08 — Hazen-Williams & Equivalent Pipe

Water Resources, CIVL318

Last revision on October 23, 2018

Introduction

We begin this module by looking at a variety of pipes that are common in installed systems or are increasingly common in new systems.

Cast Iron Pipe

Cast iron pipe showing bell end. It is coated with yellow plastic to minimize corrosion.

Cast Iron Pipe

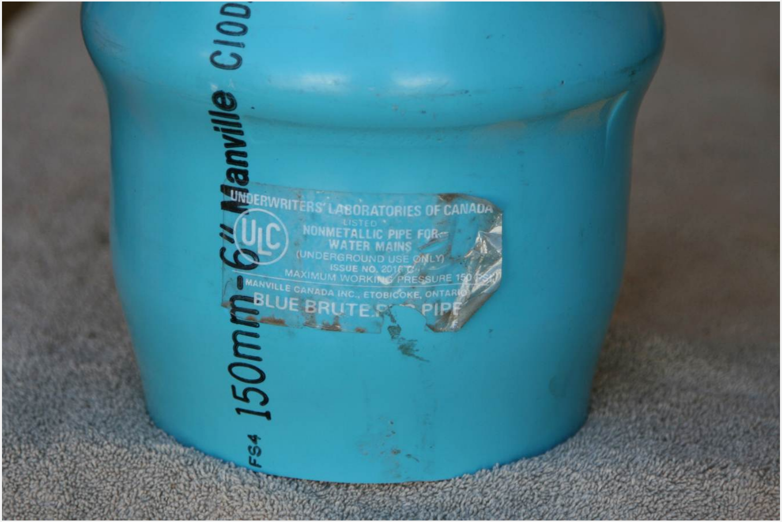
Cast iron pipe used under 16th Avenue, Bowness. Note the connection guides for correct overlap at joints.

PVC Plastic Pipe Connector

The rubber O-ring visible inside the bell is used to seal pipe connections

PVC Plastic Pipe Connector

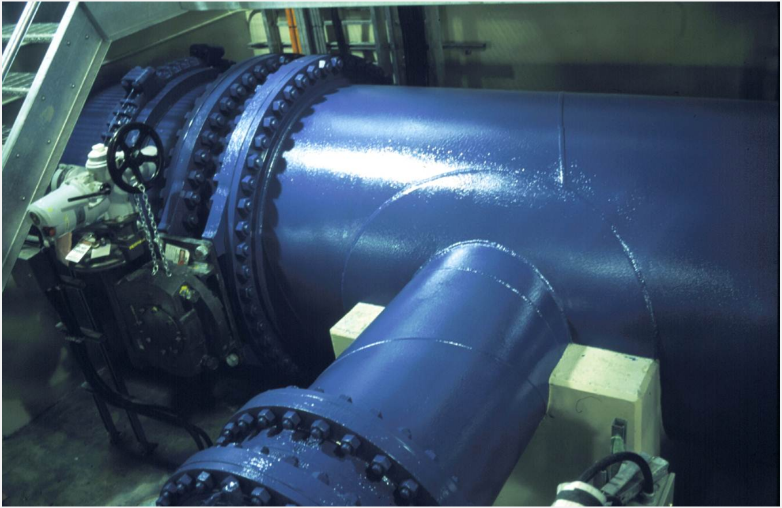
This "Blue Brute" pipe has a maximum operating pressure of 150 psi or 1000 kPa



Asbestos Cement Pipe and Connector

Not used in new installations

Inside a water distribution plant



Welded Steel Pipe

Welded steel header in a filter backwash system — Bearspaw Water Treatment Plant

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Welded Steel Pipe

Welded steel pipe fittings and connections

South Feeder from Bearspaw Plant

Welded Steel Pipe

Banff Middle Springs Plant; three pumps are to satisfy fire flow requirements

Welded Steel Pipe

Fittings - Benchlands, Canmore

Hyprescon

New Hyprescon installation, 14th St SW.

River crossing, 14th St SW

Hyprescon

Hyprescon steel pipe with cement lining and mortar coating

Mortar coating

Cement lining

Custom welded section

Custom welded section in place

Butterfly valve in Hyprescon pipe

Butterfly valve in Hyprescon pipe

Hyprescon

Concrete inspection chamber for valve maintenance

Glenmore Water Treatment Plant

Hazen-Williams Equation

We have used Darcy's equation to analyze the flow of liquids in pipes, where the volume flow rate has been provided.

It is more complicated to apply Darcy's equation to calculate flow, requiring an iterative process to find the friction factor and associated headloss.

We will now continue our analysis using the Hazen-Williams equation to solve for head loss, pipe diameter or flow in water systems.

Hazen-Williams Equation

Darcy's equation, $h_L = f \cdot \frac{L}{D} \cdot \frac{v^2}{2g}$, works for all liquids and for laminar and turbulent flow conditions.

Darcy's equation is not directly suited to calculations to determine flow rates or diameters. (We have been provided with both for all the questions we have done using Darcy's equation.)

The Hazen-Williams equation is widely used for calculations relating to the flow of water.

It is also more practical for the analysis of flow through parallel pipe systems.

Hazen-Williams Equation

Hazen-Williams Equation

$$Q = 0.849ACR^{0.63}s^{0.54}$$

where:

Q	is the volume flow rate in m^3/s
A	is the pipe area in m^2
C	is a dimensionless roughness coefficient
R	is the hydraulic radius of the pipe ($R = D/4$ for circular pipes)
s	is the hydraulic slope, the energy loss per length of pipe, h_L/L

We shall use the formula is a slightly different form...

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Hazen-Williams Equation

Since we are only concerned with circular pipes, we shall use D, the diameter of the pipe, instead of A and R:

$$\begin{aligned} Q &= 0.849 A C R^{0.63} s^{0.54} \\ &= 0.849 \left(\frac{\pi D^2}{4}\right) C \left(\frac{D}{4}\right)^{0.63} s^{0.54} \\ &= 0.27842\, C\, D^{2.63} s^{0.54} \end{aligned}$$

Hazen-Williams Equation

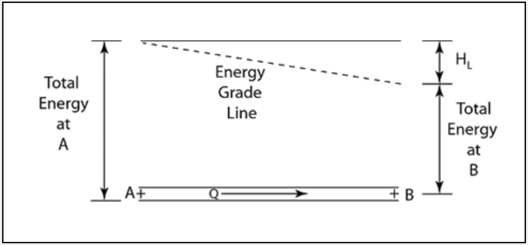
Since we are only concerned with circular pipes, we shall use D, the diameter of the pipe, instead of A and R:

$$\begin{aligned} Q &= 0.849ACR^{0.63}s^{0.54} \\ &= 0.849\left(\frac{\pi D^2}{4}\right)C\left(\frac{D}{4}\right)^{0.63}s^{0.54} \\ &= 0.27842\,C\,D^{2.63}s^{0.54} \end{aligned}$$

It is more useful to have Q in L/s and D in mm:

$$\begin{aligned} Q &= 1000 \times 0.27842\,C\left(\frac{D}{1000}\right)^{2.63}s^{0.54} \\ &= 0.0000035867\,C\,D^{2.63}s^{0.54} \\ &= \frac{C\,D^{2.63}s^{0.54}}{279000} \end{aligned}$$

Energy Grade Line



Consider a horizontal pipe from A to B; energy is lost due to friction as water flows along the pipe such that the total energy at any point is shown by the dotted line – the energy grade line.

Total energy loss between A and B is h_L, in length L of pipe, so the slope of the energy grade line is h_L/L

We will replace the s term in the Hazen-Williams with h_L/L:

$$Q = \frac{C\,D^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}$$

Hazen-Williams Equation

The Hazen-Williams is an empirical formula, derived from experimental results. The exponents 2.63 and 0.54 are to make observed results fit the formula more consistently.

Units are not consistent on sides of the equation.

$$Q = \frac{C\,D^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}$$

C is a unitless coefficient so units are L/s on the left and mm$^{2.63}$ on the right.

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Approximate Roughness Coefficients, C

Pipe Material	New Pipe	Design C
PVC	150	150
Polyethylene	140	140
Welded Steel	140	100
Cement-lined steel (Hyprescon)	150	140
Cast Iron	130	100
Cast Iron - 10 years old		110
Cast Iron - 20 years old		95
Cast Iron - 30 years old		85
Cast Iron - 40 years old		75

http://media.toro.com/Documents/Agriculture/HazenWilliamsEquation.pdf

Restrictions on Hazen-Williams

The Hazen-Williams equation works under the following conditions:

- Water flowing at approximately 10℃ - 15℃ in fully pressurised pipe
- Flow velocity not in excess of 3.0 m/s
- Flow is in pipe with a diameter larger than 50 mm and smaller than 2.0 m

Use of the Hazen-Williams under other conditions from those specified above will result in some error.

(In practice, there is always error. C is an estimated value; pipe does not stay new for long!)

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Alternative Form - Solving for Head Loss

$$Q = \frac{C\,D^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}$$

$$\Rightarrow \left(\frac{h_L}{L}\right)^{0.54} = \frac{279000\,Q}{C\,D^{2.63}}$$

$$\Rightarrow \frac{h_L}{L} = \left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{\frac{1}{0.54}}$$

$$\Rightarrow \boldsymbol{h_L = L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852}}$$

Alternative Form - Solving for Diameter

$$Q = \frac{C\,D^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}$$

$$\Rightarrow\ D^{2.63} = \frac{279000\,Q}{C\left(\frac{h_L}{L}\right)^{0.54}}$$

$$\Rightarrow\ D = \left(\frac{279000\,Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{\frac{1}{2.63}}$$

$$\boldsymbol{\Rightarrow\ D = \left(\frac{279000\,Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802}}$$

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Hazen-Williams Various Forms

Hazen-Williams Formulæ

$$Q = \frac{C\,D^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}$$

$$h_L = L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852}$$

$$D = \left(\frac{279000\,Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802}$$

where Q is in L/s, D is in mm, h_L and L are in m.

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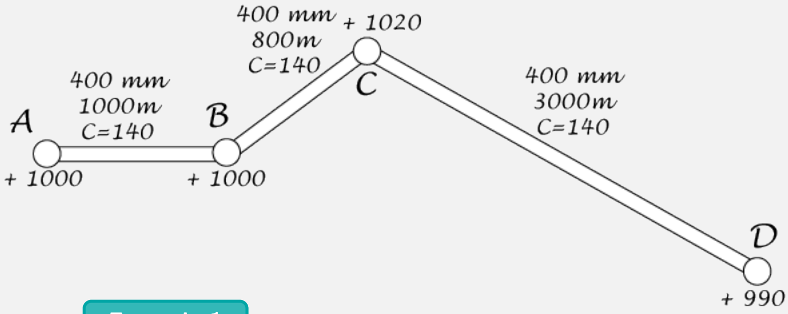
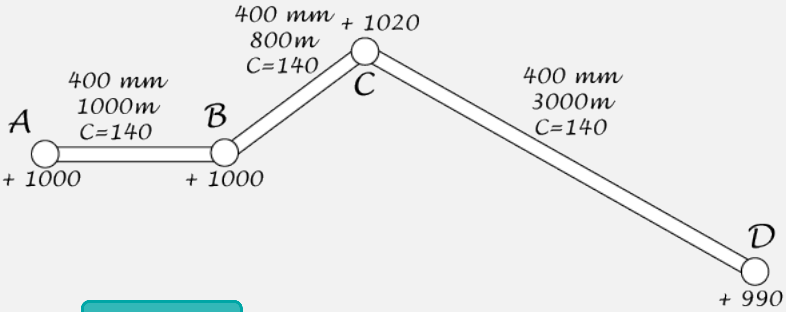


Example 1

The pipes are cement-lined Hyprescon with a diameter of 400 mm and a roughness coefficient of $C = 140$ and flow is 200 L/s. Elevations are as indicated.

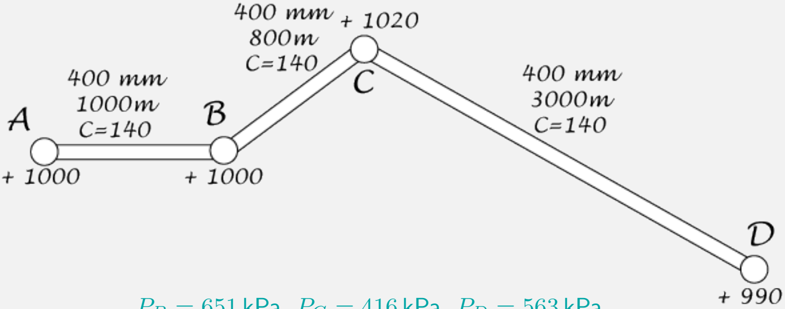
Calculate the pressure at B given that the pressure at A is 700 kPa.



Exercise 1

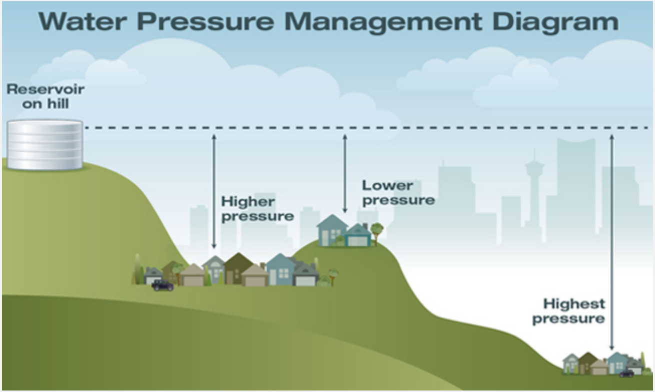
The pipes are cement-lined Hyprescon with a diameter of $400\,\mathrm{mm}$ and a roughness coefficient of $C = 140$ and flow is 200 L/s. Elevations are as indicated.

Calculate the pressures at C and D, given that the pressure at A is $700\,\mathrm{kPa}$.

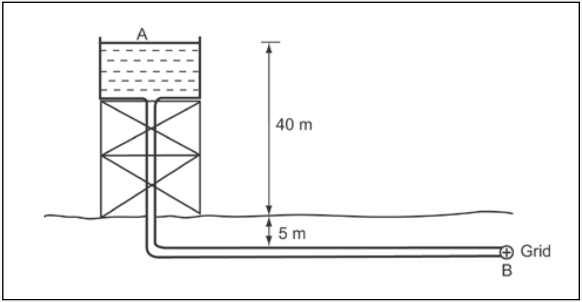


$P_B = 651\,\text{kPa}$, $P_C = 416\,\text{kPa}$, $P_D = 563\,\text{kPa}$

Pressures are lower at points farther away from the point of delivery (A is the point of delivery in this case) but the main pressure determinant is elevation, with higher elevations in a distribution system having lower pressures.



http://www.calgary.ca/UEP/Water/Pages/Drinking-water/Water-quality/Water-Pressure.aspx

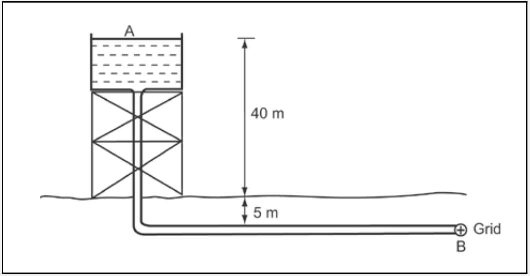


Example 2

Water flows from a storage tank through a welded steel pipe that is $1200\,\text{m}$ long and $350\,\text{mm}$ in diameter, entering a distribution grid at point B. Assume $C = 100$.

Determine:

1. The pressure at B when the flow is $150\,\text{L/s}$
2. The maximum flow rate into the grid when the minimum allowable pressure at B is $400\,\text{kPa}$.

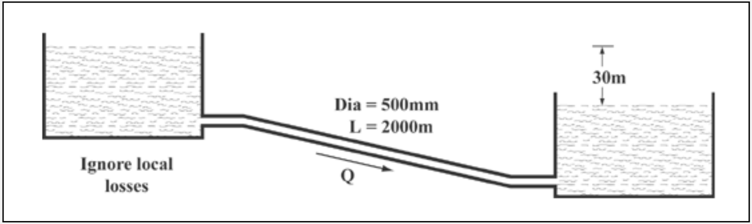


Velocity $v_B = 1.5591$ m/s in part (1) and velocity head was small (0.12389 m).

Omitting the velocity head term altogether would have changed P_B by about 0.3% of the calculated 317 kPa value. This is well within the error to be expected in estimation of the C-value.

In water distribution systems, velocities are normally designed to be less than 2 m/s since faster flow causes excessive head loss and may cause noise and vibration.

It is generally acceptable to ignore velocity head in cases where diameter or flow are unknown; this greatly simplifies some calculations.



Exercise 2

Water flows from one reservoir down to another, through a $500\,\mathrm{mm}$ diameter pipe that is $2000\,\mathrm{m}$ in length. The difference in elevation between the surfaces of the two reservoirs is $30\,\mathrm{m}$.

Determine:

1. The flow with high density polyethylene pipe (HDPE) with $C = 140$
2. The flow with welded steel with $C = 100$
3. The diameter of HDPE pipe required for a flow of $1200\,\mathrm{L/s}$

Neglect minor losses.

Hazen-Williams and Minor Losses

In our earlier study of minor losses, we calculated minor head losses through valves using

$$h_L = k \cdot \frac{v^2}{2g}$$

which required knowledge of both Q and D to calculate v.

We defined the 'equivalent length ratio,' L_e/D, of a valve or fitting where the equivalent length, L_e, is defined as the length of pipe that would produce the same (frictional) loss as the valve or fitting (minor) losses.

Thus, to determine the losses due to a fully open globe valve (with $L_e/D = 340$), we can instead calculate the frictional losses due to length L_e of pipe.

(Note that we will still need the diameter D of the pipe)

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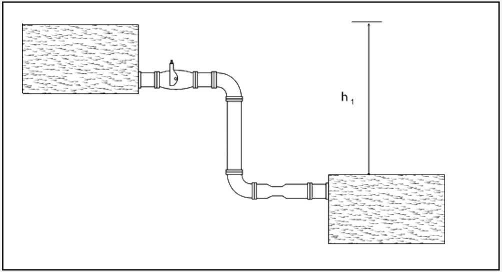


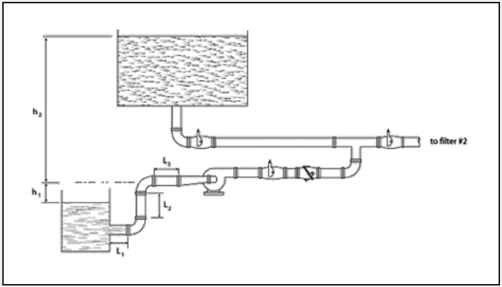
Example 3

In a water treatment plant, water flows from a filter down to a clear well through the pipe system shown. The pipe is welded steel with a diameter of 300 mm and roughness coefficient $C = 130$. The total length of pipe is 50 m. Elevation difference h_1 between the tanks is 5.0 m.

Equivalent length ratios, L_e/D, are:

Entrance and exit losses:	50	Butterfly valve:	35
Large radius elbows:	25	Venturi meter:	100

Determine the flow through the system.



Example 4

In a water treatment plant, backwash water is pumped from the clear well through the pipe system shown to the filter. The required backwash flow is $10\,\mathrm{L/s}$ per square meter of filter area (the filter dimensions are $10\,\mathrm{m}$ by $15\,\mathrm{m}$. The inlet pipe is made of welded steel $(C = 130)$, has a diameter of $1000\,\mathrm{mm}$ and a total length $(L_1 + L_2 + L_3)$ of $10\,\mathrm{m}$.

The outlet pipe, from the pump to the filter, is also welded steel, has a diameter of $700\,\mathrm{mm}$ and a length of $70\,\mathrm{m}$.

The two elevation differences are $h_1 = 2\,\mathrm{m}$ and $h_2 = 10\,\mathrm{m}$.

Cont'd. . .

Equivalent length ratios, L_e/D, are:

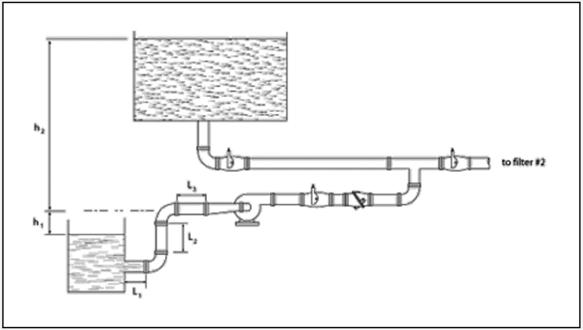
Entrance:	10	Elbow (inlet):	25
Eccentric Reducer:	2	Butterfly Valve:	40
Check Valve:	120	Elbow (outlet):	35
Tee Connection (through):	60		

Cont'd. . .

Determine:

1. The head losses on the inlet side (clear well to pump)
2. The head losses on the outlet side (pump to filter)

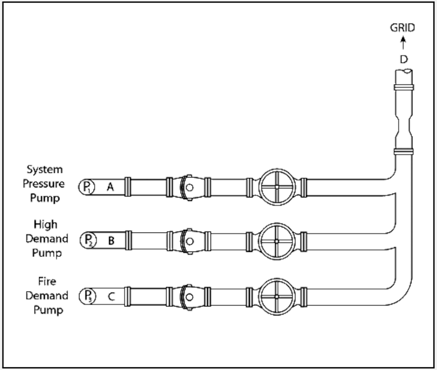
Neglect exit losses into the filter.



Exercise 3

This exercise is a continuation of the previous example. Determine:

3. The head added by the pump
4. The pressure at the pump outlet



Example 5

The pumps and piping system are used to supply a municipal grid. Pump P_1 (P_2 and P_3 are not running at this time) runs continuously and maintains the basic pressure in the distribution grid beyond point D. The elevations are the same at the pump and the discharge point D.

The outlet pipe, from the pump to point D, is welded steel $(C = 130)$ with a diameter of 200 mm and a total length between fittings of 10 m.

The minimum pressure required at D is 500 kPa for a design flow of 150 L/s.

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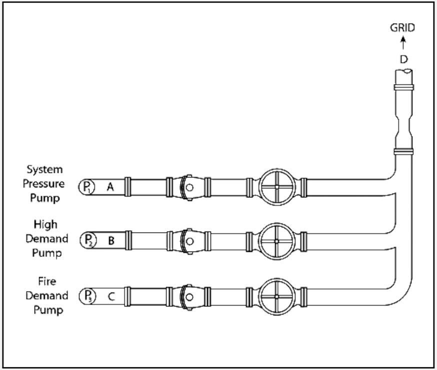

Example 5 cont'd. . .

Check Valve:	120	Gate Valve:	15
Tee Connection (through):	60	Venturi Meter:	100

There is no flow from pumps P_2 and P_3. Determine:

1. the head losses between A and D
2. the pressure at A required for the required pressure and flow at D

Determining Flow Through Pipes in Series

We have used the Hazen-Williams Equation to find flow through a single pipe but how can we find the flow through a system of pipes in series (such as pipes AB, BC and CD shown above)?

Note that, **for pipes in series**, the volume flow rate is constant through each pipe and that the head loss is cumulative.

Pipes in Series

$$Q_{AB} = Q_{BC} = Q_{CD}$$

$$h_{AD} = h_{AB} + h_{BC} + h_{CD}$$

These are **important** properties of pipes in series!

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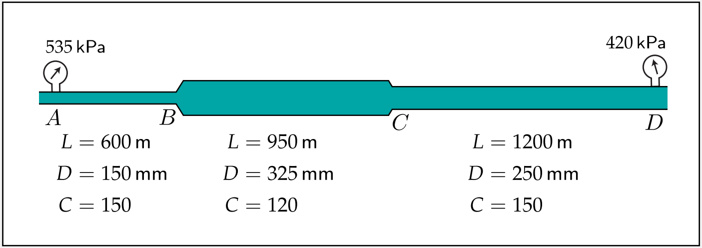

Example 5

Determine Q, the volume flow rate from A to D, through the system shown. Ignore minor losses and assume that A and D are at the same elevation.

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Equivalent Pipes

1. Two pipes and/or pipe systems are **hydraulically equivalent** if each pipe or system has the same headloss at the same flow.
2. An imaginary, or virtual, pipe that is hydraulically equivalent to a real pipe or system is known as an **equivalent pipe**.
3. An equivalent pipe is determined by three properties (length, diameter, resistance coefficient). By setting two of these to convenient values (e.g., length $L = 1000$ m and resistance coefficient $C = 100$), the diameter of the equivalent pipe is uniquely defined.
4. An equivalent pipe does not physically exist; it is a tool to simplify pipe network analysis.

Example 6

a. Determine the diameter of a pipe with length $L = 1000\,\text{m}$ and resistance coefficient $C = 100$ that is equivalent to $785\,\text{m}$ of new Schedule 40 12-in steel pipe $(D = 303.2\,\text{mm}, C = 130)$.

b. Verify that this equivalent pipe has the same headloss as the 12-in steel pipe for two arbitrary flows (choose a couple of flows at random, different from the flow used in part a).

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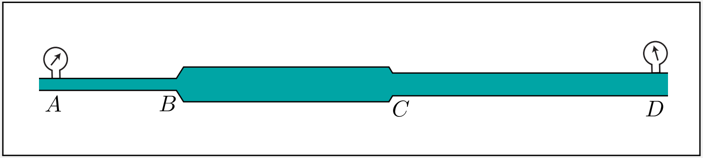
Using Equivalent Pipe Technique to Analyze Pipes in Series



1. Choose an arbitrary flow rate though the 'real' system and use the Hazen-Williams to find the head loss for each of the pipes for this flow. We use $Q = 100\,\mathsf{L/s}$
2. Sum the headlosses to get the total headloss through all the series pipes.
3. Find the diameter of an equivalent pipe, with length $L = 1000\,\mathrm{m}$ and resistance coefficient $C = 100$.

   (Note that the values for L and C are arbitrary and are chosen for convenience and consistency; the final result of an analysis will be the same using any length or resistance coefficient.)
4. Use this single equivalent pipe instead of the multiple pipes in series to analyze the system (usually using the GEE).

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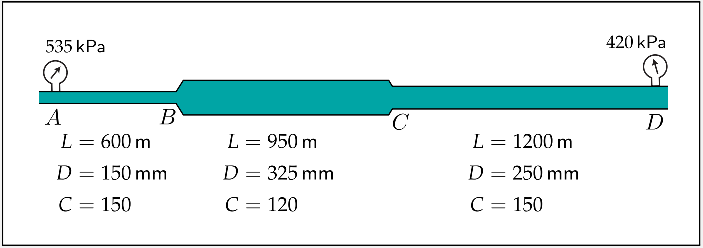

Example 7

Use the equivalent pipe technique to determine Q, the volume flow rate from A to D, through the system shown. Ignore minor losses and assume that A and D are at the same elevation.

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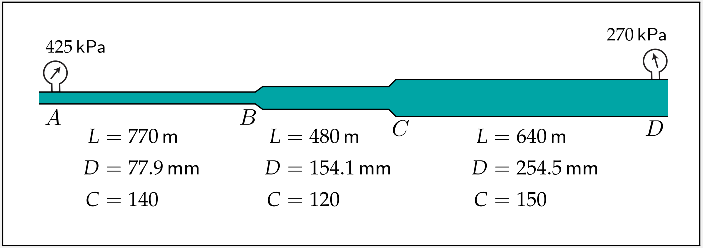
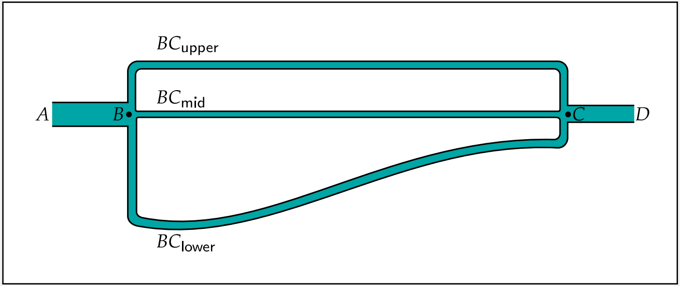


Exercise 4

Use the equivalent pipe technique to determine Q, the volume flow rate from A to D, through the system shown. Ignore minor losses and assume that A and D are at the same elevation.

Systems with Pipes in Parallel



- Suppose that the flow through the system shown is $18\,\text{L/s}$. Then $Q_{AB} = 18\,\text{L/s}$ and $Q_{CD} = 18\,\text{L/s}$.
- There are three pipes from B to C; these pipes are in **parallel** (i.e., they each start at the same node B and end at the same node C). The system flow of $18\,\text{L/s}$ is split between these three pipes.
- We don't know the flow is in each of these three pipes but, since no water is added or diverted away from the system, the entire flow through AD is divided in some fashion such that:

$$Q_{AB} = Q_{BC_{\text{upper}}} + Q_{BC_{\text{mid}}} + Q_{BC_{\text{lower}}} = Q_{CD}$$

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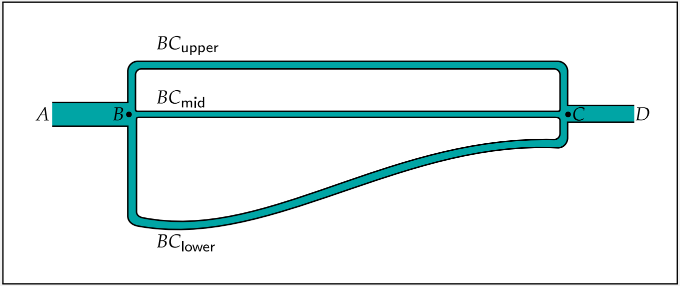
Systems with Pipes in Parallel

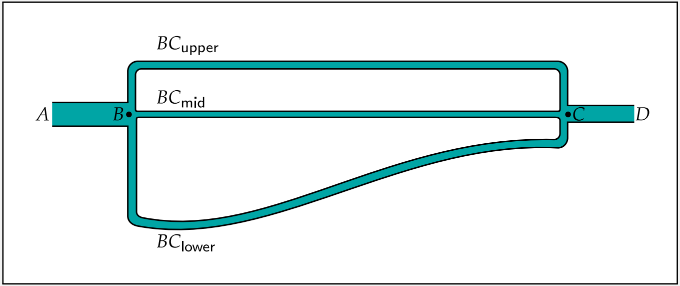


- Now, suppose that the pressure at B is $P_B = 600\,\text{kPa}$ and the pressure at C is $P_C = 450\,\text{kPa}$.
- The drop in pressure between B and C is $150\,\text{kPa}$, or approximately $15\,\text{m}$ of headloss.
- Each of the three parallel pipes has the same pressure drop and headloss. In particular:

$$h_{L_{BC_{\text{upper}}}} = h_{L_{BC_{\text{mid}}}} = h_{L_{BC_{\text{lower}}}}$$

- **Important**: Each of the three parallel pipes has a headloss of $15\,\text{m}$, and the **total headloss between B and C is $15\,\text{m}$** - do not add them!

Systems with Pipes in Parallel



Parallel Pipes Results

$$Q_{AB} = Q_{BC_{\text{upper}}} + Q_{BC_{\text{mid}}} + Q_{BC_{\text{lower}}} = Q_{CD}$$

$$h_{L_{BC_{\text{upper}}}} = h_{L_{BC_{\text{mid}}}} = h_{L_{BC_{\text{lower}}}}$$

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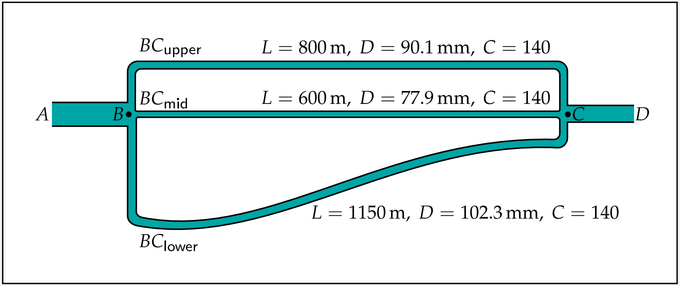
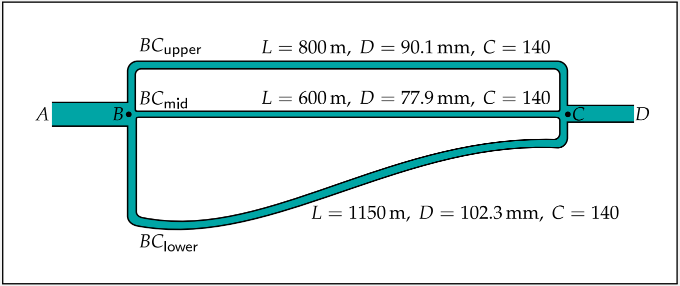


Example 8

Given a flow of $18\,\text{L/s}$ and ignoring minor losses:

- a Determine the volume flow rate through each of the parallel pipes between B and C.
- b Determine the headloss due to friction between B and C.

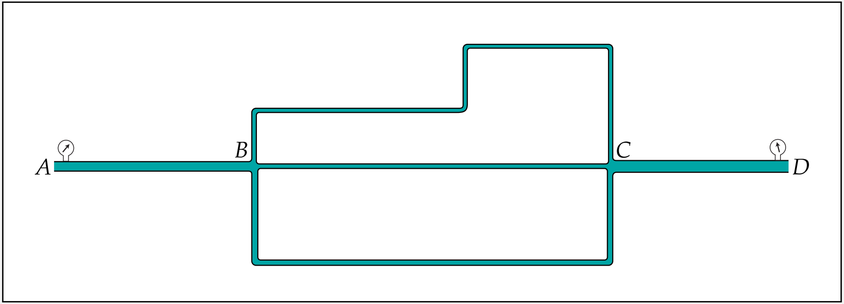


Example 9

Given a flow of $18\,\text{L/s}$ and ignoring minor losses:

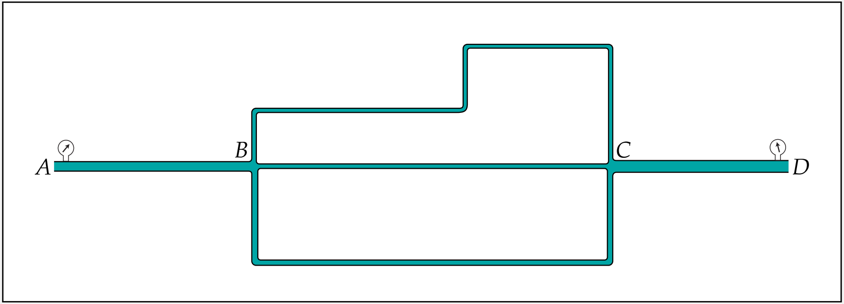
a. Determine the proportion of the flow that goes through each parallel pipe by choosing a convenient headloss between B and C.
b. Determine the volume flow rate through each of the parallel pipes.

Determining Flow Through Parallel Pipes Using an Equivalent Pipe



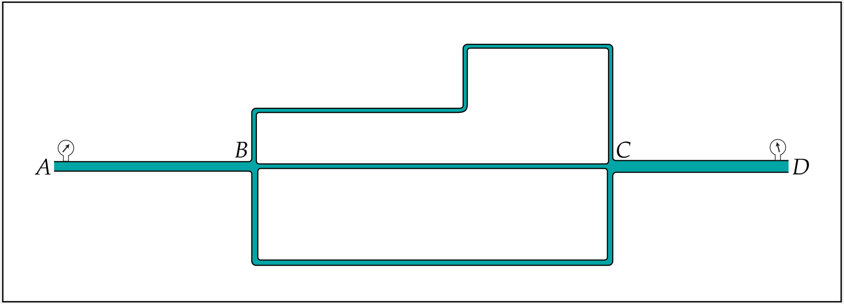
If we are given the properties (length, diameter, resistance coefficient) for each pipe and the pressures at A and D, how do we determine the volume flow rate through the system?

Determining Flow Through Parallel Pipes Using an Equivalent Pipe



1. Find an equivalent pipe, BC_{equiv}, that replaces the three parallel pipes between B and C:

Determining Flow Through Parallel Pipes Using an Equivalent Pipe



1. Find an equivalent pipe, BC_{equiv}, that replaces the three parallel pipes between B and C:
   a. Assume a headloss of $10\,\text{m}$ between B and C.

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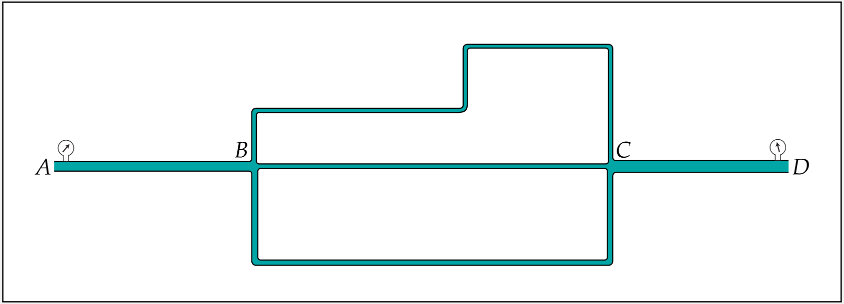
Determining Flow Through Parallel Pipes Using an Equivalent Pipe



1. Find an equivalent pipe, BC_{equiv}, that replaces the three parallel pipes between B and C:
   a. Assume a headloss of $10\,\text{m}$ between B and C.
   b. Calculate the flow through each parallel pipe for this headloss of $10\,\text{m}$. Sum these flows.

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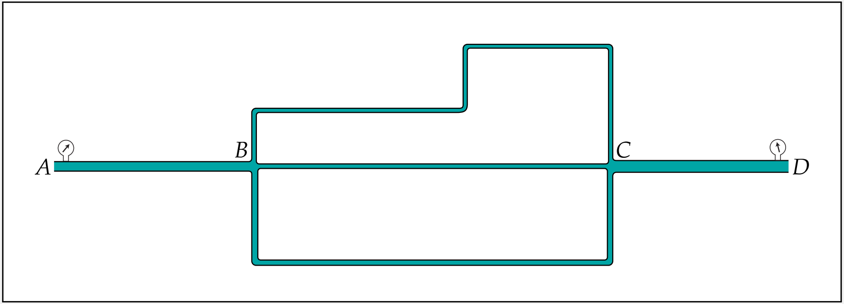
Determining Flow Through Parallel Pipes Using an Equivalent Pipe



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   c. Now we have the total flow between B and C for a headloss of $10\,\text{m}$. Use these to find BC_{equiv}.

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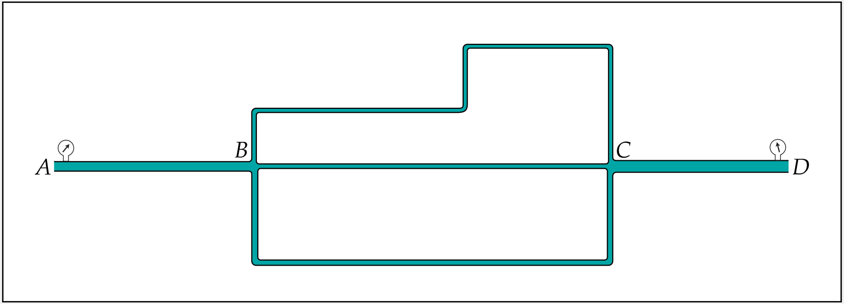

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   a. Assume a headloss of $10\,\text{m}$ between B and C.
   b. Calculate the flow through each parallel pipe for this headloss of $10\,\text{m}$. Sum these flows.
   c. Now we have the total flow between B and C for a headloss of $10\,\text{m}$. Use these to find BC_{equiv}.
2. Solve the system of pipes in series (AB, BC_{equiv}, CD) as in Example 8.

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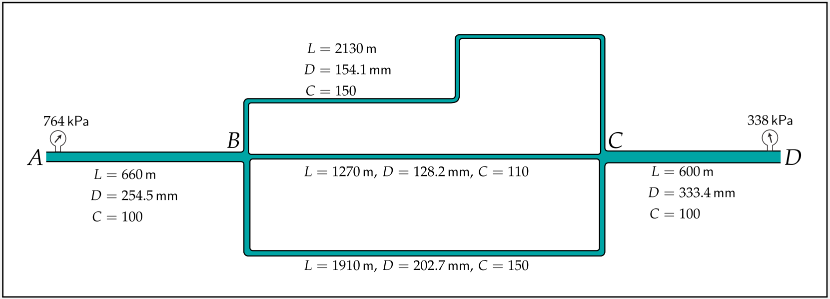

Example 10

A, B, C and D are at the same elevation. Determine the flow through the system from A to D. (Ignore minor losses.)

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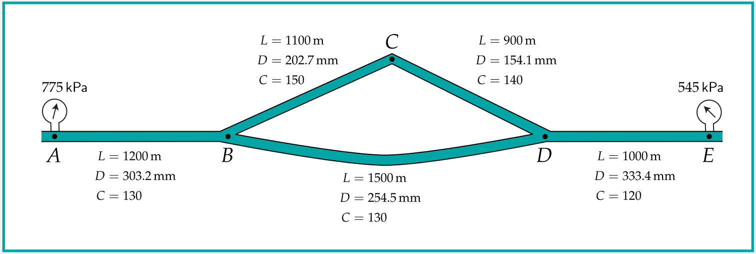
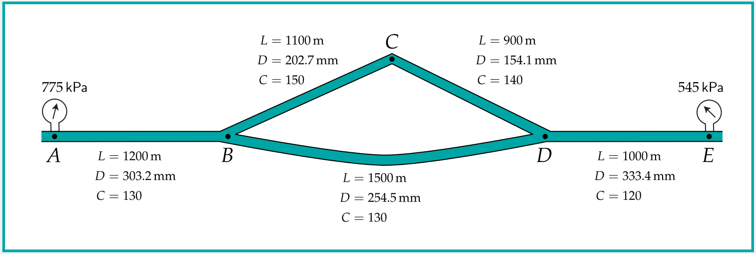


Exercise 5

Determine Q. All pipes are horizontal. Disregard minor losses.



Exercise 5

Determine Q. All pipes are horizontal. Disregard minor losses.

The process:

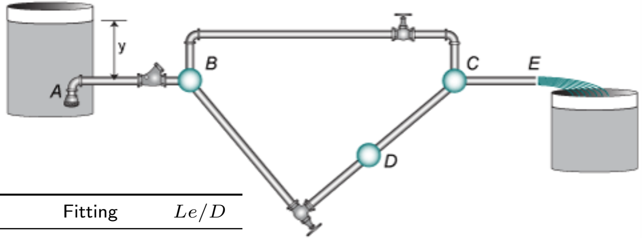
1. Replace series pipes BC and CD with a single equivalent pipe BCD
2. Replace parallel pipes BCD and BD with a single equivalent pipe $BD2$
3. Replace series pipes AD, $BD2$ and DE with a single equivalent pipe AE
4. Calculate the headloss for the system
5. Determine the flow through the system

Minor Losses

It is simple to incorporate minor losses into the equivalent pipe methods we have used.

For pipes with valves or fittings, calculate and use the effective length of the pipes rather than the actual length. All other calculations remain the same.

Exercise 6



	Fitting	Le/D
	Angle Valve	150
	Check Valve	100
	Elbow	50
	Foot Valve	75
	Gate Valve	35

Pipe	Length (m)	diam (mm)	C
AB	10	500	125
BC	2000	275	150
BD	1500	250	100
DC	1000	300	100
CE	10	500	125

Given that $y = 6.7\,\text{m}$, determine the flow through the system.
(Nodes B and E are at the same elevation. Disregard exit losses.)

Summary

1. The most important lessons to be learned about system analysis are that:
   a. Flow is constant through pipes in series.
   b. Parallel pipes have the same headloss, and that the headloss across a system of parallel pipes is the same as the headloss in each pipe: do not add these headlosses!
2. The methods described above do not work for all systems (or even for most systems): not all systems can be split into series and/or parallel pipes as we have done.
3. There are a number of iterative techniques (Hardy-Cross, Newton-Raphson, Gradient, Linear ...) for analysing more complex hydraulic networks. These are mathematically intensive. Instead, we shall use software to analyse such networks.

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