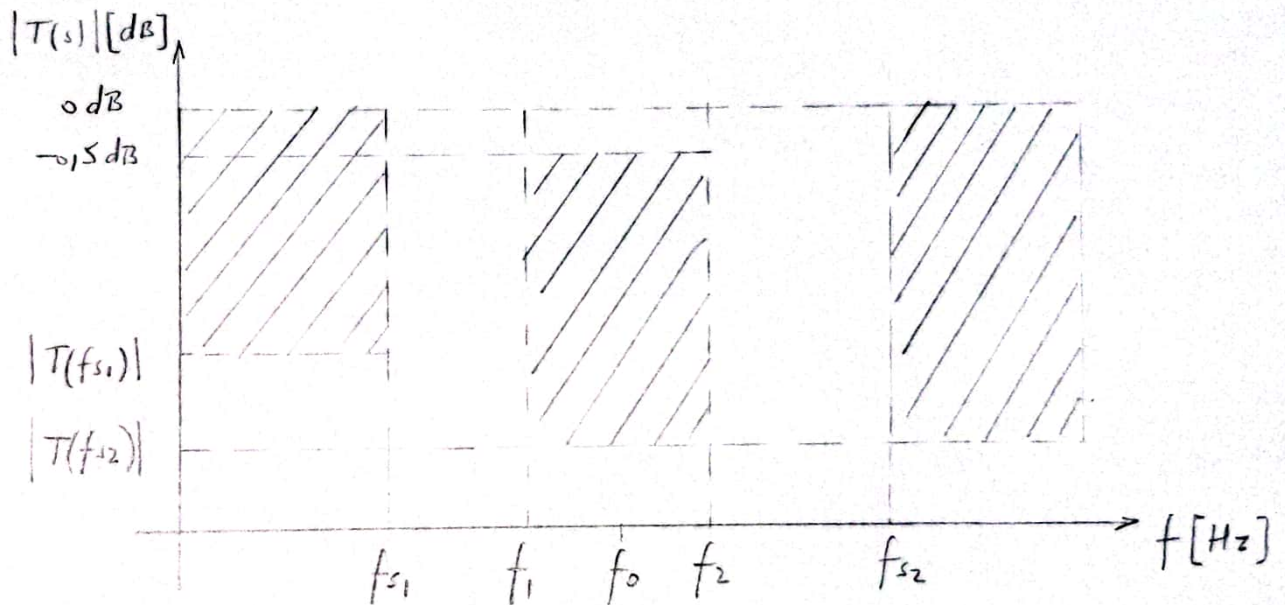


$$\omega_0 = 2\pi \cdot 22 \text{ KHz}, \quad q = 5$$

Aproximação Chebyshev com ripple de 0,5 dB

$$|T(f_{s1})| = -16 \text{ dB para } f_{s1} = 17 \text{ KHz}$$

$$|T(f_{s2})| = -24 \text{ dB para } f_{s2} = 36 \text{ KHz}$$



Schaumann: "Lowpass-to-Bandpass Transformation", Cap 9.2,
 Pág 360-361

$$1. \quad \omega_0 = \sqrt{\omega_1 \omega_2} \quad ; \quad \omega_2 = \frac{\omega_0^2}{\omega_1} \quad (1) \quad \left\{ \begin{array}{l} \frac{\omega_0^2}{\omega_1} = \frac{\omega_0}{q} + \omega_1 \\ \frac{\omega_0^2}{\omega_1} - \omega_1 = \frac{\omega_0}{q} \end{array} \right.$$

$$\text{Multiplico por } \omega_1 \text{ mais: } \omega_0^2 - \omega_1^2 = \frac{\omega_0}{q} \omega_1$$

$$\omega_1^2 + \frac{\omega_0}{q} \omega_1 - \omega_0^2 = 0$$

$$\omega_1^2 + \frac{2\pi \cdot 22 \text{ KHz}}{s} \omega_1 - (2\pi \cdot 22 \text{ KHz})^2 = 0$$

$$\omega_1^2 + 8800\pi \left[\frac{\text{rad}}{s} \right] \omega_1 - 1,936 \cdot 10^9 \pi^2 \left[\frac{\text{rad}}{s} \right]^2 = 0$$

$$\omega_{11} = 125,0965 \cdot 10^3 \frac{\text{rad}}{s} \quad ; \quad \omega_{12} = - \frac{\text{rad}}{s}$$

Descartado.

Reemplazando $\omega_{11} = \omega_1 = 125,0965 \cdot 10^3 \frac{\text{rad}}{s}$ en (1):

$$\omega_2 = \frac{\omega_0^2}{\omega_1} = \frac{(2\pi \cdot 22 \text{ KHz})^2}{125,0965 \cdot 10^3 \frac{\text{rad}}{s}} = 152,7425 \cdot 10^3 \frac{\text{rad}}{s}$$

$$2. \quad \Omega\omega = \omega_0 = 2\pi \cdot 22 \text{ KHz}$$

$$\omega_{1-n} = \frac{\omega_1}{\Omega\omega} = \frac{\omega_1}{\omega_0} = \frac{125,0965 \cdot 10^3 \text{ rad/s}}{2\pi \cdot 22 \text{ KHz}} \approx 0,9050$$

$$\omega_{2-n} = \frac{\omega_2}{\Omega\omega} = \frac{\omega_2}{\omega_0} = \frac{152,7425 \cdot 10^3 \text{ rad/s}}{2\pi \cdot 22 \text{ KHz}} \approx 1,1050$$

$$\omega_{s1-n} = \frac{\omega_{s1}}{\Omega\omega} = \frac{\omega_{s1}}{\omega_0} = \frac{2\pi \cdot f_{s1}}{2\pi \cdot 22 \text{ KHz}} = \frac{2\pi \cdot 17 \text{ KHz}}{2\pi \cdot 22 \text{ KHz}} \approx 0,7727$$

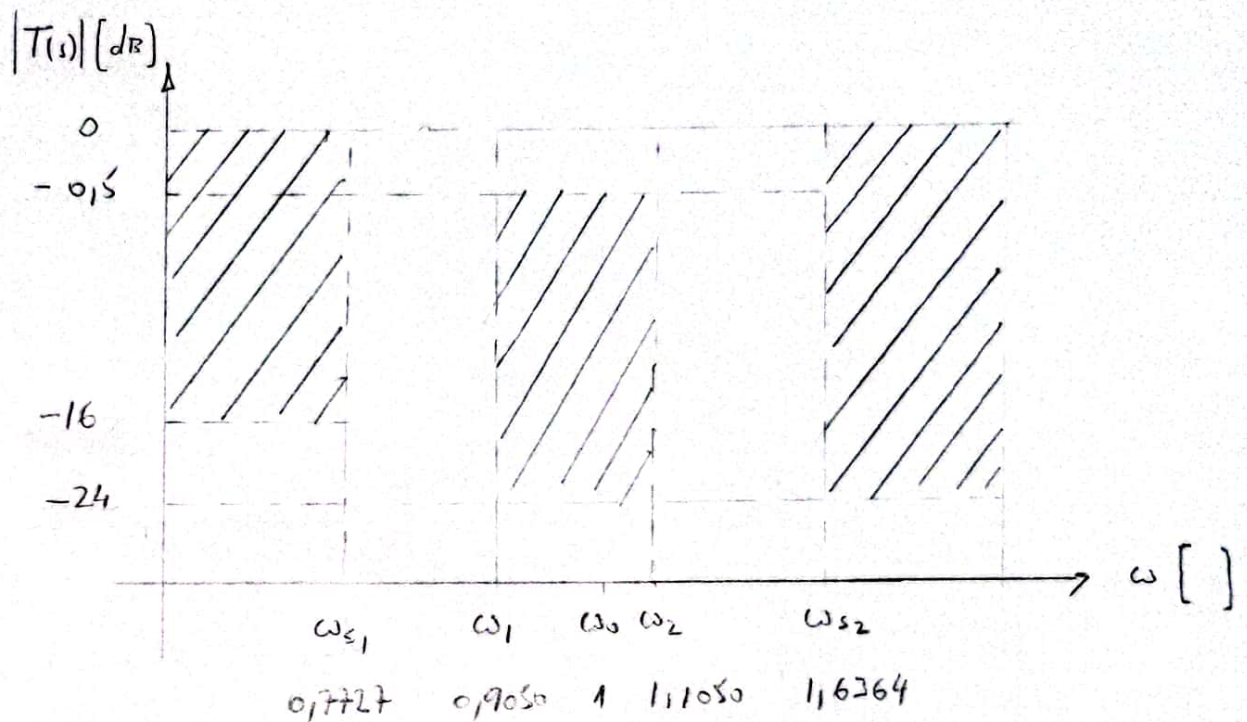
$$\omega_{s2-n} = \frac{\omega_{s2}}{\Omega\omega} = \frac{\omega_{s2}}{\omega_0} = \frac{2\pi \cdot f_{s2}}{2\pi \cdot 22 \text{ KHz}} = \frac{2\pi \cdot 36 \text{ KHz}}{2\pi \cdot 22 \text{ KHz}} \approx 1,6364$$

$$\omega_{0-n} = \frac{\omega_0}{\Omega\omega} = \frac{\omega_0}{\omega_0} = 1$$

En adelante, omito el subíndice n pero recordando que se trata

de pulsaciones (o bien frecuencias) normalizadas.

Plantilla de diseño para banda normalizada:



3.

$$\Omega_{s1} = 9 \frac{(\omega_{s1}^2 - 1)}{\omega_{s1}} = 5 \frac{(0.7727^2 - 1)}{0.7727} = -2.6073$$

Como $|T_{LP}|$ se trata de una función par, desestimamos el signo (-).

$$\Omega_{s2} = 9 \frac{(\omega_{s2}^2 - 1)}{\omega_{s2}} = 5 \frac{(1.6364^2 - 1)}{1.6364} = 5.1265$$

$$\begin{aligned}
 5. \quad |T_{LP}(j\Omega)|^2 &= T_{LP}(j\Omega) \cdot T_{LP}(-j\Omega) = T_{LP}(s) T_{LP}(-s) \Big|_{s=j\Omega} \\
 &= \frac{1}{1 + \xi^2 C_n^2(\Omega)}
 \end{aligned}$$

$$|\alpha|^2 = 1 + \xi^2 C_n^2(\Omega) ; |\alpha|_{dB} \triangleq \alpha_{dB} ;$$

$$\alpha_{dB} = 10 \log [1 + \xi^2 C_n^2(\Omega)]$$

$$\text{Despejando } \xi^2 : \xi^2 = \frac{10^{\alpha_{dB}/10} - 1}{C_n^2(\Omega)}$$

Para (α_{\min}, f_p) , o bien, $(\alpha_{\min}, \Omega_{pn})$, obtengo ξ^2 :

$$\xi^2 = \frac{10^{\alpha_{\min}/10} - 1}{C_n^2(\Omega_{pn})} = \frac{10^{0,5/10} - 1}{1} = 0,122 ; \xi = 0,3493$$

Para (α_{\min}, f_{s1}) , o bien, $(\alpha_{\min}, \Omega_{s1})$, itero n veces

hasta encontrar un $n \in \mathbb{Z}$ que cumpla con $\alpha_{\min} \geq 16 \text{ dB}$:

$$\alpha_{\min \text{ dB}} = 10 \log [1 + \xi^2 C_n^2(\Omega_{s1})]$$

$$\alpha_{\min \text{ dB}} = 10 \log \left\{ 1 + \xi^2 \cosh^2 \left[n \cdot \cosh^{-1}(\Omega_{s1}) \right] \right\}$$

$$n=1 : \alpha_{\min} \text{ dB} = 10 \log \left\{ 1 + 0,122 \cosh^2 \left[1 \cdot \cosh^{-1}(2,6073) \right] \right\}$$

$$= 2,623 \text{ dB}$$

$$n=2 : \alpha_{\min} \text{ dB} = 10 \log \left\{ 1 + 0,122 \cosh^2 \left[2 \cdot \cosh^{-1}(2,6073) \right] \right\}$$

$$= 13,088 \text{ dB}$$

$$n=3 : \alpha_{\min} \text{ dB} = 10 \log \left\{ 1 + 0,122 \cosh^2 \left[3 \cdot \cosh^{-1}(2,6073) \right] \right\}$$

$$= 26,870 \text{ dB} \geq 16 \text{ dB}$$

Para (α_{\min}, f_{s2}) , o bien, $(\alpha_{\min}, \Omega_{s2})$, itero n veces hasta encontrar un $n \in \mathbb{Z}$ que cumpla con $\alpha_{\min} \geq 24 \text{ dB}$:

$$\alpha_{\min} \text{ dB} = 10 \log \left\{ 1 + \xi^2 \cosh^2 \left[n \cdot \cosh^{-1}(\Omega_{s2}) \right] \right\}$$

$$n=1 : \alpha_{\min} \text{ dB} = 10 \log \left\{ 1 + 0,122 \cosh^2 \left[1 \cdot \cosh^{-1}(5,1265) \right] \right\}$$

$$= 6,239 \text{ dB}$$

$$n=2 : \alpha_{\min} \text{ dB} = 10 \log \left\{ 1 + 0,122 \cosh^2 \left[2 \cdot \cosh^{-1}(5,1265) \right] \right\}$$

$$= 25,124 \text{ dB} \geq 24 \text{ dB}$$

Nos que daures con $n=3$ de forma que satisfaga ambos requerimientos.

$$6. \quad C_0(\Omega) = 1$$

$$C_1(\Omega) = \Omega$$

$$C_n(\Omega) = 2\Omega \cdot C_{n-1}(\Omega) - C_{n-2}(\Omega)$$

$$C_2(\Omega) = 2\Omega \cdot \Omega - 1 = 2\Omega^2 - 1$$

$$C_3(\Omega) = 2\Omega(2\Omega^2 - 1) - \Omega = 4\Omega^3 - 2\Omega - \Omega = 4\Omega^3 - 3\Omega$$

$$|T_{LP}(j\Omega)|^2 = \frac{1}{1 + \xi^2 C_3^2(\Omega)} = \frac{1}{1 + \xi^2 (4\Omega^3 - 3\Omega)^2}$$

$$= \frac{1}{1 + \xi^2 (4\Omega^3 - 3\Omega)(4\Omega^3 - 3\Omega)}$$

$$= \frac{1}{1 + \xi^2 (16\Omega^6 - 12\Omega^4 - 12\Omega^4 + 9\Omega^2)}$$

$$= \frac{1}{16\xi^2\Omega^6 - 24\xi^2\Omega^4 + 9\xi^2\Omega^2 + 1}$$

Divido numerador y denominador por $16\xi^2$:

$$|T_{LP}(j\Omega)|^2 = \frac{a}{\Omega^6 - b\Omega^4 + c\Omega^2 + a} ; \quad b = \frac{24\cancel{\Omega^2}}{16\cancel{\Omega^2}} = \frac{3}{2}$$

$$c = \frac{9\cancel{\Omega^2}}{16\cancel{\Omega^2}} = \frac{9}{16}$$

Reemplazo Ω por $\frac{s}{j}$:

$$a = \frac{1}{16\cancel{\Omega^2}}$$

$$|T_{LP}(s)|^2 = \frac{a}{\left(\frac{s}{j}\right)^6 - b\left(\frac{s}{j}\right)^4 + c\left(\frac{s}{j}\right)^2 + a}$$

$$= \frac{a}{-s^6 - b s^4 - c s^2 + a}$$

$$|T_{LP}(s)|^2 = T_{LP}(s) \cdot T_{LP}(-s) = \underbrace{\frac{\alpha}{s^3 + \beta s^2 + \gamma s + \alpha}}_{T_{LP}(s)} \underbrace{\frac{\alpha}{-s^3 + \beta s^2 - \gamma s + \alpha}}_{T_{LP}(-s)}$$

$$= \frac{\alpha^2}{-s^6 + \beta s^5 - \gamma s^4 + \alpha s^3 - \beta s^5 + \beta^2 s^4 - \beta \gamma s^3 + \alpha \beta s^2}$$

$$\rightarrow \frac{-\gamma s^4 + \beta \gamma s^3 - \gamma^2 s^2 + \alpha \gamma s - \alpha s^3 + \alpha \beta s^2 - \alpha \gamma s + \alpha^2}{-s^6 + \beta s^5 - \gamma s^4 + \alpha s^3 - \beta s^5 + \beta^2 s^4 - \beta \gamma s^3 + \alpha \beta s^2}$$

$$= \frac{\alpha^2}{-s^6 + (\beta - \beta)s^5 + (-\gamma + \beta^2 - \gamma)s^4 + (\alpha - \beta\gamma + \beta\gamma - \alpha)s^2 +}$$

$$\frac{(\alpha\beta - \gamma^2 + \alpha\beta)s^2 + (\alpha\gamma - \alpha\gamma)s + \alpha^2}{\alpha^2}$$

$$= \frac{\alpha^2}{-s^6 + \underbrace{(\beta^2 - 2\gamma)}_{-b}s^4 + \underbrace{(2\alpha\beta - \gamma^2)}_{-c}s^2 + \underbrace{\alpha^2}_a}$$

$$a = \alpha^2 = \frac{1}{16\gamma^2} ; \alpha = \frac{1}{4\gamma} = \frac{1}{4 \cdot 0,3493} = 0,7157$$

$$-b = \beta^2 - 2\gamma = -\frac{3}{2} ; \beta = \sqrt{2\gamma - \frac{3}{2}} ; \gamma = \frac{\beta^2}{2} + \frac{3}{4} \quad (1)$$

$$-c = 2\alpha\beta - \gamma^2 = -\frac{9}{16} ; \gamma = \sqrt{2\alpha\beta + \frac{9}{16}} ; \beta = \frac{\gamma^2}{2\alpha} - \frac{9}{32\alpha}$$

$$\gamma = \sqrt{2 \cdot 0,7157 \cdot \beta + \frac{9}{16}} ; \beta = \frac{\gamma^2}{2 \cdot 0,7157} - \frac{9}{32 \cdot 0,7157}$$

$$\gamma = \sqrt{1,4314\beta + \frac{9}{16}} \quad (3) ; \beta = \frac{\gamma^2}{1,4314} - 0,3930 \quad (2)$$

Reemplazo (1) en (2):

$$\beta = \frac{\left(\frac{\beta^2}{2} + \frac{3}{4}\right)^2}{1,4314} - 0,3930 = \frac{\left(\frac{\beta^4}{4} + 2\frac{\beta^2}{2}\frac{3}{4} + \frac{9}{16}\right)}{1,4314} - 0,3930$$

$$\beta = 0,6986 \left(\frac{\beta^4}{4} + \frac{3\beta^2}{4} + \frac{9}{16} \right) - 0,3930 =$$

$$0,6986 \frac{\beta^4}{4} + 0,6986 \frac{3\beta^2}{4} - \beta + 0,6986 \frac{9}{16} - 0,3930 =$$

$$0,1747 \beta^4 + 0,5239 \beta^2 - \beta + 0,3930 - 0,3930 =$$

$$\beta^4 + 2,9988 \beta^2 - 5,17241 \beta = 0 ; \beta (\beta^3 + 2,9988 \beta - 5,17241) = 0$$

$$\beta_1 = 0, \beta_2 = 1,2529, \beta_3, \beta_4, \text{ raíces complejas conjugadas}$$

Reemplazando $\beta_2 = \beta$ en (1):

$$Y = \frac{1,2529^2}{2} + \frac{3}{4} = 1,5349$$

$$\text{Verificación: } \beta^2 - 2Y = -b ; 1,2529^2 - 2 \cdot 1,5349 = -\frac{3}{2} \checkmark$$

$$2\alpha\beta - Y^2 = -c ; 2 \cdot 0,7157 \cdot 1,2529 - 1,5349^2 = -\frac{9}{16} \checkmark$$

$$T_{LP}(s) = \frac{\alpha}{s^3 + \beta s^2 + \gamma s + \alpha} = \frac{0,7157}{s^3 + 1,2529 s^2 + 1,5349 s + 0,7157}$$

7. $T_{LP}(f) \Big|_{f=K(s)} = T_{BP}(s)$

$$f = K(s) = \frac{1}{B} \frac{s^2+1}{s}$$

Reemplazando el núcleo de la transformación $K(s)$ en $T_{LP}(f)$:

$$T_{BP}(s) = \frac{0,7157}{\left(\frac{1}{B} \frac{s^2+1}{s}\right)^3 + 1,2529 \left(\frac{1}{B} \frac{s^2+1}{s}\right)^2 + 1,5349 \left(\frac{1}{B} \frac{s^2+1}{s}\right) + 0,7157}$$

$$= \frac{0,7157}{\frac{1}{B^3} \frac{(s^2+1)^3}{s^3} + 1,2529 \frac{1}{B^2} \frac{(s^2+1)^2}{s^2} + 1,5349 \frac{1}{B} \frac{s^2+1}{s} + 0,7157}$$

$$= \frac{0,7157}{\frac{(s^2+1)(s^2+1)(s^2+1)}{B^3 s^3} + 1,2529 \frac{1}{B^2} \frac{s^4+2s^2+1}{s^2} + 1,5349 \frac{1}{B} \frac{s^2+1}{s} + 0,7157}$$

$$= \frac{0,7157}{\frac{(s^4+2s^2+1)(s^2+1)}{B^3 s^3} + \dots}$$

$$= \frac{s^6 + s^4 + 2s^4 + 2s^2 + s^2 + 1}{B^3 s^3} + \dots$$

$$= \frac{s^6 + 3s^4 + 3s^2 + 1}{B^3 s^3} + \frac{1,2529 (s^4 + 2s^2 + 1)}{B^2 s^2} + \frac{1,5349 (s^2 + 1)}{B \cdot s} + 0,7157$$

$$= \frac{s^6 + 3s^4 + 3s^2 + 1 + [1,2529 (s^4 + 2s^2 + 1) B s] + [1,5349 (s^2 + 1) B^2 s^2] + 0,7157 B^3 s^3}{B^3 s^3}$$

$$= \frac{s^6 + 3s^4 + 3s^2 + 1 + [1,2529 (s^4 + 2s^2 + 1) B s] + [1,5349 (s^2 + 1) B^2 s^2] + 0,7157 B^3 s^3}{B^3 s^3}$$

$$= \frac{s^6 + 3s^4 + 3s^2 + 1 + 1,2529 \cdot B s^5 + 2,5058 \cdot B s^3 + 1,2529 \cdot B s + 1,5349 B^2 s^4 + \dots}{B^3 s^3}$$

$$> 1,5349 B^2 s^2 + 0,7157 B^3 s^3$$

$$= \frac{0,7157 B^3 s^3}{s^6 + 1,2529 B s^5 + (3 + 1,5349 B^2) s^4 + (2,5058 \cdot B + 0,7157 B^3) s^3 +}$$

$$\rightarrow \frac{(3 + 1,5349 B^2) s^2 + 1,2529 B s + 1}{}$$

$$= \frac{0,7157 \cdot \left(\frac{1}{s}\right)^3 s^3}{s^6 + 1,2529 \cdot \frac{1}{s} s^5 + \left[3 + 1,5349 \left(\frac{1}{s}\right)^2\right] s^4 + \left[2,5058 \frac{1}{s} + 0,7157 \left(\frac{1}{s}\right)^3\right] s^3 +}$$

$$\rightarrow \frac{\left[3 + 1,5349 \left(\frac{1}{s}\right)^2\right] s^2 + 1,2529 \frac{1}{s} s + 1}{}$$

$$= \frac{5,7256 \cdot 10^{-2} s^3}{s^6 + 0,2506 s^5 + 3,0614 s^4 + 0,5069 s^3 + 3,0614 s^2 + 0,2506 s + 1}$$

den

raices-den = numpy.roots(den)

$$p_1 = -0,0347 + j1,1069$$

$$p_4 = -0,0624 - j0,9981$$

$$p_2 = -0,0347 - j1,1069$$

$$p_5 = -0,0283 + j0,9026$$

$$p_3 = -0,0624 + j0,9981$$

$$p_6 = -0,0283 - j0,9026$$

$$T_{BP}(s) = \frac{5,7256 \cdot 10^{-3} s^3}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)(s-p_5)(s-p_6)}$$

$$T_{BP}(s) = \frac{5,7256 \cdot 10^{-3} s^3}{\left[s - (-0,0347 + j1,1069)\right] \left[s - (-0,0347 - j1,1069)\right]}$$

$$\left[s - (-0,0624 + j0,9981)\right] \left[s - (-0,0624 - j0,9981)\right]$$

$$\left[s - (-0,0283 + j0,9026)\right] \left[s - (-0,0283 - j0,9026)\right]$$

$$T_{BP}(s) = \frac{5,7256 \cdot 10^{-3} s^3}{\left(s^2 + 0,0347s + j1,1069s + 0,0347s + 0,0347^2 + \right. \\ \left. j0,0347 \cdot 1,1069 - j1,1069s - j1,1069 \cdot 0,0347 - j^2 1,1069^2\right) \\ \left(s^2 + 0,0624s + j0,9981s + 0,0624s + 0,0624^2 + \right. \\ \left. j0,0624 \cdot 0,9981 - j0,9981s - j0,9981 \cdot 0,0624 - j^2 0,9981^2\right) \\ \left(s^2 + 0,0283s + j0,9026s + 0,0283s + 0,0283^2 + \right. \\ \left. j0,0283 \cdot 0,9026 - j0,9026s - j0,9026 \cdot 0,0283 - j^2 0,9026^2\right)}$$

$$T_{eq}(s) = \frac{5,7256 \cdot 10^{-3} s^3}{\left[s^2 + (0,0347 + 0,0347) s + 0,0347^2 + 1,1069^2 \right]}$$

$$\frac{\rightarrow}{\left[s^2 + (0,0624 + 0,0624) s + 0,0624^2 + 0,9981^2 \right]}$$

$$\frac{\rightarrow}{\left[s^2 + (0,0283 + 0,0283) s + 0,0283^2 + 0,9026^2 \right]}$$

$$T_{BP}(s) = \frac{5,7256 \cdot 10^{-3} s^3}{\left(s^2 + 0,0694 s + 1,2264 \right) \left(s^2 + 0,1248 s + 1 \right)}$$

$$\frac{\rightarrow}{\left(s^2 + 0,0566 s + 0,8155 \right)}$$

$$T_{BP}(s) = \frac{h_1 \frac{\omega_{01}}{q_1} s}{s^2 + \frac{\omega_{01}}{q_1} s + \omega_{01}^2} \cdot \frac{h_2 \frac{\omega_{02}}{q_2} s}{s^2 + \frac{\omega_{02}}{q_2} s + \omega_{02}^2} \cdot \frac{h_3 \frac{\omega_{03}}{q_3} s}{s^2 + \frac{\omega_{03}}{q_3} s + \omega_{03}^2}$$

$$\omega_{01}^2 = 1,2264$$

$$\omega_{01} = 1,1074$$

$$\frac{\omega_{01}}{q_1} = 0,0694$$

$$q_1 = 15,9568$$

$$\omega_{02}^2 = 1$$

$$\omega_{02} = 1$$

$$\frac{\omega_{02}}{q_2} = 0,1248$$

$$q_2 = 8,0128$$

$$\omega_{03}^2 = 0,8155$$

$$\omega_{03} = 0,9031$$

$$\frac{\omega_{03}}{q_3} = 0,0566$$

$$q_3 = 15,9558$$

$$T_{BP}(s) = \frac{h_1 \frac{1,11074}{15,9568} s}{s^2 + \frac{1,11074}{15,9568} s + 1,2264} \cdot \frac{h_2 \frac{1}{8,0128} s}{s^2 + \frac{1}{8,0128} s + 1} \cdot \frac{h_3 \frac{0,9031}{15,9588} s}{s^2 + \frac{0,9031}{15,9588} s + 0,8155}$$

$$h_1 \frac{1,11074}{15,9568} = h_1 0,0694 = \sqrt[3]{5,7256 \cdot 10^{-3}} ; h_1 = 2,5778$$

$$h_2 \frac{1}{8,0128} = h_2 0,1248 = \sqrt[3]{5,7256 \cdot 10^{-3}} ; h_2 = 1,4335$$

$$h_3 \frac{0,9031}{15,9588} = h_3 0,0566 = \sqrt[3]{5,7256 \cdot 10^{-3}} ; h_3 = 3,1608$$

$$T_{BP}(s) = \frac{2,5778 \cdot 0,0694 \cdot s}{s^2 + 0,0694 s + 1,2264} \cdot \frac{1,4335 \cdot 0,1248 s}{s^2 + 0,1248 s + 1} \cdot \frac{3,1608 \cdot 0,0566 \cdot s}{s^2 + 0,0566 s + 0,8155}$$

Conviene reescribirla como:

$$T_{BP}(s) = \frac{1,4335 \cdot 0,1248 s}{s^2 + 0,1248 s + 1} \cdot \frac{2,5778 \cdot 0,0694 s}{s^2 + 0,0694 s + 1,2264} \cdot \frac{3,1608 \cdot 0,0566 s}{s^2 + 0,0566 s + 0,8155}$$

$$= \frac{1,4335 \frac{1}{8,0128} s}{s^2 + \frac{1}{8,0128} s + 1} \cdot \frac{2,5778 \cdot \frac{1,11074}{15,9568} s}{s^2 + \frac{1,11074}{15,9568} s + 1,2264} \cdot \frac{3,1608 \frac{0,9031}{15,9588} s}{s^2 + \frac{0,9031}{15,9588} s + 0,8155}$$

$$\omega_{01} \cdot \omega_{03} = 1 \wedge \gamma_1 \approx \gamma_3$$