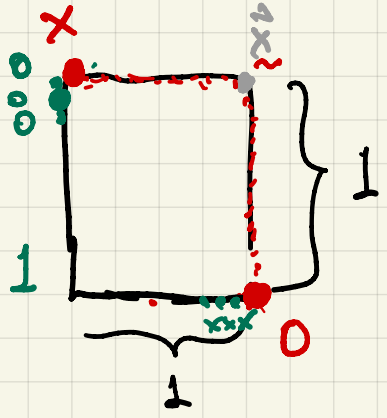
k-NN Summary:

- K-NN is a good classifier if the distance metric really reflects similarity among data points.

- **Downside**

① As $d \gg 0$, we are cursed.

As $d \gg 0$, $dist(\vec{x}, \vec{p}) \to 1$



② Inefficient algorithms.

Testing

large n

Resource consumption
- Space use $O(n)$
- Time $O(nd)$

Perceptron

The Perceptron (Frank Rosenblat 1957)

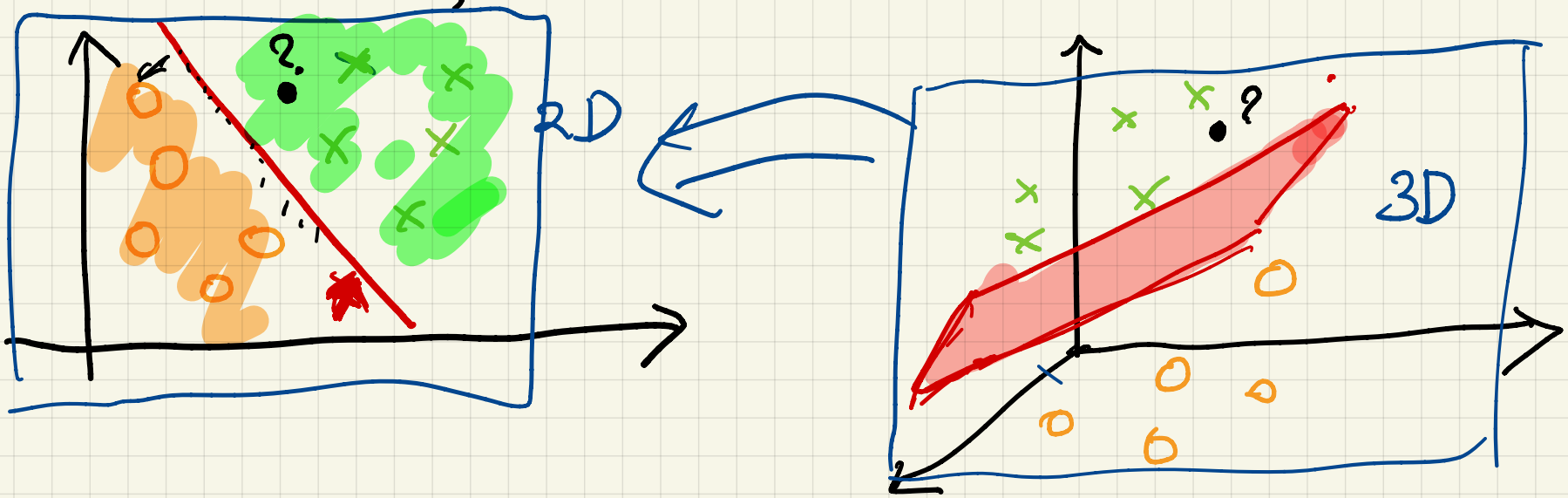
- The first ML algorithm
- Assume that the task is binary classification

$$Y = \{-1, +1\}$$

O X

hyperplane

- For illustration, we assume $X = \mathbb{R}^2$



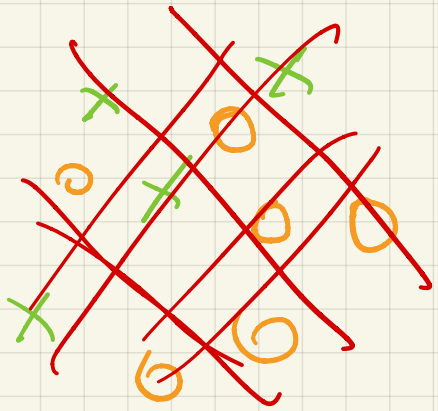
Hyperplane

- A 2D hyperplane is a line.
- A 3D hyperplane is a plane.

Def: A hyperplane is a subspace whose dimension is one less than that of its ambient space.

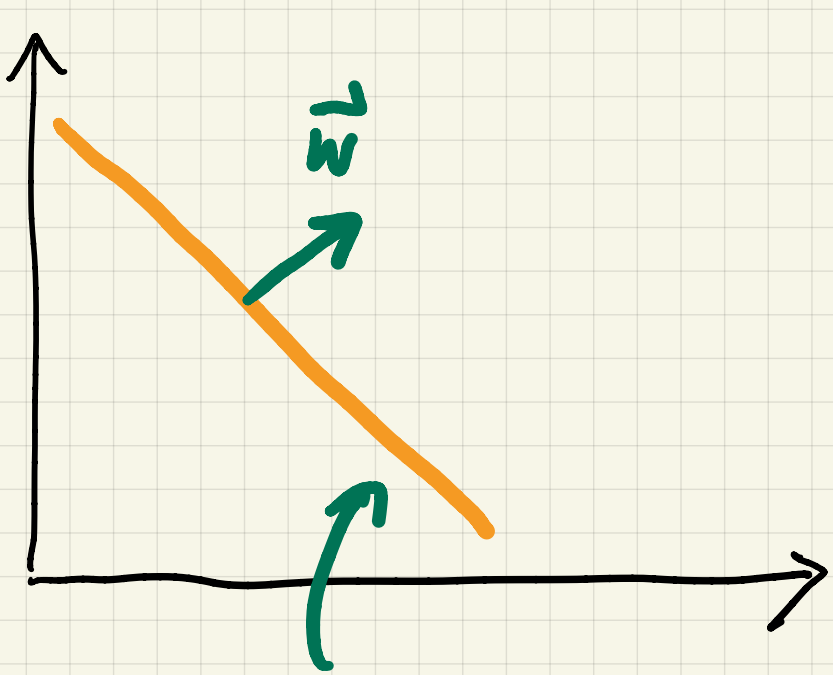
Perceptron's assumption:

There is a hyperplane that separates data points of one class of label from another.



Remark: For high dimensional data, there almost always exists such hyperplane.

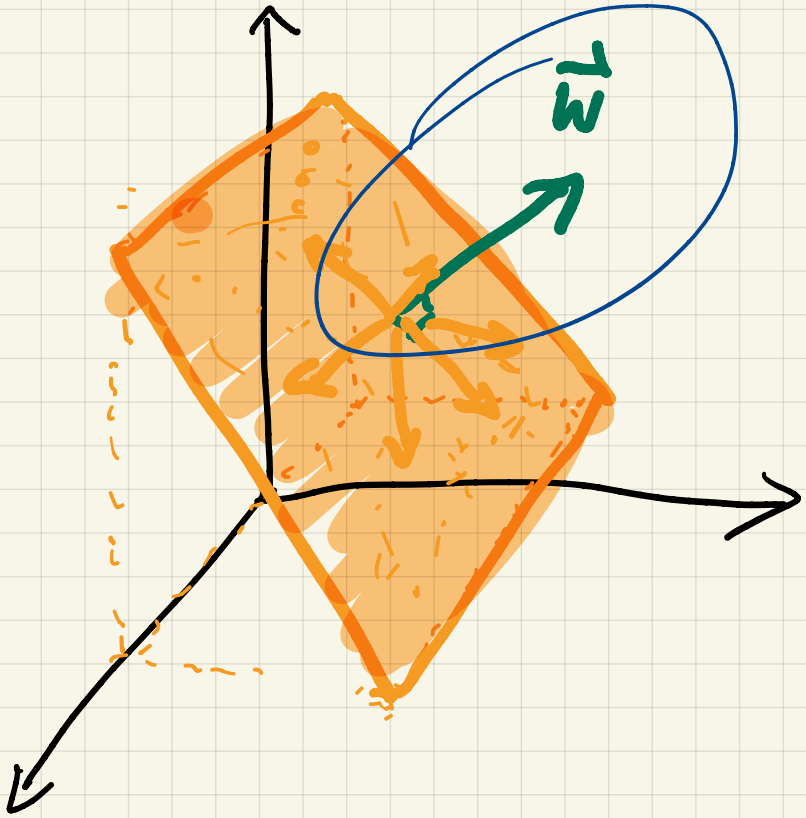
Perceptron's job: To find the hyperplane.



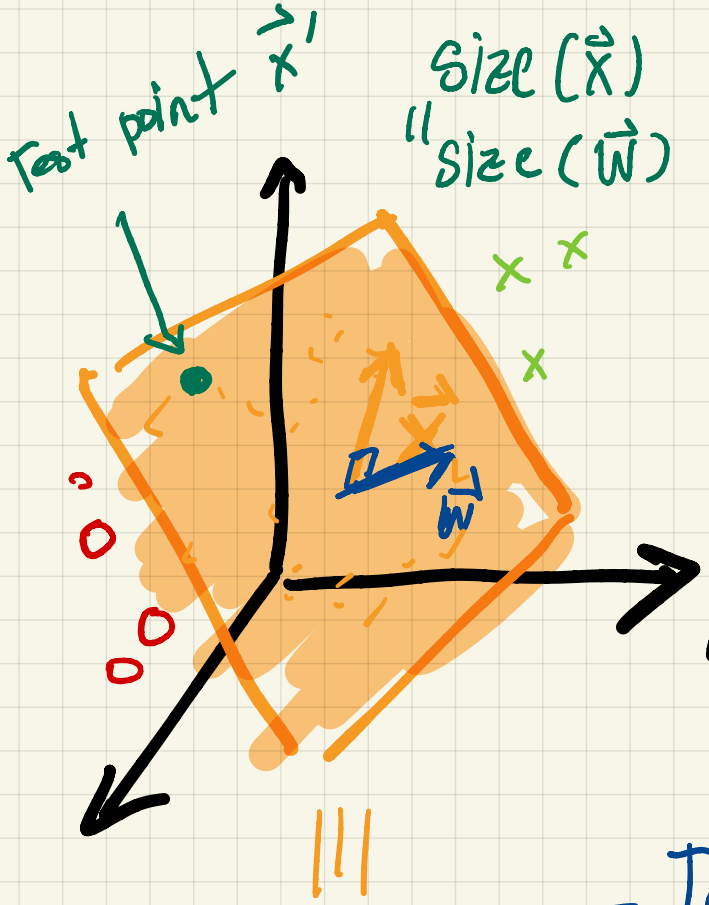
$y = mx + b$

slope

offset



We wish to find the vector $\vec{w}$ that defines our hyperplane.



$$\vec{w}^T \vec{x} = 0$$

$$y = \{-1, +1\}$$

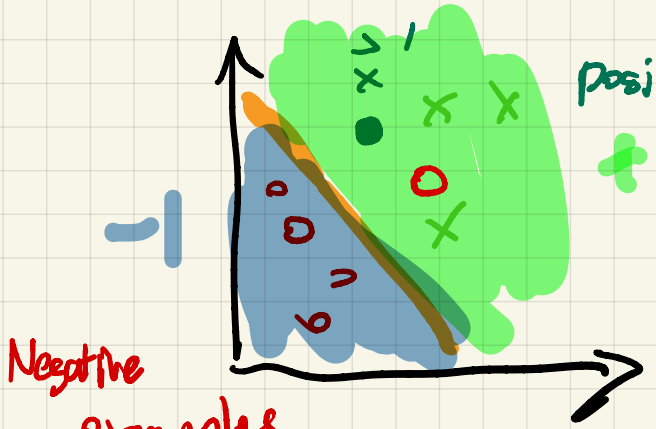
$$x \in R^d$$

- Training: Find the vector $\vec{w}$ that defines a hyperplane

$$H = \{ \vec{x} \mid (\vec{w}^T)\vec{x} + b = 0 \}$$

$O(d)$

- Testing: sign $((\vec{w}^T)\vec{x}' + b)$



$(\vec{w}^T)\vec{x}' + b > 0$

$O(d)$

$\vec{w}^T \vec{x}' + b < 0$

Quiz II

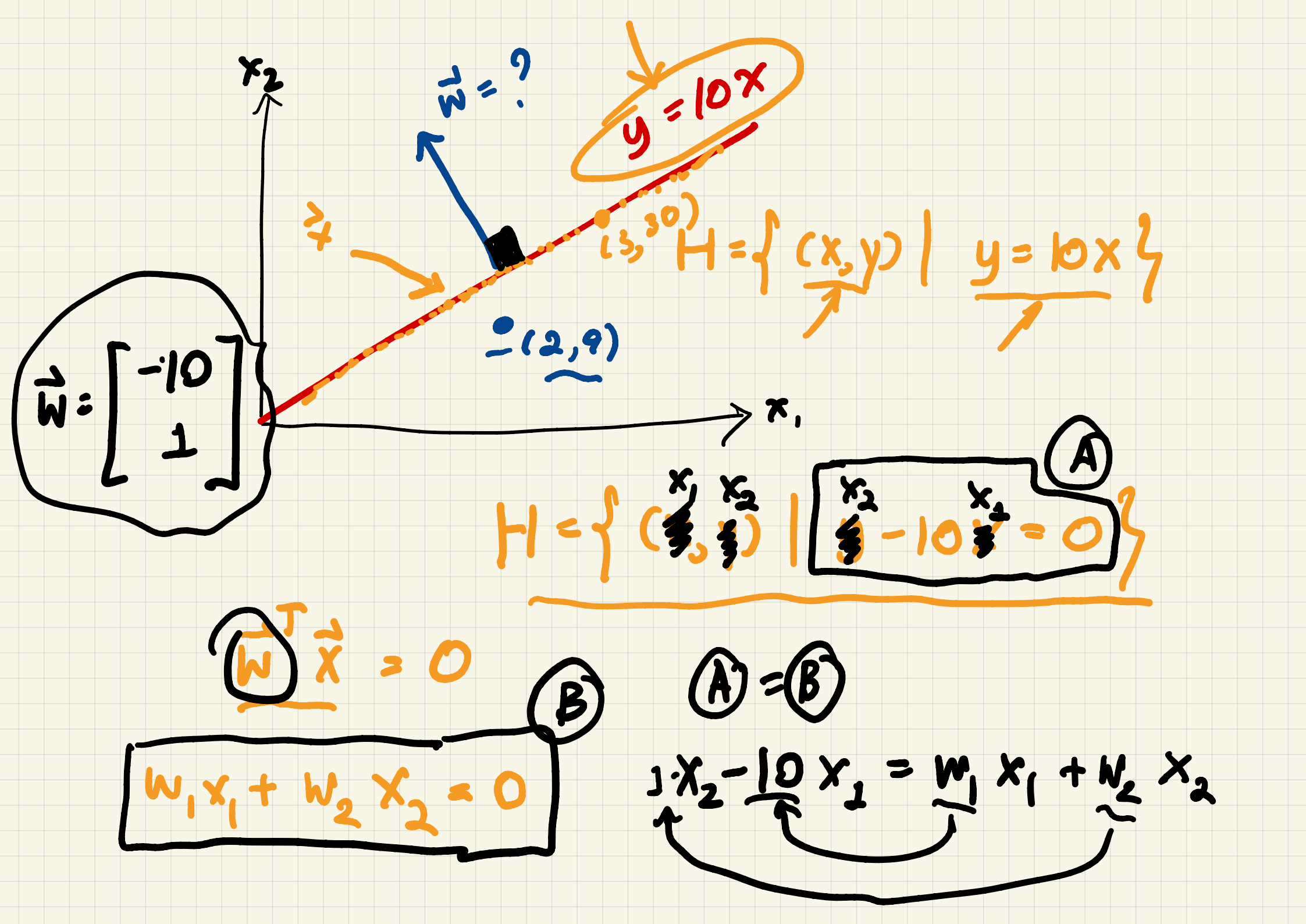
(1) Find the vector $\vec{w}$ and offset b that defines the line (2D-hyperplane) $y = -2x + 10$.

(2) If we have a test point $(2, 3)$, what would it be classified regarding the hyperplane in (1)?

(3) Find the vector $\vec{w}$ and offset b that defines the plane (3D-hyperplane)

$$2x + 4y - 5z + 10 = 0$$

(4) If we have a test point $(-5, -2.5, -2)$, what would it be classified regarding the hyperplane in (3)?


x_2
$\vec{w} = ?$
$y = 10x$
$\vec{x}$
$(3, 30)$
$H = \{ (x, y) \mid y = 10x \}$
$(2, 9)$
$\vec{w} = \begin{bmatrix} -10 \\ 1 \end{bmatrix}$
x_1
$H = \{ (x_1, x_2) \mid x_2 - 10 x_1 = 0 \}$
A
$\vec{w}^T \vec{x} = 0$
B
$w_1 x_1 + w_2 x_2 = 0$
A = B
$1 \cdot x_2 - 10 x_1 = w_1 x_1 + w_2 x_2$