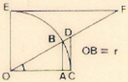
CIENCIA Y TECNICA REUNIDAS

TRIGONOMETRIA

B Z B

DEFINICION Y REPRESENTACION DE FUNCIONES TRIGONOMETRICAS



		Si $r=1$
Sen	$\frac{AB}{OB}$	AB
Cos	$\frac{OA}{OB}$	OA
Tag	$\frac{AB}{OA}$	DC
Cosec	$\frac{OB}{AB}$	OF
Sec	$\frac{OB}{OA}$	OD
Cotg	$\frac{OA}{AB}$	EF

VALOR DE UNA RAZON EN FUNCION DE LAS DEMAS

	Sen	Cos	Tag	Cosec	Sec	Cotg
Sen	—	$\sqrt{1-Cos^2}$	$\frac{tg}{\sqrt{1+tg^2}}$	$\frac{1}{Cosec}$	$\frac{\sqrt{Sec^2-1}}{Sec}$	$\frac{1}{\sqrt{1+Cotg^2}}$
Cos	$\sqrt{1-Sen^2}$	—	$\frac{1}{\sqrt{1+tg^2}}$	$\frac{\sqrt{Cosec^2-1}}{Cosec}$	$\frac{1}{Sec}$	$\frac{Cotg}{\sqrt{1+Cotg^2}}$
Tag	$\frac{Sen}{\sqrt{1-Sen^2}}$	$\frac{\sqrt{1-Cos^2}}{Cos}$	—	$\frac{1}{\sqrt{Cosec^2-1}}$	$\sqrt{Sec^2-1}$	$\frac{1}{Cotg}$
Cosec	$\frac{1}{Sen}$	$\frac{1}{\sqrt{1-Cos^2}}$	$\frac{\sqrt{1+tg^2}}{tg}$	—	$\frac{Sec}{\sqrt{Sec^2-1}}$	$\sqrt{1+Cotg^2}$
Sec	$\frac{1}{\sqrt{1-Sen^2}}$	$\frac{1}{Cos}$	$\sqrt{1+tg^2}$	$\frac{Cosec}{\sqrt{Cosec^2-1}}$	—	$\frac{\sqrt{1+Cotg^2}}{Cotg}$
Cotg	$\frac{\sqrt{1-Sen^2}}{Sen}$	$\frac{Cos}{\sqrt{1-Cos^2}}$	$\frac{1}{Tag}$	$\sqrt{Cosec^2-1}$	$\frac{1}{\sqrt{Sec^2-1}}$	—

ANGULOS NEGATIVOS, COMPLEMENTARIOS, SUPLEMENTARIOS, ETC.

	$-a$	$90 \pm a$	$180 \pm a$	$270 \pm a$	$n360 \pm a$
Sen	$-$ Sen a	$+$ Cos a	$\mp$ Sen a	$-$ Cos a	$-$ Sen a
Cos	$+$ Cos a	$\mp$ Sen a	$-$ Cos a	$\pm$ Sen a	$+$ Cos a
Tag	$-$ Tag a	$\mp$ Cotg a	$\pm$ Tag a	$\mp$ Cotg a	$\pm$ Tag a
Cosec	$-$ Cotg a	$\mp$ Tag a	$\pm$ Cotg a	Tag a	Cotg a
Sec	$+$ Sec a	Cosec a	$-$ Sec a	Cosec a	$+$ Sec a
Cotg	$-$ Cotg a	Tag a	$\pm$ Cotg a	$\mp$ Tag a	$\pm$ Cotg a

VARIACION DE LAS FUNCIONES SEGUN EL CUADRANTE

	CUADRANTES			
	1° 0 – 90	2° 90 – 180	3° 180 – 270	4° 270 – 360
Sen	$0 \to 1$	$1 \to 0$	$0 \to -1$	$-1 \to 0$
Cos	$1 \to 0$	$0 \to -1$	$-1 \to 0$	$0 \to 1$
Tag	$0 \to +\infty$	$-\infty \to 0$	$0 \to +\infty$	$-\infty \to 0$
Cosec	$+\infty \to 1$	$1 \to +\infty$	$-\infty \to -1$	$-1 \to -\infty$
Sec	$1 \to +\infty$	$-\infty \to -1$	$-1 \to -\infty$	$+\infty \to 1$
Cotg	$+\infty \to 0$	$0 \to -\infty$	$+\infty \to 0$	$0 \to -\infty$

RADIAN Angulo cuyo arco es igual al radio

EQUIVALENCIA EN RADIANES Y VALOR DE LAS FUNCIONES DE ANGULOS USUALES

	rad	Sen	Cos	Tag	Cosec	Sec	Cotg
0	0	0	1	0	∞	1	∞
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\sqrt{3}/2$	$1/\sqrt{3}$	2	$2/\sqrt{3}$	$\sqrt{3}$
45	$\frac{\pi}{4}$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	$2/\sqrt{2}$	$2/\sqrt{2}$	1
60	$\frac{\pi}{3}$	$\sqrt{3}/2$	$\frac{1}{2}$	$\sqrt{3}$	$2/\sqrt{3}$	2	$1/\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	∞	1	∞	0
180	π	0	-1	0	∞	-1	0
270	$\frac{3\pi}{2}$	-1	0	∞	-1	∞	0
360	2π	0	1	0	∞	1	∞

RELACIONES FUNDAMENTALES

$Sen \cdot Cosec = 1$

$Cos \cdot Sec = 1$

$Tg \cdot Cotg = 1$

$Sen^2 + Cos^2 = 1$

$Tg = \frac{Sen}{Cos}$

$Cotg = \frac{Cos}{Sen}$

$1 + tg^2 = Sec^2$

$1 + Cotg^2 = Cosec^2$

$Sen(a \pm b) = Sen\ a\ Cos\ b \pm Cos\ a\ Sen\ b$

$Cos(a \pm b) = Cos\ a\ Cos\ b \mp Sen\ a\ Sen\ b$

$Tg(a \pm b) = \frac{tg\ a \pm tg\ b}{1 \mp tg\ a\ tg\ b}$

$$
\left.\begin{aligned}
&Sen\ a + Sen\ b = 2\ Sen\ A\ Cos\ B \\
&Sen\ a - Sen\ b = 2\ Cos\ A\ Sen\ B \\
&Cos\ a + Sen\ b = 2\ Sen(\tfrac{\pi}{4} - B)\, Cos(\tfrac{\pi}{4} - A) \\
&Cos\ a - Sen\ b = 2\ Sen(\tfrac{\pi}{4} - A)\, Cos(\tfrac{\pi}{4} - B) \\
&Cos\ a + Cos\ b = 2\ Cos\ A\ Cos\ B \\
&Cos\ a - Cos\ b = -2\ Sen\ A\ Sen\ B \\
&Tga + Tgb = \frac{Sen(2A)}{Cos\ a\ Cos\ b} \\
&Tga - Tgb = \frac{Sen(2B)}{Cos\ a\ Cos\ b} \\
&Cotg\ a + Tg\ b = \frac{Cos(2B)}{Sen\ a\ Cos\ b} \\
&Cotg\ a - Tg\ b = \frac{Cos(2A)}{Sen\ a\ Cos\ b} \\
&Cotg\ a + Cotg\ b = \frac{Sen(2A)}{Sen\ a\ Sen\ b} \\
&Cotg\ a - Cotg\ b = \frac{Sen(2B)}{Sen\ a\ Sen\ b}
\end{aligned}\right\}
$$

con: $A = \frac{a+b}{2}$, $B = \frac{a-b}{2}$

$Sen(a+b) + Sen(a-b) = 2 Sen\ a\ Cos\ b$

$Sen(a+b) - Sen(a-b) = 2 Cos\ a\ Sen\ b$

$Cos(a-b) + Cos(a+b) = 2 Cos\ a\ Cos\ b$

$Cos(a-b) - Cos(a-b) = 2 Sen\ a\ Sen\ b$

$$
\left\{\begin{aligned}
&1 + Cos(a+b) = 2 Cos\ A \\
&1 - Cos(a-b) = 2\ Sen\ A
\end{aligned}\right.
$$

$Sen\ 2a = 2 Sen\ a\ Cos\ a$

$Cos\ 2a = Cos^2 a - Sen^2 a = 2 Cos^2 a - 1 = 1 - 2\ Sen^2 a$

$Tg\ 2a = \frac{2\ tg\ a}{1 - tg^2 a}$

$Sen\ 3a = 3 Sen\ a - 4\ Sen^3 a$

$Cos\ 3a = 4 Cos^3 a - 3 Cos\ a$

$Sen\ na = 2\ Sen\ [(n-1)\, a]\ Cos\ a - Sen\ [(n-2)\, a]$

$Cos\ na = 2\ Cos\ [(n-1)\, a]\ Cos\ a - Cos\ [(n-2)\, a]$

$Sen \frac{a}{2} = \pm\sqrt{\frac{1 - Cos\ a}{2}}$ $\begin{cases} +\text{Si } \frac{a}{2} \text{ en cuadrantes } 1° \text{ ó } 2° \\ -\text{Si } \frac{a}{2} \text{ en } \quad \cdot\cdot \quad 3° \text{ ó } 4° \end{cases}$

$Cos \frac{a}{2} = \pm\sqrt{\frac{1 + Cos\ a}{2}}$ $\begin{cases} +\text{Si } \frac{a}{2} \text{ en cuadrantes } 1° \text{ ó } 4° \\ -\text{Si } \frac{a}{2} \text{ en } \quad \cdot\cdot \quad 2° \text{ ó } 3° \end{cases}$

$Tg \frac{a}{2} = \pm\sqrt{\frac{1 - Cos\ a}{1 + Cos\ a}} = \frac{1 - Cos\ a}{Sen\ a} = \frac{Sen\ a}{1 + Cos\ a}$ $\begin{cases} +\text{Si } \frac{a}{2} \text{ en cuadrantes } 1° \text{ ó } 3° \\ -\text{Si } \frac{a}{2} \text{ en } \quad \cdot\cdot \quad 2° \text{ ó } 4° \end{cases}$

Le será de utilidad conocer TODA la colección "BZB"

temas B Z B CENTRAL PARA ESPAÑA: Salamanca, Apdo. 345 DELEGACION: Barcelona (15), Calabria 29 Tel.: 2 23 28 20

Dividido por	$\text{Sen a} + \text{Sen b}$	$\text{Sen a} - \text{Sen b}$	$\text{Cos a} + \text{Cos b}$	$\text{Cos a} - \text{Cos b}$
$\text{Sen a} + \text{Sen b}$	1	$\frac{\text{tg B}}{\text{tg A}}$	Cotg A	tg A
$\text{Sen a} - \text{Sen b}$	$\frac{\text{tg A}}{\text{tg B}}$	1	Cotg B	tg B
$\text{Cos a} + \text{Cos b}$	tg A	tg B	1	tg A tg B
$\text{Cos a} - \text{Cos b}$	Cotg B	Cotg A	Cotg A Cotg B	1

$A = \frac{a+b}{2}$; $B = \frac{a-b}{2}$

Dividido por	tg a + tg b	tg a − tg b	Cotg a + Cotg b	Cotg a − Cotg b
tg a + tg b	1	$\frac{\text{Sen}(2B)}{\text{Sen}(2A)}$	Cotg a Cotg b	Cotg(2A) · Cotg a
tg a − tg b	$\frac{\text{Sen}(2A)}{\text{Sen}(2B)}$	1	Cotg(2B) Cotg a	$\frac{\text{Cos}(2A)}{\text{Cos}(2B)}$ Cotg a
Cotg a + Cotg b	tg a · tg b	tg(2B) tg a	1	$\frac{\text{Cos}(2A)}{\text{Cos}(2B)}$
Cotg a − Cotg b	$\frac{\text{Sen}(2A)}{\text{Sen}(2B)}$ tg a tg b	tg a tg b	$\frac{\text{Cos}(2B)}{\text{Cos}(2A)}$	1

TRIANGULOS OBLICUANGULOS

		$\frac{A}{2}$	$\frac{B}{2}$	$\frac{C}{2}$
Sen		$\sqrt{\frac{(p-b)(p-c)}{bc}}$	$\sqrt{\frac{(p-a)(p-c)}{ac}}$	$\sqrt{\frac{(p-a)(p-b)}{ab}}$
		$\frac{a}{b+c}\text{Cos}\frac{B-C}{2}$	$\frac{b}{c+a}\text{Cos}\frac{C-A}{2}$	$\frac{c}{a+b}\text{Cos}\frac{A-B}{2}$
Cos		$\sqrt{\frac{p(p-a)}{bc}}$	$\sqrt{\frac{p(p-b)}{ac}}$	$\sqrt{\frac{p(p-c)}{ab}}$
		$\frac{a}{b-c}\text{Sen}\frac{B-C}{2}$	$\frac{b}{c-a}\text{Sen}\frac{C-A}{2}$	$\frac{c}{a-b}\text{Sen}\frac{A-B}{2}$
Tg		$\sqrt{\frac{(p-b)(p-c)}{p(p-a)}}$	$\sqrt{\frac{(p-a)(p-c)}{p(p-b)}}$	$\sqrt{\frac{(p-a)(p-b)}{p(p-c)}}$
		$\frac{b-c}{b+c}\text{Cotg}\frac{B-C}{2}$	$\frac{c-a}{c+a}\text{Cotg}\frac{C-A}{2}$	$\frac{a-b}{a+b}\text{Cotg}\frac{A-B}{2}$
Cotg		$\frac{b+c}{b-c}\text{tg}\frac{B-C}{2}$	$\frac{c+a}{c-a}\text{tg}\frac{C-A}{2}$	$\frac{a+b}{a-b}\text{tg}\frac{A-B}{2}$
		$\sqrt{\frac{p(p-a)}{(p-b)(p-c)}}$	$\sqrt{\frac{p(p-b)}{(p-a)(p-c)}}$	$\sqrt{\frac{p(p-c)}{(p-a)(p-b)}}$

OTRAS FORMULAS

$\text{Sen a} = \frac{2\,\text{tg}\frac{a}{2}}{1+\text{tg}^2\frac{a}{2}}$; $\text{Cos a} = \frac{1-\text{tg}^2\frac{a}{2}}{1+\text{tg}^2\frac{a}{2}}$; $\text{Tg a} = \frac{2\,\text{tg}\frac{a}{2}}{1-\text{tg}^2\frac{a}{2}}$

$\frac{\text{Cos}(a+b)}{\text{Cos}(a-b)} = \frac{1-\text{tg a}+\text{tg b}}{1+\text{tg a tg b}}$

$\text{Sen}^2(a+b)-\text{Sen}^2(a-b) = 3\,\text{Sen 2a Sen 2b}$

$\text{Cos}^2(a+b)-\text{Cos}^2(a-b) = -\text{Sen 2a Sen 2b}$

$\text{Sen}^2 a-\text{Sen}^2 b = \text{Sen}(a+b)\,\text{Sen}(a-b)$

$\text{Cos}^2 a-\text{Sen}^2 b = \text{Cos}(a+b)\,\text{Cos}(a-b)$

Relaciones para triangulos:

$\frac{a+b}{a-b} = \frac{\text{tg}\frac{A+B}{2}}{\text{tg}\frac{A-B}{2}}$ $\quad \frac{b+c}{b-c} = \frac{\text{tg}\frac{B+C}{2}}{\text{tg}\frac{B-C}{2}}$ $\quad \frac{c+a}{c-a} = \frac{\text{tg}\frac{C+A}{2}}{\text{tg}\frac{C-A}{2}}$

si $a+b+c = 180°$:

$\text{Sen a}+\text{Sen b}+\text{Sen c} = 4\,\text{Cos}\frac{a}{2}\,\text{Cos}\frac{b}{2}\,\text{Cos}\frac{c}{2}$

$\text{Sen}^2 a+\text{Sen}^2 b+\text{Sen}^2 c = 2\,\text{Cos a Cos b Cos c}+2$

$\text{Cos a}+\text{Cos b}+\text{Cos c} = 4\,\text{Sen}\frac{a}{2}\,\text{Sen}\frac{b}{2}\,\text{Sen}\frac{c}{2}+1$

$\text{Cos}^2 a+\text{Cos}^2 b+\text{Cos}^2 c = 1-2\,\text{Cos a Cos b Cos c}$

$\text{Cotg a Cotg b}+\text{Cotg a Cotg c}+\text{Cotg b Cotg c} = 1$

$\text{tga}+\text{tgb}+\text{tgc} = \text{tga}\cdot\text{tgb}\cdot\text{tgc}$

$\text{Cotg}\frac{a}{2}+\text{Cotg}\frac{b}{2}+\text{Cotg}\frac{c}{2} = \text{Cotg}\frac{a}{2}\cdot\text{Cotg}\frac{b}{2}\,\text{Cotg}\frac{c}{2}$

RESOLUCION TRIANGULOS RECTANGULOS

DATOS	INCOGNITAS					
	a	b	c	B	C	Area
bc	$\sqrt{b^2+c^2}$	–	–	$\text{tg B} = \frac{b}{c}$	$\text{tg C} = \frac{c}{b}$	$\frac{b\cdot c}{2}$
ac	–	$\sqrt{a^2-c^2}$	–	$\text{Cos B} = \frac{c}{a}$	$\text{Sen C} = \frac{c}{a}$	$\frac{b}{2}\sqrt{a^2-c^2}$
aB	–	a Sen B	a Cos B	–	C=90−B	$\frac{a^2}{4}\text{Sen}(2B)$
bB	$\frac{b}{\text{Sen B}}$	–	b Cotg B	–	C=90−B	$\frac{b^2}{2}\text{Cotg B}$
bc	$\frac{b}{\text{Cos C}}$	–	–	B=90−C	–	$\frac{b}{2}\text{tg C}$

$a^2 = b^2+c^2$ $\quad B+C = 90°$ $\quad a = \frac{b}{\text{Sen B}} = \frac{c}{\text{Sen C}}$

$\text{Sen B} = \text{Cos C} = \frac{b}{c}$ $\quad \text{Sen C} = \text{Cos B} = \frac{c}{a}$

$\text{Tg B} = \text{Cotg C} = \frac{b}{c}$ $\quad \text{Tg C} = \text{Cotg B} = \frac{c}{b}$

$\text{Tg}\frac{B}{2} = \sqrt{\frac{a-c}{a+c}}$ $\quad \text{Tg}\frac{C}{2} = \sqrt{\frac{a-b}{a+b}}$

RESOLUCION TRIANGULOS OBLICUANGULOS

$\text{Sen A} = \frac{2}{bc}\sqrt{p(p-a)(p-b)(p-c)}$

$\text{Sen B} = \frac{2}{ac}\sqrt{p(p-a)(p-b)(p-c)}$

$\text{Sen C} = \frac{2}{ab}\sqrt{p(p-a)(p-b)(p-c)}$

$\frac{a}{\text{Sen A}} = \frac{b}{\text{Sen B}} = \frac{c}{\text{Sen C}}$

$A+B+C = 180$

•	$B = \frac{B+C}{2}+\frac{B-C}{2}$
••	$C = \frac{B+C}{2}-\frac{B-C}{2}$

$\text{Cotg A} = \frac{b-a\,\text{Cos C}}{a\,\text{Sen C}} = \frac{c-a\,\text{Cos B}}{a\,\text{Sen B}}$

$\text{Cotg B} = \frac{c-b\,\text{Cos A}}{b\,\text{Sen A}} = \frac{a-b\,\text{Cos C}}{b\,\text{Sen C}}$

$\text{Cotg C} = \frac{a-c\,\text{Cos B}}{c\,\text{Sen B}} = \frac{b-c\,\text{Cos A}}{c\,\text{Sen A}}$

DATOS	INCOGNITAS					
	a	b	c	A	B	C
abc	–	–	–	$\text{tg}\frac{A}{2} = \sqrt{\frac{(p-b)(p-c)}{p(p-a)}}$	$\text{tg}\frac{B}{2} = \sqrt{\frac{(p-a)(p-c)}{p(p-b)}}$	$\text{tg}\frac{C}{2} = \sqrt{\frac{(p-a)(p-b)}{p(p-c)}}$
bcA	$a^2=b^2+c^2-2bc\,\text{Cos A}$	–	–	–	$A+B+C=180$ • $\text{Tg}\frac{B-C}{2} = \frac{b-c}{b+c}\,\text{tg}\frac{B+C}{2}$	$A+B+C=180$ •• $\text{Tg}\frac{B-C}{2} = \frac{b-c}{b+c}\,\text{tg}\frac{B+C}{2}$
abA	–	–	$\frac{a}{\text{Sen A}} = \frac{c}{\text{Sen C}}$	–	$\text{Sen B} = \frac{b}{a}\text{Sen A}$	A+B+C=180
aBC	–	$b = \frac{a\,\text{Sen B}}{\text{Sen }180-B-C}$	$\frac{a}{\text{Sen A}} = \frac{c}{\text{Sen C}}$	A=180− B+C	–	–
aAB	–	$\frac{b}{\text{Sen B}} = \frac{a}{\text{Sen A}}$	$\frac{c}{\text{Sen C}} = \frac{a}{\text{Sen A}}$	–	–	C=180 − A+B

BZB Marca REG. *Patentes TRAM.* CON LICENCIA FISCAL Dep. Legal B-37504 -72