

# COMP336 — Big Data

## Week 10 Lecture 1: Frequent Itemsets

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COMP336 2018H1

# Programme

- 1 The Market-Basket Model
- 2 Finding Frequent Itemsets
- 3 Scaling Up to Big Data

## Reading

- Leskovec, Rajaraman, Ullman (2014): Mining of Massive Datasets, Chapter 6. <http://www.mmds.org/>

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- 1 The Market-Basket Model
  - Association Rules
  - Frequent Itemsets
  - Finding Association Rules
- 2 Finding Frequent Itemsets
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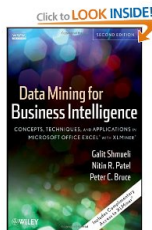
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# What Are Association Rules?

- Study of “what goes with what”:
  - “Customers who bought X also bought Y”.
  - What symptoms go with what diagnosis.
- Transaction-based or event-based.
- Also called “market basket analysis” and “affinity analysis”.
- Originated with study of customer transactions databases to determine associations among items purchased.

# Used in Many Recommender Systems



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## Customers Who Bought This Item Also Bought



**Data Mining Techniques:  
For Marketing, Sales,....** by  
Gordon S. Linoff

★★★★☆ (33)

**\$31.50**



**Data Warehousing 101:  
Concepts and  
Implementation** by Arshad  
Khan

★★★★☆ (1)

**\$13.17**



**Data Analysis and Decision  
Making (with...)** by S. Christian  
Albright

★★★★☆ (4)

**\$165.77**



**Customer Relationship  
Management: A Database  
Approach** by V. Kumar

★★★★☆ (2)

**\$63.13**



**Business Process Change,  
Second Edition: A Guide  
f...** by Paul Harmon

★★★★☆ (12)

**\$34.33**

# A Working Example

## Purchase of phone faceplates

A store that sells accessories for mobile phones runs a promotion on faceplates. Customers who purchase multiple faceplates from a choice of six different colours get a discount. The store managers, who would like to know what colours of faceplates customers are likely to purchase together, collected a transaction database.

Transaction	Faceplate colours purchased				
1	red	white	green		
2	white	orange			
3	white	blue			
4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				



# Candidate Rules

## Example of a rule

IF {red, white} THEN {green}

## Terminology

Antecedent {red, white}.

Consequent {green}.

Item set {red, white, green, orange, blue, yellow}.

The antecedent and the consequent are **disjoint**:

- They have no items in common.



# Many Rules are Possible

For example: Transaction 1 supports several rules, such as

- ① “If red, then white” (“If a red faceplate is purchased, then so is a white one”).
- ② “If white, then red”.
- ③ “If red and white, then green”.
- ④ ...

# Frequent Item Sets

- Ideally, we want to create all possible combinations of items.
- **Problem:** computation time grows exponentially as # items increases.
- **Solution:** consider only “frequent item sets”.
- Criterion for frequent: **support**.

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# Definition of Frequent Itemsets

- A set of items that appears in many baskets is said to be “frequent”.
- Assume there is a number  $s$ , called the **support threshold**.
- If  $I$  is a set of items, the **support** for  $I$  is the number of baskets for which  $I$  is a subset.
- We then say that  $I$  is frequent if its support is  $s$  or more.

## In our Working Example

The support for the item set {red, white} is 4

Transaction	Faceplate colours purchased			
1	red	white	green	
2	white	orange		
3	white	blue		
4	red	white	orange	
5	red	blue		
6	white	blue		
7	white	orange		
8	red	white	blue	green
9	red	white	blue	
10	yellow			



# Applications of Frequent Itemsets I

## Brick-and-Mortar Retailing

**Items:** Products.

**Baskets:** Sets of products someone bought in one trip to the store.

- By finding frequent itemsets, a retailer can learn what is commonly bought together and arrange the marketing and sales strategy accordingly.
- A famous example is the itemset “diapers and beer”.
  - **Strategy:** run a sale on diapers and simultaneously raise the price of beer.
  - **Strategy:** place cans of beer in the aisle for baby products.

# Applications of Frequent Itemsets II

## Plagiarism Detection

Items: Documents.

Baskets: Sentences.

## Clinical Analysis

Items Drugs and side-effects.

Baskets: Patients.

- Has been used to detect combinations of drugs that result in particular side-effects.
- This approach requires an extension: Absence of an item needs to be observed as well as presence.

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# Measures of Performance I

## Confidence

The % of antecedent transactions that also have the consequent item set.

- Confidence is the conditional probability of taking the consequent given that we have taken the antecedent:

$$\begin{aligned}\text{Confidence} &= P(\text{consequent}|\text{antecedent}) \\ &= \frac{P(\text{antecedent}\&\text{consequent})}{P(\text{antecedent})}\end{aligned}$$

- However, if the antecedent or consequent have strong support, this value is not useful.

## Measures of Performance II

### Lift Ratio

$$\text{Lift ratio} = \frac{\text{Confidence}}{\text{Benchmark confidence}}$$

- The benchmark confidence is the percentage of transactions in the entire dataset that have the consequent:

$$\text{Benchmark confidence} = \frac{\text{no. transactions with consequent item set}}{\text{no. transactions in database}}$$

- Lift ratio  $> 1$  indicates that the rule is more useful than just selecting transactions randomly.

# Process of Rule Selection I

Generate all rules that meet specified support & confidence.

- 1 Find frequent item sets (those with sufficient support — see above).
- 2 From these item sets, generate rules with sufficient confidence (or lift ratio, whatever criteria we set).

## Process of Rule Selection II

### Example

Rules from {red, white} item set.

- Support for {red,white} = 40%; Support for {red} = 50%; Support for {white} = 80%.

The rules are:

- 1 IF {red} THEN {white}
  - Confidence =  $40\% / 50\% = 80\%$ .
  - Lift ratio =  $80\% / 80\% = 1$ .
- 2 IF {white} THEN {red}
  - Support for {red,white} = 40%.
  - Confidence =  $40\% / 80\% = 50\%$ .
  - Lift ratio =  $50\% / 50\% = 1$ .

If confidence cutoff is 70%, report only rule 1. If lift ratio  $> 1$ , none of these rules are useful.

# Interpretation

- **Lift ratio** shows how effective the rule is in finding consequents (useful if finding particular consequents is important).
- **Confidence** shows the rate at which consequents will be found (useful in learning costs of promotion).
- **Support** measures overall impact.

## Caution: The Role of Chance

- Random data can generate apparently interesting association rules.
- The more rules you produce, the greater this danger.
- Rules based on large numbers of records are less subject to this danger.

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# Finding Frequent Itemsets

- A naïve approach to find frequent itemsets of length  $k$  would use a nested loop that generates all possible itemsets.
- But in a real scenario there may be many possible different items.
- This approach would be too time-consuming and may exceed memory capacity.
  - Suppose  $10^5$  items, counts are 4-byte integers.
  - Number of pairs of items:  $10^5(10^5 - 1)/2 = 5 * 10^9$ .
  - This would require  $2 * 10^{10}$  bytes (20Gb of memory).
  - What about counting triples??

# Intuition of the A-Priori Algorithm

## Monotonicity

- If a set of items  $I$  appears at least  $s$  times, **so does every subset  $J$  of  $I$ .**
- Consequently: if a set of items  $J$  has a support below  $s$ , **so does every set  $I$  containing  $J$ .**
- Applied to a pair of items: If an item  $i$  does not appear in  $s$  baskets, then no pair including  $i$  can appear in  $s$  baskets.

# Generating Frequent Item Sets

## The A-Priori Algorithm

For  $k$  products ...

- 1 User sets a minimum support criterion.
- 2 **First pass:** generate list of one-item sets that meet the support criterion.
- 3 **Second pass:** use the list of one-item sets to generate list of two-item sets that meet the support criterion.
- 4 **Third pass:** use list of two-item sets to generate list of three-item sets.
- 5 ...
- 6 Continue up through  $k$ -item sets.

# Example with the Phone Faceplates Dataset

## The Dataset

Transaction	Faceplate colours purchased				
1	red	white	green		
2	white	orange			
3	white	blue			
4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

## Two-item sets $\geq 3$

Set	Support
red, white	4
red, orange	1
red, blue	3
white, orange	3
white, blue	4
orange, blue	0

## One-item sets with support $\geq 3$

Set	Support
red	5
white	8
green	2
orange	3
blue	5
yellow	1

## Three-item sets

Set	Support
red, white, blue	2

# Example with the Phone Faceplates Dataset

## The Dataset

Transaction	Faceplate colours purchased				
1	red	white	green		
2	white	orange			
3	white	blue			
4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

## Two-item sets $\geq 3$

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red, white	4
red, orange	1
red, blue	3
white, orange	3
white, blue	4
orange, blue	0

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Set	Support
red	5
white	8
green	2
orange	3
blue	5
yellow	1

## Three-item sets

Set	Support
red, white, blue	2

# Example with the Phone Faceplates Dataset

## The Dataset

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1	red	white	green		
2	white	orange			
3	white	blue			
4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

## Two-item sets $\geq 3$

Set	Support
red, white	4
red, orange	1
red, blue	3
white, orange	3
white, blue	4
orange, blue	0

## One-item sets with support $\geq 3$

Set	Support
red	5
white	8
green	2
orange	3
blue	5
yellow	1

## Three-item sets

Set	Support
red, white, blue	2

# Example with the Phone Faceplates Dataset

## The Dataset

Transaction	Faceplate colours purchased				
1	red	white	green		
2	white	orange			
3	white	blue			
4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

## Two-item sets $\geq 3$

Set	Support
red, white	4
red, orange	1
red, blue	3
white, orange	3
white, blue	4
orange, blue	0

## One-item sets with support $\geq 3$

Set	Support
red	5
white	8
green	2
orange	3
blue	5
yellow	1

## Three-item sets

Set	Support
red, white, blue	2

# Example with the Phone Faceplates Dataset

## The Dataset

Transaction	Faceplate colours purchased				
1	red	white	green		
2	white	orange			
3	white	blue			
4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

## Two-item sets $\geq 3$

Set	Support
red, white	4
red, orange	1
red, blue	3
white, orange	3
white, blue	4
orange, blue	0

## One-item sets with support $\geq 3$

Set	Support
red	5
white	8
green	2
orange	3
blue	5
yellow	1

## Three-item sets

Set	Support
red, white, blue	2



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## Considerations for Efficiency

- We assume that there are too many baskets to fit in main memory.
- So the file of baskets must be read from disk.
- We also assume that the number of items in each basket is relatively small.
- It will generally take much longer time to read a basket than to compute pairs or triples of items in the basket.
- There will normally be very few itemsets with size larger than 3 and support over the threshold.

We'd better read the basket file sequentially, and as few times as possible.

## A-Priori Algorithm — Pass 1

- 1 Initialise counters for all items to zero.
- 2 FOR each basket:
  - 3 Read basket from disk.
  - 4 FOR each item in basket:
    - 5 Increment item's count.

# Pass 1 — Example

## The Dataset

Transaction	Faceplate colours purchased				
1	red	white	green		
2	white	orange			
3	white	blue			
4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

## One-item sets with support $\geq 3$

Set	Support
red	1
white	1
green	1
orange	0
blue	0
yellow	0

# Pass 1 — Example

## The Dataset

Transaction	Faceplate colours purchased			
1	red	white	green	
2	white	orange		
3	white	blue		
4	red	white	orange	
5	red	blue		
6	white	blue		
7	white	orange		
8	red	white	blue	green
9	red	white	blue	
10	yellow			

## One-item sets with support $\geq 3$

Set	Support
red	1
white	2
green	1
orange	1
blue	0
yellow	0

# Pass 1 — Example

## The Dataset

Transaction	Faceplate colours purchased			
1	red	white	green	
2	white	orange		
3	white	blue		
4	red	white	orange	
5	red	blue		
6	white	blue		
7	white	orange		
8	red	white	blue	green
9	red	white	blue	
10	yellow			

## One-item sets with support $\geq 3$

Set	Support
red	1
white	3
green	1
orange	1
blue	1
yellow	0

# Pass 1 — Example

## The Dataset

Transaction	Faceplate colours purchased			
1	red	white	green	
2	white	orange		
3	white	blue		
4	red	white	orange	
5	red	blue		
6	white	blue		
7	white	orange		
8	red	white	blue	green
9	red	white	blue	
10	yellow			

## One-item sets with support $\geq 3$

Set	Support
red	2
white	4
green	1
orange	2
blue	1
yellow	0

# Pass 1 — Example

## The Dataset

Transaction	Faceplate colours purchased				
1	red	white	green		
2	white	orange			
3	white	blue			
4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

## One-item sets with support $\geq 3$

Set	Support
red	3
white	4
green	1
orange	2
blue	2
yellow	0



# Pass 1 — Example

## The Dataset

Transaction	Faceplate colours purchased			
1	red	white	green	
2	white	orange		
3	white	blue		
4	red	white	orange	
5	red	blue		
6	white	blue		
7	white	orange		
8	red	white	blue	green
9	red	white	blue	
10	yellow			

## One-item sets with support $\geq 3$

Set	Support
red	3
white	5
green	1
orange	2
blue	3
yellow	0

# Pass 1 — Example

## The Dataset

Transaction	Faceplate colours purchased			
1	red	white	green	
2	white	orange		
3	white	blue		
4	red	white	orange	
5	red	blue		
6	white	blue		
7	white	orange		
8	red	white	blue	green
9	red	white	blue	
10	yellow			

## One-item sets with support $\geq 3$

Set	Support
red	3
white	6
green	1
orange	3
blue	3
yellow	0

# Pass 1 — Example

## The Dataset

Transaction	Faceplate colours purchased			
1	red	white	green	
2	white	orange		
3	white	blue		
4	red	white	orange	
5	red	blue		
6	white	blue		
7	white	orange		
8	red	white	blue	green
9	red	white	blue	
10	yellow			

## One-item sets with support $\geq 3$

Set	Support
red	4
white	7
green	2
orange	3
blue	4
yellow	0

# Pass 1 — Example

## The Dataset

Transaction	Faceplate colours purchased			
1	red	white	green	
2	white	orange		
3	white	blue		
4	red	white	orange	
5	red	blue		
6	white	blue		
7	white	orange		
8	red	white	blue	green
9	red	white	blue	
10	yellow			

## One-item sets with support $\geq 3$

Set	Support
red	5
white	8
green	2
orange	3
blue	5
yellow	0

# Pass 1 — Example

## The Dataset

Transaction	Faceplate colours purchased			
1	red	white	green	
2	white	orange		
3	white	blue		
4	red	white	orange	
5	red	blue		
6	white	blue		
7	white	orange		
8	red	white	blue	green
9	red	white	blue	
10	yellow			

## One-item sets with support $\geq 3$

Set	Support
red	5
white	8
green	2
orange	3
blue	5
yellow	1

# Pass 1 — Example

## The Dataset

Transaction	Faceplate colours purchased			
1	red	white	green	
2	white	orange		
3	white	blue		
4	red	white	orange	
5	red	blue		
6	white	blue		
7	white	orange		
8	red	white	blue	green
9	red	white	blue	
10	yellow			

## One-item sets with support $\geq 3$

Set	Support
red	5
white	8
green	2
orange	3
blue	5
yellow	1

## A-Priori Algorithm — Storage Considerations for Pass 2

- Storing all possible pairs of items might not fit on memory.
- So, we only store pairs of items that we are counting.
- Pairs of items are stored as triples:

$$(i, j, c)$$

$i$ : Item 1

$j$ : Item 2

$c$ : Count (initialised to zero)

## A-Priori Algorithm — Pass 2

- ➊ FOR each basket:
- ➋     Read basket from disk.
- ➌     FOR each pair  $(i, j)$  of items in basket:
- ➍         IF both  $i$  and  $j$  are frequent (according to first pass):
- ➎         IF  $(i, j)$  has been counted:
- ➏             Increment count for  $(i, j)$
- ➐         ELSE:
- ➑             Add counter for  $(i, j)$  initialised to 1.



# Pass 2 — Example

## The Dataset

Transaction	Faceplate colours purchased			
1	red	white	green	
2	white	orange		
3	white	blue		
4	red	white	orange	
5	red	blue		
6	white	blue		
7	white	orange		
8	red	white	blue	green
9	red	white	blue	
10	yellow			

## Two-item sets $\geq 3$

<i>i</i>	<i>j</i>	<i>c</i>
red	white	1

Only 5 pairs were counted  
from a total of 15.

## One-item sets with support $\geq 3$

Set	Support
red	5
white	8
orange	3
blue	5

## Pass 2 — Example

### The Dataset

Transaction	Faceplate colours purchased				
1	red	white	green		
2	white	orange			
3	white	blue			
4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

### Two-item sets $\geq 3$

<i>i</i>	<i>j</i>	<i>c</i>
red	white	1
white	orange	1

Only 5 pairs were counted  
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### One-item sets with support $\geq 3$

Set	Support
red	5
white	8
orange	3
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## Pass 2 — Example

### The Dataset

Transaction	Faceplate colours purchased				
1	red	white	green		
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4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

### Two-item sets $\geq 3$

<i>i</i>	<i>j</i>	<i>c</i>
red	white	1
white	orange	1
white	blue	1

Only 5 pairs were counted  
from a total of 15.

### One-item sets with support $\geq 3$

Set	Support
red	5
white	8
orange	3
blue	5

# Pass 2 — Example

## The Dataset

Transaction	Faceplate colours purchased				
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6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

## Two-item sets $\geq 3$

<i>i</i>	<i>j</i>	<i>c</i>
red	white	2
white	orange	2
white	blue	1
red	orange	1

Only 5 pairs were counted  
from a total of 15.

## One-item sets with support $\geq 3$

Set	Support
red	5
white	8
orange	3
blue	5

## Pass 2 — Example

### The Dataset

Transaction	Faceplate colours purchased				
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4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

### Two-item sets $\geq 3$

<i>i</i>	<i>j</i>	<i>c</i>
red	white	2
white	orange	2
white	blue	1
red	orange	1
red	blue	1

Only 5 pairs were counted  
from a total of 15.

### One-item sets with support $\geq 3$

Set	Support
red	5
white	8
orange	3
blue	5

# Pass 2 — Example

## The Dataset

Transaction	Faceplate colours purchased				
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7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

## Two-item sets $\geq 3$

<i>i</i>	<i>j</i>	<i>c</i>
red	white	2
white	orange	2
white	blue	2
red	orange	1
red	blue	1

Only 5 pairs were counted  
from a total of 15.

## One-item sets with support $\geq 3$

Set	Support
red	5
white	8
orange	3
blue	5

## Pass 2 — Example

### The Dataset

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10	yellow				

### Two-item sets $\geq 3$

<i>i</i>	<i>j</i>	<i>c</i>
red	white	2
white	orange	3
white	blue	2
red	orange	1
red	blue	1

Only 5 pairs were counted  
from a total of 15.

### One-item sets with support $\geq 3$

Set	Support
red	5
white	8
orange	3
blue	5

## Pass 2 — Example

### The Dataset

Transaction	Faceplate colours purchased				
1	red	white	green		
2	white	orange			
3	white	blue			
4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

### Two-item sets $\geq 3$

<i>i</i>	<i>j</i>	<i>c</i>
red	white	3
white	orange	3
white	blue	3
red	orange	1
red	blue	2

Only 5 pairs were counted  
from a total of 15.

### One-item sets with support $\geq 3$

Set	Support
red	5
white	8
orange	3
blue	5



# Pass 2 — Example

## The Dataset

Transaction	Faceplate colours purchased				
1	red	white	green		
2	white	orange			
3	white	blue			
4	red	white	orange		
5	red	blue			
6	white	blue			
7	white	orange			
8	red	white	blue	green	
9	red	white	blue		
10	yellow				

## Two-item sets $\geq 3$

<i>i</i>	<i>j</i>	<i>c</i>
red	white	4
white	orange	3
white	blue	4
red	orange	1
red	blue	3

Only 5 pairs were counted  
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10	yellow				

## Two-item sets $\geq 3$

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red	white	4
white	orange	3
white	blue	4
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Only 5 pairs were counted  
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## Two-item sets $\geq 3$

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red	white	4
white	orange	3
white	blue	4
red	orange	1
red	blue	3

Only 5 pairs were counted  
from a total of 15.

## One-item sets with support $\geq 3$

Set	Support
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# Programme

- 1 The Market-Basket Model
- 2 Finding Frequent Itemsets
- 3 Scaling Up to Big Data
  - PCY Algorithm
  - Multistage Algorithm

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# Park-Chen-Yu Algorithm

## Observation of A-Priori algorithm

- In pass 1, most memory is idle.
  - Pass 2 will usually be the most memory-consuming.
- ⇒ Can we use the idle memory to reduce memory required in pass 2?

## Solution by PCY

At pass 1, while counting items, maintain a hash table that will count pairs of items.

# PCY Algorithm — First Pass

## Algorithm

- 1 FOR each basket:
- 2     FOR each item in basket:
- 3         Add 1 to item's count.
- 4     FOR each pair of items in basket:
- 5         Hash the pair to a bucket.
- 6         Add 1 to the count for that bucket.

## Notes

- 1 We want to use as many hash buckets as we can fit in memory.
- 2 Generalisation of a Bloom filter: We need to count the pairs so that we can check if their support is at least  $s$ .

## PCY Algorithm — Second Pass

### Algorithm

- 1 FOR each basket:
- 2     FOR each pair in the basket:
- 3         IF both items in the pair are frequent:
- 4             Use hash function of pass 1 to hash the pair to a bucket.
- 5             IF the count for that bucket is  $\geq s$ :
- 6                 Count the pair.

### Note

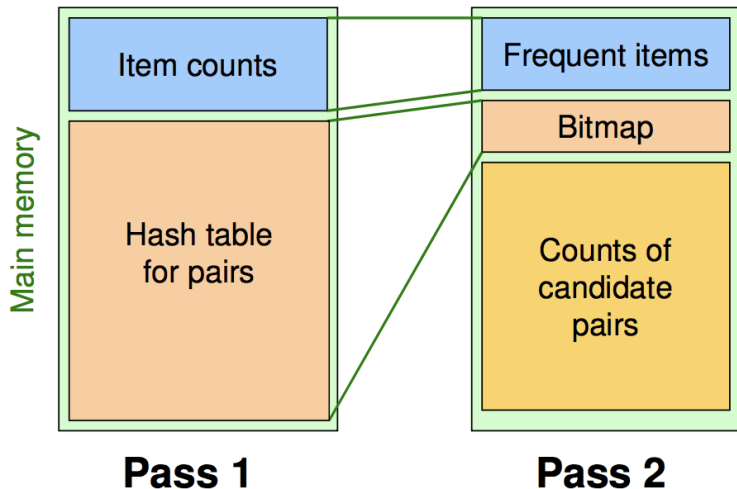
- Several pairs might have hashed to the same bucket.
- ⇒ Even if a pair hashes to a bucket with count  $\geq s$ , it might not be frequent.
- ⇒ If a pair does not hash to a bucket with count  $\geq s$ , we are certain that it is not frequent.



# What is the Advantage of Hash Tables?

- Like in the Bloom filter, the space to store the buckets in memory is smaller than the space of storing pairs of items and their counts.
- This is so because the number of buckets is less than the number of pairs of items.
- We only need to know what buckets are over a threshold, so after the first pass we can compact the hash table into a bit vector.
  - Bit = 1 if bucket is  $\geq s$ .
- We can actually chain hash tables to reduce the number of pairs to count... see below.

## Main Memory: Picture of PCY



# Programme

- 1 The Market-Basket Model
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# The Multistage Algorithm

- The multistage algorithm attempts to reduce the number of pairs to be counted.
- It does this by adding a new pass through the file of baskets.

## Pass 1

Count individual items and maintain a hash table of counts of pairs as in PCY algorithm.

## Pass 2

Create a new hash table, using an independent hash function, that counts pairs whose items were hashed as frequent in pass 1 **and both items in the pair are frequent as determined by pass 1.**

## Pass 3

Count pairs that hashed to frequent buckets **in both hashes.**

# Multistage Algorithm — Pass 1

## Algorithm

- 1 FOR each basket:
- 2     FOR each item in basket:
- 3         Add 1 to item's count.
- 4     FOR each pair of items in basket:
- 5         Use hash function 1 to hash the pair to a bucket.
- 6         Add 1 to the count for that bucket.

## Note

This is exactly the same as pass 1 of PCY.

## Multistage Algorithm — Pass 2

### Algorithm

- 1 FOR each basket:
- 2     FOR each pair in the basket:
- 3         IF both items in the pair are frequent:
- 4             Use hash function 1 to hash the pair to a bucket.
- 5             IF the count for that bucket is  $\geq s$ :
- 6                 Use hash function 2 to hash the pair to a bucket.
- 7                 Add 1 to the count for that bucket..

### Note

We keep two separate hash tables, one per hash function.

# Multistage Algorithm — Pass 3

## Algorithm

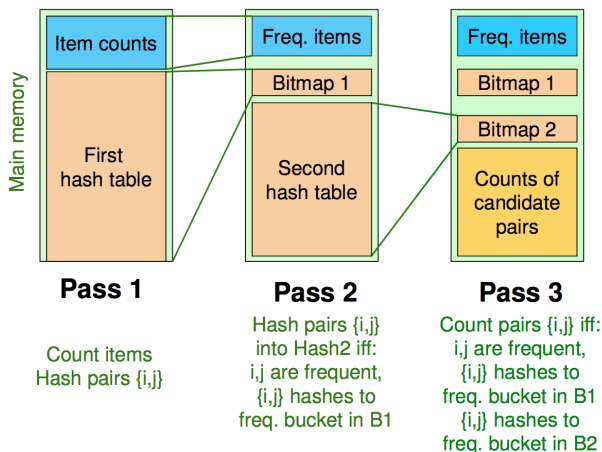
- 1 FOR each basket:
- 2     FOR each pair in the basket:
- 3         IF both items in the pair are frequent:
- 4             Use hash function 1 to hash the pair to a bucket.
- 5             Use hash function 2 to hash the pair to a bucket.
- 6             IF the count for both buckets is  $\geq s$ :
- 7                 Count the pair.

## Do we really need to maintain both hashes in pass 3?

- It would be tempting to avoid checking the first hash in pass 3.
- But there may be pairs that hash to a frequent bucket in hash 2 but do not hash to a frequent bucket in pass 1.
- Remember: due to collisions, an infrequent pair may end up hashed in the same bucket as a frequent pair.
- We therefore need to make sure that the two hash functions are independent.



# Main Memory: Multistage



<http://www.mmds.org>

# Take-home Messages

- The Marked-Basket model.
- General approach to find association rules.
- Counting item pairs are the costliest part of finding association rules.
- The A-Priori algorithm.
- Efficient implementation of A-Priori.
- Variants for big data.

# What's Next

## Week 11

- Large-Scale Machine Learning (I)