RELACIÓN II - EDIP

Grupo B5

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1)

a)

X \ Y	1	2	3	4	5	6
1	0	1	0	0	0	3
2	1	0	1	0	1	0
3	1	0	0	2	1	1
4	2	3	0	0	0	1
5	2	1	1	0	0	0
6	0	1	0	0	1	0

b) $\bar{X} = \sum_{i=1}^{6} f_{i\cdot} \cdot x_i = 3{,}375$ es la media aritmética de la varia

X. Como se trata de dados y los números deben ser enteros hablaremos de que $\bar{X}$ es aproximadamente 3, usando 3.375 para los calculos

$\bar{y} = \sum_{j=1}^{6} f_{\cdot j} \cdot y_j = 3{,}208\overline{3}$ es la media aritmética de la variable

Y, Nuevamente diremos que $\bar{y}$ es aproximadamente 3, usando para los calcul

$3{,}208\overline{3}$

Para ver la homogeneidad recurriremos a el C.V, para lo que primero calcularemos la desviación típica de cada variable.

$$\sigma_x = \sqrt{\sum_{i=1}^{6} f_{i\cdot} (x_i - \bar{x})^2} = 1{,}5224$$

$$\sigma_y = \sqrt{\sum_{j=1}^{6} f_{\cdot j} \cdot (y_j - \bar{y})^2} = 1{,}8479$$

$\Longrightarrow$

$$CV(X) = \frac{\sigma_x}{|\bar{x}|} = 0{,}4510\,1$$

$$CV(Y) = \frac{\sigma_y}{|\bar{y}|} = 0{,}576$$

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X	n_{i2}
1	1
2	0
3	0
4	3
5	1
6	1

X	n_{i5}
1	0
2	1
3	1
4	0
5	0
6	1

Serían las distribuciones ~~con~~ de X condicionadas a $Y_j = 2$ y $Y_j = 5$. Uniendo ambas tablas llegamos a:

X	n_{ij}
1	1
2	1
3	1
4	3
5	1
6	2
	$n_{\cdot j}$ $j=2,5$

Con la tabla obtenida calcularemos la Mediana para una subpoblación n_j donde $j=2,5$, es decir

$n_j = n_{\cdot 2} + n_{\cdot 5} = 9$

$N_{ij} = \frac{n_j}{2} = 4,5$ ningún X_i tiene N_i igual a 4,5 en la tabla anterior, luego al tratarse de variable discreta tomaremos que $Me = X_i$ tal que $N_i > \frac{n_j}{2}$, es decir $Me = 4$

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Luego las puntuaciones del dado X son más homogéneas.

~~c) Para ver que resultado es más frecuente cuando $X_i = 3$ haremos la distribución marginal de Y, tal que $\{y_j, n_{\cdot j}\}_{j=1,\dots,6}$ para $i=3$~~

Y	$n_{\cdot j}$
1	1
2	0
3	0
4	2
5	1
6	1

c) Para ver que resultado es más frecuente cuando $X_i = 3$ haremos la distribución condicionada de Y a la modalidad X_3.

Y	n_{3j}
1	1
2	0
3	0
4	2
5	1
6	1

Claramente la modalidad de Y cuyo frecuencia absoluta n_{3j} es la mayor es Y_4 con $n_{34} = 2$

d) Nuevamente haremos la distribución de X condicionada a $y_j = 2$ y la distribución de X condicionada a $y_j = 5$ dando lugar a las siguientes tablas de frecuencia.

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2:

X\Y	160	162	164	166	168	170	$n_{i\cdot}$	$n_{i\cdot} \cdot x_i$	$n_{i\cdot}(x_i - \bar{x})^2$
48	3	2	2	1	0	0	8	384	304,694
51	2	3	4	2	2	1	14	714	140,807
54	1	3	6	8	5	1	24	1296	0,705
57	0	0	1	2	8	3	14	798	24,003
60	0	0	0	2	4	4	10	600	339,225
$n_{\cdot j}$	6	8	13	15	19	9	70	3792	809,927
$n_{\cdot j} \cdot y_j$	960	1296	2132	2490	3192	1530	11600		
$n_{\cdot j}(y_j - \bar{y})^2$	195,912	110,362	38,2	1,225	99,273	165,313	670,2847		

a)

$\bar{x} = \frac{3792}{70} = 54,1714$

$\bar{y} = \frac{11600}{70} = 165,7142$

$\sigma_x^2 = \frac{809,927}{70} = 11,5704 \quad \sigma_x = \sqrt{11,5704} = 3,4015 \Rightarrow CV(x) = \frac{\sigma_x}{|\bar{x}|} = 0,0628$

$\sigma_y^2 = \frac{670,2847}{70} = 8,7184 \quad \sigma_y = \sqrt{8,7184} = 2,953 \Rightarrow CV(y) = \frac{\sigma_y}{|\bar{y}|} = 0,01782$

Es más representativo la altura.

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b)

x\y	166	168	170	$n_{i.}$
48	1	0	0	1
51	2	2	1	5
54	8	5	1	14
				20

$\% = \frac{20}{70} \cdot 100 = \frac{2}{7} \cdot 100 = 28{,}5714\ \%$

c)

x\y	166	168	170	$n_{i.}$
54	8	5	1	14
57	2	8	3	13
60	2	4	4	10
				37

n para más de 165 es 43

$\% = \frac{37}{43} \cdot 100 = 86{,}047\ \%$

d)

Es bimodal $M_{o_1} = 166$ y $M_{o_2} = 168$ ya que las frecuencias absolutas coinciden. $n_{34} = n_{45}$

e)

$\bar{X}_{164} = \frac{48 \cdot 2 + 51 \cdot 4 + 54 \cdot 6 + 57 \cdot 1}{13} = 52{,}3846$ $\quad \sigma_x^2 = 6{,}3905$ $\quad CV(x) = \frac{\sigma_x}{|\bar{x}|} = 0{,}048$

$\bar{X}_{168} = \frac{51 \cdot 2 + 54 \cdot 5 + 57 \cdot 8 + 60 \cdot 4}{19} = 56{,}21$ $\quad \sigma_x^2 = 7{,}4294$ $\quad CV(x) = \frac{\sigma_x}{|\bar{x}|} = 0{,}048$

Es más representativa la de los individuos que miden 164.

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3)

X\Y	1	2	3	4
1	7	0	0	0
2	10	2	0	0
3	11	5	1	0
4	10	6	6	0
5	8	6	4	2
6	1	2	3	1
7	1	0	0	1
8	0	0	1	1

$n = 89$

Para el cálculo de la recta de regresión de Y sobre X usaremos,

$$y = \underbrace{\frac{\sigma_{xy}}{\sigma_x^2}}_{b} x + \underbrace{\bar{y} - \frac{\sigma_{xy}}{\sigma_x^2}\bar{x}}_{a} \qquad (y = bx + a)$$

$$\cancel{b = \frac{m_{11} - m_{10} \cdot m_{01}}{m_0}} \qquad b = \frac{m_{11} - m_{01} m_{10}}{m_{20} - m_{10}^2}$$

$$a = m_{01} - m_{10}\left(\frac{m_{11} - m_{01} \cdot m_{10}}{m_{20} - m_{10}^2}\right)$$

Luego calcularemos la función, la para la que calcularemos a y b

$$y = \frac{\sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij} x_i y_j - \left(\sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij} y_j\right)\cdot\left(\sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij} x_i\right)}{\sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij} x_i^2 - \left(\sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij} x_i\right)^2} \cdot x + \bar{y} - \frac{\sigma_{xy}}{\sigma_x^2}\bar{x}$$

Recordemos que $m_{10} = \bar{x}$ y $m_{01} = \bar{y}$

Luego los calcularemos.

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$$\bar{X} = \sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij} x_i = \frac{1}{n}\cdot(342) = 3,8427$$

$$\bar{y} = \sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij} y_j = \frac{1}{n}\cdot(155) = 1,7416$$

b) $b = \dfrac{m_{11} - \bar{y}\cdot\bar{X}}{m_{20} - \bar{X}^2}$ $\qquad a = \underset{\bar{y}}{m_{01}} - \underset{\bar{X}}{m_{10}} \cdot \underbrace{\left(\dfrac{m_{11} - \bar{y}\cdot\bar{X}}{m_{20} - \bar{X}^2}\right)}_{b}$

$$m_{11} = \sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij}\cdot x_i \cdot y_j = \frac{1}{n}(666) = 7,4831$$

$$m_{20} = \sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij}\, x_i^2 = 17,2809$$

$$b = \frac{7,4831 - 3,8427\cdot 1,7416}{17,2809 - (3,8427)^2} = \frac{0,7906}{2,5145} = 0,3144$$

$$a = 1,7416 - 3,8427\cdot(0,3144) = 0,5334$$

La curva de regresión será

$y = 0,3144\cdot X + 0,5334$ es la recta de regresión de Y sobre X

b) Calcularemos la varianza residual

$$\sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij}\left[y_j - f(x_i)\right]^2 = 0,6171 = \sigma_{ry}^2$$

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$$\cancel{\sigma_{ey}^2 = \sum_{i=1}^{8}\sum_{j=1}^{4} f_{ij}(\hat{y}_j - \bar{y})^2}$$

Conocida la varianza residual calcularemos σ^2_y y haremos la correlación

$$\sigma_y^2 = \sum_{i=1}^{p}\sum_{j=1}^{4} f_{ij}(y_j - \bar{y})^2 = 0{,}8658$$

Ahora calcularemos la correlación y si sale próxima a ~~0~~ 1 será una buena recta de regresión.

$$\eta^2_{Y/X} = 1 \Longleftrightarrow \frac{\sigma_{ry}^2}{\sigma_y^2} = 0 \Longrightarrow 1 - \frac{\sigma_{ry}}{\sigma_y^2} = \eta_{XY} = 1 - 0{,}7127 = 0{,}287$$

$$\frac{\sigma_{ry}^2}{\sigma_y} = \frac{0{,}6171}{0{,}8658} = 0{,}7127$$ luego está próximo a uno y por tanto

el coeficiente de ~~rela~~ correlación será próximo a cero, luego no es apropiado usar una relación lineal

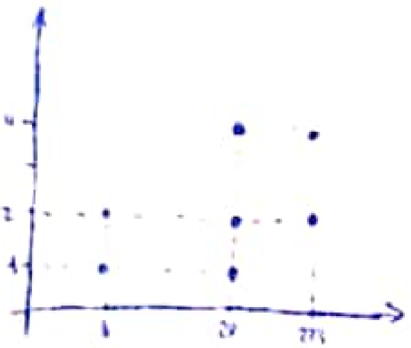
EDSP

Relación 2

Y = tensión de vapor de agua en ml de Hg
X = temperatura en °C

4

X | Y	(0'5, 1'5]	(1'5, 2'5]	(2'5, 5'5]	c_i	n_i
(1, 15]	4	2	0	8	6
(15, 25]	1	4	2	20	7
(25, 30]	0	3	5	27'5	8
c_j	1	2	4		21
n_j	5	9	7	21	



Hallamos la recta de regresión de Y sobre X, ya que nos dicen estudiar el comportamiento de la tensión de vapor en términos de la temperatura.

Hallamos las medias marginales de cada variable.

$$\bar{x} = \frac{6 \cdot 8 + 7 \cdot 20 + 8 \cdot 27'5}{21} = 19'4286 \text{ °C}$$

$$\bar{y} = \frac{5 \cdot 1 + 9 \cdot 2 + 7 \cdot 4}{21} = 2'4286 \text{ ml Hg}$$

Hallamos la desviación típica de X

$$\sigma_x = \sqrt{\frac{6(8-19'4286)^2 + 7(20-19'4286)^2 + 8(27'5-19'4286)^2}{21}} = 7'8895 \text{ °C}$$

$$\sigma_{xy} = \mu_{11} = \sum_{i=1}^{k}\sum_{j=1}^{p} f_{ij}(x_i-\bar{x})(y_j-\bar{y})$$

$$= \frac{4(8-19'4286)(1-2'4286) + 2(8-19'4286)(2-2'4286) + (20-19'4286)(1-2'4286)}{21}$$

$$+ \frac{4(20-19'4286)(2-2'4286) + 2(20-19'4286)(4-2'4286) + 3(27'5-19'4286)(2-2'4286)}{21}$$

$$+ \frac{5(27'5-19'4286)(4-2'4286)}{21} = 6'1020$$

$$\sigma_y^2 = \frac{5(1-2'4286)^2 + 9(2-2'4286)^2 + 7(4-2'4286)^2}{21}$$

$$= 1'3878$$

$$\frac{\sigma_{xy}}{\sigma_x^2} = \frac{6'1020}{7'8875^2} = 0'098$$

$$r^2 = \frac{\sigma_{xy}^2}{\sigma_x^2\sigma_y^2} = \frac{6'1020^2}{1'3878 \cdot 7'8875} = 0'4310$$

$$y = 0'098x + 2'4286 - 0'098 \cdot 19'4286$$

$$= \boxed{0'098x + 0'5246}$$

5.

$$f^j = f_{i,j} = \frac{n_{ij}}{n_{\cdot j}}$$

X|Y	1	2	3	4	5	$n_{i\cdot}$	f^1	f^2	f^3	f^4	f^5
10	2	4	6	10	8	30	2/10	4/20	6/30	10/50	8/40
20	1	2	3	5	4	15	1/10	2/20	3/30	5/50	4/40
30	3	6	9	15	12	45	3/10	6/20	9/30	15/50	12/40
40	4	8	12	20	16	60	4/10	8/20	12/30	20/50	16/40
$n_{\cdot j}$	10	20	30	50	40	150					

X|Y	1	2	3	$n_{i\cdot}$	f^1	f^2	f^3
-1	0	1	0	1	0	1/2	0
0	1	0	1	2	1	0	1
1	0	1	0	1	0	1/2	0
$n_{\cdot j}$	1	2	1	4			

$$\frac{n_{i1}}{n_{\cdot 1}} = \frac{n_{i2}}{n_{\cdot 2}} = \ldots$$

$$\frac{2}{10} = \frac{4}{20} = \frac{6}{30} = \frac{10}{50} = \frac{8}{40} = \frac{30}{150} \qquad \frac{n_{i\cdot}}{n} = \frac{30}{150}$$

$$\frac{1}{10} = \frac{2}{20} = \frac{3}{30} = \frac{5}{50} = \frac{4}{40} = \frac{15}{150}$$

$$\frac{3}{10} = \frac{6}{20} = \frac{9}{30} = \frac{15}{50} = \frac{12}{40} = \frac{45}{150}$$

$$\frac{4}{10} = \frac{8}{20} = \frac{12}{30} = \frac{20}{50} = \frac{16}{40} = \frac{60}{150}$$

Luego X es independiente estadísticamente de Y

$0 \neq \frac{1}{2} \neq \frac{1}{4}$

$1 \neq 0 \neq \frac{2}{4}$

$0 \neq \frac{1}{2} \neq \frac{1}{4}$

Luego X depende de Y estadísticamente

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5)

X\Y	1	2	3	4	5	
10	2	4	6	10	8	$30 = n_{1\cdot}$
20	1	2	3	5	4	$15 = n_{2\cdot}$
30	3	6	9	15	12	$45 = n_{3\cdot}$
40	4	8	12	20	16	$60 = n_{4\cdot}$
	10	20	30	50	40	
	$= n_{\cdot 1}$	$= n_{\cdot 2}$	$= n_{\cdot 3}$	$= n_{\cdot 4}$	$= n_{\cdot 5}$	

Por la definición de ~~variab~~ independencia estadística ~~g~~ Y ~~X~~ será independiente estadísticamente de X si

$$\frac{n_{1j}}{n_{1\cdot}} = \frac{n_{2j}}{n_{2\cdot}} = \cdots = \frac{n_{ij}}{n_{i\cdot}} = \cdots = \frac{n_{kj}}{n_{k\cdot}} \quad \forall j = 1, 2, \ldots, p.$$

lo cual se verifica

$$\frac{2}{30} = \frac{1}{15} = \frac{3}{45} = \frac{4}{60} \quad \text{para } j = 1$$

$$\frac{4}{30} = \frac{2}{15} = \frac{6}{45} = \frac{8}{60} \quad \text{para } j = 2$$

$$\frac{6}{30} = \frac{3}{15} = \frac{9}{45} = \frac{12}{60} \quad \text{para } j = 3$$

$$\frac{10}{30} = \frac{5}{15} = \frac{15}{45} = \frac{20}{60} \quad \text{para } j = 4$$

$$\frac{8}{30} = \frac{4}{15} = \frac{12}{45} = \frac{16}{60} \quad \text{para } j = 5$$

luego Y es independiente de X y por ser la independencia una propiedad recíproca X es independiente de Y

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X \ Y	1	2	3	
-1	0	1	0	1
0	1	0	1	2
1	0	1	0	1
	1	2	1	

Puesto que a y_i le corresponden dos modalidades de X y a x_2 le correspon dos modalidades de Y no hay dependencia funcional del carácter X de Y ni viceversa.

Ahora probaremos si hay independencia

$$\frac{n_{1j}}{n_{1.}} = \frac{n_{2j}}{n_{2.}} = \cdots = \frac{n_{Hj}}{n_{H.}} \quad \forall j = 1, \ldots, K$$

$$\frac{0}{1} \neq \frac{1}{1} \neq \frac{0}{1} \Longrightarrow \text{no hay independencia estadística}$$

Luego hay dependencia pero no funcional.

En el primer caso no tiene sentido hacer la curva de regresión pues los puntos serán todos iguales.

Calcularemos las distribuciones de Y condicionadas a X_i $\forall i = 1, \ldots, 3$

$x_i = -1$

y_j	
1	0
2	1
3	0

$x_i = 0$

y_j	
1	1
2	0
3	1

$x_i = 1$

y_j	
1	0
2	1
3	0

$\bar{Y}_1 = 2$ $\quad \bar{Y}_2 = 2$ $\quad \bar{Y}_3 = 2$

Dibujaremos la curva que pase por

$(-1, 2)$, $(0, 2)$, $(1, 2)$

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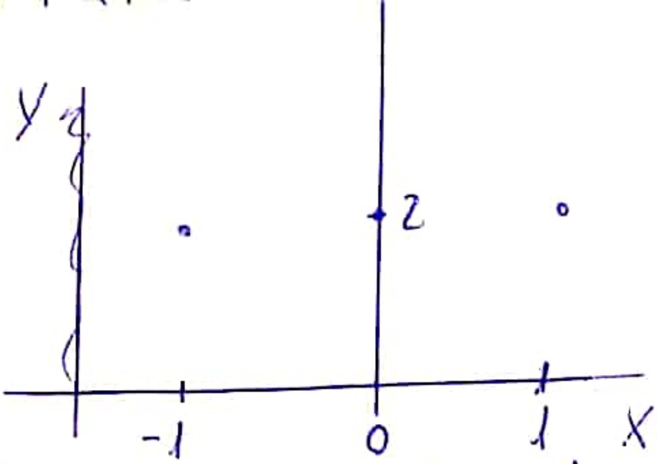

los tres puntos son iguales, luego al final se trata de una recta constante,
lo mismo ocurre con la ~~recta~~ curva de regresión de tipo I de X/Y

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6:

X\Y	1	2	3	4	$n_{i\cdot}$	$n_{i\cdot} x_i$	$n_{i\cdot}(x_i-\bar{x})^2$	$\sum_{i=1}\sum_{j=1} n_{ij}\, x_i\, y_j$	$\sum_{j=1} n_{ij}(y_j-\bar{y}_i)^2$
10	1	3	0	0	4	40	31,36	10	0,75
12	0	1	4	3	8	96	5,12	312	3,5
14	2	0	0	2	4	56	5,76	140	9
16	4	0	0	0	4	64	40,96	64	9
$n_{\cdot j}$	7	4	4	5	20	256	83,2	586	22,25
$n_{\cdot j}\, y_j$	7	8	12	20	47				
$n_{\cdot j}(y_j-\bar{y})^2$	12,7575	0,49	1,69	13,613	28,5505				
$\sum_{i=1}^{k} n_{ij}(x_i-\bar{x}_j)^2$	29,714	3	0	4,8	37,514				

a) $\frac{1}{7} \neq \frac{3}{4} \neq \frac{0}{4}$ Son dependientes pero no funcionalmente

b)

Para X/Y

$\bar{x}_1 = \frac{10+14\cdot 2+16\cdot 4}{7} = 14,571$

$\bar{x}_2 = \frac{3\cdot 10+12}{4} = 10,5$

$\bar{x}_3 = \frac{12\cdot 4}{4} = 12$

$\bar{x}_4 = \frac{3\cdot 12+2\cdot 14}{5} = 12,8$

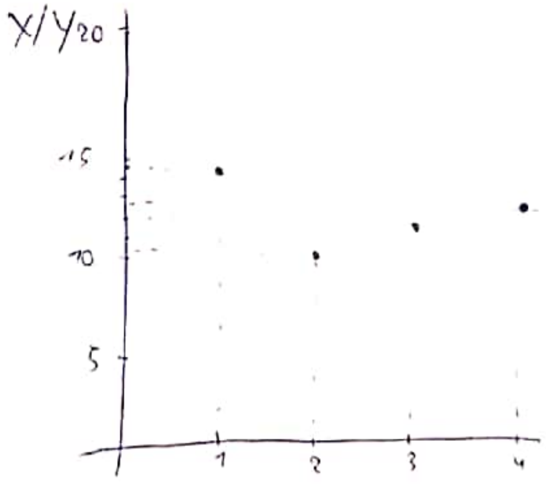
Para Y/X

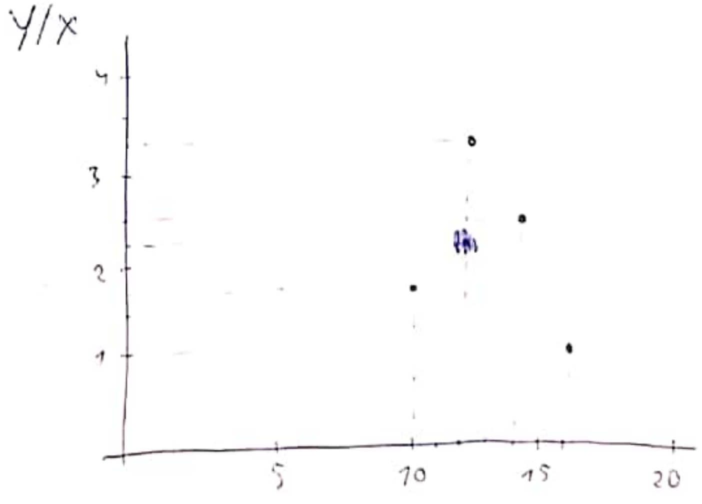
$\bar{y}_1 = \frac{1+3\cdot 2}{4} = \frac{7}{4} = 1,75$

$\bar{y}_2 = \frac{1\cdot 2+4\cdot 3+3\cdot 4}{8} = 3,25$

$\bar{y}_3 = \frac{2\cdot 1+2\cdot 4}{4} = \frac{10}{4} = \frac{5}{2} = 2,5$

$\bar{y}_4 = \frac{4}{4} = 1$





c) $\sigma^2_{ry} = \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{p} n_{ij}(y_{ij} - \bar{y}_i)^2 = \frac{22,25}{20} = 1,1125$

$\sigma^2_{ey} = \sigma^2_y - \sigma^2_{ry} = 1,4275 - 1,1125 = 0,315$

$\eta^2 y/x = \frac{\sigma^2_{ey}}{\sigma^2_y} = \frac{0,315}{1,4275} = 0,2207$

$\sigma^2_{rx} = \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{p} n_{ij}(x_{ij} - \bar{x}_j)^2 = \frac{37,514}{20} = 2,2843$

$\sigma^2_{ex} = \sigma^2_x - \sigma^2_{rx} = 4,16 - 1,8757 = 2,2843$

$\eta^2 x/y = \frac{\sigma^2_{ex}}{\sigma^2_x} = \frac{2,2843}{4,16} = 0,5491$

d)

$m_{01} = \bar{y} = \frac{47}{20} = 2,35$ $\sigma^2_y = \frac{28,5505}{20} = 1,4275$

$m_{10} = \bar{x} = \frac{256}{20} = 12,8$ $\sigma^2_x = \frac{83,2}{20} = 4,16$

$m_{11} = \frac{586}{20} = 29,3$ $\sigma_{xy} = m_{11} - m_{01}m_{10} = -0,78$

$y - 2,35 = \frac{-0,78}{4,16}(x - 12,8)$ $x - 12,8 = \frac{-0,78}{1,4275}(y - 2,35)$

$\boxed{y = -0,1875x + 4,75}$ $\boxed{x = -0,5464y + 14,084}$

$\eta^2 x/y = a \cdot a' = 0,1024 = r^2$

$r = 0,32$ no están correladas linealmente

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$r_Y = \frac{\ }{\sigma^2_x} = \frac{\ }{4,16}$

7.-

a) <u>Distribución A</u>

Y depende funcionalmente de X.

<u>Distribución B</u>

X depende funcionalmente de Y

Y depende funcionalmente de X

<u>Distribución C</u>

X depende funcionalmente de Y

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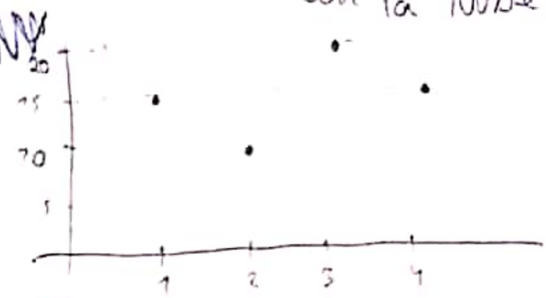
b)

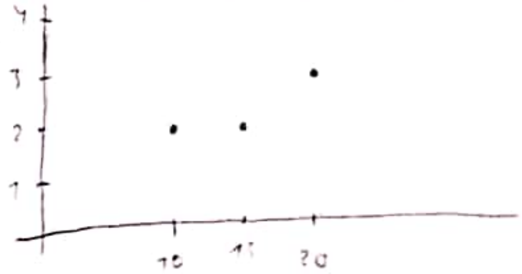
X\Y	10	15	20	$n_{i\cdot}$	$\bar{y}_i$
1	0	2	0	2	15
2	1	0	0	1	10
3	0	0	3	3	20
4	0	1	0	1	15
$n_{\cdot j}$	1	3	3	7	
$\bar{x}_j$	2	2	3		

Y/X

Coincide con la Nube de puntos



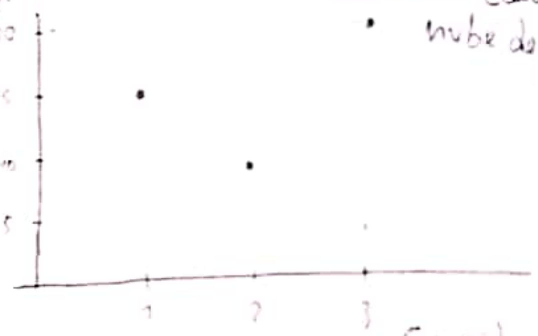
X/Y



X\Y	10	15	20	$n_{i\cdot}$	$\bar{y}_i$
1	0	2	0	2	15
2	1	0	0	1	10
3	0	0	3	3	20
$n_{\cdot j}$	1	2	3	6	
$\bar{x}_j$	2	1	3		

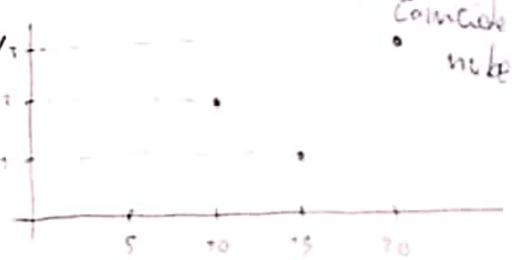
Y/x

Coincide con la nube de puntos



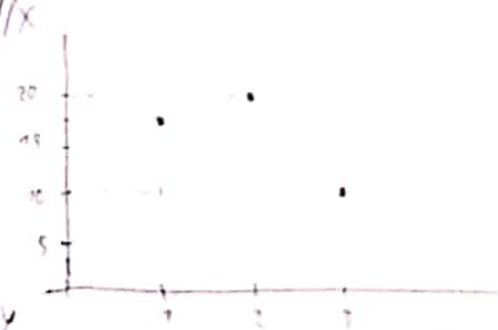
X/Y

Coincide con la nube de puntos



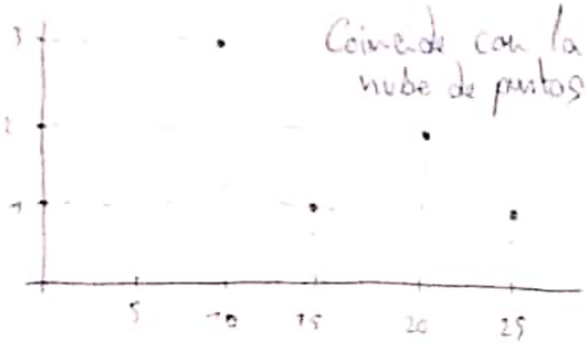
X\Y	10	15	20	25	$n_{i\cdot}$	$\bar{y}_i$
1	0	3	0	1	4	17,5
2	0	0	1	0	1	20
3	2	0	0	0	2	10
$n_{\cdot j}$	2	3	1	1		
$\bar{x}_j$	3	1	2	1		

Y/x



X/Y

Coincide con la nube de puntos



8.

X\Y	1	2	3	4	$n_{i\cdot}$	$x_i \cdot n_{i\cdot}$	$n_{i\cdot}(x_i-\bar{x})^2$	$\sum_{j=1}^{p} n_{ij} x_i y_j$
1	1	2	0	0	3	3	8,3337	5
2	1	2	3	1	7	14	3,1114	36
3	0	1	2	6	9	27	0,9998	96
4	0	0	2	3	5	20	8,8884	72
$n_{\cdot j}$	2	5	7	10	24	64	21,3333	209
$y_j \cdot n_{\cdot j}$	2	10	21	40	73			
$n_{\cdot j}(y_j-\bar{y})^2$	8,336	5,426	0,012	9,183	22,957			

a)

$m_{10} = \bar{x} = \frac{64}{24} = 2,6667$ $\sigma_x^2 = \frac{21,3333}{24} = 0,8889$

$m_{01} = \bar{y} = \frac{73}{24} = 3,0417$ $\sigma_y^2 = \frac{22,957}{24} = 0,9566$

$m_{11} = \frac{209}{24} = 8,7083$ $\sigma_{xy} = 8,7083 - 2,6667 \cdot 3,0417 = 0,597$

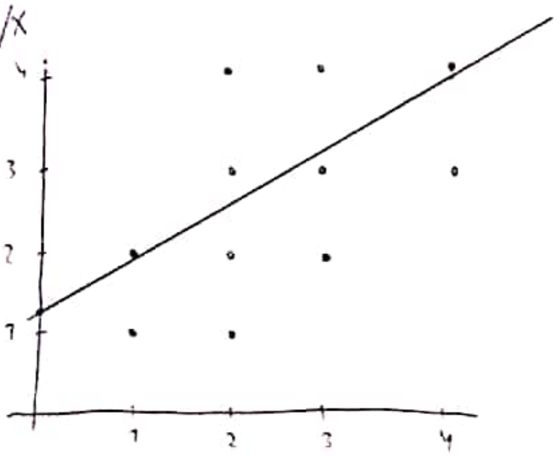
Y/X $y - 3,0417 = \frac{0,597}{0,8889}(x - 2,6667)$

$y = 0,6717x + 1,251$

X/Y $x - 2,6667 = \frac{0,597}{0,9566}(y - 3,0417)$

$x = 0,624y + 0,7684$

b) Y/X



$\eta^2_{X/Y} = a \cdot a' = 0,6717 \cdot 0,624 =$
$= 0,4191 = r^2$

$r = 0,6474$

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c)

$x = 0{,}624 \cdot 6 + 0{,}7684 =$
$= 4{,}5124$

Podemos decir que el número de balauzas se encuentra alrededor de 4,5124.

Pero la predicción no es fiable ya que puede ocurrir cualquier cosa, no podemos predecir algo ~~de una modalidad que no aparece~~ sin tener datos de una modalidad.

9.-

X\Y	(10,20]	(20,25]	(25,30]	(30,35]	(35,40]	$n_{i\cdot}$	c_i	$n_{i\cdot}c_i$	$n_{i\cdot}(c_i-\bar{x})^2$	$\sum n_{ij} \cdot y_j$
(15,18]	3	2	3	0	0	8	16,5	132	213,005	2846,2
(18,21]	0	4	2	2	0	8	19,5	156	37,325	4095
(21,24]	0	7	10	6	1	24	22,5	540	16,934	14962,5
(24,27]	0	0	2	5	3	10	25,5	255	147,456	8415
$n_{\cdot j}$	3	13	17	13	4	50		1083	414,72	30318,75
c_j	15	22,5	27,5	32,5	37,5					
$n_{\cdot j}c_j$	45	292,5	467,5	422,5	150	1377,5				
$n_{\cdot j}(y_j-\bar{y})^2$	472,508	331,533	0,043	318,533	396,01	1518,627				

$m_{10} = \bar{x} = \frac{1083}{50} = 21{,}66$ $\quad \sigma_x^2 = \frac{414{,}72}{50} = 8{,}294$ $\quad m_{11} = \frac{30318{,}75}{50} = 606{,}375$

$m_{01} = \bar{y} = \frac{1377{,}5}{50} = 27{,}55$ $\quad \sigma_y^2 = \frac{1518{,}627}{50} = 30{,}373$ $\quad \sigma_{xy} = m_{11} - m_{01}m_{10} = 9{,}642$

Y/X $\quad y - 27{,}55 = \frac{9{,}642}{8{,}294}(x - 21{,}66)$ $\qquad$ X/Y $\quad x - 21{,}66 = \frac{9{,}642}{30{,}373}(y - 27{,}55)$

$x = 0{,}317y + 12{,}914$

$y = 1{,}163x + 2{,}37$

$r^2 = 1{,}163 \cdot 0{,}317 = 0{,}3687 \quad r = 0{,}607$

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CDSP (ejercicios)

Relación 2

10. $x+4y=1$

$x+5y=2$

Suponemos que la primera ecuación es la recta de regresión Y/X y la segunda, de X/Y. Entonces:

$y=\frac{1}{4}-\frac{1}{4}x$ $\qquad r^2=\frac{\sigma_{xy}^2}{\sigma_x^2\sigma_y^2}=\frac{\sigma_{xy}}{\sigma_x^2}\cdot\frac{\sigma_{xy}}{\sigma_y^2}=a\cdot a'=-\frac{1}{4}\cdot(-5)=\frac{5}{4}$

$x=2-5y$

$a=-\frac{1}{4}$

$a'=-5$

Pero como sabemos, $0\leq r^2\leq 1$ por lo que la suposición es incorrecta.

Suponemos que la primera ecuación es la recta de regresión de X/Y y la segunda de Y/X. Entonces:

$x=1-4y$ $\qquad a=-4$

$y=\frac{2}{5}-\frac{1}{5}x$ $\qquad a'=-\frac{1}{5}$ $\qquad r^2=-4\cdot(-\frac{1}{5})=\frac{4}{5}$

Como vemos, el signo de las pendientes es el mismo, por lo que las rectas son correctas y además, $0<r^2<1$ luego la suposición es correcta.

11.

x_i	n_i	x_in_i	$x_i^2n_i$
3	5	15	45
5	1	5	25
8	2	16	128
9	1	9	81
	9	45	279

Recta de regresión de Y sobre X

$y=5x-20=ax+b$ donde $a=\frac{\sigma_{xy}}{\sigma_x^2}x+b\left(\frac{\sigma_{xy}}{\sigma_x^2}\bar{x}-\bar{y}\right)$

$\sum y_j^2 n_{\cdot j}=3240$

$\bar{x}=\frac{45}{9}=5$

$\sigma_x^2=m_{20}=\sum_{i}^{4}(x_i-\bar{x})=\frac{(3-5)^2\,5+(5-5)^2\,1+2(8-5)^2+(9-5)^2}{9}=$

$=6$

Hallar $x=a'y+b'$

$a'=\frac{\sigma_{xy}}{\sigma_y^2}$

$a=\frac{\sigma_{xy}}{\sigma_x^2}\Rightarrow\sigma_{xy}=a\cdot\sigma_x^2=5\cdot 6=30$

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$\bar{y} = 5\bar{x} - 20 = 5 \cdot 5 - 20 = 5$

$$\left(\begin{array}{l} m_{10} = a' + b' m_{01} \\ m_{11} = a' m_{02} + b' m_{01} \end{array} \Leftrightarrow \begin{array}{l} 5 = a' + 5b' \\ 55 = 5a' + 316' \end{array}\right)$$

$m_{10} = \bar{x} = 5$

$m_{01} = \bar{y} = 5$

$m_{11} = \sum \sum f_{ij} x_i y_j$

$\sigma_{xy} = m_{11} - m_{10} m_{01}$

$m_{11} = \sigma_{xy} + m_{10} m_{01} = \sigma_{xy} + \bar{x}\bar{y}$

$= 30 + 5 \cdot 5 = 55 \rightarrow \boxed{m_{11} = 55}$

$m_{20} = \sum f_{ij} x_i^2 =$

$= \frac{1}{n} \sum n_{i\cdot} x_i^2$

$= \frac{1}{9}(97 + \frac{279}{9} = 131 = m_{20}$

$m_{02} = (\sum_1^r n_{\cdot j} y_j^2) \frac{1}{n} = \frac{3240}{\frac{60}{9}} = \boxed{360 = m_{02}}$

$$\varphi(a', b') = \sum^r \sum^s (x_i - (a' y_j + b'))^2$$

$$\frac{\partial \varphi}{\partial a'} = 2 \sum^r \sum^s (x_i - (a' y_j + b'))(-y_j) = 0$$

$$\sum^r \sum^s f_{ij} x_i y_j - a' \sum^r \sum^s y_j^2 - b' \sum \sum y_j = 0$$

$$m_{11} - a' m_{02} - b' m_{01} = 0$$

$$m_{11} = a' m_{02} + b' m_{01}$$

$$\frac{\partial \varphi}{\partial b'} = 2 \sum^r \sum^s f_{ij} (x_i - (a' y_j + b')) = 0$$

$$\sum^r \sum^s f_{ij} x_i - a' \sum^r \sum^s f_{ij} y_j - b' = 0$$

$$m_{10} - a' m_{01} - b' = 0$$

$$m_{10} = a' m_{01} + b'$$

$$\left.\begin{array}{l} m_{11} = a' m_{02} + b' m_{01} \\ m_{10} = a' m_{01} + b' \end{array}\right\} \Leftrightarrow \left.\begin{array}{l} 55 = a' \cdot 360 + b' \cdot 5 \\ 5 = a' 5 + b' \end{array}\right\} \quad b' = 5 - 5a'$$

$$55 = 360a' \cdot 5(5 - 5a')$$

$$55 = 360a' - 25a' + 25$$

$$30 = 335a'$$

$$a' = 0'0896 \qquad b' = 5 - 5(0'0896) = 4'552$$

$$\boxed{x = 0'0896y + 4'552}$$

$a' = 0'0896 = \frac{\sigma_{xy}}{\sigma_y^2} \Leftrightarrow \sigma_y^2 = \frac{\sigma_{xy}}{0'0896} = \frac{30}{0'0896} = 334'82$

Hallamos la bondad de los ajustes lineales.

$$\eta'_{y/x} \cdot \eta'_{x/y} = r^2 = \frac{\sigma_{xy}^2}{\sigma_x^2 \sigma_y^2} = \frac{30^2}{6 \cdot 334'82} = \boxed{0'448}$$

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$r = 0{,}667$

12:

a) $y - \bar{y} = \frac{\sigma_{xy}}{\sigma_x^2}(x - \bar{x})$

$m_{10} = \frac{31{,}47}{24} = 1{,}31125$ $\quad$ $m_{11} = \frac{349{,}486}{24} = 14{,}562$

$m_{01} = \frac{219{,}719}{24} = 9{,}155$ $\quad$ $\sigma_{xy} = 2{,}5574$ $\quad$ $m_{02} = \frac{2396{,}504}{24} = 99{,}8543$

$m_{20} = \frac{51{,}075}{24} = 2{,}1281$ $\quad$ $\sigma_x^2 = 2{,}1281 - 1{,}31125^2 = 0{,}4087$

$y - 9{,}155 = 6{,}2574(x - 1{,}3125)$ $\quad$ $\sigma_y^2 = 99{,}8543 - 9{,}155^2 = 16{,}04$

$\boxed{y = 6{,}2574x + 0{,}9422}$

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$y = 6,2574 \cdot 0,5 + 0,9422 =$

$= 4,0709 \qquad 4,0709 \cdot 100 = 407,09$

$y = 6,2574 + 0,9422 = 7,1996$

$7,1996 \cdot 200 = 1.439,92$

$y = 6,2274 \cdot 2 + 0,9402 = 13,397 \qquad Y_{Total} = 3.186,71$

$13,397 \cdot 100 = 1.339,7$

$r^2 = \frac{\sigma_{xy}^2}{\sigma_x^2 \sigma_y^2} = 0,9976$ Es muy fiable.

b)

$$\psi(a,b,c) = \sum_{i=1}^{k} \sum_{j=1}^{p} f_{ij} [y_j - (a + bx_i + cx_i^2)]^2$$

$$\frac{d\psi}{da} = 0 \Rightarrow -2 \sum_{i=1}^{k} \sum_{j=1}^{p} f_{ij} (y_j - a - bx_i - cx_i^2) = 0$$

$$m_{01} - a - bm_{10} - cm_{20} = 0$$

$$\frac{d\psi}{db} = 0 \Rightarrow \sum_{i=1}^{k} \sum_{j=1}^{p} f_{ij} (y_j - a - bx_i - cx_i^2)^2 = 0$$

$$\sum_{i=1}^{k} \sum_{j=1}^{p} 2 f_{ij} (y_j - a - bx_i - cx_i^2)(-x_i) = 0$$

$$m_{11} - am_{10} - bm_{20} - cm_{30} = 0$$

$$\frac{d\psi}{dc} = 0 \Rightarrow \sum_{i=1}^{k} \sum_{j=1}^{p} f_{ij} (y_j - a - bx_i - cx_i^2)^2 = 0$$

$$\sum_{i=1}^{k} \sum_{j=1}^{p} 2 f_{ij} (y_j - a - bx_i - cx_i^2)(-x_i^2) = 0$$

$$m_{21} - am_{30} - bm_{30} - cm_{40} = 0$$

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$$\begin{rcases} m_{01} - a - b m_{10} - c m_{20} = 0 \\ m_{11} - a m_{10} - b m_{20} - c m_{30} = 0 \\ m_{21} - a m_{20} - b m_{30} - c m_{40} = 0 \end{rcases} \quad \begin{aligned} m_{30} &= 3,9 \\ m_{21} &= 26,4164 \\ m_{40} &= 7,624 \end{aligned}$$

$$\begin{rcases} 4,755 - a - b\,1,37725 - c\,2,1287 = 0 \\ 14,562 - a\,1,37725 - b\,2,1287 - c\,3,9 = 0 \\ 26,4164 - a\,2,1287 - b\,3,9 - c\,7,624 = 0 \end{rcases}$$

$a = 0,7951$

$b = 6,5512$

$c = -0,1082$

$$\boxed{y = 0,7951 + 6,5512x - 0,1082x^2}$$

$y_{100} = 4,0436\$$

$y_{100} = 7,2387$ $\qquad y_{total} = 3748,45\$$ combustible

$y_{700} = 13,4647$

c)

$$\sigma^2_{ry} = \sum_{i=1}^{k}\sum_{j=1}^{p} f_{ij}\left[y_j - f(x_i)\right]^2 = \qquad \boxed{\begin{aligned}(a+bx+cx^2)(a+bx+cx^2) &= a^2 + abx + acx^2 + \\ &+ abx + b^2x^2 + bcx^3 + acx^2 + cbx^3 + c^2x^4 = \end{aligned}}$$

$$= \sum_{i=1}^{k}\sum_{j=1}^{p} f_{ij}\left(y_j^2 - 2y_j f(x_i) + f(x_i)^2\right) =$$

$$= m_{02} - 2\overset{m_{01}}{a} - 2b\,m_{11} - 2c\,m_{21} + \sum\sum f_{ij}\, f(x_i)^2 = \qquad = a^2 + 2abx + 2acx^2 + b^2x^2 + 2bcx^3 + c^2x^4$$

$$= m_{02} - 2am_{01} - 2bm_{11} - 2cm_{21} + a^2 + 2ab\,m_{10} + 2ac\,m_{20} + b^2m_{20} + 2bc\,m_{30} + c^2m_{40} =$$

$$= 49,8543 - 14,5582 - 190,797 + 5,7765 + 0,6322 + 13,66 - 0,3662 +$$

$$+ 97,334 - 5,53 + 0,8249 = 0,7705$$

$$\eta^2_{Y/X} = 1 - \frac{0,7705}{16,04} = 0,9519$$

Se ajusta mejor la primera ya que $0'9976 > 0'9519$

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X	y	1/x	$1/x_i^2 \cdot n_i$	$\Sigma n_{ij} y_i$	$z_i \Sigma n_{ij} y_i$
10	50	1/10	1/100	50	5
12'5	90	1/12'5	1/156'25	90	7'2
20	160	1/20	1/400	160	8
25	180	1/25	1/625	180	7'2
	480	0'27	0'0205	480	27'4

$$\boxed{y = a + \frac{b}{x}} \longrightarrow \text{Ecuacion}$$

$$Z = 1/x \longrightarrow y = a + bz$$

$$a = \bar{y} - \frac{\sigma_{yz}}{\sigma_z^2} \cdot \bar{z} \qquad b = \frac{\sigma_{yz}}{\sigma_z^2}$$

$$\sigma_z^2 = \frac{\Sigma z_i^2 \cdot f_i}{\Sigma f_i} - \bar{z}^2$$

Fórmulas

$$\bar{z} = \frac{0'27}{4} = 0'0675 \qquad \bar{y} = \frac{480}{4} = 120$$

$$\sigma_{yz} = \frac{\Sigma n_{ij} y_j z_i}{n} - \bar{z} \cdot \bar{y} = \frac{27'4}{4} - 0'0675 \cdot 120 = -1'25$$

$$\sigma_z^2 = \frac{0'0205}{4} - 0'0675^2 = 0'0005688$$

$$a = 120 - \left(\frac{-1'25}{0'0005688}\right) \cdot 0'0675 = 268'3386$$

$$b = \frac{-1'25}{0'0005688} = -2197'609$$

$$y = 268'3386 - \frac{2197'609}{x}$$

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$y_i'^2$	y	x	$\Sigma x_i y_i f_i$	$\Sigma x_i^2 f_i$	$\Sigma y_i^2 f_i$	z	$z^2 \cdot n_i$	$z_i \Sigma n_{ij} y_i$	y'	x'	$\Sigma x_i y'_i$	$\Sigma x'_i y'_i$	$x_i'^2$
11'566801	30	9	270	81	900	1/9	1/81	10/3	3'401	2'197	30'609	7'472	4'8268
15'303744	50	10	500	100	2500	1/10	1/100	5	3'912	2'302	39'12	9'0054	5'2992
18'045504	70	12	840	144	4900	1/12	1/144	35/6	4'248	2'485	50'976	10'55628	6'175225
19'2019244	80	15	1200	225	6400	1/15	1/225	16/3	4'382	2'708	65'73	11'86646	7'333264
22'915369	120	22	2640	484	14400	1/22	1/484	60/11	4'787	3'091	105'314	14'7966	9'554281
24'423364	140	32	4480	1024	19600	1/32	1/1024	35/8	4'942	3'466	158'144	17'12897	12'03156
111'456706	490	100	9930	2058	487000	0'4378	0'0368	29'3295	25'672	16'249	448'893	70'8257	45'201939

0'4378 → 0'4378156566

0'0368 → 0'03677772461

29'3295 → 29'32954545

a) Relación lineal:

$$y-\bar{y} = \frac{\sigma_{xy}}{\sigma_x^2}(x-\bar{x}).$$

$$\sigma_{xy} = \frac{1}{N}\cdot\sum_i\sum_j x_i y_j f_{ij} - \bar{x}\cdot\bar{y} \qquad \bar{x} = \frac{\Sigma x_i}{N} = \frac{100}{6} = 16'667$$

$$\sigma_{xy} = \frac{1}{6}\cdot 9930 - 16'667\cdot 81'667 = \qquad \bar{y} = \frac{\Sigma y_j}{N} = \frac{490}{6} = 81'667$$

$$= 293'856$$

$$\sigma_x^2 = \frac{\Sigma x_i^2 f_i}{\Sigma f_i} - \bar{x}^2 = \frac{2058}{6} - 16'667^2 = 65'211$$

$$y - 81'667 = \frac{293'856}{65'211}\cdot(x - 16'667) \rightarrow$$

$$\rightarrow y = 4'5062x + 6'5616$$

$$\sigma_y^2 = \frac{\Sigma y_i^2 f_i}{\Sigma f_i} - \bar{y}^2 = \frac{487000}{6} - 81'667^2 =$$

$$= \cancel{74497'16778}\ 1447'16778$$

$$r = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{293'856}{\sqrt{65'211}\cdot\sqrt{\cancel{74497'16778}\ 1447'16778}} = 0'9566$$

c) ③ Curva potencial

$y = ax^b \rightarrow$ ecuación curva potencial.

Transformación: $\ln y = \ln a + b \ln x$

Cambio de variable: $y' = \ln y$
$x' = \ln x$

$y' = a' + bx'$

$$a' = \bar{y}' - \frac{\sigma_{y'x'}}{\sigma^2_{x'}} \cdot \bar{x}' \qquad b = \frac{\sigma_{x'y'}}{\sigma^2_{x'}}$$

$$\sigma^2_{x'} = \frac{\sum x_i'^2}{N} - \bar{x}'^2 \qquad \sigma^2_{y'} = \frac{\sum y_i'^2}{N} - \bar{y}'^2$$

$$\bar{x}' = \frac{\sum x_i'}{N} \qquad \bar{y}' = \frac{\sum y'_i}{N}$$

Fórmulas

$$\bar{x}' = \frac{16'249}{6} = 2'708166667 \qquad \bar{y}' = \frac{25'672}{6} = 4'27866667$$

$$\sigma^2_{x'} = \frac{45'201939}{6} - 2'708166667^2 = 0'1994898$$

$$\sigma^2_{y'} = \frac{111'456706}{6} - 4'27866667^2 = 0'2691292$$

$$\sigma_{x'y'} = \frac{\sum y_i' \cdot x_i'}{N} - \bar{x}' \cdot \bar{y}' = \frac{70'8257}{6} - 2'708166667 \cdot 4'27866667 =$$

$$= 0'216941$$

$$a' = 4'27866667 - \frac{0'216941}{0'1994898} \cdot 2'708166667 = 1'33359186$$

$$a = e^{a'} = 3'79465$$

$$b = \frac{0'216941}{0'1994898} = 1'087479159$$

$$\boxed{y = 3'79465 \cdot x^{1'087479159}}$$

$$r = \frac{\sigma_{x'y'}}{\sigma_{x'}\sigma_{y'}} = \frac{0'216941}{\sqrt{\sigma^2_{x'}} \cdot \sqrt{\sigma^2_{y'}}} = 0'9363$$

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b) Hipérbola equilátera.

Sea la ecuación de la hipérbola equilátera:

$$y = a + \frac{b}{x}$$

Sea $z = 1/x \rightarrow y = a + bz$

$$a = \bar{y} - \frac{\sigma_{yz}}{\sigma_{z^2}} \cdot \bar{z} \qquad b = \frac{\sigma_{yz}}{\sigma_{z^2}}$$

$$\sigma_{z^2} = \frac{\sum z_i^2 \cdot f_i}{\sum f_i} - \bar{z}^2 \qquad \bar{z} = \frac{\sum z_i f_i}{N}$$

$$\bar{y} = \frac{\sum y_i f_i}{N}$$

Fórmulas

$$\bar{z} = \frac{0'4378066}{6} = 0'0729692761 \qquad \bar{y} = \frac{490}{6} = 81'6666667$$

$$\sigma_{xy} = \frac{\sum n_i y_i z_i}{N} - \bar{z} \cdot \bar{y} = \frac{24'32954545}{6} - 0'0729692761 \cdot 81'6666667 =$$

$$= -1'07093684 9$$

$$\sigma_{z^2} = \frac{0'0367772461}{6} - 0'0729692761^2 = 0'00080502576$$

$$a = 81'6666667 - \left(\frac{-1'070936849}{0'00080502576}\right) \cdot 0'0729692761^2 = 178'7387$$

$$b = \frac{-1'070936849}{0'00080502576} = -1330'31364$$

$$\boxed{y = 178'7387 - \frac{1330'31364}{x}}$$

$$r = \frac{\sigma_{yz}}{\sigma_y \sigma_z} = \frac{-1'070936849}{\sqrt{1447'16778} \cdot \sqrt{0'00080502576}} = -0'9922$$

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d) Curva exponencial

Ecuación curva exponencial $\rightarrow y = a \cdot b^x$

Transformación : $\ln y = \ln a + x \ln b$

Cambio de variable : $y' = \ln y$

$x' = x$

$\bar{x} = 16'66667 \qquad \bar{y}' = 4'27866667$

$\sigma_x^2 = 65'211111 \qquad \sigma_{y'}^2 = 0'2691292$

$$\sigma_{xy'} = \frac{448'893}{6} - 16'66667 \cdot 4'27866667 = 3'504374571$$

$$a' = \bar{y}' - \frac{\sigma_{xy'}}{\sigma^2 x} \cdot \bar{x} = 4'27866667 - \frac{3'504374571}{65'211111} \cdot 16'66667 =$$

$= 3'383$

$a = e^{3'383} = 29'45901573$

$$b' = \frac{\sigma_{xy'}}{\sigma^2 x} = \frac{3'504374571}{65'211111} = 0'0537389$$

$b = e^{0'0537389} = 1'0552$

$$\boxed{y = 29'45901573 \cdot 1'0552^x}$$

$$r = \frac{\sigma_{xy'}}{\sigma_x \sigma_{y'}} = \frac{3'504374571}{\sqrt{65'211111} \cdot \sqrt{0'2691292}} = 0'83651$$

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- ¿Qué ajuste es más adecuado?

Para ello, calculamos la varianza residual.

- Relación lineal.

$i=1 \to (30 - f(9))^2 = \cancel{+279}\,3'0053828$

$i=2 \to (50 - f(10))^2 = 2'63607696$

$i=3 \to (70 - f(12))^2 = 87'684496$

$i=4 \to (80 - f(15))^2 = 34'16870116$

$i=5 \to (120 - f(22))^2 = 204'547204$

$i=6 \to (140 - f(32))^2 = 115'7776$

$\Sigma = 737'8194619$

- σ_{ey}

$i=1 \to (f(9) - 81'667)^2 = \cancel{1690'130765}\ \cancel{902'6058836}\ 1193'67486$

$i=2 \to (f(10) - 81'667)^2 = 902'6058836$

$i=3 \to (f(12) - 81'667)^2 = 442'302961$

$i=4 \to (f(15) - 81'667)^2 = 56'4361537 6$

$i=5 \to (f(22) - 81'667)^2 = 577'488961$

$i=6 \to (f(32) - 81'667)^2 = 4773'842649$

$\Sigma = 7946'351468$

$\sigma^2_y = 8684'17093$

$$\eta^2_{y|x} = 1 - \frac{737'8194619}{8684'17093} = 0'9150385837$$

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– Hipérbola equilátera

- Residuos

$i=1 \rightarrow (30 - f(9))^2 = 87942'68935$

$i=2 \rightarrow (50 - f(10))^2 = 68523'56641$

$i=3 \rightarrow (70 - f(12))^2 = 48223'35627$

$i=4 \rightarrow (80 - f(15))^2 = 35128'60894$

$i=5 \rightarrow (120 - f(22))^2 = 14210'42849$

$i=6 \rightarrow (140 - f(32))^2 = 6449'856922$

$\Sigma = 260488'5064$

- σey

$i=1 \rightarrow (f(9) - 81'66667)^2 = 2574'611195$

$i=2 \rightarrow (f(10) - 81'66667)^2 = 1293'075859$

$i=3 \rightarrow (f(12) - 81'66667)^2 = 190'094329$

$i=4 \rightarrow (f(15) - 81'66667)^2 = 70'29856581$

$i=5 \rightarrow (f(22) - 81'66667)^2 = 1339'794117$

$i=6 \rightarrow (f(32) - 81'66667)^2 = 3080'216561$

$\Sigma = 8548'090629$

$$\sigma^2 y = \sigma^2 ey + \sigma^2 ry = 269036'597$$

$$\eta^2_{y/x} = 1 - \frac{260488'5064}{269036'597} = 0'0317729658 2$$

- Curva potencial

  - Residuos

$i=1 \rightarrow (30 - f(9))^2 = 129'7215494$

$i=2 \rightarrow (50 - f(10))^2 = 12'85795048$

$i=3 \rightarrow (70 - f(12))^2 = 179'7611988$

$i=4 \rightarrow (80 - f(15))^2 = 61'85712231$

$i=5 \rightarrow (120 - f(22))^2 = 112'3007782$

$i=6 \rightarrow (140 - f(32))^2 = \cancel{936'1390242}\ 597'0080696$

$\Sigma = \cancel{1432'687623}\ 1093'50667$

  - σ_{ey}

$i=1 \rightarrow (f(9) - 81'667)^2 = 1622'24719$

$i=2 \rightarrow (f(10) - 81'667)^2 = 1242'7363$

$i=3 \rightarrow (f(12) - 81'667)^2 = 628'714101$

$i=4 \rightarrow (f(15) - 81'667)^2 = 90'851331$

$i=5 \rightarrow f(22) - 81'667)^2 = 769'292751$

$i=6 \rightarrow f(32) - 81'667)^2 = 6850'38985$

$\Sigma = 11204'2315$

$\sigma_y^2 = 1093'50667 + 11204'2315 = 12297'7382$

$$\eta^2_{y/x} = 1 - \frac{1093'50667}{12297'7382} = 0'91108067$$

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- Curva exponencial

- Residuos

$i=1 \to (30 - f(9))^2 = 316'070228$
$i=2 \to (50 - f(10))^2 = 0'17283121$
$i=3 \to (70 - f(12))^2 = 192'23143$
$i=4 \to (80 - f(15))^2 = 197'295105$
$i=5 \to (120 - f(22))^2 = 572'690876$
$i=6 \to (140 - f(24))^2 = 595'904364$

$\Sigma = 1874'36483$

- σ_{ey}

$i=1 \to (f(9) - 81'66667)^2 = 1237'39528$
$i=2 \to (f(10) - 81'66667)^2 = 1073'16177$
$i=3 \to (f(12) - 81'66667)^2 = 788'480241$
$i=4 \to (f(15) - 81'66667)^2 = 347'82844$
$i=5 \to (f(22) - 81'6667)^2 = 67'124249$
$i=6 \to (f(32) - 81'6667)^2 = 4558'57448$

$\Sigma = 8042'56446$

$\sigma_y^2 = 1874'36483 + 8042'56446 = 9916'9293.$

$$\eta_{y/x} = 1 - \frac{1874'36483}{9916'9293} = 0'81099343$$

Como conclusión, ~~llegamos~~ obtenemos que el ajuste más adecuado ~~el~~ es la relación lineal, ya que es la que tiene el coeficiente de determinación más cercano a 1.

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