ECO 2302: Networks in Trade and Macroeconomics

Problem Set 4

Due date: 5:00 PM, 9 April 2021

In this assignment, we will use a simple sector-level production network model to study the effects of government policy in the presence of trade costs and market distortions.

Model Setup

There are two sectors, 1 and 2, in a closed economy. There is a representative household that supplies 1 unit of labor inelastically and that has Cobb-Douglas preferences over goods from the two sectors given by:

$$U = \left(\frac{C_1}{\alpha_1}\right)^{\alpha_1} \left(\frac{C_2}{\alpha_2}\right)^{\alpha_2} \tag{1}$$

where $\alpha_1 + \alpha_2 = 1$. Output in sector i is produced under perfect competition by combining labor L_i and a composite intermediate input M_i using a Cobb-Douglas technology with labor share λ_i:

$$X_i = \left(\frac{L_i}{\lambda_i}\right)^{\lambda_i} \left(\frac{M_i}{1-\lambda_i}\right)^{1-\lambda_i} \tag{2}$$

The composite intermediate M_i is produced by combining intermediates from both sectors using a Cobb-Douglas technology:

$$M_i = \left(\frac{X_{i1}}{\beta_{i1}}\right)^{\beta_{i1}} \left(\frac{X_{i2}}{\beta_{i2}}\right)^{\beta_{i2}} \tag{3}$$

where X_{ij} is the quantity of sector j inputs purchased by sector i, and $\beta_{i1} + \beta_{i2} = 1$. In what follows, we will take the wage as the numeraire and denote the price of sector i output by P_i.

To simplify the analysis, we will assume that the sectors are symmetric with the following parameters:

$$\alpha_1 = \alpha_2 = \frac{1}{2}$$
$$\lambda_1 = \lambda_2 = \frac{1}{3}$$
$$\beta_{ij} = \frac{1}{2},\ \forall i, j \in \{1, 2\}$$

Benchmark Equilibrium

First, as a benchmark, we will solve for the equilibrium of the model described above.

1. Solve the cost-minimization problem for producers in each sector. What is the equilibrium price of each sector's output, $\{P_i\}_{i=1}^2$?

2. Write down the market clearing conditions for each sector's output. What is the equilibrium output of each sector, $\{X_i\}_{i=1}^2$?

3. Solve the household's utility maximization problem. What is the value of household utility, U?

Policy in the Benchmark Equilibrium

Now suppose that a social planner decides to impose a subsidy of s on cross-sector intermediate input purchases. Specifically, for every dollar of sector j inputs purchased by firms in sector i with $i \neq j$, firms in sector i receive a subsidy of s dollars. The cost minimization problem for firms in sector i can hence be written as:

$$P_i = \min_{L_i, X_{ii}, X_{ij}} \{L_i + P_i X_{ii} + (1 - s) P_j X_{ij}\} \tag{4}$$
$$\text{s.t. } X_i = 1 \tag{5}$$

To finance the subsidies, the social planner levies a lump-sum tax T from the household, which must satisfy:

$$T = sP_2 X_{12} + sP_1 X_{21} \tag{6}$$

Now recall that we are taking the wage as the numeraire, the household supplies one unit of labor, and firms earn zero profits. Hence, the after-tax income of the representative household is:

$$I = 1 - T \tag{7}$$

4. Solve the cost-minimization problem for producers in each sector. What is the equilibrium price of each sector's output, $\{P_i\}_{i=1}^2$? How do prices depend on the subsidy s?

5. Write down the market clearing conditions for each sector's output. What is the equilibrium output of each sector, $\{X_i\}_{i=1}^2$, given the tax T and subsidy s?

6. Using equation (6) and your answers from parts (4) and (5), solve for the tax T as a function of the subsidy s. How does the tax value depend on s?

7. Solve the household's utility maximization problem. What is the value of household utility, U, given the subsidy s?

8. What is the optimal subsidy s that maximizes the value of household welfare? Explain the intuition for your answer.

Policy in an Equilibrium with Trade Costs

Now suppose that cross-sector purchases incur an iceberg trade cost of $\tau > 0$. Specifically, delivering one unit of sector j's output to sector i with $i \neq j$ requires shipping $1 + \tau$ units of sector j's output. As before, suppose that the social planner imposes a subsidy of s on cross-sector intermediate input purchases. Hence, the cost minimization problem for firms in sector i can now be written as:

$$P_i = \min_{L_i, X_{ii}, X_{ij}} \{L_i + P_i X_{ii} + (1 + \tau - s) P_j X_{ij}\} \tag{8}$$

$$\text{s.t. } X_i = 1 \tag{9}$$

The government budget balance and household income equations (6)-(7) remain the same. The only other change in the equilibrium conditions is that market clearing now requires:

$$X_1 = C_1 + X_{11} + (1 + \tau) X_{21} \tag{10}$$

$$X_2 = C_2 + X_{22} + (1 + \tau) X_{12} \tag{11}$$

9. Repeat the analysis in parts (4)-(8) and show that given a subsidy s, household welfare is equal to:

$$U = \frac{1 + \tau - 2s}{(1 + \tau - s)^2} \tag{12}$$

10. What is the optimal subsidy s that maximizes the value of household welfare? Explain the intuition for your answer.

Policy in a Distorted Equilibrium

Now suppose that τ is not an iceberg trade cost, but instead represents a market distortion. Specifically, suppose that for every dollar of sector j inputs purchased by firms in sector i with $i \neq j$, firms in sector i capture τ dollars as rent. Hence, firms in sector i again choose inputs to solve the following problem:

$$\min_{L_i, X_{ii}, X_{ij}} \{L_i + P_i X_{ii} + (1 + \tau - s) P_j X_{ij}\} \tag{13}$$

$$\text{s.t. } X_i = 1 \tag{14}$$

However, the rent earned from these distortions is now rebated to the household, and hence household income is:

$$I = 1 - T + \tau P_2 X_{12} + \tau P_1 X_{21} \tag{15}$$

Furthermore, the market clearing conditions are the same as in the benchmark equilibrium.

11. What is the optimal subsidy s that maximizes the value of household welfare? Explain the intuition for your answer.