Death Matters: Endogenous Mortality and Economic Growth

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1 Introduction

In so far as population growth is a determinant of economic growth, mortality must be a determinant of economic growth. This follows naturally from the fact that population growth is itself a function of mortality. Despite the importance of mortality in driving economic growth, there has been little to no work on *endogenous* mortality models. Part of this is likely due to the fact that some portion of mortality is fundamentally outside of our control. People die whether we want them to or not. However, simply because we cannot completely control mortality does not entail that it ought to be treated as exogenous. In fact, much of human activity is devoted precisely to the task of endogenizing mortality. We toil to avoid death and, besides modern sanitation, the most effective means we have developed is healthcare.

When economic agents invest in healthcare, they do so in order to prevent an untimely death. This simple fact explains the large investments in healthcare the world over. Given that economic agents do in fact endogenize mortality, it seems fitting that endogenous growth models should likewise capture this endogenous relationship between healthcare investment and mortality. Despite its intuitive appeal, much of the work on endogenous growth theory has eschewed endogenous mortality models. In fact, when the growth rate is endogenized, it is usually births which are deemed endogenous, relegating mortality to an exogenous force. We have reversed this picture. This is not because we deem attempts to endogenize births as wrongheaded, but simply in order to maintain a manageable level of tractability of the model. We hope to extend our model in the future towards both endogenous births and endogenous deaths, but for now will focus only on the latter issue. As we will demonstrate, an endogenous account of mortality that incorporates healthcare investment will yield an important insight into the relationship between economic growth and population growth. Attempts to stave off the inevitable can successfully prolong life but come at a cost. Namely, the reduction in non-healthcare investment, which can have retarding effect on growth. The trick is to balance a reduction in mortality and a strong growth rate. Our model will effectively capture this trade-off and offer insight into how best to manage these competing goals.

In this paper, we will first review the scant literature surround the endogenization of mortality within the context of economic growth models. Proceeding from there, we will introduce several models to examine the relationship between death and economic outcomes. We will begin with a generic Solow model with a representative agent and exogenous population dynamics. We will then proceed into a Solow model with endogenous population dynamics that will demonstrate the limits of using such an exogenous growth model. Finally, we will conclude with our proposed model which uses endogenous mortality as well as endogenous technological change as proposed in Romer ('86). We will then move our paper into a brief examination of optimization within our model done on the part of the government and a relationship between taxes, population dynamics, and growth.

2 Literature Review

The literature in endogenous growth has been for the most part silent on endogenizing mortality. There seems to be, from a survey of the literature, three main approaches to endogenous death. These are (I) Potential Extension, (II) Healthcare, and (III) Pollution. The main approach has been to propose extending existing endogenous growth models to incorporate endogenous mortality. This approach is best summarized by Barro and Sala-i-Martin who write:

> We [also] do not allow *d* [the death rate] to depend on family or public expenditures on medical care, sanitation, and so on, although these influences on the mortality rate would be an *important extension of the model*. (p.412, emphasis added)

Although some could argue that simply proposing endogenous mortality as an extension is hardly an approach to modeling the phenomenon, it certainly highlights the importance of this project.

The literature has also explored the impact of healthcare on mortality. When incorporated into an endogenous growth framework, authors are able to examine the impact of household and or public expenditure on mortality rates. That being said, the literature here is quite sparse. Authors such as Leung and Wang have examined the impact of healthcare on life expectancy and traced this relationship for its impact on growth (Leung and Wang, 2010). However, this project has been done within a neoclassical growth framework. Others, such as Kalemli-Ozcan have explored similar dynamics but within a partial equilibrium context (Kalemli-Ozcan, et al. 2000). The result is that there has been a paucity of work on endogenous mortality within an endogenous general equilibrium framework.

The final avenue of research on endogenous mortality has been motivated by pollution. As pollution is taken to be, at least partially, within the control of individuals and governments as well as a determinant of mortality, it has been considered a viable pathway for endogenizing death. Reis explores the possibility of eliminating pollution all together given hoped for technological advances. She then explores the impact of this breakthrough on growth, in part via a reduced mortality rate (Reis, 2001). Peretto and Valente trace the impact of growth on pollution and then the feedback loop in which increased pollution increases mortality (Peretto and Valente, working paper).

What becomes clear from a survey of the literature is that it is possible to endogenize mortality by *at least* two different avenues: healthcare and pollution. What unifies these approaches is that they are within the control of relevant economic agents, be they private or governmental. An interesting possible extension suggested by the literature would be to combine healthcare and pollution into a single model of endogenous mortality. Pollution, caused by economic growth, increases mortality thus spurring the need for increased investment in healthcare. This in turn will have an effect on growth, thus changing the development of pollution within an economy. At present, no model has explored the interrelations between these two endogenizing pathways. For the present, we have focused exclusively on healthcare, but that does not mean that pollution is ever far from our minds.

3 The Model

We have constructed a one-sector dynamic general equilibrium model which produces a commodity using labor and capital. These inputs are supplied by identical households which face an intertemporal choice between labor and leisure. All labor is directly supplied to firms and firms rent assets directly from households as capital. In order to ensure the proper functioning of intertemporal choices, we have included a financial market into the model. For simplicity we have normalized the price of consumption, investment, and output to 1. Employing a representative agent model, all identical individuals are endowed with time, initial wealth, and the ability to perform labor. All wages are determined endogenously through the model via the relationship between utility of consumption and utility derived from leisure. Population growth is a non-linear function of population and mortality. In the second model, endogenized mortality is controlled by healthcare investment by the government funded by a wage tax. The representative firm follows the Solow ('57) model of exogenous technological change. After demonstrating the inherent shortcomings of a Solow approach to growth models, we move the math into an endogenous growth model based on the framework proposed by Romer ('86). Our models generate market clearing equilibria when they reach the steady-state.

$$(1)\ P_C = P_I = P_Y = 1$$

3.1 Base Model

In this section we will introduce three models to examine the role of mortality in economic growth. Our first model will be a basic model with exogenous mortality. We will then introduce an endogenous component in the form of health care, but within the context of an exogenous growth model. The results of this will demonstrate the necessity of incorporating endogenous growth such as the model proposed by Romer ('86), which will form our final proposed model.

3.11 The Household

3.111 Population Dynamics

We employ a *generic* model of population growth which is determined by population, birthrate, and mortality. This is represented as:

$$(2)\ \dot{L}/L = m(L, n, d)$$

In this equation L represents the labor force for the economy and N represents the total population. Importantly we have set L = N, thereby making all members of the population workers in the economy. This assumption removes differences associated with age and life-stages as we have erased childhood and retirement. The variable n represents the constant exogenous birth rate and d represents the endogenous death rate, which maintains an exogenous component.

The endogenous death rate is a function of healthcare investment. As healthcare investment increases, the good Health increases. Note, that in model one, Health is a good belonging to each individual, denoted by h. For reasons of tractability, we have normalized the price of h to one. This increase in h causes a decrease in the death rate d, but with diminishing marginal returns. Thus, in so far as healthcare investment is determined within the model, the death rate is endogenous. In the Exogenous Wage Tax Model, healthcare investment will be funded by the government which will produce h using tax revenue. As such, in the Base Model, we can treat d as exogenous while in the Exogenous Wage Tax Model it becomes endogenous.

$$(2.1)\ d = \phi - \psi(h)$$

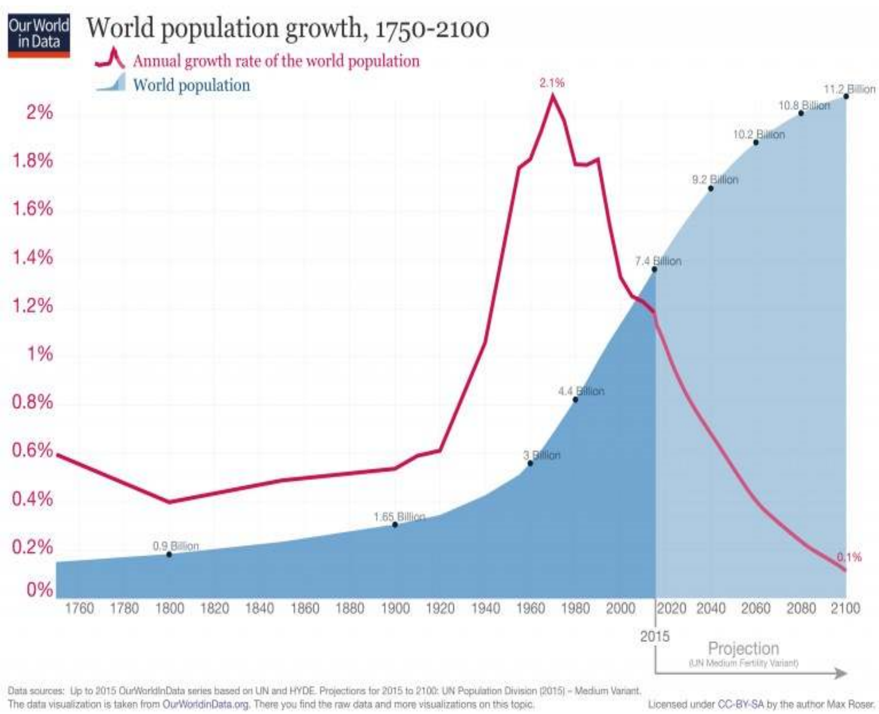
Here ϕ represents the "death intercept" or the base rate level of mortality without healthcare investment. The parameter ψ represents the impact of healthcare on mortality. Critically, ψ is a decreasing function, entailing that when h increases, d will decrease. This implies that increased health will prevent death.

The rate of population growth is, as shown above, a function of n and d. Importantly, it is increasing in n and decreasing in d. This can be interpreted to mean that when the birth rate of a population is increasing, the population growth rate is also increasing. Likewise, when the mortality rate for a population increases, the population growth rate decreases. This naturally coheres with our a priori understanding of the relationship between birth and death and population growth.

The relationship between population and time can best be understood as a process with three distinct phases. In the first phase, the population growth rate is very small and population remains nearly constant over time. This phase coheres with the times before the agricultural revolution. The second phase involves an increasing population growth rate, which entails a drastically increasing population over time. This period coheres with the agricultural revolution and the subsequent centuries. The final phase of our dynamics involves a drastic decrease in the population growth rate as population growth approaches zero. This final phase can be understood by the fact that h exhibits decreasing marginal returns. In order to avoid the population reaching infinity in finite time, we must also introduce a hard carrying capacity for population. Such a carrying capacity has been explored in Lotka-Volterra models such as the following:

$$(2.2)\ \dot{L} = nL(1 - L/M) - dL$$

Here M represents the maximum carrying capacity for the population. Due to tractability concerns, we have used a generic function rather than implementing this precise function. Nevertheless, when we examine optimization, we will see that the population dynamics in our model must be governed by an equation of this type. Importantly, our overall dynamics exhibit a high degree of fit with observed population dynamics, as shown below (Roser and Ortiz-Ospina, 2017).



3.112 Utility

Consumption patterns of the representative agent is dictated by preference structure, which can be represented using utility functions:

$$(3)\ U = \int_0^\infty e^{-(\rho)t} U(c, 1 - l)dt$$

Note, this utility function only captures the utility of the individual thereby ignoring the overall welfare of households. This entails a break with the standard Benthamite representation of utility. In this equation, the variable ρ represents the intertemporal discount rate. The variable *c* represents per-capita consumption. And, l represents the per-capita fraction of time spent working. The variables *c* and *l* are choice variables for the individual.

3.113 Constraints

The representative agent faces a variety of constraints which dictate their behavior patterns:

(i) Time – Given a finite lifespan, agents must allocate their time so as to maximize their utility. This involves choosing between labor and leisure at any given moment. We represent this choice as the fraction of labor and leisure. Given that these are taken to be an exhaustive list of possible uses of time, we can define leisure as 1-*l*. The benefits from this choice are the wages accrued from labor, represented by: *wl*. The costs derive from forgone leisure.

$$(4)\ 0 \leq l \leq 1$$

(ii) Budget Constraint – Individuals are endowed with a finite strictly positive initial wealth allocation. As individuals are members of households, each time a new member of the household is born, their wealth is diluted. On the other hand, when members of the household die, the rate of accumulation is increased as new wealth is dispersed amongst fewer individuals. This results in the following representation for wealth accumulation, which functions as our budget constraint:

$$(5)\ \dot{a} = [r - m(L, n, d)]a + wl - c$$

Given, as stated above, given *m* is decreasing in *d*, as the mortality rate decreases wealth accumulation increases.

(iii) Solvency Constraint – Individual consumption cannot exceed the representative agents' ability to finance said consumption. This effectively eliminates the possibility of a Ponzi Scheme. It can be represented as follows:

$$(6)\ a_0 > 0$$

(iv) Transversality Condition – Individuals face a transversality condition which compels them to consume all of their wealth at infinite time. Thereby leaving no wealth unconsumed. This guarantees that the household will consume their wealth over time, but actually has little bearing on the individual's consumption of wealth. While we use a representative agent model, incorporating a finite lifetime for each agent significantly increases mathematical demands and would indeed be justifiable as its own extension of the current economic growth models.

$$(7)\ \lim_{t\to\infty} a\,(t)e^{-\int_0^t[r(v)-n]dv} \geq 0$$

3.114 Maximization

In order to guarantee an interior solution, we impose the following conditions on the marginal utility of consumption:

$$i)\ MU_c > 0\ at\ an\ increasing\ rate$$

$$ii)\ MU_c = 0\ at\ c = 0$$

$$iii)\ MU_c = \infty\ at\ c = \infty$$

This condition entails that the marginal utility of consumption is greater than zero at an increasing rate. This can be understood as a non-satiation condition.

The representative agent seeks to maximize utility subject to the constraints detailed above. In order to do so, we set up the following Present Value Hamiltonian:

$$(8)\ PVH = \int_0^\infty e^{-(\rho)t}U(c, 1-l)dt + \alpha([r - m(L,n,d)]a + wl - c)$$

Here α represents the shadow value of wealth. Using the first-order partial derivates of the Present Value Hamiltonian, we are able to generate the labor supply equation as follows:

$$(9)\ U_c/U_{1-l} = 1/w$$

This demonstrates that the labor supply at any given time is the inverse of real wages, given our previous assumption about the price of consumption.

Assuming a separable utility function and setting the elasticity of consumption equal to one, we derive the following equation for the consumption path:

$$(10)\ r = \rho + \dot{c}/c$$

Here we define *r* as the reservation rate of assets that an individual demands in order to forgo current consumption. The above equation shows that this reservation rate should be equal to at least the parameter that defines an individual's intertemporal consumption choice, ρ, plus some compensation for having an unequal consumption path. If we take *r* as exogenous, then we are able to derive the path of consumption. Let the average rate of interest in a period be denoted by $\overline{r_t} = 1/t \int_0^t r(s)ds$ This can be represented as follows:

$$(11)\ c_{(t)} = c_{(0)}e^{(\overline{r_t}-\rho)t}$$

By equating the present discounted value of consumption to the present discounted value of wealth, we can see that consumption today is proportional to total wealth. This is shown in the following representation:

$$(12)\ c_{(0)} = \mu_{(0)}\left(a_{(0)} + b_{(0)}\right)$$

where $a_{(0)}$ is initial wealth, $b_{(0)}$ is human wealth denoted by $\int_0^\infty e^{-\overline{r_t}t} - m(L,n,d)t\mathrm{w}_t l_t\, dt.$, and $\mu_{(0)}$ is the marginal propensity to consume out of wealth represented as $1/\mu_{(0)} = \int_0^\infty e^{-\rho t} - m(L,n,d)t\, dt.$ If we institute an exogenous shock to the death rate, which decreases *d*, *b0* will increase. Furthermore, said exogenous shock will decrease $\mu_{(0)}$. The combined effect is that initial consumption, $c_{(0)}$, will decrease. This can be understood as saying that when the death rate decreases, initial consumption decreases because agents now need to stretch their consumption over their more extended lives. It is evident that the marginal propensity to consume out of wealth is time invariant. This follows from the earlier assumption that the elasticity of consumption is one.

3.12 The Firm

3.121 Production Function

The representative firm in this model is characterized by the following production function:

$$(13)\ Y = F(K, XL)$$

Here Y represents total output, K represents capital, L represents labor, and X represents labor augmenting technology. The variable X grows at the following rate:

$$(14)\ \dot{X}/X = x$$

Here *x* represents the rate of growth of labor augmenting technology. Capital depreciates at the following rate:

$$(15)\ \dot{K} = I - \delta K$$

The variable δ represents the depreciation rate for capital and *I* represents investment. Investment is unbounded by construction, but due to resource constraints in the economy, as represented by the household budget constraint, there cannot be infinite investment. As can be seen, labor and capital are flow variables.

<u>3.122 Firm Maximization</u>

The firm maximizes profits subject to initial capital levels, the rate of capital accumulation, and the transversality condition listed below. This can be represented by the following Current Value Hamiltonian:

$$(16)\ CVH = F(K, XL) - wL - I + q(I - \delta K)$$

Here q represents the marginal benefit of investment. The representative firms' transversality condition is as follows:

$$(17)\ \lim_{t\to\infty} a\,(t)e^{-\bar{r}_t}q_{(t)}K_{(t)} = 0$$

From the first-order conditions we are able to derive the equation for the demand for investment, which is represented as follows:

$$(18)\ r = P_Y F(K, XL)/q - \delta + \dot{q}/q$$

Here *r* represents the firms' willingness to pay for an asset. Note that *q* is equal to the price of investment and this signifies market trade.

Solving the maximization problem yields the function for capital per worker. This in turn can be represent as follows:

$$(19)\ \dot{k}/k = sf(k)/k - \big(\delta + m(L, n, d)\big)$$

Here we take s to represent the marginal propensity to save. If we were to institute an exogenous shock which decreases d, then the capital accumulation per worker would decrease. Effectively, as there are now more workers in the economy, the amount of capital must be shared amongst more people resulting in the capital per worker to decrease.

With the introduction of labor augmenting technology X, the capital accumulation per *effective* worker further decreases with an exogenous shock to the death rate, as shown in the following equation:

$$(19.1)\ \dot{\tilde{k}}/\tilde{k} = sf(k)/\tilde{k} - (\delta + m(L, n, d) + x)$$

The greater magnitude for this decrease in capital accumulation per effective worker is due to the fact that workers are now more efficient then when we left out labor augmenting technology. Thus, workers require less capital to produce the same level of capital. The labor augmenting technology does not change the impact of the exogenous shock to the death rate but is an important determinant of the capital accumulation per effective worker.

<u>3.13 Equilibrium Conditions</u>

Recall that the representative household seeks to maximize utility and the representative firm seeks to maximize profits. Given these two maximization goals, the model will be in equilibrium under the following condition. Total household wealth is equal to total firm capital. This involves households *directly* renting their wealth to firms as capital. Consequently, per capita wealth is equal to per capita capital.

$$(20)\ a = k$$

The focus of this component of the model is on the effective worker, due to the introduction of labor augmenting technology. This is signified by the introduction of a tilde symbol- ~ - over the relevant variables.

Given the nature of our model, the rate of growth of output per capita will equal the rate of growth of labor augmenting technology. This is a direct inference from the equation below: $for\ \dot{\tilde{k}} = 0, sf(k)/\tilde{k} = \delta + m(L, n, d) + x$. Given an exogenous shock to the death rate, decreasing d, there will be an increase on both sides of the equation. This will result in an increase in capital per effective worker in the steady-state of the model. $\dot{\tilde{c}}/\tilde{c} = r - \rho - x$

$where\ \tilde{c}\ is\ the\ consumption\ per\ effective\ worker\ \ implies, at\ \tilde{c} = 0, r = \rho - x$. The rate of consumption per effective worker now also decreases by *x*. The death rate does not factor into the rate of growth of consumption per effective worker. The only effect of a change in the death rate is on the initial level of consumption, as seen earlier. As can be seen, this is a model *with* growth rather than a model *of* growth. This is intentional in order to facilitate the tractability of the model and to highlight the importance of mortality on the growth rate.

3.2 Exogenous Wage Tax Model

Here we amend the base model by explicitly modeling investment in healthcare. This results in an additional determinant of overall growth. As healthcare investment increases, mortality is taken to decrease resulting in an increasing population growth rate. This healthcare good will be treated as a public good and as such will not figure into the utility function of the representative individual. It is only efficacious in so far as it reduces mortality. However, healthcare cannot be treated as mana from heaven. It needs to be paid for. In this model we introduce the government and stipulate that their sole function is to provide healthcare to the citizenry. This healthcare is paid for by a tax imposed on the wages of our representative individual. As the wage tax rate increases, the government is able to provide more healthcare, thus reducing mortality. However, the investment in healthcare faces diminishing marginal returns, with a positive first derivative but negative second derivative- death cannot be staved off indefinitely. At the same time, this increased wage tax rate reduces the overall wealth of the population. Wages in this model are still a function of the interaction between utility of leisure and consumption.

We can represent the wage tax rate imposed by the government on the individuals within the household as ε. All of the revenue generated from the wage tax is directly funneled into the production and distribution of our public good, h. Importantly, we have assumed that the government faces a budget constraint requiring all expenditures to be equal to tax revenue. As the wage tax is the only tax revenue for the government and the production and distribution of *h* is the only expenditure, we can safely presume that the following holds:

$$(20.1)\ \varepsilon w = h \Rightarrow d = \phi - \psi(\varepsilon w)$$

The "direct effect" will be the result of a reduction in wealth due to the imposition of the tax. The "indirect effect" will be the result in a decrease in the death rate due to increased healthcare investment. The Exogenous Wage Tax Model will preserve all elements of the Base Model unless explicitly stated otherwise.

3.121 The Household

The representative household for this model faces the same constraints *except* for the budget constraint. As such we will replace Equation (5) with the following:

$$(21)\ \dot{a} = [r - m(L, n, d)]a + w(1 - \varepsilon)l - c$$

As the wage tax rate increases, the accumulation of wealth is affected in two ways. First, the increase in taxes reduces the amount of wealth accumulated from wages, thus reducing overall accumulation of wealth. Furthermore, an increase in the wage tax rate increases the amount of *h* produced by the government, resulting in a lower mortality rate *d*. This decrease in *d* likewise results in a decrease in the accumulation of wealth. The net result is a decrease in the accumulation of wealth as the wage tax rate is increased.

When maximizing the utility function subject to the new budget constraint and all other unchanged constraints, the following change to the labor supply path occurs:

$$(22)\ U_c/U_{1-l} = 1/(1 - \varepsilon)w$$

As the wage tax rate increases, the wages received by workers decreases. This results in a decrease in the supply of labor.

Importantly, the consumption path does *not* change with the introduction of the exogenous wage tax. This entails that the forward integration remains the same as in the Base Model. However, when we equate the Present Discounted Value of Consumption to the Present Discounted Value of Wealth, we generate a different relationship than that in Equation (12). Instead, we generate the following:

$$(23)\ c_{(0)} = \mu_{(0)}\left(a_{(0)} + b_{(0)}\right)$$

$Where\ a_{(0)}\ is\ in$itial wealth, $b_{(0)}\ is\ human\ wealth\ denoted\ by\ the\ function$

$$\int_0^{\infty} (e^{-\bar{r}_t t} - m(L,n,d)t)(1- \ \varepsilon)\, \mathrm{w_t}\, \mathrm{l_t}\, dt$$

And $\mu_{(0)}$ is the marginal propensity to consume out of wealth represented as

$$1/\mu_{(0)} = \int_0^{\infty} e^{-\rho t} - m(L,n,d)tdt$$

If we introduce an exogenous shock increasing ε there will be two effects on initial consumption. The direct effect will be a decrease in *b0* and no change in *μ0*. The indirect effect, mediated by the decrease in *d*, will entail an increase in *b0* and a decrease in *μ0*. The net result will be a decrease in initial consumption which may or may not be greater than the decrease exhibited in the Base Model. $\Delta c_{(0)}$ from model 2 ≥ $\Delta c_{(0)}$ from model 1.

3.122 The Firm

The introduction of the wage tax will induce a direct and indirect effect on the firm. An exogenous increase in ε will result in a direct change to the labor supply path. The tax hike will disincentivize work, decreasing the relative costs associated with leisure. This will result in a decrease in production. The indirect effect will result from the decrease in *d* due to the increased tax revenue needed to generate *h*. This will result in longer lifespans for the individuals, thus leading to higher output. As a result, the net rate of capital accumulation per effective worker will depend on ψ as shown below:

$$(24)\ \dot{\tilde{k}}/\tilde{k} = sf(k)/\tilde{k} - (\delta + m(L,n,d) + x + w\varepsilon l)$$

The direct effect of increasing ε results in an increase in the rate of capital accumulation because higher wage taxes reduces labor supply, thus initiating the need for greater capital per effective worker. Likewise, the indirect effect of increasing ε results in a decrease in *d*. This leads to a higher population and thus a decrease in capital per effective worker. Whether the direct or indirect effect will dominate depends on the effectiveness of healthcare investment, which we defined as ψ. A further extension of the model could involve calibrating ψ using real world data from the healthcare sector.

3.123 The Equilibrium

As in the Base Model, we have set household assets equal to capital. This is because the households directly rent their assets to firms in the form of capital. In the steady-state, we set capital accumulation per effective worker equal to zero, much as we did in the Base Model. However, the introduction of ε results in the following: for $\dot{\tilde{k}} = 0$, $sf(k)/\tilde{k} = \delta + m(L, n, d) + x + w\varepsilon l$. We can evaluate the equilibrium by considering two exhaustive cases. In the first case, the direct effect of increasing the wage tax rate dominates. This results in the amount of capital per effective worker increasing. Likewise, the case in which the indirect effect dominates, the amount of capital per effective worker increases. Thus, regardless of whether the direct effect or the indirect effect dominates in Equation (24), capital per effective worker will increase. The only difference between these cases comes down to the *rate* of capital per effective worker accumulation, not whether there is capital accumulation.

The rate of consumption per effective worker is invariant to changes in the wage tax rate. At $\tilde{c} = 0$, $r = \rho - x$. However, the initial level of consumption will decrease. The magnitude of the decrease will be greater than or equal to that exhibited in the Base Model in which only the death rate changed. At a more holistic level, as is seen with exogenous growth models generally, the government cannot affect growth in per capita output. This is solely determined by the rate of growth of labor augmenting technology, which in the case of the second model was exogenous. All the government can influence are the starting points of the growth trajectory for the economy, while the slope *x* remains the same. Therefore, it is easy to see that in this model very little can be done to change the dynamics of the model. In order to fully capture the intertemporal trade-offs associated with investments in healthcare, we propose a model using endogenous growth.

3.3 Endogenous Mortality Model

With the goal of balancing an endogenous mortality rate and economic growth, we modify the Exogenous Wage Tax Model to now include a technology component that augments labor. This is once again denoted by X; however, now it is determined within the system. Using a setup like the Romer ’86 model, we assume imperfect competition and increasing returns to scale so that X is a non-rival good. This means that ideas that are produced locally can be used globally.

3.31 Ideas and Technology

Therefore, there are two aspects of technology that bear looking into, namely the diffusion and creation of new idea. We examine each of these below.

3.311 Diffusion

In order to capture the diffusion and creation dynamics, we must abandon the representative firm. This results in disaggregation throughout the economy equivalent to one unit of production. The production function now is represented as follows, although firms still produce homogenous goods. $Y_i = F(K_i, X_i L_i)$ where $i = 1, 2, \dots, N$ equals the number of firms. Note that F is perfectly symmetrical and homogeneity in K and L is preserved. This means that X is the source of increasing returns for the firm. Here, units of capital and labor used by one firm cannot be used by another.

There is an instantaneous diffusion of knowledge and ideas in this model, represented as:

$$(25)\ X_i = Z_i + \Sigma_{i \neq j} \sigma Z_j$$

Where Z_i are the ideas generated by firm i, Z_j are the stock of ideas generated by the other $N-1$ firms and σ is the subset of ideas generated by other firms used by firm i at no additional costs. Note that σ has no subscript. This preserves symmetry and can be thought of as representing legal excludability like patents or usefulness of ideas of other firms to firm *i*. If $\sigma = 0$ in an economy, then it kills any incentive for diffusion of ideas.

3.312 Creation

Incorporating the concept of learning by doing, we are able to model the incremental accumulation of knowledge by firm *i* as a function of the accumulation of capital. This can be mathematically represented as follows:

$$(26)\ \dot{Z_i} = \theta \dot{K_i}$$

Here, θ represents the creation of ideas within a firm. Thus, knowledge accumulation is a by-product of capital accumulation and can be thought of as an investment process. The firms in the model divert resources from production to capital accumulation in order to gain knowledge. For each firm, this process can then be represented by the following equation:

$$(27)\ \dot{K_i} = I_i - \delta K_i$$

Note that, once again, in order to preserve symmetry in the model, we assume that the depreciation rate is homogenous across firms. This implies $X_i = \theta\left[K_i + \sigma\Sigma_{i\neq j}K_j\right]$. A higher value of θ implies that the economic environment is more conducive to generating ideas rather than accumulating capital. In this case there is more idea creation by each firm. Similarly, a higher value of σ indicates a higher scope of interaction between firms, which then motivates the diffusion of new ideas. Overall, firm *i* gets to utilize the capital accumulated by other firms.

Note that if K represents the aggregate stock of capital in the economy at any point in time, and all firms reach the same level of capital stock at equilibrium, then the following holds: $K = \sum_{i=1}^{N} K_i \Rightarrow K/N = K_i$. Similarly, $L/N = L_i$, where $L = \sum_{i=1}^{N} L_i$. Combined, these then generate equation (28):

$$(28)\ X_i = \theta K/N[1 + \sigma(N-1)]$$

It can be seen in the equation above that none of the variables on the right-hand side have subscripts. This implies that the marginal contribution from the diffusion of ideas by firm *i* can be viewed as negligible when compared to the overall ideas X. Hence, the firm takes ideas and knowledge to be beyond its control. As a result, X becomes the effect of the cross fertilization of ideas and capital accumulation, representing a multiplier effect. As the economy becomes larger and the number of firms grow, this effect becomes very large.

3.312 Aggregating Behavior

Since firms operate in a market with homogenous goods, $K = \sum_{i=1}^{N} Y_i = \sum_{i=1}^{N} F\,(K_i, X_i L_i)$. Using linear homogeneity properties, this can be rewritten as:

$$(29)\ Y = KF\big(1, \theta\big(1 + \sigma(N-1)\big)L/N\big)$$

Thus, labor remains essential in the economy and growth comes from labor augmentation. If all other factors are held constant, the aggregate output i.e. the GDP of the economy is proportional to aggregate capital. This implies $y = kF\big(1, \theta\big(1 + \sigma(N-1)\big)L/N\big)$. Similarly, if all other factors are held constant, the per capita output *y* is proportional to per capita capital *k.* Assuming that there is perfect and instantaneous diffusion of ideas across firms in the economy, we model σ = 1. In this case, the equation becomes $y = kF(1, \theta L)$.

3.313 Equilibrium

With no changes to the household from the Exogenous Wage Tax Model, we can equate the wealth of households to the output generated, denoted by Y, in order to close the model. Assuming, once again that the savings rate s is a constant fraction of income, we generate the following equation:

$$(30)\ \dot{k}/k = sF(1, \theta L) - \delta - w\varepsilon l$$

In this case, the taxes ε factor in directly and indirectly through L, which now represents the equilibrium level of employment in the economy. A higher population leads to an increase in the scope for idea implementation via labor augmenting technology. This means that everyone in the firm uses the idea and this results in a change in the growth rate of output per capita. Additionally, the ratio of consumption to capital in the economy remains the fixed. While growth in capital per effective worker has been slowed with the introduction of the tax, the increase in the number of workers- and therefore in output and investment- can offset this effect. In order to determine which effect dominates, and if there is an optimal level of tax in order to stimulate growth, we examine an optimization problem in the following section.

4 Optimization

In this section we undertake an examination to understand some of the dynamics of the model. We assume that the government has an objective function that they seek to maximize. We determine that the level of taxation the government will choose- and therefore investment in health- will depend on what they seek to optimize. To demonstrate how a government might choose a level of tax, we assume the government seeks to optimize the path of growth in this economy.

4.1 Optimal Growth

The government faces a growth of output function given by equation (31).

$$(31)\ \dot{Y} = \dot{K}F(1, \theta L) + KF_L\theta\dot{L}$$

Here we see that output grows related to an increase in technology, but also grows as a function of population growth, $\dot{L}$. If we assume that the initial starting point is one of equilibrium- including a scenario in which the exogenous death rate and the exogenous birth rate are equal, then growth in the economy is driven entirely by investment in capital, since growth of labor will be zero. In period zero, the government introduces a wage tax to invest in healthcare. This investment will set of an immediate growth of L. Since our model has assumed that the entrance and exit of population into the labor-force is instantaneous- newborns work immediately and there is no retirement- there is an immediate increase in output. However, given that the dynamics of K are governed by equation (30), that means the growth rate of output has also decreased from ε. By taking the derivative of equation (31) with respect to ε we yield equation (32).

$$(32)\ d\dot{Y}/d\varepsilon = \dot{K}F_L\theta L_\varepsilon + \dot{K}_\varepsilon F + KF_L\theta \dot{L_\varepsilon}$$

Equation 32 suggests that the effect of changes in taxation on output will be related to three competing effects. First, the tax rate changes the amount of labor available, L_ε, and the effect of this change on capital accumulation. Second, the effect of taxes on the accumulation of capital. Finally, how the tax rate affects the rate of population growth. In equation (32) it is important to note that an introduction of a tax rate, ε, affects both the amount of labor available and the growth of labor through the health channel, so it's affects are indirect. However, since we have assumed that there is no loss in the conversion of taxes to healthcare investment, the effect is easily understood. By setting equation (32) equal to zero and subbing in the various components, one can generate an optimal level of ε so as to maximize growth of output. This is given in equation (33)

$$(33)\ \varepsilon *= \frac{sY - \delta K + sF(1,\theta L) - \dfrac{LF(1,\theta L)}{\theta L_\varepsilon} + K\dfrac{\dot{L_\varepsilon}}{L_\varepsilon}}{wL + \dfrac{F(1,\theta L)}{\theta}}$$

This rather complicated equation has some relatively simple intuition- notably, that if the population growth set off by the investment in healthcare is overly large (or grows to be infinite), then ε* will also grow infinitely. Since we noted at the beginning of this section that growth is positively correlated with increases in population, this result is not unexpected. As a result, for this equation to make sense, population must be subject to a Malthusian check, such as the one in equation (2.2), for the existence of a positive, finite ε* to be guaranteed.

5 Conclusion

This paper has sought to understand the effects of mortality on economic growth. By working to endogenize the decision faced by ages surrounding investments in decreasing death, we have shown that models of endogenous mortality are feasible in economic models. We began by examining the relatively scant literature surrounding endogenous death within economic growth models. We then introduced three models. We began with an exogenous mortality model as a baseline. We then introduced an endogenous mortality model, but one that included exogenous growth. By showing that the growth path of this economy remained unchanged, we demonstrated a need to include endogenous growth within the model itself. Therefore, we presented our final model which included both endogenous mortality as well as endogenous growth in the form proposed by Romer ('86). By producing a model with endogenous death within the context of wider endogenous growth models, we hope to spawn future research questions.

We believe there are several plausible excellent extensions to this model. We would like to see a future model that includes agent utility functions that account for the disutility of death- both their own and those of the members of their households. In the current iteration of the model, members of the household are treated as interchangeable. Agents enter and exit and they do not take into account the disutility of death. Solving this highly complicated problem is probably necessary before models can be designed that examine the trade-off between improvements in fertility versus decreases in mortality, but that is another possible area of extension. We believe this model can and should also be extended to include an endogenously determined level of taxation that seeks to maximize agent's utility function rather than growth- if representative agents collectively bargain the tax rate, they will seek to maximize their own utility, which includes a component of growth but also includes a higher emphasis on the intertemporal trade-offs of consumption. We would also like to see a model that fully examines the impact of economic growth on M in the Lotka-Volterra model of population dynamics. If for example, a model incorporates pollution and its impact on environmental quality, this could conceivable negatively impact the carrying capacity of population and would involve an additional trade-off between consumption and population.

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