Lista 04 Victor Vieira de Melo

① a) $N_1 = |v_1| + |v_2| + |v_3| + \dots + |v_n|$

$|3| + |0| + |2| + |9| + |1| + |7| + |1| + |0| + |1| = 24$

b) Norma infinita

$\|v\|_\infty = \max(|v_i|)$

$|9| = 9$

c) Norma 2

$\|v_i\|_2 = \sqrt{|v_1|^2 + \dots + |v_n|^2}$

$\sqrt{(3)^2 + (0)^2 + (2)^2 + (9)^2 + (1)^2 + (7)^2 + (1)^2 + (0)^2 + (1)^2} =$

$\sqrt{9 + 4 + 81 + 49 + 1 + 1} = \sqrt{146}$

d) Norma frobenius = Norma 2.

② a) $M_{1024 \times 768} = U_{1024 \times k} \times \Sigma_{k \times k} \times V^T_{k \times 768}$

$1024 \times 1 + 1 + 1 \times 768 = 1793$

$k\ max = \left\lfloor \frac{1024 \times 768}{1793} \right\rfloor = 438$

b) Sendo m a quantidade de imagens e n o tamanho dos vetores: $m \times k$, $k \times k$, $k \times n$

$10 \times 1 + 1 + 1 \times 786432 = 786443$

$1000 \times 1 + 1 + 1 \times 786432 = 787433$

③ $A = \begin{pmatrix} 3 & 4 \\ 0 & 5 \end{pmatrix}$ $A = U \Sigma V^T$

① $A^T A = V \Sigma^T \Sigma V^T$ ② $A A^T = U \Sigma \Sigma^T U$

① $\begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 9 & 12 \\ 12 & 41 \end{pmatrix}$ ② $\begin{pmatrix} 3 & 4 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 25 & 20 \\ 20 & 25 \end{pmatrix}$

$\begin{pmatrix} 9-\lambda & 12 \\ 12 & 41-\lambda \end{pmatrix}$ $(9-\lambda)(41-\lambda) - 144 = 0$ $\lambda' = 5$

$369 - 9\lambda - 41\lambda + \lambda^2 - 144 = 0$ $\lambda'' = 45$

$\lambda^2 - 50\lambda + 225 = 0$

$\begin{pmatrix} 9-5 & 12 \\ 12 & 41-5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 & 12 \\ 12 & 36 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 & 12 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ $4x + 12y = 0$

$x = -3y$

$v_1^T = (-3, 1)$

$$\begin{pmatrix} 25-5 & 20 \\ 20 & 25-5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 20 & 20 \\ 20 & 20 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = x+y=0$$

$$x=-y$$

$V_{12}^T = (-1, 1)$

$$\begin{pmatrix} 9-45 & 12 \\ 12 & 41-45 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -36 & 12 \\ 12 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 & -4 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 12x-4y=$$

$$3x=y$$

$V_{21}^T = (1, 3)$

$$\begin{pmatrix} 25-45 & 20 \\ 20 & 25-45 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -20 & 20 \\ 20 & -20 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -x+y=0$$

$$y=x$$

$V_{22}^T = (1, 1)$

$$A = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 5 & 0 \\ 0 & 45 \end{pmatrix}\begin{pmatrix} -3 & 1 \\ 1 & 3 \end{pmatrix}$$

(4) $A = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$

(1) $A^TA = U\Sigma^T\Sigma U^T$

(2) $AA^T = U\Sigma\Sigma^T U$

(1) $$A^TA = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix}\begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

(2) $$AA^T = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(-\lambda)(4-\lambda)(9-\lambda)=0$$

$\lambda' = 0 \quad \lambda'' = 4 \quad \lambda''' = 9$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

(5) $$AA^TA = U\Sigma \overbrace{V^TV}^{I} \Sigma^T\Sigma V^T$$

$$= U\Sigma\Sigma^T\Sigma V^T$$

$$= U\Sigma^3V^T$$

(6) $A = \begin{pmatrix} \cos\theta & -\text{sen}\,\theta \\ \text{sen}\,\theta & \cos\theta \end{pmatrix}$ posto 1

$\|A - B\| \geq \|A - \sigma_k U_k V_k^T\|$

$\left\| \begin{pmatrix} \cos\theta & -\text{sen}\,\theta \\ \text{sen}\,\theta & \cos\theta \end{pmatrix} - \begin{pmatrix} \cos\theta & \text{sen}\,\theta \\ \text{sen}\,\theta & -\cos\theta \end{pmatrix} \right\| \geq 0$

(7) Matrizes de rank 3 que

$\|A - A_1\|_2 = \|A - A_2\|_2$ sendo $A_2 = \sigma_1 U_1 V_1^T + \sigma_2 U_2 V_2^T$ então $A_1 = A_2$,

mas $A_1 = \sigma_1 U_1 V_1^T$ e como rank $= 3$, temos $\sigma_1 \geq \sigma_2 \geq \sigma_3 > 0$

Logo, $\sigma_2 U_2 V_2^T = 0$ e $\sigma_2 \neq 0$, o que mostra $U \perp V^T$

(8) $\sqrt{|v_1|^2 + |v_2|^2 + \dots + |v_m|^2} \leq \max |v_i| \sqrt{m}$

$|v_1|^2 + |v_2|^2 + \dots + |v_m|^2 \leq (\max |v_i| \sqrt{m})^2$

$|v_1|^2 + |v_2|^2 + \dots + |v_m|^2 \leq |v_i|_1^2 + |v_i|_2^2 + \dots + |v_i|_m^2$

Como v_i é o maior de todos os v's ele é o maior

$\|v\|_1 \leq \sqrt{m}\, \|m\|_3$

$\|x y^T\| \leq \|x\| \|y\|$

$\|v \cdot w^T\| \leq \|v\| \|w\|$

$|v_1| + |v_2| + \dots + |v_m| \leq \sqrt{|v_1|^2 + \dots + |v_m|^2} \cdot \sqrt{1^2 + \dots + 1^2}$

$|v_1| + |v_2| + \dots + |v_m| \leq \sqrt{|v_1|^2 + \dots + |v_m|^2} \cdot \sqrt{m}$

$\|v\|_1 \leq \sqrt{m} \cdot \|v\|_2$