

The Rate, Composition and Volume Method: A Zero-Loss Approach to Decomposing Electoral and Compositional Shifts

Donald Moratz
dmoratz@sas.upenn.edu



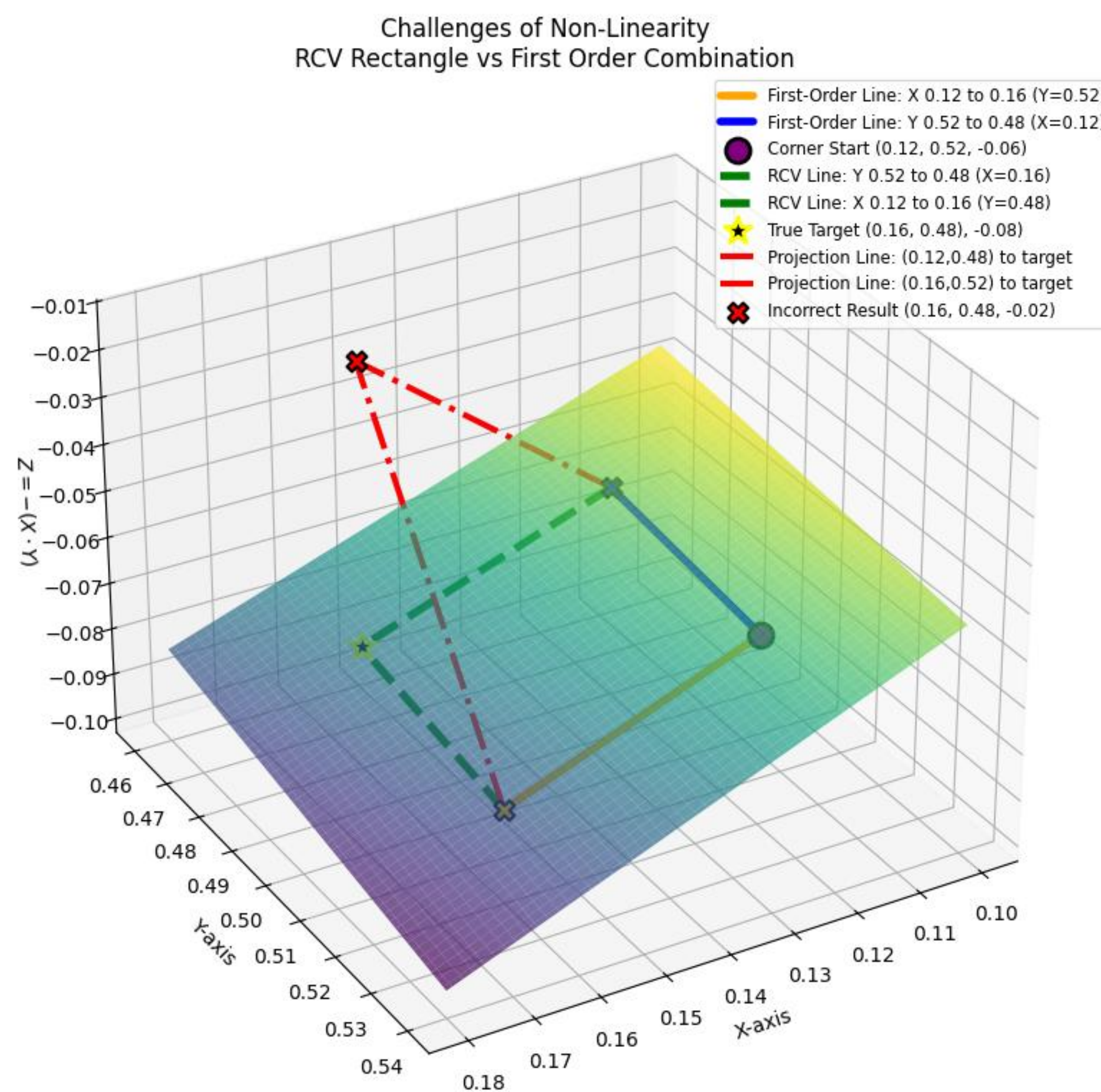
Abstract

Existing methods for decomposing electoral change (e.g., derivative-based, regression) often fail to fully account for total vote shifts and complex compositional dynamics due to their inability to capture cross-partial derivatives. This work introduces the **Rate, Composition, and Volume (RCV) Decomposition**, a novel, zero-loss approach that precisely quantifies within-group and cross-group compositional changes. RCV offers a more accurate, interpretable, and consistent framework for analyzing electoral outcomes.

Problem With Existing Approaches

Traditional methods, such as the derivative-based approach, conflate composition and volume effects. Critically, both derivative-based and regression models fail to capture the interactive effects (cross-partial derivatives) between rate, composition, and volume changes. Traditional approaches often treat these as independent, missing crucial interaction terms. This leads to:

- Inaccurate accounting for total vote shifts.
- Misrepresentation of individual component contributions (rate, composition, volume).
- Biased estimations that can overstate or understate actual electoral dynamics.



This 3D graph illustrates how neglecting cross-partial derivatives, even with only two interacting components, leads to significant miscalculations in the overall change. The **First-Order Line** deviates from the **RCV Line** and **True Target**, demonstrating the accurate decomposition provided by RCV. This issue is compounded when considering three components (Rate, Composition, Volume).

The RCV Decomposition

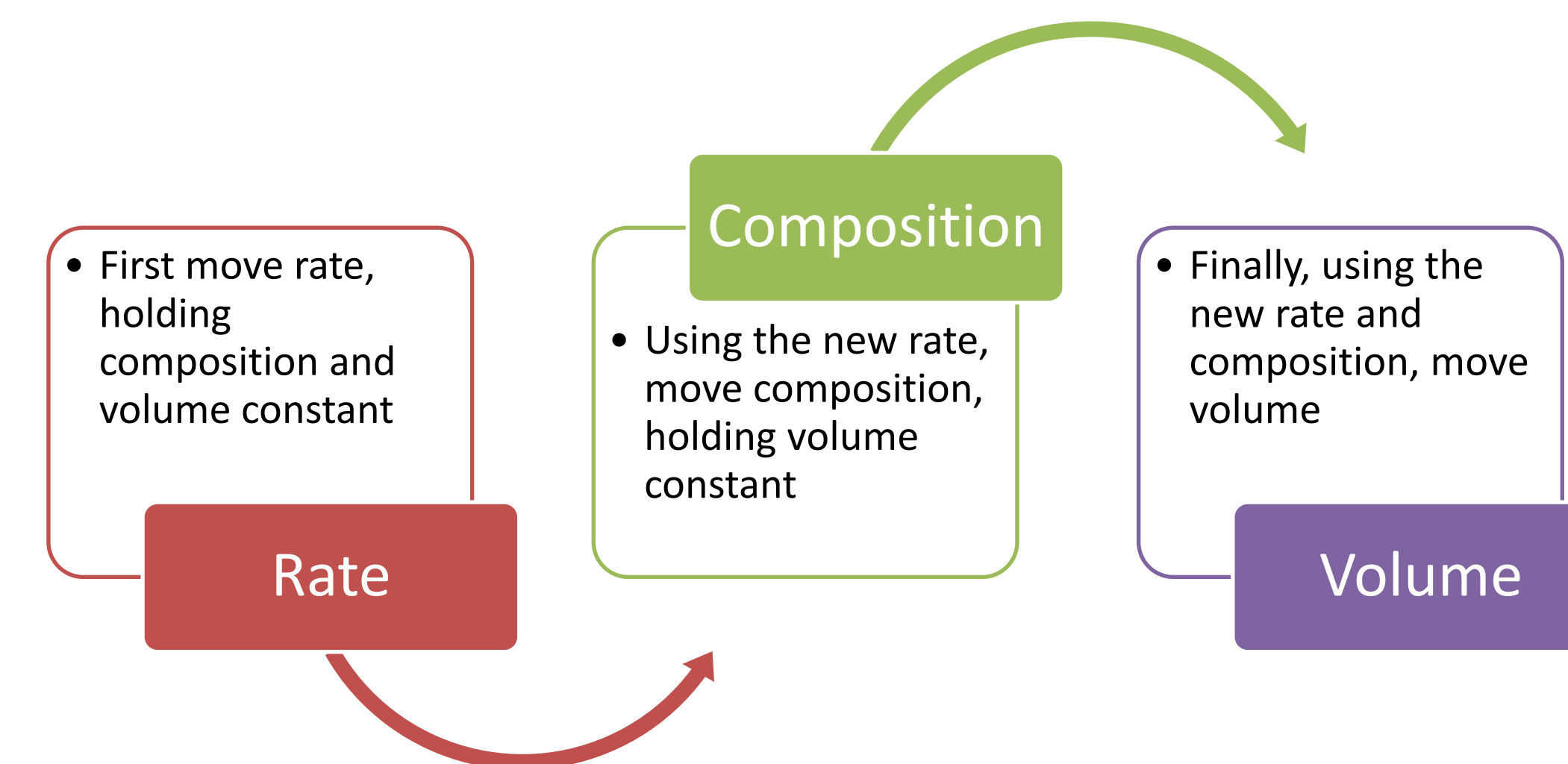
RCV accurately decomposes total vote change (ΔZ) into distinct, interpretable components:

- **Rate (ΔR):** Change in a group's vote choice.
- **Composition (ΔC):** Change in a group's share of the electorate.
- **Volume (ΔV):** Change in the overall electorate size.
- The RCV approach is a zero-loss, non-linear solution that ensures the sum of decomposed changes equals the total observed change ($\Delta Z = \Delta R + \Delta C + \Delta V$). It explicitly accounts for complex sequential interactions between these components.

Core Principle: $\Delta Z = Z(t+1) - Z(t)$

Key Equation

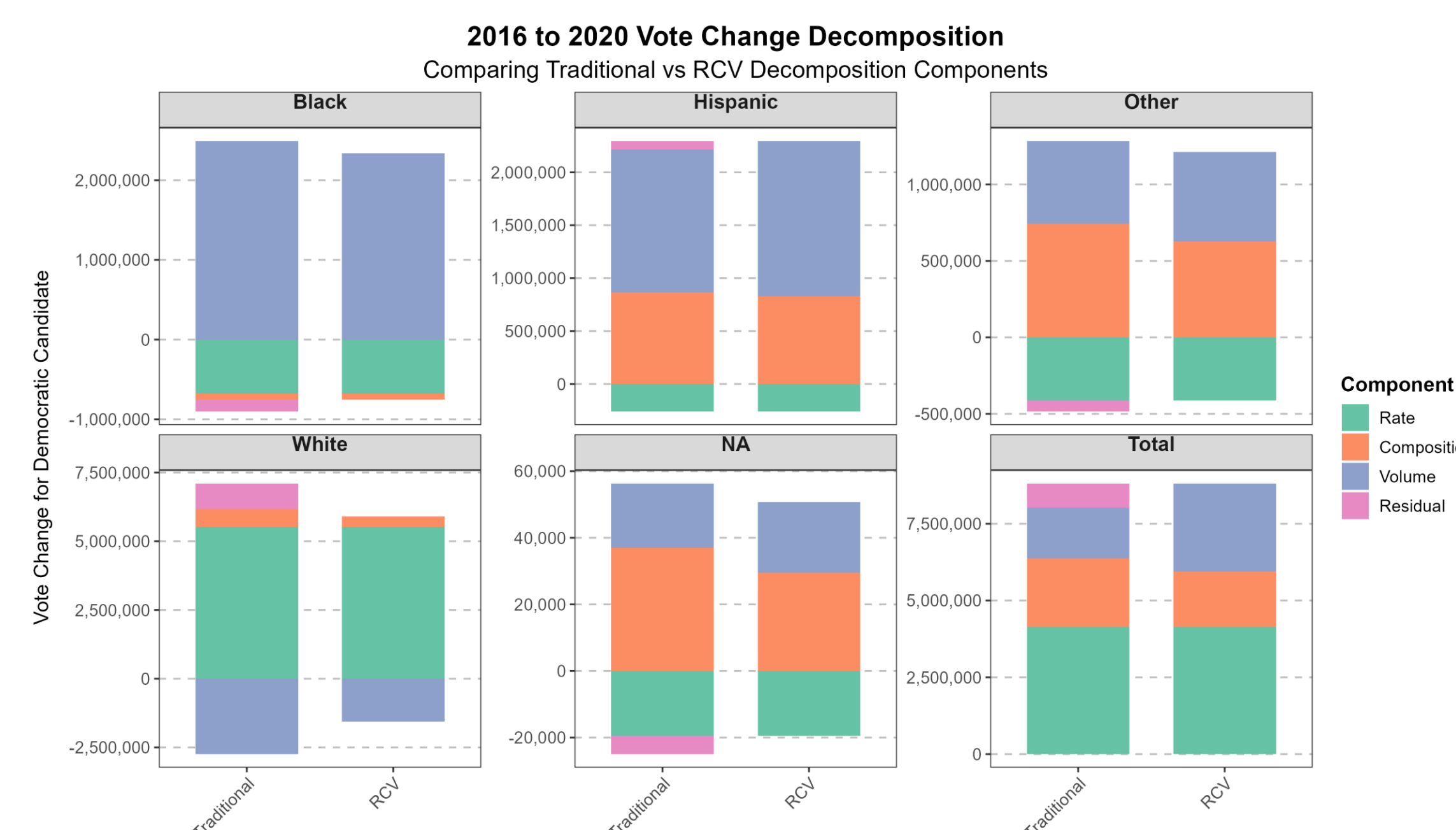
$$\Delta Z = [r_{t+1} - r_t]c_t v_t + r_{t+1}[c_{t+1} - c_t]v_t + r_{t+1}c_{t+1}[v_{t+1} - v_t]$$



RCV in Action: U.S. Elections

US Election Data demonstrates RCV's superior accuracy. Unlike traditional methods, RCV consistently:

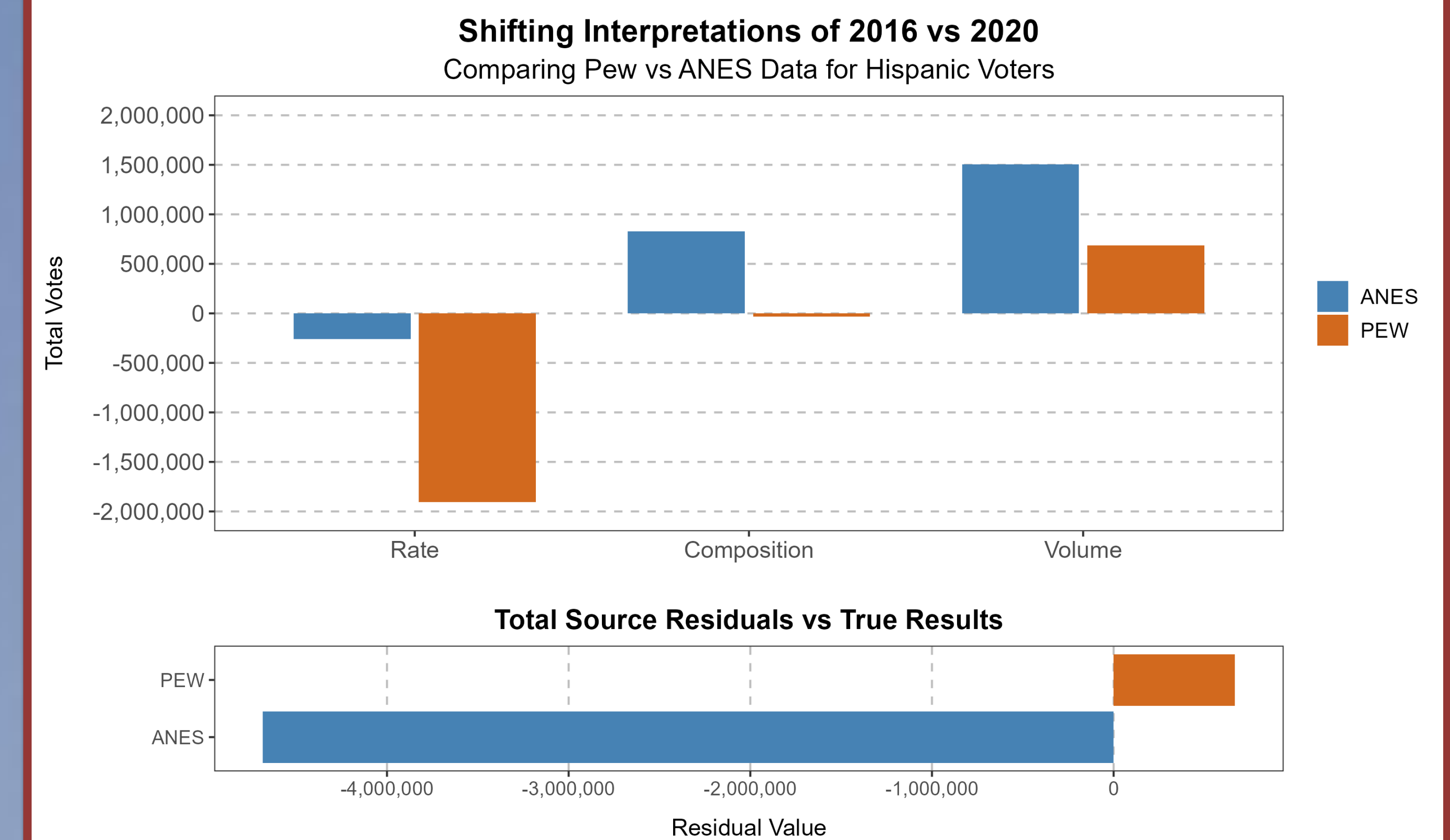
- Produces perfectly additive decompositions, ensuring zero-loss.
- Correctly attributes changes to their respective Rate, Composition, and Volume components, even under complex, interacting scenarios.
- Exposes how reliance on other methods can lead to biased estimations of electoral shifts, particularly when group sizes and voting patterns change simultaneously.



RCV in Action: U.S. Elections

Applying RCV to the 2016 and 2020 U.S. presidential elections reveals critical insights often missed by other methods:

- **Data Validation:** RCV can expose inconsistencies or errors in underlying survey data (e.g., differences between ANES vs. PEW estimates), allowing researchers to identify data quality issues before drawing conclusions
- **Hispanic Vote Shift:** RCV analysis accurately highlighted a significant shift in support among Hispanic voters, which was a key factor in the 2020 results and provided prescient insights for 2024.
- This provides a robust framework for understanding nuanced electoral dynamics that are crucial for contemporary political analysis.



Conclusion & Implications

The RCV Decomposition provides a mathematically rigorous and practically valuable tool for political scientists. It offers:
Unparalleled Accuracy: Zero-loss decomposition for comprehensive electoral change analysis.

- **Enhanced Interpretability:** Clear, distinct contributions of Rate, Composition, and Volume shifts.
- **Robust Data Evaluation:** A framework for identifying data inconsistencies.
- **Improved Predictive Power:** Better understanding of underlying trends for future electoral outcomes.
- RCV is essential for advancing our understanding of demographic and electoral change in modern political science.

Key References

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