

# Optimizing Energy Market Trading with Conformal Predictions and Conditional Value at Risk

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**Abstract**—We optimized the market trading of a renewable energy generation operator with conditional value at risk based on probabilistic forecasts made with a conformalized Long Short-term Memory (LSTM) recurrent neural network. This work demonstrates an end-to-end workflow of how field data can be ingested, analyzed, and exploited to reduce risk exposure for the operator. This is financially beneficial to the individual operator, but taken to a large scale this methodology increases the incentive for renewable generation participation which should drive cost and emissions down. This work used only eight features with favorable results, so it is expected that further studies and more advanced models with the same architecture would provide better yield.

**Index Terms**—Julia, Flux, LSTM, Conformal Prediction, conditional value at risk, renewable energy, energy markets, optimization, revenue, risk management

## I. INTRODUCTION

The IEEE Hybrid Energy Forecasting and Trading Competition [1] challenges participants to make day-ahead, half-hourly probabilistic forecasts of solar and wind energy production for a solar farm and Hornsea-1 Wind Farm in the east of England with a combined 3.6 GW capacity and then maximize revenue through commitment in the day-ahead market. Any difference between the committed energy and actual energy is traded at the single settlement price (SSP). The implied task is to also forecast the market prices so that the operator can reduce their risk exposure from both the energy production and market prices.

### A. Motivation

This project serves as a fitting capstone for this course as it applies many topics covered ranging from unit commitment and energy market trading to advanced predictive and prescriptive analytics for complex and uncertain events. It is an interesting and practical opportunity to wrestle with the available resources to make the best decisions for the operator. Lastly, we find it a compelling problem because reducing the risk for renewable energy generation operators will encourage more participation and be of a net benefit to investors, consumers, and the environment.

### B. Objectives

Our primary objective is to maximize the operator's revenue by making bids on the day-ahead market which minimizes risk based on the forecasted energy production and market prices. We achieve this by training a Long Short-Term Memory

(LSTM) model to make point predictions of these values for the next day. We then employ methods to quantify the uncertainty of these predictions. First, we use Conformal Prediction (CP) to calculate prediction intervals which give some assurance that our bid will be within some range above or below the predicted value and evaluate the revenues from this. Then, we use Conditional Value at Risk (CVaR) by generating scenarios that simulate the errors in our predictions and set a bid that maximizes revenue while minimizing the risk of a worst-case scenario.

### C. Literature Review

Conformal prediction, or conformal inference is a user-friendly method to quantify uncertainty intervals for models. These intervals are distribution-free in the sense that they have explicit, non-asymptotic guarantees without distribution or model assumptions [2]. This is useful in the implementation of our project because it is used to determine the uncertainty without assuming an underlying distribution, potentially skewing the confidence intervals. It uses a calibration dataset to make accurate inferences about the uncertainty of the model that was trained on a training data set. Another trading approach is to use scenarios to simulate future eventualities and evaluate the outcome of different decisions in those scenarios. This approach uses an optimization model to optimize a bidding curve that allows the operator to stabilize revenue and avoid risks [3]. This approach aims to avoid risky bidding policies for the operator to protect against worst-case scenarios by making more conservative bids to minimize risk. Both these approaches are areas of research that we implement and compare the results.

## II. DATA ANALYSIS

We have obtained datasets from two sources: the competition itself which provides the energy production data through the Rebase API [4] and the VisualCrossing API [5] which provides the weather data. The energy data details the solar production, wind production, DAP, and SSP in half-hourly increments. The weather data is treated as historic for the period preceding a given forecast and as a weather forecast for the forecast horizon. If deployed, the model would need to operate only using forecasted weather data. This approach is acceptable for this study as 48-hour-ahead weather forecasts are typically very accurate and we are only incorporating basic weather features.

### A. Data Insights

The tables below provide summary statistics for the energy and weather data used throughout this report. We have the

amount of power produced from solar, wind, and combined total power as well as DAP and SSP. The DAP is the Intermittent Market Reference Price published by the Low Carbon Contracts Company. It is the weighted average of the prices from Great Britain's two-day-ahead auctions, operated by NordPool and EPEX Spot. The SSP is calculated and distributed by Elexon and is the price paid for energy imbalances in the market. As our task is to make bids on the total energy rather than the solar or wind specifically, we combine these two as Total Energy. This quantity is less volatile than the individual sources as adding two random variables or convoluting them will reduce the variance. This effect is observed as we see the median total energy is greater than the sum of the median solar and wind energy and the mean value is closer to the median value. For the market prices we see similar mean and median values in the DAP and SSP but the domain of the SSP is much larger. Also, it is notable that both have negative values indicating there are times of an energy surplus being penalized by the market.

Table I: Rebase Energy Data Summary

	variable	mean	min	median	max
	String	Float32	Float32	Float64	Float32
1	Solar MW/hr	276.497	0.0	7.85413	1853.73
2	Wind MW/hr	543.425	0.0	698.674	826.254
3	Total Energy MW/hr	819.922	0.0	778.34	2367.19
4	DAP GBP/MWh	56.0618	-23.77	61.565	112.23
5	SSP GBP/MWh	55.2193	-88.0	55.595	177.71

Looking at the tabular summary of the weather data, we observe that the mean and medians are roughly in the middle of the feature ranges for all, but cloud cover. With a mean of 71% and a median of 92%, we see that the data is heavily skewed towards being more cloudy.

Table II: Weather Data Summary

	variable	mean	min	median	max
	String	Float32	Float32	Float64	Float32
1	Temperature (°C)	8.54791	-2.3	8.0	19.0
2	Wind Speed (kph)	16.9168	0.9	16.3	46.4
3	Wind Direction (°)	191.088	2.0	200.0	359.0
4	Cloud Cover (%)	71.4654	0.0	91.6	100.0
5	Visibility (km)	14.4783	0.0	14.9	29.8

## B. Data Visualizations

We first examine the histograms of our weather and energy features. The weather data is consistent with what could be ascertained from the summary statistics.

The histograms of the energy data provide a new context for our analysis. We observe how the Total Energy resembles a Normal distribution compared to the Exponential decay of the Solar Energy and the Beta distribution of the Wind Energy. The market prices also resemble a normal distribution with two notable exceptions. They both have fat tails which indicates there are times when very low prices are observed with relative frequency. This is a primary source of risk for trading. The second difference is that the SSP exhibits a bimodal distribution. The fat left tail of the SSP is associated with energy surplus, the right peak is associated with energy deficit, and the central peak is the price when the system is relatively balanced. This sets up several distinct states of the system which could greatly alter

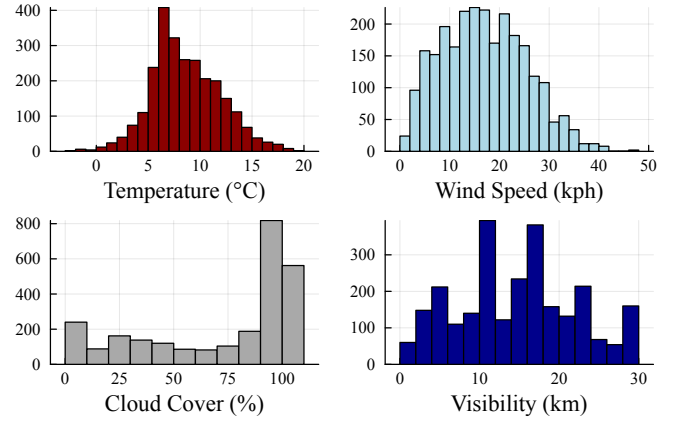


Fig. 1: Weather Data Histograms

the best trading bid regardless of the actual energy production. For instance, if the system is in a deficit, a high-risk operator could commit no energy and trade everything produced at a price that is possibly much higher than the day ahead price. In reality, this likely requires more knowledge of the future and control of the system than is available to any operator and any arbitrage situation would quickly be eliminated by the market.

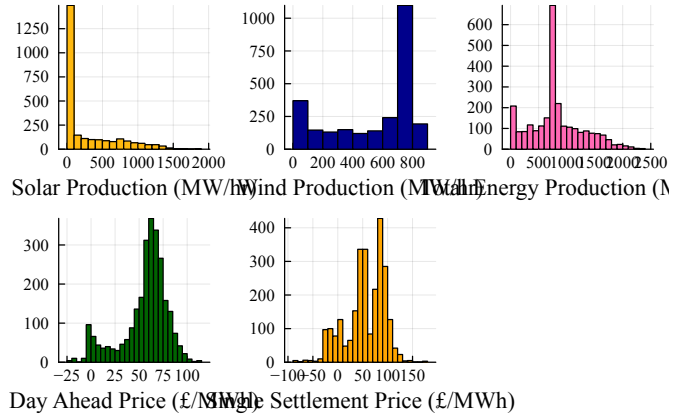


Fig. 2: Energy Data Histograms

Next, we study the day-ahead and then the single settlement prices as time histories and their correlations to weather and daily seasonality. In each group of plots, the top plot shows the entire history of the data while the second provides a closer look at a single week. The bottom plots show the prices vs. temperature and total energy production with the right side showing the daily seasonality. The relative variability of the SSP compared to the DAP is highlighted by the scales being the same in the first and second plots.

For the DAP, we see a clear pattern with a few random dips in the price but eventually returning to the seasonal mean. The week plot provides a clearer look at the typical pattern. The daily seasonal plot shows the duck curve associated with high prices in the morning and evening which is driven by both energy production and cyclical demand. The shape of the curves is there is a concentrated group at the upper end of the prices, but there are many times when price curves are much lower. In the temperature plot, we see that the price tends to show the same general fluctuations regardless of the

temperature. There are two days around April 9th which seem to have low costs associated with the warmest days. This relative price indifference to temperature is unexpected but may be explained by the type of weather typical for this region and the current lack of air conditioning systems. The bottom plot shows how the price changes with total energy production from solar and wind. We cannot pick an obvious pattern looking at the monthly plot, but we can see that prices are generally higher at times of day when production is higher. There are two drivers which are somewhat counteracting each other. The first is that demand is typically going to be higher because people are awake and using electricity. At the same time, production is higher because the sun is up and powering solar panels. The dynamics in the system are complex as price, demand, weather, and production are inextricably linked.

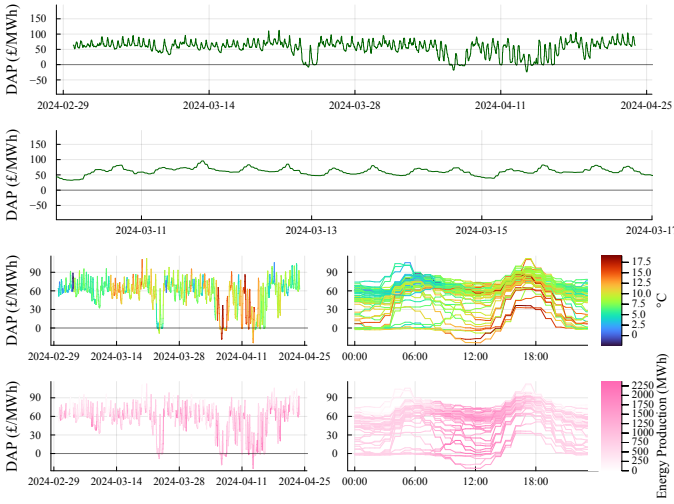


Fig. 3: Day Ahead Prices

We see different characteristics with the SSP as the signal is hardly distinguishable from a random walk. The domain of the SSP is The SSP is a different story as the time series is twice that of the DAP and it traverses from low values to high values in cycles that are not obvious. The only seasonality observed is that around 18:00, the SSP typically constricts to a range between about 25 and 100. The curves themselves look like spaghetti which further indicates that the change in SSP is practically random. As mentioned above, if the SSP became predictable, the market would eliminate the arbitrage opportunity.

The next data visualization depicts a statistical summary of the energy production and market prices for each day in the period as well as grouped by day of the week and hour of the day. The violin plots indicate the density of observations near that level for the given group. The boxplots quantify the spread by showing the mean, quartiles, and outliers. These plots capture the trend over the entire time and the daily seasonality better than a single time series plot. Where before the SSP looked like a random walk, we can observe some correlation to the DAP. Still, we note that the variance of the SSP is much wider. The duck curve for the DAP is more pronounced and we see there is a fairly right upper and lower quartiles with outliers far beyond this range. Lastly, we note that the energy

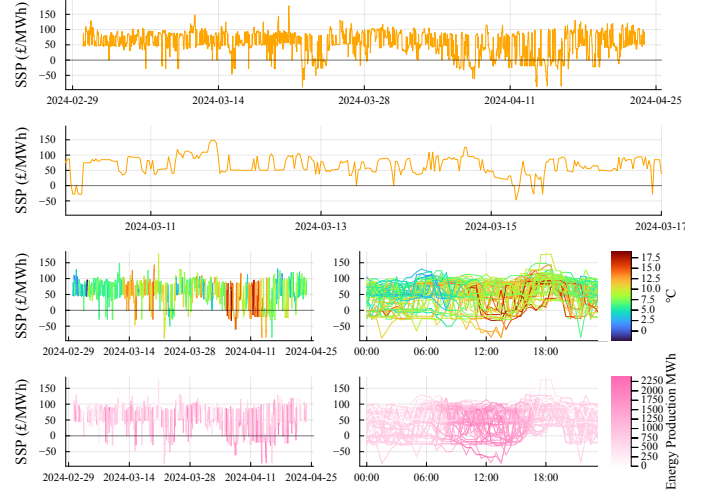


Fig. 4: Single Settlement Prices

production is somewhat random over a longer period, but fairly consistent on a daily cycle.

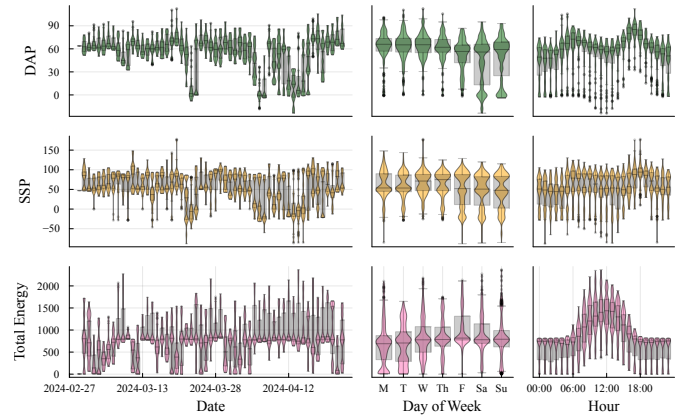


Fig. 5: Single Day, Weekly, and Daily Trends

The pairplot shows the direct correlation or lack thereof among the key variables.

The group of marginal kernel density estimate (KDE) plots offers a final visualization of the correlation among the key features. At the top left we see a slight positive correlation between Total Energy and Wind Speed. At the top right, we observe that the DAP and Total Energy are perpendicular to each other, indicating no correlation. The same observation is true for the Total Energy and SSP in the bottom left. In the bottom right we see two correlated regions. Most significantly, we see that the very negative SSP prices are associated with the lowest DAP prices. Then we see a general correlation for the main region of prices but the two upper peaks of the SSP are independent of the DAP.

### C. Data Conclusions

The analysis of the time series and correlations among the data support our hypothesis that there are predictable dynamics at play which would allow for accurate predictions. However, the dynamics are not straightforward and highly nonlinear. We consider this system to be an ideal one to model with a neural

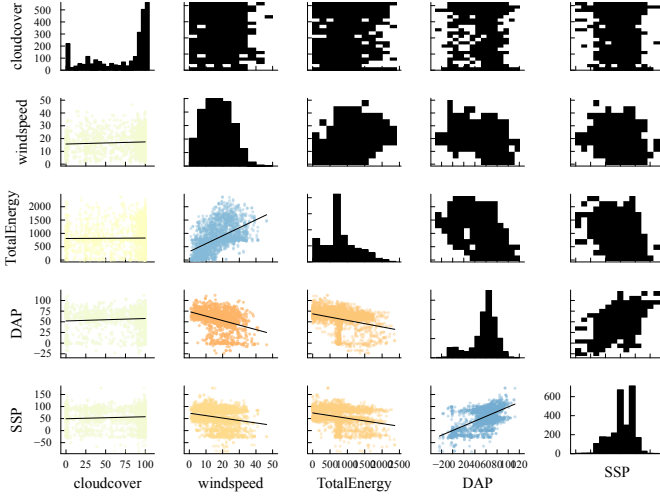


Fig. 6: Correlation PairPlot

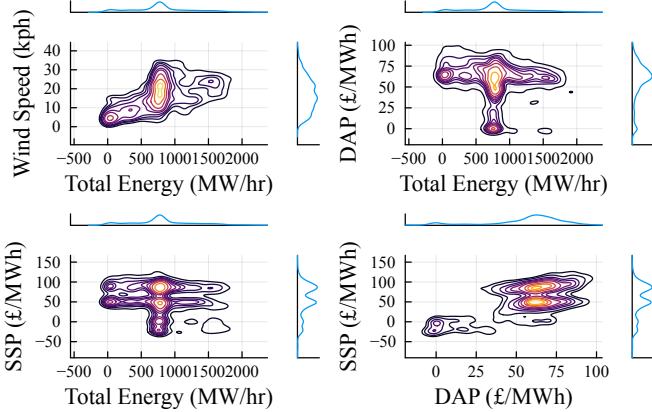


Fig. 7: Marginal KDEs

network as it can approximate any function. While we lose explainability and we are only left with a point forecast, we can use the methods mentioned at the outset to use the predictions in probabilistic trading strategies. The next section will detail the LSTM model we used to make these predictions.

### III. LONG SHORT-TERM MEMORY MODEL

We implemented an LSTM recurrent neural network multi-target regressor to predict the solar, wind, total energy, DAP, and SSP for the next time step. We reasoned that an LSTM solves the vanishing gradient problem associated with generic recurrent neural networks and is built to work with time-series data. The complex dynamics among the features can be approximated by the dense output layer. Outputting all targets at once is computationally efficient and provides the model with additional information when updating the parameters. The seasonality of the solar production and DAP were provided directly as sinusoidal waves with periods of 24 hours and 12 hours, respectively. All inputs and outputs were normalized so that the different scales of the multi-target regression targets would not affect the model's learning.

#### A. Training

We used Julia's deep learning library, Flux [6], to build and train the LSTM. The workflow is as follows:

1. Define the loss function. We used mean squared error as we wanted to penalize outliers.
2. Build the LSTM. Our LSTM consists of two LSTM layers followed by a dense output layer with a sigmoid activation. The input size, twelve, is the length of the weather and energy columns plus two for the seasonality components. The output length, five, is the length of the energy columns. The hidden dimension size is eight. This results in a model with 1,293 trainable parameters.
3. Set up the optimizer. We used the Adam optimizer with a learning rate of  $1e-4$ .
4. Train the model on the training data. A special note for training LSTMs is that it can be important to reset the LSTM and then call it on the first batch of data before training it on the subsequent batches. This primes the LSTM's hidden state so that gradients are based on the recent data and not some arbitrary state. We trained the model for 128 epochs on 45% of the available data with batch sizes of 32.

#### B. Performance

We simulated the trading environment by making predictions up to 48 hours out from a given time. For instance, we would run the model on data up to 8:00 AM on Monday, the time at which we must submit bids for the period of 8:30 AM Tuesday to 8:00 AM Wednesday. Beginning at 8:30 AM on Monday, the model pulls "forecasted" weather data and its own latest predictions to predict the next outputs. This continues until the prediction period is complete. We observe the performance on the testing data below.

Qualitatively, we observe the LSTM predictions are sensible and follow the general characteristics of the target variables. The predicted values are plotted in color over the actual values in grey. The solar and wind predictions capture the general moving average but are not as accurate individually as the total energy predictions. The DAP prediction also generally captures the trend and seasonality of the actual values. The SSP predictions are, as expected, less precise than the other predictions. It is unable to capture the random fluctuations in the values but it stays in the general range of the actual values.

We check the Mean Absolute and Root Mean Squared Error for all three testing sets to ensure the model is not overfitting. The results are shown in the tables below.

	feature	Train	Calib	Test
	Symbol	Float64	Float64	Float64
1	Solar	148.681	149.791	210.36
2	Wind	181.207	216.488	193.169
3	TotalEnergy	294.591	252.243	318.488
4	DAP	17.853	22.0404	21.6718
5	SSP	32.5873	37.1497	41.2682

The error values are fairly consistent across datasets for the two error measures indicating that we have not overfit the model.



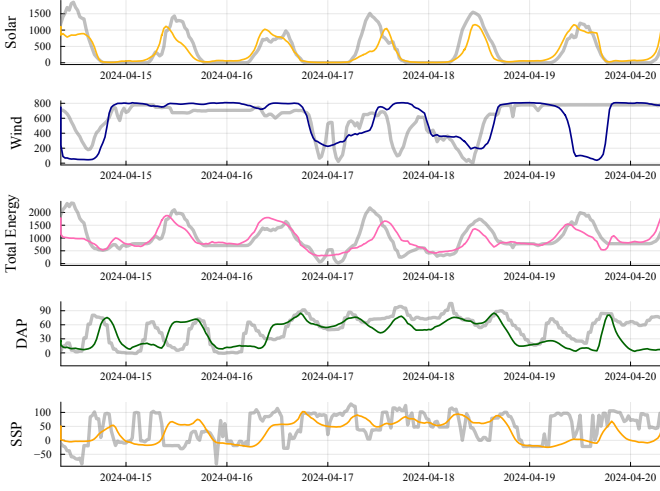


Fig. 8: Predictions

Table IV: Root Mean Square Error

	features	Train	Calib	Test
	Symbol	Float64	Float64	Float64
1	Solar	331.406	242.563	331.406
2	Wind	270.718	327.327	270.718
3	TotalEnergy	426.21	351.866	426.21
4	DAP	29.0975	28.7697	29.0975
5	SSP	51.722	47.0617	51.722

Looking at the residual histograms below, we see they resemble Normal distributions centered on zero. This indicates we will be able to conformalize the model to obtain a probabilistic forecast. Solar residuals have most values at zero because the model correctly predicts no solar energy production at night. We get around this by using the combined energy as we don't explicitly have to know the solar and the wind energy production.

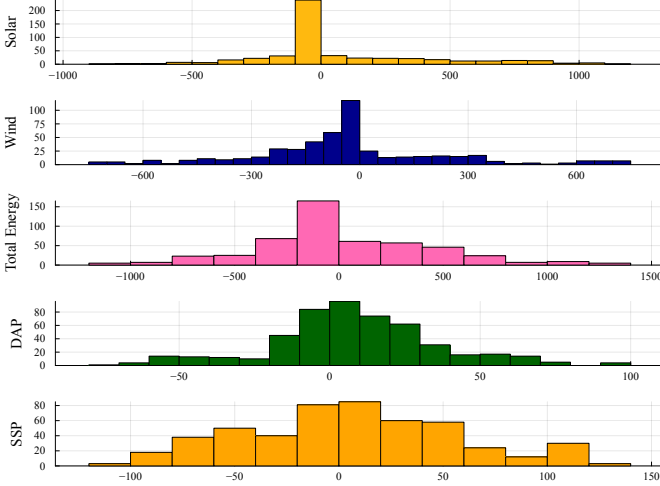


Fig. 9: Calibration Data Residuals

#### IV. CONFORMALIZING LSTM

We are satisfied with the performance of the LSTM, but there is an unquantified risk in using it because it only outputs a point forecast. Our first method to deal with this is to conformalize

the model with calibration data. This will allow us to get confidence intervals around the prediction which can be used in many ways by the operator. This is a powerful and simple method to extend the utility of a model like the LSTM.

#### A. Implementation

The process is essentially accomplished in two steps. First, non-conformity scores are calculated for each prediction in the calibration data. Second, a desired quantile is retrieved which represents the probability that the true value will be within a certain amount of non-conformity with that quantile. For our purposes, nonconformity scores are just the mean error and they are indexed by time.

#### B. Performance

The heatmaps of the non-conformity of the Total Energy, DAP, and SSP show the results. The x-axis is the time of day, and the y-axis is the probability of the true value being within the non-conformity score indicated by the color bar. Blue values are desirable as they indicate a low non-conformity score while red indicates a high non-conformity score. For instance, at 3:00 AM, the Total Energy score was low with a value of around 200 with probability 1. So we can be very confident the true value will be within 200 MWh of the prediction. Conversely, the DAP at 7:00 AM has a relatively high non-conformity score that dips down below the 0.5 quantile. This means that we cannot even be sure that true DAP will be within 40 GBP/MWh with better than 50% probability.

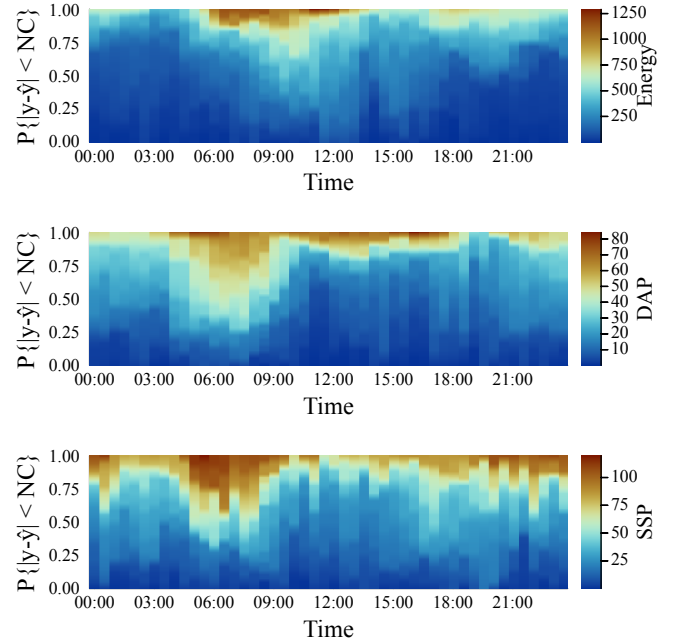


Fig. 10: Nonconformity Scores Probabilities

#### V. MARKET TRADING

The predictions of the energy production and market prices are used to inform the bidding for the following day using

different strategies for comparison. The operator cannot store energy or commit to negative energy production. The strategies explored are:

1. Commit the Point Forecast
2. Commit the Point Forecast - a 10% confidence interval obtained from Conformal Prediction
3. Conditional Value at Risk

These strategies are compared to a benchmark of the revenue generated from perfectly predicting and committing the total energy produced. We consider this to be the most reasonable benchmark as anything better amounts to timing the SSP market. Revenue is calculated for this competition according to this formula(1) which approximates the impact of the difference between the energy prediction and actual production on the settlement price.

$$Revenue = Trade * DAP + \Delta_E(SSP - 0.07\Delta_E) \quad (1)$$

$$\Delta_E = Actual - Trade$$

The difference between actual and traded energy is penalized by the SSP and its square multiplied by 0.07. This is the piece that penalizes for both over and underproduction when SSP is positive which is the norm. While there are times when SSP is negative, they tend to be at times of overproduction so there is not a realistic way to exploit an arbitrage scenario by over-promising and under-delivering when prices are negative.

#### A. Revenue from Point Prediction

The predicted total energy is used as the traded amount in the day ahead market. Revenue is calculated as shown above using the actual total energy and the actual market prices. This does not consider the forecasts for the market prices and is the most straightforward to implement. Since there is no quantification of risk, there is no indication of the expected value of this energy commitment or possible undesirable outcomes.

#### B. Revenue from Conformal Prediction

We concluded that as a general rule, it is possible to reduce risk by under-committing energy rather than over-committing in the day-ahead market. The revenue formula penalizes failing to meet the commitment more. It is a delicate balance and we found that reducing the point prediction by the non-conformity score at the 10% quantile can have a favorable impact on the revenue. This approach could be justified by a game-theory approach which would consider playing the game indefinitely. Currently, this value is arbitrary without a strong justification but is included as an example of how the conformal prediction can and cannot be used.

#### C. Revenue from Conditional Value at Risk

The shortcomings of the above methods are primarily that they do not consider the market prices or the joint probability of the predictions and the correlation of the errors.

1) *Overview*: Stochastic optimization using Conditional Value at Risk allows us to incorporate this information into the trading strategy. In this situation, we are risk averse and our goal is to maximize the expected value of some lower quartile of possible outcomes. In this study, we opted for 33% to indicate that more likely than not, in the worst 33% of cases, we will have a certain amount of revenue. This is a more sophisticated method than the previous two and is a more realistic representation of the trading environment. Future work could include a specific risk profile that would specify that, for example, the probability of losing X in any one time period or day must be less than Y% or the probability of ever falling below a certain threshold must be less than Z%.

2) *Implementation*: We implemented CVaR by sampling residuals from the calibration data and adding them to the point forecast. At each prediction step we did the following:

1. Sample 250 residuals from the calibration data. These are indexed by time which captures two key elements simultaneously. First, is that the model may just be more accurate for a given feature at some time. This could be based on weather patterns, demand, or any number of factors. Secondly, it captures that at the 9:00 AM prediction, we are only 49 steps into the future while at the 7:30 AM prediction, we are at 97 steps. We know that error accumulates over time so this is an important factor to consider.
2. Add the residuals to the point forecast to get 250 scenarios of the total energy, DAP, and SSP.
3. Evaluate the revenue gained for each scenario at all available trade amounts. The true energy production is taken as the point forecast plus the scenario energy production residual.
4. At each available trade amount, calculate the mean of the lower 33% of the revenues in the scenarios. This is the CVaR.
5. Find the trade amount with the maximum CVaR.

The results of this process are depicted below. The multi-colored curves represent different scenarios evaluated for each available trade amount. The color represents how far off the predicted energy production was from the actual energy production. The solid black curve is the mean, or expected value of trading that amount of energy for those scenarios. The dashed black curve is the CVaR. For this optimization problem with a single decision variable, how much to trade, we can graphically see the maximum is at about 700 MWh. So the decision is made to trade that amount and the actual revenue is shown as the green diamond. Also plotted are the “+” showing the revenue from a perfect prediction and commitment of the energy production and the “o” showing the point prediction and associated revenue. The fact that the CVaR revenue is higher than the CVaR curve indicates the true outcome was not in the worst 33% of scenarios. In this case, the maximal CVaR happens to be close to the maximal mean, but it is not always the case. Typically, we would see the CVaR peak at a point off of the peak of the mean curve. The point is to maximize the expectation of the worst outcomes, but this comes at the cost of not maximizing the outcome. If one had infinite funds

and could assume infinite risk, the optimal strategy would be to maximize the expected value. In the real world, we prefer to limit our risk exposure. Lastly, we note that CVaR performs better than the point forecast. With CVaR we have accounted for our uncertainty about the energy production and market prices while the point forecast is only considering production and no uncertainty. This highlights the importance of considering the uncertainty in the predictions and trading.

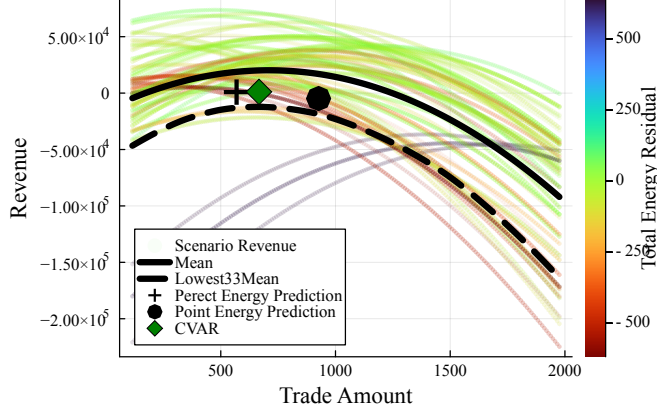


Fig. 11: Conditional Value at Risk

#### D. Performance

The plot of cumulative revenue over the testing period for each trading strategy is shown below. The perfect forecast is a point of reference for what would be achieved if the operator could perfectly predict the production and not trade on the real-time market at all. As expected, this is the highest value even though it does not consider market prices. Next, we see that CP-adjusted commitment outperforms the other strategies. As alluded to earlier, this may be a case of gambling and getting lucky as there is no justification for this reduction other than that it happened to perform well. There may be a strong underlying reason which would explain why this is a good strategy. Next, we see that CVaR is just behind the CP-adjusted strategy. Lastly, the point forecast performed the worst as it suffered the biggest loss in the first week and never fully recovered to the level of the other two strategies.

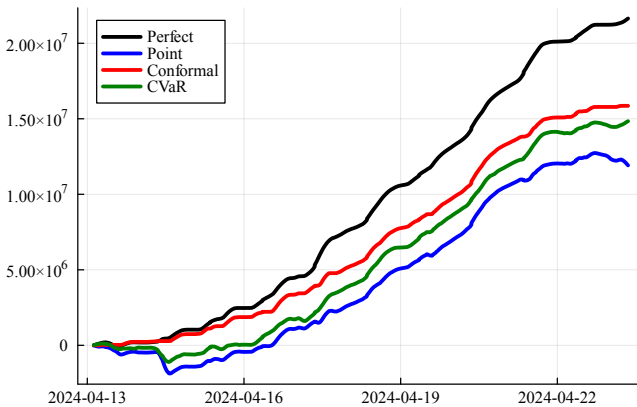


Fig. 12: Revenue vs. Time

## VI. CONCLUSIONS

The revenue from all three strategies built from the LSTM are on the same order of magnitude and off by a factor of two from the perfect energy production revenue. This indicates that our model and strategies are promising in their ability to capture the highly nonlinear and complex dynamics of the system, time dependency, and uncertainties. It also indicates there is room for improvement. We have identified the areas below as good places to start for future work:

1. Incorporate more data. We have purposely used a small subset of the available data sources, features, and history to build and test this model as a simplistic starting point and proof of concept. Specifically, demand forecasts and weather from more nearby locations would be useful.
2. Feed longer sub-sequences to the LSTM. Currently, the LSTM is updating its state only on the last step and then predicting the next step. It would be better to provide a lookback window of at least 24 hours so the seasonality would be learned implicitly rather than provided explicitly.
3. Predict optimal (or near-optimal) trade amounts directly. Once forecasts have been created for the features, train a model that takes those probabilistic forecasts to predict the optimal trade amount. This may offer a more direct path to maximizing revenue, although it would not be explainable as CVaR or other strategies are.
4. Test the CVaR on different risk profiles.

The problems addressed in this study are practical, real-world decisions that renewable energy production operators must answer every day. This report shows how advanced technologies and methodologies that rely on probabilistic approaches are within reach and can be used to limit risk for these operators. In the long run, reducing this risk will make investment in renewable energy more palatable and bring more systems online which will be a net benefit for the environment and the economy.

- [1] "Hybrid energy forecasting and trading competition." IEEE DataPort [Online]. Available: <https://ieee-dataport.org/competitions/hybrid-energy-forecasting-and-trading-competition>
- [2] A. N. Angelopoulos and S. Bates, "A gentle introduction to conformal prediction and distribution-free uncertainty quantification," *CoRR*, vol. abs/2107.07511, 2021 [Online]. Available: <https://arxiv.org/abs/2107.07511>
- [3] W. Xu, P. Zhang, and D. Wen, "Decision-making model of electricity purchasing for electricity retailers based on conditional value-atRisk in day-ahead market," in *2020 12th IEEE PES Asia-Pacific power and energy engineering conference (APPEEC)*, 2020.
- [4] "Hybrid energy forecasting and trading competition." Rebase Energy [Online]. Available: <https://www.rebase.energy/challenges/heftcom2024>
- [5] "Weather data & API." Visual Crossing [Online]. Available: <https://www.visualcrossing.com/weather-api>
- [6] M. Innes *et al.*, "Fashionable modelling with flux," *CoRR*, vol. abs/1811.01457, 2018 [Online]. Available: <https://arxiv.org/abs/1811.01457>