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Lista 2

⑥ $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ $AB = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ $BA = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$

$\det(A - \lambda I) = 0$

$\det(A - \lambda I) = (1-\lambda)^2 = 0$

$\lambda_A = 1$

$\det(AB - \lambda I) = 0$

$\det(AB - \lambda I) = (1-\lambda)(3-\lambda) - 2 = 0$

$= \lambda^2 - 4\lambda + 1 = 0$

$\Delta = 12 \quad \lambda = \frac{4 \pm \sqrt{12}}{2} = \begin{cases} 2-\sqrt{3} \\ 2+\sqrt{3} \end{cases}$

$\det(B - \lambda I) = 0$

$\det(B - \lambda I) = (1-\lambda)^2$

$\lambda_B = 1$

$\lambda'_{AB} = 2 - \sqrt{3}$

$\lambda''_{AB} = 2 + \sqrt{3}$

$\det(BA - \lambda I) = 0$

$\det(BA - \lambda I) = (3-\lambda)(1-\lambda) - 2$

$= \lambda'_{BA} = 2 - \sqrt{3}$

$\lambda''_{BA} = 2 + \sqrt{3}$

a) Não são iguais, pois $\lambda_A = \lambda_B = 1$ e $\lambda_{AB} = 2 \pm \sqrt{3}$

b) Sim, tanto AB, quanto BA têm autovalores iguais

⑦ $A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$

$\det(A - tI) = -t^3 + 12t - 36t + 32 = 0$

$t_1 = 2$

$t_2 = 2$

$t_3 = 8$

$\left(\begin{array}{ccc|cc} 4-t & 2 & 2 & 4-t & 2 \\ 2 & 4-t & 2 & 2 & 4-t \\ 2 & 2 & 4-t & 2 & 2 \end{array}\right)$

$\begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$x = -(\alpha + \beta)$

$y = \alpha$

$z = \beta$

$\alpha(-1, 1, 0) + \beta(-1, 0, 1)$

$V_1 = (-1, 1, 0)$

$V_2 = (-1, 0, 1)$

$\begin{pmatrix} -4 & 2 & 2 \\ 2 & -4 & 2 \\ 2 & 2 & -4 \end{pmatrix}$

$2x - 2z = 0$

$y - z = 0$

$x = z$

$y = z$

$V_3 = (1, 1, 1)$

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8) a) $\det(A - tI) = \det(A^t - tI)$?

$$A = \left(\begin{array}{ccc|cc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}\right)$$

$a_{11} \cdot a_{22} \cdot a_{33} + a_{12} \cdot a_{23} \cdot a_{31} + a_{13} \cdot a_{21} \cdot a_{32}$
$- (a_{12} \cdot a_{21} \cdot a_{33} + a_{11} \cdot a_{23} \cdot a_{32} + a_{13} \cdot a_{22} \cdot a_{31})$

$$A^t = \left(\begin{array}{ccc|cc} a_{11} & a_{21} & a_{31} & a_{11} & a_{21} \\ a_{12} & a_{22} & a_{32} & a_{12} & a_{22} \\ a_{13} & a_{23} & a_{33} & a_{13} & a_{23} \end{array}\right)$$

$a_{11} \cdot a_{22} \cdot a_{33} + a_{21} \cdot a_{32} \cdot a_{13} + a_{31} \cdot a_{12} \cdot a_{23} -$
$(a_{21} \cdot a_{12} \cdot a_{33} + a_{11} \cdot a_{32} \cdot a_{23} + a_{31} \cdot a_{22} \cdot a_{13})$

b) É válido pois a diagonal principal de uma matriz é mantida após transpor-la.

1) $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $A \cdot A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\text{rank}(A) = 1$ $\text{rank}(A^2) = 0$

$\text{rank}(A) > \text{rank}(A^2)$

$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow \text{rank}(A^T A) = 1$

2) $C(A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $r(A) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $N(A) = Ax = 0 \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

3) $r = m = n$ $m, n = 2$

$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\text{rank}(A_1) = 2$

$r = n < m$ $m = 3, n = 2$

$A_4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$ $\text{rank}(A_4) = 2$

$r = m < n$ $n = 3$ e $m = 2$

$A_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\text{rank}(A_2) = 2$

$r < m, r < n$ $m, n = 2$

$A_3 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $\text{rank}(A_3) = 1$

5 $w^{+}w = I \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $w^{-1} = w^{+} \to \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$

6) $w = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 1 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 & -1 & 0 & -1 \end{bmatrix}$