ECO 2302: Networks in Trade and Macroeconomics

Problem Set 1

Due date: 5:00 PM, 5 February 2021

Part I: simulating and analyzing random networks

In this part of the assignment, we will practice simulating simple undirected Poisson random networks and analyzing some of their basic properties. To do so, we will use the Matlab function **network.m**, which is provided for you on the course Quercus website. The function takes two arguments:

- N: the number of nodes in the network
- p: the edge formation probability

The output of the function is a structure with the following fields:

- M: the $N \times N$ adjacency matrix of the network, where the ij-element is equal to 1 if there is an edge between nodes i and j
- D: an $N \times N$ matrix with ij-element equal to the distance between nodes i and j
- C: a $K \times 1$ cell array, where K is the number of components of the network and the k-element of C is a vector of nodes in the k^{th} component

Now, using the function **network.m**, study the following for $N = 1000$ nodes and different values of the edge formation probability, $p \in [10^{-4}, 10^{-1}]$.[1]

1. Network density.

   Plot the sparsity pattern of the adjacency matrix using the **spy** command. How does the density of the network vary with p?

[1] Feel free to play around with different values of p.

2. Degree distribution.

   Compute the degree of each node in the network. Plot the degree distribution of the network as well as the corresponding Poisson approximation. How does the accuracy of the Poisson approximation change as p increases?

3. Connectedness.

   Compute the fraction of nodes belonging to the giant component of each network and plot this against p. You should find that there is a threshold value of p^* above which all nodes in the network become connected. This is known as a *phase transition*, which is when some feature of a network emerges with probability 1 as $N \to \infty$ for values of $p \geq p^*(N)$ for some threshold function p^*.[2]

4. Distance.

   Compute the average distance between all pairs of nodes in the giant component of the network and plot this against p. How does the average distance of the network vary with p?

5. Eigenvector centrality.

   Compute the eigenvector centralities of all nodes in the giant component of the network. Plot the distribution of eigenvector centralities relative to the mean of the distribution. How does the distribution of centralities change as p increases?

[2]In this case, the threshold function for the Poisson network to be fully connected is known analytically, and is given by $p^*(N) = \log(N)/N$.

Part II: Hulten's Theorem and Aggregate Fluctuations

Consider an economy with two sectors. A representative household supplies 1 unit of labor inelastically and has Cobb-Douglas preferences over consumption from each sector. We take the wage as numeraire. The household's utility maximization problem is:

$$Y = \max_{C_1, C_2} \left(\frac{C_1}{\alpha_1}\right)^{\alpha_1} \left(\frac{C_2}{\alpha_2}\right)^{\alpha_2}$$
$$\text{s.t. } P_1 C_1 + P_2 C_2 = 1$$

- C_i is consumption of sector i output
- P_i is the price of sector i output
- α_i is the final consumption share of sector i

Production in each sector i combines labor with intermediate inputs using a Cobb-Douglas technology. The market structure is perfect competition, and the profit maximization problem for producers in sector i is:

$$\max_{L_i, X_{ij}} \left\{ P_i X_i - L_i - \sum_{j=1}^{N} P_j X_{ij} \right\}$$
$$\text{s.t. } X_i = T_i \left(\frac{L_i}{\lambda}\right)^{\lambda} \left(\frac{M_i}{1-\lambda}\right)^{1-\lambda}$$
$$M_i = \left(\frac{X_{i1}}{\beta_{i1}}\right)^{\beta_{i1}} \left(\frac{X_{i2}}{\beta_{i2}}\right)^{\beta_{i2}}$$

- L_i is labor hired in sector i
- X_{ij} is the quantity of sector j output used in sector i production
- M_i is the composite intermediate input quantity used in sector i
- T_i is TFP in sector i
- λ is the labor share
- β_{ij} is the share of sector j output in sector i's intermediate purchases

In what follows, assume that the sectors are symmetric in all aspects except possibly for sector TFP:

$$\alpha_1 = \alpha_2 = \frac{1}{2}$$
$$\beta_{11} = \beta_{22} = \beta$$
$$\beta_{12} = \beta_{21} = 1 - \beta$$

Note that β denotes each sector's share in its own intermediate purchases.

1. Solve the household's utility maximization problem. What is log final consumption $y \equiv \log Y$ given log sector prices $p_i \equiv \log P_i$?

2. Solve the cost minimization problem in sector i. What is the log price of sector i output given log TFP in each sector, $\epsilon_i \equiv \log T_i$?

3. What is the elasticity of final consumption Y with respect to sector i TFP T_i? How does this depend on β and λ?

4. Show that Hulten's theorem applies in this model, namely that $\frac{\partial y}{\partial \epsilon_i} = R_i$ where R_i is total sales of sector i.