ECO 2302 - Networks in Trade and Macroeconomics

Lecture 6 - Networks and International Trade

Motivation

- Trade is an inherent feature of production networks
  - i.e. firm/sector i trades some output to firm/sector j
- But so far we have not really emphasized...
  - the distinction between international vs. domestic trade
- Today we will examine role of networks in *international* trade specifically
- Tintelnot et al (2019), "Trade and Domestic Production Networks"
  - how do domestic firm-to-firm linkages matter for the gains from international trade?
- Chaney (2013), "The Network Structure of International Trade"
  - to what extent can dynamics of firm customer networks explain geographic distribution of firm exports?

Trade and Domestic Production Networks

- Tintelnot, Kikkawa, Mogstad, and Dhyne (2019), "Trade and Domestic Production Networks"
- Most firms do not engage in international trade directly
  - but may trade indirectly via other firms
- Do domestic production networks matter for the gains from trade?
- Study high-quality firm-to-firm transactions data from Belgium
- Develop model of firm-to-firm production networks
- Study effects of international trade shocks
  - on firm production costs and welfare
  - with both exogenous and endogenous networks

Data

- Three main sources of data from National Bank of Belgium
- B2B transactions data:
  - all transactions >€250 annually are recorded
  - data for 896,000 VAT-IDs, 860,000 firms
  - also studied by Bernard et al (2018)
- Annual accounts data:
  - detailed information from firm balance sheets
  - revenues, capital, labor, intermediates
  - 4-digit NACE industry codes
  - geographic identifiers at zip code level
  - ownership shares in other enterprises (use this to identify unique firms)
- Customs records and intra-EU trade declarations:
  - import and export data by firm-year, product, and origin/destination

Sample Selection

- Restrict analysis to firms:
  - in private and non-financial sectors
  - with positive labor cost and at least one full-time-equivalent employee
  - with positive output
  - with tangible assets of more than 100 Euros
  - with positive total assets in at least one year during sample period
- Applying these criteria reduces the number of firms significantly
  - e.g. in 2012, no. of firms drops from around 860,000 to 98,745
  - mostly driven by exclusion of firms without employees (750,100 in 2012)

Sample Selection

Year	GDP	Output	Imports	Exports	Selected sample				
	(Excl.	Gov. & Fin.)			Count	V.A.	Sales	Imports	Exports
2002	182	458	178	193	88,301	119	604	175	185
2007	230	593	254	267	95,941	152	782	277	265
2012	248	671	317	319	98,745	164	874	292	292

Notes: All numbers except for Count are denominated in billion Euro in current prices. Belgian GDP and output are for all sectors excluding public and financial sector. See Appendix C.2 for the same statistics for the total economy. Data for Belgian GDP, output, imports, and exports are from Eurostat. Firms' value added is from the reported values from the annual accounts. This reported value added is not necessarily the same as the value added measure used later in the quantitative analysis, which is the sales value minus imports and purchases from other Belgian firms in the sample.

Source: Tintelnot et al (2019)

Indirect Exposure to Imports

- Let s_{Fi} denote firm i's direct exposure to imported inputs:

$$s_{Fi} = \frac{(\text{imported inputs})_i}{(\text{labor costs})_i + (\text{domestic inputs})_i + (\text{imported inputs})_i}$$

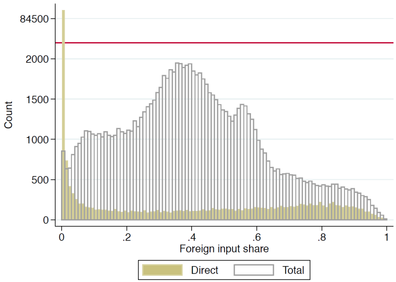
- Note that if firm i does not import directly, s_{Fi} will be zero
- Now define s_{Fi}^{total} as firm i's total exposure to imported inputs:

$$s_{Fi}^{total} = s_{Fi} + \sum_{j \in \Omega_i^{in}} s_{ji} s_{Fj}^{total}$$

  - Ω_i^{in}: set of firms that supply inputs to firm i
  - s_{ji}: share of firm i's inputs purchased from firm j
- Given data on s_{Fi} and s_{ij}, can easily compute s_{Fi}^{total} for each firm
- Note that linear specification will be rationalized by CES production model

Indirect Exposure to Imports

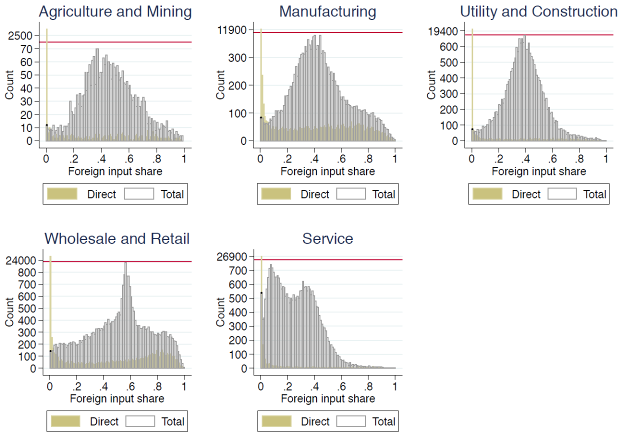
(a) Direct and total foreign input share



Source: Tintelnot et al (2019)

- Only 19% of firms import directly
  - but almost all firms obtain some foreign inputs either directly or indirectly
- Total foreign input share of median firm = 39%
  - but there is substantial variation in total foreign input share

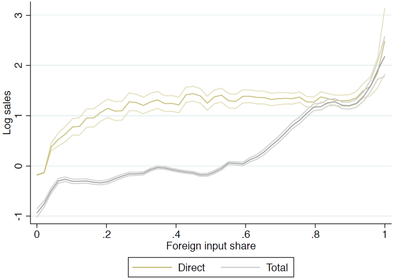
Indirect Exposure to Imports



Source: Tintelnot et al (2019)

Indirect Exposure to Imports

(a) Foreign inputs



Source: Tintelnot et al (2019)

- Log sales demeaned by industry average
- Firm size increasing in both direct and indirect import shares
  - but direct importers are much larger than firms that import only indirectly
- Suggests that firms do not have to be very large to rely heavily on foreign inputs
  - contrary to conventional wisdom based on direct input shares alone

Indirect Exposure to Exports

- Let r_{iF} denote firm i's direct exposure to exports:

$$r_{iF} = \frac{(\text{export revenue})_i}{(\text{total revenue})_i}$$

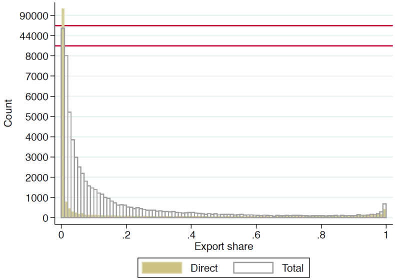
- Note that if firm i does not export directly, r_{iF} will be zero
- Now define r_{iF}^{total} as firm i's total exposure to exports:

$$r_{iF}^{total} = r_{iF} + \sum_{j \in \Omega_i^{out}} r_{ij} r_{jF}^{total}$$

  - Ω_i^{out}: set of firms that purchase inputs from firm i
  - r_{ij}: share of firm i's revenues from sales to firm j
- Given data on r_{iF} and r_{ij}, can easily compute r_{iF}^{total} for each firm
- Note that linear specification will be rationalized by CES production model

Indirect Exposure to Exports

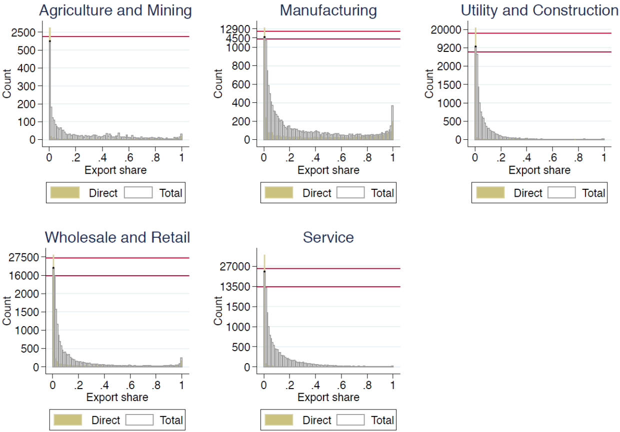
(b) Direct and total export share



Source: Tintelnot et al (2019)

- Only 12% of firms export directly
  - but 88% of firms export either directly or indirectly
- Total foreign export share of median firm = 2%
  - exporting highly concentrated even taking into account indirect exposure
  - most firms have many domestic suppliers but few domestic customers

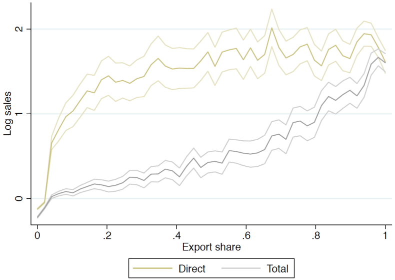
Indirect Exposure to Exports



Source: Tintelnot et al (2019)

Indirect Exposure to Exports

(b) Exports



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- Log sales demeaned by industry average
- Firm size increasing in both direct and indirect export shares
  - but direct exporters are much larger than firms that export only indirectly
- Suggests that firms do not have to be very large to rely heavily on exports
  - contrary to conventional wisdom based on direct export shares alone

Exogenous Network Model Overview

- Now develop model of production networks with international trade
- Model Belgium as a small open economy
  - i.e. foreign variables (e.g. import prices) are taken as exogenous
- First take the production network between Belgian firms as exogenous
- Households:
  - supply labor inelastically
  - have CES preferences over final consumption goods
- Discrete set of heterogeneous firms:
  - produce differentiated products
  - using CES technology combining labor and domestic/foreign intermediates
- Use model to study effect of changes in foreign input prices
  - on firm production costs and household welfare

Demand

- Representative household supplies one unit of labor inelastically
- CES preferences over final consumption goods:

$$U = \left[\sum_{i \in \Omega} (\beta_{iH} q_{iH})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

  - Ω: set of firms
  - β_{iH} : exogenous taste shifter for firm i
  - q_{iH} : quantity of firm i's output consumed by household
  - $\sigma > 1$: elasticity of substitution across products

Demand

- Final domestic demand:

$$p_{iH} q_{iH} = E \left(\frac{p_{iH}/\beta_{iH}}{P} \right)^{1-\sigma}$$

  - E: total household expenditure
  - p_{iH}: price charged by firm i to the household
- Consumer price index inherits CES form:

$$P = \left[\sum_{k \in \Omega} (p_{iH}/\beta_{iH})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

Demand

- Also assume that foreign final demand is derived from CES preferences:

$$p_{iF} q_{iF} = E_F \left(\frac{p_{iF}/\beta_{iF}}{P_F} \right)^{1-\sigma}$$

  - E_F: total foreign expenditure
  - p_{iF}: price charged by firm i to foreign buyers
  - β_{iF} : foreign taste shifter for firm i's output
  - P_F : foreign consumer price index
- Assume that $\{E_F, P_F\}$ are exogenous (small open economy)

Production

- Firms produce using CES technology combining labor and inputs:

$$X_i = \phi_i \left[(\alpha_{Li} l_i)^{\frac{\rho-1}{\rho}} + \sum_{j \in \Omega_i^{in}} (\alpha_{ji} q_{ji})^{\frac{\rho-1}{\rho}} + I_{Fi} (\alpha_{Fi} q_{Fi})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

  - ϕ_i : TFP
  - $\{\alpha_{Li}, \alpha_{ji}, \alpha_{Fi}\}$: input productivity shifters
  - l_i: labor input
  - q_{ji} :quantity of intermediates from domestic supplier j purchased by firm i
  - q_{Fi}: quantity of foreign intermediates purchased by firm i
  - Ω_i^{in}: set of domestic suppliers for firm i
  - I_{Fi}: dummy indicator $= 1$ if firm imports directly (exogenous)
  - $\rho \in (1, \sigma)$: elasticity of substitution across inputs

Production

- Marginal cost inherits CES form:

$$c_i = \frac{1}{\phi_i}\left[(w/\alpha_{Li})^{1-\rho} + \sum_{j\in\Omega_i^{in}} (p_{ji}/\alpha_{ji})^{1-\rho} + I_{Fi}\,(p_{Fi}/\alpha_{Fi})^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

  - w: wage
  - p_{ji}: price charged by firm j to firm i
  - p_{Fi}: foreign input price for firm i (exogenous)

Production

- Share of variable costs for firm i spent on labor:

$$s_{Li} = \left(\frac{w/\alpha_{Li}}{P_i}\right)^{1-\rho}$$

- Share of variable costs for firm i spent on inputs from firm j:

$$s_{ji} = \left(\frac{p_{ji}/\alpha_{ji}}{P_i}\right)^{1-\rho}$$

- Share of variable costs for firm i spent on imported inputs:

$$s_{Fi} = I_{Fi}\left(\frac{p_{Fi}/\alpha_{Fi}}{P_i}\right)^{1-\rho}$$

- Input price index for firm i:

$$P_i = \left[(w/\alpha_{Li})^{1-\rho} + \sum_{j\in\Omega_i^{in}} (p_{ji}/\alpha_{ji})^{1-\rho} + I_{Fi}\,(p_{Fi}/\alpha_{Fi})^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

Market Structure

- For simplicity, assume that all firms charge prices equal to marginal cost
  - hence markups are zero, with $p_{ij} = p_{iH} = c_i$
  - "rationalized" by assuming that buyers have all bargaining power
  - important for tractability of endogenous network model
- In the paper, allow firms to charge CES markup $\mu \equiv \frac{\sigma}{\sigma-1}$ for final sales
  - but we will abstract from this here for exposition

Effects of Foreign Price Shocks

- Now suppose that there is a uniform change in the foreign input price
  - i.e. a common change in p_{Fi} for all direct importers i
- What is the effect of this change on firms' marginal costs, c_i?
- For any variable x:
  - let $\tilde{x}$ denote new value after change in p_{Fi}
  - define $\hat{x} \equiv \tilde{x}/x$ as relative change
- Claim - changes in marginal costs are given by:

$$\hat{c}_i^{1-\rho} = \left(1 - s_{Fi}^{total}\right) \hat{w}^{1-\rho} + s_{Fi}^{total} \hat{p}_F^{1-\rho}$$

- For small (first-order) changes:

$$\frac{dc_i}{c_i} = \left(1 - s_{Fi}^{total}\right) \frac{dw}{w} + s_{Fi}^{total} \frac{dp_F}{p_F}$$

- Hence, to calculate effects of foreign price shock on firm costs...
  - need to observe/compute total foreign input share s_{Fi}

Proof of Claim

- Can write firm i's marginal cost as:

$$c_i^{1-\rho} = \left(\frac{w}{\phi_i \alpha_{Li}}\right)^{1-\rho} + \sum_{j \in \Omega_i^{in}} \left(\frac{c_j}{\phi_i \alpha_{ji}}\right)^{1-\rho} + I_{Fi} \left(\frac{p_{Fi}}{\phi_i \alpha_{Fi}}\right)^{1-\rho}$$

- With new value of p_{Fi}, costs change to $\tilde{c}_i$ and wages change to $\tilde{w}$:

$$\tilde{c}_i^{1-\rho} = \left(\frac{\tilde{w}}{\phi_i \alpha_{Li}}\right)^{1-\rho} + \sum_{j \in \Omega_i^{in}} \left(\frac{\tilde{c}_j}{\phi_i \alpha_{ji}}\right)^{1-\rho} + I_{Fi} \left(\frac{\tilde{p}_{Fi}}{\phi_i \alpha_{Fi}}\right)^{1-\rho}$$

- In relative changes:

$$\hat{c}_i^{1-\rho} = s_{Li} \hat{w}^{1-\rho} + \sum_{j \in \Omega_i^{in}} s_{ji} \hat{c}_j^{1-\rho} + s_{Fi} \hat{p}_F^{1-\rho}$$

- In matrix form:

$$\hat{c}^{1-\rho} = s_L \hat{w}^{1-\rho} + S' \hat{c}^{1-\rho} + s_F \hat{p}_F^{1-\rho}$$

- Solving for $\hat{c}^{1-\rho}$:

$$\hat{c}^{1-\rho} = \left[I - S'\right]^{-1} s_L \hat{w}^{1-\rho} + \left[I - S'\right]^{-1} s_F \hat{p}_F^{1-\rho}$$

Proof of Claim

- Now recall that total import shares satisfy:

$$s_F^{total} = s_F + S' s_F^{total}$$

- Solving for s_F^{total}:

$$s_F^{total} = \left[I - S'\right]^{-1} s_F$$

- Since input shares must sum to 1, we have:

$$1 = s_{Li} + \sum_{j \in \Omega_i^{in}} s_{ji} + s_{Fi}$$

- In matrix form:

$$\left(I - S'\right) \mathbf{1} = s_L + s_F$$

- Multiply both sides by $[I - S']^{-1}$:

$$\mathbf{1} = \left[I - S'\right]^{-1} s_L + \left[I - S'\right]^{-1} s_F$$

- Hence:

$$\left[I - S'\right]^{-1} s_L = \mathbf{1} - s_F^{total}$$

- Substituting into expression for $\hat{c}^{1-\rho}$ completes the proof

Effects of Foreign Price Shocks

- Now for simplicity, assume that wage is exogenous
  - paper has results for endogenous wages in general equilibrium
- Then for first-order changes, $dw/w = 0$, and changes in costs are:

$$\frac{dc_i}{c_i} = s_{Fi}^{total} \frac{dp_F}{p_F}$$

- Now change in household welfare is:

$$\begin{aligned} \frac{dU}{U} &= -\sum_{i \in \Omega} s_{iH} \frac{dc_i}{c_i} \\ &= -\left(\sum_{i \in \Omega} s_{iH} s_{Fi}^{total} \right) \frac{dp_F}{p_F} \end{aligned}$$

- Claim - can also rewrite this as:

$$\frac{dU}{U} = -\frac{M}{VA} \sum_{i \in \Omega} s_{iF} \left(1 - s_{Fi}^{total}\right) \frac{dp_F}{p_F}$$

- Note that to calculate aggregate welfare effect of trade shock...
  - need to know s_{Fi}^{total} (and hence domestic production network)

Proof of Claim

- Let R_{ij} denote sales from i to j
- Under perfect competition, total production costs are equal to total sales
- Hence, total imports are given by:

$$M = \sum_{i \in \Omega} R_i s_{Fi}$$

- Substituting the relation between direct and total import shares:

$$\begin{aligned} M &= \sum_{i \in \Omega} R_i \left(s_{Fi}^{total} - \sum_{j \in \Omega} s_{ji} s_{Fj}^{total} \right) \\ &= \sum_{i \in \Omega} R_i s_{Fi}^{total} - \sum_{i \in \Omega} \sum_{j \in \Omega} R_{ji} s_{Fj}^{total} \\ &= \sum_{i \in \Omega} R_i s_{Fi}^{total} - \sum_{i \in \Omega} \sum_{j \in \Omega} R_{ij} s_{Fi}^{total} \\ &= \sum_{i \in \Omega} \left(R_{iH} + R_{iF} \right) s_{Fi}^{total} \end{aligned}$$

Proof of Claim

- Dividing by value added (i.e. by w):

$$\frac{M}{VA} = \sum_{i \in \Omega} s_{iH} s_{Fi}^{total} + \frac{X}{VA} \sum_{i \in \Omega} s_{iF} s_{Fi}^{total}$$

  where X denotes exports
- Imposing trade balance ($M = X$):

$$\frac{M}{VA} \sum_{i \in \Omega} s_{iF} \left(1 - s_{Fi}^{total}\right) = \sum_{i \in \Omega} s_{iH} s_{Fi}^{total}$$

- Substituting this into the expression for dU/U completes the proof

Effects of Foreign Price Shocks

- How does this welfare result relate to Hulten's theorem?
- Given the production network (and no markups)...
  - model is like standard multi-sector model with I-O linkages
- Hence, old results should (and do) apply, e.g. Hulten's theorem:
  - "network is irrelevant for aggregate effects of idiosyncratic cost shocks"
- So why does domestic network matter for effects of trade?

Effects of Foreign Price Shocks

- Effect of import price shock on welfare:

$$\text{import price shock} \underbrace{\rightarrow}_{(1)} \text{firm costs} \underbrace{\rightarrow}_{(2)} \text{domestic welfare}$$

- Fixed domestic production network is irrelevant for (2):

$$\underbrace{dU/U}_{\text{welfare change}} = -\sum_i \underbrace{s_{iH}}_{\text{Domar weight}} \times \underbrace{dc_i/c_i}_{\text{firm-level cost shocks}}$$

but it matters for (1) because foreign shock is not fully absorbed domestically:

$$\underbrace{dU/U}_{\text{welfare change}} = \underbrace{M/VA}_{\text{import share of value-added}} \times \underbrace{\sum_i \left(1 - s_{Fi}^{total}\right) s_{iF}}_{\text{domestic content of exports}} \times \underbrace{-dp_F/p_F}_{\text{foreign price shock}}$$

- None of this is specific to networks being at firm- vs. industry/sector-level
- With a *fixed* domestic firm-level network, implications for trade are thus:
  - not a matter of new theory, but rather...
  - a matter of **better measurement** of VA trade

General Equilibrium

- Model is closed by imposing:
  - labor market clearing
  - goods market clearing
  - trade balance
- Since foreign variables are taken as exogenous...
  - solving the model with fixed networks is fairly straightforward
  - similar to solving a model with sector-level input-output networks

Counterfactuals

- Now study effects of two kinds of trade counterfactuals:
  - 10% increase in foreign input prices
  - infinite foreign input prices (autarky)
- Use model to quantify effects on:
  - firm costs
  - household welfare
- Compare predictions with two “simpler” models:
  - simple roundabout production (c.f. Eaton and Kortum (2002))

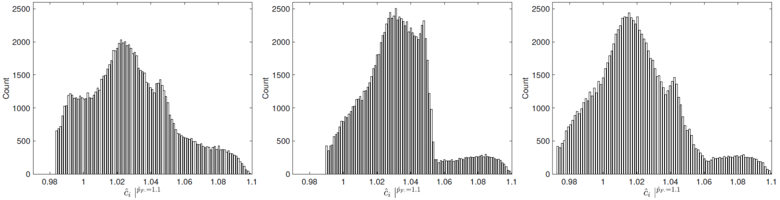
$$c_i = \frac{1}{\phi_i}\left[(w/\alpha_{Li})^{1-\rho} + (\alpha_{Di}/P)^{1-\rho} + I_{Fi}\left(p_{Fi}/\alpha_{Fi}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

  - sectoral roundabout production (c.f. Caliendo and Parro (2015))

$$c_i = \frac{1}{\phi_i}\left[(w/\alpha_{Li})^{1-\rho} + \left(\alpha_{Di}/\prod_v P_v^{\gamma_{vu(j)}}\right)^{1-\rho} + I_{Fi}\left(p_{Fi}/\alpha_{Fi}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

- In baseline counterfactuals, use 2012 data and assume $\sigma = 4$, $\rho = 2$

Counterfactuals



(a) Baseline model (b) Simple Roundabout (c) Sectoral Roundabout

Source: Tintelnot et al (2019)

- Roundabout models predict more compressed distribution of firm cost changes

Decomposition of Welfare Effects

(a) Changes in real wage upon 10% increase in foreign price

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{w}}{\hat{P}}\big\vert_{\hat{p}_{F\cdot}=1.1}$	0.940	0.941	0.931

(b) Changes in real wage upon autarky

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{w}}{\hat{P}}\big\vert_{\hat{p}_{F\cdot}\to\infty}$	0.558	0.596	0.441

Source: Tintelnot et al (2019)

- For small changes in p_F:
  - roundabout models predict similar welfare changes vis-a-vis network model
- For large changes (going to autarky):
  - differences in welfare predictions are larger

Decomposition of Welfare Effects

	Total	Direct	Indirect
10% increase in p_F.	0.940	0.968	0.973
Autarky	0.558	0.818	0.795

Source: Tintelnot et al (2019)

- Can also decompose welfare effects into direct and indirect effects
  - direct effect: replace s_{Fi}^{total} with s_{Fi} in welfare expression
  - indirect effect: replace s_{Fi}^{total} with $s_{Fi}^{total} - s_{Fi}$ in welfare expression
- About half of welfare effect is driven by indirect effect

Endogenous Network Model Overview

- In response to trade shocks, should expect firms to adjust network linkages
- To model endogenous networks, now assume that relationships are costly
  - specifically, $i \to j$ costs $f_{ij} w$
- Since firm-to-firm sales are priced at marginal cost (zero profits)...
  - need to assume that relationship costs are paid by buyers
- Incentive to form relationships with suppliers is driven by:
  - love of variety in the production function
  - positive markups for final sales (i.e. $p_{iH} = \frac{\sigma}{\sigma - 1} c_i$)
- With a discrete set of firms, solving for optimal relationships is very challenging:
  - for each firm, selecting suppliers is a large discrete choice problem
  - firms' costs are interrelated and determined in equilibrium
- To solve this problem, rely on two things:
  - a clever sourcing algorithm
  - restrictions on the set of potential buyer-seller linkages that can form

Sourcing Algorithm

- First, consider the problem of a single buyer with a set of potential suppliers
  - and take suppliers' marginal costs as given
- Can show that with $\rho < \sigma$ (inputs more substitutable than final consumption):
  - marginal benefit of adding a supplier is increasing in set of suppliers
  - i.e. suppliers are complements to one another
- Key insight:
  - suppose current supplier set for buyer i is Ω_i^{in}
  - suppose also that it is optimal for buyer i to add a new supplier j
  - if suppliers are complements, then must also be optimal to add supplier j...
  - when supplier set is $\hat{\Omega}_i^{in} \supset \Omega_i^{in}$
- This greatly reduces the dimensionality of the optimal sourcing problem
- Algorithm was first developed by Jia (2008)
  - and subsequently applied in international trade by Antras et al (2018)

Sourcing Algorithm

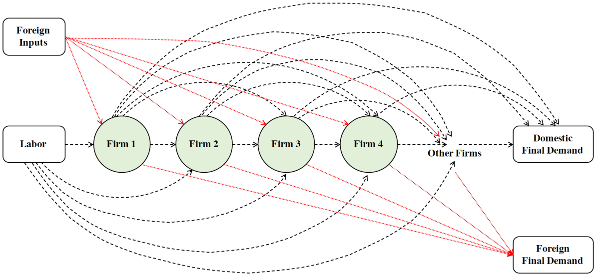
- To compute a **lower bound** for the optimal supplier set for firm i:
  1. start with a guess of no connections to any suppliers
  2. for each potential supplier, evaluate whether it is optimal to add supplier
  3. if optimal, add supplier to guess of optimal supplier set
  4. repeat 2-3 until no new suppliers are added
- To compute an **upper bound** for the optimal supplier set for firm i:
  1. start with a guess of connections to all potential suppliers
  2. for each supplier, evaluate whether it is optimal to remove supplier
  3. if optimal, remove supplier from guess of optimal supplier set
  4. repeat 2-3 until no new suppliers are removed
- Given the upper and lower bounds for the optimal supplier set:
  - can search in between the bounds for the actual optimal set
- In practice, lower and upper bounds are identical in 99% of cases (conveniently)

Network Restrictions

- Note that in the sourcing algorithm...
  - optimality of adding/dropping suppliers depends on marginal costs
- But in general equilibrium, these marginal costs depend on:
  - optimal sourcing of suppliers
  - optimal sourcing of suppliers of suppliers
  - ...
- Hence, in order to apply the sourcing algorithm:
  - need to restrict the set of potential linkages that can be formed

Network Restrictions

- First, order firms in a sequence $\{1, 2, \cdots, N\}$
- Assume that:
  - firm 1 can only source from labor
  - firm 2 can only source from labor and firm 1
  - firm 3 can only source from labor and firms $\{1, 2\}$
  - ...
- In other words, impose restriction that network has to be **acyclic**



Source: Tintelnot et al (2019)

Network Restrictions

- Sourcing decision for firm n only depends on choices by firms $\{1, \cdots, n-1\}$
  - because firms earn zero profits from firm-to-firm sales (this is key)
- Hence, sourcing problems can be solved sequentially, starting with firm 1
- Also need to restrict total number of potential suppliers per firm:
  - assume that maximum no. of suppliers is 300
  - for firm n, drawn randomly from firms $\{1, \cdots, n-1\}$
- Also need to solve for:
  - general equilibrium variables (e.g. wages)
  - export participation decision (with fixed cost of exporting)

Network Restrictions

- How acyclic is the actual production network in Belgium?
- To assess this, need to determine the "optimal" ordering of firms
- Let $\nu(i)$ for $i \in \Omega$ be a mapping of firms to positive integers in $\{1, \cdots, N\}$
- Want to solve the following problem:

$$\min_{\nu(\cdot)} \sum_{i \in \Omega} \sum_{j \in \Omega} \mathbf{1}_{[i \text{ supplies } j]} \mathbf{1}_{[\nu(i) > \nu(j)]}$$

  - i.e. objective function is number of edges that violate acyclicality
- This is known as the **feedback arc set** problem
  - there are standard algorithms for finding local minima
  - e.g. Eades, Lin, and Smyth (1993)
  - find that at most 18% of edges violate acyclicality
- Can also weight objective function by firm-to-firm sales
  - find that at most 23% of firm-to-firm sales violate acyclicality

Model Estimation

- Simulate model for $100,000$ Belgian firms
- Normalizations:
  - labor productivity shifters $\alpha_{Li} = 1$
  - final demand shifters $\beta_{iH} = 1$
- Assume independent log-normal distributions for:
  - firm TFP ϕ_i
  - firm-to-firm productivity shifters α_{ij}
  - foreign input productivity shifters α_{Fi}
  - foreign demand shifters β_{iF}
  - fixed costs of firm-to-firm sales, importing, and exporting
- Parameters estimated via simulated method of moments, targeting...
  - sourcing moments: in- and out-degree distributions; firm labor shares; firm-to-firm input shares
  - trade moments: share of firms that import and export; import and export shares (conditional on importing and exporting)
  - macro moments: ratio of aggregate exports to aggregate final demand, import content of domestic final demand

Effects of Foreign Price Shocks

- Again, study effect of changes in foreign input prices on firm costs and welfare
- Note that if equilibrium is efficient, allowing for network adjustment must:
  - dampen effects of negative shocks
  - amplify effects of positive shocks
- However, can show that market equilibrium is not efficient:
  - without firm-to-firm markups, there is no multiple marginalization
  - but there are still network extenalities in relationship formation

Effects of Foreign Price Shocks

	Endogenous Network	Fixed Network
10% increase ($\hat{p}_{F\cdot} = 1.1$)	0.9388	0.9321
Autarky ($\hat{p}_{F\cdot} \to \infty$)	0.4844	0.4157
10% decrease ($\hat{p}_{F\cdot} = 0.9$)	1.0681	1.0881
50% decrease ($\hat{p}_{F\cdot} = 0.5$)	2.2779	2.0170

Source: Tintelnot et al (2019)

- In response to **higher** foreign input prices:
  - allowing for adjustment of domestic network dampens effect of shocks
  - e.g. autarky reduces welfare by 7 p.p. more under fixed network
- In response to **lower** foreign input prices:
  - welfare gain is *lower* with endogenous networks for small declines in p_F
  - welfare gain is *higher* with endogenous networks for large declines in p_F
- Based on numerical simulations, generally find that endogenous networks:
  - dampen effects of large negative shocks
  - amplify effects of large positive shocks

The Network Structure of International Trade

- Chaney (2013), "The Network Structure of International Trade"
- Study exports by French firms and document that:
  - firms follow a history-dependent process when exporting
  - i.e. where a firm exports today affects where it will export tomorrow
- To explain these facts, develop a theory of trade frictions
  - firms can only trade by accumulating customers in locations over time
  - customers are acquired through direct search...
  - and by indirect search via existing customers

Data

- Firm-level export data for French exporters, 1986-1992
  - observe set of countries that each French firm exports to
- Data on export markets:
  - size (nominal GDP)
  - distance between countries
  - aggregate bilateral trade between countries

Motivating Evidence

- Notation
  - $I_{i,c,t}$: dummy indicator $= 1$ if French firm i exports to country c at time t
  - $d_{c,c'}$: function of distance between countries c and c'
  - $\Delta E_{c',c't}$: growth rate of exports from country c' to c between t and $t+1$
- Estimate probit regression of $I_{i,c,t+1}$ on the following
  - number of export markets: $\alpha \sum_{c'} I_{i,c',t}$
  - lagged export status: $\delta I_{i,c,t}$
  - distance from France: $\beta_1 d_{F,c}$
  - avg. distance between existing export markets and c: $\beta_2 \frac{\sum I_{i,c',t} d_{c',c}}{\sum_{c'} I_{i,c',t}}$
  - import growth of market c: $\gamma_1 \sum_{c'} \Delta E_{c',c,t}$
  - export growth of existing export markets to c: $\gamma_2 \sum_{c'} I_{i,c',t} \Delta E_{c',c,t}$
- All marginal effects are positive and significant, with expected signs:
  - $\alpha, \delta, \gamma_1, \gamma_2 > 0$ and $\beta_1, \beta_2 < 0$
- In particular, a firm is more likely to expand into market c if...
  - it already exports to countries that are near c ($\beta_2 < 0$)
  - it already exports to countries that export a lot to c ($\gamma_2 > 0$)

Model Environment

- Set of locations $x \in S = \mathbb{Z}$, discrete time
- Unit continuum of firms in each location
- Without loss of generality, consider firms at the origin $x = 0$
- Each firm searches for customers in other locations in two ways
- Local search:
  - in each period, draw random number of new customers with mean μ
  - probability that a new customer is in location $x = g(x)$
- Remote search:
  - in each period, for each existing customer in $y \in S$...
  - draw random number of new customers with mean $\pi\mu$
  - probability that a new customer is in location $x = g(|y - x|)$

Solving for the Customer Distribution

- Notation
  - $f_t(x)$: average number of customers firms at origin have in location x
  - m_t : average number of customers firms at origin have in all locations
- Law of motion for $f_t(x)$:

$$f_{t+1}(x) = f_t(x) + \mu g(x) + \pi\mu \sum_{y \in S} f_t(y)\, g(|y - x|)$$

- Aggregating across locations:

$$m_{t+1} = m_t + \mu + \pi\mu m_t$$

- First-order linear difference equation in m_t, with a closed-form solution:

$$m_t = \frac{1}{\pi}\left[(1 + \pi\mu)^t - 1\right]$$

- Note that the distribution function g...
  - only affects where customers are, not how many of them there are in total

Solving for the Customer Distribution

- To solve for the geographic distribution of customers f_t...
  - use a clever mathematical trick
- The convolution of two functions f and g is defined as:

$$(f * g)(x) \equiv \sum_{y \in S} f(y)\, g(x - y)$$

- The Fourier transform of a function f is defined as:

$$\hat{f}(\omega) = \sum_{x \in S} f(x)\, e^{-i\omega x}$$

- The **convolution theorem** states that:
  - the Fourier transform of the convolution of f and g...
  - is the product of the Fourier transforms of f and g
- (Note: these mathematical tools are used extensively in physics and engineering)

Solving for the Customer Distribution

- Applying the convolution theorem to the law of motion for f_t:

$$\hat{f}_{t+1} = \hat{f}_t + \mu\hat{g}_t + \pi\mu\hat{f}_t\hat{g}_t$$

- Now this is a first-order linear difference equation in $\hat{f}_t$, with solution:

$$\hat{f}_t = \frac{1}{\pi}\left[(1 + \pi\mu\hat{g})^t - 1\right]$$

- Now apply the convolution theorem again:

$$f_t(x) = \frac{1}{\pi}\left[(\delta(x) + \pi\mu g(x))^{*t} - \delta(x)\right]$$

  - δ: Dirac delta function ($\delta(0) = 1$ and $\delta(x) = 0\ \forall x \neq 0$)
  - $*t$: denotes convolution of a function with itself t times

Model Estimation

- With analytic solutions for m_t and f_t:
  - can characterize patterns of extensive margin of trade
- To estimate model parameters, first assume functional form for g:

$$g(x-y) = e^{-|x-y|/\gamma}$$

- Model parameters $\{\mu, \pi, \gamma\}$ are estimated via simulated method of moments:
  - using actual world geography (i.e. distances between countries)
- Targeted moments:
  - fraction of firms that export to $1, \cdots, \geq 70$
  - avg. distance of exports for firms that export to $1, \cdots, \geq 50$ countries

Estimation Results

	(1986)	(1987)	(1988)	(1989)	(1990)	(1991)	(1992)
π	2.420	2.495	2.479	2.499	2.574	2.633	2.401
	(0.187)	(0.114)	(0.150)	(0.066)	(0.114)	(0.130)	(0.200)
μ	0.371	0.368	0.384	0.362	0.357	0.338	0.384
	(0.022)	(0.013)	(0.021)	(0.010)	(0.013)	(0.014)	(0.027)
Parameter for $g(\|\|x-y\|\|)=\frac{1}{\lambda}e^{-\|\|x-y\|\|/\lambda}$:							
λ	3.419	3.398	3.448	2.906	3.515	3.418	3.513
	(0.131)	(0.145)	(0.130)	(0.403)	(0.177)	(0.132)	(0.135)

Notes: This table presents the SMM estimates of μ, π, and λ. The parameters μ and π govern the acquisition of the number of new consumers, while the parameter λ governs the geographic location of those consumers. Data: all French exporters, 1986–1992. Bootstrapped standard errors are in parentheses. All coefficients are statistically different from zero at the 1 percent level of significance.

Source: Chaney (2014)

- Remote search is estimated to be more than twice as important as direct search

Model Fit

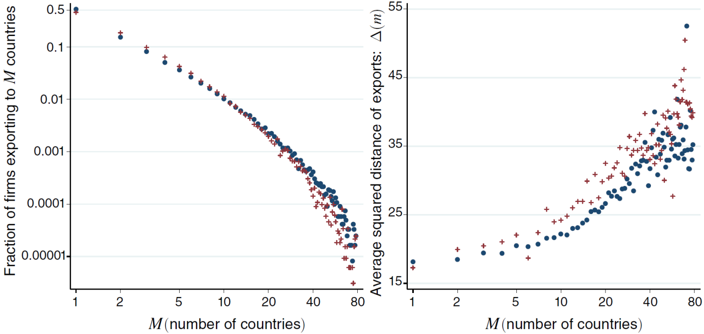


FIGURE 3. THE NUMBER AND GEOGRAPHY OF EXPORTS (*SMM estimates*)

Notes: Left panel: fraction of firms that export to M different countries. Right panel: average squared distance to a firm's export destinations, among firms exporting to M destinations, as defined in equation (8); distances are calculated in thousands of kilometers. Dots: data, all French exporters in 1992. Plus signs: simulated data; $\pi = 2.401$ (0.200), $\mu = 0.384$ (0.027) and $\lambda = 3.513$ (0.135) are estimated by simulated method of moments.

Source: Chaney (2014)

Summary and Related Papers

- Studied two applications of network theory to international trade
- Domestic firm-to-firm linkages matter for effects of trade shocks:
  - because they determine the domestic absorption of foreign shocks
- Evolution of customer networks through direct/remote search:
  - helps to explain path of firm-level expansion into export markets
- In terms of modeling, two very different approaches:
  - costly relationships + sourcing algorithm + exogenous network restriction
  - extensive margin only + use of Fourier transform for analytic solutions
- Related papers:
  - Kikkawa et al (2018) - endogenous network formation with variable markups (oligopoly)
  - Huneeus (2018) - international trade model with sticky relationships using Chilean VAT data
- Next week: networks and industrial policy