ECO 2302 - Networks in Trade and Macroeconomics

Lecture 4 - Firms and Production Networks

Motivation

- Last week, we studied buyer-seller relationship formation in bipartite networks
- Today, we will study firm-level trade in more general production networks
  - specifically, want to allow firms to be both buyers and sellers
- First, we will look at importance of network heterogeneity for firm size
  - Bernard, Dhyne, Magerman, Manova, and Moxnes (2019), "The Origins of Firm Heterogeneity: A Production Network Approach"
  - study high-quality production network data for Belgium
  - develop (exogenous) model of firm-level production network
  - use model to decompose firm size into firm/network components
- Then we will look at two general classes of endogenous network models
  - costly relationship models: Lim (2019), "Endogenous Production Networks and the Business Cycle"
  - extreme value models: Oberfield (2019), "A Theory of Input-Output Architecture"

Buyer-Seller Data

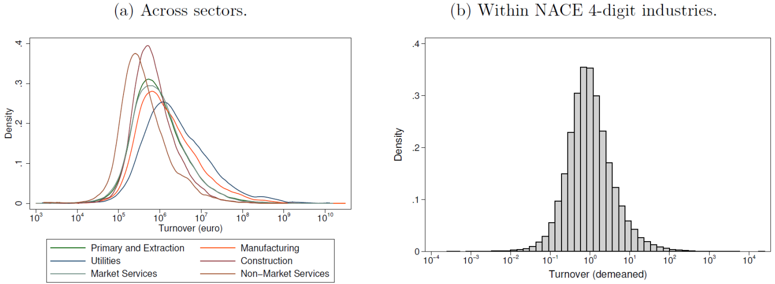
- Bernard, Dhyne, Magerman, Manova, and Moxnes (2019), "The Origins of Firm Heterogeneity: A Production Network Approach"
- Study buyer-seller data from NBB B2B Transactions Dataset (2002-2014)
  - administered by the National Bank of Belgium (NBB)
- Records extensive/intensive margins of domestic buyer-supplier relationships
  - reports sales between any two VAT-liable enterprises
  - covers all economic activities
  - all relationships with annual sales of at least 250 euros must be reported
  - penalties for late and erroneous reporting ensure high data quality
- In 2014, dataset contains information for:
  - $859,733$ Belgian firms
  - 17.3 million relationships

Firm-level Data

- Firm-level data obtained from firm annual accounts (2002-2014):
  - maintained by Central Balance Sheet Office (CBSO) at the NBB
  - sales, input purchases, employment, labor costs, main industry, etc.
- Firms' sales to final demand:
  - difference between total sales and sum of B2B sales
  - includes sales to domestic final consumers and exports
- Firms' purchases from outside observed production network:
  - difference between total input costs and sum of B2B purchases
  - includes imports

Stylized Facts

Fact 1: Firm sales, degree, and firm-to-firm sales exhibit high dispersion and skewness.



Source: Bernard et al (2018).

- Largest firms are several orders of magnitude larger than industry mean

Stylized Facts

Fact 1: Firm sales, degree, and firm-to-firm sales exhibit high dispersion and skewness.

Sector	NACE	N	Mean	St Dev	10th	50th	90th	95th	99th
Primary & Extraction	01-09	3,061	12.0	432.6	0.2	0.8	4.8	9.5	52.0
Manufacturing	10-33	18,077	14.4	250.8	0.2	1.1	13.8	34.6	201.8
Utilities	35-39	897	39.2	442.9	0.3	1.9	25.7	68.6	495.6
Construction	41-43	20,201	2.3	13.4	0.2	0.6	3.6	6.9	25.9
Market Services	45-82	65,175	5.5	79.9	0.2	0.8	6.3	13.4	63.9
Non-Market Services	84-99	2,328	2.2	26.3	0.1	0.3	2.6	5.5	24.9
All		109,739	6.8	145.1	0.2	0.8	6.6	14.3	78.4

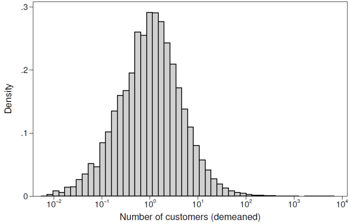
Note: Summary statistics for the matched CBSO-B2B data. 10th, 50th, etc. refers to values at the 10th, 50th, etc. percentile of the distribution.

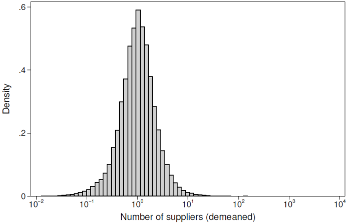
Source: Bernard et al (2018).

- 90/10 ratio for sales = 34
- Top 10% of firms account for 84% of total sales

Stylized Facts

Fact 1: Firm sales, degree, and firm-to-firm sales exhibit high dispersion and skewness.





	N	Mean	St Dev	10th	25th	50th	75th	90th	95th	99th
# of buyers	590,271	29.3	394.0	1	1	4	11	42	98	400
# of suppliers	840,607	20.6	49.5	1	3	9	22	46	71	177

Source: Bernard et al (2018).

- Out-degree distribution more dispersed than in-degree distribution
  - similar to patterns in Acemoglu et al (2012) for the US sector-level production network

Stylized Facts

Fact 1: Firm sales, degree, and firm-to-firm sales exhibit high dispersion and skewness.

Sector	N	Mean	St Dev	10th	25th	50th	75th	90th	95th	99th
Primary & Extraction	613,868	39,898	5,409,863	419	840	2,490	9,150	33,789	81,626	387,573
Manufacturing	2,755,457	44,303	2,007,421	359	613	1,661	6,185	25,436	63,467	411,379
Utilities	526,932	59,953	7,410,682	366	615	1,388	3,744	11,560	28,382	281,181
Construction	1,529,078	24,500	386,201	375	676	1,926	7,000	27,186	64,585	339,523
Market Services	11,562,445	24,373	2,886,213	341	546	1,266	4,060	15,579	37,960	224,363
Non-Market Services	315,529	8,044	319,407	315	472	998	2739	8,395	18,908	92,879
All	17,304,408	28,893	2,988,881	348	571	1,392	4,669	18,280	44,770	269,153

Note: Summary statistics for the B2B data. 10th, 25th, etc. refers to values at the 10th, 25th, etc. percentile of the distribution. Industry refers to the main industry of activity of the seller.

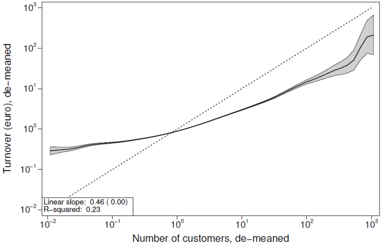
Source: Bernard et al (2018).

- Top 10% of relationships account for 92% of transaction values

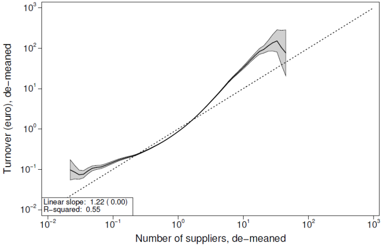
Stylized Facts

Fact 2: Larger firms have more buyers and suppliers.

(a) Firm sales and number of buyers.



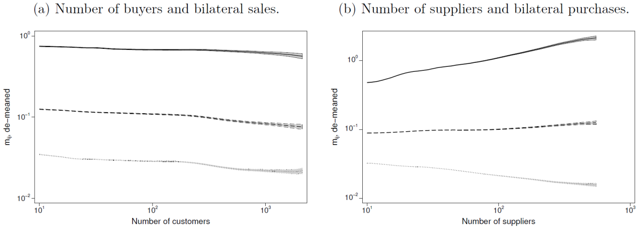
(b) Firm sales and number of suppliers.



Source: Bernard et al (2018).

Stylized Facts

Fact 3: The distribution of sales across buyers does not vary with no. of buyers.
The distribution of purchases across suppliers widens with the no. of suppliers.



Source: Bernard et al (2018).

- Suggests that large firms are large primarily because they sell to more customers
- Firms with more suppliers buy more from larger suppliers and buy less from smaller suppliers

Stylized Facts

- Stylized facts signal an important role for:
  - downstream input demand relative to final output demand
  - number of buyers and suppliers of a firm
  - seller and buyer firm characteristics
  - seller-buyer match characteristics
- Now develop theoretical framework that accommodates these features:
  - with two-sided firm heterogeneity in an input-output production network
- Model will enable decomposition of firm size into factors related to:
  - firms' own characteristics
  - upstream network characteristics
  - downstream network characteristics
  - final demand characteristics

Explaining Buyer-Seller Transactions

- Step 1: Estimate two-way fixed effects regression of firm-to-firm sales

$$\underbrace{\log \bar{m}_{ij}}_{\substack{\text{log sales} \\ \text{from i to j}}} = \underbrace{\log \bar{\psi}_i}_{\substack{\text{seller} \\ \text{effect}}} + \underbrace{\log \bar{\theta}_j}_{\substack{\text{buyer} \\ \text{effect}}} + \underbrace{\log \omega_{ij}}_{\substack{\text{match} \\ \text{effect}}}$$

- Note: all variables are demeaned by respective grand means
  - so that all fixed effects have zero mean
- Interpretation of fixed effects
  - seller effect ψ_i: high if i has high average market share amongst customers
  - buyer effect θ_j : high if j has high average share of suppliers' output
  - match effect ω_{ij}: reflects match-specific characteristics
- In structural model, $\{\psi_i, \theta_j, \omega_{ij}\}$ will have structural interpretation

Explaining Buyer-Seller Transactions

- Note that estimation requires firms to have multiple connections
  - e.g. identifying seller effect requires a firm to have at least two customers
- Hence firms with ≤ 1 out- or in-degree are dropped
  - but dropping these firms may result in more firms with ≤ 1 degree
- Iterative dropping of firms continues until all remaining firms have > 1 degree
  - procedure is known as "avalanching", result is a "mobility group"

Full Sample			Estimation Sample			
# Links	# Sellers	# Buyers	Links	Value	Sellers	Buyers
17,304,408	590,271	840,607	99%	95%	74%	88%

Note: Summary statistics for firm-to-firm transactions in the raw B2B data and in the estimation sample.

Source: Bernard et al (2018).

Explaining Buyer-Seller Transactions

	N	$\frac{var(\ln\psi_i)}{var(\ln\psi_i+\ln\theta_j)}$	$\frac{var(\ln\theta_j)}{var(\ln\psi_i+\ln\theta_j)}$	$\frac{2cov(\ln\psi_i,\ln\theta_j)}{var(\ln\psi_i+\ln\theta_j)}$	Adjusted R^2
$\ln m_{ij}$	17,054,274	0.66	0.32	0.02	0.39

Note: The table reports the (co)variances of the estimated seller and buyer fixed effects from equation (1). Estimation is based on the high-dimensional fixed effects estimator from Correia (2016).

Source: Bernard et al (2018).

- Buyer and seller FEs explain large share of variation in firm-to-firm sales
- Seller FE exhibits greater variance than buyer FE
- Correlation between seller and buyer FE is close to zero
  - high ψ_i sellers match with both high and low θ_j buyers

Explaining Firm Size

- Step 2: use seller FE, buyer FE, and match residual to decompose firm sales
- Total sales of firm i are given by:

$$S_i = \sum_{j \in \Omega_i^C} m_{ij} + F_i$$

  - m_{ij}: (un-demeaned) sales from i to j
  - Ω_i^C: set of customers for firm i
  - F_i: final sales by firm i (outside network)

Explaining Firm Size

- Hence, total sales can be decomposed as follows:

$$\log S_i = \text{const.} + \underbrace{\log \bar{\psi}_i}_{\text{upstream}} + \underbrace{\log \bar{\xi}_i}_{\text{downstream}} + \underbrace{\log \beta_i}_{\text{final demand}}$$

  - $\bar{\psi}_i$ captures attractiveness of firm as seller
  - $\bar{\xi}_i \equiv \sum_{j \in C_i} \bar{\theta}_j \omega_{ij}$ captures having more/better customers
  - $\beta_i \equiv \frac{1}{1-F_i/S_i}$ increasing in share of sales to final consumers
- Now regress each RHS factor on log firm sales
  - same variance decomposition as in Bernard et al (2018, ReStat)

Explaining Firm Size

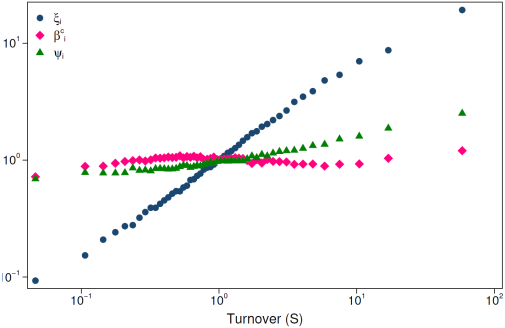
	N	Upstream $\ln\psi_i$	Downstream $\ln\xi_i$	Final Demand $\ln\beta_i$
$\ln S_i$	94,330	.18***	.81***	.01**
		(.00)	(.00)	(.00)

Note: The table reports coefficient estimates from OLS regressions of a firm size margin (as indicated in the column heading) on total firm sales. All variables are first demeaned by their 4-digit NACE industry average. Standard errors in parentheses. Significance: * $<$ 5%, ** $<$ 1%, *** $<$0.1%.

Source: Bernard et al (2018).

- Downstream factor ξ_i accounts for majority of variation in firm size
- Small role for upstream factor ψ_i implies:
  - average market share not strongly correlated with total firm sales
- Supply factors orthogonal to ψ_i (e.g. marketing efficiency) can still be important

Explaining Firm Size



Source: Bernard et al (2018).

- Findings suggest that in order to explain firm size...
  - need to understand how firms match and transact with buyers

Model

- Now develop structural model of a firm-level production network
  - will give structural interpretation to fixed effects
  - allows further decomposition of fixed effects
  - can be used to simulate counterfactuals
- Basic features of the model:
  - firms are heterogeneous in productivity/quality
  - firms are connected to heterogeneous sets of buyers/sellers
  - firms also sell to final consumers
  - set of existing firm-to-firm relationships is taken as given

Production

- Production function for firm i:

$$y_i = z_i l_i^{\alpha_i} \left(u_i^{1-\gamma_i} v_i^{\gamma_i} \right)^{1-\alpha_i}$$

  - z_i: productivity
  - l_i: labor
  - u_i: inputs purchased outside domestic network
  - v_i: inputs purchased within domestic network
  - α_i: labor share
  - γ_i: network share of inputs

Production

- Domestic network input bundle:

$$v_i = \left[\sum_{k \in \Omega_i^S} \left(\phi_{ki} \nu_{ki} \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}$$

  - Ω_i^S: set of suppliers for firm i
  - ν_{ki}: quantity of inputs purchased from seller k by buyer i
  - ϕ_{ki}: match-specific quality
  - σ: elasticity of substitution (common across all firms)

Costs

- Marginal cost for firm i:

$$c_i = \frac{1}{z_i} w_i^{\alpha} \left(\tilde{w}_i^{1-\gamma_i} P_i^{\gamma_i} \right)^{1-\alpha}$$

  - w_i: wage
  - $\tilde{w}_i$: price of non-network inputs
  - P_i: network input price index

- Price of network input bundle:

$$P_i = \left[\sum_{k \in \Omega_i^S} \left(p_{ki} / \phi_{ki} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

  - p_{ki}: price charged by seller k to buyer i

- Prices are some markup over marginal costs adjusted for trade costs:

$$p_{ij} = \tau_{ij} c_j$$

- Note that in principle, τ_{ij} depends on market structure

Firm-to-firm Sales

- Sales by seller i to buyer j:

$$m_{ij} = \underbrace{M_j}_{\substack{\text{total input}\\ \text{expenditures}}} \times \underbrace{\gamma_j}_{\substack{\text{network input}\\ \text{share}}} \times \underbrace{\left(\frac{p_{ij}/\phi_{ij}}{P_j}\right)^{1-\sigma}}_{\substack{\text{share of } i \text{ in } j\text{'s}\\ \text{network inputs}}}$$

- Now, want to decompose m_{ij} into seller, buyer, and match components
- First write match quality as:

$$\phi_{ij} = \phi_i \tilde{\phi}_{ij}$$

and write markups / trade costs as:

$$\tau_{ij} = \tau_i \tilde{\tau}_{ij}$$

Firm-to-firm Sales

- Then firm-to-firm sales can again be written as:

$$\underbrace{\log m_{ij}}_{\substack{\text{log sales} \\ \text{from i to j}}} = \underbrace{\log \psi_i}_{\substack{\text{seller} \\ \text{effect}}} + \underbrace{\log \theta_j}_{\substack{\text{buyer} \\ \text{effect}}} + \underbrace{\log \omega_{ij}}_{\substack{\text{match} \\ \text{effect}}}$$

- Seller effect depends on quality-adjusted costs:

$$\psi_i \equiv \left(\frac{\phi_i}{\tau_i c_i}\right)^{1-\sigma}$$

- Buyer effect depends on total purchases and access to inputs:

$$\theta_j \equiv M_j \gamma_j P_j^{\sigma-1}$$

- Match effect depends on match-specific quality and markups / trade costs:

$$\omega_{ij} \equiv \left(\frac{\tilde{\phi}_{ij}}{\tilde{\tau}_{ij}}\right)^{\sigma-1}$$

Production Capability and Input Price Index

- Note that $\{\psi_i, \theta_i\}$ depend on firm characteristics *and* supplier prices through P_i
- However, firm characteristics can be isolated via the following transformation:

$$\psi_i \left(\frac{\theta_i}{\gamma_i M_i} \right)^{\gamma_i(1-\alpha_i)} = \left[\frac{\phi_i z_i / \tau_i}{w_i^{\alpha_i} \tilde{w}_i^{(1-\alpha_i)(1-\gamma_i)}} \right]^{\sigma-1} \equiv Z_i$$

  - call Z_i the *production capability* of firm i
- To recover estimate of input price index $P_i^{\sigma-1}$, use:
  - estimate of buyer effect $\theta_i \equiv M_i \gamma_i P_i^{\sigma-1}$
  - data on total network input purchases $M_i \gamma_i$

Correlations

Firm Size Component	$\ln S_i$	$\ln \psi_i$	$\ln \xi_i$	$\ln P_i^{1-\sigma}$	$\ln n_i^c$	$\ln n_i^s$	$\ln Z_i$
Total Sales, $\ln S_i$	1						
Upstream, $\ln \psi_i$	.23	1					
Downstream, $\ln \xi_i$	.66	-.16	1				
Inverse input price index, $\ln P_i^{1-\sigma}$	.91	.14	.61	1			
# Customers, $\ln n_i^c$	.49	-.33	.85	.50	1		
# Suppliers, $\ln n_i^s$	.76	-.02	.63	.76	.57	1	
Production Capability, $\ln Z_i$	-.31	.79	-.51	-.40	-.59	-.51	1

Source: Bernard et al (2018).

- S_i strongly positively correlated with both ψ_i and ξ_i
  - correlation with ξ_i twice as large as with ψ_i
- ψ_i and ξ_i negatively correlated:
  - firms with many/"better" customers have smaller customer market shares
  - suggests firms are unlikely to succeed on both intensive/extensive margins
- S_i strongly positively correlated with $P_i^{1-\sigma}$:
  - larger firms tend to benefit from cheaper input prices

Correlations

Firm Size Component	$\ln S_i$	$\ln \psi_i$	$\ln \xi_i$	$\ln P_i^{1-\sigma}$	$\ln n_i^c$	$\ln n_i^s$	$\ln Z_i$
Total Sales, $\ln S_i$	1						
Upstream, $\ln \psi_i$	.23	1					
Downstream, $\ln \xi_i$	.66	-.16	1				
Inverse input price index, $\ln P_i^{1-\sigma}$	.91	.14	.61	1			
# Customers, $\ln n_i^c$	.49	-.33	.85	.50	1		
# Suppliers, $\ln n_i^s$	.76	-.02	.63	.76	.57	1	
Production Capability, $\ln Z_i$	-.31	.79	-.51	-.40	-.59	-.51	1

Source: Bernard et al (2018).

- $P_i^{1-\sigma}$ positively correlated with ψ_i:
  - firms with low input prices have higher market shares among customers
- Z_i negatively correlated with $\left\{S_i, \xi_i, P_i^{1-\sigma}\right\}$, positively correlated with ψ_i:
  - firms with higher production capability tend to sell less, have fewer/smaller customers, higher input prices, and higher market shares among customers

Downstream Decomposition

- Next, use model to further decompose downstream/upstream components
- Decomposition of downstream component $\xi_i \equiv \sum_{j\in\Omega_i^C} \theta_j \omega_{ij}$:

$$\log \xi_i = \underbrace{\log N_i^C}_{\text{no. of customers}} + \underbrace{\log \bar{\theta}_i}_{\text{avg. customer capability}} + \underbrace{\log \Gamma_i^C}_{\text{covariance term}}$$

- Average customer capability:

$$\bar{\theta}_i \equiv \prod_{j\in\Omega_i^C} (\theta_j)^{1/N_i^C}$$

- Covariance between customer capability and match effect:

$$\Gamma_i^C \equiv \frac{1}{N_i^C} \sum_{j\in\Omega_i^C} \omega_{ij} \frac{\theta_j}{\bar{\theta}_i}$$

- Firms face higher network demand ($\xi_i \uparrow$) if...
  - they have more customers ($N_i^c \uparrow$)
  - their customers have higher capability ($\bar{\theta}_i \uparrow$)
  - customers with high capability are also better matches ($\Gamma_i^C \uparrow$)

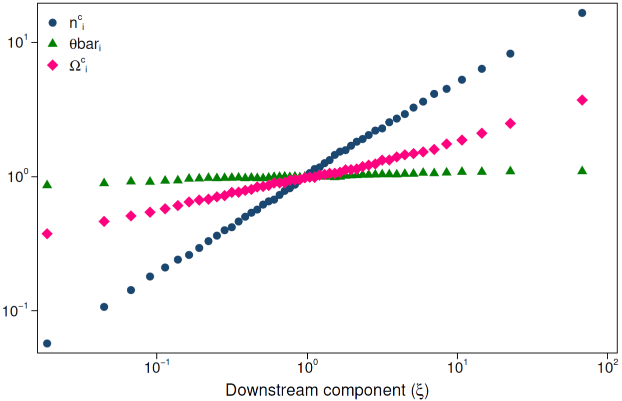
Downstream Decomposition

	# Customers $\ln n_i^c$	Avg Customer Capability $\ln \bar{\theta}_i$	Customer Covariance $\ln \Omega_i^c$
$\ln \xi_i$	$.71^{***}$	$.03^{***}$	$.26^{***}$
	(.00)	(.00)	(.00)

Source: Bernard et al (2018).

- Most variation in downstream component explained by number of customers
- Covariance term is also important:
  - bigger firms sell to buyers with high capability that are also good matches
- Negligible role for average customer capability:
  - large firms do not match with more capable buyers on average

Downstream Decomposition



Source: Bernard et al (2018).

Upstream Decomposition

- Decomposition of upstream component ψ_i:

$$\log \psi_i = \log Z_i + \gamma_i (1 - \alpha_i) \Big[\underbrace{\log N_i^S}_{\text{no. of suppliers}} + \underbrace{\log \bar{\psi}_i}_{\text{avg. supplier capability}} + \underbrace{\log \Gamma_i^S}_{\text{covariance term}} \Big]$$

- Average supplier capability:

$$\bar{\psi}_i \equiv \prod_{j \in \Omega_i^S} (\psi_j)^{1/N_i^S}$$

- Covariance between supplier capability and match effect:

$$\Gamma_i^S \equiv \frac{1}{N_i^S} \sum_{j \in \Omega_i^S} \omega_{ji} \frac{\psi_j}{\bar{\psi}_i}$$

- Firms have higher market share amongst customers ($\psi_i \uparrow$) if...
  - own production capability is higher ($Z_i \uparrow$)
  - they have more suppliers ($N_i^S \uparrow$)
  - their suppliers have higher capability ($\bar{\psi}_i \uparrow$)
  - suppliers with high capability are also better matches ($\Gamma_i^S \uparrow$)

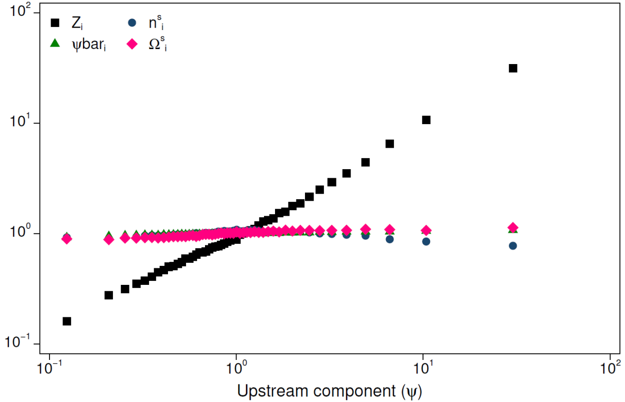
Upstream Decomposition

	Own Prod Capability $\ln Z_i$	# Suppliers $\ln n_i^s$	Avg Suppl. Capability $\ln \bar{\psi}_i$	Suppl. Cov. $\ln \Omega_i^s$
$\ln \psi_i$	.93***	-.01***	.03***	.06***
	(.00)	(.00)	(.00)	(.00)

Source: Bernard et al (2018).

- Own production capability explains almost all variation in upstream component
  - differences in average market shares among customers explained primarily by inherent firm characteristics (e.g. productivity/quality)

Upstream Decomposition



Source: Bernard et al (2018).

Costly Relationship Models

- Lim (2019), "Endogenous Production Networks and the Business Cycle"
- Develops model of endogenous firm-level production networks
  - assumptions about production technology generate incentive to form links
  - firms trade off benefits of link formation against cost of each link
- Model features:
  - many-to-many matching between heterogeneous firms
  - firm-level heterogeneity in sets of customers and suppliers
  - dynamic adjustment of production network in response to shocks
- Empirical application:
  - simulate model counterfactuals to quantify importance of network adjustment for aggregate fluctuations

Basic Environment

- First study model with fixed production network
  - key problem: given buyer-seller network, how much do firms trade?
- Exogenous unit continuum of firms producing differentiated goods
- Firms heterogeneous over states $\chi \equiv \{\phi, \delta\}$
  - ϕ: **fundamental productivity** (labor input more productive)
  - δ: **fundamental demand** (household prefers product more)
  - exogenous distribution function F_χ and support $S_\chi \subset \mathbb{R}^2_+$
- Firm output produced by combining labor and inputs purchased from other firms
- Representative household:
  - supplies L units of labor inelastically
  - demands positive quantities of all goods in the economy

Final Demand

- Household has CES preferences over all goods in the economy:

$$U = \left[\int_{S_\chi} \left[\delta x_H(\chi) \right]^{\frac{\sigma-1}{\sigma}} dF_\chi(\chi) \right]^{\frac{\sigma}{\sigma-1}}$$

- Final demand for χ-firm output:

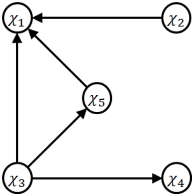
$$x_H(\chi) = \Delta_H \delta^{\sigma-1} p_H(\chi)^{-\sigma}$$

  - $p_H(\chi)$: price charged by χ-firms for final sales
- Demand shifter $\Delta_H \equiv U P_H^\sigma$ is determined in general equilibrium
- Consumer price index P_H inherits CES structure:

$$P_H = \left[\int_{S_\chi} \left[p_H(\chi) / \delta \right]^{1-\sigma} dF_\chi(\chi) \right]^{\frac{1}{1-\sigma}}$$

- Conditional on prices, final demand $x_H(\chi)$ is greater for firms with higher δ

Production Technology



- Firm-to-firm trade characterized by **production network**
- Network fully specified by **matching function** m**:**
  - $m(\chi, \chi') =$probability that χ-firm buys from χ'-firm
- Given continuum of firms, $m(\chi, \chi')$ is also...
  - fraction of $\chi - \chi'$ buyer-seller pairs that match

Production Technology

- Given matching function, production is CES in labor and supplier inputs:

$$X(\chi) = \left[[\phi l(\chi)]^{\frac{\sigma-1}{\sigma}} + \int_{S_\chi} m(\chi, \chi') \left[\alpha x(\chi, \chi') \right]^{\frac{\sigma-1}{\sigma}} dF_\chi(\chi') \right]^{\frac{\sigma}{\sigma-1}}$$

  - $l(\chi)$: quantity of labor hired
  - $x(\chi, \chi')$: quantity of inputs purchased from χ'-sellers
  - $\alpha \in (0, 1)$: input suitability

Production Technology

- Marginal cost $\eta(\chi)$ inherits CES structure:

$$\eta(\chi) = \left[\phi^{\sigma-1} + \alpha^{\sigma-1} \int_{S_\chi} m(\chi, \chi') \left[p(\chi, \chi')\right]^{1-\sigma} dF_\chi(\chi')\right]^{\frac{1}{1-\sigma}}$$

  - $p(\chi, \chi')$: price charged by χ'-sellers to χ-buyers
- Conditional on prices, firms with higher ϕ have lower marginal cost $\eta(\chi)$
- Labor demand (taking wage as numeraire):

$$l(\chi) = X(\chi)\, \eta(\chi)^\sigma\, \phi^{\sigma-1}$$

- Intermediate demand by χ-buyers from χ'-sellers:

$$x\left(\chi, \chi^{''}\right) = X(\chi)\, \eta(\chi)^\sigma\, \alpha^{\sigma-1} p\left(\chi, \chi^{'}\right)^{-\sigma}$$

Market Structure

- Market structure: **monopolistic competition**
- With continuum of sellers for each buyer, implies that...
  - all firms charge constant CES markup, $\mu = \frac{\sigma}{\sigma - 1}$
- Hence prices are given by:

$$p_H(\chi) = \mu \eta(\chi)$$
$$p\left(\chi', \chi\right) = \mu \eta(\chi)$$

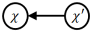
- Essential for model tractability

Market Structure

- In principle, network structure could affect:
  - firm-to-firm markups
  - efficiency of the market equilibrium
- Possible alternative market structures with variable markups:
  - oligopolistic competition
  - multilateral bargaining
- However, existing research (e.g. Kikkawa et al (2018)) suggests that variable markups in production networks are:
  - computationally intractable
  - quantitatively unimportant for shock aggregation/propagation

Sourcing and Selling Decisions

- Given network, how much do firms buy and sell?



Sourcing and Selling Decisions

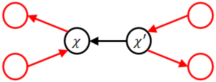
- Given network, how much do firms buy and sell?

$$\chi \longleftarrow \chi'$$

  - firm-to-firm trade depends on fundamental $\{\phi, \delta\}$ of buyer/seller...

Sourcing and Selling Decisions

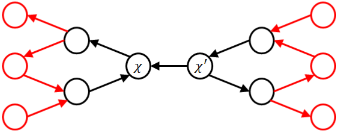
- Given network, how much do firms buy and sell?



  - firm-to-firm trade depends on fundamental $\{\phi, \delta\}$ of buyer/seller...
  - but also on $\{\phi, \delta\}$ of buyers/sellers of buyer/seller...

Sourcing and Selling Decisions

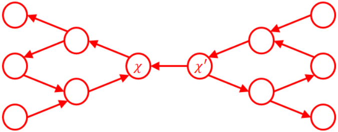
- Given network, how much do firms buy and sell?



  - firm-to-firm trade depends on fundamental $\{\phi, \delta\}$ of buyer/seller...
  - but also on $\{\phi, \delta\}$ of buyers/sellers of buyer/seller...
  - and on $\{\phi, \delta\}$ of buyers/sellers of buyers/sellers of buyer/seller...

Sourcing and Selling Decisions

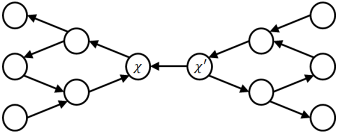
- Given network, how much do firms buy and sell?



  - firm-to-firm trade depends on fundamental $\{\phi, \delta\}$ of buyer/seller...
  - but also on $\{\phi, \delta\}$ of buyers/sellers of buyer/seller...
  - and on $\{\phi, \delta\}$ of buyers/sellers of buyers/sellers of buyer/seller...
  - aggregate state variable $=$ entire network?

Sourcing and Selling Decisions

- Given network, how much do firms buy and sell?



  - firm-to-firm trade depends on fundamental $\{\phi, \delta\}$ of buyer/seller...
  - but also on $\{\phi, \delta\}$ of buyers/sellers of buyer/seller...
  - and on $\{\phi, \delta\}$ of buyers/sellers of buyers/sellers of buyer/seller...
  - aggregate state variable = entire network?

- Solution: characterize firms in terms of **network productivity and quality**

$$\Phi(\chi) \equiv \eta(\chi)^{1-\sigma} \qquad \text{(inverse marginal cost)}$$

$$\Delta(\chi) \equiv X(\chi)\,\eta(\chi)^{\sigma} \qquad \text{(intermediate demand shifter)}$$

Firm Network Characteristics

- From solving cost minimization problem for each firm...

$$\Phi(\chi) = \phi^{\sigma-1} + \mu^{1-\sigma}\alpha^{\sigma-1}\int_{S_\chi} m(\chi,\chi')\,\Phi(\chi')\,dF_\chi(\chi')$$

  - costs depend on fundamental productivity and costs of suppliers
- From output market clearing condition for each firm...

$$\Delta(\chi) = \mu^{-\sigma}\Delta_H\delta^{\sigma-1} + \mu^{-\sigma}\alpha^{\sigma-1}\int_{S_\chi} m(\chi',\chi)\,\Delta(\chi')\,dF_\chi(\chi')$$

  - demand depends on fundamental demand and demand of customers
- These are **functional equations** in $\Phi(\cdot)$ and $\Delta(\cdot)$
  - but how do we know that there is a unique solution for $\{\Phi,\Delta\}$?

Firm Network Characteristics

- Proof relies on the **contraction mapping theorem**
  - a fundamental theorem often used in dynamic programming
  - e.g. for studying dynamic macro models
- Let (X, d) be a *metric space*
  - e.g. $X = [0,1]$ and $d(x,y) = |x - y|$ for all $x, y \in [0,1]$
- Let $T : X \to X$ be a mapping from X to itself
  - e.g. $T(x) = x$
- Then T is a **contraction** if there exists some $\beta \in (0,1)$ such that:

$$d\left(T(x), T(y)\right) \leq \beta d(x,y)$$

  - e.g. $T(x) = \frac{1+x}{2}$ is a contraction (with $\beta = \frac{1}{2}$)
- Intuitively, a contraction brings points in X closer to each other

Firm Network Characteristics

- The **contraction mapping theorem** states that:
  - any contraction T on a complete metric space (X, d)...
  - has a **unique fixed point** in X, i.e. some $x^* \in X$ such that $T(x^*) = x^*$
  - and starting from any $x \in X$, the sequence $T^n(x)$ converges to x^*
- Can show that $\{\Phi, \Delta\}$ equations are (decoupled) contraction mappings
  - specifically, on the space of bounded functions on S_χ
- Then the contraction mapping theorem guarantees...
  - existence and uniqueness of a solution for $\{\Phi, \Delta\}$
  - iterating on equations from any initial guess will converge to the solution
- This is the analogue of the adjacency matrix invertibility condition...
  - in a continuous space

Firm Network Characteristics

- Now given firm network characteristics, all other variables of interest are known:

$$
\begin{aligned}
\text{firm revenue:} &\quad R(\chi) \propto \Delta(\chi)\,\Phi(\chi) \\
\text{firm profit:} &\quad \Pi(\chi) \propto \Delta(\chi)\,\Phi(\chi) \\
\text{firm-to-firm sales:} &\quad r\left(\chi, \chi^{'}\right) \propto \Delta(\chi)\,\Phi\left(\chi^{'}\right) \\
\text{firm-to-firm profit:} &\quad \pi\left(\chi, \chi^{'}\right) \propto \Delta(\chi)\,\Phi\left(\chi^{'}\right)
\end{aligned}
$$

- Regardless of how complex network is...
  - can solve for all firm and firm-to-firm variables in seconds on standard PC

Dynamic Network Formation

- Now ask: which relationships do firms choose to form?
- Dynamic model environment:
  - discrete time
  - linear household preferences
  - firm fundamental characteristics $\{\phi_t, \delta_t\}$ are time-varying
- CES production technology generates incentives to form links:
  - constant marginal cost $\Rightarrow$ more customers desirable
  - finite, positive substitution elasticity $\Rightarrow$ more suppliers desirable
- Counterbalance incentives by assuming that:
  - each active relationship requires ξ_t units of labor
  - ξ_t follows an arbitrary stochastic process for each relationship
  - but is independent across relationships and from $\{\phi_t, \delta_t\}$

Relationship Selection

- When do firms optimally choose to form relationships?
- Assume that selling firm pays full share of relationship cost:
  - optimal pricing is the same as before
  - buying firm is always agreeable to any trading relationship
  - c.f. Bernard et al (2018, ReStat)
- Static variable profit earned by a $\chi^{'}$-firm from selling to χ-firm at date t:

$$\pi_t\left(\chi,\chi^{'}\right)\propto\Delta_t\left(\chi\right)\Phi_t\left(\chi^{'}\right)$$

- **Acceptance function** - probability that a relationship is selected:

$$a_t\left(\chi,\chi^{'}\right)=\Pr\left[\pi_t\left(\chi,\chi^{'}\right)\geq\xi_t\right]=F_{\xi,t}\left[\pi_t\left(\chi,\chi^{'}\right)\right]$$

where $F_{\xi,t}$ is the unconditional distribution of ξ_t

Relationship Selection

- Law of motion for the matching function:

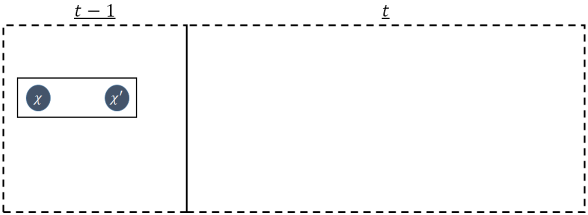
$$m_t = \underbrace{m_{t-1}}_{\text{existing relationships}} + \underbrace{a_t\left(1 - m_{t-1}\right)}_{\text{newly-created relationships}} - \underbrace{\left(1 - a_t\right) m_{t-1}}_{\text{terminated relationships}}$$

- Under these assumptions, matching and acceptance functions are identical
- To solve the model with endogenous production networks:
  - start with a guess of $\{\Phi, \Delta\}$
  - compute $\pi\left(\chi, \chi'\right) \propto \Delta\left(\chi\right)\Phi\left(\chi'\right)$
  - compute $a = m = F_{\xi}\left(\pi\right)$
  - compute new guess of $\{\Phi, \Delta\}$ from network characteristic equations
  - iterate until convergence
- Model closed by imposing labor market clearing condition

Extension with Sticky Relationships

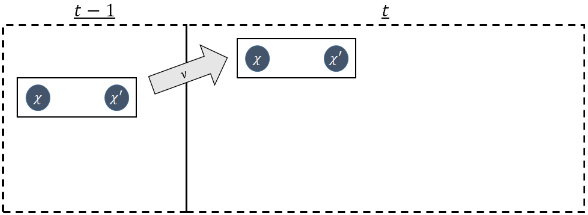
- Also possible to extend model by introducing dynamic frictions
- Suppose that relationships are **sticky**
  - c.f. sticky price models in macro
- With probability $1-\nu$, each relationship receives option to switch status
  - i.e. activate if not active, terminate if active
- This introduces *option values* into relationship formation
  - forming a relationship is valuable in part because...
  - it allows the relationship to exist in the future
- For simplicity, assume that:
  - firm fundamental characteristics $\{\delta, \phi\}$ are time invariant
  - relationship cost shocks ξ_t are *iid* with mean $\bar{\xi}$

Sticky Relationships



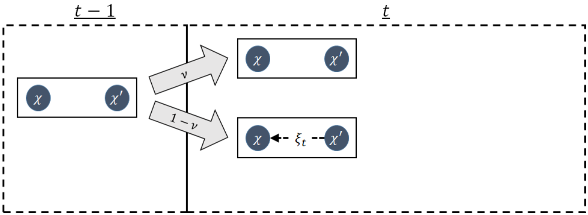
- Inactive $\chi - \chi'$ relationship at date $t-1$

Sticky Relationships



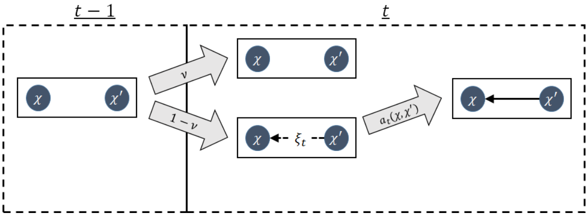
- Inactive $\chi - \chi'$ relationship at date $t-1$
  - with probability ν, no reset shock received: relationship remains inactive

Sticky Relationships



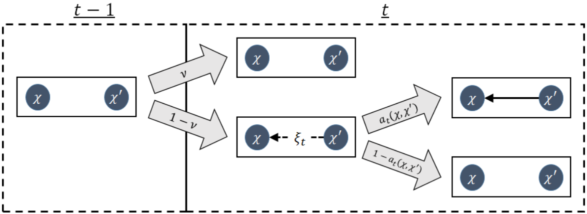
- Inactive $\chi - \chi^{'}$ relationship at date $t-1$
  - with probability ν, no reset shock received: relationship remains inactive
  - with probability $1-\nu$, reset shock received: firms select based on cost ξ_t

Sticky Relationships



- Inactive $\chi - \chi'$ relationship at date $t-1$
  - with probability ν, no reset shock received: relationship remains inactive
  - with probability $1-\nu$, reset shock received: firms select based on cost ξ_t
    - with probability $a_t\left(\chi, \chi'\right)$, relationship activated

Sticky Relationships



- Inactive $\chi - \chi'$ relationship at date $t-1$
  - with probability ν, no reset shock received: relationship remains inactive
  - with probability $1-\nu$, reset shock received: firms select based on cost ξ_t
    - with probability $a_t\left(\chi,\chi'\right)$, relationship activated
    - with probability $1-a_t\left(\chi,\chi'\right)$, relationship rejected

Relationship Selection

- Value of selling:

$$V_t^+\left(\chi,\chi^{'}|\xi_t\right)=\underbrace{\pi_t\left(\chi,\chi^{'}\right)-\xi_t}_{\text{static profit}}+\underbrace{\beta\nu\mathbb{E}_t\left[V_{t+1}^+\left(\chi,\chi^{'}|\xi_{t+1}\right)\right]}_{\text{stuck-in value}}+\underbrace{\beta\left(1-\nu\right)\mathbb{E}_t\left[V_{t+1}^O\left(\chi,\chi^{'}|\xi_{t+1}\right)\right]}_{\text{reset option value}}$$

- Value of not selling:

$$V_t^-\left(\chi,\chi^{'}\right)=\underbrace{\beta\nu V_{t+1}^-\left(\chi,\chi^{'}\right)}_{\text{stuck-out value}}+\underbrace{\beta\left(1-\nu\right)\mathbb{E}_t\left[V_{t+1}^O\left(\chi,\chi^{'}|\xi_{t+1}\right)\right]}_{\text{reset option value}}$$

- Reset option value:

$$V_t^O\left(\chi,\chi^{'}|\xi_t\right)=\max\left\{V_t^+\left(\chi,\chi^{'}|\xi_t\right),V_t^-\left(\chi,\chi^{'}\right)\right\}$$

Relationship Selection

- Selling premium equals EPV of profits before relationship can be reset:

$$V_t^+\left(\chi,\chi^{'}|\xi_t\right)-V_t^-\left(\chi,\chi^{'}\right)=\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta\nu)^s\left[\pi_{t+s}\left(\chi,\chi^{'}\right)-\xi_{t+s}\right]\right]$$

- Acceptance function with forward-looking firms:

$$a_t\left(\chi,\chi^{'}\right)=F_\xi\left[\sum_{s=0}^{\infty}(\beta\nu)^s\left[\pi_{t+s}\left(\chi,\chi^{'}\right)-\beta\nu\bar{\xi}\right]\right]$$

- Need to iterate on guess of path for profit functions
  - takes about 1 hour on standard PC

Relationship Selection

- In steady-state:

$$a\left(\chi,\chi'\right)=F_{\xi}\left[\frac{\pi\left(\chi,\chi'\right)-\beta\nu\bar{\xi}}{1-\beta\nu}\right]$$

- Forward-looking firm decisions imply:
  - temporarily unprofitable relationships may be activated if $\pi\left(\chi,\chi'\right)>\bar{\xi}$
  - temporarily profitable relationships may not be activated if $\pi\left(\chi,\chi'\right)<\bar{\xi}$
  - firm pairs will never trade in steady-state if $\pi\left(\chi,\chi'\right)<\beta\nu\bar{\xi}$
- Stickiness parameter is estimated by Huneeus (2018) using Chilean VAT data
  - with an extension to multiple locations of production

Model Properties and Predictions

- Existence and uniqueness
  - exogenous network equilibrium is unique (contraction mapping theorem)
  - endogenous network equilibrium is unique (Tarski fixed point theorem)
- Efficiency
  - exogenous network equilibrium is inefficient: multiple marginalization
  - with endogenous networks, additional source of “network externality”
- Model generates analytic predictions about:
  - firm-level revenue and degree distributions
  - assortativity of matching between firms (negative degree matching)
  - dynamics of relationships
  - how production network matters for welfare

Extreme-value Models

- Oberfield (2019), "A Theory of Input-Output Architecture"
- Model of an endogenous production network with the following features:
  - continuum of firms
  - firms produce using labor and intermediate inputs from one supplier
  - potential relationships (techniques) for intermediates arrive randomly
  - productivity of each potential relationship is random
  - firms choose best supplier amongst available potential suppliers

Model Environment

- Continuum of firms produce using labor and one supplier's input
- Production function for firm i if it uses firm j's good as an input:

$$y_i = z_{ij} \left(\frac{x_{ij}}{\alpha}\right)^{\alpha} \left(\frac{l_i}{1-\alpha}\right)^{1-\alpha}$$

  - y_i: output of firm i
  - z_{ij}: relationship-specific productivity
  - x_{ij}: quantity of intermediate input
  - l_i: quantity of labor
  - α: intermediate share

- Marginal cost of firm i from using this production technology:

$$c_i = \frac{1}{z_{ij}} p_{ij}^{\alpha} w^{1-\alpha}$$

where p_{ij} is the price charged by j to i and w is the wage

Pricing and Market Structure

- Now suppose that all firms charge marginal cost, so that $p_{ij} = c_j$
- Can be rationalized by:
  - bilateral two-part pricing (ad valorem price plus lump-sum transfer)
  - pairwise stable equilibrium concept (no pair of firms wants to deviate)
- Then marginal cost is:

$$c_i = \frac{1}{z_{ij}} c_j^{\alpha} w^{1-\alpha}$$

or equivalently in terms of **efficiency** $q_i \equiv \frac{w}{c_i}$:

$$q_i = z_{ij} q_j^{\alpha}$$

Number of Techniques

- Now suppose that # of potential techniques for a firm is Poisson with mean M:

$$\Pr\left[n \text{ techniques}\right] = \frac{M^n e^{-M}}{n!}$$

- For each technique:
  - identity of potential supplier is random
  - productivity of technique z_{ij} is drawn from distribution H (exogenous)
- Let F denote equilibrium distribution of q across firms (endogenous)
- Key problem: what is F?

Solving for the Efficiency Distribution

- Efficiency from using a technique:

$$q_i = z_{ij} q_j^{\alpha}$$

- Conditional on $z_{ij} = z$, probability that random supplier delivers efficiency $\leq q$:

$$G(q|z) = F\left[(q/z)^{1/\alpha}\right]$$

- Unconditional probability that technique delivers efficiency $\leq q$:

$$G(q) = \int_{S_z} G(q|z) dH(z)$$

- Conditional on n techniques, probability that firm has efficiency $\leq q$:

$$F(q|n) = G(q)^n$$

- Unconditional probability that a firm has efficiency $\leq q$ at date t:

$$\begin{aligned} F(q) &= \sum_{n=0}^{\infty} \frac{M^n e^{-M}}{n!} F(q|n) \\ &= e^{-M[1-G(q)]} \end{aligned}$$

Solving for the Efficiency Distribution

- Now put everything together:

$$F(q) = e^{-M\left[\int_{S_z}\left[1-F\left[(q/z)^{1/\alpha}\right]\right]dH(z)\right]}$$

- Given M and H, this is a functional equation in firm efficiency distribution F
  - this is the heart of the model
- Computationally, easy to iterate and solve
- To get analytic solutions:
  - assume technique productivity is **Pareto**, $H(z) = 1 - \left(\frac{\underline{z}}{z}\right)^{\theta}$
- Implies that arrival rate of techniques with efficiency $> z$ is $M\underline{z}^{\theta}z^{-\theta}$
- Now consider limit in which $M \to \infty$, $\underline{z} \to 0$ with $M\underline{z}^{\theta} \equiv m$ constant
  - i.e. more techniques, lower average productivity of each technique, constant average productivity of all techniques

Solving for the Efficiency Distribution

- Then, easy to show that efficiency distribution is:

$$F(q) = e^{-Tq^{-\theta}}$$

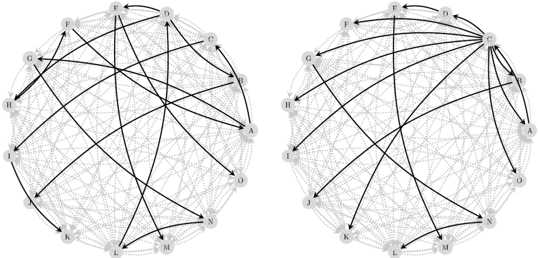
  - i.e. a *Frechet distribution*
- Same trick as in Kortum (1997) growth model
  - which in fact underlies the Frechet distribution in EK (2002)
- Scale parameter of the distribution is $T = [\Gamma(1-\alpha)\, m]^{\frac{1}{1-\alpha}}$
  - more links $\Rightarrow$ higher average efficiency
- Shape parameter θ is inherited from shape parameter of z_{ij} draws
- Easy to extend to dynamic setting (growth):
  - suppose techniques arrive over time
  - according to non-homogeneous Poisson process with rate $\mu(t)$
  - then # of techniques at date t is Poisson with mean $M_t = \int_0^t \mu(t)\, dt$

Model Properties - Degree Distribution

- Can show that among firms with efficiency q, distribution of no. of customers is:

$$d^C(q) \sim \text{Poisson}\left(\frac{m}{T} q^{\alpha\theta}\right)$$

- Hence:
  - larger firms (higher q) have more customers on average
  - larger input share (higher α) makes this positive relationship stronger



(a) $\alpha = 0.1$ (b) $\alpha = 0.9$

FIGURE 3.—Equilibrium supply chains and α. This figure shows entrepreneurs' choices of techniques. The set of techniques, Φ, is held fixed; the only difference is the value of α. The dark edges represent techniques that are used. $M = 15$ and $H(z) = 1 - z^{-2}$ for $z \geq 1$.

Source: Oberfield (2018).

Model Properties - Assortativity

- Can also show that matching patterns are characterized by:
  - zero correlation between efficiencies of buyer and seller
- Since average degree depends only on firm efficiency:
  - out-degrees of buyer and seller are also uncorrelated
  - i.e. model predicts neutral degree assortativity
- However, since model does not generate many-to-many matching...
  - cannot think about assortativity in same terms as Bernard et al (2018)

Open Economy Extension

- Eaton, Kortum, and Kramarz (2018) extend the model to setting with:
  - multiple inputs
  - multiple countries
- Basic framework is similar to EK (2002)
  - but now firms need to produce a finite set of tasks to produce
  - each task can be produced in-house using labor or outsourced
  - technology for outsourcing of each task is similar to Oberfield (2018)...
  - but suppliers can reside in different countries from the buyer
- Use model to rationalize variation observed in French export data
  - where identify of both French seller and foreign buyer are observed
- Potential to introduce Bertrand competition (BEJK (2003)):
  - would generate variable markups in a seemingly tractable way

Summary and Related Papers

- Studied importance of network heterogeneity for firm size
  - downstream factors more important than upstream factors
  - network heterogeneity explains more than half of firm size heterogeneity
- Studied two classes of endogenous firm-level production network models
- Costly relationship models:
  - with many-to-many matching, can better model extensive margin
  - but hard to get closed-form solutions
- Extreme value models:
  - with distributional assumptions, can get closed-form solutions
  - but extensive margin of relationships is only partially captured
- Open challenge:
  - modeling variable markups in networks in a tractable way
- Related papers:
  - Magerman et al (2018) - studies role of firm-level production networks in explaining aggregate volatility
  - Taschereau-Dumouchel (2018) - endogenous network formation in environment where set of potential links is exogenously restricted
- Next week: production networks and international trade