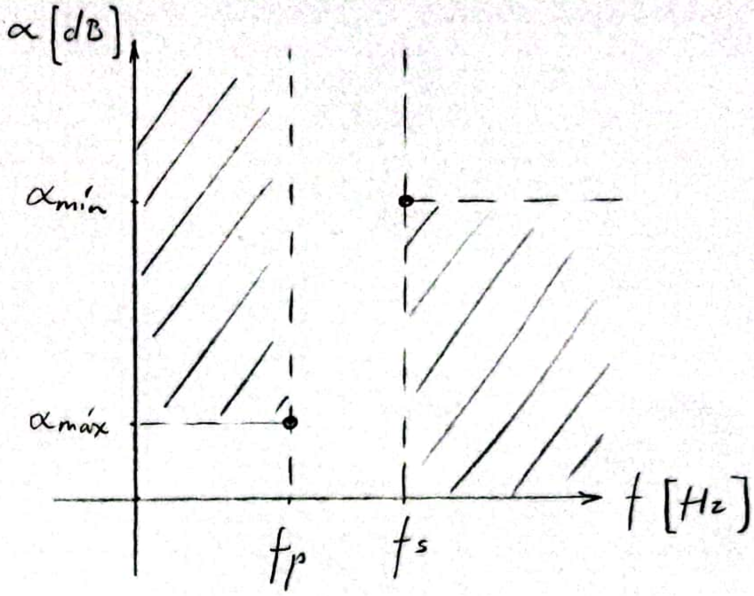

$\alpha_{máx} = 1\,dB$ $\quad f_p = 1500\,Hz$ $\quad \Omega_\omega = \omega_p = 2\pi f_p \;;\; \omega_{pn} = \dfrac{\omega_p}{\Omega_\omega} = 1$

$\alpha_{mín} = 12\,dB$ $\quad f_s = 3000\,Hz$ $\quad \omega_{sn} = \dfrac{\omega_s}{\Omega_\omega} = \dfrac{2\pi f_s}{\Omega\omega} = \dfrac{2\pi \cdot 2 f_p}{\Omega\omega} = 2\omega_{pn} = 2$

$$\left| T(j\omega) \right|^2 = T(j\omega) \cdot T(-j\omega) = T(s) \cdot T(-s) \Big|_{s=j\omega} = \frac{1}{1 + \underline{\underline{\xi^2 \omega^{2n}}}}$$

$$|\alpha|^2 = 1 + \xi^2 \omega^{2n} \;;\; |\alpha|_{dB} \triangleq \alpha dB \;;\; \underbrace{\alpha dB = 10 \log \left(1 + \xi^2 \omega^{2n}\right)}\; (1)$$

Despejando: $\xi^2 = \dfrac{10^{\frac{\alpha dB}{10}} - 1}{\omega^{2n}}$ (2)

Para $(\alpha_{máx}, f_p)$, o bien, $(\alpha_{máx}, \omega_{pn})$, reemplazo valores en (2):

$$\xi^2 = 10^{\frac{\alpha_{máx}}{10}} - 1 = 0{,}2589 \;;\; \xi = 0{,}5088$$

Escaneado con CamScanner

Para $(\alpha_{mín}, f_s)$, o bien, $(\alpha_{mín}, \omega_{s_n})$, itero en (1)

hasta encontrar un $n \in \mathbb{Z}$ que cumpla con $\alpha_{mín} \geq 12\,dB$:

$$10 \log \left(1+ \xi^2 \omega_{s_n}^{2n}\right) = \alpha_{mín}$$

$n=1$: $\alpha_{mín} = 10 \log \left(1+ 0{,}2589 \cdot 2^{2\cdot 1}\right) \simeq 3{,}0869\,dB$

$n=2$: $\alpha_{mín} = 10 \log \left(1+ 0{,}2589 \cdot 2^{2\cdot 2}\right) \simeq 7{,}1116\,dB$

$\underline{n=3}$: $\alpha_{mín} = 10 \log \left(1+ 0{,}2589 \cdot 2^{2\cdot 3}\right) \simeq \underline{12{,}4476\,dB > 12\,dB}$

$$|T(j\omega)|^2 = \frac{1}{1+\xi^2 \omega^{2\cdot 3}} = \frac{1}{1+\xi^2 \omega^6}\bigg|_{\omega=\frac{s}{j}} = T(s)\cdot T(-s)\bigg|_{s=j\omega}$$

$$= \frac{1}{1+\xi^2\left(\frac{s}{j}\right)^6} = \frac{1}{1+\xi^2 \frac{s^6}{(-1)}} = \frac{1}{1-\xi^2 s^6} = \frac{1/\xi^2}{1/\xi^2 - s^6}$$

$$= \frac{c}{(s^3+a\cdot s^2+b\cdot s+c)}\frac{c}{(-s^3+a\cdot s^2-b\cdot s+c)}$$

$$= \frac{c^2}{-s^6+as^5-bs^4+c\cdot s^3-as^5+a^2s^4-abs^3+acs^2-bs^4+abs^3 -b^2s^2+bcs-cs^3+acs^2-bcs+c^2}$$

$$= \frac{c^2}{-s^6+(a-a)s^5+(-b+a^2-b)s^4+(c-ab+ab-c)s^3+(ac-b^2+ac)s^2 \rightarrow}$$

$$\xrightarrow{\qquad} + (bc-bc)s + c^2$$

$$= \frac{c^2}{-s^6+(a-a)s^5+(a^2-2b)s^4+(c-ab+ab-c)a^3+(2ac-b^2)s^2+(bc-bc)s+c^2}$$

Igualo términos: $\frac{1}{\xi^2} = c^2 \longrightarrow$ Término orden 0

$0 = bc-bc \longrightarrow$ " " 1

$0 = 2ac-b^2 \longrightarrow$ " " 2

$0 = c-ab+ab-c \longrightarrow$ " " 3

$0 = a^2-2b \longrightarrow$ " " 4

$0 = a-a \longrightarrow$ " " 5

$-1 = -1 \longrightarrow$ " " 6

Ecuaciones que aportan información:

$c^2 = \frac{1}{\xi^2} \simeq 3{,}8625$; $c \simeq 1{,}9653$

$2ac - b^2 = 0$; $2ac = b^2$

$a^2 - 2b = 0$; $a^2 = 2b$; $a = \sqrt{2b}$

$2\sqrt{2b}\,c = b^2$; elevo al cuadrado mam

$8\cancel{b}c^2 = b^{\cancel{4}3}$

$b = \sqrt[3]{8c^2} \simeq 3{,}1379$

$a = \sqrt{2b} \simeq 2{,}5051$

$$T(s) = \frac{c}{s^3 + a s^2 + b \cdot s + c} = \frac{1{,}9653}{s^3 + 2{,}5051 s^2 + 3{,}1379 s + 1{,}9653}$$