ECO 2302 - Problem Set 3

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1 Brute force solution

1. In a network with N firms, a firm i can choose whether to source or not from other $N-1$ firms. This gives 2^{N-1} possible unique values of vectors $\{e_{ij}\}_{j=1}^{N}$.
There are N total firms. As a result, the total number of unique networks is $2^{(N-1)N}$. (Another the way to think is that the total elements of a $N \times N$ matrix is N^2, but N diagonal elements are set to 0. This leaves the choice between 0 and 1 for $N^2 - N$ remaining elements, so the total number of choices is $2^{(N-1)N}$).

2. I use Julia instead in this problem set. Please see the code for this part. I find a single equilibrium which is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

3. With $f = 0.15$ and 0.25, there also exist a single equilibrium.

For $f = 0.15$, the equilibrium matrix is $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

For $f = 0.25$, the equilibrium matrix is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Increasing f will make sourcing more expensive and consequently decreases the number of connections in network.

2 Multiple Equilibria

4. When changing $A_i = 1$, there are 2 equilibria which are $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

5. There are two stable networks in this part because firms are symmetric and the fixed cost is not high compared to gain from sourcing. As a result, if firm 1 sources, its productivity will be higher and this encourages firm 2 to source to benefit from more productive supplier. Firm 2 will choose to source because the fixed cost is lower than benefit from sourcing. On the other hand, as two firms are similar, if none sources, they have the same productivity and no incentive to source.

3 The Curse of Dimensionality

6. Running the code with $N = 2$ takes 0.000316 seconds.

7. Running the code with $N = 3$ takes 0.005390 seconds.
Running the code with $N = 4$ takes 0.690682 seconds.
Running the code with $N = 5$ takes 65.184888 seconds.
The time scales really fast because the number of possible matrices grow at exponential rate.