Module 8: Hazen Williams Equation and Equivalent Pipes (CIVL 318)

Hazen-Williams Equations

$$Q = \frac{C\,D^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}, \qquad h_L = L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852}, \qquad D = \left(\frac{279000\,Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802}$$

Equivalent-Length Ratios for Fittings

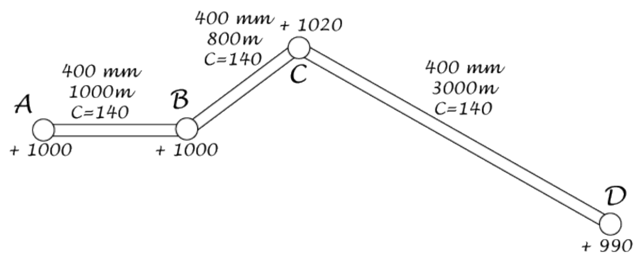
Type	L_e/D
Globe valve — fully open	340
Angle valve — fully open	150
Gate valve — fully open	8
— 3/4 open	35
— 1/2 open	160
— 1/4 open	900
Check valve — swing type	100
Check valve — ball type	150
Butterfly valve — fully open — 2-8"	45
— 10-14"	35
— 16-24"	25
Foot valve — poppet disc type	420
Foot valve — hinged disc type	75
90° standard elbow	30
90° long radius elbow	20
90° street elbow	50
45° standard elbow	16
45° street elbow	26
Close return bend	50
Standard tee — flow through run	20
Standard tee — flow through branch	60
Gradual enlargement — 15° cone angle	8
Gradual enlargement — 20° cone angle	15
Gradual enlargement — 30° cone angle	23
Gradual reduction — 15° to 40° cone angle	2
Pipe entrance — inward projecting	50
Pipe entrance — square	25
Pipe entrance — rounded	10
Venturi meter	100

Example 1:

For the pipeline shown, calculate the pressure at B, given that the pressure at A is 700 kPa.

The pipes are cement-lined Hyprescon with a diameter of 400 mm and a roughness coefficient of $C = 140$. Flow through the system is 200 L/s.

Elevations are as indicated.



Solution:

First, apply the Hazen-Williams:

$$
\begin{aligned}
h_{L_{AB}} &= L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852} \\
&= 1000\left(\frac{279000 \times 200}{140 \times 400^{2.63}}\right)^{1.852} \\
&= 4.9903\,\text{m}
\end{aligned}
$$

Now, apply the GEE:

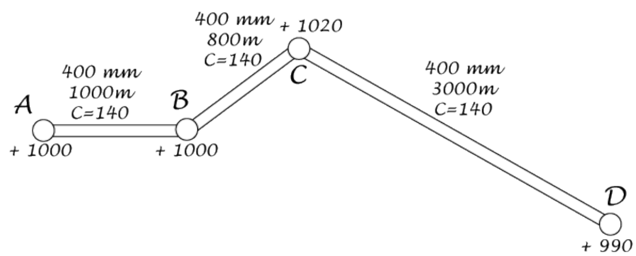
$$
\begin{aligned}
\frac{P_A}{\gamma} + \cancel{z_A} + \cancel{\frac{v_A^2}{2g}} - h_L &= \frac{P_B}{\gamma} + \cancel{z_B} + \cancel{\frac{v_B^2}{2g}} \\
\frac{700}{9.81} - 4.9903 &= \frac{P_B}{9.81} \\
\\
P_B &= 651.05\,\text{kPa} \\
\boldsymbol{P_B} &= \mathbf{651\,kPa}
\end{aligned}
$$

Exercise 1:

For the pipeline shown, calculate the pressure at C and D, given that the pressure at A is 700 kPa.

The pipes are cement-lined Hyprescon with a diameter of 400 mm and a roughness coefficient of $C = 140$. Flow through the system is 200 L/s.

Elevations are as indicated.



Solution:

First, apply the Hazen-Williams:

$$
\begin{aligned}
h_{L_{BC}} &= L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852} \\
&= 800\left(\frac{279000 \times 200}{140 \times 400^{2.63}}\right)^{1.852} \\
&= 3.9922\,\text{m}
\end{aligned}
$$

$$
\begin{aligned}
h_{L_{CD}} &= L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852} \\
&= 3000\left(\frac{279000 \times 200}{140 \times 400^{2.63}}\right)^{1.852} \\
&= 14.971\,\text{m}
\end{aligned}
$$

Now, apply the GEE:

$$
\begin{aligned}
\frac{P_B}{\gamma} + z_B + \cancel{\frac{v_B^2}{2g}} - h_L &= \frac{P_C}{\gamma} + z_C + \cancel{\frac{v_C^2}{2g}} \\
\frac{651.05}{9.81} - 3.9922 &= \frac{P_C}{9.81} + 20
\end{aligned}
$$

$$
P_C = 415.69\,\text{kPa}
$$

$$
\boldsymbol{P_C = 416\,\textbf{kPa}}
$$

$$
\begin{aligned}
\frac{P_C}{\gamma} + z_C + \cancel{\frac{v_C^2}{2g}} - h_L &= \frac{P_D}{\gamma} + z_D + \cancel{\frac{v_D^2}{2g}} \\
\frac{415.69}{9.81} + 30 - 14.971 &= \frac{P_D}{9.81}
\end{aligned}
$$

$$
P_D = 563.12\,\text{kPa}
$$

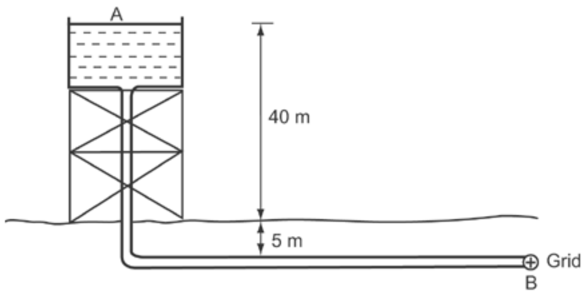
$$
\boldsymbol{P_D = 563\,\textbf{kPa}}
$$

Example 2:

Water flows from a storage tank through a welded steel pipe that is 1200 m long and 350 mm in diameter, entering a distribution grid at point 'B'. Assume C=100. Determine:

(1) The pressure at 'B' when the flow is 150 L/s
(2) The maximum flow rate into the grid when the minimum allowable pressure at 'B' is 400 kPa.

Minor losses are negligible compared to friction losses.



Solution (1):

$$h_L = L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852}$$
$$= 1200\left(\frac{279000\times 150}{100\times 350^{2.63}}\right)^{1.852}$$
$$= 12.561\,\text{m}$$

$$v = \frac{Q}{A}$$
$$= \frac{0.150}{\pi(0.350)^2/4}$$
$$= 1.5591\,\text{m/s}$$

$$\frac{v^2}{2g} = 0.12389\,\text{m}$$

G.E.E.:

$$\cancelto{0}{\frac{P_A}{\gamma}} + z_A + \cancelto{0}{\frac{v_A^2}{2g}} - h_L = \frac{P_B}{\gamma} + \cancelto{0}{z_B} + \frac{v_B^2}{2g}$$
$$45 - 12.561 = \frac{P_B}{9.81} + 0.12389$$

$$P_B = 317.01\,\text{kPa}$$
$$\boldsymbol{P_B = 317\,\text{kPa}}$$

Notice that if we recalculated the pressure at B omitting the velocity head, then $P_B = 318.2\,\text{kPa}$, not very different from including it.

Solution (2):

What flow/headloss will give a pressure of 400 kPa at B?

$$\cancelto{0}{\frac{P_A}{\gamma}} + z_A + \cancelto{0}{\frac{v_A^2}{2g}} - h_L = \frac{P_B}{\gamma} + \cancelto{0}{z_B} + \frac{v_B^2}{2g}$$
$$45 - h_L = \frac{400}{9.81} + \frac{v_B^2}{2g}$$

One equation and two unknowns! We could solve it iteratively, guessing at a flow and seeing what P_B is for this flow, then trying another flow until we converge on a pressure of 400 kPa at B.

But the velocity head had an effect of about 0.3% in part (1); it will be less here as we need less velocity/headloss to keep the pressure higher. So, in problems of this type, we **simply ignore the velocity head term...**

$$45 - h_L = \frac{400}{9.81} + \cancelto{0}{\frac{v_B^2}{2g}}$$
$$h_L = 4.2253\,\text{m}$$

What flow will give this headloss?

$$Q = \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}$$
$$= \frac{100\times 350^{2.63}\left(\frac{4.2253}{1200}\right)^{0.54}}{279000}$$
$$= 83.272\,\text{L/s}$$
$$\boldsymbol{Q = 83.3\,\text{L/s}}$$

Let's look at the value of the velocity head we discarded...

$$v = \frac{Q}{A}$$
$$= \frac{0.083272}{\pi(0.350)^2/4}$$
$$= 0.86551\,\text{m/s}$$

$$\frac{v^2}{2g} = 0.038181\,\text{m}$$

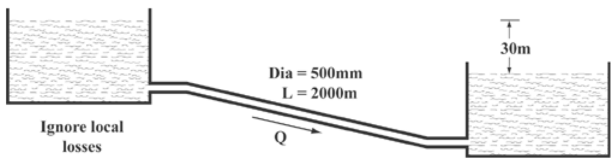
The velocity head is small enough that we can disregard it. Any error from not omitting the headloss is negligible compared with error in estimating the C-value.

Exercise 2:
Water flows from one reservoir down to another, through a 500 mm diameter pipe that is 2000 m in length. The difference in elevation between the surfaces of the two reservoirs is 30 m.

Determine:

(1) The flow with high density polyethylene pipe (HDPE) with $C = 140$
(2) The flow with welded steel with $C = 100$
(3) The diameter of HDPE pipe required for a flow of 1200 L/s

Disregard minor losses.



Note: At the surfaces of both reservoirs, pressure and velocity head are 0 so the GEE reduces to

$$30 - h_L = 0$$

Solution (1): For HDPE,

$$\begin{aligned} Q &= \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000} \\ &= \frac{140(500)^{2.63}\left(\frac{30}{2000}\right)^{0.54}}{279000} \\ &= 651.48\,\mathrm{L/s} \\ \boldsymbol{Q} &= \mathbf{651\,L/s} \end{aligned}$$

Solution (2): For welded steel,

$$\begin{aligned} Q &= \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000} \\ &= \frac{100(500)^{2.63}\left(\frac{30}{2000}\right)^{0.54}}{279000} \\ &= 465.35\,\mathrm{L/s} \\ \boldsymbol{Q} &= \mathbf{465\,L/s} \end{aligned}$$

Solution (3): Diameter for a flow of 1200 L/s with HDPE,

$$\begin{aligned} D &= \left(\frac{279000\,Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802} \\ &= \left(\frac{279000 \times 1200}{140\left(\frac{30}{2000}\right)^{0.54}}\right)^{0.3802} \\ &= 630.42\,\mathrm{mm} \\ \boldsymbol{D} &= \mathbf{630\,mm} \end{aligned}$$

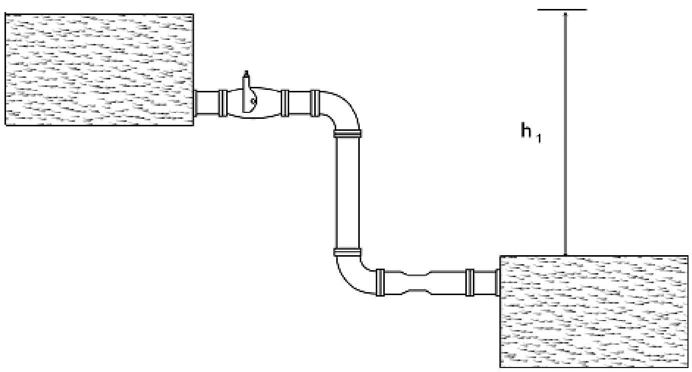
Example 3:

In a water treatment plant, water flows from a filter down to a clear well through the pipe system shown. The pipe is welded steel with a diameter of 300 mm and roughness coefficient $C = 130$. The total length of pipe is 50 m. Elevation difference h_1 between the tanks is 5 m.

Equivalent length ratios, L_e/D, are:

Entrance and exit losses:	50	Butterfly valve:	35
Large radius elbows:	25	Venturi meter:	100

Determine the flow through the system.



Solution:

Effective length of the pipe: (length and diameter in metres!)

$$\begin{aligned} L_{\text{eff}} &= \text{Actual pipe length} + D\left(\frac{L_e}{D}\right) \\ &= 50 + 0.3(50 + 35 + 25 + 25 + 100 + 50) \\ &= 50 + 85.5 \\ &= 135.5\,\text{m} \end{aligned}$$

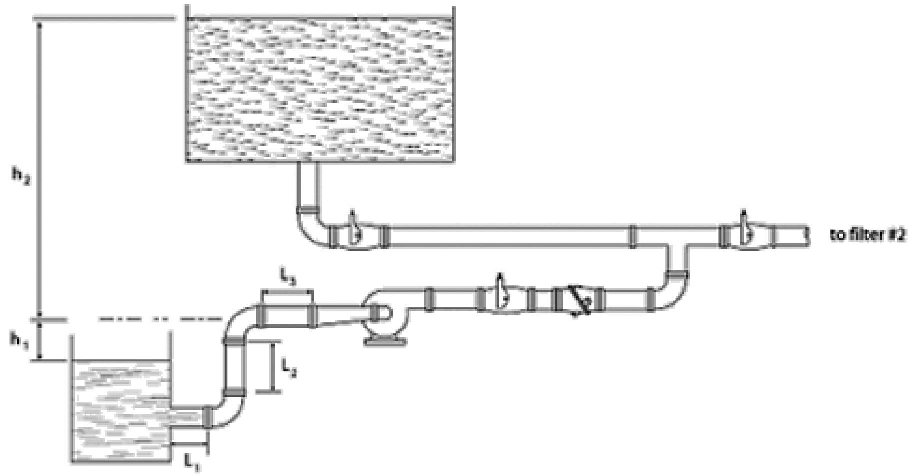
As earlier, headloss between the two surfaces is just the elevation difference:

$$h_L = 5\,\text{m}$$

Find the flow:

$$\begin{aligned} Q &= \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000} \\ &= \frac{130(300)^{2.63}\left(\frac{5}{135.5}\right)^{0.54}}{279000} \\ &= 256.66\,\text{L/s} \\ \boldsymbol{Q} &= \mathbf{257\,L/s} \end{aligned}$$

Example 4:



In a water treatment plant, backwash water is pumped from the clear well through the pipe system shown to the filter. The required backwash flow is 10 L/s per square meter of filter area (the filter dimensions are 10 m by 15 m. The inlet pipe is made of welded steel ($C = 130$), has a diameter of 1000 mm and a total length ($L_1 + L_2 + L_3$) of 10 m. The outlet pipe, from the pump to the filter, is also welded steel, has a diameter of 700 mm and a length of 70 m.

The two elevation differences are $h_1 = 2$ m and $h_2 = 10$ m.

Equivalent length ratios, L_e/D, are:

Entrance:	10	Elbow (inlet):	25
Eccentric Reducer:	2	Butterfly Valve:	40
Check Valve:	120	Elbow (outlet):	35
Tee Connection:	60		

Determine:

(1) The head losses on the inlet side (clear well to pump)
(2) The head losses on the outlet side (pump to filter)

Neglect exit losses into the filter.

Solution:

Q required for backwash in the filter:

$$Q = 10\,\text{m} \times 15\,\text{m} \times 0.01\,\text{m}^3/\text{s} = 1.5000\,\text{m}^3/\text{s}$$

(1)

$$\begin{aligned} L_{eff} &= 10 + 1(10 + 25 + 25 + 2) = 72.000\,\text{m} \\ h_L &= 72\left(\frac{279000 \times 1500}{130(1000)^{2.63}}\right)^{1.852} \\ &= 0.19834\,\text{m} \\ \boldsymbol{h_{L_{(in)}}} &= \mathbf{0.1983\,m} \end{aligned}$$

(2)

$$\begin{aligned} L_{eff} &= 70 + 0.7(40 + 120 + 35 + 60 + 40 + 35) \\ &= 301.00\,\text{m} \\ h_L &= 301\left(\frac{279000 \times 1500}{130(700)^{2.63}}\right)^{1.852} \\ &= 4.7112\,\text{m} \\ \boldsymbol{h_{L_{(out)}}} &= \mathbf{4.71\,m} \end{aligned}$$

Exercise 3:

This exercise is a continuation of the previous example. Determine:

(3) The head added by the pump
(4) The pressure at the pump outlet

Solution:

(3)

Apply the GEE between the surface of the clear well (W) and the surface of the filter (F):

$$\frac{P_W}{\gamma} + z_W + \frac{v_W^2}{2g} + h_A - h_L = \frac{P_F}{\gamma} + z_F + \frac{v_F^2}{2g}$$
$$h_A - (0.19834 + 4.7112) = 12$$
$$h_A = 16.910\,\text{m}$$
$$\boldsymbol{h_A = 16.91\,\text{m}}$$

(4)

Apply the GEE between the pump (P) and the surface of the filter (F):

$$\frac{P_P}{\gamma} + z_P + \frac{v_P^2}{2g} - h_L = \frac{P_F}{\gamma} + z_F + \frac{v_F^2}{2g}$$
$$\frac{P_P}{9.81} + 0 + 0.77430 - 4.7112 = 0 + 10 + 0$$
$$\Rightarrow P_P = 136.72\,\text{kPa}$$
$$\boldsymbol{P_P = 136.7\,\text{kPa}}$$

Example 6:

The pumps and piping system are used to supply a municipal grid.

Pump P_1 runs continuously and maintains the basic pressure in the distribution grid beyond point D.

The elevations are the same at the pump and the discharge point D.

The outlet pipe, from the pump to point D, is welded steel $(C = 130)$ with a diameter of 200 mm and a total length between fittings of 10 m.

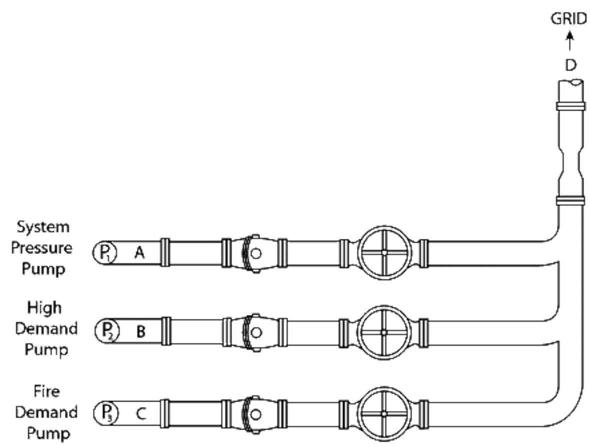
The minimum pressure required at D is 500 kPa for a design flow of 150 L/s.

Equivalent length ratios, L_e/D, are:

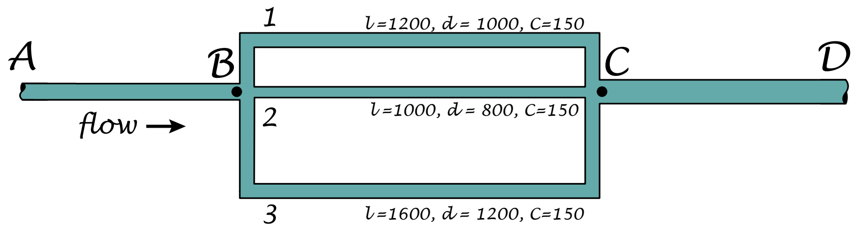
Check Valve:	120	Gate Valve:	15
Tee Connection:	60	Venturi Meter:	100

There is no flow from pumps P_2 and P_3. Determine:

(1) the head losses between A and D
(2) the pressure at A required for the required pressure and flow at D



Example 7:



The flow through the system from A to D is $Q = 3.3\,\mathrm{m}^3/\mathrm{s}$. Pipe lengths are given in m, diameters are in mm, and C is the Hazen-Williams roughness coefficient.
Determine the flow through Q_{BC1}, Q_{BC2} and Q_{BC3} and the head loss between B and C. (You may assume that all losses are due to friction.)

Solution:

$$h_{L_1} = h_{L_2} = h_{L_3}$$

$$L_1 \left(\frac{279000 Q_1}{C_1 D_1^{2.63}} \right)^{1.85} = L_2 \left(\frac{279000 Q_2}{C_2 D_2^{2.63}} \right)^{1.85} = L_3 \left(\frac{279000 Q_3}{C_3 D_3^{2.63}} \right)^{1.85}$$

$$1200 \left(\frac{279000 Q_1}{150 \times 1000^{2.63}} \right)^{1.85} = 1000 \left(\frac{279000 Q_2}{150 \times 800^{2.63}} \right)^{1.85} = 1600 \left(\frac{279000 Q_3}{150 \times 1200^{2.63}} \right)^{1.85}$$

Simplify - divide by $100 \left(\frac{279000}{150} \right)^{1.85}$

$$12 \left(\frac{Q_1}{1000^{2.63}} \right)^{1.85} = 10 \left(\frac{Q_2}{800^{2.63}} \right)^{1.85} = 16 \left(\frac{Q_3}{1200^{2.63}} \right)^{1.85}$$

Simplify - raise everything to the power $1/1.85$

$$12^{1/1.85} \cdot \frac{Q_1}{1000^{2.63}} = 10^{1/1.85} \cdot \frac{Q_2}{800^{2.63}} = 16^{1/1.85} \cdot \frac{Q_3}{1200^{2.63}}$$

$$\frac{Q_1}{2.0261 \times 10^7} = \frac{Q_2}{1.2433 \times 10^7} = \frac{Q_3}{2.8014 \times 10^7}$$

Then

$$Q_2 = \frac{1.2433}{2.0261} \cdot Q_1 = 0.61364 Q_1$$

$$Q_3 = \frac{2.8014}{2.0261} \cdot Q_1 = 1.3827 Q_1$$

Total flow through the three pipes is $3.3\,\mathrm{m}^3/\mathrm{s}$ so

$$Q_1 + Q_2 + Q_3 = 3300\,\mathrm{L/s}$$

$$Q_1 + 0.61364Q_1 + 1.3827Q_1 = 3300\,\mathrm{L/s}$$

$$Q_1 = 1101.3\,\mathrm{L/s}$$

$$Q_2 = 0.61364(1101.3) = 675.83\,\mathrm{L/s}$$

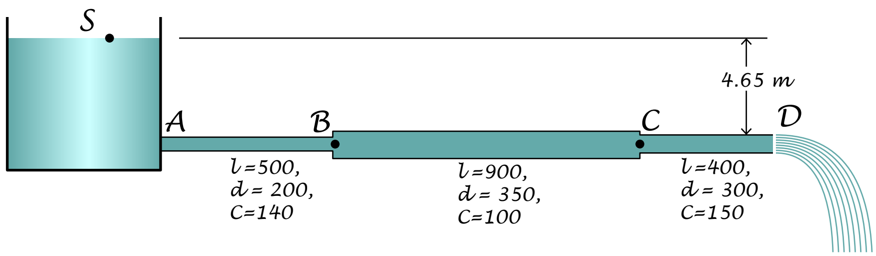
$$Q_3 = 1.3827(1101.3) = 1522.8\,\mathrm{L/s}$$

The headloss between B and C can be calculated from any one of the three pipes.

$$h_{L_1} = 1200\left(\frac{279000 \times 1101.3}{150 \times 1000^{2.63}}\right)^{1.85}$$

$$= 1.4415\,\mathrm{m}$$

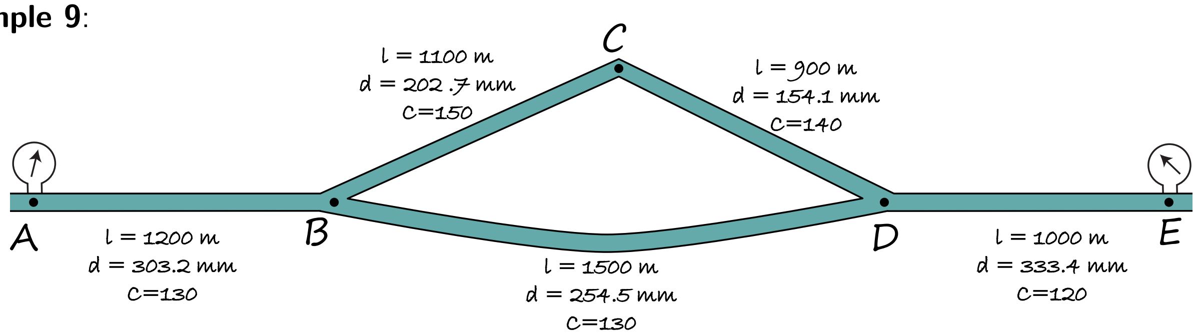
Example 8:



Determine the flow through the system shown above and the average flow velocity in the smallest pipe. (Neglect any minor losses and the velocity head at the exit.)

Solution:

Example 9:



Determine the velocity of the flow through each of the five pipes if there is a pressure difference of 295 kPa between A and E. (Neglect any minor losses.)

Solution: The process:

(1) Replace series pipes BC and CD with a single equivalent pipe BCD
(2) Replace parallel pipes BCD and BD with a single equivalent pipe $BD2$
(3) Replace series pipes AD, $BD2$ and DE with a single equivalent pipe AE
(4) Determine the flow through the system
(5) Calculate the velocities for the flow through each pipe

(1) Equivalent Pipe BCD

Find the head losses in BC and CD for a flow of 100 L/s:

$$h_L = L\left(\frac{279000Q}{CD^{2.63}}\right)^{1.852}$$

$$h_{L_{BC}} = 1100\left(\frac{279000 \times 100}{150(202.7)^{2.63}}\right)^{1.852}$$
$$= 36.677\,\text{m}$$

$$h_{L_{CD}} = 900\left(\frac{279000 \times 100}{140(154.1)^{2.63}}\right)^{1.852}$$
$$= 129.60\,\text{m}$$

Find the diameter of the equivalent pipe:

$$D = \left(\frac{279000Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802}$$

$$D_{BCD} = \left(\frac{279000 \times 100}{100\left(\frac{36.677+129.60}{1000}\right)^{0.54}}\right)^{0.3802}$$
$$= 169.98\,\text{mm}$$

(2) Equivalent Pipe $BD2$

Find the flows in BCD and BD for a headloss of 10 m:

$$Q = \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}$$

$$\begin{aligned}Q_{BCD} &= \frac{100(169.98)^{2.63}\left(\frac{10}{1000}\right)^{0.54}}{279000}\\ &= 21.895\,\text{L/s}\end{aligned}$$

$$\begin{aligned}Q_{BD} &= \frac{130(254.5)^{2.63}\left(\frac{10}{1500}\right)^{0.54}}{279000}\\ &= 66.101\,\text{L/s}\end{aligned}$$

Percentage of flow through BCD:

$$\begin{aligned}\frac{Q_{BCD}}{Q}\% &= \frac{21.895}{21.895+66.101}\times 100\%\\ &= 24.882\%\end{aligned}$$

Headloss at $Q = 100$ L/s:

$$h_L = L\left(\frac{279000Q}{CD^{2.63}}\right)^{1.852}$$

$$\begin{aligned}h_{L_{BCD}} &= 1000\left(\frac{279000\times 24.882}{100(169.98)^{2.63}}\right)^{1.852}\\ &= 12.667\,\text{m}\end{aligned}$$

Repeat with BD to check

$$\begin{aligned}h_{L_{BD}} &= 1500\left(\frac{279000\times 75.118}{130(254.5)^{2.63}}\right)^{1.852}\\ &= 12.667\,\text{m}\end{aligned}$$

Find diameter of equivalent pipe:

$$D = \left(\frac{279000Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802}$$

$$\begin{aligned}D_{BD2} &= \left(\frac{279000\times 100}{100\left(\frac{12.667}{1000}\right)^{0.54}}\right)^{0.3802}\\ &= 288.37\,\text{mm}\end{aligned}$$

(3) Equivalent Pipe AE

Find the head losses in AB, $BD2$ and DE for a flow of 100 L/s:

$$h_L = L\left(\frac{279000Q}{CD^{2.63}}\right)^{1.852}$$

$$\begin{aligned} h_{L_{AB}} &= 1200\left(\frac{279000 \times 100}{130(303.2)^{2.63}}\right)^{1.852} \\ &= 7.3369\,\text{m} \end{aligned}$$

$$\begin{aligned} h_{L_{BD2}} &= 1000\left(\frac{279000 \times 100}{100(288.37)^{2.63}}\right)^{1.852} \\ &= 12.689\,\text{m} \quad \text{(we already have this, 12.667, above!)} \end{aligned}$$

$$\begin{aligned} h_{L_{DE}} &= 1000\left(\frac{279000 \times 100}{120(333.4)^{2.63}}\right)^{1.852} \\ &= 4.4653\,\text{m} \quad \text{(we already have this, 12.667, above!)} \end{aligned}$$

$$\begin{aligned} h_{L_{AE}} &= 7.3369 + 12.689 + 4.4653 \\ &= 24.491\,\text{m} \end{aligned}$$

Find the diameter of the equivalent pipe:

$$D = \left(\frac{279000Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802}$$

$$\begin{aligned} D_{BCD} &= \left(\frac{279000 \times 100}{100\left(\frac{24.491}{1000}\right)^{0.54}}\right)^{0.3802} \\ &= 251.86\,\text{mm} \end{aligned}$$

(4) Flow through the system

Find the head loss associated with a pressure drop of 296 kPa
(GEE with no elevation or velocity head terms):

$$\Delta P = \gamma h_L$$
$$295 = 9.81 \times h_L$$
$$h_L = 30.071\,\mathrm{m}$$

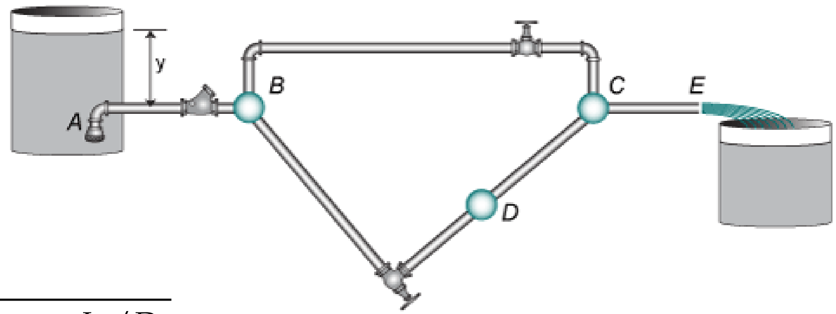
Find the flow for this headloss:

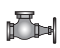
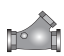
$$Q_{AE} = \frac{100(251.86)^{2.63}\left(\frac{30.071}{1000}\right)^{0.54}}{279000}$$
$$= 111.59\,\mathrm{L/s}$$

(5) Velocities are:

$$V_{AB} = \frac{0.11159\,\mathrm{m^3/s}}{\pi(0.3032\,\mathrm{m})^2/4} = 1.5545\,\mathrm{m/s}$$
$$V_{BC} = \frac{0.11159 \times 0.24882\,\mathrm{m^3/s}}{\pi(0.2027\,\mathrm{m})^2/4} = 0.86043\,\mathrm{m/s}$$
$$V_{CD} = \frac{0.11159 \times 0.24882\,\mathrm{m^3/s}}{\pi(0.1541\,\mathrm{m})^2/4} = 1.4887\,\mathrm{m/s}$$
$$V_{BD} = \frac{0.11159 \times 0.75118\,\mathrm{m^3/s}}{\pi(0.2545\,\mathrm{m})^2/4} = 1.6478\,\mathrm{m/s}$$
$$V_{BD} = \frac{0.11159\,\mathrm{m^3/s}}{\pi(0.3334\,\mathrm{m})^2/4} = 1.2782\,\mathrm{m/s}$$

Exercise 10:
Nodes B, C, D and E are all at the same elevation. Given that $y = 6.7\,\mathrm{m}$, determine the flow through the system (disregard exit losses).



	Fitting	Le/D
	Angle Valve	150
	Check Valve	100
	Elbow	50
	Foot Valve	75
	Gate Valve	35

Pipe	Length (m)	diam (mm)	C
AB	10	500	125
BC	2000	275	150
BD	1500	250	100
DC	1000	300	100
CE	10	500	125

Solution

The objective is to replace the system of five pipes AB, BC, CD, DC and CE with a single hydraulically equivalent pipe from A to E. Then we can use the Hazen-Williams equation to find flow through the system.

Our process will be as follows:

(1) Find the effective lengths of the pipes that have valves or fittings.
(2) Consider the series system of two pipes BD and DC: find a hydraulically equivalent pipe BDC.
(3) Consider the parallel system of two pipes BDC (just found above) and BC: find a hydraulically equivalent pipe $BC2$ for flow from B to C.
(4) Consider the series system of three pipes AB, $BC2$, and CE: find a hydraulically equivalent pipe AE.
(5) Use the General Energy Equation and the Hazen-Williams equation to find Q.

(1) Find the effective lengths of the pipes that have valves or fittings.

$$L_{\text{eff}} = \text{Actual Length} + \text{Diameter} \times \left(\Sigma \frac{L_e}{D}\right)$$

$$\begin{aligned}
L_{\text{eff}(AB)} &= 10 + 0.5(75 + 50 + 100) = 122.5\,\text{m} \\
L_{\text{eff}(BC)} &= 2000 + 0.275(50 + 35 + 50) = 2037.1\,\text{m} \\
L_{\text{eff}(BD)} &= 1500 + 0.250(150) = 1537.5\,\text{m}
\end{aligned}$$

(2) Find a single equivalent pipe BDC for the two pipes BD and DC in series.

Find the headloss between B and C for a flow of 100 L/s though BD and DC, using the effective lengths of the pipes.

$$h_L = L\left(\frac{279000Q}{CD^{2.63}}\right)^{1.852}$$

$$\begin{aligned}
h_{L(BD)} &= 1537.5\left(\frac{279000 \times 100}{100 \times 250^{2.63}}\right)^{1.852} = 39.110\,\text{m} \\
h_{L(DC)} &= 1000\left(\frac{279000 \times 100}{100 \times 300^{2.63}}\right)^{1.852} = 10.467\,\text{m}
\end{aligned}$$

$$h_{L(BDC)} = 39.110 + 10.467 = 49.577\,\text{m}$$

Find an equivalent pipe for BDC (i.e., a pipe that has a head loss of 49.577 m for a flow of 100 L/s). We use an equivalent pipe with length 1000 m and resistance coefficient 100, and find the equivalent pipe diameter.

$$D = \left(\frac{279000Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802}$$

$$D_{BDC} = \left(\frac{279000 \times 100}{100\left(\frac{49.577}{1000}\right)^{0.54}}\right)^{0.3802} = 217.92\,\text{mm}$$

(3) Now, consider pipe BC and the pipe BDC just found as a parallel system. Find an equivalent pipe for these two parallel pipes.

Assume a headloss of 10 m between B and C and find the flow through each of BC and BDC:

$$Q = \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}$$

$$\begin{aligned}
Q_{BC} &= \frac{150 \times 275^{2.63}\left(\frac{10}{2037.1}\right)^{0.54}}{279000} = 79.262\,\mathrm{L/s} \\
Q_{BDC} &= \frac{100 \times 217.92^{2.63}\left(\frac{10}{1000}\right)^{0.54}}{279000} = 42.084\,\mathrm{L/s} \\
Q_{BC \text{ and } BDC} &= 79.262 + 42.084 = 121.35\,\mathrm{L/s}
\end{aligned}$$

A flow of 121.35 L/s between B and C through both pipes BC and BDC produce a headloss of 10 m. Find an equivalent pipe that replaces these two parallel pipes:

$$D_{BC\text{equiv}} = \left(\frac{279000 \times 121.35}{100\left(\frac{10}{1000}\right)^{0.54}}\right)^{0.3802} = 325.83\,\mathrm{mm}$$

(4) Now we have a series system: AB, BCequiv and CE. Assume a flow of 100 L/s from A to E, find the total headloss for this flow and then replace these three pipes with a single equivalent pipe.

$$h_{L(AB)} = 122.5 \left(\frac{279000 \times 100}{125 \times 500^{2.63}} \right)^{1.852} = 0.070450\,\text{m}$$
$$h_{L(BC\text{equiv})} = 1000 \left(\frac{279000 \times 100}{100 \times 325.83^{2.63}} \right)^{1.852} = 7.0000\,\text{m}$$
$$h_{L(CE)} = 10 \left(\frac{279000 \times 100}{125 \times 500^{2.63}} \right)^{1.852} = 0.0057510\,\text{m}$$

$$h_{L(AE)} = 0.070450 + 7.0000 + 0.0057510 = 7.0762\,\text{m}$$

Find the equivalent pipe that has a headloss of 7.0762 m for a flow of 100 L/s:

$$D_{AE} = \left(\frac{279000 \times 100}{100 \left(\frac{7.0762}{1000}\right)^{0.54}} \right)^{0.3802} = 324.99\,\text{mm}$$

(5) From the General Enery Equation (disregarding velocity head at E), headloss through the actual system is 6.7 m. We just need the flow that generates this headloss:

$$Q_{AE} = \frac{100 \times 324.99^{2.63} \left(\frac{6.7}{1000}\right)^{0.54}}{279000} = 96.983\,\text{L/s}$$