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INTRODUCTION

Productivity change is conventionally defined as the residual growth of real product not accounted for by the growth of real factor input.[1] The contribution of productivity change as a source of economic growth is then measured by the percentage of the growth rate of real product " explained " by the productivity residual, i.e. by the ratio of the latter to the former. Unfortunately, this method of estimating the contribution of productivity change to economic growth is rather misleading and tends to misstate its true impact.[2] The problem lies in the fact that some inputs are themselves outputs of the productive process: capital and intermediate input. Increased factor efficiency will, in general, lead to increased output, and thus to increases in the quantity of produced inputs available for production. In any post-mortem assessment of the sources of growth, this induced expansion in produced inputs must be recognized as having been the result of productivity change. That is, the growth rate of total factor productivity must be adjusted for the additional input available as a result of the increased factor efficiency. This adjustment is made for the induced accumulation of capital in an earlier paper, and the adjustment is found nearly to double the importance of technical progress as a source of growth.[3]

The present paper studies the interaction of productivity change and intermediate input.[4] The expansion in the production of intermediate goods occurring because of increased factor efficiency makes it important to distinguish between productivity change *originating in* a sector and the *impact* of productivity change *on* the sector. Productivity change in the first sense refers to the shift in the sectoral technology and is measured by the conventional productivity residual. Productivity change in the second sense measures the equilibrium response to the shifting sectoral technologies, and includes (*a*) the induced reallocation of factor input between sectors, and (*b*) the induced expansion in intermediate input, which serves to magnify the effect of technical change. In assessing the importance of productivity change as a source of growth, it is the second sense which is relevant, since it is the impact of productivity change which affects the evolution of the sector, and not the change in factor efficiency occurring within that sector.

The distinction between the two aspects of productivity change is analogous to the distinction between nominal and effective tax incidence. An ad-valorem excise tax imposed at a given statutory rate may be regarded as shifting the commodity supply curve upward; the distributional impact of the tax depends on the equilibrium adjustment to this shift. The well-known Harberger (1962) model of tax incidence was, in fact, specifically intended to take these adjustments into account. In an essentially parallel vein, this paper develops the distinction between nominal and effective rates of technical change from the point of view of productivity analysis.

Effective rates of productivity change are defined in this paper using the reduced form of an N-sector growth model, where the rate of change of total factor supply, and the rate of change of technological efficiency are assumed to be given exogenously. The growth rates of the endogenous variables—prices and quantities—are expressed as linear combinations of the exogenous growth rates, and the effective rate of productivity change is defined as the total impact (via the reduced form) of efficiency change on the growth rate of sectoral final demand. The following result is then obtained: the weighted average of the sectoral effective rates is equal to the rate of change of the social production possibility frontier, which is, by definition, the aggregate rate of productivity change. The aggregate rate of productivity change is also shown to be the weighted sum of the conventional productivity residuals. This last result validates the aggregation procedure proposed in Domar (1961).

The paper has the following organization. The aggregate rate of productivity change is derived in Section 1. In Section 2, the convention residuals are defined for each sector. The aggregate rate is then shown, in Section 3, to equal the Domar weighted sum of the conventional sectoral residuals. In Section 4, the effective rate of productivity change for each sector is defined, conditions are derived under which the aggregate rate of Section 1 is the weighted average of the effective rates. An alternative and more general derivation of the effective rate is given in Section 5.

1. AGGREGATE PRODUCTIVITY CHANGE

Aggregate productivity change can be thought of as the expansion in the social production possibility set, holding real factor input constant.[5] Letting $Y = (Y_1, ..., Y_N)$ denote the vector of real final demand (consumption and investment), and $J = (J_1, ..., J_K)$ denote the vector of total primary factor supply, the social production possibility frontier is defined implicitly by

$$F(Y, J, t) = 0. \qquad \text{...(1)}$$

We assume that (1) is continuously differentiable and homogeneous of degree zero in Y and J, implying

$$\sum_{i=1}^{N} \frac{\partial F}{\partial Y_i} Y_i + \sum_{k=1}^{K} \frac{\partial F}{\partial J_k} J_k = 0. \qquad \text{...(2)}$$

We assume also that the economy is in competitive equilibrium:

$$\begin{aligned} -\frac{\partial F/\partial Y_i}{\partial F/\partial Y_1} &= p_i \quad i = 2, ..., N \\ \frac{\partial F/\partial J_k}{\partial F/\partial Y_1} &= w_k \quad k = 1, ..., K \end{aligned} \qquad \text{...(3)}$$

where $p = (1, p_2, ..., p_N)$ and $w = (w_1, ..., w_K)$ are the normalized vectors of product and factor prices respectively.

Aggregate productivity change can now be defined as the rate of change of $F(\cdot)$ with respect to time, holding primary input J constant. Total differentiation of (1) yields

$$\sum_{i=1}^{N} \frac{\partial F}{\partial Y_i} \dot{Y}_i + \sum_{k=1}^{K} \frac{\partial F}{\partial J_k} \dot{J}_k + \dot{F} = 0$$

where dots over variables denote derivatives with respect to time.[6] It follows from (2) that

$$\sum_{i=1}^{N} \frac{\frac{\partial F}{\partial Y_i} Y_i}{\sum \frac{\partial F}{\partial Y_i} Y_i} \frac{\dot{Y}_i}{Y_i} + \sum_{k=1}^{K} \frac{\frac{\partial F}{\partial J_k} J_k}{\sum \frac{\partial F}{\partial J_k} J_k} \frac{\dot{J}_k}{J_k} + \frac{\dot{F}}{\sum \frac{\partial F}{\partial Y_i} Y_i} = 0. \qquad \text{...(4)}$$

The last term on the left-hand side of (4) is the rate of change of (1) with respect to the tangent at Y. In view of (3), it can be written

$$T = \frac{\dot{F}}{\sum \frac{\partial F}{\partial Y_i} Y_i} = \sum_{i=1}^{N} \frac{p_i Y_i}{\Sigma p_i Y_i} \frac{\dot{Y}_i}{Y_i} - \sum_{k=1}^{K} \frac{w_k J_k}{\Sigma w_k J_k} \frac{\dot{J}_k}{J_k} \qquad \ldots(5)$$

i.e. as the difference between the Divisia index of final demand and the Divisia index of total primary input. It is worth noting that the aggregate productivity index can be calculated from price and quantity data.

2. SECTORAL PRODUCTIVITY CHANGE

We assume now that production is non-joint, so that the technology of each sector can be characterized by a constant returns to scale production function of the form[7]

$$Q_i = F^i(X^i, J^i, t) \quad i = 1, \ldots, N \qquad \ldots(6)$$

where Q_i is sectoral gross output and $X^i = (X_{1i}, \ldots, X_{Ni})$ and $J^i = (J_{1i}, \ldots, J_{Ki})$ are the vectors of intermediate and primary input used in the ith sector. We will have occasion below to further restrict the technology to the case of Hicks-neutrality:

$$Q_i = A_i F^i(X^i, J^i) \quad i = 1, \ldots, N. \qquad \ldots(6')$$

The necessary conditions for sectoral equilibrium are

$$\frac{\partial Q_i}{\partial X_{ji}} = \frac{p_j}{p_i}, \quad \frac{\partial Q_i}{\partial J_{ki}} = \frac{w_k}{p_i} \quad \begin{matrix} i, j = 1, \ldots, N \\ k = 1, \ldots, K. \end{matrix} \qquad \ldots(7)$$

Sectoral rates of productivity change are derived by logarithmic differentiation of (6) with respect to time. This yields

$$\frac{\dot{Q}_i}{Q_i} = \frac{\dot{F}^i}{F^i} + \sum_{j=1}^{N} \left(\frac{\partial Q_i}{\partial X_{ji}} \frac{X_{ji}}{Q_i}\right) \frac{\dot{X}_{ji}}{X_{ji}} + \sum_{k=1}^{K} \left(\frac{\partial Q_i}{\partial J_{ki}} \frac{J_{ki}}{Q_i}\right) \frac{\dot{J}_{ki}}{J_{ki}} \quad i = 1, \ldots, N$$

which implies from the marginal productivity conditions (7) that

$$\frac{\dot{F}^i}{F^i} = \frac{\dot{Q}_i}{Q_i} - \sum_{j=1}^{N} \frac{p_j X_{ji}}{p_i Q_i} \frac{\dot{X}_{ji}}{X_{ji}} - \sum_{k=1}^{K} \frac{w_k J_{ki}}{p_i Q_i} \frac{\dot{J}_{ki}}{J_{ki}} \quad i = 1, \ldots, N. \qquad \ldots(8)$$

This is the well-known " residual " of productivity analysis.[8] It relates the shift in the technologies (6) to the growth rate of real product not explained by the share-weighted growth rates of the real factor inputs. It thus measures the change in factor efficiency occurring within a sector, but does not measure the impact of the change in efficiency on the growth of that sector.

3. DOMAR AGGREGATION

We now consider the relationship between the aggregate rate of productivity change (5) and the individual sectoral rate (8). It will turn out that the former is the weighted sum of the latter, with the weights being those first proposed by Domar (1961).

In deriving this result, we note that the balance of supply and demand in the product and factor markets requires that

$$Q_i = Y_i + \sum_{j=1}^{N} X_{ij} \quad i = 1, \ldots, N \qquad \ldots(9)$$

and

$$J_k = J_{k1} + \ldots + J_{kN} \quad k = 1, \ldots, K. \qquad \ldots(10)$$

Logarithmic differentiation of the balance equations yields

$$\frac{\dot{Q}_i}{Q_i} = \frac{p_i Y_i}{p_i Q_i}\frac{\dot{Y}_i}{Y_i} + \sum_{j=1}^{N} \frac{p_i X_{ij}}{p_i Q_i}\frac{\dot{X}_{ij}}{X_{ij}} \quad i = 1, \ldots, N \qquad \ldots(11)$$

and

$$\frac{\dot{J}_k}{J_k} = \sum_{k=1}^{K} \frac{w_k J_{ki}}{w_k J_k}\frac{\dot{J}_{ki}}{J_{ki}} \quad i = 1, \ldots, K. \qquad \ldots(12)$$

Substitution of (8) and (11) into the definition of aggregate productivity change (5) results in

$$T = \sum_{i=1}^{N} \frac{p_i Q_i}{\Sigma p_i Y_i}\frac{\dot{F}^i}{F^i} + \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{p_j X_{ji}}{\Sigma p_i Y_i}\frac{\dot{X}_{ji}}{X_{ji}} + \sum_{i=1}^{N}\sum_{k=1}^{K} \frac{w_k J_{ki}}{\Sigma p_i Y_i}\frac{\dot{J}_{ki}}{J_{ki}}$$
$$- \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{p_i X_{ij}}{\Sigma p_i Y_i}\frac{\dot{X}_{ij}}{X_{ij}} - \sum_{k=1}^{K} \frac{w_k J_k}{\Sigma w_k J_k}\frac{\dot{J}_k}{J_k} \qquad \ldots(13)$$

from which it follows, using (12), that

$$T = \sum_{i=1}^{N} \frac{p_i Q_i}{\Sigma p_i Y_i}\frac{\dot{F}^i}{F^i}. \qquad \ldots(14)$$

This is essentially the aggregation rule proposed by Domar (1961). The equality in (14) indicates that Domar's procedure is equivalent to the rate of change of the social production possibility frontier holding primary input constant. This is one of the basic results of the paper.

The sum of the weights in (14) is greater than or equal to one, since $p_i Q_i \geqq p_i Y_i$ by (9). The intuitive reason is that the change in sectoral factor efficiency creates, in general, extra output Q_i, which serves to increase both final demand Y_i and intermediate deliveries X_{ij}. The increase in intermediate deliveries, however, serves further to increase output in those sectors using the intermediate good, and this further increases output, and so on. The total effect of sectoral productivity change on the social production possibility set is thus greater than the direct effects of the $\dot{F}_i/F_i$, and this is reflected in the weighting scheme of (14).

4. THE EFFECTIVE RATE OF PRODUCTIVITY CHANGE

The above interpretation of (14) can be justified by considering an alternative characterization of sectoral productivity change. Rather than defining productivity change in terms of its magnitude, we will define it in terms of its impact on the growth rate of final demand in each sector. It will then be shown that the weighted *average* of these " effective rates " of productivity change is equivalent to the Domar aggregation procedure, i.e. that

$$T = \sum_{i=1}^{N} \frac{p_i Y_i}{\Sigma p_i Y_i}\frac{\dot{Z}_i}{Z_i} = \sum_{i=1}^{N} \frac{p_i Q_i}{\Sigma p_i Y_i}\frac{\dot{F}^i}{F^i} \qquad \ldots(15)$$

where $\dot{Z}_i/Z_i$ is the effective rate of productivity change in the ith sector. The implication of (15) for Domar aggregation is that the Domar weights produce the same result as productivity change measured in terms of its effect.

The effective rates of productivity change are derived from the reduced form of the N-sector model of Section 2. The prices p and w, and the quantities Y, X^i, J^i, and $Q = (Q_1, \ldots, Q_N)$ are assumed to be uniquely determined at each moment in time by the exogenously given supply of primary factors, J, and the exogenously determined level of technology. We assume further that technical change in each sector is Hicks-neutral, so that the level of technology can be parameterized by the $A = (A_1, \ldots, A_N)$ defined in (6′). Finally, to close the model we assume the existence of demand functions for each good:

$$Y_i = \Gamma^i(p_2, \ldots, p_N, w_1, \ldots, w_K) \quad i = 2, \ldots, N. \qquad \ldots(16)$$

The logarithmic differential equations (8), (11), and (12), along with the logarithmic differentials of (16) and (7) constitute a system of $N(N+3)+K(N+1)-1$ linear equations in the growth rates of the endogenous and exogenous variables. Assuming that

$$N(N+3)+K(N+1)-1$$

endogenous variables, are uniquely determined by the $N+K$ exogenous growth rates, a reduced form can be derived in which the growth rates of the endogenous variables depend linearly on the growth rates of A and J.

Since we are interested in the impact of the $\dot{A}_i/A_i$ on sectoral final demand, we focus attention on the reduced form equation for Y:

$$\frac{\dot{Y}_i}{Y_i} = \sum_{j=1}^{N} \gamma_{ij} \frac{\dot{A}_j}{A_j} + \sum_{k=1}^{K} \eta_{ik} \frac{\dot{J}_k}{J_k} \quad i = 1, \ldots, N. \qquad \ldots(17)$$

The γ_{ij} are partial elasticities which indicate the full equilibrium response in final demand in the ith sector to the changing technology of the jth sector. These partial elasticities therefore have the property that they fully account for the impact of efficiency change in all sectors on the growth of final demand in each sector. This corresponds to the definition of effective rate of productivity change discussed in the introduction. Formally, this effective rate is given by

$$\frac{\dot{Z}_i}{Z_i} = \gamma_{i1} \frac{\dot{A}_1}{A_1} + \ldots + \gamma_{iN} \frac{\dot{A}_N}{A_N} \quad i = 1, \ldots, N. \qquad \ldots(18)$$

This formulation does not, in general, have empirical content since the γ_{ij} are complicated functions of various substitution elasticities and commodity shares.[9]

It turns out, however, that the partial elasticities η_{ik} can be related to prices and quantities, so that (15) can be derived from (17) and (18). From the linearity of (17) in the rates of growth $\dot{A}_i/A_i$ and $\dot{J}_k/J_k$,

$$\eta_{ik} = \left. \frac{\dot{Y}_i/Y_i}{\dot{J}_k/J_k} \right|_{A_i \text{ and } J_j \neq J_k, \text{ constant}} \quad \begin{matrix} i = 1, \ldots, N \\ j, k = 1, \ldots, K \end{matrix} \qquad \ldots(19)$$

η_{ik} is thus a multiplier associated with $\dot{J}_k/J_k$. Now, having assumed that final demand is uniquely determined by technology, A, and total factor supply, J, (1) can be written

$$F(Y(J, A), J, A) = 0. \qquad \ldots(20)$$

Total differentiation of (20) with respect to J_k, holding A and J_i, $i \neq k$, constant, results in

$$\sum_{i=1}^{N} \left. \frac{\partial F}{\partial Y_i} \frac{dY_i}{dJ_k} \right|_{A, J_i \neq J_k, \text{ constant}} dJ_k + \frac{\partial F}{\partial J_k} dJ_k = 0. \qquad \ldots(21)$$

Or, in view of (19),

$$\sum_{i=1}^{N} \frac{\partial F/\partial Y_i}{\partial F/\partial J_k} \frac{Y_i}{J_k} \eta_{ik} + 1 = 0 \qquad \ldots(22)$$

which, from (2) implies

$$\sum_{i=1}^{N} \frac{p_i Y_i}{w_k J_k} \eta_{ik} = 1 \qquad \ldots(23)$$

Equations (17) and (18) can now be substituted in (4) to yield

$$T = \sum_{i=1}^{N} \frac{p_i Y_i}{\Sigma p_i Y_i} \frac{\dot{Z}_i}{Z_i} + \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{p_i Y_i \eta_{ij}}{\Sigma p_i Y_i} \frac{\dot{J}_k}{J_k} - \sum_{k=1}^{K} \frac{w_k J_k}{\Sigma p_i Y_i} \frac{\dot{J}_k}{J_k}. \qquad \ldots(24)$$

The first equality in (15) follows directly from (23).

5. ALTERNATIVE DERIVATION

The equivalence between the aggregate rate (5) and the weighted average of the effective rates is intuitively plausible. Since we have accounted for the impact of productivity change on final demand in each sector through the $\dot{Z}_i/Z_i$, we would expect that the share-weighted average of the $\dot{Z}_i/Z_i$ would itself provide a reasonable definition of aggregate productivity change. One would, in fact, expect the result to hold without the restrictive assumption of Hicks-neutral technical change. This is indeed the case. It is also the case that the assumption of non-joint production can be relaxed.

Associated with the production possibility frontier (1) and the equilibrium conditions (3) is a restricted profit function $\Pi(p, J, t)$ which is homogeneous of degree one in p given J and t, and homogeneous of degree one in J given p and t. Furthermore, from Shephard's lemma for restricted profit functions,[10]

$$Y = \frac{\partial \Pi}{\partial p}(p, J, t) = f(p, J, t) \qquad \text{...(25)}$$

$$w = \frac{\partial \Pi}{\partial J}(p, J, t) = g(p, J, t).$$

Since Π is homogeneous of degree one in p, f is homogeneous of degree zero, implying

$$Y = f(p, J, t) = f(p/I, J, t) \qquad \text{...(26)}$$

where $I = \Sigma w_k J_k$ denotes aggregate consumers' income. Given that total factor supplies are exogenously determined, the demand functions (16) can be written

$$Y = \Gamma(p, I) \qquad \text{...(27)}$$

since the J_k are, by hypothesis, invariant to p and w.[11] Since (16) is homogeneous of degree zero in p and w, (27) is homogeneous of degree zero in p and I, implying that, in equilibrium, $\Gamma(p/I) = f(p/I, J, t)$. We assume that this equilibrium relationship uniquely determines the normalized prices p/I:

$$\frac{p}{I} = \phi(J, t). \qquad \text{...(28)}$$

so that:

$$Y = f(p, J, t) = f(p/I, J, t) = f(\phi(J, t), J, t) \qquad \text{...(29)}$$

implying that Y is a function of J and t alone, and that (29) is a reduced form equation for Y given the exogeneity of primary factor supply, J, and technology. The effective rate of productivity change in the ith sector can therefore be defined as the rate of change of Y, in (29), holding J constant:

$$\begin{aligned}\frac{\partial Y_i/\partial t}{Y_i} &= \frac{\dot{Y}_i}{Y_i} - \sum_{j=1}^{N} \frac{\partial f^i}{\partial p_j} \sum_{k=1}^{K} \frac{\partial \phi^j}{\partial J_k} \frac{\dot{J}_K}{Y_i} - \sum_{k=1}^{K} \frac{\partial f^i}{\partial J_k} \frac{\dot{J}_k}{Y_i} \\ &= \frac{\dot{Y}_i}{Y_i} - \sum_{j=1}^{N} \frac{\partial^2 \Pi}{\partial p_i \partial p_j} \sum_{k=1}^{K} \frac{\partial \phi^j}{\partial J_k} \frac{\dot{J}_k}{Y_i} - \sum_{k=1}^{K} \frac{\partial^2 \Pi}{\partial p_i \partial J_k} \frac{\dot{J}_k}{Y_i} \quad i = 1, \ldots, N \end{aligned} \qquad \text{...(30)}$$

This is the analogue of (18), but without the assumption of Hicks-neutral technical change. Consider, now, an aggregate rate of productivity change defined as the share-weighted average of the effective rates (30):

$$T^* = \sum_{i=1}^{N} \frac{p_i Y_i}{\Sigma p_i Y_i} \frac{\partial Y_i/\partial t}{Y_i} \qquad \text{...(31)}$$

Because $\partial\Pi/\partial p_j$ is homogeneous of degree zero in p,

$$\sum_{i=1}^{N} p_i \frac{\partial^2 \Pi}{\partial p_i \partial p_j} = 0 \qquad \ldots(32)$$

for every j. Furthermore, Π is homogeneous of degree one in p given J, implying that $\partial\Pi/\partial J_k$ is also homogeneous of degree one in p, so that

$$\sum_{i=1}^{N} p_i \frac{\partial^2 \Pi}{\partial p_i \partial J_k} = \frac{\partial \Pi}{\partial J_k} = w_k \qquad \ldots(33)$$

Substitution of (30) into (31) then implies $T^* = T$ by (32) and (33). In other words, T^* is equivalent to the rate of change of (1) holding J constant.

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NOTES

1. The literature on productivity change includes, among others, Christensen and Jorgenson (1969, 1970), Denison (1962, 1967), Jorgenson and Griliches (1967), Kendrick (1961, 1970), and Solow (1957). For studies which explicitly allow for intermediate goods, see Star (1974) and Watanabe (1971).

2. We will generally use the term " productivity change " instead of " technical change " since the main focus of the paper is on changes in factor efficiency. " Productivity change " is also used to denote changes in total factor productivity, which, if correctly defined, should be equivalent to efficiency change.

3. See Hulten (1975), which uses data from the Christensen–Jorgenson (1969, 1970) studies. The adjusted residual is found to be 2·67% per annum and to account for nearly 64% of the growth rate of output. The average annual Christensen–Jorgenson residual is 1·42% (for the period 1948-66), and explains only 34% of the growth rate of output.

4. Technical change with intermediate input has been studied extensively by those primarily interested in the implications for international trade. See, for example, Casas (1972*a*, 1972*b*) and Melvin (1969*b*), and references contained therein. The paper by Melvin discusses some of the difficulties which are encountered in a two-sector model of technical change with intermediate goods.

5. An alternative formulation, due to Solow (1957), can be given in terms of an aggregate production function. However, when separate sectoral production functions are assumed to exist, the social production possibility frontier reduces to the aggregate production function only under highly restrictive assumptions on the sectoral technologies (see Green (1966), and Hall (1973)).

6. All variables are assumed to be smooth functions of time (e.g. $Y_i(t)$, $J_k(t)$, etc.).

7. The technologies are also assumed to be strictly quasi concave and continuously differentiable. These restrictions are not, however, sufficient to guarantee that the production possibility set is non-empty and bounded (see Melvin (1969*a*)). In the two sector—one primary input case, Melvin shows that additional restrictions include (a) $F^i > 0$ implies $X^i > 0$ and $J^i > 0$, and (b) that the isoquants of F^i do not intersect the X and J axes.

8. A more common form of the residual involves only primary inputs and uses value added as the measure of real product. If intermediate goods are in fact used in production, this approach is appropriate if, and only if, the relevant technologies are of the form

$$Q_i = F^i(g_i(J_{i1}, J_{iK}, t), X_{i1}, \ldots, X_{iN}).$$

9. Results have, however, been obtained in special cases. For example, in discussing the Findlay–Grubert Theorem, Casas (1972a) obtains an exact expression which is equivalent to (18) for the case of two sectors, two primary inputs, and Hicks-neutral technical change in one sector.

10. For references to restricted profit functions, see Gorman (1968), Diewert (1974), and Lau (1976).

11. This assumes a single consumer. For the case of many consumers, the homotheticity of preferences must be added to insure invariance to the distribution of the fixed J_k between consumers.

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