Test Functions

- Unimodal Functions (5):
  - Shifted Sphere Function
  - Shifted Schwefel's Problem 1.2
  - Shifted Rotated High Conditioned Elliptic Function
  - Shifted Schwefel's Problem 1.2 with Noise in Fitness
  - Schwefel's Problem 2.6 with Global Optimum on Bounds

- Multimodal Functions (20):
  - Basic Functions (7):
    - Shifted Rosenbrock's Function
    - Shifted Rotated Griewank's Function without Bounds
    - Shifted Rotated Ackley's Function with Global Optimum on Bounds
    - Shifted Rastrigin's Function
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    - Shifted Rotated Weierstrass Function
    - Schwefel's Problem 2.13
  - Expanded Functions (2):
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    - Expanded Rotated Extended Scaffer's F6
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    - Rotated Hybrid Composition Function 2 with a Narrow Basin for the Global Optimum
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    - Non-Continuous Rotated Hybrid Composition Function 3
    - Rotated Hybrid Composition Function 4
    - Rotated Hybrid Composition Function 4 without Bounds
  - **Pseudo-Real Problems: Available from** `http://www.cs.colostate.edu/~genitor/functions.html`. If you have any queries on these problems, please contact Professor Darrell Whitley. Email: whitley@CS.ColoState.EDU

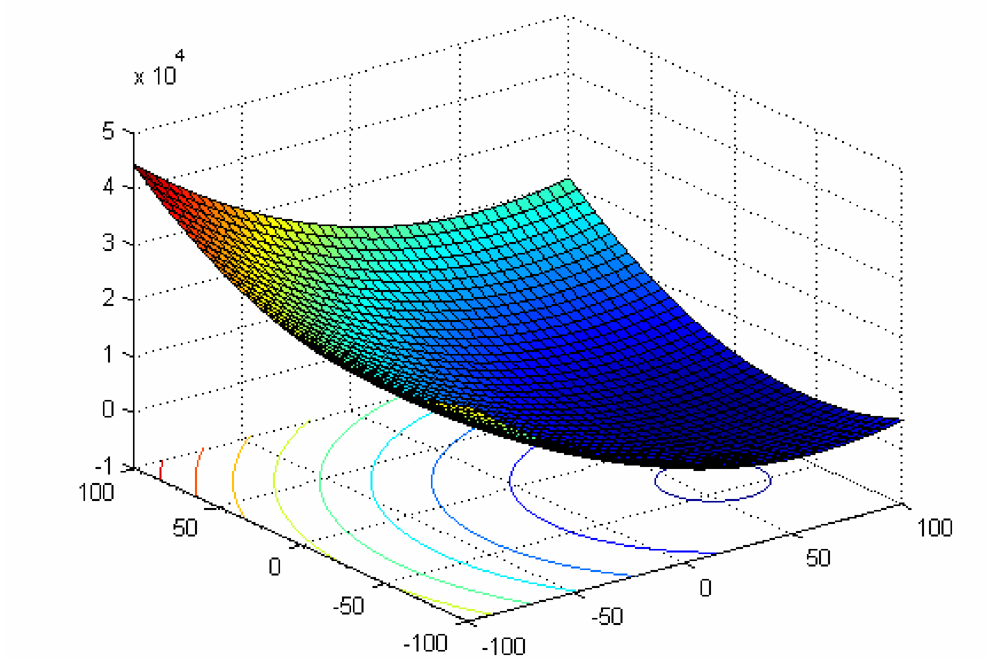
Introduction to the test functions:

1 Shifted Sphere Function

$$f(x)=\sum_{i=1}^{D} z_i^{2}+f\_bias\,,\ z=x-o\,,\ x=[x_1,x_2,...,x_D]$$

D: dimension

$o=[o_1,o_2,...,o_D]$: the shifted global optimum, to avoid the global optimum is on the origin.



Properties:

- Unimodal
- Shifted
- Separable
- Scalable
- $x\in[-100,100]^D$, Global optimum: $x^*=o$, $f(x^*)=f\_bias$ (1)= - 450

Data file:

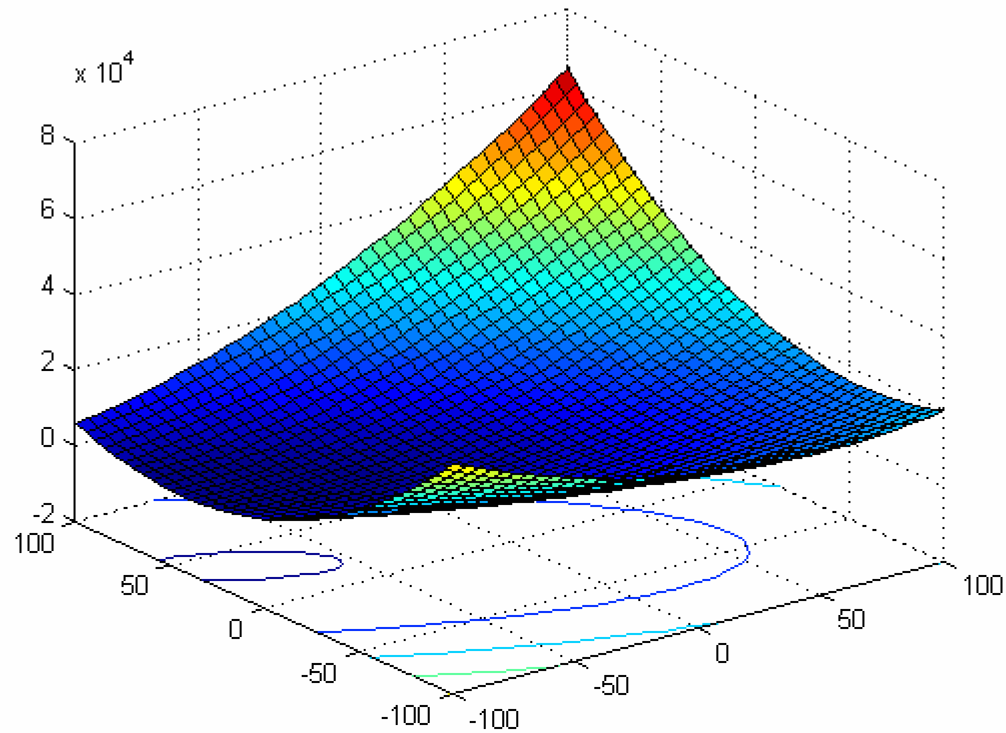
Name:	sphere_func_data.mat		
	sphere_func_data.txt		
Variable:	o	1*100 vector	the shifted global optimum
	When using, cut o=o(1:D)		
Name:	fbias_data.mat		
	fbias_data.txt		
Variable:	f_bias	1*25 vector, record all the 25 function's f_bias	

2 Shifted Schwefel's Problem 1.2

$$f(x) = \sum_{i=1}^{D} (\sum_{j=1}^{i} z_j)^2 + f\_bias, \quad z = x - o, x = [x_1, x_2, ..., x_D]$$

D: dimension

$o = [o_1, o_2, ..., o_D]$: the shifted global optimum



Properties:

- Unimodal
- Shifted
- Non-separable
- Scalable
- $x \in [-100,100]^D$, Global optimum $x^* = o$, $f(x^*) = f\_bias$ (2)= - 450

Data file:

Name:	schwefel_102_data.mat		
	schwefel_102_data.txt		
Variable:	o	1*100 vector	the shifted global optimum
	When using, cut o=o(1:D)		

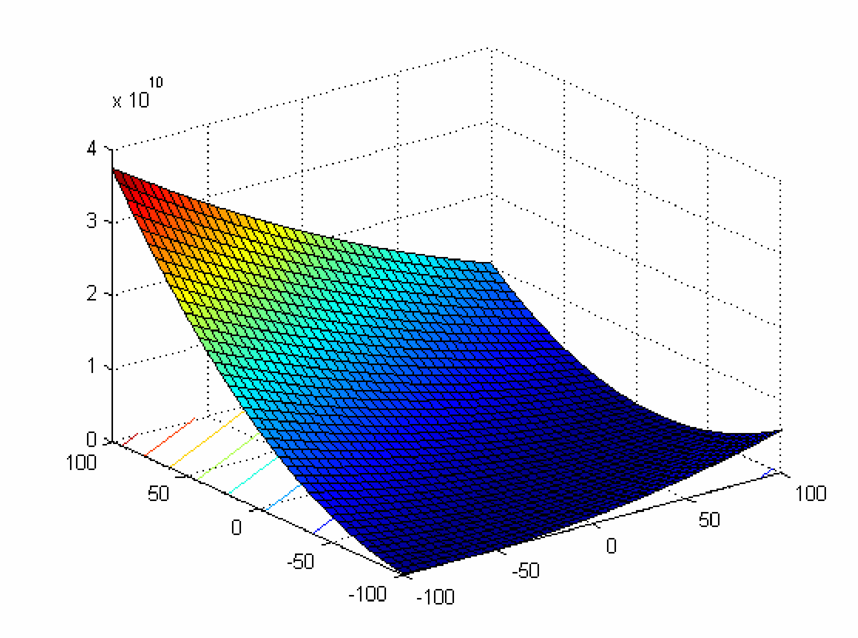
3 Shifted Rotated High Conditioned Elliptic Function

$$f(x)=\sum_{i=1}^{D}(10^6)^{\frac{i-1}{D-1}} z_i^2 + f\_bias\,,\ \ z=(x-o)*M\,,\ x=[x_1,x_2,...,x_D]$$

D: dimension

$o=[o_1,o_2,...,o_D]$: the shifted global optimum

M: orthogonal matrix



Properties:

- Unimodal
- Shifted
- Rotated
- Non-separable
- Scalable
- $x \in [-100,100]^D$, Global optimum $x^* = o$, $f(x^*) = f\_bias(3)$= - 450

Data file:

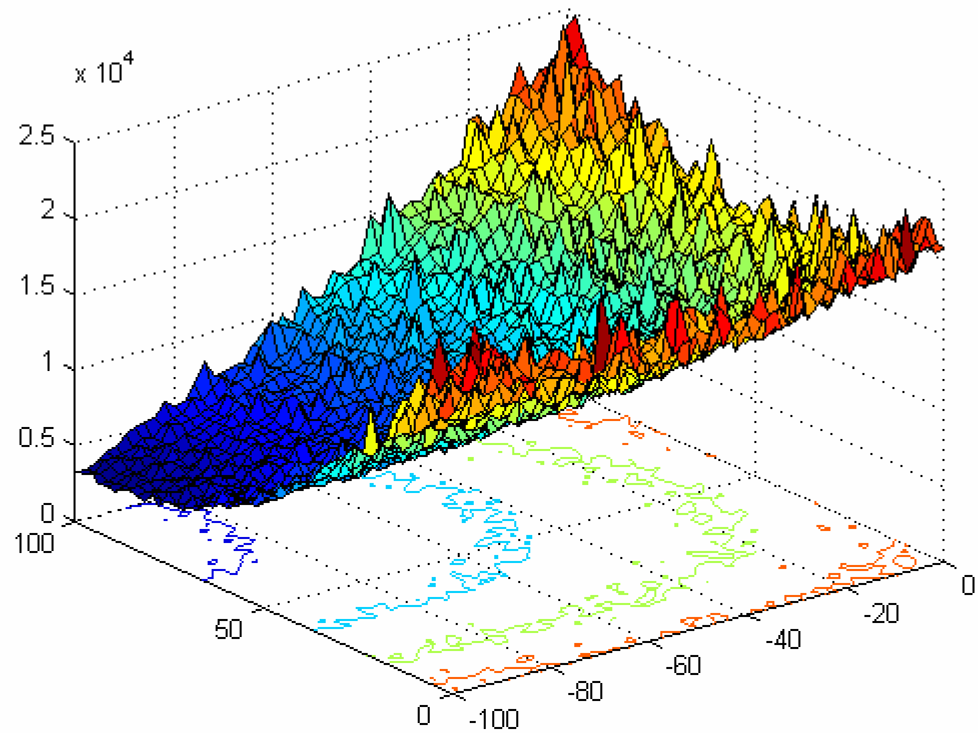
Name:	high_cond_elliptic_rot_data.mat high_cond_elliptic_rot_data.txt	
Variable:	o 1*100 vector When using, cut o=o(1:D)	the shifted global optimum
Name:	elliptic_M_D10 .mat	elliptic_M_D10 .txt
Variable:	M 10*10 matrix	
Name:	elliptic_M_D30 .mat	elliptic_M_D30 .txt
Variable:	M 30*30 matrix	
Name:	elliptic_M_D50 .mat	elliptic_M_D50 .txt
Variable:	M 50*50 matrix	

4 Shifted Schwefel's Problem 1.2 with Noise in Fitness

$$f(x) = (\sum_{i=1}^{D}(\sum_{j=1}^{i} z_j)^2) * (1 + 0.4|N(0,1)|) + f\_bias, \quad z = x - o, \; x = [x_1, x_2, ..., x_D]$$

D: dimension

$o = [o_1, o_2, ..., o_D]$: the shifted global optimum



Properties:

- Unimodal
- Shifted
- Non-separable
- Scalable
- Noise in fitness
- $x \in [-100,100]^D$, Global optimum $x^* = o$, $f(x^*) = f\_bias(4)$= - 450

Data file:

Name: schwefel_102_data.mat
schwefel_102_data.txt

Variable: o 1*100 vector the shifted global optimum
When using, cut o=o(1:D)

5 Schwefel's Problem 2.6 with Global Optimum on Bounds

$f(x) = \max\{|x_1 + 2x_2 - 7|, |2x_1 + x_2 - 5|\}, i = 1,...,n$, $x^* = [1,3]$, $f(x^*) = 0$

Extend to D dimensions:

$f(x) = \max\{|A_i x - B_i|\} + f\_bias, i = 1,...,D$, $x = [x_1, x_2, ..., x_D]$

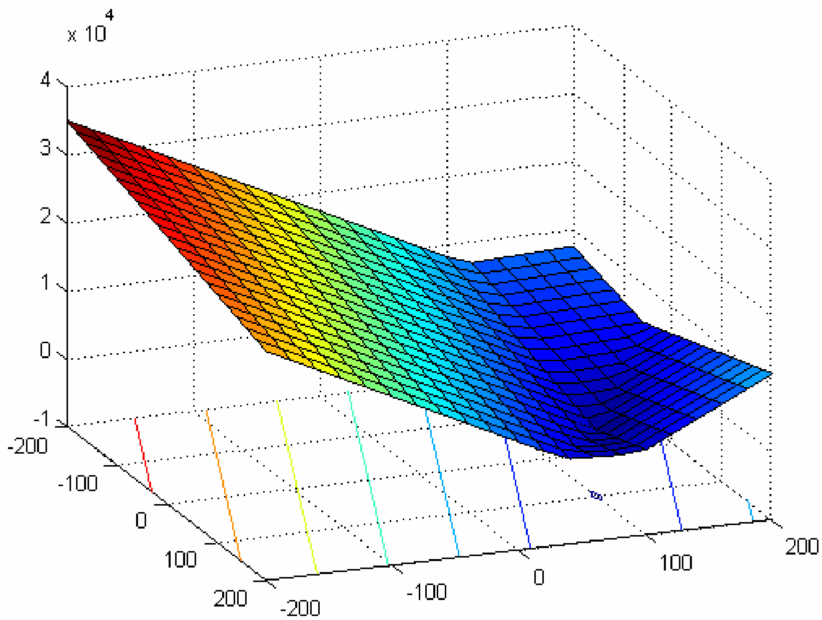
D: dimension

A is a D*D matrix, a_{ij} are integer random numbers in the range [-*500*, *500*],

$\det(A) \neq 0$, A_i is the ith row of A.

$B_i = A_i * o$, o is a D*1 vector, o_i are random number in the range [-100,100]

After load the data file, set $o_i = -100$, for $i = 1, 2, ..., \lceil D/4 \rceil$, $o_i = 100$, for $i = \lfloor 3D/4 \rfloor, ..., D$



Properties:

- ➢ Unimodal
- ➢ Non-separable
- ➢ Scalable
- ➢ If the initialization procedure initializes the population at the bounds, this problem will be solved easily.
- ➢ $x \in [-100, 100]^D$, Global optimum $x^* = o$, $f(x^*) = f\_bias$ (5)= - 310

Data file:

Name:	schwefel_206_data.mat		
	schwefel_206_data.txt		
Variable:	o	1*100 vector	the shifted global optimum
	A	100*100 matrix	

When using, cut o=o(1:D) A=A(1:D,1:D)

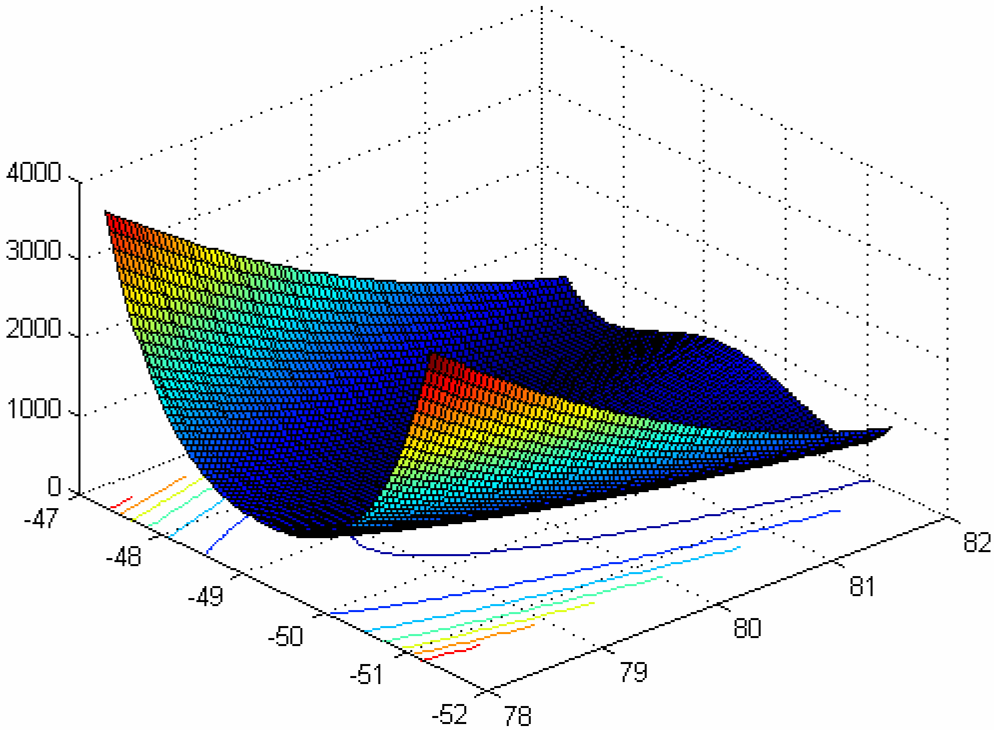
In schwefel_206_data.txt ,the first line is o (1*100 vector),and line2-line101 is A(100*100 matrix)

6 Shifted Rosenbrock's Function

$$f(x) = \sum_{i=1}^{D-1} (100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2) + f\_bias\text{,}\quad z = x - o + 1,\ x = [x_1, x_2, ..., x_D]$$

D: dimension

$o = [o_1, o_2, ..., o_D]$: the shifted global optimum



Properties:

- Multi-modal
- Shifted
- Non-separable
- Scalable
- Having a very narrow valley from local optimum to global optimum
- $x \in [-100,100]^D$, Global optimum $x^* = o$, $f(x^*) = f\_bias(6) = 390$

Data file:

Name:	rosenbrock_func_data.mat		
	rosenbrock_func_data.txt		
Variable:	o	1*100 vector	the shifted global optimum
	When using, cut o=o(1:D)		

7 Shifted Rotated Griewank's Function without Bounds

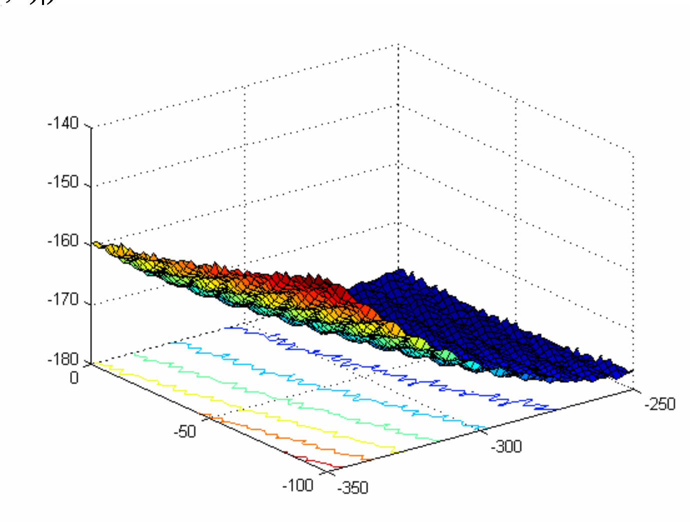
$$f(x)=\sum_{i=1}^{D}\frac{z_i^{2}}{4000}-\prod_{i=1}^{D}\cos\left(\frac{z_i}{\sqrt{i}}\right)+1+f\_bias\ ,\ z=(x-o)*M\ ,\ x=[x_1,x_2,...,x_D]$$

D: dimension

$o=[o_1,o_2,...,o_D]$: the shifted global optimum

M': linear transformation matrix, condition number=3

M =*M'*(1+0.3|N(0,1)|)



Properties:

- Multi-modal
- Rotated
- Shifted
- Non-separable
- Scalable
- No bounds for variables *x*
- Initialize population in $[0,600]^D$, Global optimum $x^*=o$ is outside of the initialization range, $f(x^*)=f\_bias$ (7)= -180

Data file:

Name:	griewank_func_data.mat		
	griewank_func_data.txt		
Variable:	o	1*100 vector	the shifted global optimum
	When using, cut o=o(1:D)		
Name:	griewank_M_D10 .mat		griewank_M_D10 .txt
Variable:	M	10*10 matrix	
Name:	griewank_M_D30 .mat		griewank_M_D30 .txt
Variable:	M	30*30 matrix	
Name:	griewank_M_D50 .mat		griewank_M_D50 .txt
Variable:	M	50*50 matrix	

8 Shifted Rotated Ackley's Function with Global Optimum on Bounds

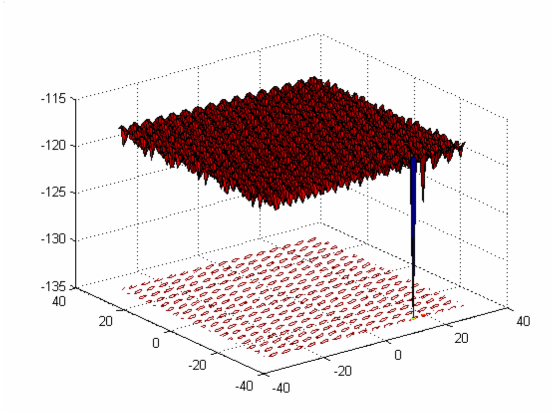
$$f(x) = -20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} z_i^2}) - \exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi z_i)) + 20 + e + f\_bias,$$

$z = (x - o) * M$, $x = [x_1, x_2, ..., x_D]$, D: dimension

$o = [o_1, o_2, ..., o_D]$: the shifted global optimum;

After load the data file, set $o_{2j-1} = -32$ o_{2j} are randomly distributed in the search range, for $j = 1, 2, ..., \lfloor D/2 \rfloor$

M: linear transformation matrix, condition number=100



Properties:

- Multi-modal
- Rotated
- Shifted
- Non-separable
- Scalable
- A's condition number Cond(A) increases with the number of variables as $O(D^2)$
- Global optimum on the bound
- If the initialization procedure initializes the population at the bounds, this problem will be solved easily.
- $x \in [-32, 32]^D$, Global optimum $x^* = o$, $f(x^*) = f\_bias(8)$= - 140

Data file:

Name:	ackley_func_data.mat		
	ackley_func_data.txt		
Variable:	o	1*100 vector	the shifted global optimum
	When using, cut o=o(1:D)		

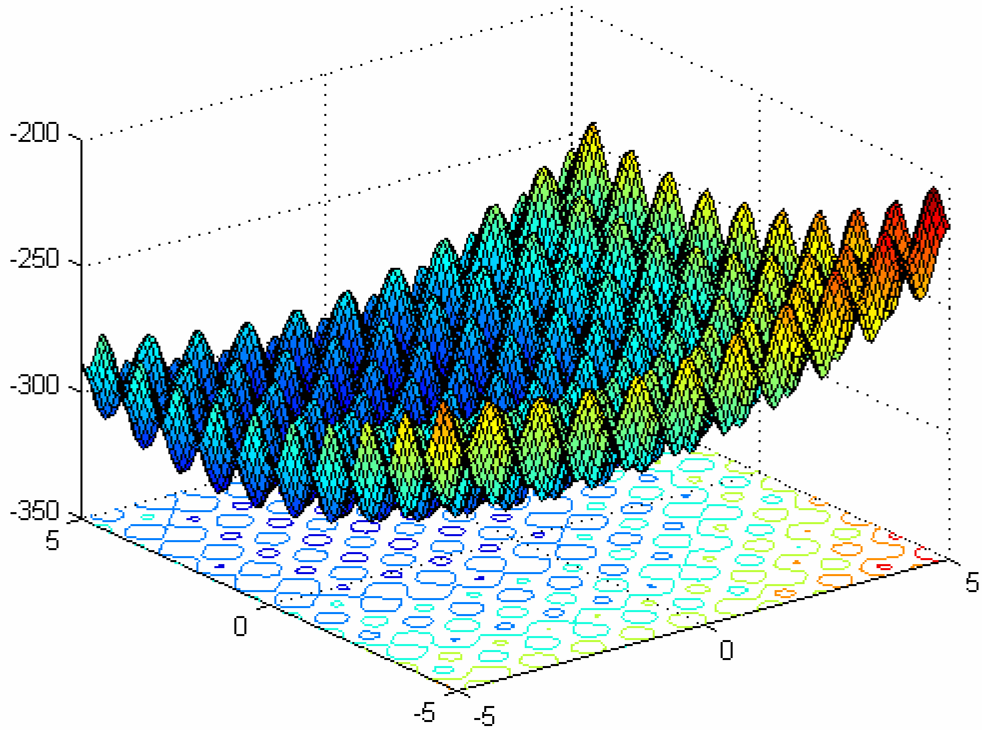
Name:	ackley_M_D10 .mat	ackley_M_D10 .txt
Variable:	M 10*10 matrix	
Name:	ackley_M_D30 .mat	ackley_M_D30 .txt
Variable:	M 30*30 matrix	
Name:	ackley_M_D50 .mat	ackley_M_D50 .txt
Variable:	M 50*50 matrix	

9 Shifted Rastrigin's Function

$$f(x)=\sum_{i=1}^{D}(z_i^{\,2}-10\cos(2\pi z_i)+10)+f\_bias,\ \ z=x-o,\ \ x=[x_1,x_2,...,x_D]$$

D: dimension

$o=[o_1,o_2,...,o_D]$: the shifted global optimum



Properties:

- Multi-modal
- Shifted
- Separable
- Scalable
- Local optima's number is huge
- $x\in[-5,5]^D$, Global optimum $x^*=o$, $f(x^*)=f\_bias$ (9)= - 330

Data file:

Name:	rastrigin_func_data.mat		
	rastrigin_func_data.txt		
Variable:	o	1*100 vector	the shifted global optimum
	When using, cut o=o(1:D)		

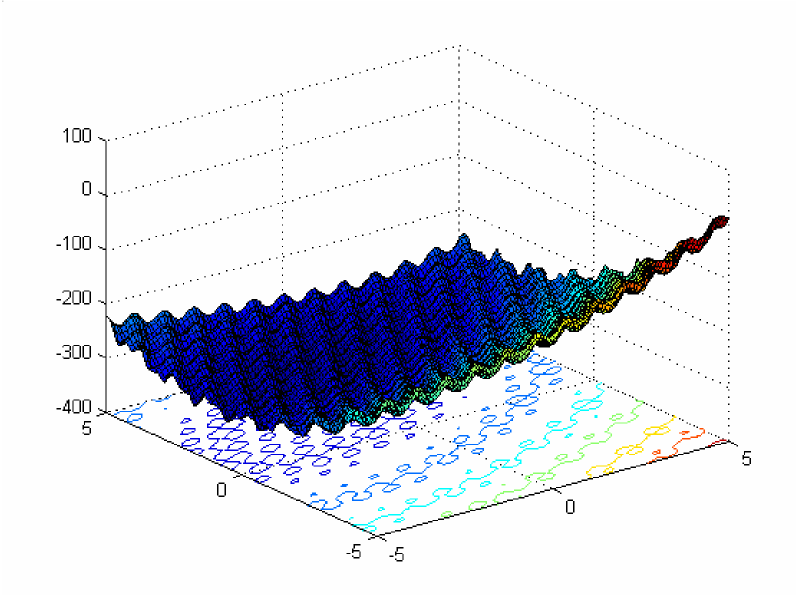
10 Shifted Rotated Rastrigin's Function

$$f(x)=\sum_{i=1}^{D}(z_i^2-10\cos(2\pi z_i)+10)+f\_bias,\quad z=(x-o)*M,\ x=[x_1,x_2,...,x_D]$$

D: dimension

$o=[o_1,o_2,...,o_D]$: the shifted global optimum

M: linear transformation matrix, condition number=2



Properties:

- Multi-modal
- Shifted
- Rotated
- Non-separable
- Scalable
- Local optima's number is huge
- $x\in[-5,5]^D$, Global optimum $x^*=o$, $f(x^*)=f\_bias(10)= -330$

Data file:

Name:	rastrigin_func_data.mat	
	rastrigin_func_data.txt	
Variable:	o 1*100 vector	the shifted global optimum
	When using, cut o=o(1:D)	
Name:	rastrigin_M_D10 .mat	rastrigin_M_D10 .txt
Variable:	M 10*10 matrix	
Name:	rastrigin_M_D30 .mat	rastrigin_M_D30 .txt
Variable:	M 30*30 matrix	
Name:	rastrigin_M_D50 .mat	rastrigin_M_D50 .txt
Variable:	M 50*50 matrix	

11 Shifted Rotated Weierstrass Function

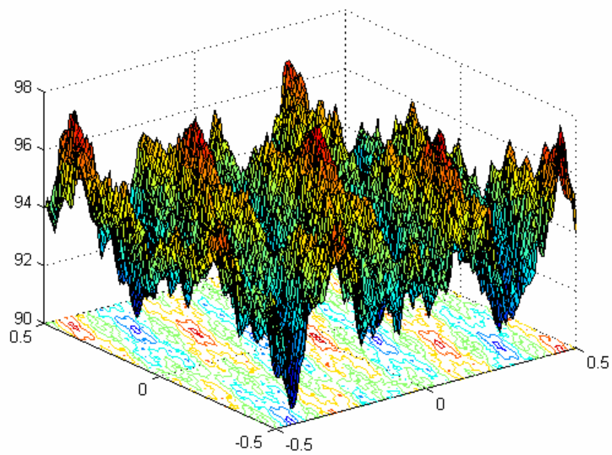
$$f(x)=\sum_{i=1}^{D}(\sum_{k=0}^{k\max}[a^k\cos(2\pi b^k(z_i+0.5))])-D\sum_{k=0}^{k\max}[a^k\cos(2\pi b^k\cdot 0.5)]+f\_bias,$$

a=0.5, b=3, k_{max}=20, $z=(x-o)*M$, $x=[x_1,x_2,...,x_D]$

D: dimension

$o=[o_1,o_2,...,o_D]$: the shifted global optimum

M: linear transformation matrix, condition number=5



Properties:

- Multi-modal
- Shifted
- Rotated
- Non-separable
- Scalable
- Continuous but differentiable only on a set of points
- $x\in[-0.5,0.5]^D$, Global optimum $x^*=o$, $f(x^*)=f\_bias(11)$= 90

Data file:

Name:	weierstrass_data.mat		
	weierstrass_data.txt		
Variable:	o	1*100 vector	the shifted global optimum
	When using, cut o=o(1:D)		
Name:	weierstrass_M_D10 .mat		weierstrass_M_D10 .txt
Variable:	M	10*10 matrix	
Name:	weierstrass_M_D30 .mat		weierstrass_M_D30 .txt
Variable:	M	30*30 matrix	
Name:	weierstrass_M_D50 .mat		weierstrass_M_D50 .txt
Variable:	M	50*50 matrix	

12 Schwefel's Problem 2.13

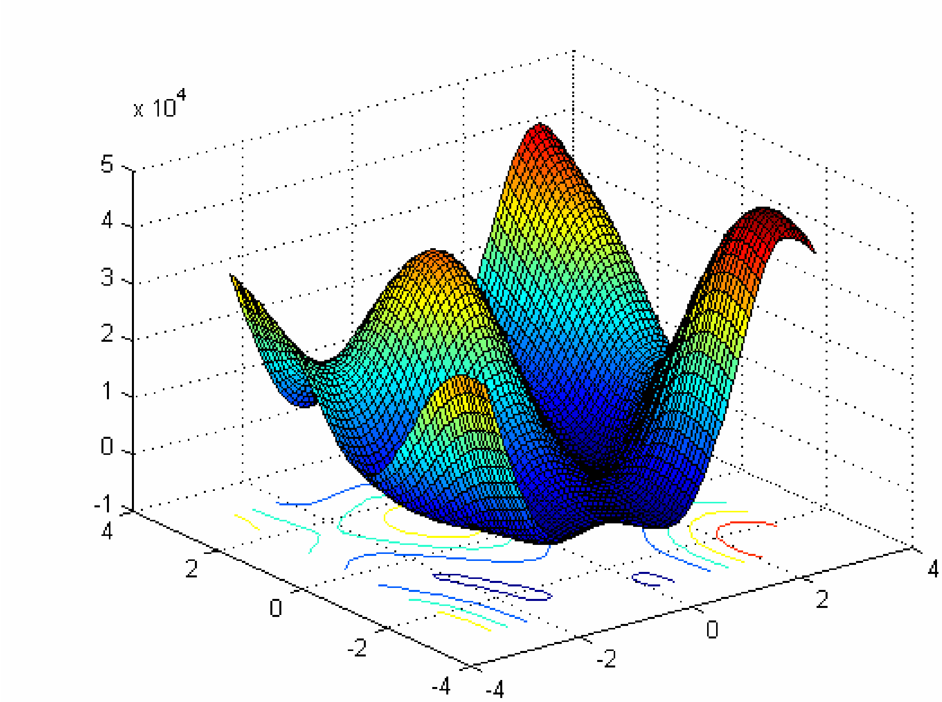
$$f(x)=\sum_{i=1}^{D}(A_i-B_i(x))^2+f\_bias\,,\ x=[x_1,x_2,...,x_D]$$

$$A_i=\sum_{j=1}^{D}(a_{ij}\sin\alpha_j+b_{ij}\cos\alpha_j)\,,\ B_i(x)=\sum_{j=1}^{D}(a_{ij}\sin x_j+b_{ij}\cos x_j)\,,\ \text{for } i=1,...,D$$

D: dimension

A, *B* are two *D***D* matrix, a_{ij}, b_{ij} are integer random numbers in the range [-100,100],

$\alpha=[\alpha_1,\alpha_2,...,\alpha_D]$, α_j are random numbers in the range $[-\pi,\pi]$.



Properties:

- Multi-modal
- Shifted
- Non-separable
- Scalable
- $x\in[-\pi,\pi]^D$, Global optimum $x^*=\alpha$, $f(x^*)=f\_bias(12)=-460$

Data file:

Name:	schwefel_213_data.mat		
	schwefel_213_data.txt		
Variable:	alpha	1*100 vector	the shifted global optimum
	a	100*100 matrix	
	b	100*100 matrix	

When using, cut alpha=alpha(1:D) a=a(1:D,1:D) b=b(1:D,1:D)

In schwefel_213_data.txt, and line1-line100 is a(100*100 matrix),and line101-line200 is b (100*100 matrix), the last line is alpha(1*100 vector),

Expanded Functions :

Use a two dimensional function $F(x, y)$ as a starting function.

The corresponding expanded function

$$EF(x_1, x_2, ..., x_D) = F(x_1, x_2) + F(x_2, x_3) + ... + F(x_{D-1}, x_D) + F(x_D, x_1)$$

13 Shifted Expanded Griewank's plus Rosenbrock's Function (F8F2)

F8: Griewank's Function: $F8(x) = \sum_{i=1}^{D} \frac{x_i^{\,2}}{4000} - \prod_{i=1}^{D} \cos(\frac{x_i}{\sqrt{i}}) + 1$

F2: Rosenbrock's Function: $F2(x) = \sum_{i=1}^{D-1} (100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2)$

$$F8F2(x_1, x_2, ..., x_D) = F8(F2(x_1, x_2)) + F8(F2(x_2, x_3)) + ... + F8(F2(x_{D-1}, x_D)) + F8(F2(x_D, x_1))$$

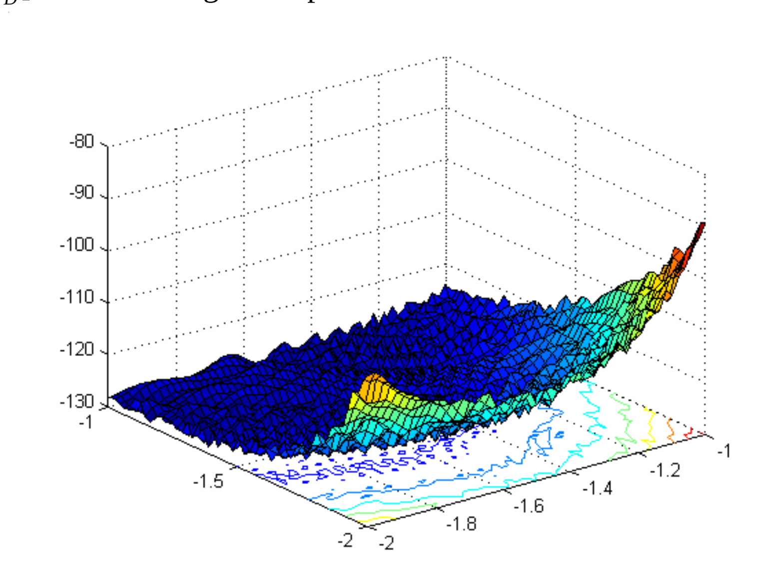
Shift to

$$f(x) = F8(F2(z_1, z_2)) + F8(F2(z_2, z_3)) + ... + F8(F2(z_{D-1}, z_D)) + F8(F2(z_D, z_1)) + f\_bias$$

$z = x - o + 1$, $x = [x_1, x_2, ..., x_D]$

D: dimension

$o = [o_1, o_2, ..., o_D]$: the shifted global optimum



Properties:

- Multi-modal
- Shifted
- Non-separable
- Scalable
- $x \in [-5, 5]^D$, Global optimum $x^* = o$, $f(x^*) = f\_bias(13)$=-130

Data file:

Name:	EF8F2_func_data.mat		
	EF8F2_func_data.txt		
Variable:	o	1*100 vector	the shifted global optimum
	When using, cut o=o(1:D)		

14 Shifted Rotated Expanded Scaffer's F6 Function

$$F(x,y)=0.5+\frac{(\sin^2(\sqrt{x^2+y^2})-0.5)}{(1+0.001(x^2+y^2))^2}$$

Expanded to

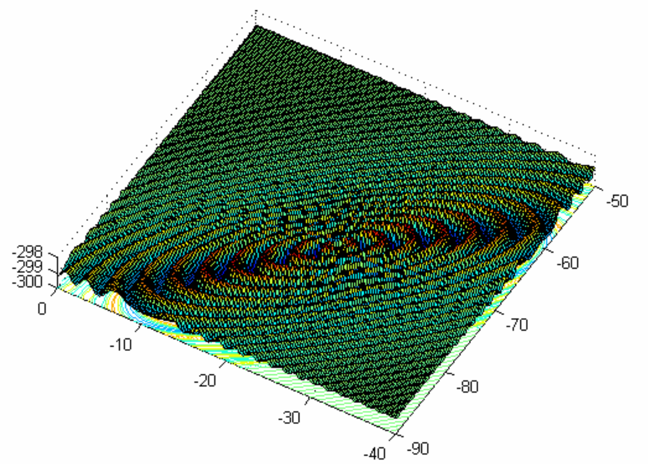
$f(x)=EF(z_1,z_2,...,z_D)=F(z_1,z_2)+F(z_2,z_3)+...+F(z_{D-1},z_D)+F(z_D,z_1)+f\_bias$,

$z=(x-o)*M$, $x=[x_1,x_2,...,x_D]$

D: dimension

$o=[o_1,o_2,...,o_D]$: the shifted global optimum

M: linear transformation matrix, condition number=3



Properties:

- Multi-modal
- Shifted
- Non-separable
- Scalable
- $x\in[-100,100]^D$, Global optimum $x^*=o$, $f(x^*)=f\_bias(14)=-300$

Data file:

Name:	E_ScafferF6_func_data.mat		
	E_ScafferF6_func_data.txt		
Variable:	o	1*100 vector	the shifted global optimum
	When using, cut o=o(1:D)		
Name:	E_ScafferF6_M_D10 .mat		E_ScafferF6_M_D10 .txt
Variable:	M	10*10 matrix	
Name:	E_ScafferF6_M_D30 .mat		E_ScafferF6_M_D30 .txt
Variable:	M	30*30 matrix	
Name:	E_ScafferF6_M_D50 .mat		E_ScafferF6_M_D50 .txt
Variable:	M	50*50 matrix	

Composition functions

$F(x)$: new composition function
$f_i(x)$: i[th] basic function used to construct the composition function
n: number of basic functions
D: dimension
M_i: linear transformation matrix for each $f_i(x)$
o_i: new shifted optimum position for each $f_i(x)$

$$F(x) = \sum_{i=1}^{n} \{w_i * [f_i'((x-o_i)/\lambda_i * M_i) + bias_i]\} + f\_bias$$

w_i: weight value for each $f_i(x)$, calculated as below:

$$w_i = \exp\left(-\frac{\sum_{k=1}^{D}(x_k - o_{ik})^2}{2D\sigma_i^2}\right),$$

$$w_i = \begin{cases} w_i & w_i == \max(w_i) \\ w_i*(1\text{-}\max(w_i).\text{^}10) & w_i \neq \max(w_i) \end{cases}$$

then normalize the weight $w_i = w_i / \sum_{i=1}^{n} w_i$

σ_i: used to control each $f_i(x)$'s coverage range, a small σ_i give a narrow range for that $f_i(x)$
λ_i : used to stretch compress the function, λ_i>1 means stretch, λ_i<1 means compress
o_i define the global and local optima's position, $bias_i$ define which optimum is global optimum. Using o_i, $bias_i$, a global optimum can be placed anywhere.

If $f_i(x)$ are different functions, different functions have different properties and height, in order to get a better mixture, estimate a biggest function value $f_{\max i}$ for 10 functions $f_i(x)$, then normalize each basic functions to similar heights as below:
$f_i'(x) = C * f_i(x) / |f_{\max i}|$, C is a predefined constant.
$|f_{\max i}|$ is estimated using $|f_{\max i}| = f_i((x'/\lambda_i) * M_i)$, x'=[5,5…,5].

In the following composition functions
Basic function number n: 10
D: dimension
o: n*D matrix, define $f_i(x)$'s global optimal positions
$bias$ =[0, 100, 200, 300, 400, 500, 600, 700, 800, 900]. Hence the first function $f_1(x)$ always the function with the global optimum.
C=2000

Pseudo Code:

Define f1-f10, σ, λ, bias, C, load data file o and rotated linear transformation matrix M1-M10

y =[5,5…,5].

For i=1:10

$$w_i = \exp\left(-\frac{\sum_{k=1}^{D}(x_k - o_{ik})^2}{2D\sigma_i^2}\right),$$

$$fit_i = f_i(((x-o_i)/\lambda_i)*M_i)$$

$$f\max_i = f_i((y/\lambda_i)*M_i),$$

$$fit_i = C*fit_i / f\max_i$$

EndFor

$$SW = \sum_{i=1}^{n} w_i$$

$$MaxW = \max(w_i)$$

For i=1:10

$$w_i = \begin{cases} w_i & w_i == MaxW \\ w_i*(1\text{-}MaxW.\hat{}10) & w_i \neq MaxW \end{cases}$$

$$w_i = w_i / SW$$

EndFor

$$F(x) = \sum_{i=1}^{n}\{w_i * [fit_i + bias_i]\}$$

$$F(x) = F(x) + f\_bias$$

15 Hybrid Composition Function 1

$f_{1-2}(x)$: Rastrigin's Function

$$f_i(x)=\sum_{i=1}^{D}(x_i^2-10\cos(2\pi x_i)+10)$$

$f_{3-4}(x)$: Weierstrass Function

$$f_i(x)=\sum_{i=1}^{D}(\sum_{k=0}^{k\max}[a^k\cos(2\pi b^k(x_i+0.5))])-D\sum_{k=0}^{k\max}[a^k\cos(2\pi b^k\cdot 0.5)],$$

a=0.5, b=3, k_{max}=20

$f_{5-6}(x)$: Griewank's Function

$$f_i(x)=\sum_{i=1}^{D}\frac{x_i^2}{4000}-\prod_{i=1}^{D}\cos(\frac{x_i}{\sqrt{i}})+1$$

$f_{7-8}(x)$: Ackley's Function

$$f_i(x)=-20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D}x_i^2})-\exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i))+20+e$$

$f_{9-10}(x)$: Sphere Function

$$f_i(x)=\sum_{i=1}^{D}x_i^2$$

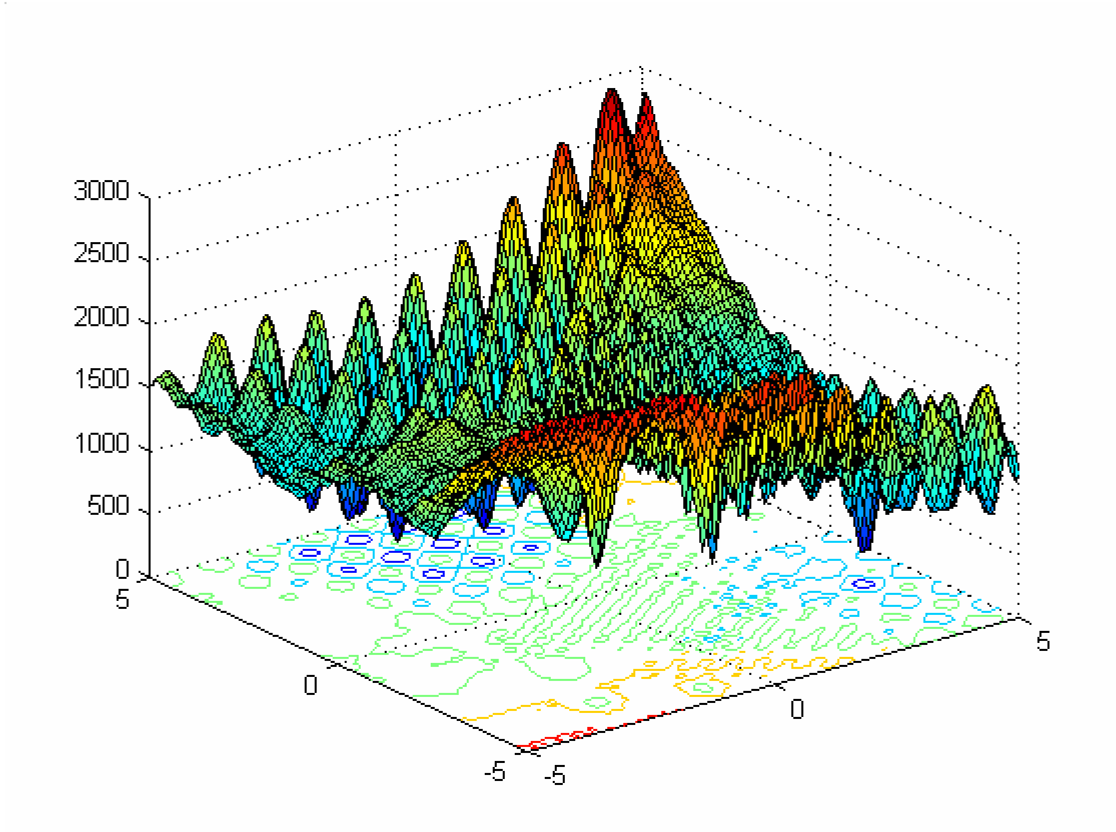
$\sigma_i=1$ for $i=1,2,...,D$

λ = [1, 1, 10, 10, 5/60, 5/60, 5/32, 5/32, 5/100, 5/100]

M_i are all identity matrices

Please notice that these formulas are just for the basic functions, no shift or rotation is included in. x here is just a variable in a function.

Take f_1 as an example, when we calculate $f_1(((x-o_1)/\lambda_1)*M_1)$, we need calculate $f_1(z)=\sum_{i=1}^{D}(z_i^2-10\cos(2\pi z_i)+10)$, $z=((x-o_1)/\lambda_1)*M_1$.



Properties:

- Multi-modal
- Separable near the global optimum (Rastrigin)
- Scalable
- A huge number of local optima
- Different function's properties are mixed together
- Sphere Functions give two flat areas for the function
- $x \in [-5,5]^D$, Global optimum $x^* = o_1$, $f(x^*) \approx f\_bias(15)$= 120

Data file:

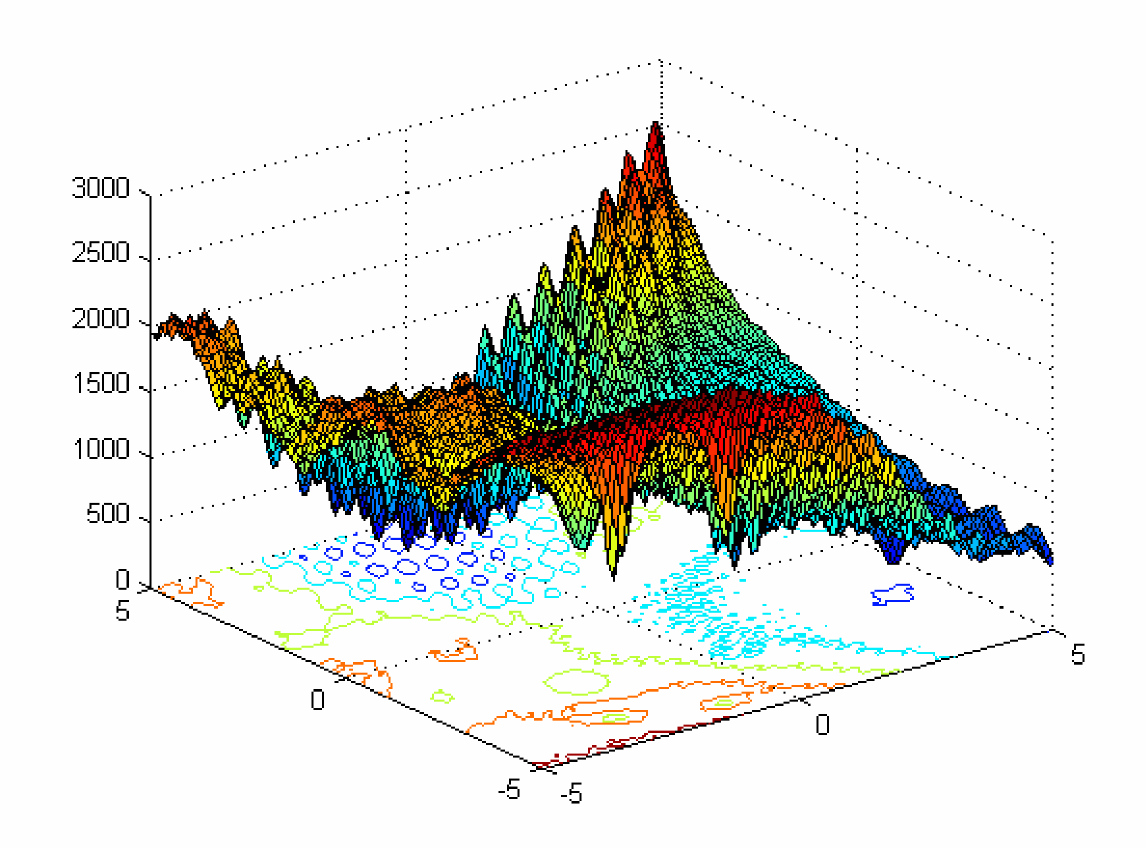
Name: hybrid_func1_data.mat
hybrid_func1_data.txt

Variable: o 10*100 vector the shifted optimum for 10 functions
When using, cut o=o(:,1:D)

16 Rotated Hybrid Composition Function 1

Except M_i are different linear transformation matrixes, condition numbers are 2

All settings are the same with Function 15: Hybrid Composition Function 1



Properties:

- Multi-modal
- Rotated
- Non-Separable
- Scalable
- A huge number of local optima
- Different function's properties are mixed together
- Sphere Functions give two flat areas for the function.
- $x \in [-5,5]^D$, Global optimum $x^* = o_1$, $f(x^*) \approx f\_bias(16)=120$

Data file:

Name: hybrid_func1_data.mat
hybrid_func1_data.txt

Variable: o 10*100 vector the shifted optima for 10 functions
When using, cut o=o(:,1:D)

Name: hybrid_func1_M_D10 .mat

Variable: M an structure variable
Contains M.M1 M.M2, … , M.M10 ten 10*10 matrixes

Name: hybrid_func1_M_D10 .txt
Variable: M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 10*10 matrixes, 1-10 lines are M1, 11-20 lines are M2,....,91-100 lines are M10

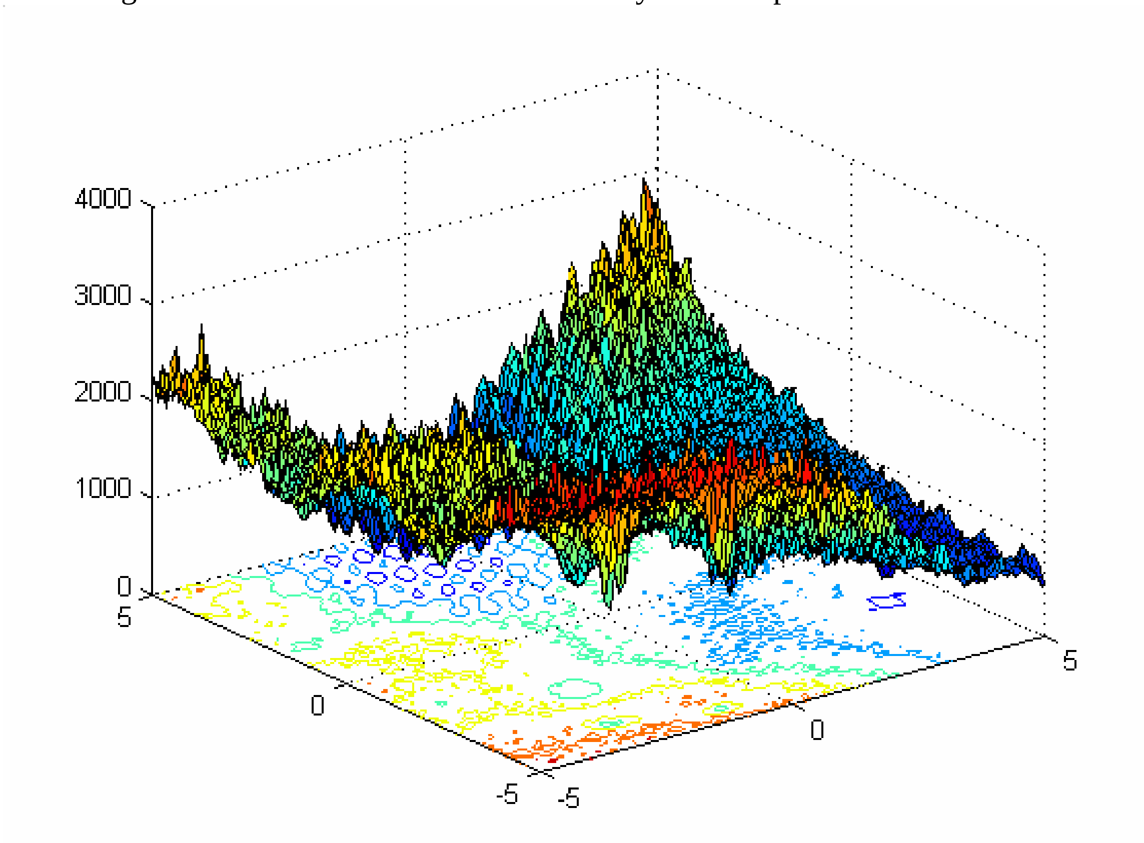
Name: hybrid_func1_M_D30 .mat
Variable: M an structure variable contains M.M1,…,M.M10 ten 30*30 matrix
Name: hybrid_func1_M_D30 .txt
Variable: M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 30*30 matrixes, 1-30 lines are M1, 31-60 lines are M2,....,271-300 lines are M10

Name: hybrid_func1_M_D50 .mat
Variable: M an structure variable contains M.M1,…,M.M10 ten 50*50 matrix
Name: hybrid_func1_M_D50 .txt
Variable: M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 50*50 matrixes, 1-50 lines are M1, 51-100 lines are M2,....,451-500 lines are M10

17 Rotated Hybrid Composition Function 1 with Noise in Fitness

Assume Function 16 Rotated Hybrid Composition Function 1 before adding f_bias is $G(x)$,then $f(x) = G(x)*(1+0.2|\mathrm{N}(0,1)|) + f\_bias$

All settings are same with Function 16 Rotated Hybrid Composition Function 1



Properties:

- Multi-modal
- Rotated
- Non-Separable
- Scalable
- A huge number of local optima
- Different function's properties are mixed together
- Sphere Functions give two flat areas for the function.
- With Gaussian noise in fitness
- $x \in [-5,5]^D$, Global optimum $x^* = o_1$, $f(x^*) \approx f\_bias(17) = 120$

Data file:

Same with Function 16 Rotated Hybrid Composition Function 1.

18 Rotated Hybrid Composition Function 2

$f_{1-2}(x)$: Ackley's Function

$$f_i(x) = -20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}) - \exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)) + 20 + e$$

$f_{3-4}(x)$: Rastrigin's Function

$$f_i(x) = \sum_{i=1}^{D}(x_i^2 - 10\cos(2\pi x_i) + 10)$$

$f_{5-6}(x)$: Sphere Function

$$f_i(x) = \sum_{i=1}^{D} x_i^2$$

$f_{7-8}(x)$: Weierstrass Function

$$f_i(x) = \sum_{i=1}^{D}(\sum_{k=0}^{k\max}[a^k\cos(2\pi b^k(x_i+0.5))]) - D\sum_{k=0}^{k\max}[a^k\cos(2\pi b^k\cdot 0.5)],$$

a=0.5, b=3, k_{max}=20

$f_{9-10}(x)$: Griewank's Function

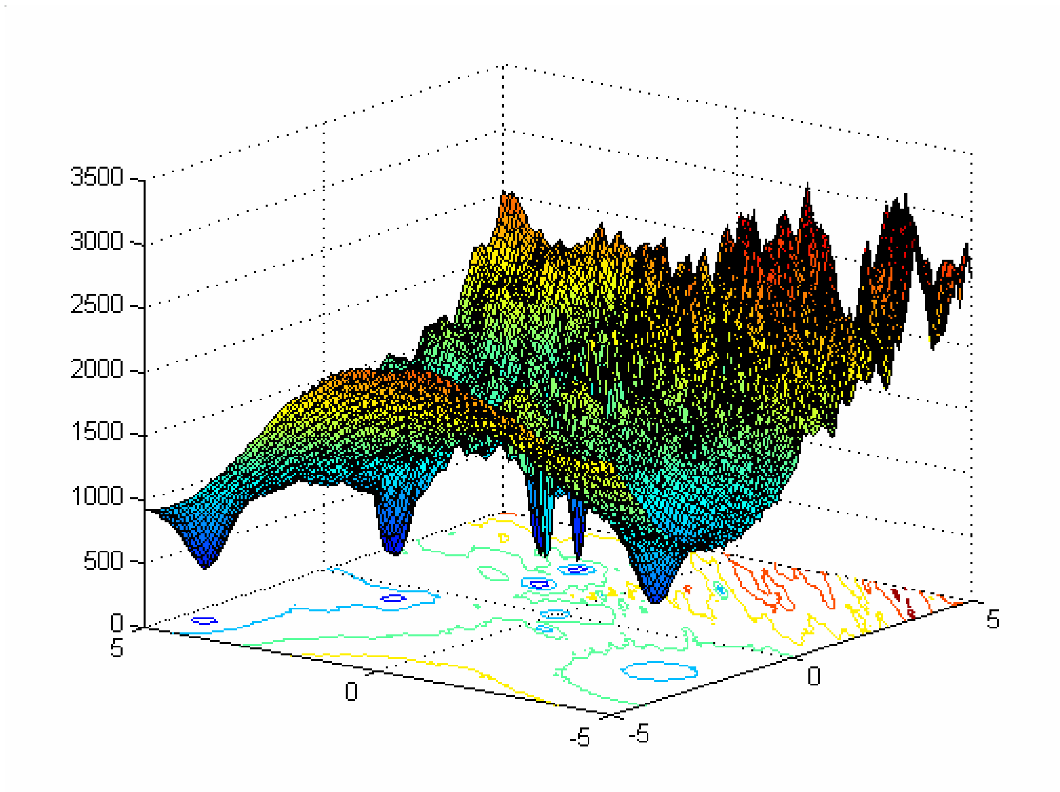
$$f_i(x) = \sum_{i=1}^{D}\frac{x_i^2}{4000} - \prod_{i=1}^{D}\cos(\frac{x_i}{\sqrt{i}}) + 1$$

σ =[1, 2, 1.5, 1.5, 1, 1, 1.5, 1.5, 2, 2];

λ = [2*5/32; 5/32; 2*1; 1; 2*5/100; 5/100; 2*10; 10; 2*5/60; 5/60]

M_i are all rotation matrices. Condition numbers are [2 3 2 3 2 3 20 30 200 300]

$o_{10} = [0, 0, ..., 0]$



Properties:

- Multi-modal
- Rotated
- Non-Separable
- Scalable
- A huge number of local optima
- Different function's properties are mixed together
- Sphere Functions give two flat areas for the function.
- A local optimum is set on the origin
- $x \in [-5,5]^D$, Global optimum $x^* = o_1$, $f(x^*) \approx f\_bias(18) = 10$

Data file:

Name: hybrid_func2_data.mat
hybrid_func2_data.txt

Variable: o 10*100 vector the shifted optima for 10 functions
When using, cut o=o(:,1:D)

Name: hybrid_func2_M_D10 .mat

Variable: M an structure variable
Contains M.M1 M.M2, … , M.M10 ten 10*10 matrixes

Name: hybrid_func2_M_D10 .txt

Variable: M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 10*10 matrixes, 1-10 lines are M1, 11-20 lines are M2,....,91-100 lines are M10

Name:	hybrid_func2_M_D30 .mat
Variable:	M an structure variable contains M.M1,…,M.M10 ten 30*30 matrix
Name:	hybrid_func2_M_D30 .txt
Variable:	M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 30*30 matrixes, 1-30 lines are M1, 31-60 lines are M2,....,271-300 lines are M10

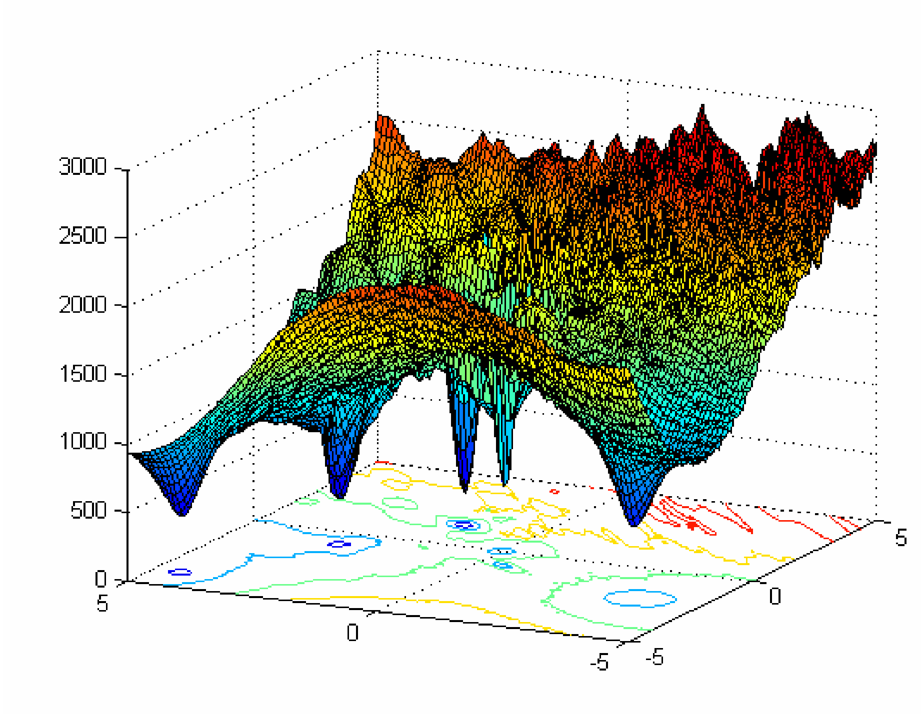
Name:	hybrid_func2_M_D50 .mat
Variable:	M an structure variable contains M.M1,…,M.M10 ten 50*50 matrix
Name:	hybrid_func2_M_D50 .txt
Variable:	M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 50*50 matrixes, 1-50 lines are M1, 51-100 lines are M2,....,451-500 lines are M10

19 Rotated Hybrid Composition Function 2 with narrow basin global optimum

All settings are the same with Function 18: Rotated Hybrid Composition Function 2 Except

σ =[0.1, 2, 1.5, 1.5, 1, 1, 1.5, 1.5, 2, 2];,

λ = [0.1*5/32; 5/32; 2*1; 1; 2*5/100; 5/100; 2*10; 10; 2*5/60; 5/60]



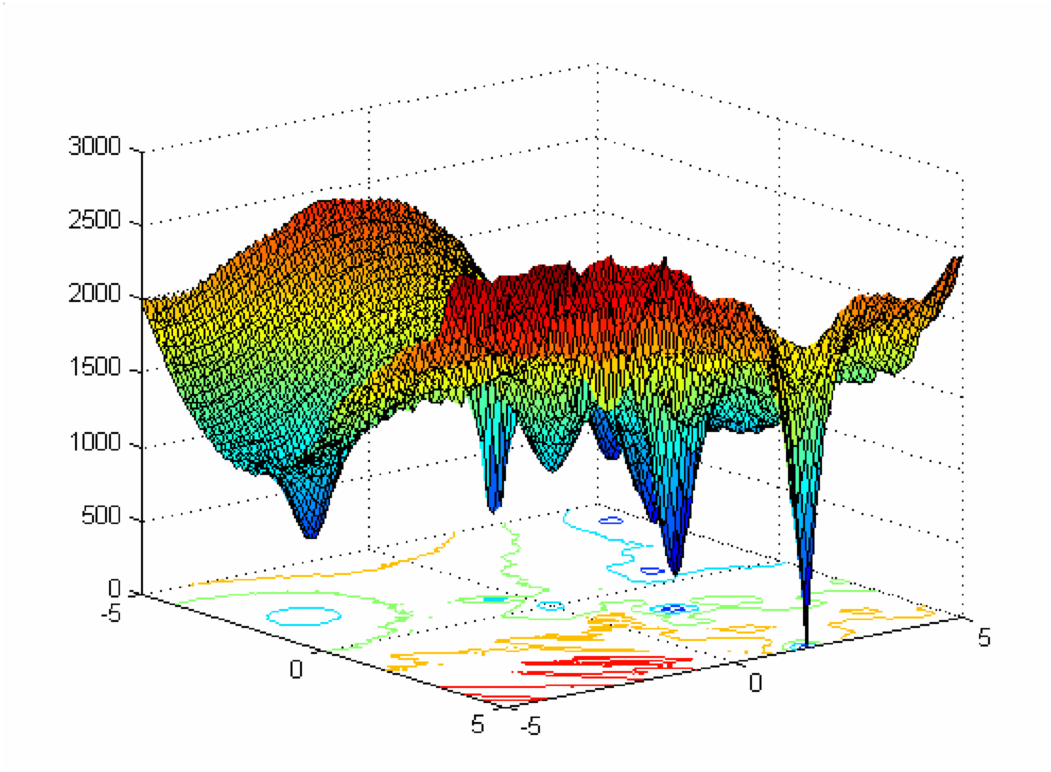
Properties:

- Multi-modal
- Non-separable
- Scalable
- A huge number of local optima
- Different function's properties are mixed together
- Sphere Functions give two flat areas for the function.
- A local optimum is set on the origin
- A narrow basin for the global optimum
- $x \in [-5,5]^D$, Global optimum $x^* = o_1$, $f(x^*) \approx f\_bias(19)$=10

Data file:

Same with Function 18 Rotated Hybrid Composition Function 2.

20 Rotated Hybrid Composition Function 2 with Global Optimum on the Bounds
All settings are the same with Function 18: Rotated Hybrid Composition Function 2
Except after load the data file, set $o_{1(2j)} = 5$, for $j = 1, 2, ..., \lfloor D/2 \rfloor$



Properties:

- Multi-modal
- Non-separable
- Scalable
- A huge number of local optima
- Different function's properties are mixed together
- Sphere Functions give two flat areas for the function.
- A local optimum is set on the origin
- Global optimum is on the bound
- If the initialization procedure initializes the population at the bounds, this problem will be solved easily.
- $x \in [-5, 5]^D$, Global optimum $x^* = o_1$, $f(x^*) \approx f\_bias(20) = 10$

Data file:
Same with Function 18 Rotated Hybrid Composition Function 2.

21 Rotated Hybrid Composition Function 3

$f_{1-2}(x)$: Rotated Expanded Scaffer's F6 Function

$$F(x,y) = 0.5 + \frac{(\sin^2(\sqrt{x^2+y^2}) - 0.5)}{(1+0.001(x^2+y^2))^2}$$

$$f_i(x) = F(x_1,x_2) + F(x_2,x_3) + ... + F(x_{D-1},x_D) + F(x_D,x_1)$$

$f_{3-4}(x)$: Rastrigin's Function

$$f_i(x) = \sum_{i=1}^{D} (x_i^2 - 10\cos(2\pi x_i) + 10)$$

$f_{5-6}(x)$: F8F2 Function

$$F8(x) = \sum_{i=1}^{D} \frac{x_i^2}{4000} - \prod_{i=1}^{D} \cos(\frac{x_i}{\sqrt{i}}) + 1$$

$$F2(x) = \sum_{i=1}^{D-1} (100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2)$$

$$f_i(x) = F8(F2(x_1,x_2)) + F8(F2(x_2,x_3)) + ... + F8(F2(x_{D-1},x_D)) + F8(F2(x_D,x_1))$$

$f_{7-8}(x)$: Weierstrass Function

$$f_i(x) = \sum_{i=1}^{D} (\sum_{k=0}^{k\max} [a^k \cos(2\pi b^k (x_i + 0.5))]) - D\sum_{k=0}^{k\max} [a^k \cos(2\pi b^k \cdot 0.5)],$$

a=0.5, b=3, k_{max}=20

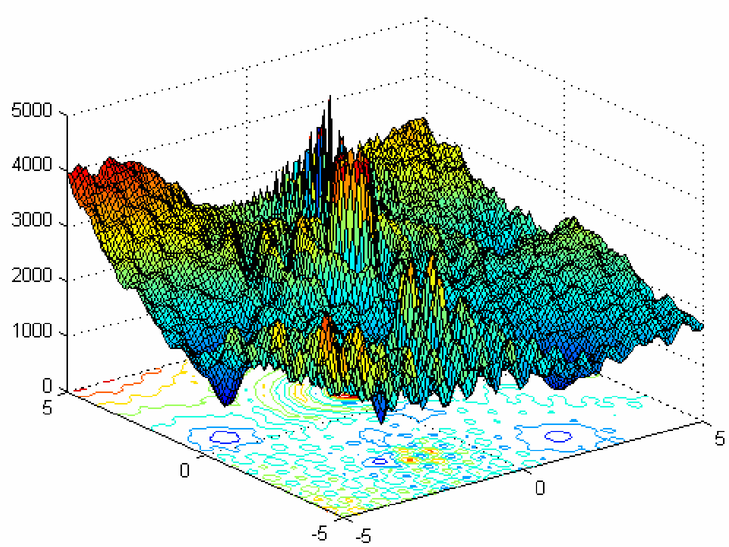
$f_{9-10}(x)$: Griewank's Function

$$f_i(x) = \sum_{i=1}^{D} \frac{x_i^2}{4000} - \prod_{i=1}^{D} \cos(\frac{x_i}{\sqrt{i}}) + 1$$

$\sigma = [1,1,1,1,1,2,2,2,2,2]$,

λ = [5*5/100; 5/100; 5*1; 1; 5*1; 1; 5*10; 10; 5*5/200; 5/200];

M_i are all orthogonal matrix



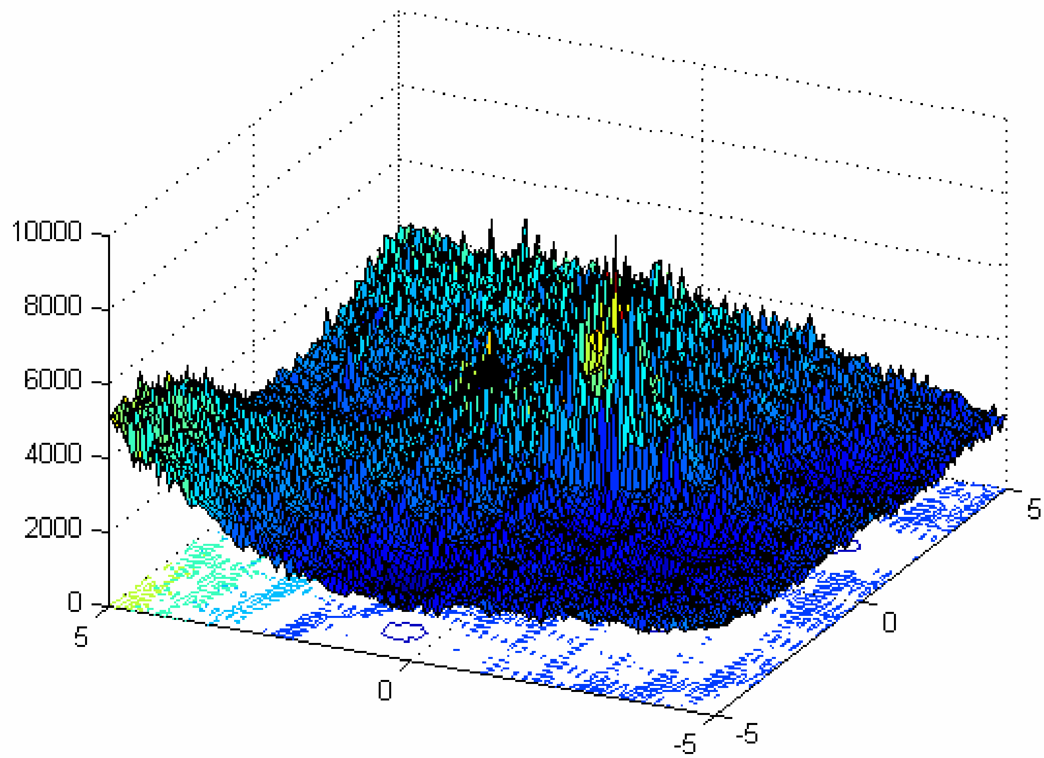
Properties:

- Multi-modal
- Rotated
- Non-Separable
- Scalable
- A huge number of local optima
- Different function's properties are mixed together
- $x \in [-5,5]^D$, Global optimum $x^* = o_1$, $f(x^*) \approx f\_bias(21)$=360

Data file:

Name:	hybrid_func3_data.mat hybrid_func3_data.txt		
Variable:	o	10*100 vector	the shifted optima for 10 functions
	When using, cut o=o(:,1:D)		
Name:	hybrid_func3_M_D10 .mat		
Variable:	M	an structure variable	
	Contains M.M1 M.M2, … , M.M10 ten 10*10 matrixes		
Name:	hybrid_func3_M_D10 .txt		
Variable:	M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 10*10 matrixes, 1-10 lines are M1, 11-20 lines are M2,....,91-100 lines are M10		
Name:	hybrid_func3_M_D30 .mat		
Variable:	M	an structure variable	contains M.M1,…,M.M10 ten 30*30 matrix
Name:	hybrid_func3_M_D30 .txt		
Variable:	M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 30*30 matrixes, 1-30 lines are M1, 31-60 lines are M2,....,271-300 lines are M10		
Name:	hybrid_func3_M_D50 .mat		
Variable:	M	an structure variable	contains M.M1,…,M.M10 ten 50*50 matrix
Name:	hybrid_func3_M_D50 .txt		
Variable:	M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 50*50 matrixes, 1-50 lines are M1, 51-100 lines are M2,....,451-500 lines are M10		

22 Rotated Hybrid Composition Function 3 with High Condition Number Matrix
All settings are the same with Function 21: Rotated Hybrid Composition Function 3
Except M_i's condition numbers are [10 20 50 100 200 1000 2000 3000 4000 5000]



Properties:

- Multi-modal
- Non-separable
- Scalable
- A huge number of local optima
- Different function's properties are mixed together
- Global optimum is on the bound
- $x \in [-5,5]^D$, Global optimum $x^* = o_1$, $f(x^*) \approx f\_bias(22)=360$

Data file:

Name:	hybrid_func3_data.mat		
	hybrid_func3_data.txt		
Variable:	o	10*100 vector	the shifted optima for 10 functions
	When using, cut o=o(:,1:D)		
Name:	hybrid_func3_HM_D10 .mat		
Variable:	M	an structure variable	
	Contains M.M1 M.M2, … , M.M10 ten 10*10 matrixes		
Name:	hybrid_func3_HM_D10 .txt		

Variable: M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 10*10 matrixes, 1-10 lines are M1, 11-20 lines are M2,....,91-100 lines are M10

Name: hybrid_func3_HM_D30 .mat
Variable: M an structure variable contains M.M1,…,M.M10 ten 30*30 matrix
Name: hybrid_func3_MH_D30 .txt
Variable: M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 30*30 matrixes, 1-30 lines are M1, 31-60 lines are M2,....,271-300 lines are M10

Name: hybrid_func3_MH_D50 .mat
Variable: M an structure variable contains M.M1,…,M.M10 ten 50*50 matrix
Name: hybrid_func3_HM_D50 .txt
Variable: M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 50*50 matrixes, 1-50 lines are M1, 51-100 lines are M2,....,451-500 lines are M10

23 Non-Continuous Rotated Hybrid Composition Function 3

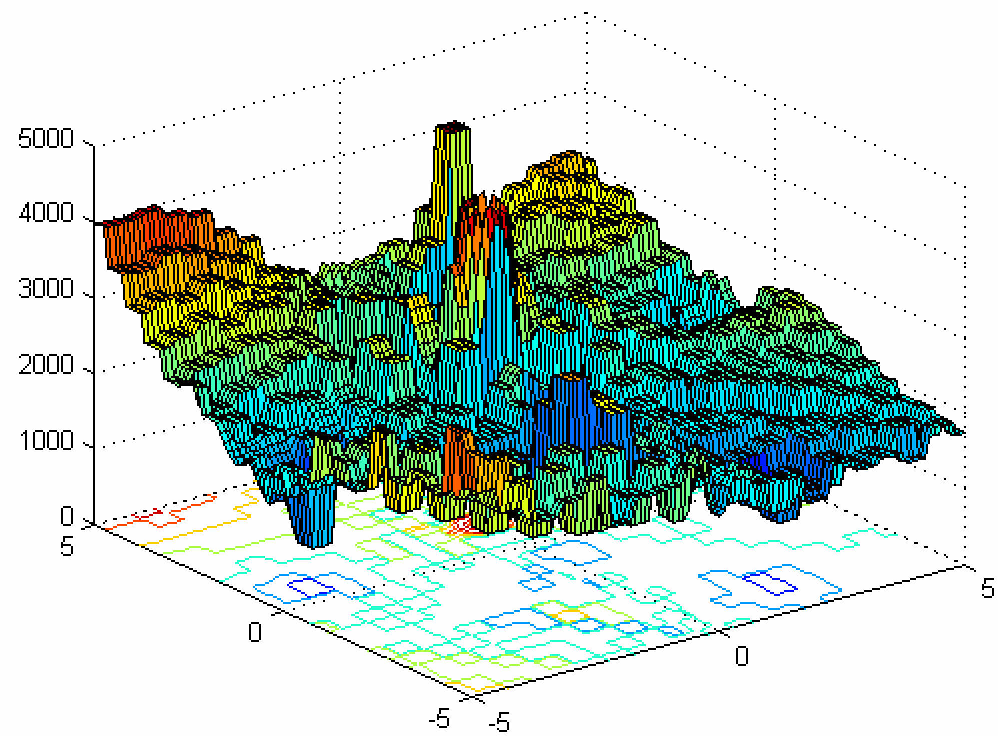
All settings are the same with Function 21: Rotated Hybrid Composition Function 3

Except $x_j = \begin{cases} x_j & |x_j - o_{1j}| < 1/2 \\ round(2x_j)/2 & |x_j - o_{1j}| >= 1/2 \end{cases}$ for $j = 1, 2, .., D$

$$round(x) = \begin{cases} a-1 & if \quad x <= 0 \,\&\, b >= 0.5 \\ a & if \quad b < 0.5 \\ a+1 & if \quad x > 0 \,\&\, b >= 0.5 \end{cases},$$

where *a* is x's integral part and *b* is x's decimal part

All "round" operators in this document use the same schedule.



Properties:

- Multi-modal
- Non-separable
- Scalable
- A huge number of local optima
- Different function's properties are mixed together
- Non-continuous
- Global optimum is on the bound
- $x \in [-5,5]^D$, Global optimum $x^* = o_1$, $f(x^*) \approx f\_bias(23)=360$

Data file:

Same with Function 21 Rotated Hybrid Composition Function 3.

24 Rotated Hybrid Composition Function 4

$f_1(x)$: Weierstrass Function

$$f_i(x)=\sum_{i=1}^{D}(\sum_{k=0}^{k\max}[a^k\cos(2\pi b^k(x_i+0.5))])-D\sum_{k=0}^{k\max}[a^k\cos(2\pi b^k 0.5)],$$

a=0.5, b=3, k_{max}=20

$f_2(x)$: Rotated Expanded Scaffer's F6 Function

$$F(x,y)=0.5+\frac{(\sin^2(\sqrt{x^2+y^2})-0.5)}{(1+0.001(x^2+y^2))^2}$$

$$f_i(x)=F(x_1,x_2)+F(x_2,x_3)+...+F(x_{D-1},x_D)+F(x_D,x_1)$$

$f_3(x)$: F8F2 Function

$$F8(x)=\sum_{i=1}^{D}\frac{x_i^{\ 2}}{4000}-\prod_{i=1}^{D}\cos(\frac{x_i}{\sqrt{i}})+1$$

$$F2(x)=\sum_{i=1}^{D-1}(100(x_i^2-x_{i+1})^2+(x_i-1)^2)$$

$$f_i(x)=F8(F2(x_1,x_2))+F8(F2(x_2,x_3))+...+F8(F2(x_{D-1},x_D))+F8(F2(x_D,x_1))$$

$f_4(x)$: Ackley's Function

$$f_i(x)=-20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D}x_i^2})-\exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i))+20+e$$

$f_5(x)$: Rastrigin's Function

$$f_i(x)=\sum_{i=1}^{D}(x_i^{\ 2}-10\cos(2\pi x_i)+10)$$

$f_6(x)$: Griewank's Function

$$f_i(x)=\sum_{i=1}^{D}\frac{x_i^{\ 2}}{4000}-\prod_{i=1}^{D}\cos(\frac{x_i}{\sqrt{i}})+1$$

$f_7(x)$: Non-Continuous Expanded Scaffer's F6 Function

$$F(x,y)=0.5+\frac{(\sin^2(\sqrt{x^2+y^2})-0.5)}{(1+0.001(x^2+y^2))^2}$$

$$f(x)=F(y_1,y_2)+F(y_2,y_3)+...+F(y_{D-1},y_D)+F(y_D,y_1)$$

$$y_j=\begin{cases} x_j & |x_j|<1/2 \\ round(2x_j)/2 & |x_j|>=1/2 \end{cases} \text{ for } j=1,2,..,D$$

$f_8(x)$: Non-Continuous Rastrigin's Function

$$f(x)=\sum_{i=1}^{D}(y_i^{\ 2}-10\cos(2\pi y_i)+10)$$

$$y_j=\begin{cases} x_j & |x_j|<1/2 \\ round(2x_j)/2 & |x_j|>=1/2 \end{cases} \text{ for } j=1,2,..,D$$

$f_9(x)$: High Conditioned Elliptic Function

$$f(x) = \sum_{i=1}^{D} (10^6)^{\frac{i-1}{D-1}} x_i^2$$

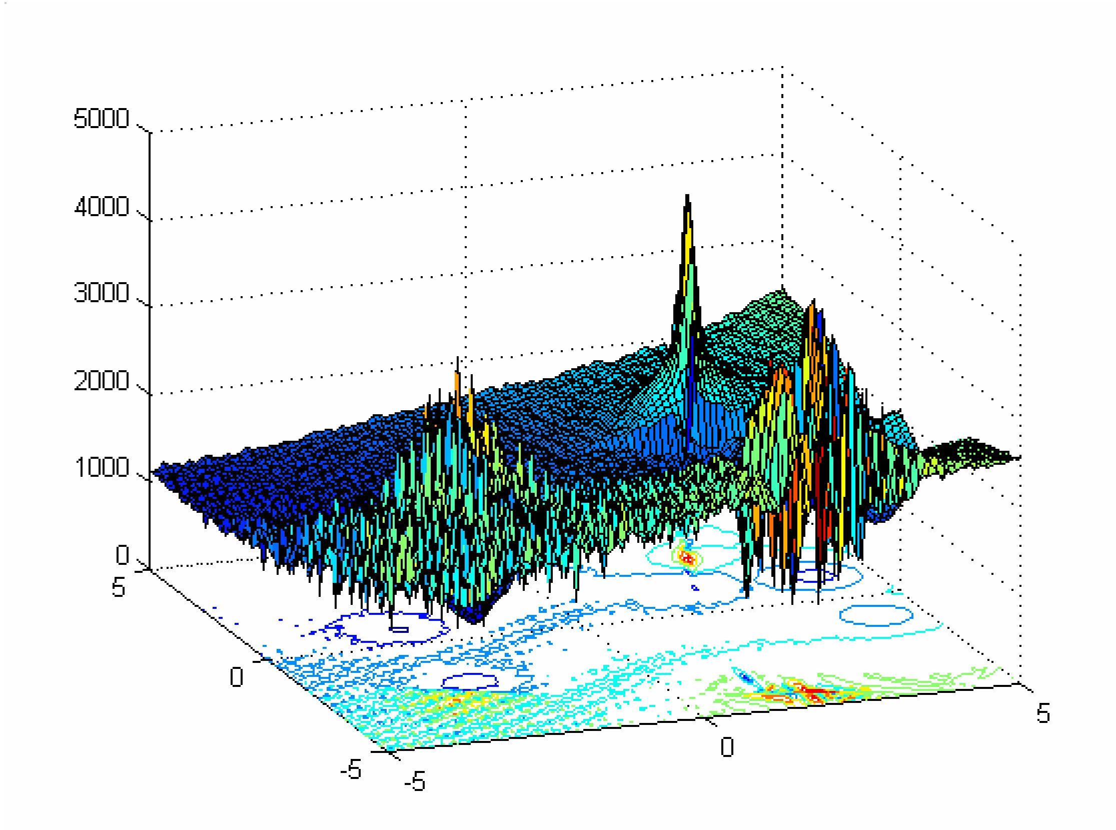
$f_{10}(x)$: Sphere Function with Noise in Fitness

$$f_i(x) = (\sum_{i=1}^{D} x_i^2)(1 + 0.1|N(0,1)|)$$

$\sigma_i = 2$,for $i = 1, 2..., D$

λ =[10; 5/20; 1; 5/32; 1; 5/100; 5/50; 1; 5/100; 5/100]

M_i are all rotation matrices, condition numbers are [100 50 30 10 5 5 4 3 2 2];



Properties:

- Multi-modal
- Rotated
- Non-Separable
- Scalable
- A huge number of local optima
- Different function's properties are mixed together
- Unimodal Functions give flat areas for the function.
- $x \in [-5, 5]^D$, Global optimum $x^* = o_1$, $f(x^*) \approx f\_bias(24)$= 260

Data file:

Name: hybrid_func4_data.mat
hybrid_func4_data.txt
Variable: o 10*100 vector the shifted optima for 10 functions
When using, cut o=o(:,1:D)

Name: hybrid_func4_M_D10 .mat
Variable: M an structure variable
Contains M.M1 M.M2, … , M.M10 ten 10*10 matrixes
Name: hybrid_func4_M_D10 .txt
Variable: M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 10*10 matrixes, 1-10 lines are M1, 11-20 lines are M2,....,91-100 lines are M10

Name: hybrid_func4_M_D30 .mat
Variable: M an structure variable contains M.M1,…,M.M10 ten 30*30 matrix
Name: hybrid_func4_M_D30 .txt
Variable: M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 30*30 matrixes, 1-30 lines are M1, 31-60 lines are M2,....,271-300 lines are M10

Name: hybrid_func4_M_D50 .mat
Variable: M an structure variable contains M.M1,…,M.M10 ten 50*50 matrix
Name: hybrid_func4_M_D50 .txt
Variable: M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 are ten 50*50 matrixes, 1-50 lines are M1, 51-100 lines are M2,....,451-500 lines are M10

25 Rotated Hybrid Composition Function 4 without bounds

All settings are the same with Function 24: Rotated Hybrid Composition Function 4
Except no exact search range set for this test function.

Properties:

- Multi-modal
- Non-separable
- Scalable
- A huge number of local optima
- Different function's properties are mixed together
- Unimodal Functions give flat areas for the function.
- Global optimum is on the bound
- No bounds
- Initialize population in $[2,5]^D$, Global optimum $x^* = o_1$ is outside of the initialization range, $f(x^*) \approx f\_bias(25)$=260

Data file:

Same with Function 24: Rotated Hybrid Composition Function 4.

Comparisons Pairs:

Different Condition Number:

- 1. Shifted Rotated Sphere Function
- 2. Shifted Schwefel's Problem 1.2
- 3. Shifted Rotated High Conditioned Elliptic Function

Function With Noise Vs Without Noise

Pair 1:

- 2. Shifted Schwefel's Problem 1.2
- 4. Shifted Schwefel's Problem 1.2 with Noise in Fitness

Pair 2:

- 16. Rotated Hybrid Composition Function 1
- 17. Rotated Hybrid Composition Function 1 with Noise in Fitness

Function without Rotation Vs With Rotation

Pair 1:

- 9. Shifted Rastrigin's Function
- 10. Shifted Rotated Rastrigin's Function

Pair 2:

- 15. Hybrid Composition Function1
- 16. Rotated Hybrid Composition Function 1

Continuous Vs Non-continuous

- 21. Rotated Hybrid Composition Function 3
- 23. Non-Continuous Rotated Hybrid Composition Function 3

Global Optimum on Bounds Vs Global Optimum on Bounds

- 18. Rotated Hybrid Composition Function 2
- 20. Rotated Hybrid Composition Function 2 with the Global Optimum on the Bounds

Wide Global Optimum Basin Vs Narrow Global Optimum Basin

- 18. Rotated Hybrid Composition Function 2
- 19. Rotated Hybrid Composition Function 2 with a Narrow Basin for the Global Optimum

Orthogonal Matrix Vs High Condition Number Matrix

- 21. Rotated Hybrid Composition Function 3
- 22. Rotated Hybrid Composition Function 3 with High Condition Number Matrix

Global Optimum in the Initialization Range Vs Global Optimum outside of the Initialization Range

- 24. Rotated Hybrid Composition Function 4
- 25. Rotated Hybrid Composition Function 4 without Bounds

Similar Groups:

Unimodal Functions

Function 1-5

Multi-modal Functions:

Function 6-25

- Single Function: Function 6-12
- Expanded Function: Function 13-14
- Hybrid Composition Function: Function 15-25

Functions with Global Optimum outside of the Initialization Range

- 7. Shifted Rotated Griewank’s Function without Bounds
- 25. Rotated Hybrid Composition Function 4 without Bounds

Functions with Global Optimum on Bounds

- 5. Schwefel’s Problem 2.6 with Global Optimum on Bounds
- 8. Shifted Rotated Ackley’s Function with Global Optimum on Bounds
- 20. Rotated Hybrid Composition Function 2 with the Global Optimum on the Bounds

Notes

Note 1: About Linear Transformation Matrix

M=P*D*Q

P, Q are two orthogonal matrixes, generated using Classical Gram-Schmidt method
D is diagonal matrix

$$u = rand(1, D)\,,\; d_{ii} = c^{\frac{u_i - \min(u)}{\max(u) - \min(u)}}$$

M's condition number Cond(M)=c

Note 2: On page 17, w_i values are sorted and raised to a higher power. The objective is to ensure that each optimum (local or global) is determined by only one function while allowing a higher degree of mixing of different functions just a very short distance away from each optimum.

Note 3: We assign different positive and negative objective function values, instead of zeros. This may influence some algorithms that make use of the objective values.

Note 4: We assign the same objective values to the comparison pairs in order to make the comparison easier.

Note 5: High condition number rotation may convert a multimodal problem into a unimodal problem. Hence, moderate condition numbers were used for multimodal.

Note 6: Additional data files will be provided with about ten 50-D coordinate positions and the corresponding fitness values in order to help the verification process during the code translation.

Note 7: It is insufficient to make any statistically meaningful conclusions on the pairs of problems as each case has at most 2 pairs. We would probably require 5 or 10 or more pairs for each case. We would consider this extension for the edited volume.

Note 8: Pseudo-real world problems are available from the web link given below. If you have any queries on these problems, please contact Professor Darrell Whitley directly. Email: whitley@CS.ColoState.EDU
`Web-link:` `http://www.cs.colostate.edu/~genitor/functions.html`.

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