TS3 con aproximación de Chevyshev.

ξ^2 y n son iguales a máxima planicidad

No así la $|T(j\omega)|^2$ que ahora es $\dfrac{1}{1+\xi^2 \underline{C_n^2(\omega)}}$

$\cosh^2(n \cdot \cosh^{-1}(\omega))$

NOTA

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$$C_0(\omega) = 1$$
$$C_1(\omega) = \omega$$
$$C_n(\omega) = 2\omega \cdot C_{n-1}(\omega) - C_{n-2}(\omega).$$

$$C_2(\omega) = 2 \cdot \omega \cdot \omega - 1 = 2\omega^2 - 1$$

$$C_3(\omega) = 2\omega\left(2\omega^2-1\right) - \omega = 4\omega^3 - 2\omega - \omega = 4\omega^3 - 3\omega$$

Schaumann Pág 279

$$\left|T(j\omega)\right|^2 = \frac{1}{1+\xi^2\left(4\omega^3-3\omega\right)^2}$$

$$= \frac{1}{1+\xi^2\left(16\omega^6 - 24\omega^4 + 9\omega^2\right)}$$

$$= \frac{1}{16\xi^2\omega^6 - 24\xi^2\omega^4 + 9\xi^2\omega^2 + 1}$$

$$= \frac{a}{\omega^6 - b\omega^4 + c\omega^2 + a}$$; totalmente arbitrario.

$$a = \frac{1}{16\xi^2}$$

$$b = \frac{24\xi^2}{16\xi^2} = \frac{3}{2}$$

$$c = \frac{9\xi^2}{16\xi^2} = \frac{9}{16}$$

$$\left|T(j\omega)\right|^2_{\omega = \frac{s}{j}} = \frac{a}{-s^6 - bs^4 - cs^2 + a}$$

$$= \frac{\alpha}{s^3+\beta s^2+\gamma s+\alpha} \cdot \frac{\alpha}{-s^3+\beta s^2-\gamma s+\alpha}$$

Resolver...

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$$= \frac{\alpha^2}{-s^6+\beta s^5-\gamma s^4+\alpha s^3-\beta s^5+\beta^2 s^4-\beta\gamma s^3+\alpha\beta s^2 \rightarrow}$$

$$\rightarrow \overline{-\gamma s^4+\beta\gamma s^3-\gamma^2 s^2+\alpha\gamma s-\alpha s^3+\alpha\beta s^2-\alpha\gamma s+\alpha^2}$$

$$= \frac{\alpha^2}{-s^6+(\beta-\beta)s^5+(-\gamma+\beta^2-\gamma)s^4+(\alpha-\beta\gamma+\beta\gamma-\alpha)s^3+ \rightarrow}$$

$$\rightarrow \overline{(\alpha\beta-\gamma^2+\alpha\beta)s^2+(\alpha\gamma-\alpha\gamma)s+\alpha^2}$$

$$= \frac{\alpha^2}{-s^6+\underbrace{(\beta^2-2\gamma)}_{-b}s^4+\underbrace{(2\alpha\beta-\gamma^2)}_{-c}s^2+\underbrace{\alpha^2}_{a}}$$

$$a=\alpha^2=\frac{1}{16\xi^2} \; ; \; \alpha=\frac{1}{4\xi}=\frac{1}{4\cdot 0{,}5088}=0{,}4913$$

$$-b=\beta^2-2\gamma=-\frac{3}{2} \; ; \; \beta=\sqrt{2\gamma-\frac{3}{2}} \; ; \; \gamma=\frac{\beta^2}{2}+\frac{3}{4} \quad (1)$$

$$-c=2\alpha\beta-\gamma^2=-\frac{9}{16} \; ; \; \gamma=\sqrt{2\alpha\beta+\frac{9}{16}} \; ; \; \beta=\frac{\gamma^2}{2\alpha}-\frac{9}{32\alpha}$$

$$\gamma=\sqrt{2\cdot 0{,}4913\cdot\beta+\frac{9}{16}} \; ; \; \beta=\frac{\gamma^2}{2\cdot 0{,}4913}-\frac{9}{32\cdot 0{,}4913}$$

$$\gamma=\sqrt{0{,}9826\beta+\frac{9}{16}} \quad (3) \; ; \; \beta=\frac{\gamma^2}{0{,}9826}-0{,}5725 \quad (2)$$

NOTA

Reemplazo (1) en (2):

$$\beta = \frac{\left(\frac{\beta^2}{2} + \frac{3}{4}\right)^2}{0{,}9826} - 0{,}5725 = \frac{\frac{\beta^4}{4} + 2\frac{\beta^2}{2}\frac{3}{4} + \frac{9}{16}}{0{,}9826} - 0{,}5725$$

$$\beta = 1{,}0177\left(\frac{\beta^4}{4} + \frac{3\beta^2}{4} + \frac{9}{16}\right) - 0{,}5725$$

$$1{,}0177\,\frac{\beta^4}{4} + 1{,}0177\,\frac{3\beta^2}{4} - \beta + 1{,}0177\,\frac{9}{16} - 0{,}5725 = 0$$

$$0{,}2544\,\beta^4 + 0{,}7633\,\beta^2 - \beta = 0$$

$$\beta^4 + 3{,}0004\,\beta^2 - 3{,}9808\,\beta = 0$$

$$\beta\left(\beta^3 + 3{,}0004\,\beta - 3{,}9808\right) = 0$$

$\beta_1 = 0$ $\qquad\beta_3 = -0{,}4984 + j\,1{,}9353$

$\beta_2 = 0{,}9967$ $\qquad\beta_4 = -0{,}4984 - j\,1{,}9353$

Reemplazando β_2 en (1):

$$\gamma = \frac{0{,}9967^2}{2} + \frac{3}{4} = 1{,}2467$$

Verificación: $\beta^2 - 2\gamma = -b$; $0{,}9967^2 - 2.1{,}2467 = -b = -\frac{3}{2}$ ✓

$2\gamma\beta - \gamma^2 = -c$; $2.0{,}4913.0{,}9967 - 1{,}2467^2 = -c$

$-0{,}5749 \approx -c = -\frac{9}{16} = -0{,}5625$

Error 2% +/-

HOJA N°

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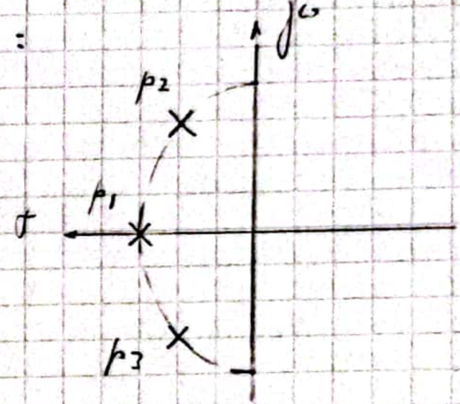
$$T(s) = \frac{\alpha}{s^3 + \beta s^2 + \gamma s + \alpha} = \frac{0{,}4913}{s^3 + 0{,}9967 s^2 + 1{,}2467 s + 0{,}4913}$$

Singularidades del denominador:

$p_1 = -0{,}4921$

$p_2 = -0{,}2523 + j0{,}9668$

$p_3 = -0{,}2523 - j0{,}9668$



$$T(s) = \frac{0{,}4913}{(s+0{,}4921)\left[s-(-0{,}2523+j0{,}9668)\right]\left[s-(-0{,}2523-j0{,}9668)\right]}$$

$$T(s) = \frac{0{,}4913}{(s+0{,}4921)(s^2 + 0{,}2523 s + j0{,}9968 s + 0{,}2523 s + 0{,}2523^2 + j0{,}2438 - j0{,}9668 s - j0{,}2438 + 0{,}9668^2}$$

$$T(s) = \frac{0{,}4913}{(s+0{,}4921)(s^2 + 0{,}5046 s + 0{,}9984)}$$

$$T(s) = \frac{0{,}4921}{(s+0{,}4921)} \frac{0{,}9984}{(s^2 + 0{,}5046 s + 0{,}9984)}$$

NOTA

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