* RELACION 5 EDIP

1) a) $\sum_{i=1}^{20} P[X=x_i] = 1 \iff K\sum_{i=1}^{20} i = 1 \iff 210K = 1 \iff K = \frac{1}{210}$

$P(x=4) = \frac{4}{210} = \frac{2}{105}$

$P(x<4) = \frac{1}{210} + \frac{2}{210} + \frac{3}{210} \cancel{+ \frac{4}{210}} = \frac{6}{210} = \frac{1}{35}$

$P(3 \leq x \leq 10) = \sum_{i=3}^{10} \frac{i}{210} = \frac{26}{105}$

$P(3 < x \leq 10) = P(3 \leq x \leq 10) - P(x=3) = \frac{7}{30}$

$P(3 < x < 10) = P(3 < x \leq 10) - P(x=10) = \frac{13}{70}$

Función de distribución:

$F_x = \sum_{i=1}^{x} K \cdot i = \sum_{i=1}^{x} \frac{i}{210}$

b) Si $x<4 \longrightarrow$ Gana 20 monedas

Si $x=4 \longrightarrow$ Gana 24 monedas

Si $x>4 \longrightarrow$ Pierde una moneda

$P(x<4) = 1/35 \longrightarrow$ Si $x_i = 20 \longrightarrow x_i P_i = 20/35$

$P(x=4) = 2/105 \longrightarrow$ Si $x_i = 24 \longrightarrow x_i P_i = 48/105$

$P(x>4) = 20/21 \longrightarrow$ Si $x_i = -1 \longrightarrow x_i P_i = -20/21$

$8/105$

$E(x) = \sum_{i=1}^{3} x_i P_i = \frac{8}{105}$.

Como $8/105 > 0 \longrightarrow$ el juego le es favorable

2) ~~X~~ X = nº de bolas B al sacar 2 con una urna con 10 bolas de las que 8 son blancas:

a) $X = \left.\begin{array}{l} P(X=0) = \frac{1}{45} \\ P(X=1) = \frac{16}{45} \\ P(X=2) = \frac{28}{45} \end{array}\right\}$ $F(x) = \begin{cases} \frac{1}{45} & \text{si } X=0 \\ \frac{17}{45} & \text{si } X=1 \\ 1 & \text{si } X \geq 2 \\ 0 & \text{si } X<0 \end{cases}$

$$\left.\begin{array}{l} P(X=0) = \dfrac{\binom{2}{2}\cdot\binom{8}{0}}{\binom{10}{2}} \\ P(X=1) = \dfrac{\binom{2}{1}\binom{8}{1}}{\binom{10}{2}} \\ P(X=2) = \dfrac{\binom{2}{0}\binom{8}{2}}{\binom{10}{2}} \end{array}\right\} \Longrightarrow P(X_{\underset{n}{\overset{\shortparallel}{2}}} = x_i) = \frac{\binom{2}{n-x_i}\binom{8}{x_i}}{\binom{10}{n}}$$

$\forall A = \{x_i : i=1,2,3\} = \{0,1,2\}$

La media de X será la suma de cada valor posible de X por la probabilidad del mismo.

$$E[X] = \sum_{i=1}^{3} x_i \, P[X=x_i] = 1{,}6$$, luego aproximadamente 2 bolas blancas se sacan

Me será aquella valor de X cuya probabilidad acumulada sea la misma por encima que por debajo, luego como ninguno ~~tiene de~~ valor de $F(x) = \frac{45}{90}$ se toma el inmediatamente superior es decir $Me = 2$

La moda será el valor más probable, luego $X=2$

c) Si consideramos intervalo intercuartílico como $[Q_1, Q_3]$, donde Q_1 y Q_3 son los valores de X tal que $F(x_i) = \frac{1}{4}$, $F(x_i) = \frac{3}{4}$ respectivamente tenemos que $\left.\begin{array}{l} Q_3 = 2 \\ Q = 1 \end{array}\right\} \Longrightarrow [1,2]$ es el intervalo intercuartílico

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EDSP (ejercicios)

Relación tema 5

3 $\underline{X}$ = nº de lanzamientos de una moneda hasta salir cara

$P(\underline{X} = x) = 2^{-x}$; $x = 1, 2, \ldots$

a) ¿$\sum_i p_i = 1$?

$x_1 = \frac{1}{2}$

$x_2 = \frac{1}{4}$

$$\sum_x P(\underline{X} = x) = \sum_x 2^{-x} = \sum_x \frac{1}{2^x} = \frac{1/2}{1-\frac{1}{2}} = \frac{1/2}{1/2} = 1$$ luego la función masa de probabilidad está bien definida.

b) $P(4 < \underline{X} < 10) = \sum_{i=5}^{9} P(\underline{X} = i) = \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8} + \frac{1}{2^9} = \frac{31}{512} = 0.0605$ $\left(\frac{1}{2^4} + \ldots + \frac{1}{2^{10}} = \frac{127}{1024} = 0.124023\right)$

$Q_1 = 1$ $Q_2 = 1.5$ $Q_3 = 2.5$

c) Moda = 1

$0.25 = 2^{-1} \Rightarrow Q_1 = 1$

$0.75 = 2^{-1} + 2^{-2} \Rightarrow [2, 3] \Rightarrow Q_3 = \frac{2+3}{2} = 2.5$

d) $$M_x(t) = E[e^{t\underline{X}}] = \sum_{i=1}^{\infty} e^{tx_i} P(\underline{X} = x_i) = \sum_{i=1}^{\infty} \frac{e^{tx_i}}{2^{x_i}} = \sum_{i=1}^{\infty} \frac{e^{ti}}{2^i}$$

$$= \frac{e^t/2}{1 - \frac{e^t}{2}} = \frac{e^t/2}{\frac{2-e^t}{2}} = \frac{e^t}{2-e^t}$$

$$m_1 = \left.\frac{d M_x(t)}{dt}\right|_{t=0} = \left.\frac{e^t(2-e^t) + e^t \cdot e^t}{(2-e^t)^2}\right|_{t=0} = \left.\frac{e^t(2-e^t) + e^{2t}}{(2-e^t)^2}\right|_{t=0} = \left.\frac{2e^t - e^{2t} + e^{2t}}{(2-e^t)^2}\right|_{t=0}$$

$$= \left.\frac{2e^t}{(2-e^t)^2}\right|_{t=0} = 2 \text{ lanzamientos}$$

$$m_2 = \frac{d'' M_x(t)}{dt} = \left.\frac{2e^t(2-e^t)^2 + 2(2-e^t) e^t \cdot 2e^t}{(2-e^t)^4}\right|_{t=0} = \left.\frac{2e^t(2-e^t)^2 + 4e^{2t}(2-e^t)}{(2-e^t)^4}\right|_{t=0}$$

$$= \frac{2e^t(2-e^t)\left((2-e^t)+2e^t\right)}{(2-e^t)^4}\Bigg|_{t=0} = \frac{2e^t\left((2-e^t)+2e^t\right)}{(2-e^t)^3}\Bigg|_{t=0} = \frac{2\cdot(1+2)}{1} = 6$$

$Var(X) = m_2 - m_1^2 = 6 - 2^2 = 2$ lanzamientos^2 .

$\sigma(X) = \sqrt{2}$ lanzamientos

$\sigma_x = \sqrt{2}$

4

$$f(x) = \begin{cases} k_1(x+1) & 0 \le x \le 4 \\ k_2 x^2 & 4 < x \le 6 \end{cases}$$

$P(0 \le x \le 4) = 2/3$

$$P(0 \le x \le 4) = \int_0^4 K_1(x+1)\,dx = K_1\left[\frac{x^2}{2}+x\right]_0^4 = K_1 \cdot 12 = \frac{2}{3} \rightarrow K_1 = \frac{1}{18}$$

$$P(4 < x \le 6) = \int_4^6 K_2 x^2\,dx = K_2\left[\frac{x^3}{3}\right]_4^6 = K_2 \cdot \frac{152}{3} = \frac{1}{3} \rightarrow K_2 = \frac{1}{152}$$

$$F_X(x) = \begin{cases} 0 & \text{si } x < 0 \\ \left[\frac{x^2}{36}+\frac{x}{18}\right]_0^x & 0 \le x \le 4 \\ 2/3 + \frac{1}{152}\cdot\left[\frac{x^3}{3}\right]_4^x & 4 < x \le 6 \\ 1 & x > 6 \end{cases}$$

5) X = "dimensión en centímetros de los tornillos que salen de cierta fábrica" 1 si $x>6$

$$f(x) = \frac{K}{x^2}, \quad 1 \leq x \leq 10$$

a) valor de K, y obtener función de distribución

$$\int_1^{10} \frac{K}{x^2}\,dx = 1 \Rightarrow K \int_1^{10} \frac{1}{x^2}\,dx = K \cdot \left(-\frac{1}{x}\right)_1^{10} =$$

$$= K \cdot \left(-\frac{1}{10} - \left(-\frac{1}{1}\right)\right) = K \cdot \left(1 - \frac{1}{10}\right) = \frac{K \cdot 9}{10} = 1 \Rightarrow K = \frac{10}{9}$$

$$F(x) = \begin{cases} \int_1^{x} \frac{10}{9 \cdot x^2} & \text{si } 1 \leq x \leq 10 \qquad 1 > 10 \\ 0 & \cancel{\text{otro caso}}\ x < 1 \end{cases}$$

b)

$$\int_2^{5} \frac{10}{9x^2}\,dx = \frac{10}{9} \int_2^{5} \frac{1}{x^2}\,dx = \frac{10}{9} \cdot \left(-\frac{1}{x}\right)_2^{5} =$$

$$= \frac{10}{9} \cdot \left(-\frac{1}{5} - \left(-\frac{1}{2}\right)\right) = \frac{10}{9}\left(\frac{1}{2} - \frac{1}{5}\right) = \frac{\cancel{10}}{9}\left(\frac{3}{\cancel{10}}\right) = \frac{3}{9} = \frac{1}{3}$$

c) ¿Me? ¿P_{95}?

$$\frac{10}{9}\int_1^{x} f(t)\,dt = \frac{10}{9} \cdot \left(-\frac{1}{t}\right)_1^{x} = 0{,}5 \Rightarrow \left(-\frac{1}{\cancel{t}}\right)^{x} = 0{,}45 \Rightarrow \left(-\frac{1}{x} + 1\right) = 0{,}45$$

$$\Rightarrow \frac{1}{x} = 0{,}55 \Rightarrow x = 1{,}8182 \text{ cm}$$

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$$\frac{10}{9}\int_1^x \frac{1}{x^2}\,dx = \frac{10}{9}\left(-\frac{1}{x}\right)_1^x = 0{,}95 \Rightarrow \left(-\frac{1}{x}\right)_1^x = 0{,}855$$

$$\Rightarrow \left(-\frac{1}{x}+1\right) = 0{,}855 \Rightarrow \frac{1}{x} = 0{,}145 \Rightarrow x = 6{,}89655 \text{ aproximadamente}$$

d) Y = "dimensión tornillos producidos en otra fábrica, con igual media y desviación típica"

$$E[X] = \int_1^{10} x\,\frac{1}{x^2}\cdot\frac{10}{9}\,dx = \frac{10}{9}\left[\ln(x)\right]_1^{10} = \frac{10}{9}\ln(10) =$$

$$= 2{,}5584 \text{ es la media de } X$$

$$E[X^2] = \int_1^{10} x^2 f(x) = \frac{10}{9}\int_1^{10} 1\,dx = \frac{10}{9}(x)_1^{10} = \frac{90}{9} = 10$$

$$Var(X) = 10 - \left(\frac{10}{9}\ln(10)\right)^2 = 10 - \frac{100}{90}\cdot\ln^2(10) = 4{,}109$$

$$\sigma_X = \sqrt{Var(X)} = 2{,}027067$$

$$E[X] = E[Y],\quad Var(X) = Var(Y) \Rightarrow E[X^2] = E[Y^2]$$

$$\Rightarrow \int_a^b f(y)\,dy \geq 0{,}99$$

$$1 - \frac{4{,}109}{k^2} = 0{,}99 \Rightarrow k = 20{,}27067$$

$$P(|X - E[X]| < 20{,}27) \geq 1 - \frac{Var[X]}{20{,}27067^2}$$

$$|X - E[X]| < 20{,}27067 \Rightarrow X < 22{,}82907$$

6)

$$f(x) = \begin{cases} \frac{2x-1}{10} & 1<x\leq 2 \\ 0{,}4 & 4<x\leq 6 \end{cases}$$

a) $P(1{,}5<X\leq 2) = \int_{1,5}^{2} \frac{2x-1}{10}\,dx = \frac{1}{10}\cdot\left(\int_{1,5}^{2} 2x\,dx - \int_{1,5}^{2} 1\,dx\right) =$

$= \frac{1}{10}\left([x^2]_{1,5}^{2} - (x]_{1,5}^{2}\right) = \frac{1}{10}(1{,}75-0{,}5) = 0{,}125$

$P(2{,}5< X\leq 3{,}5) = 0$

$P(4{,}5\leq X < 5{,}5) = \int_{4,5}^{5,5} 0{,}4\,dx = (0{,}4x)_{4,5}^{5,5} = 0{,}4$

$P(1{,}2< X \leq 5{,}2) = \int_{1,2}^{2} \frac{2x-1}{10} + \int_{4}^{5,2} 0{,}4\,dx =$

$= \frac{1}{10}\left(\int_{1,2}^{2} 2x\,dx - \int_{1,2}^{2} 1\,dx\right) + [0{,}4x]_{4}^{5,2} =$

$= \frac{1}{10}\left([x^2]_{1,2}^{2} - [x]_{1,2}^{2}\right) + [0{,}4x]_{4}^{5,2} = \frac{1}{10}(2{,}56-0{,}8) + 0{,}48 = 0{,}656$

b) $E[X] = \int_{1}^{2} x\cdot\frac{2x-1}{10}\,dx + \int_{4}^{6} 0{,}4\,x\,dx$

$= \frac{1}{10}\left(\int_{1}^{2} 2x^2 - \int_{1}^{2} x\,dx\right) + 0{,}4\left(\frac{x^2}{2}\right)_{4}^{6} =$

$= \frac{1}{10}\left(\left(\frac{2x^3}{3}\right)_{1}^{2} - \underbrace{\left(\frac{x^2}{2}\right)_{1}^{2}}_{1,5}\right) + 0{,}4\left(\frac{x^2}{2}\right)_{4}^{6} = 0{,}31667 + 4 = 4{,}31667$

c)

$$M_X[t] = E[e^{tx}] =$$

$$= \int_1^2 e^{tx} \cdot \frac{2x-1}{10}\,dx + \int_4^6 e^{tx}\, 0{,}4\,dx =$$

$$= \frac{1}{10}\left(\int_1^2 e^{tx}\, 2x - \int_1^2 e^{tx}\,dx\right) + 0{,}4 \cdot \left(\frac{e^{tx}}{t}\right)_4^6 =$$

$u = 2x \quad du = 2\,dx$

$dv = e^{tx} \quad v = \frac{e^{tx}}{t}$

$$= \frac{1}{10}\cdot\left(\left(\frac{2x\cdot e^{tx}}{t} - \frac{2e^{tx}}{t^2}\right)_1^2 - \left(\frac{e^{tx}}{t}\right)_1^2\right) + 0{,}4\cdot\left(\frac{e^{tx}}{t}\right)_4^6 =$$

$$= \frac{1}{10}\cdot\left(\left(\frac{4e^{2t}}{t} - \frac{2e^{2t}}{t^2} - \frac{2e^{t}}{t} + \frac{2e^{t}}{t^2} - \frac{e^{2t}}{t} + \frac{e^{t}}{t}\right)\right) + 0{,}4\cdot\left(\frac{e^{6t}}{t} - \frac{e^{4t}}{t}\right) =$$

$$= \frac{1}{10}\cdot\left(\frac{3e^{2t} - e^{t}}{t} - \frac{2e^{2t}}{t^2} + \frac{2e^{t}}{t^2}\right) + \frac{0{,}4\cdot e^{4t}(e^{2t}-1)}{t}$$

7 $f(x) = \frac{3}{4}(2x - x^2)$, $0 \le x \le 2$.

a) $\frac{3}{4}\int_0^t 2x - x^2\,dx = \frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]_0^t = \frac{1}{2} \rightarrow$

$\rightarrow \frac{3}{4}\left(t^2 - \frac{t^3}{3}\right) = \frac{1}{2} \Longleftrightarrow t^2 - \frac{t^3}{3} = \frac{2}{3} \longrightarrow t = 1$

$\left.\begin{array}{l} = 1+\sqrt{3} \\ = 1-\sqrt{3} \end{array}\right\} \notin [0,2]$

Por tanto, la cantidad debe ser igual a 1000 unidades

b) $f(y) = \frac{3}{4}(4y - y^2 - 3)$, $1 \le y \le 3$.

$E(x) = \cancel{\frac{3}{4}\int 4y - y^2 - 3\,dy}\ \frac{3}{4}\int_0^2 2x^2 - x^3\,dx = \frac{3}{4}\left[\frac{2}{3}x^3 - \frac{x^4}{4}\right]_0^2 = 1$

$E(y) = \frac{3}{4}\int_1^3 4y^2 - y^3 - 3y\,dy = \frac{3}{4}\left[\frac{4}{3}y^3 - \frac{y^4}{4} - \frac{3}{2}y^2\right]_1^3 = 2$

$E(x^2) = \frac{3}{4}\int_0^2 2x^3 - x^4\,dx = \cancel{\frac{3}{4}(\ldots)}\ 6/5$

$E(y^2) = \frac{3}{4}\int_1^3 4y^3 - y^4 - 3y^2\,dy = 21/5$

$$\left.\begin{aligned} Var(x) &= E(x^2) - E(x)^2 = \cancel{5}\ 6/5 - 1^2 = 0'2 \\ Var(y) &= E(y^2) - E(y)^2 = 21/5 - 2^2 = 0'2 \end{aligned}\right\} = \text{Medida de dispersión}$$

$\Uparrow$

Suposición correcta

$$CV_x = \frac{\sqrt{0'2}}{1} \qquad CV_y = \frac{\sqrt{0'2}}{2}$$

Como $CV_x \neq CV_y$, si ha afectado a la dispersión de la demanda.

P:

$P(X=-2)=\frac{1}{5}$ $P(X=-1)=\frac{1}{10}$ $P(X=0)=\frac{1}{5}$

$P(X=1)=\frac{2}{5}$ $P(X=2)=\frac{1}{10}$

$Y=X+2$ $P(Y=0)=\frac{1}{5}$ $P(Y=1)=\frac{1}{10}$ $P(Y=2)=\frac{1}{5}$

$P(Y=3)=\frac{2}{5}$ $P(Y=4)=\frac{1}{10}$

$Z=X^2$ $P(Z=4)=\frac{1}{5}$ $P(Z=1)=\frac{1}{10}$ $P(\cancel{X}=0)=\frac{1}{5}$

$P(Z=1)=\frac{2}{5}$ $P(Z=4)=\frac{1}{10}$

$P(Z=4)=\frac{3}{10}$

$P(Z=1)=\frac{1}{2}$

$P(Z=0)=\frac{1}{5}$

$E[X]=-2\cdot\frac{1}{5}+(-1)\cdot\frac{1}{10}+0\cdot\frac{1}{5}+1\cdot\frac{2}{5}+2\cdot\frac{1}{10}=$

$=\frac{1}{10}=0.1$

$E[Y]=0\cdot\frac{1}{5}+1\cdot\frac{1}{10}+2\cdot\frac{1}{5}+3\cdot\frac{2}{5}+4\cdot\frac{1}{10}=$

$=\frac{21}{10}=2.1$

$Var(X)=0.882+0.121+0.002+0.324+0.361=1.69$ $\sigma_x=\sqrt{1.69}=1.3$

$Var(Y)\overset{\not=}{=}0.882+0.121+0.002+0.324+0.361=1.69$ $\sigma_y=\sqrt{1.69}=1.3$

$CV.(X)=\frac{1.3}{0.1}=13$ $CV.(Y)=\frac{1.3}{2.1}=0.61905$

$C.V.(X)>C.V.(Y)$

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9. $Y = 2X+3$ $\qquad f_X(x) = \frac{1}{4} \quad -2 < x < 2$

$Z = |X|$ $\qquad f_Y(x) = ?$

$f_Z(x) = ?$

$\cancel{h^{-1}(x) = 2x}$ $\quad h(x) = 2x+3$

$x = \frac{h(x)-3}{2} \qquad h^{-1}(x) = \frac{x-3}{2}$

$f_Y(x) = f(\cancel{2x}\ h^{-1}(x)) \left| \frac{d\, h^{-1}(x)}{dx} \right| = \frac{1}{4} \cdot \frac{1}{2} = \boxed{\frac{1}{8}}$

$g(x) = |x| = \begin{cases} -x & si \quad x < 0 \\ x & si \quad x \geq 0 \end{cases}$

$f_Z(x) = f(g^{-1}(x)) \left| \frac{d\, g^{-1}(x)}{dx} \right| = \boxed{\frac{1}{4}} \cdot \pm 1 \left\langle \begin{matrix} \frac{1}{4} \\ -\frac{1}{4} \end{matrix} \right.$

$g^{-1}(x) = \pm x$

$(y \cancel{/} \pm x)$

10:

$$f(x) = \frac{e^{-|x|}}{2}$$

$$\cancel{\frac{1}{2}\int_{-\infty}^{+\infty} e^{-|x|}\,dx = -\frac{1}{2}e^{-|x|}\Big]_{-\infty}^{+\infty} =}$$

$$|x| = \begin{cases} x & si\ x \geq 0 \\ -x & si\ x \leq 0 \end{cases}$$

$$\frac{1}{2}\int_{-\infty}^{0} e^{x}\,dx = \frac{1}{2}e^{x}\Big]_{-\infty}^{0} = \frac{1}{2} - \frac{e^{-\infty}}{2} = \frac{1}{2}$$

$$\frac{1}{2}\int_{0}^{+\infty} e^{-x}\,dx = -\frac{1}{2}e^{-x}\Big]_{0}^{+\infty} = -\frac{1}{2\cdot e^{\infty}} + \frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2}\int_{-\infty}^{x} e^{t}\,dt = \frac{1}{2}e^{t}\Big]_{-\infty}^{x} = \frac{e^{x}}{2} - \frac{e^{-\infty}}{2} = \frac{e^{x}}{2}$$

$$\frac{1}{2}\int_{0}^{x} e^{-t}\,dt = -\frac{1}{2}e^{-t}\Big]_{0}^{x} = -\frac{1}{2e^{x}} + \frac{1}{2}$$

$$F(x) = \begin{cases} \dfrac{e^{x}}{2} & si\ x \leq 0 \\[2ex] 1 - \dfrac{1}{2e^{x}} & si\ x > 0 \end{cases}$$

$$\cancel{a)\ P(|x| \leq 2) = 1 - \frac{1}{2e^{2}} = 0.93233\ \ \frac{1}{2} - \frac{1}{2e^{2}} = 0.43233}$$

$$\cancel{b)\ P(|x| < 2}$$

a) $P(|X| \le 2) = \frac{1}{2} - \frac{e^{-2}}{2} + \frac{1}{2} - \frac{1}{2e^2} = 0.86466$

b) $P(|X| \le 2 \cup X \ge 0) = 0.86466 + \frac{1}{2} - \left(\frac{1}{2} - \frac{1}{2e^2}\right) = 0.932327$

c) $P(|X| \le 2 \cap X \le -1) = \frac{e^{-1}}{2} - \frac{e^{-2}}{2} = 0.11627$

d)

$x^3 - x^2 - x - 2 \le 0$

	1	-1	-1	-2
2		2	2	2
	1	1	1	$\boxed{0}$

$x = \frac{1 \pm \sqrt{1-4}}{2} \Rightarrow x \notin R$

$(x-2)(x^2+x+1) \le 0$, >0

$P(X \le 2) = 1 - \frac{1}{2e^2} = 0.932332$

e) $P(X \in R \setminus Q) = 1$

11:

$f(x)=1 \quad 0 \le x \le 1$

a) $Y = \dfrac{X}{1+X}$

$Y + XY = X \Rightarrow Y = X - XY \Rightarrow Y = X(1-Y) \Rightarrow X = \dfrac{Y}{1-Y}$

$h^{-1}(y) = \dfrac{y}{1-y}$

$(h^{-1})'(Y) = \dfrac{1-y-(y\cdot(-1))}{(1-y)^2} = \dfrac{1}{(1-y)^2}$

$g(y) = 1\cdot\dfrac{1}{(1-y)^2} = \dfrac{1}{(1-y)^2} \qquad 0 \le x \le \dfrac{1}{2}$

$\displaystyle\int_0^x \frac{1}{(1-y)^2}\,dy = \int_0^x (1-y)^{-2}\,dy = \left.\frac{1}{1-y}\right]_0^x = \frac{1}{1-x} - 1$

$F(X) = \begin{cases} \dfrac{1}{1-y} - 1 \text{ si } 0 \le y \le \dfrac{1}{2} \end{cases}$

b) $Z = -1 \quad x < 3/4$

$\displaystyle\int_0^{3/4} dx = x\Big]_0^{3/4} = 3/4 = P(Z=-1)$

$\displaystyle\int_{3/4}^{3/4} dx = x\Big]_{3/4}^{3/4} = 0 = P(Z=0)$

$\displaystyle\int_{3/4}^{1} dx = x\Big]_{3/4}^{1} = 1/4 = P(Z=1)$

12)

X v.a simétrica respecto al punto 2 con $CV=1$

luego $E[X]=2$ $\sqrt{Var(X)}=\sigma_x=2 \Rightarrow CV=\frac{\sigma_x}{|\bar{X}|}=1$

$Var(X)=4$

- $P(-8<X<12) = P(-8<X\leq 2)+P(2<X<12)$
- $P(-6<X<10) = P(-6<X<2)+P(2\leq X<10)$

$$P(|X-E[X]|<K)\geq 1-\frac{Var[X]}{K^2}, \forall K>0$$

$$\left.\begin{array}{l} P(|X-E[X]|<10)\geq 1-\frac{4}{10^2}=0,96 \\ P(|X-E[X]|\geq 10)\leq \frac{4}{10^2}=0,04 \end{array}\right\}$$

$$0,04<P(-8<X<12)<0,96$$

$$\left.\begin{array}{l} P(|X-E[X]|<8)\geq 1-\frac{4}{64}=0,9375 \\ P(|X-E[X]|\geq 8)\leq \frac{4}{64}=0,0625 \end{array}\right\} 0,0625<P(-6<X<10)\leq 0,9375$$