RESEARCH PROPOSAL

What is the initial state that may lead to cooperation? Is cooperation path-dependent? The idea of this research is to derive a solution concept in the class of correlated extensive games. For that purpose, we will consider three structures of the game and one extension (in networks, but needs to be more thought). In the first case we will study cooperation given an exogenous/correlated initial state of the two-players binary action simultaneous-move game and how this different configurations may change equilibrium predictions. The second game extends the first to two stages, and considers a natural "entanglement" between players after the first simultaneous-move step. Given the result in the first step, player 1 and player 2 need to reach an agreement with players 3 and 4, respectively, in "independent" two-players binary action simultaneous-move games. Finally, the third game is an extension of this two-step game to a finitely-long extensive game to study Poincarè recurrences. That means that after a sufficiently long but finite time, return to a state arbitrarily close to (for continuous state systems), or exactly the same as (for discrete state systems), their initial state. A first guess would be that the equilibrium is dependent on applying unitary operators (time evolution operators) to the game. That is, those strategies that follow from the time evolution operator (bloch evolution) are Nash Equilibrium. What could be problematic in this setting is that if this hypothesis is to happen, in each stage of a finitely-long correlated extensive-form game, players believe they consciously play but the equilibrium is path-dependent. That is, "deviations" from what is rational are needed to reach the equilibrium.

Note that a more rigorous introduction to the mathematical framework would be needed, as well as to the economic meaning of the new concepts.

Keywords: Interference; Beliefs; Networks; Recurrence; Extensive games; Entanglement

1. INTRODUCTION

Our premise is that not only how "entangled" an agent is with her interactive counterpart but also previous actions/situations determine the present of a person and so of course, the order of the events have a say in how we subsequently behave. We extend order effects to a strategic setting in which we do not allow for learning, i.e. the same game is not played until the players "know how to play the game" [1]. The aim is to understand the importance of past decisions on how to face present situations and how entangled they are. A superposition state (QPT) contains ontic uncertainty (lack of existence of a feature), whereas a probability mixture (CPT) represents epistemic uncertainty (lack of knowledge) about an existing feature. In other words, ontic uncertainty reflects the nature of the system itself, not our lack of knowledge about it and so the fact that we could question any hidden-variables approach.

1.0.1. Thesis structure

- Set the basics of quantum game theory, quatum game protocol and axiomatization of quantum principles to economics (Dutch book argument): theorems that related quantum principles to rationality. Relationship between quantum operators and and economic analogs. The hope for this approach is that it may be rationalizable in the sense that QPT is also consistent with the Dutch Book theorem of rationality (Pothos et al. 2017) [2].
- The role of the external agent in correlated equilibrium settings in defining equilibrium predictions. That is how the entanglement of the players affect equilibrium predictions. Define an equivalence between the mediator (CE) and players' entanglement. The second game in this environment consist on a first stage in which player 1 and 2 plays a simultaneous normal form game and after this, both players become entangled and then, separately, play against player 3 and 4 respectively a simultaneous normal form game. The idea is to study how different configurations of the entangled state lead to different equilibrium predictions in the second stage for players 1 and 2.
- Applications in the real world:
- Digression: the role of order effects and interference in decision-making. Theory and application. QQ equality of order effects.

[1] Also to avoid quantum Zeno effect, which in a decision setting, the frequency of intermediate judgments slows down opinion change.

[2] See (Oaksford & Chater 2007) for the consistency of CPT with the Dutch Book theorem of rationality.

2. DESCRIPTION OF THE METHODOLOGY: MOTIVATION

2.0.1. Law of total probability

What we call "interference" is merely a breakdown of the law of total probability. To explain the intuition behind this "quantum" concept, let's consider the example in Pothos & Busemeyer (2022). Suppose we disclose to a decision maker some preliminary information and then we compare two conditions. In the first condition we simply measure event B whereas in the second condition we first measure event A and then event B. Given this situation, according to the classical/bayesian probability theory (CPT), the total probability is given by:

$$P(B) = P(A\&B) + P(\sim A\&B) \tag{2.1}$$

Now suppose we study this situation under a quantum probability theory (QPT). The decision maker, after being given the preliminary information, her state is represented by the ket vector $|\psi\rangle$ [3]. In this first case, a positive outcome from the measurement [4] of B is represented by a projector P_B which projects the state vector $|\psi\rangle$ onto the subspace representing B by the matrix product $P_B \cdot |\psi\rangle$. The probability of a positive outcome to action B then equals the squared length (probability amplitude) $\| P_B \cdot |\psi\rangle \|^2$. Now consider the positive outcome of observing A. As we defined in our law of total probability the event "not A" $\sim A$, let's consider $I - P_A$ the orthogonal projector that projects onto the negative outcome for A. Thus, rewriting the projector for B we get $P_B = P_B P_A + P_B(I - P_A)$ and then:

$$\begin{aligned} \| P_B |\psi\rangle \|^2 &= \| P_B P_A |\psi\rangle + P_B(I - P_A) |\psi\rangle \|^2 \\ &= \| P_B P_A |\psi\rangle \|^2 + \| P_B(I - P_A) |\psi\rangle \|^2 + \Delta \end{aligned} \tag{2.2}$$

where Δ is the sum of cross products produced by squaring the sum, and this is what we called the interference term. It takes 0 if the measures/observations are compatible and the QPT satisfies the CPT law of total probability. If Δ is different from 0 (either positive or negative), that means the measures are incompatible [5], and the CPT law of total probability is violated. To emphasize the importance of the order, we will usually write $P(A\&thenB)$ for $P(A\&B)$. Thus, conjunction is no longer conmutative as it is in the CPT. For the sake of clarity, $\| P_B P_A |\psi\rangle \|^2$ is $P(A\&thenB)$, and $\| P_B(I - P_A) |\psi\rangle \|^2$ is $P(\sim A\&B)$.

[3] Kets in our context are simply vectors.

[4] Positive outcome means that B has been observed, that is, a player j has played action $a^j = B$.

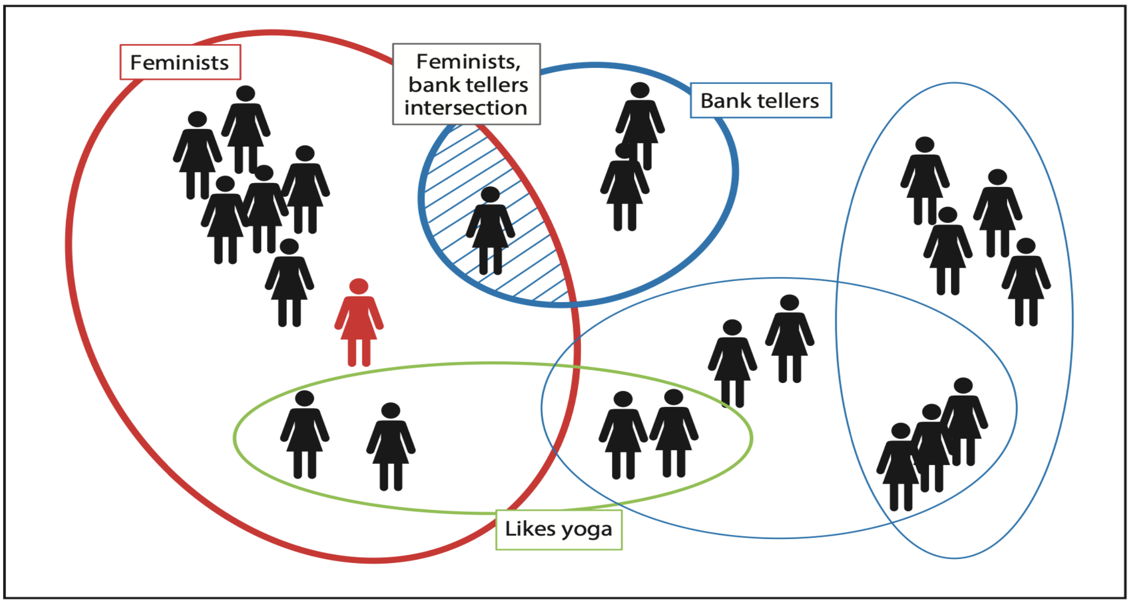
[5] Decisions (or measures) are incompatible if the presence of one decision problem alters our perception of a subsequent decision problem.

2.0.2. Quantum Probability Theory (QPT)

To motivate the use of the QPT, let's use the famous Linda problem stated in Tversky & Kahneman (1983) and extrapolated to the QPT context by Pothos & Busemeyer (2022). For this situation, CPT begins with a sample space such as:

Figure 2.1

Linda problem



Source: Pothos & Busemeyer (2022)

which contains all the various possible realizations for Linda. In the experiment, participants were told of a hypothetical person, Linda, who was described as looking like a feminist but not a bank teller. Participants were asked to rank-order the likelihood of several statements about Linda. The critical statements were that Linda is a feminist (F), Linda is a bank teller (BT), and Linda is a feminist and a bank teller. The controversy arose because the results concluded the following:

$$P(F) > P(F\&BT) > P(BT) \tag{2.3}$$

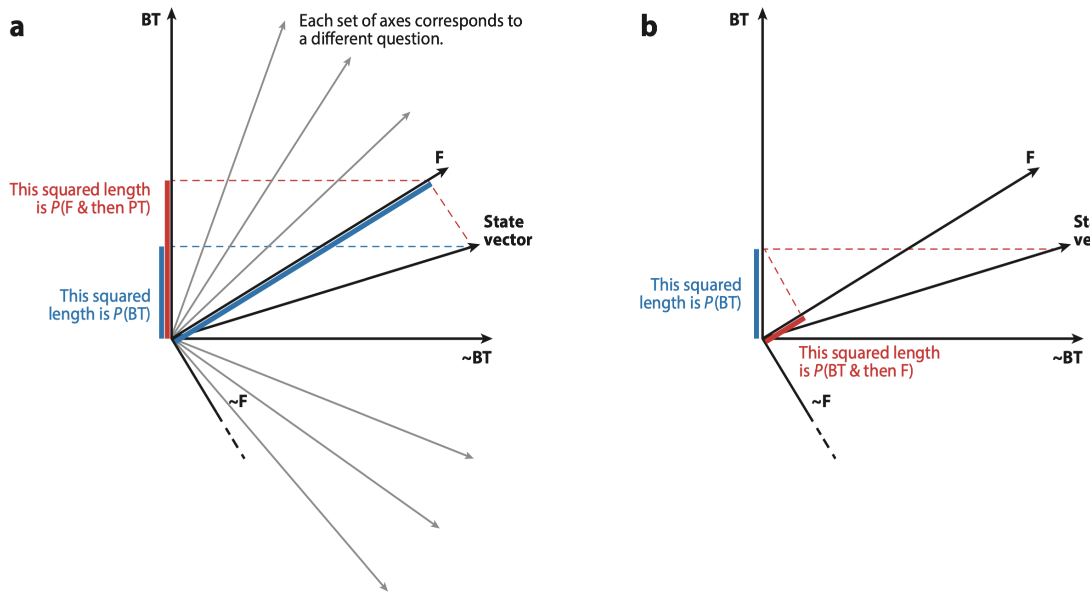
A possible outcome of a question, such as whether or not Linda is a feminist, is represented by a subset of the sample space. The outcome from a pair of questions, such as whether Linda is a feminist and bank teller, is represented by the intersection of subsets, as in the shaded region of overlap between the red and blue ellipses in Figure 2.1. The beliefs a person has about these questions are represented by a probability function that assigns a probability to each subset. For example, the probability that Linda is a feminist is the probability assigned to the red ellipse. The probabilities assigned to the union of mutually exclusive events must add. The larger the subset for a question outcome is, the more possible Lindas we can imagine consistent with this question outcome, and the more likely this question outcome will be—that is, the probability of a question outcome depends on the size of the corresponding subset.

Definition 2.0.2.1 A measurement projection is a projection from the space of all qubits to $\{0, 1\}$. We say that we measure a qubit when we apply a measurement projection to it. The result is a measurement of the said qubit.

On the other hand, QPT begins with a vector space, such as the following two-dimensional space:

Figure 2.2

Projection in a vector space



Source: Pothos & Busemeyer (2022)

Such a vector space contains all possible answers for questions about Linda. For example, for the question "Is Linda a bank teller?" there are two potential answers (yes or no) [6], represented by two unit-length vectors at a 90° angle to each other [7], forming the usual two-dimensional Cartesian plane. The answers to a different question, like "Is Linda feminist?", can be represented by a different pair -yes or no question- of orthogonal vectors rotated by some angle. Basically, QPT is a way to assign probabilities to subspaces whereas CPT assigns probabilities to subsets of elements.

The set of beliefs a person has about these questions is represented by a (unit length) state vector $|\psi\rangle$. The probability of a question outcome is obtained by projecting the state vector onto the subspace representing the answer, and then computing the squared length. For the purpose of this project, to compute the conjunction of two question outcomes (or decision-problems), we typically have to employ a sequential projection, which corresponds to resolving one question/decision-problem after the other. Suppose we are interested in $P(F\&BT)$. In this environment, we need to compute $P(F\&thenBT)$ which requires projecting the state vector onto the F ray and then, projecting this previous projection onto the BT ray. The probability amplitude of this last projection represents

[6] Where yes is BT in the y-axis and no is $\sim BT$ in the x-axis.

[7] Because these answers are mathematically and intuitively orthogonal to each other.

$P(F\&thenBT) = P(F\&BT)$. Working in the other direction, we conclude the nonconmutativity property of QPT (sequence of projections):

$$P(F\&thenBT) \neq P(BT\&thenF) \tag{2.4}$$

which subsequently leads to non-zero interference effects. Moreover, every time we resolve a question/decision-problem, the state vector has to change in a specific way; the vector collapses.

2.0.3. Superposition and the state vector

Theorem 2.0.3.1 (Superposition Principle). The net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually. In other words, the net response can be represented as a linear combination of two responses.

Recall: beliefs of a person judging Linda are represented by a state vector $|\psi\rangle$ lying in the vector space. Graphically, this state vector is not aligned with either the vector spanning the 'definitely feminist' ray or the vector spanning the 'definitely-not-feminist' ray. Instead, the state vector is a superposition state with respect to feminism, because it is formed by a linear combination of these two feminism vectors. Therefore, for a two-action problem (yes/no) called qubit:

$$|\psi\rangle = c_1 * |Yes\rangle + c_2 * |No\rangle \tag{2.5}$$

where $|Yes\rangle$ and $|No\rangle$ are the eigenvectors of each possible action/decision/answer and the value we observe is the eigenvalue of the corresponding eigenvector. The weightings c_1 and c_2 [8] are the probabilities of observing action Yes and No, respectively [9]. At the same time, this state vector can also be considered a superposition with respect to 'bank teller' formed by a linear combination of the 'bank teller' vector and the 'not-bank-teller' vector. Generally, the same state vector can represent beliefs for all decision-problems.

Definition 2.0.3.1 Suppose we have two qubits $a_1 * |Yes\rangle + a_2 * |No\rangle$ and $b_1 * |Yes\rangle + b_2 * |No\rangle$. Their joint (or entangled) state is their tensor product $a_1b_1 * |YesYes\rangle + a_1b_2 * |YesNo\rangle + a_2b_1 * |NoYes\rangle + a_2b_2 * |NoNo\rangle$.

The resolution of ontic uncertainty (using QPT) is called the collapse of the state vector, and the state vector changes in a precise way, so that it aligns with the question outcome. If the person decides that Linda is a feminist, then the new state will be aligned with the 'feminist' ray [10]. That is, making a decision changes the person's mental state.

[8] Note that not in this context, but generally, these coefficients may $c_1, c_2 \in \mathbb{C}$.

[9] These are probability amplitudes and satisfy: $|c_1|^2 + |c_2|^2 = 1$.

[10] And thus, $c_1 = 1$ and $c_2 = 0$.

2.0.4. Game setup

2.0.5. Elements of the game

1. **States:** We will restrict our setting to a two-dimensional space (i.e., two different actions) called qubit, where in Dirac notation is as mentioned before:

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{2.6}$$

where $\alpha, \beta \in \mathbb{C}$. We will work with canonical vectors:

$$|1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |0\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and so any qstate can be a complex linear combination of these two states:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \tag{2.7}$$

which represents a superposition state. Our canonical vectors play the role of being the states that represent the two actions of the agents.

2. **Measurement:** A measurement is what an outsider of the game observes once the agents have made their decision. Therefore, the measurement is a set of outcomes $(o_1, o_2, o_3, ..., o_n)$ and a corresponding complete set of orthonormal projection operators $(A_1, A_2, A_3, ..., A_n)$ such that

$$A_i A_j = \delta_{ij} A_i \qquad \sum_{i=1}^{n} A_i = \mathbb{I}$$

where δ_{ij} is the Kronecker Delta and $\mathbb{I}$ is the identity operator. That is, the measurement of an observable O of a game/system is

$$\mathcal{M}_O = \{(o_1, A_1), (o_2, A_2), ..., (o_n, A_n)\}$$

We will follow quantum insights and consider that the measurement of the game is probabilistic and not deterministic. As explained before, if a measurement of O is performed on a state $|\psi\rangle$, and the measurement outcome o_i has a corresponding projection operator A_i, then the probability that the outcome o_i will be obtained upon measurement is given by:

$$\mathbb{P}(o_i) = \langle\psi|\, A_i\, |\psi\rangle \tag{2.8}$$

where the bra $\langle\psi|$ is the Hermitian conjugate of the ket-vector $|\psi\rangle$. That is:

$$|\psi\rangle^\dagger = (|\psi\rangle^T)^* = \langle\psi|$$

where * represents complex conjugate. Due to this probabilistic description, a quantum system (in general) does not have a specific a priori value for an observable O, and any of the outcomes o_i can be obtained as a measurement result. Therefore, $|\psi\rangle$ is constrained in form by probability:

$$1 = \sum_{i=1}^{n} p(o_i) = \sum_{i=1}^{n} \langle\psi| A_i |\psi\rangle = \langle\psi| (\sum_{i=1}^{n} A_i) |\psi\rangle = \langle\psi| \mathbb{I} |\psi\rangle = \langle\psi|\psi\rangle = ||\psi||^2$$

which implies $|\psi\rangle$ is a normalized vector and $||\psi||^2$ is the norm. Particularly, for a qubit $\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$:

$$|\alpha|^2 + |\beta|^2 = 1$$

Note also that the state of the system changes under measurement and as the outcome of the measurement, this change is also random (because it depends on the outcome). Suppose o_i is the outcome of a measurement of the system O in state $|\psi\rangle$. The post-measurement state is given by (given $p(o_i)$):

$$|\psi_i\rangle = \frac{A_i |\psi\rangle}{\sqrt{p(o_i)}}$$

Upon measurement, the state $|\psi\rangle$ instantaneous and randomly collapses to state $|\psi_i\rangle$ with $p(o_i)$. Consider an example. Suppose a qubit is initially in state:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

and we want to perform a measurement on this state with outcomes and projector operators:

$$\mathcal{M} = \{(0, |0\rangle \langle 0|), (1, |1\rangle \langle 1|)\}$$

Then the probabilities of getting outcomes 0 and 1 are:

$$p(0) = \langle\psi| (|0\rangle \langle 0|) |\psi\rangle = | \langle 0|\psi\rangle |^2 = |\alpha|^2$$
$$p(1) = \langle\psi| (|1\rangle \langle 1|) |\psi\rangle = | \langle 1|\psi\rangle |^2 = |\beta|^2$$

Suppose the outcome of the measurement is 0. Then, the post-measurement state is:

$$|\psi_0\rangle = \frac{(|0\rangle \langle 0|) |\psi\rangle}{\sqrt{p(0)}} = \frac{\alpha |0\rangle}{|\alpha|} \sim |0\rangle$$

and similarly $|\psi_1\rangle \sim |1\rangle$. Therefore, a measurement in the $|0\rangle - |1\rangle$ basis, the state $|\psi\rangle$ collapses into the state $|0\rangle$ with probability $|\alpha|^2$ and into the state $|1\rangle$ with probability $|\beta|^2$, where $|\alpha|^2$ and $|\beta|^2$ are the probability amplitudes.

3. **Evolution:** This basically explains how the state evolves with time as long as it is an isolated quantum system (not interacting with the outside world, or, one which is not being measured). Then, it is described by a linear operator given that normalization is preserved. Therefore, if state $|\psi\rangle$ evolves into state $|\psi'\rangle$ then:

$$|\psi'\rangle = U\,|\psi\rangle$$

for some operator U. If $|\psi\rangle$ is normalized, then also $|\psi'\rangle$ is normalized. That implies:

$$1 = \langle\psi'|\psi'\rangle = \langle\psi|\,U^\dagger U\,|\psi\rangle$$

Since this is true for all normalized vectors, $U^\dagger U = \mathbb{I}$ holds true. Linear operators that satisfy this condition are called Unitary Operators.

4. **Composite Systems:** this specifies how to describe systems which are interacting with each other but are combinedly isolated from the rest of the universe. Suppose, a system A is interacting with a system B (the combined system AB being isolated). If, the state of the isolated system A is described by the Hilbert space $\mathcal{H}_A$ and the state of the isolated system B is described by the Hilbert space $\mathcal{H}_B$ then the state of the combined interacting system AB is described by the tensor product Hilbert space:

$$\mathcal{H}_A \otimes \mathcal{H}_B = span\Big\{|\psi\rangle_A \otimes |\psi\rangle_B \,\Big|\, |\psi\rangle_A \in \mathcal{H}_A, |\psi\rangle_B \in \mathcal{H}_B\Big\}$$

If we consider each player (not the sender) as an independent state from each other, once they start the game, they become a composite system.

Game 1

This simultaneous one-stage game (N, P, u) studies how the entanglement between the two players before the game changes equilibrium predictions with respect to a classical game theoretic approach.

- There are three players $N = \{1, 2, 3\}$ where $N = 3$ is a referee and the other two are explicit players. The pure actions of each player are: $A^1 = \{A, B\}$ and $A^2 = \{A, B\}$.
- The players share the entangled state $|\psi\rangle$ at the start of the game, and do gain access to an additional entangled qubits as play progresses. Then, we will need a mechanism to distribute new entangled states in later information sets of the game.

States are discrete. Then, the initial state is given by (this is one initial state): $|\psi\rangle = \alpha|AA\rangle + \beta|AB\rangle + \gamma|BA\rangle + \delta|BB\rangle$. That means, the first qubit (Player 1's part of the qstate, that is, what player 1 can control in the joint qstate) is in state $|A\rangle$, corresponding to Player 1 choosing action A and the second qubit is in state $|B\rangle$, corresponding to Player 2 choosing action B in the first superposed element $|AB\rangle$ in the entangled state $|\psi\rangle$. Also, $|AB\rangle$ and $|BA\rangle$ are orthogonal basis states ($\langle AB|BA\rangle = 0$) in the composite Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$ (joint qstate), where $\mathcal{H}_1$ is Player 1's Hilbert space, and $\mathcal{H}_2$ is Player 2's Hilbert space, meaning the two pure states are entirely distinct and independent. Each player manipulates only their assigned qubits by applying quantum operations like gates (preserve the probabilities of the qstate) or measurements. For example, in $|AB\rangle$, player 1 can only manipulate first qubit $|A\rangle$ and player 2 can only manipulate second qubit $|B\rangle$.

- Given ontic uncertainty, each player is in a superposition of their potential pure states. That is:

$$|\psi_1\rangle = a|A\rangle + b|B\rangle$$

$$|\psi_2\rangle = c|A\rangle + d|B\rangle$$

  That implies both players have applied a Hadamard gate, that is, randomize between actions (similar to mixed strategies but with the advantage of interference and correlation).

- The game is of complete information. The players know the structure of the game but payoffs are uncertain.

- *Definition of Quantum Correlated Equilibrium:* Let $\mathcal{G}$ be a normal form game with n players. For each player i, let A_i be the set of actions available to player i in $\mathcal{G}$. Consider a 3-tuple $(|\psi\rangle, \Gamma, Q)$ where $|\psi\rangle$ is a pure qstate, Γ is a partition of the qubits of $|\psi\rangle$ into n disjoint sets $q_1, q_2, ..., q_n$, and $Q = (Q_1, ..., Q_n)$ is a collection of n quantum circuits, where circuit Q_i takes as input the qubits q_i (as well as auxiliary $|0\rangle$ qubits) and outputs an action $a_i \in A_i$. Given such a 3-tuple, we denote $D(|\psi\rangle, \Gamma, Q)$ as the distribution resulting over outcomes of $\mathcal{G}$ when each player i applies Q_i to his qubits of $|\psi\rangle$ and plays the result, and let $u_i(D)$ be the expected utility for player i in the outcome distribution D. We say that $(|\psi\rangle, \Gamma, Q)$ is a quantum correlated equilibrium (QCE) if, for all players i and for all quantum circuits Q'_i,

$$u_i(D(|\psi\rangle, \Gamma, Q)) \geq u_i(D(|\psi\rangle), \Gamma, (Q_1, ...Q_{i-1}, Q'_i, Q_{i+1}, ..., Q_n))$$

- Payoffs: the idea of game 1 (and also game 2) is to reach cooperation; that is, both playing either A or B. If both players play (A, A), player 1 and player 2 receive, respectively $(5, 5)$ and if both play (B, B) each player receives $(8, 8)$. If either (A, B) or (B, A) occur, both players receive $(10, 0)$ and $(0, 10)$, respectively. We could use the same "winning" situation as in the CHSH game.

- The idea is to study different initial states in this first simultaneous-move game. For instance, the initial state (as if it was done by a mediator) could be prepared in such a way that players want to cooperate. The intuition is that the initial state act like a hidden "agreement" or "guidance mechanism" that the players can exploit through their local operations. We could impose cooperation and so the initial state would be $|\psi\rangle = \frac{1}{\sqrt{2}}(|AA\rangle + |BB\rangle$.

Game 2

This game is an extension of Game 1 but instead of considering one simultaneous stage, we consider two stages.

- First stage: the same game played before (same players, same action sets and same payoffs) but instead of considering an "exogenous" mediator, no initial state is considered and so entanglement between players arise naturally.
- Second stage: now player 1 plays against player 3 and player 2 against player 4 in two "independent" two-player binary action simultaneous-move game. Now players 1 and 2 need to independently reach an agreement with players 3 and 4, respectively. In this second stage: $A^1 \wedge A^3 = \{C, D\}$ and $A^2 \wedge A^4 = \{E, F\}$. We can use the same payoffs as in the first stage (I think without loss of generality).

The whole question then is, what is the initial state that may lead to cooperation? We try to derive a solution concept in this class of correlated extensive games. A first guess would be that the equilibrium is dependent on applying unitary operators (time evolution operators) to the game. That is, those strategies that follow from the time evolution operator (bloch evolution) are Nash Equilibrium. What could be problematic in this setting is that if this hypothesis is to happen, in each stage of an infinite correlated extensive-form game, players believe they consciously play but the equilibrium is path-dependent. That is, "deviations" from what is rational are needed to reach the equilibrium.

Game 3: Finitely-long extensive-form game

The idea of this game is to study the potential existence of Poincarè recurrences and thus, after a sufficiently long but finite time, return to a state arbitrarily close to (for continuous state systems), or exactly the same as (for discrete state systems), their initial state. It is interesting if we are predestined, to some extent, to play in deterministic finite cycles given the same repeated environment and an arbitrary "starting point" $P(\emptyset)$. The methodology here needs to be more developed.

Game 4: Networks

I need to develop in more detail this extension. First start with a small number of players (for computational feasibility) and a star player. To study Poincarè recurrences, would be interesting to allow the network to change with time.

3. LITERATURE REVIEW

Here I mention some papers I think are relevant for the topic. However, I consider indispensable to receive feedback about the project and bibliography to study more in depth recent advances in sequential game theory (and related, at the discretion of the expert giving me advice). Gibbons (1997) may be too old to be in the list but, not only is a reference guide for game theory but also was published almost on the date of my birth:

Deckelbaum, A. (2011). Quantum Correlated Equilibria in Classical Complete Information Games. arXiv preprint arXiv:1101.3380.

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Pothos, E. M., & Busemeyer, J. R. (2022). Quantum cognition. Annual review of psychology, 73(1), 749-778.

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