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1. INTRODUCTION

> "I learned very early the difference between knowing the name of something and knowing something." – Richard P. Feynman

> "I think it's much more interesting to live not knowing than to have answers which might be wrong. I have approximate answers and possible beliefs and different degrees of uncertainty about different things, but I am not absolutely sure of anything and there are many things I don't know anything about, such as whether it means anything to ask why we're here. I don't have to know an answer. I don't feel frightened not knowing things, by being lost in a mysterious universe without any purpose, which is the way it really is as far as I can tell." – Richard P. Feynman

Much of the existing literature implicitly assumes that agents fully understand and internalize the information they receive. While some models relax this assumption by introducing costs of information acquisition and processing, these costs are typically modeled as "finite". As a result, once an agent incurs a sufficiently "high" cost, she is assumed to be able to fully process the relevant information and make potential optimal decisions. This approach abstracts from a more fundamental question: how difficult is it for an agent to truly understand a signal or a piece of knowledge? More importantly, what are the inherent limits to agents' capacity to interpret and integrate information, even when acquisition and processing are costly but feasible/affordable?

Knowledge plays a central role in economic activity. It underpins innovation and long-run growth (Aghion & Jaravel, 2015; Bottazzi & Peri, 2007; Grossman & Helpman, 1990; Howitt & Aghion, 1998), shapes productivity within firms (Audretsch & Feldman, 1996a, 1996b; Haas & Hansen, 2007; Syverson, 2011), affects the efficiency with which information is transmitted across agents (Arenas et al., 2010; Argenziano et al., 2016; Foray, 2004; Ottaviani & Sørensen, 2006), and influences how individuals and organizations form beliefs about the state of the world (Acemoglu et al., 2016; Andreoni & Mylovanov, 2012; Cheng & Hsiaw, 2022; Kamenica & Gentzkow, 2011). From early work on technological change and the economics of ideas (Mokyr, 2011, 2016; P. Romer, 1993; P. M. Romer, 1990) to modern theories of information frictions and learning (Bergemann et al., 2015; Hellwig et al., 2012; Maćkowiak et al., 2023), economic analysis has consistently emphasized that differences in what agents know, and in how they acquire and process knowledge, are fundamental determinants of economic outcomes.

Despite its importance, knowledge is difficult to model in a satisfactory way. The most common and tractable approach treats knowledge as a scalar stock that accumulates over time (Garicano, 2000; Ide & Talamàs, 2024, 2025). While useful for many applications, this representation abstracts from essential features of knowledge, such as its internal structure and the processes governing its generation and diffusion. In particular, scalar models ignore complementarities between different pieces of information and the fact that understanding advanced concepts typically depends on mastery of more basic ones. More fundamentally, they abstract from what kind of information agents know, focusing only on how much they know. As a result, scalar representations struggle to capture realistic learning dynamics, cognitive constraints, and heterogeneity in informational content (Schlaile et al., 2020).

A richer but still tractable alternative models knowledge as a vector, where each component represents competence in a specific category or type of information, and total knowledge is often defined as the sum of its components (Bogner et al., 2018; Cowan & Jonard, 2004; Luo et al., 2015; Mueller et al., 2017). Vector representations allow for heterogeneity across knowledge dimensions and open the door to more realistic learning processes. However, even this approach frequently treats knowledge components as largely independent. This overlooks the relational nature of knowledge emphasized in recent computational and network-based studies. In such settings, it is not straightforward to characterize how different knowledge categories are related, whether acquiring information in one category positively or negatively affects others, or whether newly received information is genuinely novel to the receiver. Connections between concepts and their recombinations therefore remain ambiguous (Davies & Sankar, 2025; Tywoniak, 2007). As noted by Morone and Taylor (2010) and emphasized by Schlaile et al. (2020), "considering knowledge as a number (or a vector of numbers)...restricts our understanding of the complex structure of knowledge generation and diffusion."

This paper contributes to the literature by proposing a structured yet tractable representation of knowledge that captures interdependencies across informational components while remaining amenable to network-based learning and diffusion analysis. We model knowledge as a vector, but with an important restriction. Rather than representing general knowledge, we focus on knowledge about a single topic, denoted by θ. By imposing a bijective mapping between informational labels and real-valued activations, we ensure that what is learned is uniquely identifiable. A topic is composed of multiple pieces of information that differ in complexity and may be related to one another depending on their similarity, difficulty, and the agent's ability to connect them. This structure allows us to study learning as a constrained and interdependent process, while maintaining analytical and computational tractability. Extending the framework to general knowledge would naturally involve interactions across multiple topics, which could be modeled using a networks-of-networks approach (Schlaile et al., 2020).

Crucially, we do not interpret knowledge as the simple accumulation of independent information units. Knowledge is inherently relational (Nooteboom, 2009; Saviotti, 2011, 2019): agents form connections between different pieces of information, draw inferences, and extrapolate beyond what is explicitly communicated. At the same time, knowledge can be tacit or sticky, learning is path-dependent, and memory is imperfect (Lam et al., 1998; Sobrero & Roberts, 2001; Von Hippel, 1994). These features introduce temporal discrepancies between acquisition and retention and give rise to non-linear learning dynamics. Consequently, learning processes cannot be adequately captured by linear or purely additive rules.

We build on the idea that what individuals learn can be decomposed into smaller informational components ordered by complexity (Boschma & Frenken, 2018; Cristelli et al., 2015; Hidalgo & Hausmann, 2009). Mastery of more advanced content typically requires sufficient understanding of more fundamental elements, even though agents may display uneven knowledge profiles across dimensions. This decomposition provides a natural foundation for modeling learning as the evolution of a structured knowledge vector rather than a single aggregate measure.

More generally, when studying knowledge diffusion, several conceptual levels must be clearly defined (Schlaile et al., 2020). First, it is necessary to specify *what diffuses*. This depends on how knowledge is modeled, whether as numbers, vectors, complex entities, or networks of interrelated concepts. Second, one must specify *how knowledge diffuses*. Diffusion mechanisms depend on the nature of what diffuses and may take the form of simple contagion or complex contagion driven by homophily, social reinforcement, or learning complementarities (Tur et al., 2014; Weng, 2014; Weng et al., 2013). These mechanisms are also closely linked to assumptions about communication, ranging from barter-like exchanges that require mutual benefit (Cowan & Jonard, 2004) to unrestricted information flows (Klarl, 2014). Third, it is necessary to specify *where diffusion takes place*, which is determined by network topology. Communication may occur on regular lattices like grids (Ising model) or Von Neumann neighborhoods, random graphs, small-world networks, or scale-free networks, each implying different spatial and social constraints (Tur & Azagra-Caro, 2018). We can also give an additional interpretation to the network topology. If no structure is imposed on spatial connections, as in Erdős–Rényi random graphs for instance, communication is implicitly assumed to occur without spatial constraints and may therefore take place remotely (e.g., via digital or telecommunication technologies). In contrast, models based on spatial lattices, such as Ising-type frameworks, implicitly assume localized, in-person interactions. Finally, the fourth level concerns the *effects of diffusion and how learning outcomes are measured*, including the extent, speed, and distribution of learning outcomes across agents.

Throughout the paper, we assume truthful communication and focus on the transmission of "hard information" (Milgrom, 1981). This assumption allows us to isolate the limits imposed by cognitive constraints, memory decay, and network structure. Nevertheless, the framework can be extended to environments with persuasion, strategic communication, or "cheap talk" (Crawford & Sobel, 1982), where the content and credibility of messages become endogenous.

The objective of this research is to understand both the power and the limitations of learning when knowledge is structured, relational, and diffuses through different network topologies. By combining a structured representation of knowledge with learning mechanisms inspired by neural networks, we study the topological conditions under which agents can fully learn a topic, fail to do so, or converge to the initial knowledge states. These insights provide a foundation for future work on organizational efficiency, information processing within firms, and belief formation under uncertainty, contributing to more realistic models of economic decision-making in knowledge-intensive environments.

2. LITERATURE REVIEW

To the best of my knowledge, there are no existing studies that explicitly employ neural networks to model human learning dynamics and the diffusion of cultural knowledge in economic environments. Nevertheless, a closely related strand of literature explores the use of neural networks as learning devices in economic models and provides important insights for the present analysis.

In particular, Kuriksha (2021) models agents as neural networks that learn optimal policies from idiosyncratic past experiences. In that framework, learning is experience-based: agents use neural networks to approximate decision rules mapping states into actions over time. The model embeds these learners in a macroeconomic environment based on Aiyagari (1994), and learning is evaluated in terms of agents' ability to solve dynamic optimization problems.

This approach differs fundamentally from the one adopted here. In our setting, agents do not learn decision rules directly. Instead, they acquire explicit pieces of knowledge, which may affect their decision-making performance [1]. Moreover, rather than studying learning in a representative-agent or macroeconomic framework, the present project considers a system of interacting individual agents who learn from one another through decentralized communication and knowledge transmission.

Despite these differences, the findings in Kuriksha (2021) are highly relevant for our purposes. The paper shows that learning is intrinsically difficult and often characterized by instabilities, traps, and systematic biases, which must be mitigated through additional modeling assumptions or learning mechanisms. Importantly, these difficulties are not specific to the particular economic task being studied. Rather, they point to limitations that may be inherent to the learning process itself. This observation directly motivates the present project: if learning frictions and failures arise naturally from the structure of learning, understanding their implications is crucial for economic theory, especially in contexts where knowledge acquisition and diffusion play a central role.

A related, yet distinct, contribution is Zhao et al. (2020), who employ neural networks to model behavioral economic agents. In their framework, neural networks are used to represent agents' utility functions rather than their learning processes or decision rules. By contrast, our work abstracts entirely from preferences and utilities. Neural networks are instead used to represent how agents acquire, store, and internalize knowledge, isolating learning dynamics from incentives and choice behavior.

More broadly, there is a rapidly growing literature that applies machine learning (ML) methods in economics, particularly in macroeconomics (Fernández-Villaverde et al., 2020, 2023; Kahou et al., 2021; Maliar et al., 2019). In this strand, neural networks are primarily used as numerical tools to approximate equilibrium objects and to solve computationally demanding

[1] Though this is not explored in this study.

models with high-dimensional state and control spaces. Closely related is another line of work that uses reinforcement learning (RL) to study policy design and dynamic decision problems, such as the allocation of binary treatments (Athey & Wager, 2021) or stochastic growth models and environments with monetary and fiscal policy (Chen et al., 2021; Shi, 2021). These studies argue that neural networks can be modeling devices for bounded rationality, focusing on how agents learn to make better decisions in complex environments.

The present project departs from this literature in a fundamental way. Rather than studying learning as a means to improve decision-making, we focus on learning as the acquisition of knowledge per se. Agents do not maximize utility, face no strategic incentives, and do not misrepresent information. Communication is truthful, information flows freely, and the objective is to understand the intrinsic limitations of learning dynamics themselves. While knowledge acquisition may ultimately facilitate better decisions, decision-making is deliberately abstracted from in order to isolate the mechanics of learning and diffusion.

Because agents in our framework learn from one another, the paper is closely related to the literature on learning in social networks (Acemoglu & Ozdaglar, 2011; Acemoglu et al., 2011; Bala & Goyal, 1998; Gale & Kariv, 2003; Golub & Sadler, 2016; Mueller-Frank, 2013). In these models, information and beliefs evolve through repeated interactions among agents, and network structure plays a central role in shaping learning outcomes. Our work contributes to this literature by introducing a structured, neural-network-based representation of knowledge that allows learning dynamics to be nonlinear and path-dependent.

Furthermore, learning in our model is sequential, which connects our analysis to the literature on social learning and informational cascades (Arieli & Mueller-Frank, 2014; Banerjee, 1992; Bikhchandani et al., 1992; Çelen & Kariv, 2004; Goeree et al., 2006; Lobel & Sadler, 2015, 2016; Smith & Sørensen, 2000). These papers typically study how agents infer information from the actions or signals of others, often under assumptions about homogeneous or heterogeneous preferences. In contrast, we abstract from preferences and actions entirely and focus exclusively on the evolution of knowledge itself.

Finally, our analysis is also related to the literature on repeated linear updating and opinion dynamics, which examines how beliefs, opinions, or choices evolve when individuals repeatedly influence one another over time (Chatterjee & Seneta, 1977; DeGroot, 1974; DeMarzo et al., 2003; Lehrer & Wagner, 2012; Mossel & Tamuz, 2017). While these models typically impose linear updating rules, our framework replaces them with neural-network-based learning mechanisms. This allows for nonlinear, history-dependent learning dynamics and a richer representation of how knowledge is acquired, transformed, and diffused over time.

3. THE MODEL

3.1. General Environment

What are the topological conditions such that an agent knows everything about a topic θ after a finite period of time T? What are the topological conditions such that an agent has the same initial knowledge $\mathcal{K}_{i0}$ about a topic θ after a finite period of time T? To answer these questions, we will employ a neural network framework to characterize the families of network topologies that enable agents to fully acquire knowledge about θ or, on the other hand, converge to the initial knowledge profile.

Let K be the total knowledge existent about a certain topic θ, where we assume it is constant for all time periods. Likewise, let $K_{i,t}$ be the knowledge person i has about θ at time t. Consider that knowledge is made of pieces of information such that the knowledge set is finite and discrete $K = \{a, b, c, \ldots, z\}$, with cardinality $|K| = 26$. Note that this is an ordered set such that piece of information a represents the most basic content about θ and z the most advanced content. We also assume that the closer the letters, the more related the content.

To measure how much agent i knows about a specific piece of information, we let each "letter" $\Gamma = \gamma_i \in \{a, b, c, \ldots, z\}$ to continuously vary within the unit interval, that corresponds to the activations of each piece. Since Γ is an ordered set where order captures complexity, there is a non-order-preserving mapping $f_i : \Gamma \to [0, 1] \subset \mathbb{R}$ where the space of all such functions is $[0, 1]^{26} = \{(\gamma_a, \gamma_b, \ldots, \gamma_z) : \gamma_k \in [0, 1]\}$, so each agent's knowledge profile is a point in the 26-dimensional unit hypercube. If for some agent i, his knowledge about "b" is $b_i = 0$, it means she knows nothing about it; conversely, if $b_i = 1$ that implies she knows everything about "b".

Assumption 1 (Properties of Initial Knowledge Profiles)**.** Let $K_{i,0} \subset K_0 = K$, $\forall_{i \in \mathcal{I}}$. For any small $\epsilon > 0$:

$$\mathcal{O}(\Gamma) := \forall_{i \in \{1,\ldots,N\}} : \exists k \in \{a, \ldots, z\} \text{ s.t. } \gamma_{i,0}^{k} < 1 - \epsilon \quad \text{(No omniscience)} \tag{3.1}$$

Accordingly, we impose the constraint that an agent cannot fully comprehend a more advanced piece of information without first mastering the more fundamental ones:

$$\mathcal{H}(\Gamma) := \forall_{i \in \{1,\ldots,N\}}, \forall_{k \in \{a,\ldots,z\}} : [\gamma_{i,0}^{k} = 1-\epsilon \implies \bigwedge_{m=0}^{k-1} \gamma_{i,0}^{m} = 1-\epsilon] \quad \text{(Full knowledge hierarchy only)} \tag{3.2}$$

but could be the case that $m_i = 0.6$ and $d_i = 0.3$ (No general hierarchy), for instance. Moreover, no piece of information can be unknown for all agents at the initial knowledge profile such that

$$\mathcal{A}(\Gamma) := \forall_{k \in \{a,b,\ldots,z\}}, \quad \sum_{i=1}^{N} \gamma_{i,0}^{k} > \epsilon \quad \text{(Accessibility of knowledge)} \tag{3.3}$$

All together, the set of allowed initial knowledge profiles is given by:

$$\mathcal{S} = \left\{ \Gamma \in [0,1]^{26N} : \begin{array}{l} \gamma_{i,0}^{l} = 1-\epsilon \Rightarrow \gamma_{i,0}^{m} = 1-\epsilon, \quad \forall_{m<l},\ \forall_{i\in\{1,\dots,N\}}, \\ \gamma_{i,0} \neq \mathbf{1}_{26} - \epsilon, \quad \forall_{i\in\{1,\dots,N\}}, \\ \mathbf{1}_N^\top \Gamma_m > \epsilon, \quad \forall_{m\in\{0,\dots,25\}} \end{array} \right\} \tag{3.4}$$

If we do not impose Condition 3.3, then we allow for discoveries; in that case, agents may extrapolate from the information they possess to generate knowledge about previously "unknown" pieces of information.

Assumption 2 (Initial Knowledge Profiles Distribution)**.** Suppose there exists an order preserving bijective mapping $g : \{a, b, \dots, z\} \to \{0, 1, \dots, 25\}$ so that each letter k corresponds to an integer $g(k) \in \{0, 1, \dots, 25\}$. Given the conditions of Assumption 1, initial knowledge profiles are drawn independently from a truncated normal distribution:

$$\gamma_{i,0}^{k} \sim \mathcal{TN}(\mu_k, \sigma^2, 0, 1), \quad \text{i.i.d. across agents}$$

where

$$\mu_k = \lambda e^{-xk}, \quad 0 < \sigma < 1$$

ensures that basic pieces (small k) have higher expected knowledge than advanced pieces (large k). Note that $\lambda, x \in \mathbb{R}_+$ are constants. The joint distribution over all agents and pieces of information is then

$$f(\Gamma) = \frac{\prod_{i=1}^{N} \prod_{k=0}^{25} f_{\mathcal{TN}}(\gamma_{i,0}^{k} | \mu_k, \sigma^2, 0, 1)\mathbb{I}\{\Gamma \in \mathcal{S}\}}{Z} \tag{3.5}$$

where $\mathcal{S}$ is defined in 3.4 and Z is the normalization constant

$$Z = \int_{\mathcal{S}} \prod_{i=1}^{N} \prod_{k=0}^{25} f_{\mathcal{TN}}(\gamma_{i,0}^{k} | \mu_k, \sigma^2, 0, 1)\, d\Gamma$$

to ensure $f(\Gamma)$ is a proper probability distribution. Note that the unconstrained product distribution is $f_0(\Gamma) = \prod_{i=1}^{N} \prod_{k=0}^{25} f_{\mathcal{TN}}(\gamma_{i,0}^{k} | \mu_k, \sigma^2, 0, 1)$.

We assume that each sender–receiver interaction concerns a single piece of information. No restrictions are imposed on either party's prior knowledge: a receiver may ask about information she does not know or about information she only partially knows, while the sender may or may not possess relevant knowledge. Communication is assumed to be truthful. If the sender has no knowledge of the requested item, she transmits an activation of zero; otherwise, she transmits her entire knowledge of it, with $0 < \gamma_{j,t}^{k} \leq 1$.

Let the following linear stochastic learning-memory rules for each agent i and piece of information γ_i^k,

$$\gamma_{i,t+1}^{A_i} = f(\{\gamma_{i,t}^{A_i}\}, \{\gamma_{j,t}^{A_i \to i}\}, \{\gamma_{i,t}^{NA_i}\}) = (1-\delta_i)\gamma_{i,t}^{A_i} + \kappa_i \frac{\sum_{j \in S_{i,t}^{A_i}} \gamma_{j,t}^{A_i \to i}}{|S_{i,t}^{A_i}|} + \zeta_i \sum_{\gamma_{i,t} \in (K_{i,t} \setminus \{\gamma_{i,t}^{A_i}\})_t} \gamma_{i,t}^{NA_i} \tag{3.6}$$

$$\gamma_{i,t+1}^{NA_i^k} = f(\{\gamma_{i,t}^{NA_i^k}\}, \{\gamma_{i,t}^{A_i}\}, \{\gamma_{i,t}^{NA_i^{-k}}\}) = (1-\delta_i)\gamma_{i,t}^{NA_i^k} + \zeta_i\Bigg(\sum_{\gamma_{i,t}\in(A_i)_t}\gamma_{i,t}^{A_i} + \sum_{\gamma_{i,t}\in(NA_i\setminus\{\gamma_{i,t}^{NA_i^k}\})_t}\gamma_{i,t}^{NA_i^{-k}}\Bigg) \tag{3.7}$$

where the superscript A_i indicates the letter has been discussed by agent i, whereas NA_i means the letter has not been discussed [2]. $S_{i,t}^{A_i} \subseteq D_{i,t}$ is the set of all senders that discussed piece of information A_i with agent i in period t such that $|S_{i,t}^{A_i}| \in \mathbb{R}_+$, and $|D_{i,t}|$ is the degree of agent i at time t. Moreover, $\gamma_{j,t}^{A_i\to i}$ represents the information agent j sent to agent i about the asked piece of information.

Note that conversations between agents represent specific letters discussed between each pair of "connected agents". Each letter is randomly assigned to each "connection". Thus, if agent i is connected to three others, she "asks" three questions—each about a letter drawn uniformly at random, possibly the same or different across connections. Agent i then receives each neighbor's knowledge about the corresponding letter and, in turn, shares her own knowledge of those same letters.

In this context, $\kappa_i \in \kappa \sim \mathcal{TN}(\mu_\kappa, \sigma_\kappa^2, 0, 1)$ is the agent-specific learning rate and follows a truncated normal distribution in $[0, 1]$, whereas $\delta_i \in \delta \sim \mathcal{TN}(\mu_\delta, \sigma_\delta^2, 0, 1)$ is the agent-specific memory rate, and also follows a truncated normal distribution in $[0, 1]$. We assume $\mu_\kappa < \mu_\delta$. This makes cognitive sense since acquiring new knowledge requires active processing, while maintaining existing knowledge is more passive. Empirical evidence supports this intuition. Ebbinghaus (2013) showed that individuals retain approximately $50-60\%$ of learned material after a period without review, implying $\delta > \kappa$ and aligning with the classical forgetting curve. Moreover, this assumption ensures model stability: if κ were greater than δ, agents would learn faster than they forget, resulting in monotonic knowledge accumulation even in the absence of new inquiries.

Regarding ζ_i, it represents the agent-specific extrapolation parameter, which captures the agent's ability to relate pieces of unrelated knowledge to one another [3]. This parameter also follows a truncated normal distribution $\zeta_i \in \zeta \sim \mathcal{TN}(\mu_\zeta, \sigma_\zeta^2, -L, +L)$, where $L > 0$ means the agent is able to relate concepts; $L = 0$ means the agent has a "compartmentalized" learning; $L < 0$ means that for an agent i, learning one thing confuses her about related things or saturates her [4]. We assume $\mu_\zeta < \mu_\kappa < \mu_\delta$. Intuitively, extrapolating from one piece of knowledge to another

[2] In Equations 3.6 and 3.7 we could have allowed for exponential decay of learning by setting δ_i^t. That would have fitted most forgetting curves since they are exponential. However, we have assumed for simplicity linear decay.

[3] This may be a proxy of intelligence.

[4] Therefore, the distributions are as follows:

$$f_\kappa(\kappa_i) = \frac{\frac{1}{\sigma_\kappa}\phi\Big(\frac{\kappa_i-\mu_\kappa}{\sigma_\kappa}\Big)}{[\,\Phi(\frac{1-\mu_\kappa}{\sigma_\kappa}) - \Phi(\frac{0-\mu_\kappa}{\sigma_\kappa})\,]}, \quad 0 \le \kappa_i \le 1$$

$$f_\delta(\delta_i) = \frac{\frac{1}{\sigma_\delta}\phi\Big(\frac{\delta_i-\mu_\delta}{\sigma_\delta}\Big)}{[\,\Phi(\frac{1-\mu_\delta}{\sigma_\delta}) - \Phi(\frac{0-\mu_\delta}{\sigma_\delta})\,]}, \quad 0 \le \delta_i \le 1$$

piece is cognitively demanding and most people struggle to make these leaps without explicit guidance. For variances, we assume $\sigma_\delta^2 < \sigma_\kappa^2 < \sigma_\zeta^2$ since memory rates are more homogeneous across people than learning rates, whereas the ability to make connections between pieces of knowledge varies significantly across people [5].

Note that both equations involve the sum of a finite number of positive terms, which implies that the resulting activations $\gamma_{i,t+1}^{A_i}$ and $\gamma_{i,t+1}^{NA_i^k}$ may exceed the unit. To address this issue, we first introduce a normalization procedure. Specifically, we define a distance metric over the set of informational pieces, where smaller distances indicate greater conceptual proximity and, hence, easier cognitive integration. On top of this, we also bound the equations, ensuring that all knowledge levels remain within the unit interval.

Definition 3.1 (Distance Decay)**.** Let the weighting function $w(d_{l,m})$ be an expression that decays with the distance between two pieces of information l and m, $d_{l,m} \geq 0$ such that $w(d_{l,m}) = e^{-\lambda d_{l,m}}$ where $\lambda > 0$.

This ensures that closer pieces of information have higher weights and distant ones are more difficult to relate so have lower weights. In order to bound $\gamma_{i,t+1}^{A_i}$ and $\gamma_{i,t+1}^{NA_i^k}$, the approach we follow and which is related to how neural network models work is the logistic (sigmoid) mapping:

$$\gamma_{i,t+1}^{A_i(NA_i^k)} = \frac{1}{1 + e^{-z_{i,t+1}}} \tag{3.8}$$

where

$$z_{i,t+1} = (1-\delta_i)\gamma_{i,t}^{A_i} + \kappa_i \frac{\sum_{j \in S_{i,t}^{A_i}} \gamma_{j,t}^{A_i \to i}}{|S_{i,t}^{A_i}|} + \zeta_i \frac{\sum_{\gamma_{i,t} \in (K_{i,t} \setminus \{\gamma_{i,t}^{A_i}\})_t} w(d_{A_i,NA_i})\gamma_{i,t}^{NA_i}}{\sum_{\gamma_{i,t} \in (K_{i,t} \setminus \{\gamma_{i,t}^{A_i}\})_t} w(d_{A_i,NA_i})}, \quad \text{for } \gamma_{i,t+1}^{A_i}$$

or

$$z_{i,t+1} = (1-\delta_i)\gamma_{i,t}^{NA_i^k} + \zeta_i \frac{[\sum_{\gamma_{i,t} \in (A_i)_t} w(d_{NA_i^k,A_i})\gamma_{i,t}^{A_i} + \sum_{\gamma_{i,t} \in (NA_i \setminus \{\gamma_{i,t}^{NA_i^k}\})_t} w(d_{NA_i^k,NA_i^{-k}})\gamma_{i,t}^{NA_i^{-k}}]}{\sum_{\gamma_{i,t} \in (A_i)_t} w(d_{NA_i^k,A_i}) + \sum_{\gamma_{i,t} \in (NA_i \setminus \{\gamma_{i,t}^{NA_i^k}\})_t} w(d_{NA_i^k,NA_i^{-k}})}, \quad \text{for } \gamma_{i,t+1}^{NA_i^k}$$

where $w(d_{A_i,NA_i})$ is the distance between a discussed piece of information and any non-discussed letter, and $w(d_{NA_i^k,NA_i^{-k}})$ is the distance between a non-discussed k letter and any $-k \in$

$$f_\zeta(\zeta_i) = \frac{\frac{1}{\sigma_\zeta}\phi\left(\frac{\zeta_i - \mu_\zeta}{\sigma_\zeta}\right)}{[\,\Phi(\frac{L-\mu_\zeta}{\sigma_\zeta}) - \Phi(\frac{-L-\mu_\zeta}{\sigma_\zeta})\,]}, \quad -L \leq \zeta_i \leq +L$$

where $\phi(\cdot)$ is the standard normal PDF and $\Phi(\cdot)$ is the standard normal CDF.

[5]See Gick and Holyoak (1983), Anderson and Schooler (1991), Snow et al. (1996), Rubin and Wenzel (1996), Halford et al. (1998), Waltz et al. (1999), Heathcote et al. (2000), Wixted (2004), Oberauer et al. (2008), Newell and Rosenbloom (2013), for evidence supporting the previous assumptions.

$(NA_i \backslash \{\gamma_{i,t}^{NA_i^k}\})$. This choice is better suited to our setting than the standard Rectified Linear Unit (ReLU) activation function, $f(x) = \max\{0, x\}$, for the following reasons:

1. **Bounded Knowledge:** Knowledge levels are constrained to the unit interval $[0, 1]$. Sigmoid activation functions naturally enforce this bound, whereas ReLU functions produce unbounded-from-above outputs in $[0, \infty)$.

2. **Smooth Transitions:** Learning is modeled as a gradual process. Sigmoid functions are smooth and differentiable everywhere, while ReLU functions exhibit a sharp kink at zero, introducing non-smooth learning dynamics.

3. **Saturation Effects:** Learning exhibits diminishing returns at higher knowledge levels. Sigmoid functions saturate near the extremes whereas ReLU functions imply linear growth without saturation.

Definition 3.2. (Knowledge Dynamics) For all agents in any social network topology $\mathbf{G}_t$, the evolution of knowledge profiles $\boldsymbol{\gamma} \in [0, 1]^{N\times 26}$ follows the sequential discrete-time dynamical system:

$$\gamma_{t+1} = \mathbf{F}(\gamma_t; \boldsymbol{\kappa}, \boldsymbol{\delta}, \boldsymbol{\zeta}, \lambda, G)$$

defined by Equation 3.8.

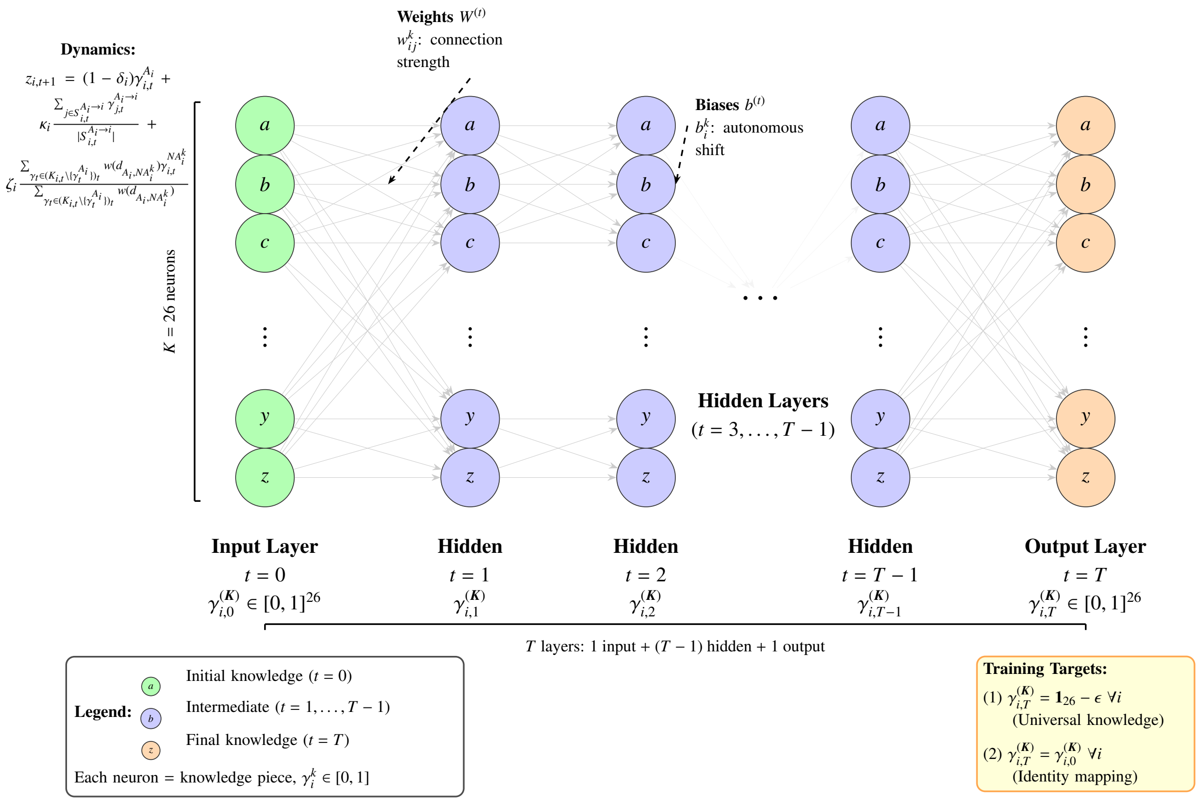
3.2. Feed-Forward Neural Network Analysis

In this framework, and according to Figure 3.1, each layer of the neural network consists of 26 neurons, labeled from a to z, representing the individual pieces of information that collectively define θ. The activations $\gamma_{i,t}^k$ of these neurons range continuously from 0 to 1, where $\gamma_{i,t}^k = 0$ indicates that agent $i \in V = \{1, \ldots, N\}$ has no knowledge about $k \in \{a, b, \ldots, z\}$, and $\gamma_{i,t}^k = 1$ indicates complete knowledge about k.

The network architecture is structured such that the input layer represents the agent's initial knowledge state at $t = 0$, while the $T - 1$ hidden layers capture the evolution of that knowledge over subsequent periods. Because we use a supervised learning framework, the output layer represents the agent's knowledge state at time T, where the activations $\gamma_{i,T}^k$ are expected to approach one for all information components. For the other case of study, activations $\gamma_{i,T}^k$ are expected to converge to the initial activations $\gamma_{i,0}^k$. We observe the initial and final states, as well as the rules governing the dynamics, and ask whether, for some network topologies, these dynamics can indeed lead to the observed endpoint.

Figure 3.1

Feed-forward Neural Network Architecture for Knowledge Dynamics.



Note that all layers have the same dimensions $L_t \in \mathbb{R}^{26}$. To generate the synthetic dataset, we begin by randomly generating the activations corresponding to the 26 neurons (a–z) for N input layers, where each input layer represents a distinct agent in the sample. The initial activation values $\boldsymbol{\gamma}_{i,0} \in [0,1]^{N\times 26}$ are drawn from the joint probability distribution stated in Assumption 2 and according to the conditions in Assumption 1. Throughout, we assume that both network connectivity and the number of agents are fixed over time.

Although agents' parameters and their initial and final knowledge profiles are independent of network topology, knowledge dynamics depend on the unobserved network structure. Consequently, we study learning dynamics across a set of candidate network topologies and unify the update rules. For an agent i with unknown degree $D_{i,t}$, each neighbor communicates exactly one piece of information per period, drawn uniformly at random from the set $K = \{a, b, \dots, z\}$. Agent i therefore receives $D_{i,t}$ informational signals per period, possibly with repetitions. This leads to a unified sigmoid update rule for each piece of information k, given by

$$z_{i,t+1}^k = (1-\delta_i)\gamma_{i,t}^k + \kappa_i \frac{\frac{1}{N-1}\sum_{-i}\gamma_{-i,t}^k}{|S_i^k|} + \zeta_i \frac{\sum_{\gamma_{i,t}\in(K_{i,t}\setminus\{\gamma_{i,t}^k\})_t} w(d_{k,-k})\gamma_{i,t}^{-k}}{\sum_{\gamma_{i,t}\in(K_{i,t}\setminus\{\gamma_{i,t}^k\})_t} w(d_{k,-k})} \tag{3.9}$$

where the second term is the learning from the network, the third term is the learning from extrapolation and $S_i^k \subseteq D_i$ where $|D_i| \approx \langle k \rangle$ such that S_i^k is the set of nodes that discuss k with agent i. For a network with N agents and expected degree $\langle k \rangle$, we randomly and uniformly assign one letter to each of the $\langle k \rangle$ expected edges of every node. Since the network is assumed to be undirected this procedure generates a total of $\frac{N\langle k \rangle}{2}$ edges, and thus letters, for each period of time.

Since we know the average number of edges but not who is connected to whom, we approximate information flows by averaging knowledge about piece k across all $-i \in (V \backslash \{i\})$ agents:

$$\frac{1}{N-1} \sum_{-i} \gamma^k_{-i,t}$$

Only the $\frac{N\langle k \rangle}{2}$ letters drawn uniformly at random in each period correspond to activations that are updated for all agents; for all remaining letters, the learning-from-the-network term is set to zero. If any of these letters is repeated, then $|S_i^k| > 1$. Regarding the set of initial network topologies that we will consider:

- **Erdős–Rényi (ER)**: $\langle k \rangle_{\text{ER}} = p(N-1) \approx pN$.
- **Small-world (Watts–Strogatz)**: $\langle k \rangle_{\text{SW}} = k_0$, where k_0 denotes the number of connections each node has at $t = 0$.
- **Scale-free (Barabási–Albert)**: $\langle k \rangle_{\text{SF}} = 2m$, where m denotes the number of existing nodes to which the new node is connected.
- **Lattice**: $\langle k \rangle_{\text{1D}} = k_0$ for 1-D rings and $\langle k \rangle_{\text{2D}} = 4$ for 2-D Von Neumann neighborhoods. $\langle k \rangle_{\text{MO}} = 8$ for Moore neighborhoods.
- **Star**: $\langle k \rangle_{\text{star}} = \frac{2(N-1)}{N} \approx 2$, for large N.
- **Complete**: $\langle k \rangle_{\text{Com}} = N - 1$.

We define convergence using a maximum time-to-convergence criterion [6] based on the sup norm:

$$\|\boldsymbol{\gamma}_{t+1} - \boldsymbol{\gamma}_t\|_\infty = \max_{i,k} |\gamma^k_{i,t+1} - \gamma^k_{i,t}| < \tau \tag{3.10}$$

where $\boldsymbol{\gamma}_t, \boldsymbol{\gamma}_{t+1} \in [0,1]^{N \times 26}$. Using a tolerance level of $\tau = 10^{-3}$, we treat the system as converged once changes fall below this threshold. We conduct the following two analyses:

1. **Communication effects**: we analyze the role of communication by fixing the number of agents at $N = 50$ and varying the average degree of degree-flexible network topologies. Specifically, we consider $\langle k \rangle \in \{2, 4, 8, 10, 12, 16\}$. For each topology, we examine both regimes of interest: (i) convergence to full learning and (ii) convergence back to the initial knowledge profiles.

[6] See Appendix A for a theoretical alternative.

2. **Population effects**: we consider different population sizes $N \in \{25, 50, 100, 200, 250\}$, while fixing the average degree at $\langle k \rangle = 4$. This choice allows us to compare all network topologies except the star and complete networks. We again examine both regimes of interest.

where the loss functions for each regime are:

$$L_{\text{full}}(\boldsymbol{\gamma}_{i,T}) = \sum_{k=1}^{K} \sum_{i=1}^{N} (\gamma_{i,T}^{k} - (1 - \epsilon))^2 \tag{3.11}$$

$$L_{\text{identity}}(\boldsymbol{\gamma}_{i,T}, \boldsymbol{\gamma}_{i,0}) = \sum_{k=1}^{K} \sum_{i=1}^{N} (\gamma_{i,T}^{k} - \gamma_{i,0}^{k})^2 \tag{3.12}$$

3.2.1. Main Result

For computational tractability, we simulate the model for 10,000 periods; extending the horizon to 1,000,000 periods does not alter the results. Our main finding (see graphs in Appendix C) is that, under these learning dynamics, the system does not converge to either full learning or to the initial knowledge profiles for all agents, regardless of network topology or population size. While some agents may individually achieve full learning, this state does not diffuse through the network, preventing the remaining agents from reaching full learning even in a complete topology. An analogous result holds for convergence to initial knowledge profiles. Instead, agents' knowledge levels generally fluctuate between their initial states and full mastery, reflecting the ongoing interplay between learning and forgetting.

On the other hand, relaxing Condition 3.3 would make convergence even more difficult to achieve. Therefore, weakening this condition does not alter the qualitative results.

4. FUTURE RESEARCH AND EXTENSIONS

A natural extension would reinterpret the knowledge vector $K_{i,t}$ not merely as acquired information, but as an agent's beliefs about topic θ. In this formulation, θ itself could represent the true state of the world, while each component γ^k represents the information necessary to accurately understand that state. This shift enables several important generalizations.

First, we could modify the loss function to explicitly account for misinterpretation—the gap between what an agent believes she knows (her self-assessed $\gamma^k_{i,t}$) and what she actually understands (her true comprehension). The objective would become:

$$L(\boldsymbol{\gamma}_{i,T}, \boldsymbol{\gamma}) = \sum_{k=1}^{K} [(\gamma^k_{i,T} - \gamma^k)^2 + \lambda|\text{self-assessment}^k_{i,T} - \gamma^k_{i,T}|]$$

where the second term penalizes the agent's metacognitive error. This formulation captures the realistic phenomenon that people often overestimate or underestimate their own knowledge, with significant implications for learning efficiency and information-seeking behavior.

More ambitiously, this framework naturally extends to strategic environments. Consider a setting where multiple agents must make decisions based on incomplete and potentially misaligned beliefs about θ. Each agent's strategy would depend on her knowledge vector $K_{i,t}$, which now functions as her subjective belief distribution over the state space. For instance, in a coordination game, agents must decide whether to adopt a new technology. Success requires sufficient understanding of its fundamental principles (basic γ^k) and its advanced applications (complex γ^k). However, agents observe only their own knowledge vectors and must infer others' states through communication, as modeled in our framework. The equilibrium would then depend on: (i) the network topology governing information flows, (ii) the distribution of initial beliefs (Assumption 2), and (iii) agents' strategic incentives to truthfully share knowledge versus withhold it.

On the other hand, the current framework imposes Accessibility of knowledge, assuming no piece of information is completely unknown to the network initially. Relaxing this constraint allows for endogenous knowledge creation or "discoveries." Under this extension, agents could extrapolate beyond existing knowledge to generate genuinely novel information. Mechanically, this would mean allowing some initial conditions where $\sum_{i=1}^{N} \gamma^k_{i,0} = 0$ for certain $k \in \{a, \ldots, z\}$. The extrapolation parameter ζ_i would then play a crucial role: agents with high ζ_i could synthesize connections between known pieces to generate positive activations for previously undiscovered pieces k.

The topology conditions would need to ensure not just efficient diffusion, but also sufficient diversity of initial knowledge profiles and cognitive abilities to enable the emergence of complete knowledge through creative recombination. Such discoveries would be path-dependent: which pieces of information get discovered, and in what order, would depend on the sequence of interactions and the cumulative knowledge state of the network. This introduces rich dynamics where different network structures may converge to different knowledge frontiers, even starting from identical initial distributions.

Finally, we have emphasized that agents in our model learn from one another. However, the underlying network topology governing these interactions is unobserved. While we generate agents' initial knowledge profiles, the structure of the interaction network itself is latent. For this reason, a Graph Neural Network (GNN) approach (Bronstein et al., 2017; Bruna et al., 2013; Gori et al., 2005; Scarselli et al., 2008) is not directly applicable in our setting, as standard GNNs require the input graph—i.e., the adjacency structure—to be known.

Nevertheless, GNNs constitute a natural and promising extension for future research. Conceptually, they are closely related to the neural-network framework employed here, but are designed to operate on non-Euclidean data. Rather than taking a fixed Euclidean dataset consisting of all agents' embeddings- vectors with the initial knowledge profiles- as input, GNNs take as input a graph together with node-, edge-, and/or global-level features. Learning proceeds through a process of message passing (Gilmer et al., 2017), in which nodes iteratively aggregate information from their neighbors according to the given topology. The output preserves the same graph structure and produces predictions at the node, edge, or graph level.

In our context, node-level attributes, specifically nodeweights summarizing agents' knowledge, would be the primary features of interest. If the interaction topology were observed, a GNN could be used to predict whether agents eventually learn the entire topic θ within a given time horizon T, or instead converge back to their initial knowledge profiles.

Importantly, while classical GNNs assume a fixed and known graph, recent extensions allow the network structure itself to be learned or updated endogenously, including graph structure learning, graph rewiring, and dynamic or temporal GNNs[7]. Incorporating such approaches would allow future work to jointly model learning dynamics and the evolution of interaction networks.

[7] Examples include graph structure learning (GSL), neural relational inference, dynamic graph learning, temporal GNNs, event-based graphs, neural point-process GNNs, and latent fully connected graphs with learned sparsity.

5. CONCLUSION

The objective of this short report was to characterize the families of initial network topologies that allow agents to fully learn a given topic θ, as well as those that lead agents to converge back to their initial knowledge profiles. More broadly, we aimed to infer global network properties from node-level knowledge dynamics by studying how learning and forgetting propagate through social interaction networks. To this end, we simulated the proposed learning dynamics over many interaction periods and across a wide range of network topologies and population sizes.

Our results show that, under the learning and memory rules considered here, the system does not converge to either full learning or to the initial knowledge profiles for all agents. This lack of convergence is robust: extending the simulation horizon from $10,000$ to $1,000,000$ periods does not alter the qualitative outcomes [8] and the result holds regardless of network topology or population size, including complete networks.

These findings suggest that, in the absence of stronger assumptions, network structure alone is insufficient to guarantee global learning outcomes. In particular, even highly connected networks do not overcome the intrinsic limitations imposed by bounded cognition, memory decay, and nonlinear learning dynamics.

If the proposed learning rules can be empirically tested or calibrated, these results may have meaningful implications for understanding why knowledge diffusion often remains incomplete in real-world social and organizational networks. More importantly, the negative result provides a clear direction for future research.

[8] Results are also robust to the outputs of the loss functions (see Appendix D).

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A. Alternative to time-to-convergence criterion: Upper bound (UNFINISHED)

From Equation 3.9, the fastest possible growth rate of knowledge is bounded by:

$$(1 - \delta_i) + \kappa_i + \zeta_i \tag{1}$$

Since $\delta_i > \kappa_i > \zeta_i$ in expectation, learning is not explosive. The reduced-form update for agent i, letter k, is

$$\gamma^k_{i,t+1} = \sigma(z_{i,t+1}) = \frac{1}{1 + e^{-z^k_{i,t+1}}}$$

where $z_{i,t+1} = (1 - \delta_i)\gamma^k_{i,t} + \kappa_i\bar{\gamma}_t + \zeta_i\hat{\gamma}_t$, $\bar{\gamma}_t = \frac{\frac{1}{N-1}\sum_{-i}\gamma^k_{-i,t}}{|S_i|}$ is the average learning from neighbors and $\hat{\gamma}_t = \frac{\sum_{\gamma_{i,t}\in(K_{i,t}\setminus\{\gamma^k_{i,t}\})_t} w(d_{k,-k})\gamma^{-k}_{i,t}}{\sum_{\gamma_{i,t}\in(K_{i,t}\setminus\{\gamma^k_{i,t}\})_t} w(d_{k,-k})}$ is the extrapolation term.

Proposition 1. *$\sigma(\cdot)$ is $\frac{1}{4}$-Lipschitz.*

Proof. A function f is Lipschitz if there exists a constant L s.t.

$$|f(x) - f(y)| \leq L|x - y| \quad \forall x, y$$

For differentiable functions, a sufficient condition is:

$$\sup_x |f'(x)| \leq L$$

Therefore, for our sigmoid

$$\sigma'(z) = \sigma(z)(1 - \sigma(z))$$

satisfying $0 \leq \sigma(z) \leq \frac{1}{4}$- the maximum at $z = 0$. Thus, $\sup_{z\in[0,1]} |\sigma'(z)| = \frac{1}{4}$ and $L = \frac{1}{4}$ s.t. $|\sigma(x) - \sigma(y)| \leq \frac{1}{4}|x - y| \quad \forall x, y$. The intuition is that the activation alone shrinks all differences by at least a factor $\frac{1}{4}$. □

Let $B_t = \kappa_i\bar{\gamma}_t + \zeta_i\hat{\gamma}_t$ and $A = (1 - \delta_i)$ so that A is a contraction force and B_t is a bounded reinforcement term.

Take two states γ_t and γ'_t that only differ in their initial condition. Consider:

$$\Delta z := z_{t+1} - z'_{t+1}$$

Substituting and by linearity and the triangle inequality,

$$|\Delta z| \leq (1 - \delta_i)|\gamma^k_{i,t} - \gamma^{k\prime}_{i,t}| + \kappa_i|\bar{\gamma}^k_t - \bar{\gamma}^{k\prime}_t| + \zeta_i|\hat{\gamma}^k_t - \hat{\gamma}^{k\prime}_t|$$

Proposition 2. *$\bar{\gamma}$ and $\hat{\gamma}$ are Lipschitz constants ≤ 1.*

Proof. Take two states γ_t and γ_t'. Let $S = |S_i|$. Then, since the topology is the same under both states:

$$\begin{aligned}|\bar{\gamma}_t^k - \bar{\gamma}_t^{k\prime}| &= \left|\frac{S}{N-1}\sum_{-i}(\gamma_{-i,t}^k - \gamma_{-i,t}^{k\prime})\right| \\ &\leq \frac{S}{N-1}\sum_{-i}\left|\gamma_{-i,t}^k - \gamma_{-i,t}^{k\prime}\right| \quad \text{(triangle inequality)} \\ &\leq \max_{-i}\left|\gamma_{-i,t}^k - \gamma_{-i,t}^{k\prime}\right|\end{aligned}$$

and taking the maximum over letters k:

$$||\bar{\gamma}_t - \bar{\gamma}_t'||_\infty \leq ||\gamma_t - \gamma_t'||_\infty$$

Thus, $\bar{\gamma}_t$ is Lipschitz constant weakly smaller than one. More precisely, equal to 1. That is, an average cannot move more than the largest thing you put into it. Basically, if no individual changes their knowledge by more than ϵ, the average cannot change by more than ϵ.

For $\hat{\gamma}_t$, since this is an extrapolation, we need to be more careful. $\hat{\gamma}_t$ in our model is a deterministic function of current knowledge, built from linear combinations and bounded in $(0, 1)$. That is, it has a generic form $\hat{\gamma}_t = H(\gamma_t)$ where where H satisfies:

$$\sum_j |H_{ij}| \leq 1 \tag{2}$$

Note that 2 runs over all the knowledge entries that are used to form the extrapolation: $-k$.

In our case, $H = \sum_{\gamma_{i,t}\in(K_{i,t}\setminus\{\gamma_{i,t}^k\})_t} \frac{w(d_{k,-k})}{\sum_{\gamma_{i,t}\in(K_{i,t}\setminus\{\gamma_{i,t}^k\})_t} w(d_{k,-k})}\gamma_{i,t}^{-k}$. Weights $w(d_{k,-k})$ depend only on semantic distances, not on γ, and the denominator is a constant given the knowledge set $K_{i,t}$. Note that a mapping F is linear if:

$$F(ax + by) = aF(x) + bF(y)$$

Given our operator H, it is linear in the vector of activations since the denominator is constant and the numerator is a weighted sum of inputs. Therefore, H is a normalized linear (affine) operator with Lipschitz constant ≤ 1. If H is linear:

$$\hat{\gamma}_t = M\gamma_t$$

and every row of M sums in absolute value to ≤ 1, then:

$$||M(\gamma_t - \gamma_t')||_\infty \leq ||\gamma_t - \gamma_t'||_\infty$$

Thus, $\hat{\gamma}_t$ is also Lipschitz constant weakly smaller than one. This is exactly the same logic as with averaging.

Extrapolation rearranges or smooths existing information — it does not invent new variation. Taking averages or making inferences cannot make society more sensitive to small belief changes than society already is.

□

By Proposition 2:

$$|\Delta z| \leq \Big[(1 - \delta_i) + \kappa_i + \zeta_i\Big] \|\gamma_t - \gamma_t'\|_\infty$$

B. Parameters

Table 1 summarizes the values of all parameters employed in our simulation exercises.

Table 1

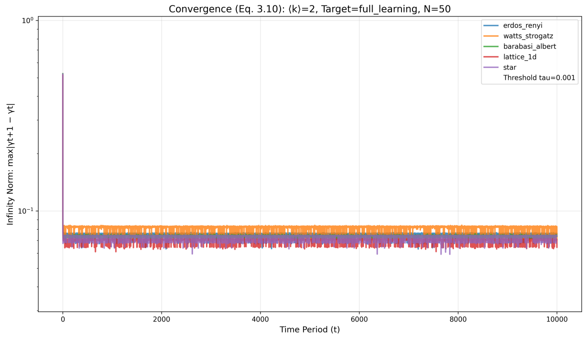
Model Parameters and Configuration

Parameter	Symbol	Description	Value
Knowledge Representation			
Number of pieces	K	Pieces of information (a–z)	26
Epsilon	ϵ	Threshold for "almost 1/0"	0.01
Convergence Criteria			
Tolerance	τ	Convergence tolerance	10^{-3}
Max iterations	$T_{\max}$	Safety limit	10,000
Initial Knowledge Distribution			
Parameters	λ_K, X_K, σ_K	Decay rate, exponential, std dev	0.8, 0.15, 0.15
Agent Parameters (Truncated Normal)			
Learning rate	$\kappa \sim \mathcal{N}(\mu_\kappa, \sigma_\kappa^2)$	Mean, std dev, support	0.3, 0.1, [0, 1]
Memory decay	$\delta \sim \mathcal{N}(\mu_\delta, \sigma_\delta^2)$	Mean, std dev, support	0.5, 0.08, [0, 1]
Extrapolation	$\zeta \sim \mathcal{N}(\mu_\zeta, \sigma_\zeta^2)$	Mean, std dev, support	0.15, 0.15, $[-0.3, 0.3]$
Distance Decay & Experimental Design			
Distance decay	λ_{dist}	Similarity decay parameter	0.5
Population sizes	N	Agents in network	25, 50, 100, 200, 250
Average degrees	$\langle k \rangle$	Connections per agent	2, 4, 8, 10, 12, 16
Network Topologies: Erdős-Rényi, Watts-Strogatz ($p_{rewire} = 0.1$), Barabási-Albert, 1D/2D Lattice, Star			

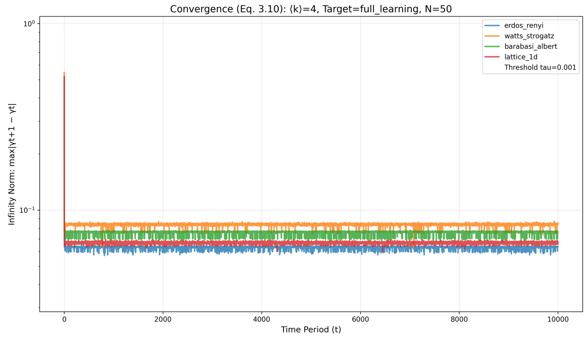
C. Results

Figure 1

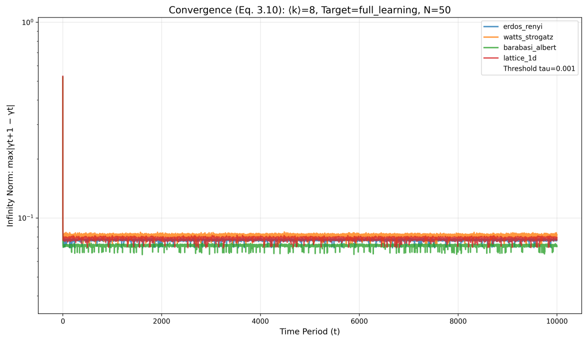
Communication effects: Full learning



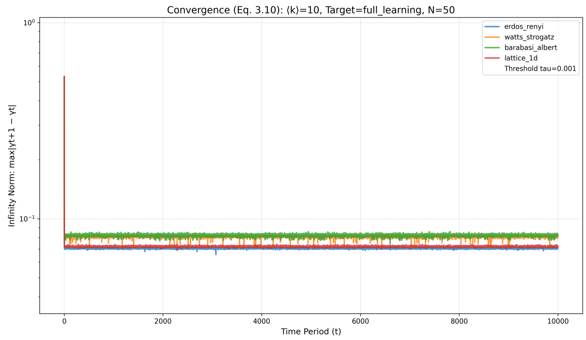
(a) *Average degree of 2*



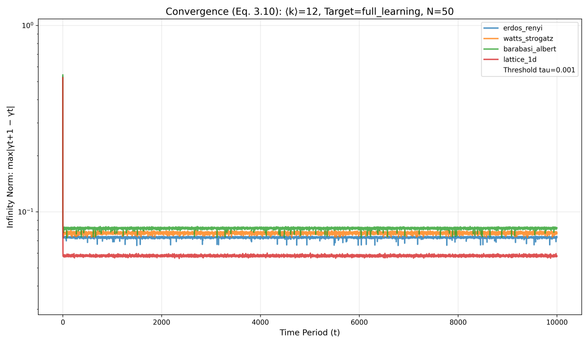
(b) *Average degree of 4*



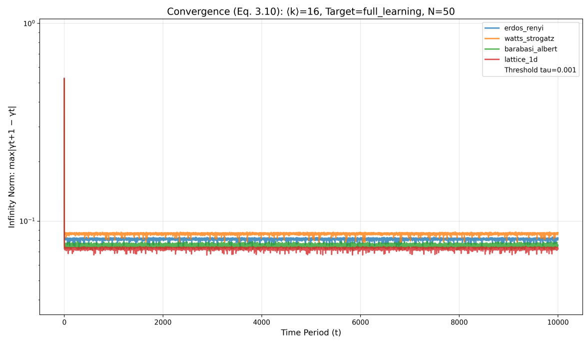
(c) *Average degree of 8*



(d) *Average degree of 10*



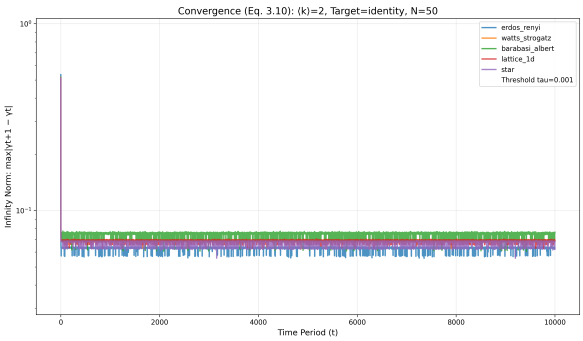
(e) *Average degree of 12*



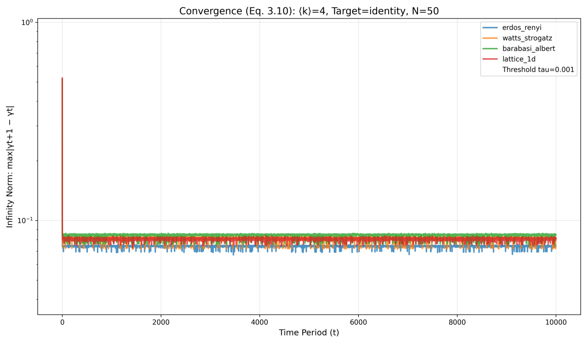
(f) *Average degree of 16*

Figure 2

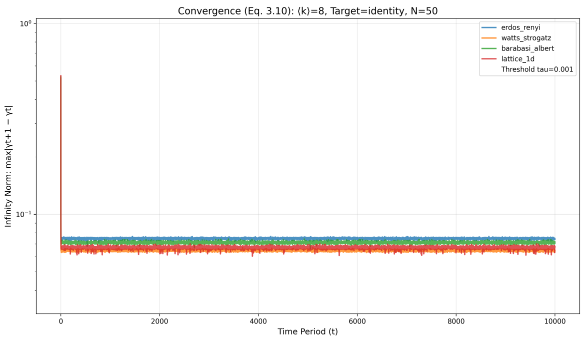
Communication effects: Identity



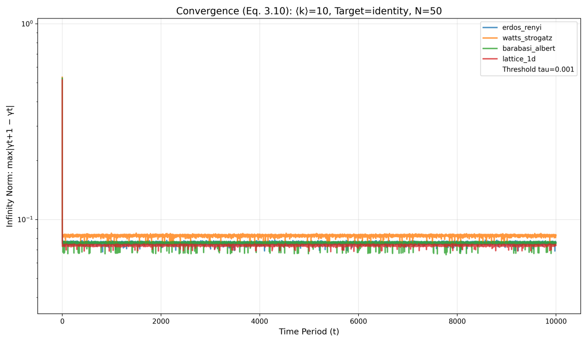
(a) *Average degree of 2*



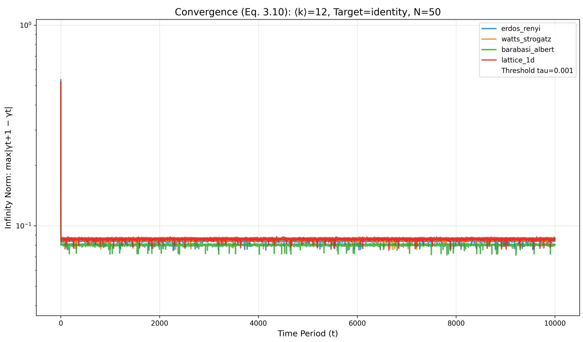
(b) *Average degree of 4*



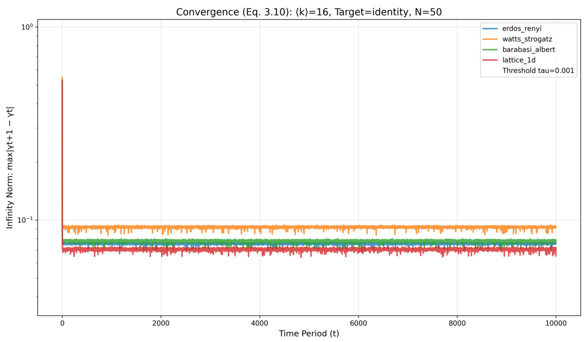
(c) *Average degree of 8*



(d) *Average degree of 10*



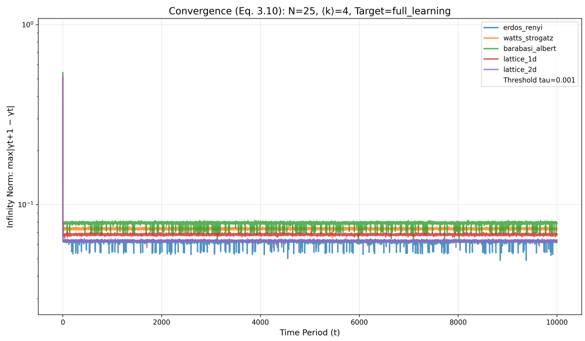
(e) *Average degree of 12*



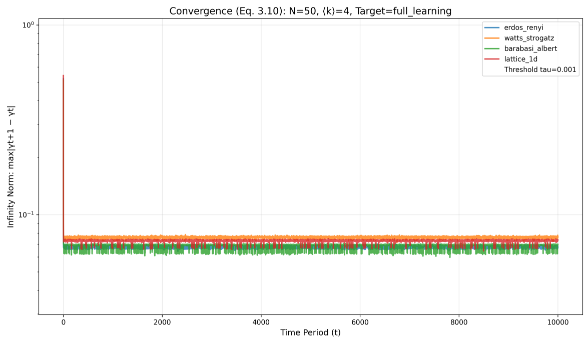
(f) *Average degree of 16*

Figure 3

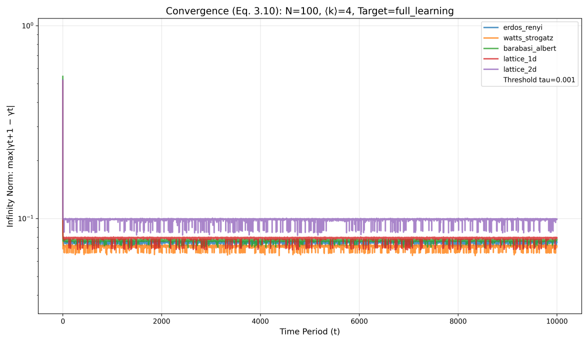
Population effects: Full learning



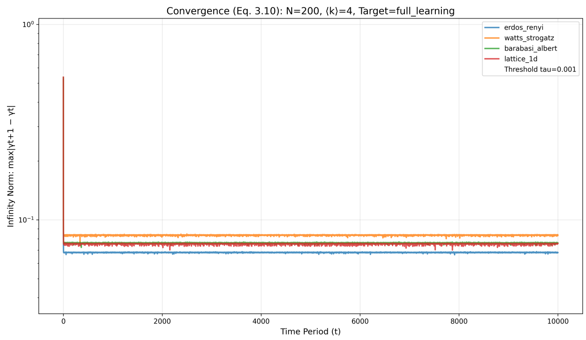
(a) *Population size* $N = 25$



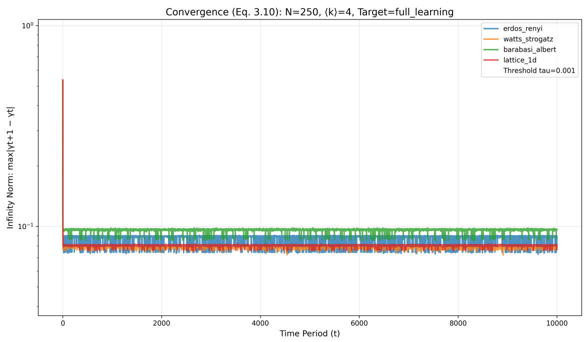
(b) *Population size* $N = 50$



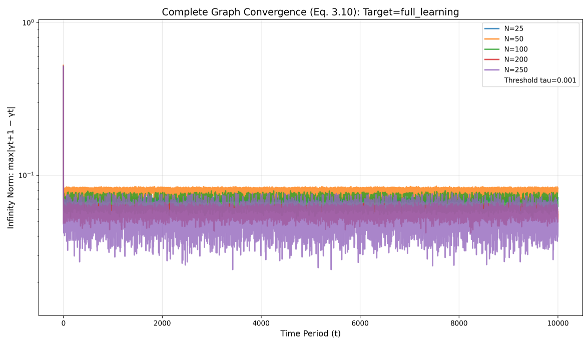
(c) *Population size* $N = 100$



(d) *Population size* $N = 200$



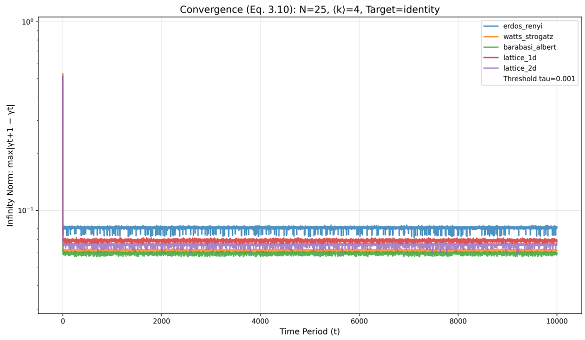
(e) *Population size* $N = 250$



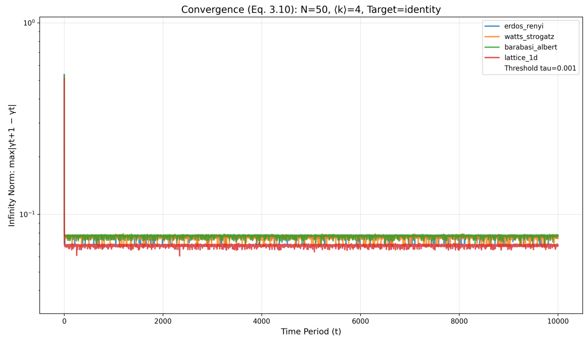
(f) *Complete topology*

Figure 4

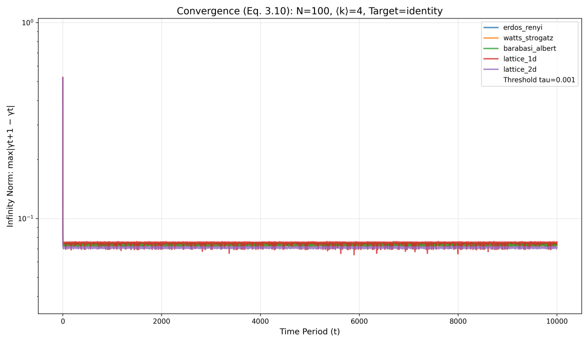
Population effects: Identity



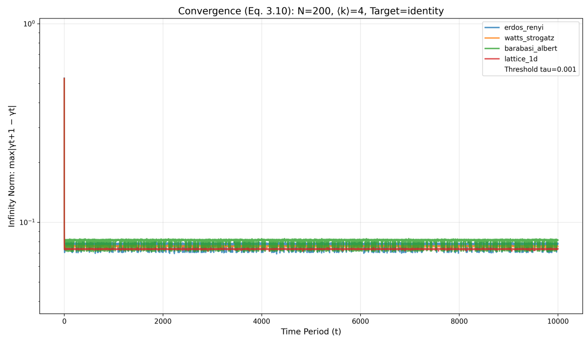
(a) *Population size* $N = 25$



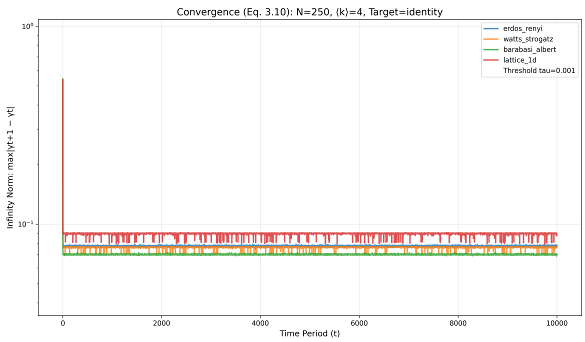
(b) *Population size* $N = 50$



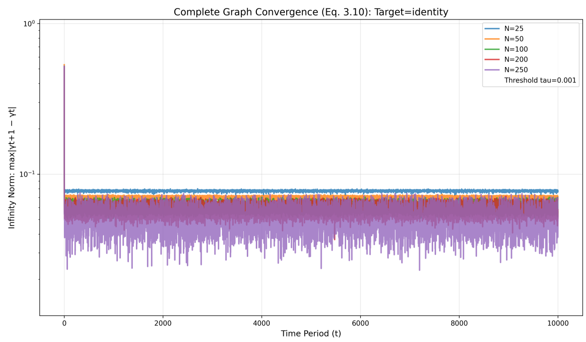
(c) *Population size* $N = 100$



(d) *Population size* $N = 200$



(e) *Population size* $N = 250$

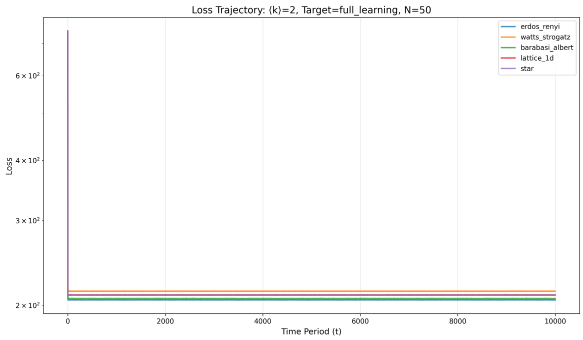


(f) *Complete topology*

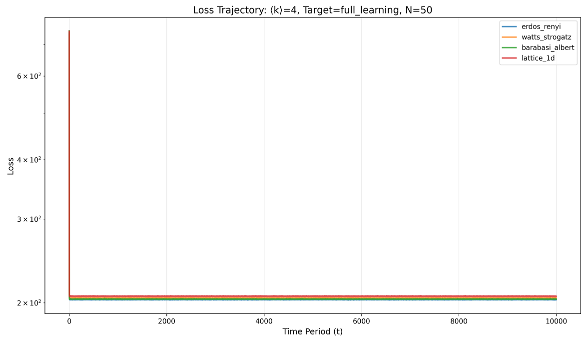
D. Loss function outcomes

Figure 5

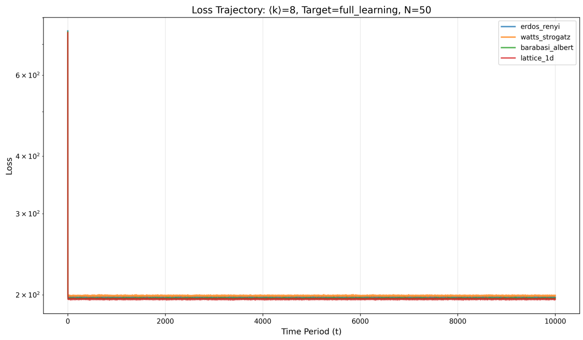
Communication effects: Full learning



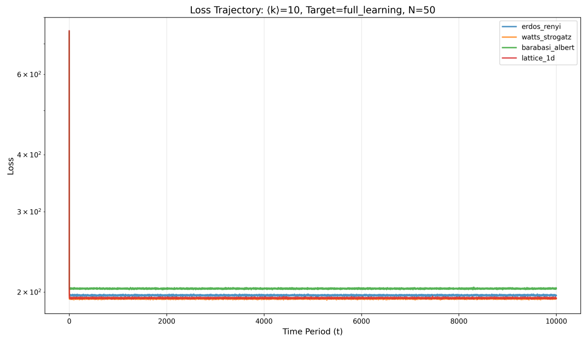
(a) *Average degree of 2*



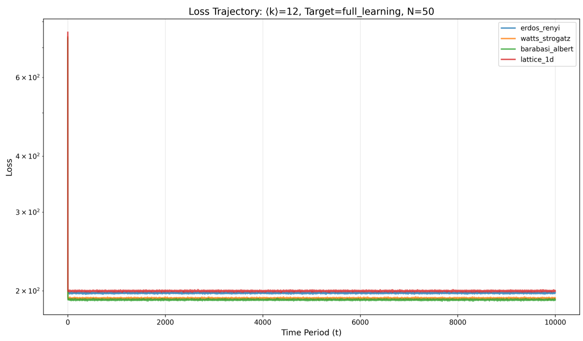
(b) *Average degree of 4*



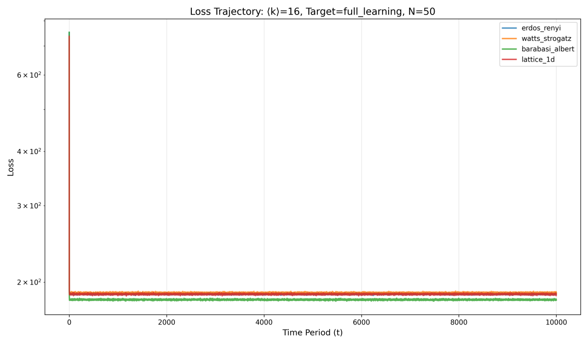
(c) *Average degree of 8*



(d) *Average degree of 10*



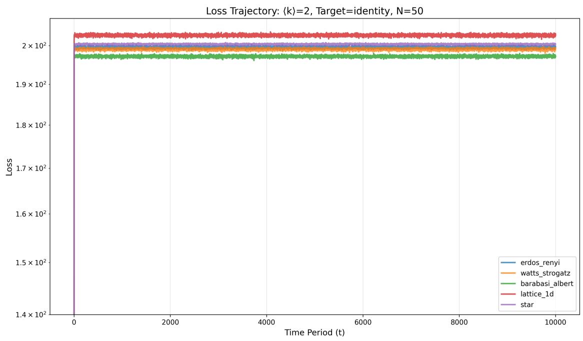
(e) *Average degree of 12*



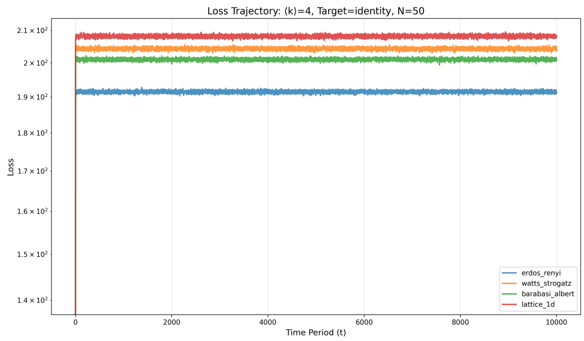
(f) *Average degree of 16*

Figure 6

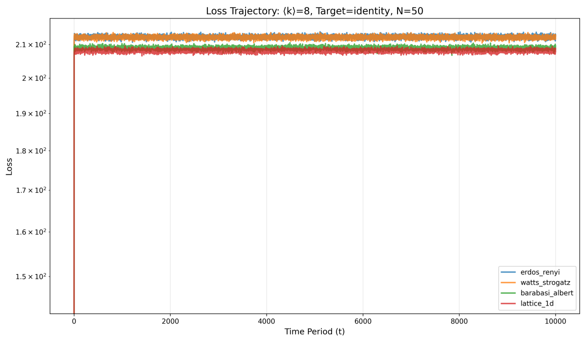
Communication effects: Identity



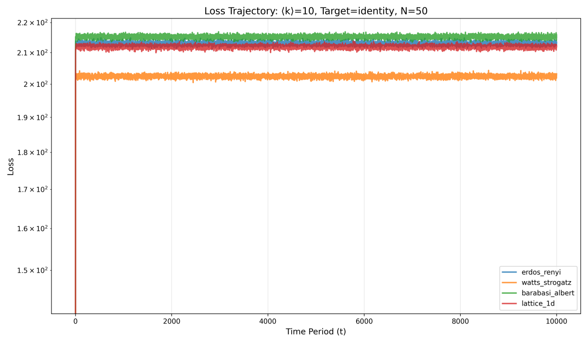
(a) *Average degree of 2*



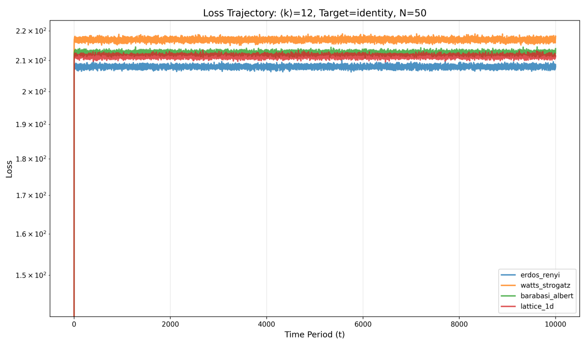
(b) *Average degree of 4*



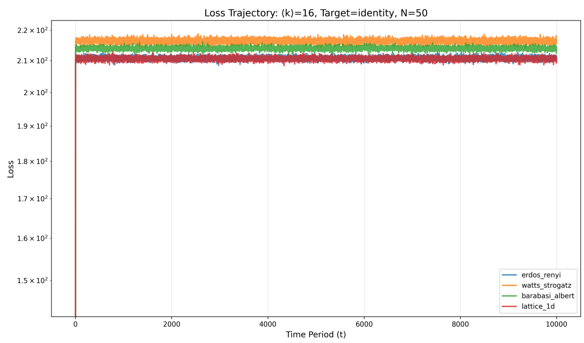
(c) *Average degree of 8*



(d) *Average degree of 10*



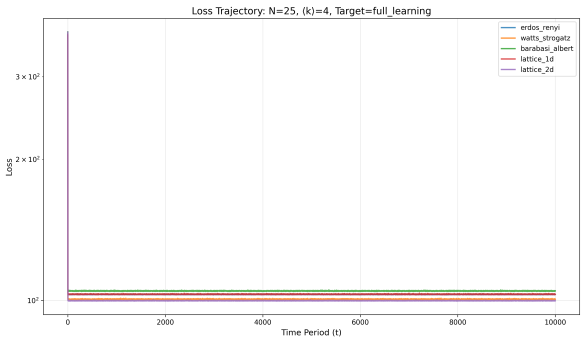
(e) *Average degree of 12*



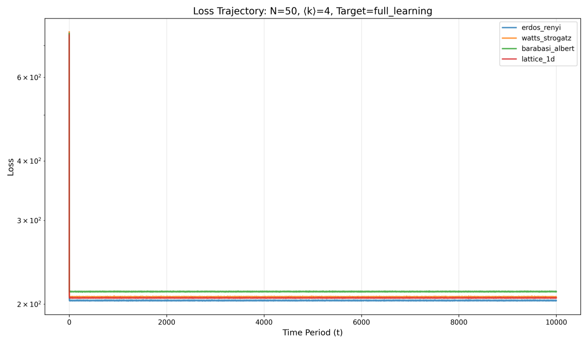
(f) *Average degree of 16*

Figure 7

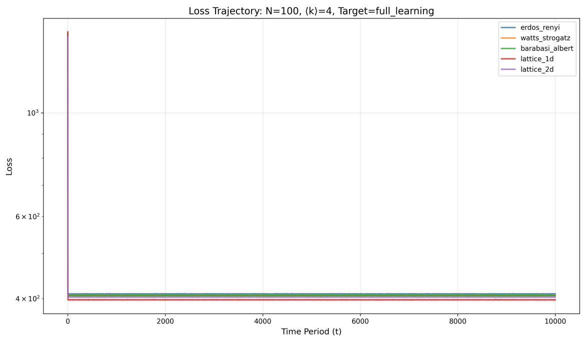
Population effects: Full learning



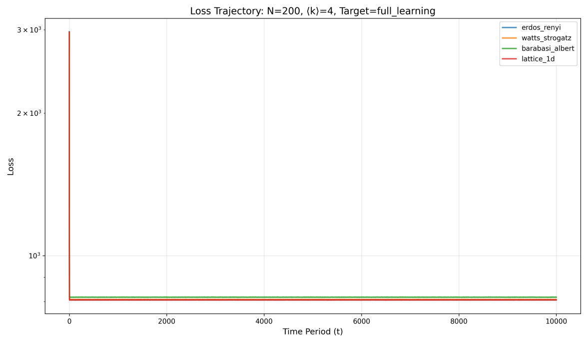
(a) *Population size* $N = 25$



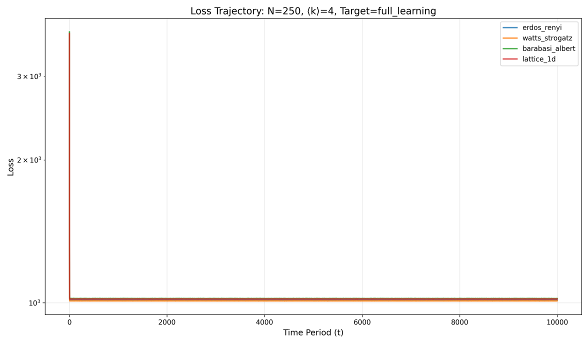
(b) *Population size* $N = 50$



(c) *Population size* $N = 100$



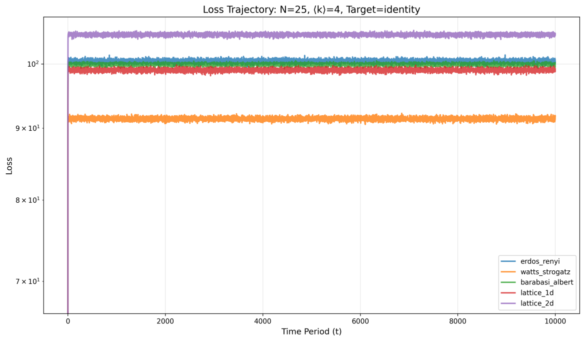
(d) *Population size* $N = 200$



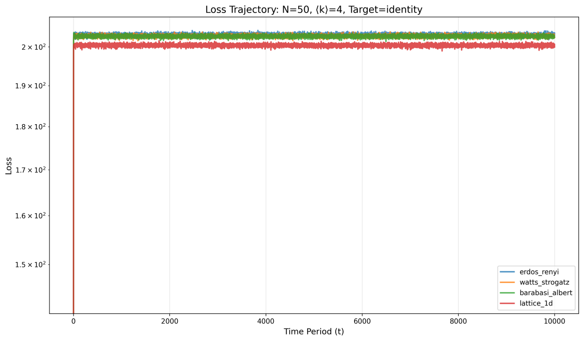
(e) *Population size* $N = 250$

Figure 8

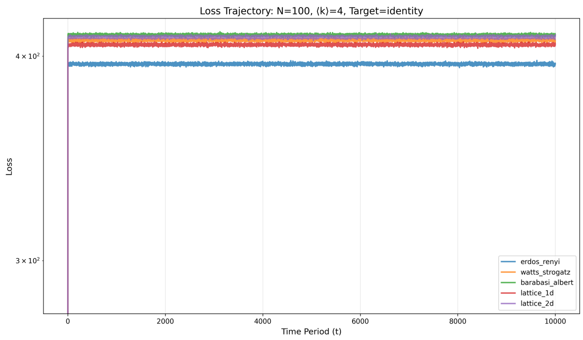
Population effects: Identity



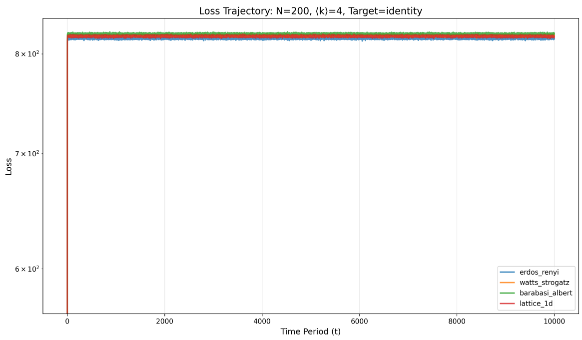
(a) *Population size N* = 25



(b) *Population size N* = 50



(c) *Population size N* = 100



(d) *Population size N* = 200

(e) *Population size N* = 250