ECO2302: Networks in Trade and Macroeconomics

Winter 2021

Final Examination

This exam has **6** pages including this cover page. It contains four questions worth 30 points each for a total of **120 points**. The questions are roughly ordered in terms of increasing difficulty. Please answer all the questions and make sure to show clearly how you arrive at your answers. Question 4 will require the use of Matlab (or some other programming tool) - please submit a copy of your code for this question.

Question 1 (30 points)

Consider a closed economy with N sectors. There is a representative household that supplies 1 unit of labor inelastically and that has preferences over goods from each sector given by:

$$U = \prod_{i=1}^{N} \left(\frac{C_i}{\alpha_i} \right)^{\alpha_i}$$

where C_i is final consumption of sector i output. Final consumption shares are symmetric across sectors, so that $\alpha_i = \frac{1}{N}$ for all i. Sector i output is produced under perfect competition using a technology that combines labor L_i and materials M_i as follows:

$$X_i = T_i \left(\frac{L_i}{\lambda} \right)^{\lambda} \left(\frac{M_i}{1-\lambda} \right)^{1-\lambda}$$

where T_i denotes TFP of sector i. Materials M_i are in turn produced by combining intermediate inputs from all sectors as follows:

$$M_i = \prod_{j=1}^{N} \left(\frac{X_{ij}}{\beta_{ij}} \right)^{\beta_{ij}}$$

where X_{ij} denotes the quantity of sector j output used as inputs in sector i. Input-output shares are given by:

$$\beta_{ij} = \begin{cases} 1-\beta, & \text{if } i = j \\ \frac{\beta}{N-1}, & \text{if } i \neq j \end{cases}$$

for some $\beta \in [0, 1]$.

Note that the model described above has only three parameters: λ, β, and N. The parameter λ measures the importance of intermediates in production relative to labor, while β measures the importance of cross-sector linkages relative to own-sector linkages. The goal is to characterize how the volatility of household welfare depends on these parameters.

(a) First, derive an expression for the price of output in sector i, P_i, in terms of prices in all other sectors $\{P_j\}_{j \neq i}$ and TFP values in all sectors $\{T_j\}_{j=1}^{N}$. (10 points)

(b) Now suppose that log TFP in each sector i (i.e. $\log T_i$) is a normal random variable with variance σ^2. Furthermore, suppose that TFPs are independent across sectors. What is the variance of log household utility (i.e. $\log U$)? (15 points)

(c) How does this variance depend on the parameters λ, β, and N? What is the intuition for this dependence? (5 points)

Question 2 (30 points)

Consider a small open economy. There is a representative household that supplies $L = 50$ units of labor inelastically and that has preferences over goods from each of $N+1$ sectors given by:

$$U = \prod_{i=1}^{N+1} \left(\frac{C_i}{\alpha_i} \right)^{\alpha_i}$$

where C_i is final consumption of sector i output and $\sum_{i=1}^{N+1} \alpha_i = 1$. Goods from sectors $1, \cdots, N$ are produced domestically, while goods from sector $N+1$ are imported from abroad at an exogenous price P^*. Output for each domestic sector $i \in \{1, \cdots, N\}$ is produced under perfect competition. The production technology combines labor L_i and materials M_i as follows:

$$X_i = T_i \left(\frac{L_i}{\lambda} \right)^{\lambda} \left(\frac{M_i}{1-\lambda} \right)^{1-\lambda}$$

where $\lambda \in (0,1)$ and T_i denotes TFP in sector i. Materials M_i are produced by combining intermediates from all sectors (including the imported sector):

$$M_i = \prod_{j=1}^{N+1} \left(\frac{X_{ij}}{\beta_{ij}} \right)^{\beta_{ij}}$$

where X_{ij} denotes the quantity of sector j output used as inputs in sector i and $\sum_{j=1}^{N+1} \beta_{ij} = 1$ for all i. In what follows, assume that the wage is exogenous and equal to 1.

(a) First, suppose that all output from domestic sectors $\{1, \cdots, N\}$ is consumed domestically and hence there are no exports. Suppose that in equilibrium, given household utility maximization and firm cost minimization, the nominal value of imports by final consumers is equal to 5 and the nominal value of imports by domestic producers is equal to 10. In this equilibrium, what is the first-order effect of a change in the import price on household welfare, $\frac{\partial \log U}{\partial \log P^*}$? Show clearly how you arrived at your answer. (20 points)

(b) Now, suppose instead that every sector exports a fixed fraction $\kappa \in (0,1)$ of its output to the rest of the world. All other parameters of the model remain the same as before. How does the first-order effect of a change in the import price on household welfare, $\frac{\partial \log U}{\partial \log P^*}$, depend on κ? Explain clearly the reasoning for your answer. (10 points)

Question 3 (30 points)

Consider a closed economy with a set of N firms, $\Omega \equiv \{1, \cdots, N\}$. These firms are potentially connected in a production network $E \equiv \{e_{ij}\}_{i,j\in\Omega}$, where $e_{ij} = 1$ if firm j supplies firm i and $e_{ij} = 0$ otherwise. Firms are also characterized by endogenous productivities $\{\phi_i\}_{i\in\Omega}$, which depend on exogenous firm technologies $\{T_i\}_{i\in\Omega}$ and the production network in the following way:

$$\phi_i = T_i + \alpha \sum_{j \neq i} e_{ij}\phi_j$$

We will refer to the parameter α as *input suitability*. Assume throughout that $\alpha < \frac{1}{N-1}$. Relationships are costly, such that each firm has to pay a fixed cost f for each active supplier. Given its productivity ϕ_i and set of suppliers, profits for firm i are then given by:

$$\pi_i = A\phi_i - \sum_{j \neq i} e_{ij} f$$

where A is an exogenous aggregate profit shifter.

Now assume that all firms have the same exogenous technology, so that $T_i = T$ for all $i \in \Omega$. We will say that a network E is *stable* if there is no firm in Ω that can achieve *strictly greater* profits by unilaterally changing its set of active suppliers, holding constant the set of active suppliers for all other firms.

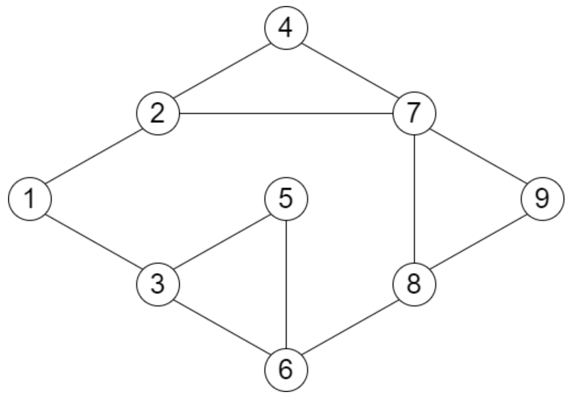
(a) Under what range of values for input suitability α is the empty network stable? How does this range depend on the parameters $\{A, T, f, N\}$ and why? (Hint: check how profits for an individual firm would change if it were to add n suppliers. This cannot yield strictly greater profits for the firm for any $n = 1, \cdots, N-1$.) (10 points)

(b) Under what range of values for input suitability α is the complete network stable? How does this range depend on the parameters $\{A, T, f, N\}$ and why? (Hint: check how profits for an individual firm would change if it were to drop n suppliers. This cannot yield strictly greater profits for the firm for any $n = 1, \cdots, N-1$. This is a harder problem than part (a).) (15 points)

(c) Under what range of values for input suitability α are both the empty and complete networks stable? For this range of values for α, which network yields greater profits for each firm? (5 points)

Question 4 (30 points)

The economy is comprised of a set of $N = 9$ locations arranged in a network as shown in the figure below:



Goods are transported between locations in this network by a set of traders. The goal of our analysis will be to characterize the cost of transporting goods from one location to another. The transportation technology is based on the following assumptions.

First, goods can only be transported between *connected* locations, where connections are represented by links in the network above. Links are bidirectional. For example, goods can only move from location 1 to locations $\{2, 3\}$ directly, while goods can only move from location 2 to locations $\{1, 4, 7\}$ directly.

Second, transporting goods across each link of the network incurs an additive transport cost of $\tau = 1$. This cost is the same for all links and is independent of the direction being traveled. For example, transporting goods from location 1 to location 6 through the trade route $1 \to 3 \to 5 \to 6$ incurs a total transport cost of 3τ.

Third, traders are not very sophisticated in choosing their trade routes and move about the network at *random*. Specifically, suppose that a trader is tasked with transporting goods from some location i to some other location $j \neq i$. When the trader first leaves i, she selects the next location k to move to from among the set of locations that are connected to i with *equal probability*. If $k = j$, the trader delivers the good and the path is terminated. If $k \neq j$, the trader then moves to another location connected to k, again with equal probability over the set of locations connected to k. This process repeats until the trader reaches j.

Under the above assumptions, the total transportation cost incurred in shipping goods from i to j is a random variable. We are now interested in characterizing the *expected* transportation cost of shipping from i to j, denoted by T_{ij}. (We assume $T_{ii} = 0$.) To specify this formally, let Ω^{ij} denote the set of all possible paths for shipping from i to j. For a path $\omega \in \Omega^{ij}$, let t^{ij}_{ω} denote the total trade cost incurred along the path and let p^{ij}_{ω} denote the probability that this path is chosen by a trader who is shipping from i to j. Then, the expected trade cost of shipping from i to j for $j \neq i$ is:

$$T_{ij} = \sum_{\omega \in \Omega^{ij}} p_{\omega}^{ij} t_{\omega}^{ij}$$

As an example, consider the problem of shipping from location 1 to 6. Two possible paths through which this trade might occur are:

- Path A: $1 \to 3 \to 6$.
  - Starting from 1, the trader chooses $1 \to 3$ with probability $\frac{1}{2}$.
  - Conditional on reaching 3, the trader chooses $3 \to 6$ with probability $\frac{1}{3}$.
  - Hence, the probability that path A is chosen is $p_A^{16} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.
  - The total trade cost incurred on this path is $t_A^{16} = 2\tau$.
- Path B: $1 \to 3 \to 5 \to 6$.
  - Starting from 1, the trader chooses $1 \to 3$ with probability $\frac{1}{2}$.
  - Conditional on reaching 3, the trader chooses $3 \to 5$ with probability $\frac{1}{3}$.
  - Conditional on reaching 5, the trader chooses $5 \to 6$ with probability $\frac{1}{2}$.
  - Hence, the probability that path B is chosen is $p_B^{16} = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{12}$.
  - The total trade cost incurred on this path is $t_B^{16} = 3\tau$.

However, there are many other paths that could also be chosen (for example, $1 \to 3 \to 5 \to 3 \to 6$). In fact, since we allow traders to retrace their steps, it should be easy to see that there is an *infinite* number of possible paths for transporting goods between any pair of locations in the network. Hence, computing expected trade costs manually is infeasible. However, this can be easily done using approaches that we covered in the course (hint: think *recursively*) and some programming.

(a) First, compute the expected cost of trading between every pair of locations in the network, T_{ij}. Explain clearly your approach to doing this. Which location has the highest average expected trade cost to other locations, $\bar{T}_i \equiv \frac{1}{N-1} \sum_{j \neq i} T_{ij}$? Which location has the lowest average expected trade cost to other locations? (15 points)

(b) Next, let $\Theta \equiv \frac{1}{N} \sum_{i=1}^{N} \bar{T}_i$ denote the average expected trade cost between all pairs of locations in the network. Define the *importance* of a link in the network as the increase in the average trade cost Θ when that link is removed from the network. Which link in the network above has the highest importance? (15 points)