BME 105
Spring 2023

Homework 1

<div align="center">Representing negative numbers in binary: "Two's Complement"</div>

Instructions:
Answer Problems 1 - 5 below, showing all of your steps. Submit your solutions on BlackBoard in the form of a single PDF file.

Due date: Friday, February 3rd at 1:00 PM EST.

We have seen how positive integers are stored in binary form. For example, the binary number 1010_2 is equivalent to:

$\mathbf{1} \times 2^3 + \mathbf{0} \times 2^2 + \mathbf{1} \times 2^1 + \mathbf{0} \times 2^0 = 8 + 2 = 10_{10}$

An interesting question is then how to represent *negative* numbers. In this homework assignment, you are introduced to so-called "Two's Complement" notation. In this scheme, the coefficient of the most significant bit enters the equation above with a negative sign. Let's demonstrate with an example. In Two's Complement form, the number 1010_2 is given by:

$\mathbf{-1} \times 2^3 + \mathbf{0} \times 2^2 + \mathbf{1} \times 2^1 + \mathbf{0} \times 2^0 = -8 + 2 = 6_{10}$

Note the minus sign on the most significant bit. The result above states that, in Two's Complement notation, the binary sequence 1010_2 is equivalent to the decimal number 6_{10}. As another example, the number 1011_2 represents:

$\mathbf{-1} \times 2^3 + \mathbf{0} \times 2^2 + \mathbf{1} \times 2^1 + \mathbf{1} \times 2^0 = -8 + 2 + 1 = 5_{10}$

In Two's Complement notation, negative numbers begin with a "1", while positive numbers begin with a 0. For example, the number 0011_2 is given by:

$\mathbf{-0} \times 2^3 + \mathbf{0} \times 2^2 + \mathbf{1} \times 2^1 + \mathbf{1} \times 2^0 = 3_{10}$

Problems:

1. Consider a **6**-bit binary sequence. Assuming Two's Complement notation, what is the decimal number represented by the sequence 101010_2 ?

2. What is the binary form of the integer 22_{10} ?

3. Take your solution to (2) above and perform the following steps:

    a. Invert each bit (0's become 1's, 1's become 0's).
    b. Add 1 to the number. *Hint: $001_2 + 001_2 = 010_2$*
    c. Convert the resulting number to decimal notation **assuming Two's Complement notation**.

    What do you notice about the result?

4. What is the negative number with the largest magnitude (absolute value) that can be represented with 6 bits in Two's Complement form? *State the answer in both binary and decimal formats.*

5. What is the positive number with the largest magnitude that can be represented with 6 bits in Two's Complement form? *State the answer in both binary and decimal formats.*