  - Λ: demand shifter
  - θ: price elasticity of final demand (> 1)

Benchmark Model

- Production function for firm B:

$$y_t = Aq_t$$

  - q_t: quantity of inputs purchased from supplier S
  - A: productivity of firm B

- Production function for firm S:

$$q_t = a_t^{\gamma} x_t$$

  - a_t: relationship-specific capital stock
  - γ: returns to relationship-specific capital (< 1)
  - x_t: primary input

- Seller's input x_t is purchased at exogenous time-varying cost w_t
  - follows some stochastic process on $[\underline{w}, \infty)$ with $\underline{w} > 0$
- Key object of interest: price charged by seller to buyer, p_t

Buyer's Problem

- Given supplier price p at each date, profit maximization problem for firm B is:

$$\max_{p^f} \left\{ p^f y - pq \right\}$$

$$\text{s.t. } y = \Lambda \left(p^f \right)^{-\theta}$$

$$y = Aq$$

- Optimal final price is a constant markup over marginal cost:

$$p^f (p) = \frac{\theta}{\theta - 1} \left(\frac{p}{A} \right)$$

- Optimal quantity purchased is declining with the supplier's price:

$$q (p) = \left(\frac{\theta}{\theta - 1} \right)^{-\theta} \Lambda A^{\theta - 1} p^{-\theta}$$

- Static profit for firm B is declining with the supplier's price:

$$\pi^B (p) = \frac{1}{\theta} \left(\frac{\theta}{\theta - 1} \right)^{1-\theta} \Lambda \left(\frac{A}{p} \right)^{\theta - 1}$$

Evolution of Relationship Capital

- In initial period, a_0 is drawn from some exogenous distribution on $[0, \infty)$
- In each subsequent period, relationship capital depreciates at rate δ
  - e.g. wear and tear of customized machines
  - e.g. turnover of employees that breaks personal bonds
- Relationship capital also increases by amount proportional to quantity traded q_t
  - e.g. learning by doing
  - e.g. higher incentive to invest in customized equipment

Evolution of Relationship Capital

- Law of motion for relationship capital:

$$a_{t+1} = (1-\delta)\,a_t + \rho q\,(p_t) + \epsilon_{t+1}$$

  - ρ: constant measuring effect of trade on capital accumulation
  - ϵ_t : random shock $\mathcal{N}\left(0, \sigma_\epsilon^2\right)$
- Key mechanism:
  - by varying the price p_t of the relationship...
  - the seller also affects the future path of relationship-specific capital

Seller's Problem

- Assume that the seller sets the price p_t
  - i.e. abstract from bargaining over prices between buyer and seller
- Static profit for firm S given p and $\{a, w\}$ at each date

$$\pi^S(p|a,w) = (p - w/a^\gamma)\, q(p)$$

- If seller were to choose p to maximize static profits alone:
  - since $q(p)$ has constant elasticity $-\theta$...
  - optimal price is also constant markup over marginal cost

$$\tilde{p} = \frac{\theta}{\theta - 1}\left(\frac{w}{a^\gamma}\right)$$

- However, seller also cares about the effect of p on evolution of a
  - hence $\tilde{p}$ is in general *not* the optimal price

Seller's Problem

- Note that the **state** of the relationship at date t is $\{a_t, w_t\}$
- Let $V^S(a, w)$ denote the value of the relationship to the seller when:
  - relationship capital stock is a
  - cost of primary input is w
- Seller's problem can then be written as:

$$V^S(a_0, w_0) = \max_{\{p_t\}_{t=0}^{\infty}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \pi^S(p_t | a_t, w_t)\right]$$
$$\text{s.t. } a_{t+1} = (1-\delta) a_t + \rho q(p_t) + \epsilon_{t+1}$$

  - price p_t is contingent on the state $\{a_t, w_t\}$ at each date
  - expectation $\mathbb{E}$ is over exogenous shocks to $\{a_t, w_t\}$
  - β: discount factor
- Similar to a standard capital accumulation problem

Seller's Problem

- *Recursive* formulation of the seller's problem:

$$V^S(a,w) = \max_p \left\{ \pi^S(p|a,w) + \beta\mathbb{E}\left[V^S(a',w')\right]\right\}$$
$$\text{s.t. } a' = (1-\delta)a + \rho q(p) + \epsilon'$$

- First-order condition implies that optimal price satisfies:

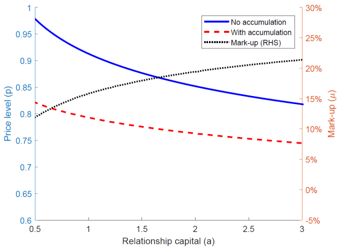
$$p = \frac{\theta}{\theta-1}\left[w/a^\gamma - \beta\rho\mathbb{E}\left[V_a^S(a',w')\right]\right]$$

where $V_a^S(a,w) \equiv \frac{\partial V^S(a,w)}{\partial a}$ is the marginal value of relationship capital

- Key implication: optimal price is *lower* than price that maximizes static profits $\tilde{p}$
  - selling firm trades off higher profits today...
  - against accumulating more relationship capital in the future

Equilibrium Prices and Markups

(a) Price and Mark-Up as Function of Capital

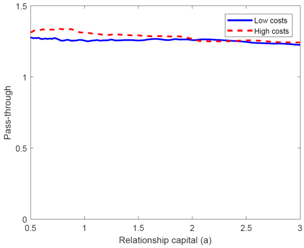


Source: Heise (2018).

- Can show that price p is *decreasing* in relationship capital
  - as a increases, relationship becomes more productive, so price falls
- But markup $\frac{p}{w/a^{\gamma}}$ is *increasing* in relationship capital
  - as a increases, marginal value of relationship capital falls (because $\gamma < 1$)
  - so incentive to lower price to increase a falls, and $\mu \to \frac{\theta}{\theta - 1}$

Equilibrium Pass-through

(b) Pass-Through as Function of Capital

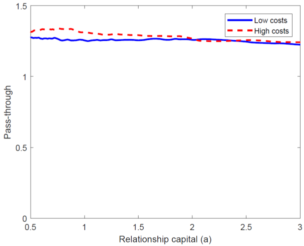


Source: Heise (2018).

- What happens when primary input cost w increases?
  - since supplier cost is higher, intermediate input price p also increases
  - furthermore, higher w lowers marginal value of relationship capital
  - hence p increases more than one-for-one with w (pass-through $> 100\%$)

Equilibrium Pass-through

(b) Pass-Through as Function of Capital



Source: Heise (2018).

- However, pass-through actually *declines* slightly with relationship capital
  - because higher w lowers marginal value of capital $\frac{\partial V}{\partial a}$ more when a is low

Limited Commitment

- To rationalize why pass-through *increases* with relationship age...
  - now introduce **limited commitment** in the relationship
- Key idea: buyer and seller cannot commit to remaining in relationship forever
  - if value of relationship is not "good enough"...
  - either buyer or seller will terminate the relationship
- Now suppose that if the buyer decides to leave the relationship...
  - buyer receives outside option $U^B(w)$
- Similarly, if seller decides to leave the relationship
  - seller receives outside option $U^S(w)$
- Note that U^B and U^S are independent of a...
  - because capital is fully specific to the relationship

Buyer's Value

- Value of the relationship with state (a, w) to the buyer given supplier price p:

$$V^B(a, w|p) = \pi^B(p) + \beta\mathbb{E}\left[I' V^B(a', w'|p(a', w')) + (1 - I') U^B(w')\right]$$

  - $I(a, w) = 1$ if relationship is continued in state $\{a, w\}$
  - $p(a, w)$ is optimal supplier price in state $\{a, w\}$

Seller's Problem

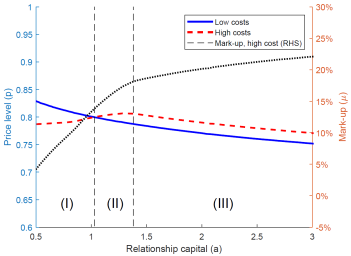
- Seller's problem:

$$V^S(a,w) = \max_p \left\{ \pi^S(p|a,w) + \beta\mathbb{E}\left[\max\left\{V^S(a',w'), U^S(w')\right\}\right]\right\}$$
$$\text{s..t } a' = (1-\delta)a + \rho q(p) + \epsilon'$$
$$V^B(a,w|p) \geq U^B(w)$$

- Note that with limited commitment...
  - choice of p is now also subject to the buyer's participation constraint
- If a is low or w is high, the buyer's constraint might be binding
  - seller then has to charge a lower price p...
  - so as to incentivize the buyer to stay in the relationship
- Relationship is continued as long as $V^S(a,w) \geq U^S(w)$
  - i.e. as long as $a \geq \underline{a}(w)$ for some cutoff value $\underline{a}$ that is increasing in w

Equilibrium Prices and Markups

(a) Price and Mark-Up as Function of Capital

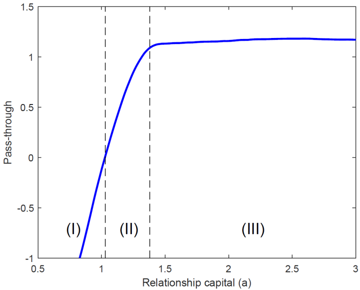


Source: Heise (2018).

- When w is low and/or a is high, the buyer is unconstrained:
  - hence price p is decreasing in relationship capital as before
- However, when w is high and buyer's constraint binds:
  - higher relationship capital relaxes the constraint
  - hence seller is able to charge higher prices, and p is *increasing* with a

Equilibrium Pass-through

(b) Pass-Through as Function of Capital

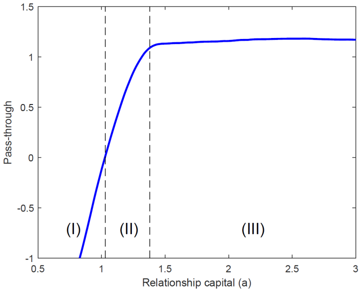


Source: Heise (2018).

- What happens when primary input cost w increases?
- In region (I), pass-through is negative:
  - because seller has to lower p to keep buyer in relationship

Equilibrium Pass-through

(b) Pass-Through as Function of Capital

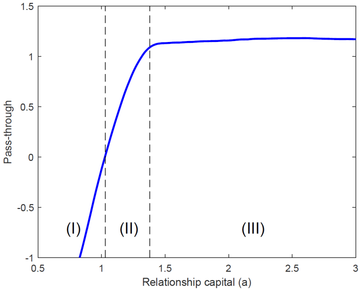


Source: Heise (2018).

- What happens when primary input cost w increases?
- In region (II), pass-through is positive and increasing with a:
  - because higher a relaxes the constraint and allows seller to charge higher p

Equilibrium Pass-through

(b) Pass-Through as Function of Capital



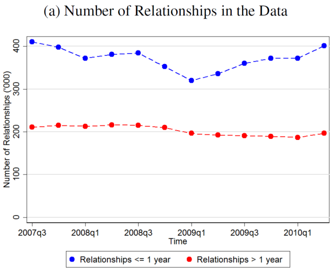
Source: Heise (2018).

- What happens when primary input cost w increases?
- In region (III), pass-through is the same as in unconstrained case

Quantitative Analysis

- Can embed this simple model in a more complex trade model, with:
  - multiple buyers and sellers
  - search and matching frictions between buyers and sellers
- Model estimation:
  - parameters structurally estimated using LFTTD data
  - primary input cost process estimated using exchange rate data
- Application: effect of relationship age distribution on aggregate pass-through

The Great Recession

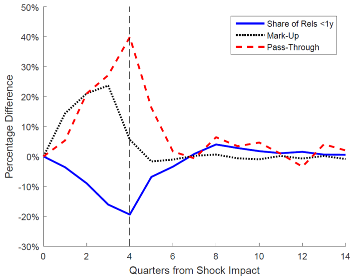


Source: Heise (2018).

- During great recession of 2008-09:
  - number of relationships of age less than one year fell by 20
  - number of older relationships fell by only 2%

Model Counterfactual

(b) Impact of Shock to Matching Probability



Source: Heise (2018).

- Model is simulated to match this change in age distribution
  - mark-ups rise by 23%, pass-through rises by 40%
- Consistent with previous empirical findings that pass-through is *countercyclical*
  - i.e. shock propagation is stronger in recessions

Summary and Related Papers

- Hulten's theorem:
  - under general assumptions in a competitive economy...
  - network is irrelevant for first-order aggregate effects of sectoral shocks
- Beyond Hulten's theorem:
  - production network structure matters for higher-order effects
  - except for knife-edge case with Cobb-Douglas technology
- Networks and aggregate fluctuations
  - production network structure matters for aggregate volatility
- Shock propagation
  - challenging to identify network propagation effects
  - buyer-seller linkages seem to matter for propagation
  - input specificity and relationship age matter for strength of propagation
- Related papers:
  - Grassi (2018) - sectoral production networks with oligopoly
  - Baqaee (2018, Ecma) - propagation of exit/entry shocks across sectors
  - Carvalho et al (2016), Boehm et al (2017) - supply disruptions due to 2011 Tōhoku earthquake
- Next week: buyer-seller relationships (bipartite networks)

Homogeneous Functions

- A function $f\left(\{x_i\}_{i=1}^N\right)$ that is homogeneous of degree d satisfies:

$$f\left(\{kx_i\}_{i=1}^N\right) = k^d f\left(\{x_i\}_{i=1}^N\right)$$

  for any real number k

- In other words, multiplying all function inputs by a constant k...
  - results in k^d times the function output

back

Euler's Homogeneous Function Theorem

- Let $f\left(\{x_i\}_{i=1}^{N}\right)$ be a homogeneous function of degree d
- Then Euler's Theorem states that:

$$nf = \sum_{i=1}^{N} x_i \frac{\partial f_i}{\partial x_i}$$

back

The Envelope Theorem

- Consider a general constrained optimization problem:

$$\max_x f(x, \alpha)$$
$$\text{s.t. } g_i(x, \alpha) \geq 0, \ \ \forall i = 1, \cdots, N$$

  - f: objective function
  - x: choice variable(s)
  - α: parameter(s) of the model
  - $\{g_i\}_{i=1}^N$: constraints

- Given the parameters α:
  - let $x^*(\alpha)$ denote the optimal solution for x
  - let $\lambda^*(\alpha)$ denote the corresponding vector of Lagrange multipliers

The Envelope Theorem

- The Lagrangian for the problem at the optimal solution is:

$$\mathscr{L}(\alpha) = f(x^*(\alpha), \alpha) + \lambda^*(\alpha) \cdot g(x^*(\alpha), \alpha)$$

- Then the envelope theorem characterizes the derivative of the Lagrangian with respect to the parameters:

$$\frac{\partial \mathscr{L}}{\partial \alpha} = \frac{\partial f(x^*(\alpha), \alpha)}{\partial \alpha} + \lambda^*(\alpha) \cdot \frac{\partial g(x^*(\alpha), \alpha)}{\partial \alpha}$$

- In other words, we can ignore terms involving $\frac{\partial x^*(\alpha)}{\partial \alpha}$ and $\frac{\partial \lambda^*(\alpha)}{\partial \alpha}$

back

The Gershgorin Circle Theorem

- Let A be an $N \times N$ matrix with ij-element a_{ij}
- Let $R_i \equiv \sum_{j \neq i} \left| a_{ij} \right|$ denote the i^{th} row sum excluding a_{ii}
- Let $D_i \equiv [a_{ii} - R_i, a_{ii} + R_i]$ denote the i^{th} *Gershgorin disc*
- Then every eigenvalue λ_i of A satisfies:

$$\lambda_i \in D_j$$

for at least one $j \in \{1, \cdots, N\}$

back

The Lindeberg-Lévy Central Limit Theorem

- Let $\{X_1, \cdots, X_N\}$ be a sequence of *iid* random variables
  - with finite mean μ and finite variance σ^2
- Let S_N denote the sample average:

$$S_N \equiv \frac{1}{N} \sum_{i=1}^{N} X_i$$

- Then as $N \to \infty$:

$$\sqrt{N} \left(S_N - \mu \right) \xrightarrow{d} \mathcal{N} \left(0, \sigma^2 \right)$$

i.e. the random variable $\sqrt{N} \left(S_N - \mu \right)$ converges in distribution to a normal random variable with mean 0 and variance σ^2

back