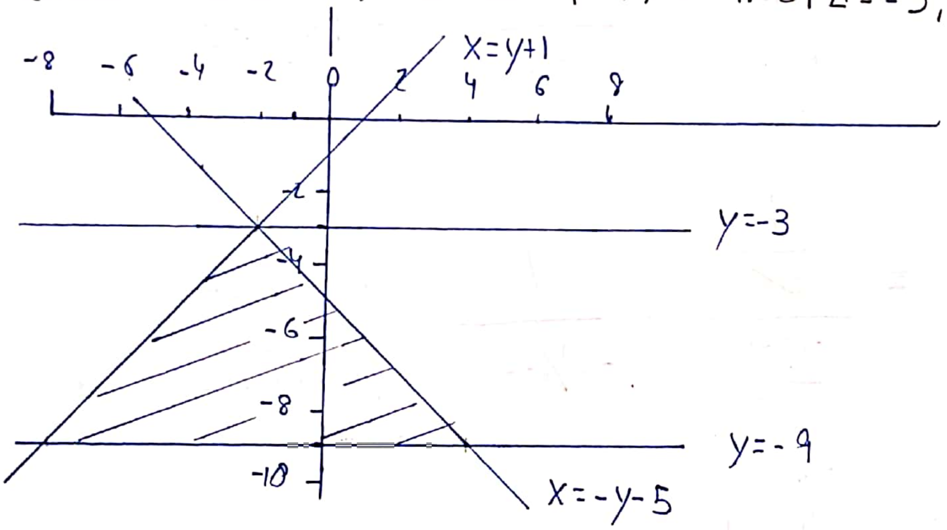
2) DANIEL MONJAS MIGUÉLEZ.

70274432-W

Sea (X,Y) un vector aleatorio continuo con la función de densidad conjunta que se muestra a continuación

$$f(x,y)=\frac{1}{36} \quad y+1<x<-5-y \quad , -9<y<-3$$

Obtener la función de densidad de Y condicionada a un valor x_0, así como la función de densidad de x condicionada a un valor y_0. A través de esas funciones de densidad condicionadas, calcular $P(Y<-4.82 \mid X=-3,75)$ y $P(X>-3,75 \mid Y=-4,82)$



$$f_X(x_0)=\begin{cases}\displaystyle\int_{-9}^{x-1}\frac{1}{36}\,dy=\frac{x+8}{36} & \text{si } -8<x_0\leq -2\\[2ex] \displaystyle\int_{-9}^{-x-5}\frac{1}{36}\,dy=\frac{-5-x+9}{36}=\frac{4-x}{36} & \text{si } -2\leq x<4\end{cases}$$

$$f_{Y/X=x_0}=\frac{f(x_0,y)}{f_X(x_0)}$$

$$f_{Y|X=x_0} = \frac{f(x_0,y)}{f_X(x_0)} = \begin{cases} \dfrac{\frac{1}{\cancel{36}}}{\frac{x_0+8}{\cancel{36}}} = \dfrac{1}{x_0+8} & \text{si } \begin{array}{l} -8 < x_0 \leq -2 \\ -9 < y < x_0 - 1 \end{array} \\[2em] \dfrac{\frac{1}{\cancel{36}}}{\frac{4-x_0}{\cancel{36}}} = \dfrac{1}{4-x_0} & \text{si } \begin{array}{l} -2 \leq x_0 < 4 \\ -9 < y < -5 - x_0 \end{array} \end{cases}$$

$$f_Y(y) = \int_{y+1}^{-5-y} \frac{1}{36}\, dx = \frac{-5-y-y-1}{36} = \frac{-2y-6}{36} \qquad -9 < y < -3$$

$$f_{X|Y=y_0} = \frac{f(x,y_0)}{f_Y(y_0)} = \frac{\frac{1}{\cancel{36}}}{\frac{-2y_0-6}{\cancel{36}}} = \frac{1}{-2y_0-6} \qquad \begin{array}{l} -9 < y_0 < -3 \\ y_0 + 1 < x < -5 - y_0 \end{array}$$

$$P[Y < -4{,}82 \mid X = -3{,}75] = \int_{-9}^{-4{,}82} \frac{1}{4{,}25}\, dy = \frac{-4{,}82+9}{4{,}25} = 0{,}9835294$$

$$P[X > -3{,}75 \mid Y = -4{,}82] = \int_{-3{,}75}^{0{,}18} \frac{1}{3{,}64}\, dx = \frac{-0{,}18+3{,}75}{3{,}64} = 0{,}98076923$$