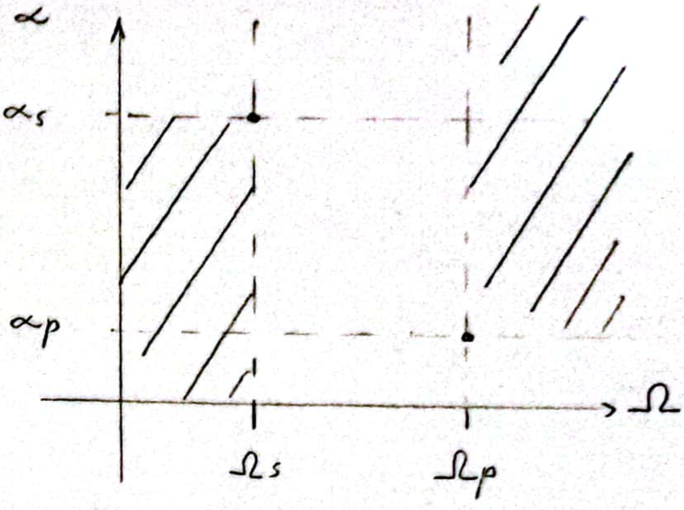
#01.

$\Omega_p = 1$

$\Omega_s = \frac{1}{3}$

$\alpha_p = 0{,}5\,dB$

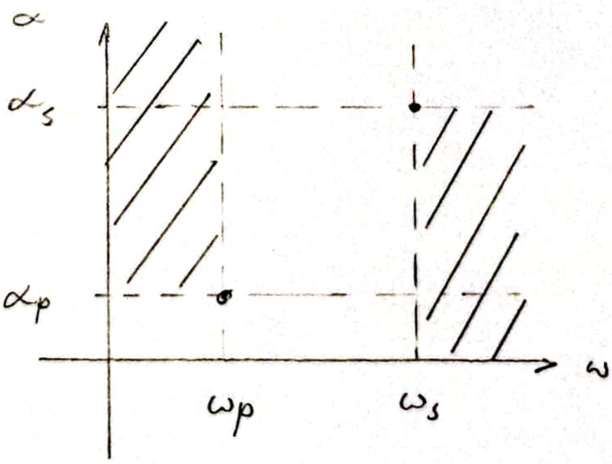
$\alpha_s = 16\,dB$




Plantilla Pasa Altos
Objetivo.

Aplicamos el núcleo de transformación $k_{HP}(\omega) = \Omega = -\frac{1}{\omega}$ para obtener la plantilla pasa bajos prototipo:

Plantilla
Pasa Bajos
Prototipo




$\omega_p = \frac{1}{\Omega_p} = 1$

$\omega_s = \frac{1}{\Omega_s} = 3$

Para máxima planicidad:

$$|T(j\omega)|^2 = |T(j\omega)|\,|T(-j\omega)| = T(s)\cdot T(-s)\Big|_{s=j\omega} = \frac{1}{1+\xi^2\omega^{2n}}$$

$$|\alpha|^2 = 1+\xi^2\omega^{2n} \;;\; |\alpha|_{dB} \triangleq \alpha_{dB} \;;\; \alpha_{dB} = 10\log\left(1+\xi^2\omega^{2n}\right) \quad (1)$$

Despejando $\xi^2$ de 1, obtenemos:

$$\xi^2 = \frac{10^{\alpha dB/10} - 1}{\omega^{2n}} \quad 2$$

Reemplazando valores en 2:

Para $(\alpha_p, \omega_p)$: $\xi^2 = \dfrac{10^{\alpha p/10} - 1}{\omega p} = \dfrac{10^{0,5/10} - 1}{1} = 0,1220$.

Para $(\alpha_s, \omega_s)$, itero en 1 hasta encontrar un $n \in \mathbb{Z}$ que cumpla con $\alpha_{min} \geq 16$ dB:

$$\alpha_{min} = 10 \log\left(1 + \xi^2 \omega_s^{2n}\right)$$

$n = 1$: $\alpha_{min} = 10 \log\left(1 + 0,1220 \cdot 3^{2.1}\right) \simeq 3,218$ dB

$n = 2$: $\alpha_{min} = 10 \log\left(1 + 0,1220 \cdot 3^{2.2}\right) \simeq 10,368$ dB

$\underline{n = 3}$: $\alpha_{min} = 10 \log\left(1 + 0,1220 \cdot 3^{2.3}\right) \simeq 19,540$ dB

$$|T(j\omega)|^2 = \frac{1}{1 + \xi^2 \omega^{2.3}} = \frac{1}{1 + \xi^2 \omega^6} = T(s) \cdot T(-s)\Big|_{s = j\omega}$$

$$|T(j\omega)|^2 = \frac{1}{1+\xi^2\left(\frac{s}{j}\right)^6} = \frac{1}{1-\xi^2 s^6} = \frac{\frac{1}{\xi^2}}{\frac{1}{\xi^2}-s^6}$$

$$|T(j\omega)|^2 = \underbrace{\frac{c}{s^3+as^2+bs+c}}_{T(s)}\ \underbrace{\frac{c}{-s^3+as^2-bs+c}}_{T(-s)}$$

$$|T(j\omega)|^2 = \frac{c^2}{-s^6+as^5-bs^4+cs^3-as^5+a^2s^4-abs^3+acs^2-bs^4+abs^3-b^2s^2+bcs-cs^3+acs^2-bcs+c^2}$$

$$|T(j\omega)|^2 = \frac{c^2}{-s^6+(a-a)s^5+(-b+a^2-b)s^4+(c-ab+ab-c)s^3+(ac-b^2+ac)s^2+(bc-bc)s+c^2}$$

$$|T(j\omega)|^2 = \frac{c^2}{-s^6+(a^2-2b)s^4+(2ac-b^2)s^2+c^2}$$

$a^2-2b=0$ ; $a^2=2b$ ; $a=\sqrt{2b}$ (3) $\qquad c^2=\frac{1}{\xi^2}\simeq 8{,}1967$

$2ac-b^2=0$ ; $2ac=b^2$ (4) $\qquad c=2{,}8630$

Reemplazo (3) en (4) : $2\sqrt{2b}\,c=b^2$ (5)

Elevo al cuadrado [illegible] en (5) : $8bc^2=b^4$ ${}^3$

Escaneado con CamScanner

$$b = \sqrt[3]{8c^2} = 2\sqrt[3]{c^2} \simeq 4,0325$$

Reemplazando en 3 : $a = \sqrt{2b} \simeq 2,8399$

$$T_{LP}(s) = \frac{c}{s^3 + as^2 + bs + c} = \frac{2,8630}{s^3 + 2,8399\,s^2 + 4,0325\,s + 2,8630}$$

Aplicamos el núcleo de la transformación para obtener $T_{HP}(s)$:

$$T_{HP}(s) = \frac{2,8630}{\left(\frac{1}{s}\right)^3 + 2,8399\left(\frac{1}{s}\right)^2 + 4,0325\left(\frac{1}{s}\right) + 2,8630}$$

$$T_{HP}(s) = \frac{2,8630\,s^3}{1 + 2,8399\,s + 4,0325\,s^2 + 2,8630\,s^3}$$

$$T_{HP}(s) = \frac{s^3}{s^3 + \frac{4,0325}{2,8630}s^2 + \frac{2,8399}{2,8630}s + \frac{1}{2,8630}}$$

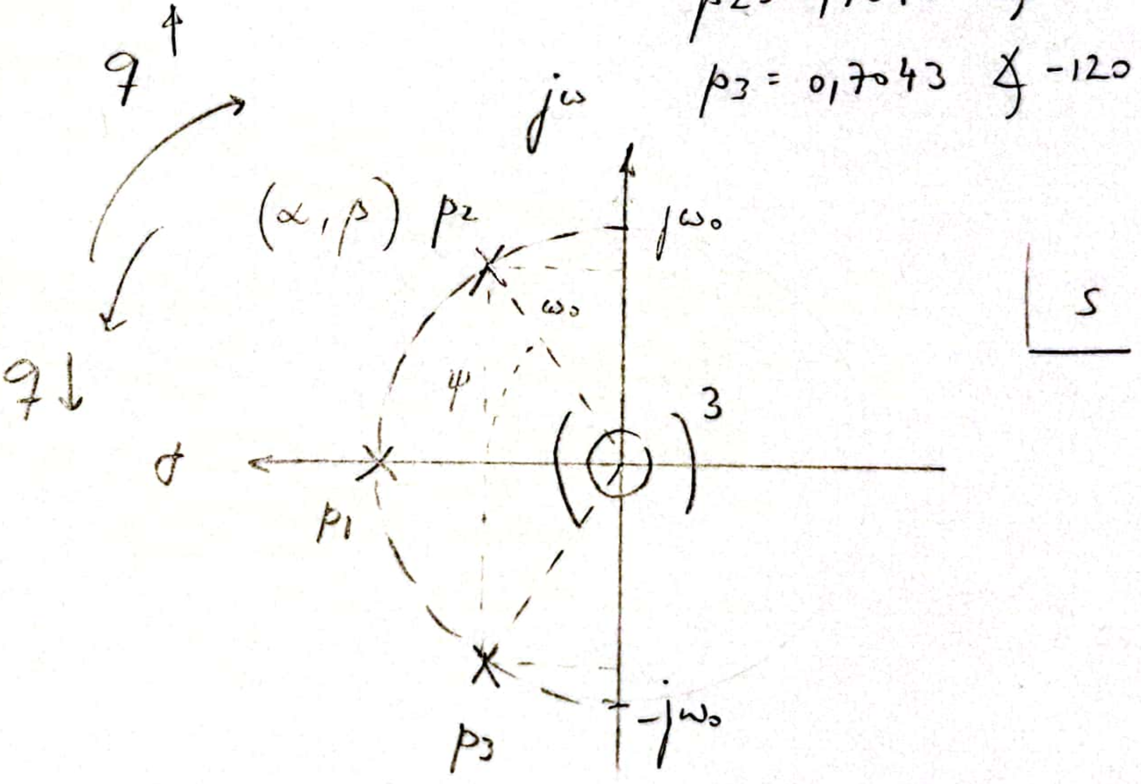
$$T_{HP}(s) = \frac{s^3}{s^3 + 1,4085\,s^2 + 0,9919\,s + 0,3493}$$ ; Punto a

Singularidades del numerador: $z_1 = z_2 = z_3 = 0$

Singularidades del denominador:

$$\left.\begin{aligned} p_1 &= -0{,}7043 \\ p_2 &= -0{,}3521 + j\,0{,}6099 \\ p_3 &= -0{,}3521 - j\,0{,}6099 \end{aligned}\right\} \alpha + j\beta$$

$$\left.\begin{aligned} p_1 &= 0{,}7043 \;\measuredangle\; 180^\circ \\ p_2 &= 0{,}7043 \;\measuredangle\; 120^\circ \\ p_3 &= 0{,}7043 \;\measuredangle\; -120^\circ \end{aligned}\right\} |\,|, \measuredangle$$




Además: $\dfrac{\alpha}{\omega_0} = \cos\varphi$ ; $2\dfrac{\alpha}{\omega_0} = 2\cos\varphi$

$$2\alpha = 2\cos\varphi \cdot \omega_0 = \frac{\omega_0}{q} \; ; \; 2\cos\varphi = \frac{1}{q}$$

$$q = \frac{1}{2\cos\varphi}$$

$$T_{HP}(s) = \frac{s^3}{(s-p_1)(s-p_2)(s-p_3)}$$

$$T_{HP}(s) = \frac{s^3}{[s-(-0,7043)][s-(-0,3521+j\,0,6099)][s-(-0,3521-j\,0,6099)]}$$

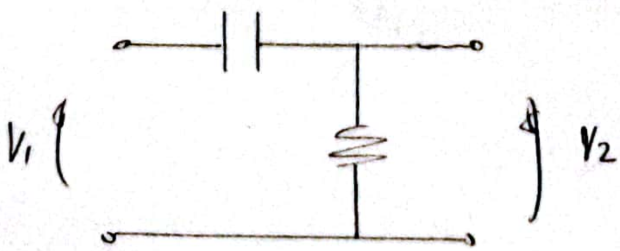
$$T_{HP}(s) = \frac{s^3}{(s+0,7043)(s^2+0,3521\,s+\cancel{j\,0,6099\,s}+0,3521\,s+0,3521^2+\cancel{j\,0,3521\cdot 0,6099}-\cancel{j\,0,6099\,s}-\cancel{j\,0,6099\cdot 0,3521}-j^2\,0,6099^2)}$$

$$T_{HP}(s) = \frac{s^3}{(s+0,7043)[s^2+(0,3521+0,3521)s+0,3521^2+0,6099^2]}$$

$$T_{HP}(s) = \underbrace{\frac{s}{s+\underbrace{0,7043}_{\omega_0}}}_{FOS}\;\underbrace{\frac{s^2}{s^2+\underbrace{0,7042}_{\frac{\omega_0}{q}}s+\underbrace{0,4959}_{\omega_0^2}}}_{SOS}$$

Punto b.

FOS:

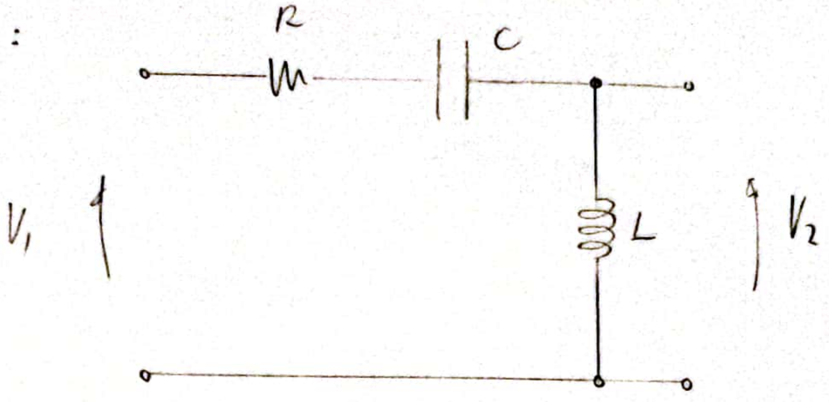



$$T_{FOS}(s) = \frac{V_2}{V_1} = \frac{R}{\frac{1}{sC}+R} = \frac{sRC}{sRC+1}$$

$$= \frac{s}{s+1/RC}$$

$\omega_0 = \frac{1}{RC}$ ; $\Omega_z = R$ ; $R = 1$ ; $C = \frac{1}{R\omega_0}$ ; $C = \frac{1}{0{,}7043} = 1{,}42$

sos :

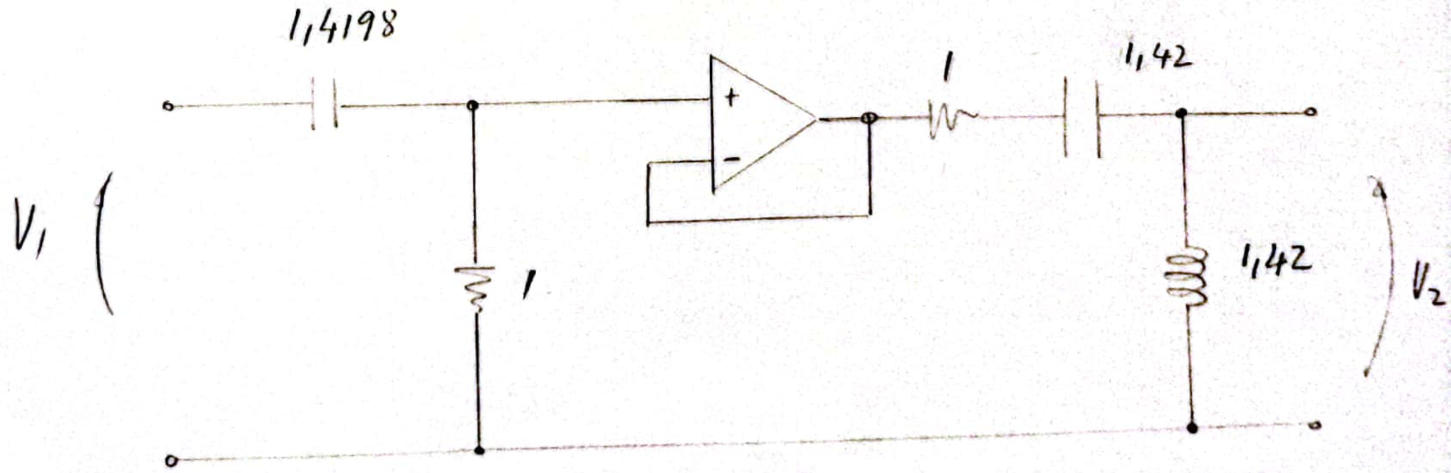



$$T_{sos}(s) = \frac{sL}{R + sL + \frac{1}{sC}} = \frac{s^2LC}{sRC + s^2LC + 1} = \frac{s^2}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

$\omega_0^2 = 0{,}4959$ ; $\omega_0 = 0{,}7043$

$\frac{\omega_0}{q} = \frac{R}{L}$ ; $\Omega_z = R$ ; $R = 1$ ; $L = R\frac{q}{\omega_0} = R\frac{1}{\omega_0/q} = \frac{q}{\omega_0} = 1{,}42$

$\omega_0^2 = \frac{1}{LC}$ ; $C = \frac{1}{L\omega_0^2} = \frac{1}{1{,}42 \cdot 0{,}4909} = 1{,}42$



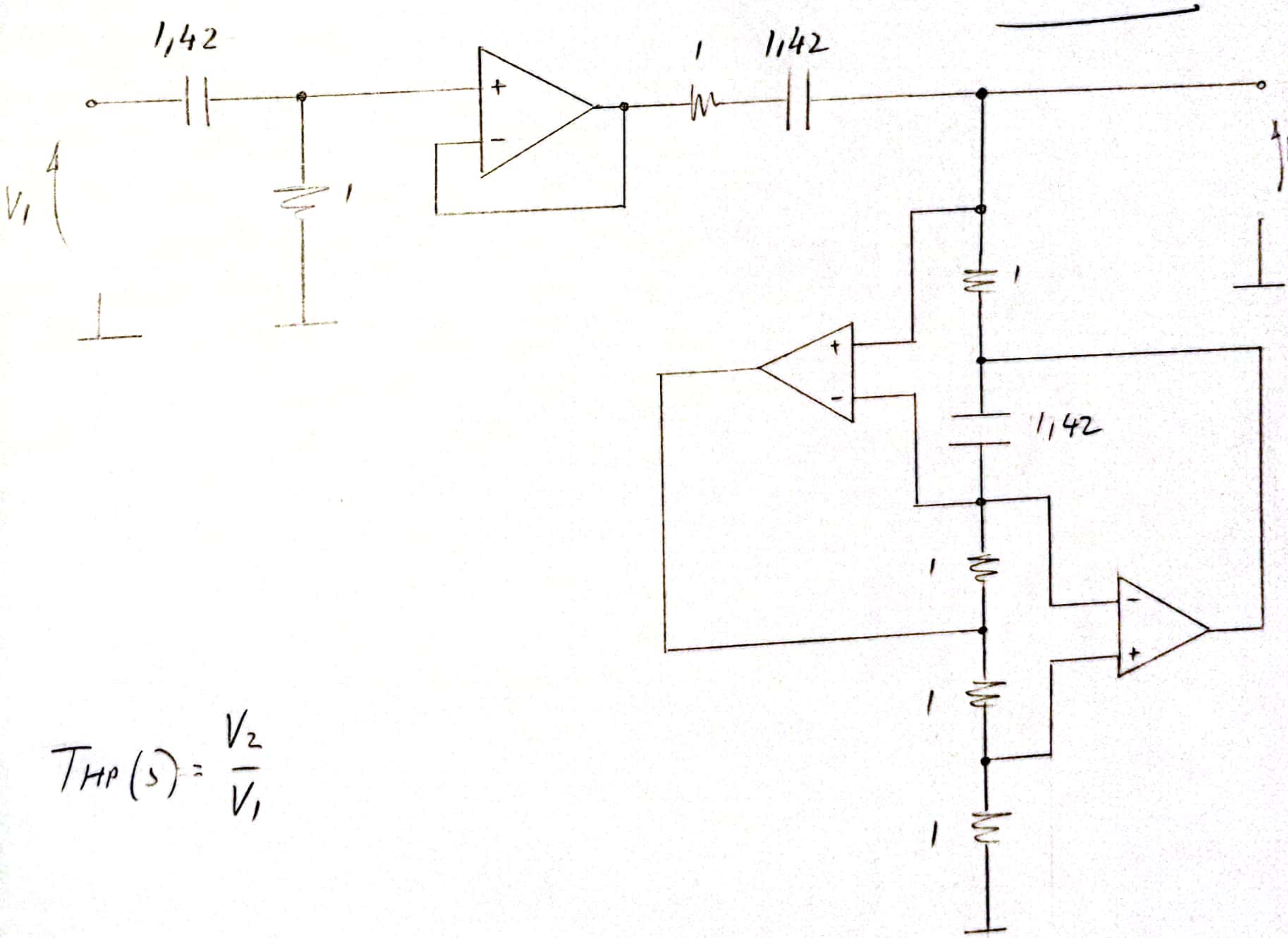

$$T_{HP}(s) = \frac{V_2}{V_1}$$

Implementando el GIC de Antoniou para reemplazar el inductor:

Recordando que $Z = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$ y tomando $Z_1 = Z_3 = Z_4 = Z_5 = R$ y $Z_2 = \frac{1}{sC}$,

obtenemos $Z = \frac{R.R.\cancel{R}}{\frac{1}{sC}\cancel{R}} = sCR^2 = sL_{eq}$ con $L_{eq} = CR^2 = 1{,}42$

Punto c



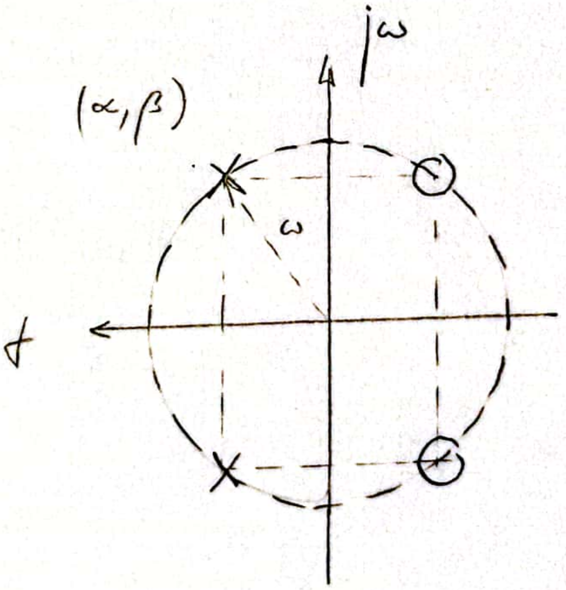

$T_{HP}(s) = \frac{V_2}{V_1}$

#02.

$$\varphi_{Total} = \varphi_{Num} - \varphi_{Den} = -2\pi$$

Propongo un filtro pasa todo de orden 2 de forma tal de cumplir con el requerimiento de fase:

$$T_{AP}(s)\Big|_{N=2} = k\,\frac{s^2 - s\frac{\omega_0}{q} + \omega_0^2}{s^2 + s\frac{\omega_0}{q} + \omega_0^2}$$



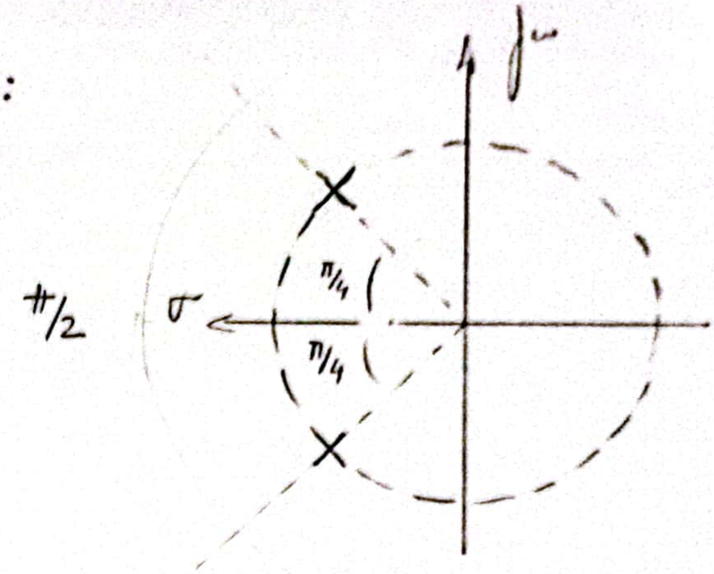

$$\varphi(\omega) = (+/-)\ \mathrm{arctg}\left(\frac{\omega \pm \beta}{\alpha}\right)$$

$\rightarrow$ Polos $\measuredangle$ $(+)$

$\rightarrow$ Ceros $\measuredangle$ $(-)$

$$\varphi_{Total} = \underbrace{\varphi_{Num}}_{(-)} - \underbrace{\varphi_{Den}}_{(+)} = -\pi - (+\pi) = -2\pi$$

Máxima planicidad :

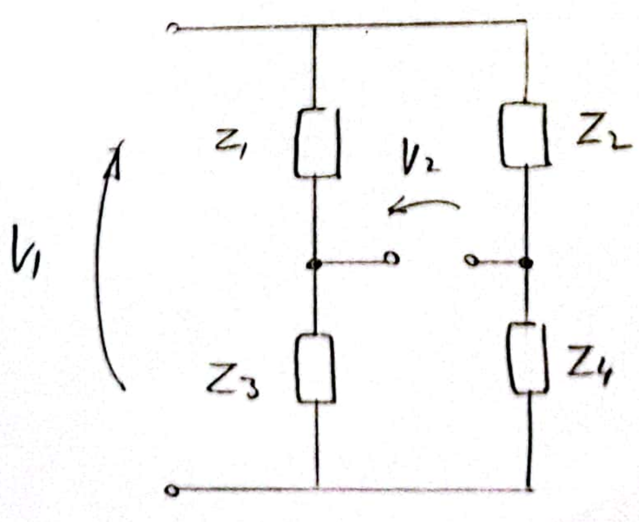



Considerando $\omega_0 = 1$ : $q = \dfrac{1}{2\cos\varphi} = \dfrac{1}{2\cos \pi/4} = \dfrac{1}{\sqrt{2}}$

$$\frac{\omega_0}{q} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2} \; ; \quad T_{AP}(s)\Big|_{N=2} = h\,\frac{s^2 - s\sqrt{2} + 1}{s^2 + s\sqrt{2} + 1}$$

Punto a.

Implementamos el Lattice de forma genérica




$$\frac{V_2}{V_1} = \frac{Z_3}{Z_3 + Z_1} - \frac{Z_4}{Z_2 + Z_4}$$

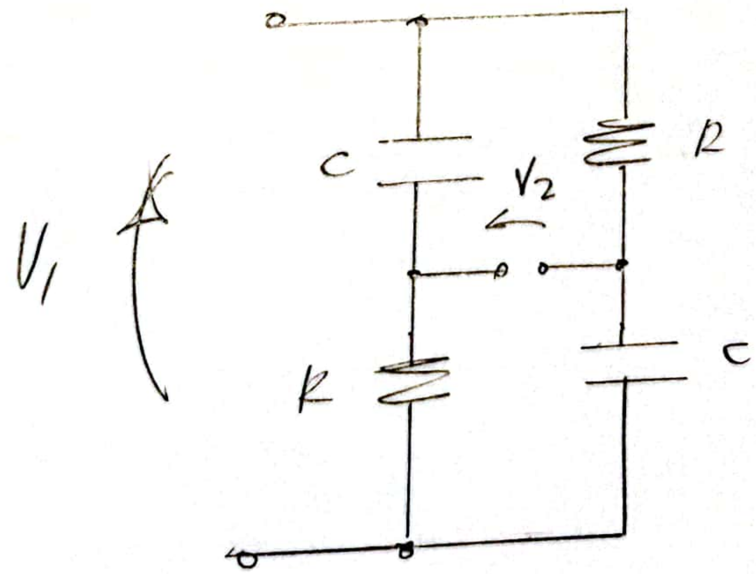
$$= \frac{Z_3(Z_2 + Z_4) - Z_4(Z_3 + Z_1)}{(Z_3 + Z_1)(Z_2 + Z_4)}$$

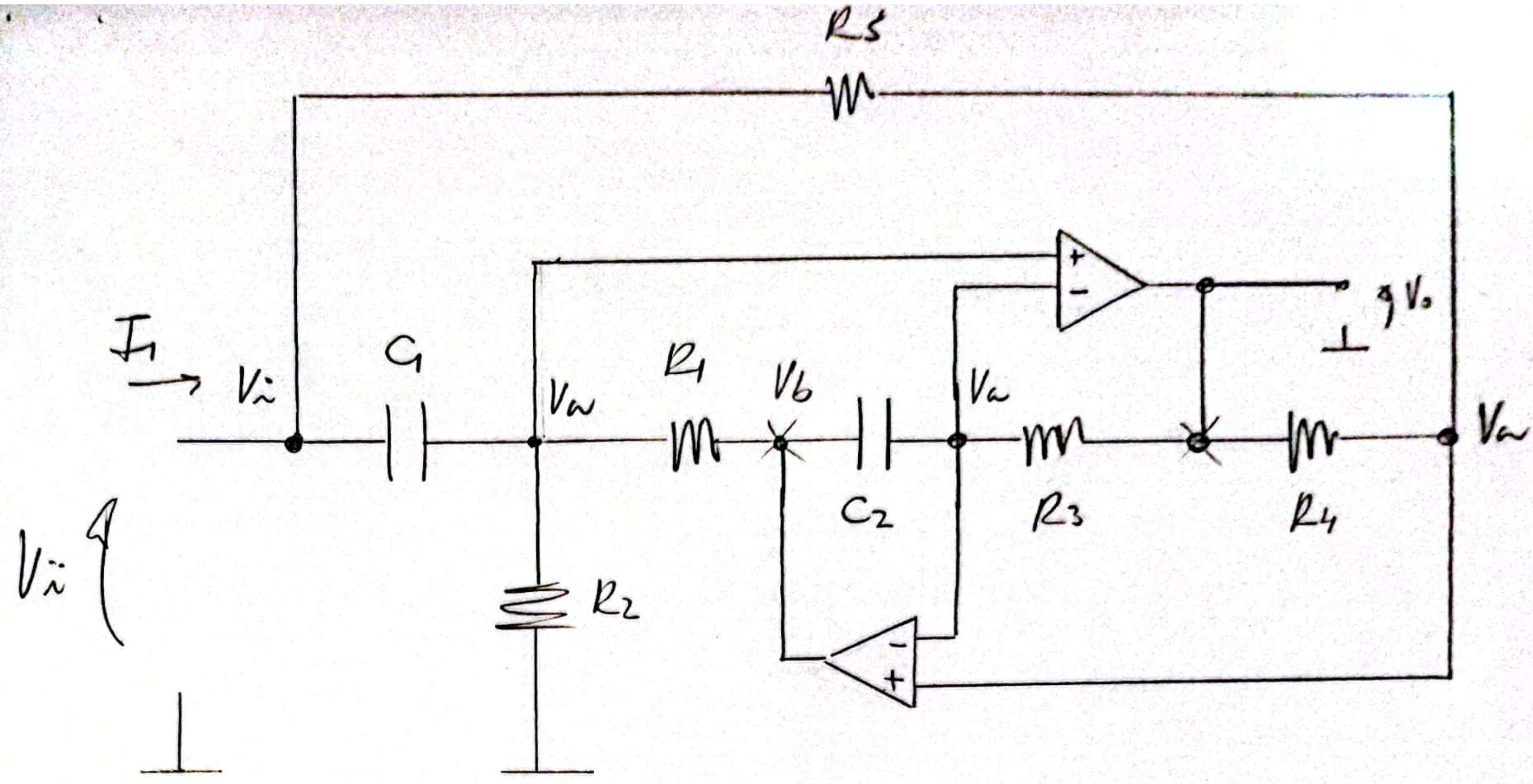
Si $Z_1 = Z_4 \wedge Z_2 = Z_3$

$$\frac{V_2}{V_1} = \frac{Z_2(Z_2+Z_1) - Z_1(Z_2+Z_1)}{(Z_1+Z_2)(Z_2+Z_1)} = \frac{(Z_2+Z_1)(Z_2-Z_1)}{(Z_1+Z_2)^2}$$

$$= \frac{Z_2-Z_1}{Z_1+Z_2}$$

; si $Z_1 = \frac{1}{sC}$ y $Z_2 = R$, obtenemos un filtro pasa todo de orden 2 implementado con un Lattice simétrico que conforma una estructura pasiva $\longrightarrow$ Punto (b.)






$$I_1 = V_i\,(G_5 + sC_1) - V_a\, sC_1 \qquad (1)$$

$$0 = V_a\,(sC_1 + G_1 + G_2) - V_i\, sC_1 - V_b\, G_1 \qquad (2)$$

$$0 = V_a\,(sC_2 + G_3) - V_b\, sC_2 - V_o\, G_3 \qquad (3)$$

$$0 = V_a\,(G_4 + G_5) - V_i\, G_5 - V_o\, G_4 \qquad (4)$$

Despejo $V_a$ de (4): $V_a = \dfrac{V_i G_5 + V_o G_4}{G_4 + G_5}$ (5)

Despejo $V_b$ de (3) y reemplazo (5)

$$V_b = \frac{V_a\,(sC_2 + G_3) - V_o\, G_3}{sC_2} =$$

$$V_b = \frac{V_i G_5 + V_o G_4}{G_4 + G_5} \frac{(sC_2 + G_3)}{sC_2} - \frac{V_o G_3}{sC_2} \quad (6)$$

Reemplazo (5) y (6) en (2):

$$0 = \frac{V_i G_5 + V_o G_4}{G_4 + G_5} (sC_1 + G_1 + G_2) - V_i sC_1$$

$$- \left[ \frac{V_i G_5 + V_o G_4}{G_4 + G_5} \frac{(sC_2 + G_3)}{sC_2} - \frac{V_o G_3}{sC_2} \right] G_1$$

$$0 = V_i \frac{G_5 (sC_1 + G_1 + G_2)}{G_4 + G_5} + V_o \frac{G_4 (sC_1 + G_1 + G_2)}{G_4 + G_5} - V_i sC_1$$

$$- V_i \frac{G_5 G_1 (sC_2 + G_3)}{(G_4 + G_5) sC_2} - V_o \frac{G_4 G_1 (sC_2 + G_3)}{(G_4 + G_5) sC_2} + V_o \frac{G_3 G_1}{sC_2}$$

$$V_o \left[ \frac{G_4 G_1 (sC_2 + G_3)}{(G_4 + G_5) sC_2} - \frac{G_3 G_1}{sC_2} - \frac{G_4 (sC_1 + G_1 + G_2)}{G_4 + G_5} \right] =$$

$$V_i \left[ \frac{G_5 (sC_1 + G_1 + G_2)}{G_4 + G_5} - sC_1 - \frac{G_5 G_1 (sC_2 + G_3)}{(G_4 + G_5) sC_2} \right]$$

$$V_o \quad \frac{G_4 G_1 (sC_2 + G_3) - G_3 G_1 (G_4 + G_5) - G_4 (sC_1 + G_1 + G_2) sC_2}{\cancel{(G_4 + G_5)}\, sC_2} =$$

$$V_i \quad \frac{G_5 (sC_1 + G_1 + G_2) sC_2 - sC_1 (G_4 + G_5) sC_2 - G_5 G_1 (sC_2 + G_3)}{\cancel{(G_4 + G_5)}\, sC_2}$$

$$\frac{V_o}{V_i} = \frac{G_5 (sC_1 + G_1 + G_2) sC_2 - sC_1 (G_4 + G_5) sC_2 - G_5 G_1 (sC_2 + G_3)}{G_4 G_1 (sC_2 + G_3) - G_3 G_1 (G_4 + G_5) - G_4 (sC_1 + G_1 + G_2) sC_2}$$

$$\frac{V_o}{V_i} = \frac{\cancel{s^2 G_5 C_1 C_2} + \cancel{s G_1 G_5 C_2} + s G_2 G_5 C_2 - s^2 G_4 C_1 C_2 - \cancel{s^2 G_5 C_1 C_2} \rightarrow}{s \cancel{G_1 G_4 C_2} + \cancel{G_1 G_3 G_4} - \cancel{G_1 G_3 G_4} - G_1 G_3 G_5}$$

$$\frac{\rightarrow - \cancel{s G_1 G_5 C_2} - G_1 G_3 G_5}{-s^2 G_4 C_1 C_2 - \cancel{s G_1 G_4 C_2} - s G_2 G_4 C_2}$$

$$\frac{V_o}{V_i} = \frac{-s^2 G_4 C_1 C_2 + s G_2 G_5 C_2 - G_1 G_3 G_5}{-s^2 G_4 C_1 C_2 - s G_2 G_4 C_2 - G_1 G_3 G_5}$$

$$\frac{V_o}{V_i} = \frac{+\cancel{G_4}\cancel{C_1}\cancel{C_2}}{+\cancel{G_4}\cancel{C_1}\cancel{C_2}} \quad \frac{s^2 - s\,\frac{G_2\cancel{G_4}\cancel{C_2}}{\cancel{G_4}C_1\cancel{C_2}} + \frac{G_1G_3G_5}{G_4C_1C_2}}{s^2 + s\,\frac{G_2\cancel{G_4}\cancel{C_2}}{\cancel{G_4}C_1\cancel{C_2}} + \frac{G_1G_3G_5}{G_4C_1C_2}}$$

$$\frac{V_o}{V_i} = \frac{s^2 - s\,\frac{G_2}{C_1} + \frac{G_1G_3G_5}{G_4C_1C_2}}{s^2 + s\,\frac{G_2}{C_1} + \frac{G_1G_3G_5}{G_4C_1C_2}} = \frac{s^2 - s\,\frac{\omega_0}{q} + \omega_0^2}{s^2 + s\,\frac{\omega_0}{q} + \omega_0^2}$$

Punto d.

Puntos c, e, f, no adjunto desarrollo. —

10/jul/2023 Son 15 (quince) hojas

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Legajo: 150.152-5