ECO 2302 - Problem Set 1

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1 Part I: simulating and analyzing random networks

I wrote the code in Python for this problem set.

1.1 Network density

The density increases as p increases.

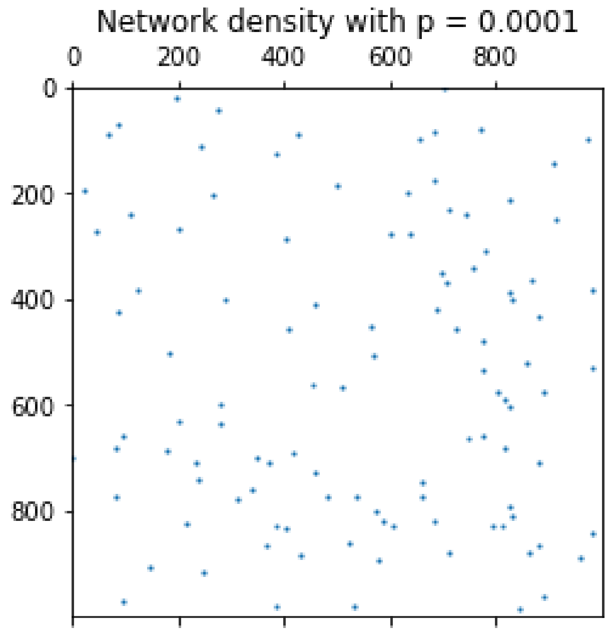

Figure 1: Network density - p = 0.0001

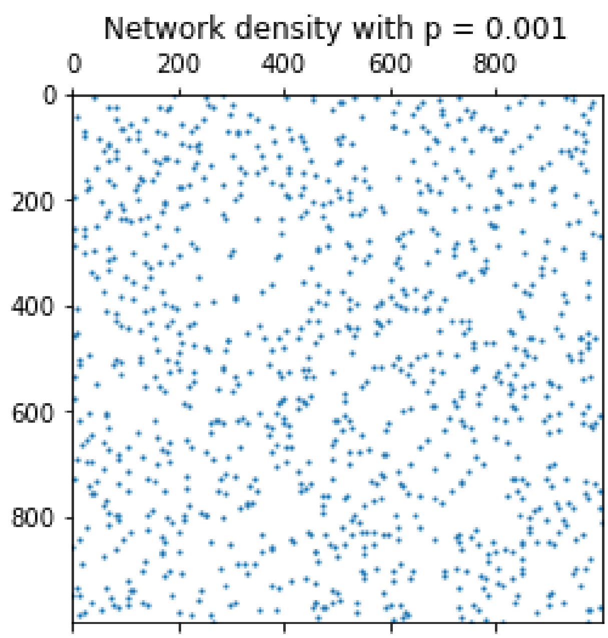

Figure 2: Network density - p = 0.001

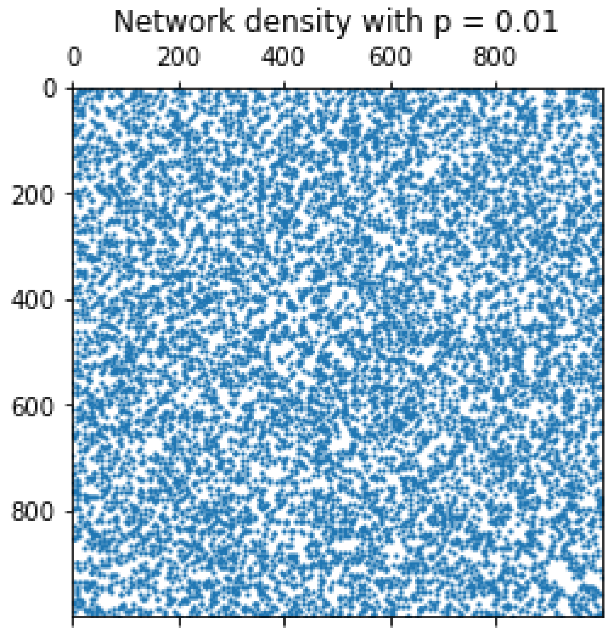

Figure 3: Network density - p = 0.01

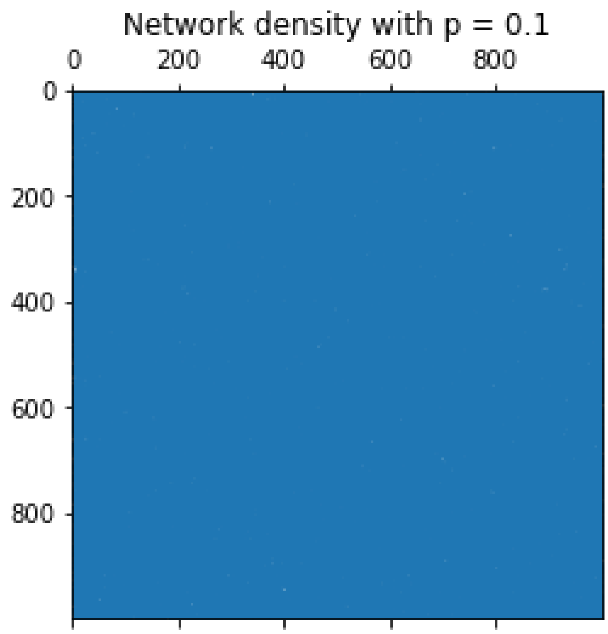

Figure 4: Network density - p = 0.1

1.2 Degree distribution

As p rises, the accuracy of Poisson approximation falls.

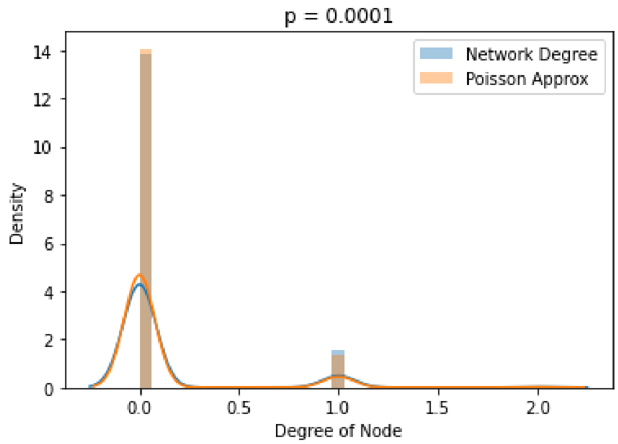

Figure 5: Network density - p = 0.0001

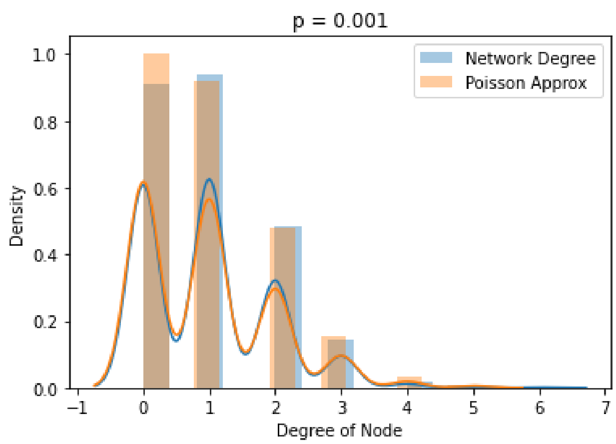

Figure 6: Network density - p = 0.001

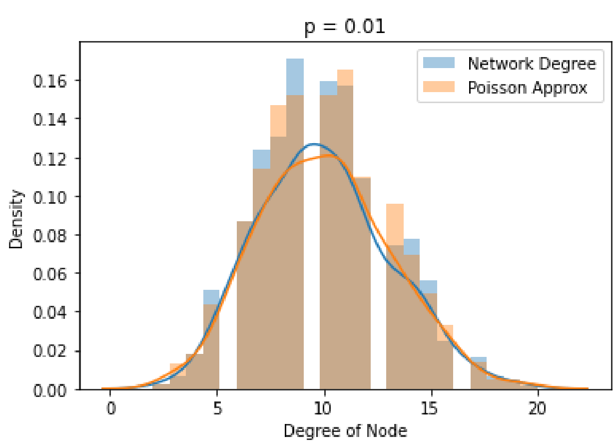

Figure 7: Network density - p = 0.01

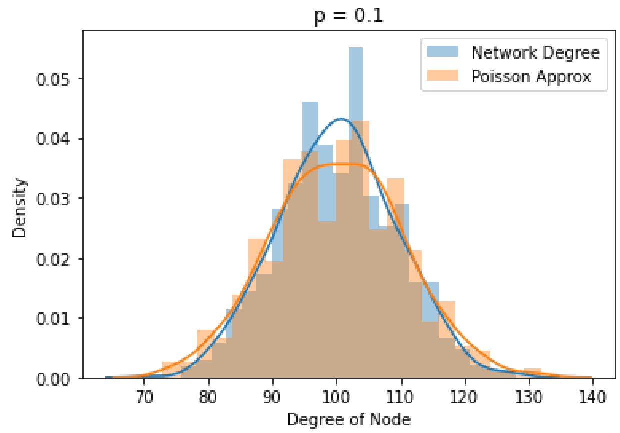

Figure 8: Network density - p = 0.1

1.3 Connectedness

There's a threshold $p^* \approx 0.005$ above which all nodes in the network are connected.

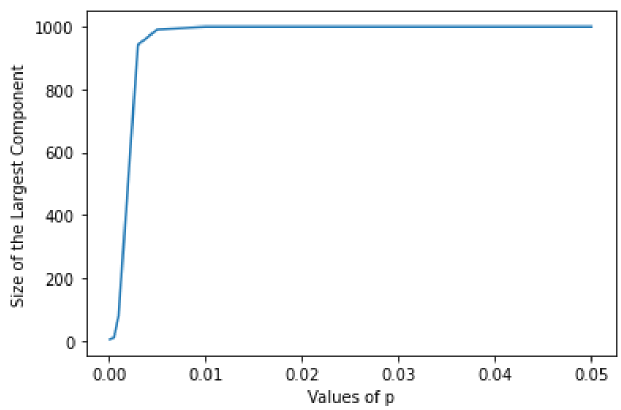

Figure 9: Network density - p = 0.1

1.4 Distance

The average distance in the giant compenent of the network rises when p takes small values between 0 and $p^* \approx= 0.0001$. This is because at values of p close to 0, the giant network has very few nodes and the number of nodes rise as p increases. This results in an increase in distance when p takes small values. However, for p above that threshold, the average distance decreases when p rises because the number of nodes in giant networks does not change much and the nodes within the networks are more connected.

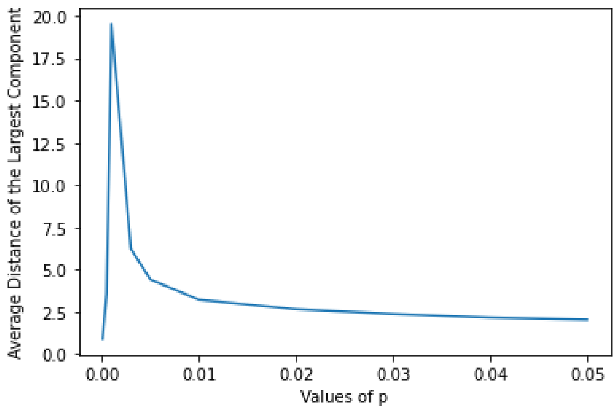

Figure 10: Network density - p = 0.1

1.5 Eigenvector centrality

The eigenvector centralities of all nodes in the giant component of the network disperses more as p increases. This is because as p rises, more nodes are connected so the eigenvector centralities becomes less concentrated in a few nodes.

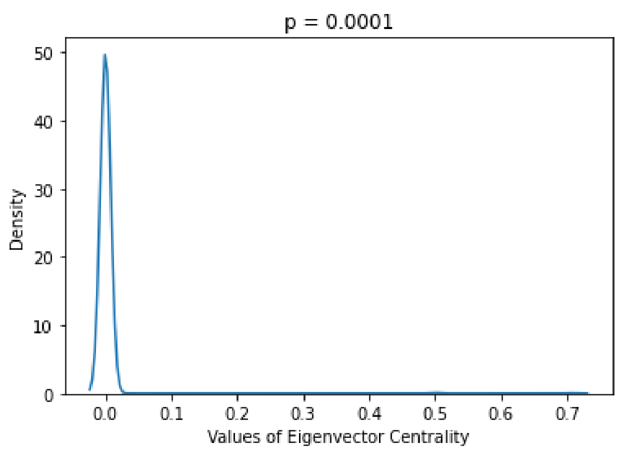

Figure 11: Network density - p = 0.0001

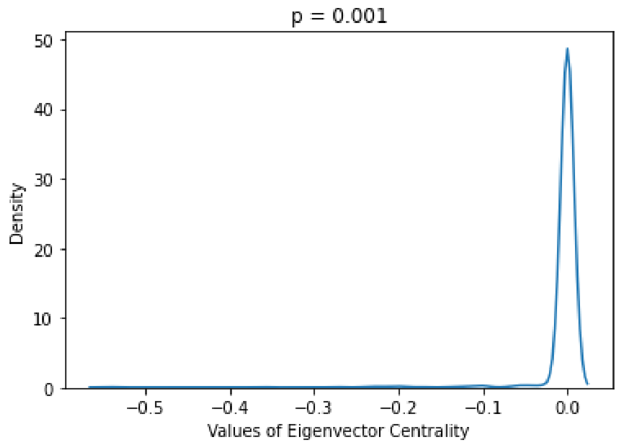

Figure 12: Network density - p = 0.001

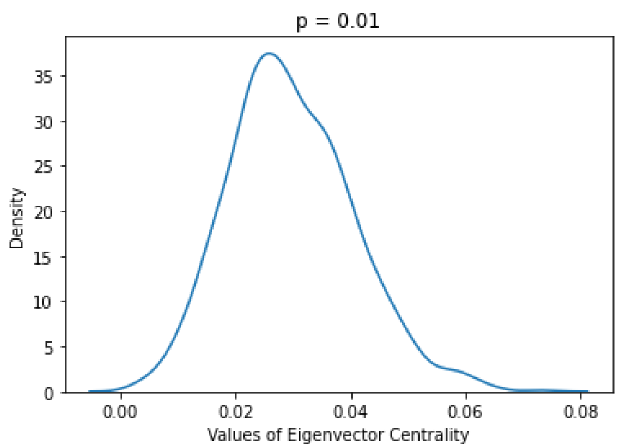

Figure 13: Network density - p = 0.01

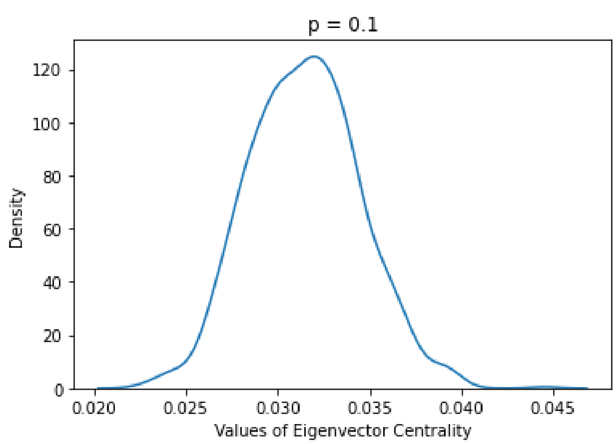

Figure 14: Network density - p = 0.1

2 Part II: Hulten's Theorem and Aggregate Fluctuations

1. Household's utility maximization problem

$$\underset{C_1,C_2}{max}\, Y = \left(\frac{C_1}{\alpha}\right)^{\alpha}\left(\frac{C_2}{1-\alpha}\right)^{1-\alpha} \quad \text{s.t } P_1C_1 + P_2C_2 = 1$$

As $\alpha = 0.5$, FOC implies that $P_1C_1 = P_2C_2 = \frac{1}{2}$ or equivalently $C_i = \frac{1}{2P_i}$. Substitute C_i into Y function, we have

$$\begin{aligned} log\, Y &= \alpha\, log\left(\frac{C_1}{\alpha}\right) + (1-\alpha)\, log\left(\frac{C_2}{1-\alpha}\right) \\ &= \alpha\, log\left(\frac{1}{2\alpha P_1}\right) + (1-\alpha)\, log\left(\frac{1}{2(1-\alpha)P_2}\right) \\ &= \frac{1}{2}\, log\left(\frac{1}{P_1}\right) + \frac{1}{2}\, log\left(\frac{1}{P_2}\right) \\ &= -\frac{log\, P_1 + log\, P_2}{2} \end{aligned}$$

Equivalently, we have

$$y = -\frac{p_1 + p_2}{2} \tag{1}$$

2. Cost minimization problem in sector i

$$\begin{aligned} &\underset{L_i,X_{ij}}{max}\left(P_iX_i - L_i - \sum_{j=1}^{2} P_jX_{ij}\right) \\ \text{s.t } X_i &= T_i\left(\frac{L_i}{\lambda}\right)^{\lambda}\left(\frac{M_i}{1-\lambda}\right)^{1-\lambda} \\ M_i &= \left(\frac{X_{ij}}{\beta_{ij}}\right)^{\beta_{ij}}\left(\frac{X_{ii}}{\beta_{ii}}\right)^{\beta_{ii}} \end{aligned}$$

FOCs are

$$L_i = \lambda P_iX_i = \lambda R_i \tag{2}$$

$$P_jX_{ij} = (1-\lambda)\beta_{ij}P_iX_i \tag{3}$$

From equation (2) and labor market clearing condition, we have: $\lambda(R_1 + R_2) = 1$ or equivalently $R_1 + R_2 = \frac{1}{\lambda}$.

From equation (3), we have

$$X_{ij} = \frac{(1-\lambda)\beta_{ij}}{P_j} P_i X_i$$

Substituting this equation and 2 into production function yields

$$\begin{aligned} X_i &= T_i (P_i X_i)^{\lambda} \left[\left(\frac{P_i X_i}{P_j} \right)^{\beta_{ij}} \left(\frac{P_i X_i}{P_i} \right)^{\beta_{ii}} \right]^{1-\lambda} \\ &= T_i P_i X_i P_j^{-(1-\lambda)\beta_{ij}} P_i^{-(1-\lambda)\beta_{ii}} \end{aligned}$$

Further simplification yields

$$\begin{aligned} P_i &= \frac{1}{T_i} P_j^{(1-\lambda)\beta_{ij}} P_i^{(1-\lambda)\beta_{ii}} \\ &= \frac{1}{T_i} P_j^{(1-\lambda)(1-\beta)} P_i^{(1-\lambda)\beta} \end{aligned}$$

Taking log of both sides yields

$$p_i = -\epsilon_i + (1-\lambda)(1-\beta) p_j + (1-\lambda)\beta p_i \tag{4}$$

With two sectors, we have a linear system of 2 equations and 2 variables (p_1 and p_2). The solution for p_i is

$$p_i = \frac{1}{2} \left[\left(\frac{1}{1+(1-\lambda)(1-2\beta)} - \frac{1}{\lambda} \right) \epsilon_j - \left(\frac{1}{1+(1-\lambda)(1-2\beta)} - \frac{1}{\lambda} \right) \epsilon_i \right] \tag{5}$$

3. Summing equation (4) across $i = 1, 2$ gives

$$p_1 + p_2 = -\frac{\epsilon_1 + \epsilon_2}{\lambda}$$

Combining this with equation 1, we have

$$y = -\frac{p_1 + p_2}{2} = \frac{\epsilon_1 + \epsilon_2}{2\lambda} \tag{6}$$

4. From equation 6, we can derive elasticity of final consumption Y to ϵ_i is $\frac{\partial y}{\partial \epsilon_i} = \frac{1}{2\lambda}$.
To prove that Hulten's theorem holds, we will next solve for R_i.
From goods market clearing condition, we have $R_i = P_i C_i + P_i X_{ii} + P_i X_{ji}$. Combining

with equation (3), we have

$$R_i = \frac{1}{2} + (1-\lambda)(1-\beta)R_j + (1-\lambda)R_i \tag{7}$$

We can then derive $R_1 = R_2$. We also know from earlier part that $R_1 + R_2 = \frac{1}{\lambda}$. As a result, $R_1 = R_2 = \frac{1}{2\lambda}$. Therefore, we can show that $\frac{\partial y}{\partial \epsilon_i} = R_i$ and Hulten's theorem applies in this model.