Module 5: Friction Losses (CIVL 318)

Reynolds Number:

$$N_R = \frac{vD\rho}{\eta}$$

$$N_R < 2000 \Rightarrow \text{laminar}$$

$$2000 < N_R < 4000 \Rightarrow \text{critical flow}$$

$$N_R > 4000 \Rightarrow \text{turbulent flow}$$

Laminar flow:

$$f = \frac{64}{N_R}$$

Turbulent flow::

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_R^{0.9}}\right)\right]^2}$$

Darcy's Equation:

$$h_L = f \times \frac{L}{D} \times \frac{v^2}{2g}$$

Roughness, ϵ:

Material (new, clean)	ϵ (m)
Glass	Smooth
Plastic	3.0×10^{-7}
Copper, brass, lead (tubing)	1.5×10^{-6}
Commercial steel, welded steel	4.6×10^{-5}
Wrought iron	4.6×10^{-5}
Ductile Iron - coated	1.2×10^{-4}
Ductile Iron - uncoated	2.4×10^{-4}
Concrete	1.2×10^{-4}
Riveted steel	1.8×10^{-3}

Example 1: Flow is said to be in the **critical region**, with neither fully laminar or fully turbulent flow, if the Reynolds number for the flow is between 2000 and 4000.

Determine the range of velocities and volume flow rates for which flow is in the critical region for:

(1) water at 5°C flowing in 1/2-in copper tubing
(2) water at 95°C flowing in 1/2-in copper tubing
(3) fuel oil at 10°C (sg $= 0.94$, $\eta = 2.4$ Pa $\cdot$ s),
flowing in 12-in Schedule 40 steel pipe

Solution:
(1) From tables in the text or provided, $\rho = 1000$ kg/m^3, $\eta = 1.52 \times 10^{-3}$ Pa $\cdot$ s and $D = 13.39$ mm so:

$$\begin{aligned}
2000 &= \frac{v_{2000}(0.01339)(1000)}{1.52 \times 10^{-3}} \\
v_{2000} &= 0.22704 \text{ m/s} \\
4000 &= \frac{v_{4000}(0.01339)(1000)}{1.52 \times 10^{-3}} \\
v_{4000} &= 0.45407 \text{ m/s}
\end{aligned}$$

$$\begin{aligned}
Q_{2000} &= \pi(0.01339 \text{ m})^2/4 \times 0.22704 \text{ m/s} \\
&= 3.1971 \times 10^{-5} \text{m}^3/\text{s} \\
&= 0.031971 \text{ L/s} \\
Q_{4000} &= 0.063942 \text{ L/s}
\end{aligned}$$

For flow to remain in the critical region:

$$\begin{aligned}
0.22704 \text{ m/s} < &v < 0.45407 \text{ m/s} \\
0.031971 \text{ L/s} < &Q < 0.063942 \text{ L/s}
\end{aligned}$$

(2) $\rho = 962$ kg/m^3 and $\eta = 2.92 \times 10^{-4}$ Pa $\cdot$ s so:

$$\begin{aligned}
N_R &= \frac{vD\rho}{\eta} \\
2000 &= \frac{v_{2000}(0.01339)(962)}{2.92 \times 10^{-4}} \\
v_{2000} &= 0.045337 \text{ m/s} \\
v_{4000} &= 0.090675 \text{ m/s}
\end{aligned}$$

$$\begin{aligned}
Q_{2000} &= \pi(0.01339 \text{ m})^2/4 \times 0.045337 \text{ m/s} \\
&= 6.3842 \times 10^{-6} \text{m}^3/\text{s} \\
&= 0.0063842 \text{ L/s} \\
Q_{4000} &= 0.012765 \text{ L/s}
\end{aligned}$$

For most situations, water flow is fully turbulent.

(3) $\rho = 940$ kg/m^3, $\eta = 2.4$ Pa $\cdot$ s and $D = 303.2$ mm so:

$$\begin{aligned}
2000 &= \frac{v_{2000}(0.3032)(962)}{2.4} \\
v_{2000} &= 16.842 \text{ m/s} \\
v_{4000} &= 33.683 \text{ m/s}
\end{aligned}$$

$$\begin{aligned}
Q_{2000} &= \pi(0.3032 \text{ m})^2/4 \times 16.842 \text{ m/s} \\
&= 7.5715 \times 10^{-6} \text{m}^3/\text{s} \\
&= 1.2160 \text{ m}^3/\text{s} \\
Q_{4000} &= 2.4320 \text{ m}^3/\text{s}
\end{aligned}$$

Example 2:

Determine the headloss due to friction in fuel oil at 10°C flowing through 125 m of 12-in Schedule 40 steel pipe with an average flow velocity of 4.5 m/s.

Then determine the headloss if the average flow velocity is reduced to 2.25 m/s.

(sg $= 0.94$, $\eta = 2.4$ Pa · s).

Solution: First, we must calculate the Reynolds number:

$$\begin{aligned} N_R &= \frac{vD\rho}{\eta} \\ &= \frac{4.5 \times 0.3032 \times 940}{2.4} \\ &= 534.39 \end{aligned}$$

Flow is laminar ($N_R < 2000$) so $f = \dfrac{64}{N_R} = 0.11976$

Use Darcy's Equation to calculate the head loss:

$$\begin{aligned} h_L &= f \times \frac{L}{D} \times \frac{v^2}{2g} \\ &= 0.11976 \times \frac{125}{0.3032} \times \frac{4.5^2}{2g} \\ &= 50.960 \text{ m} \end{aligned}$$

Now, calculate the headloss at 2.25 m/s

$$\begin{aligned} N_R &= \frac{2.25 \times 0.3032 \times 940}{2.4} \\ &= 267.20 \end{aligned}$$

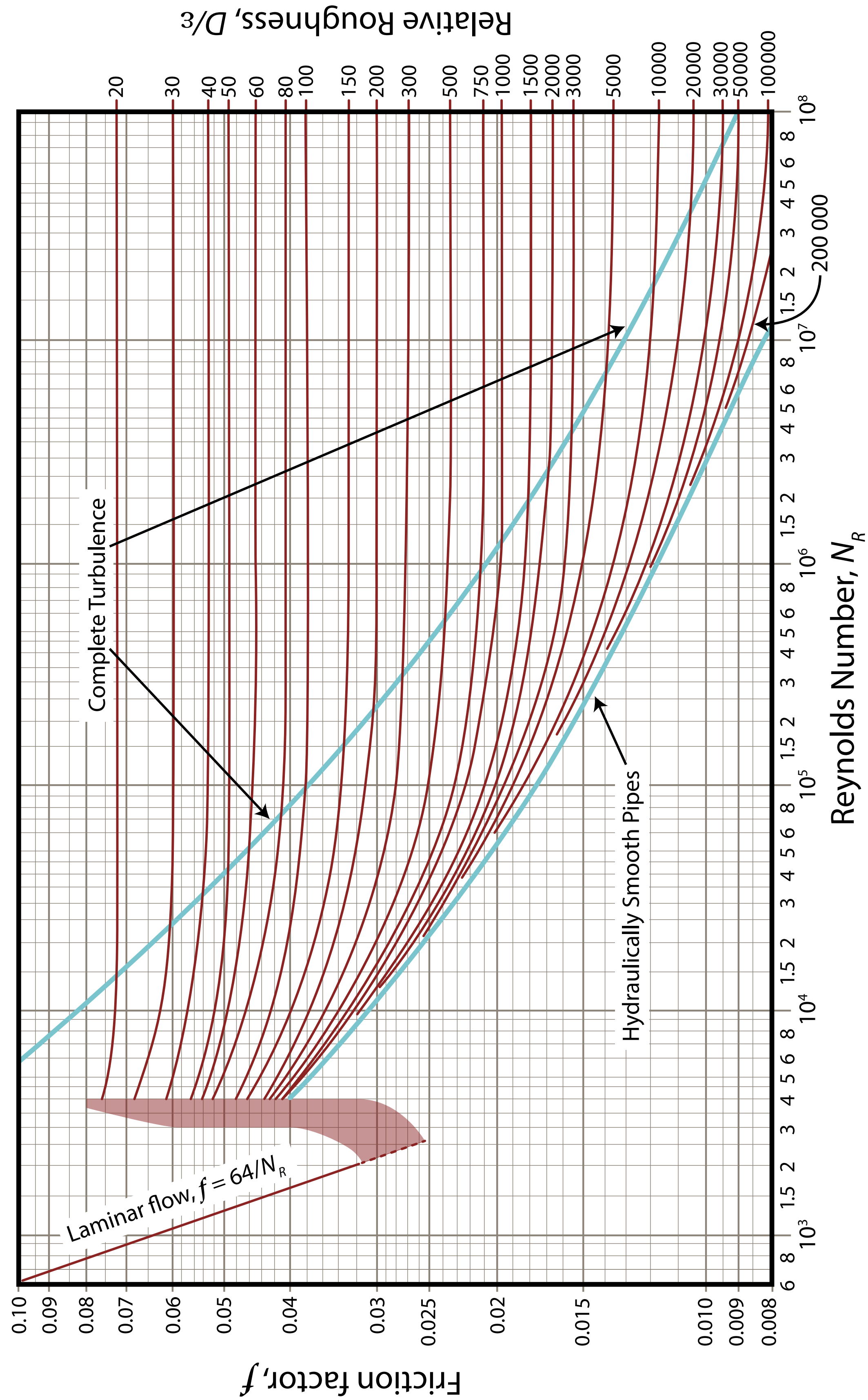
$$f = \frac{64}{167.20} = 0.23952$$

$$\begin{aligned} h_L &= 0.23952 \times \frac{125}{0.3032} \times \frac{2.25^2}{2g} \\ &= 25.479 \text{ m} \end{aligned}$$

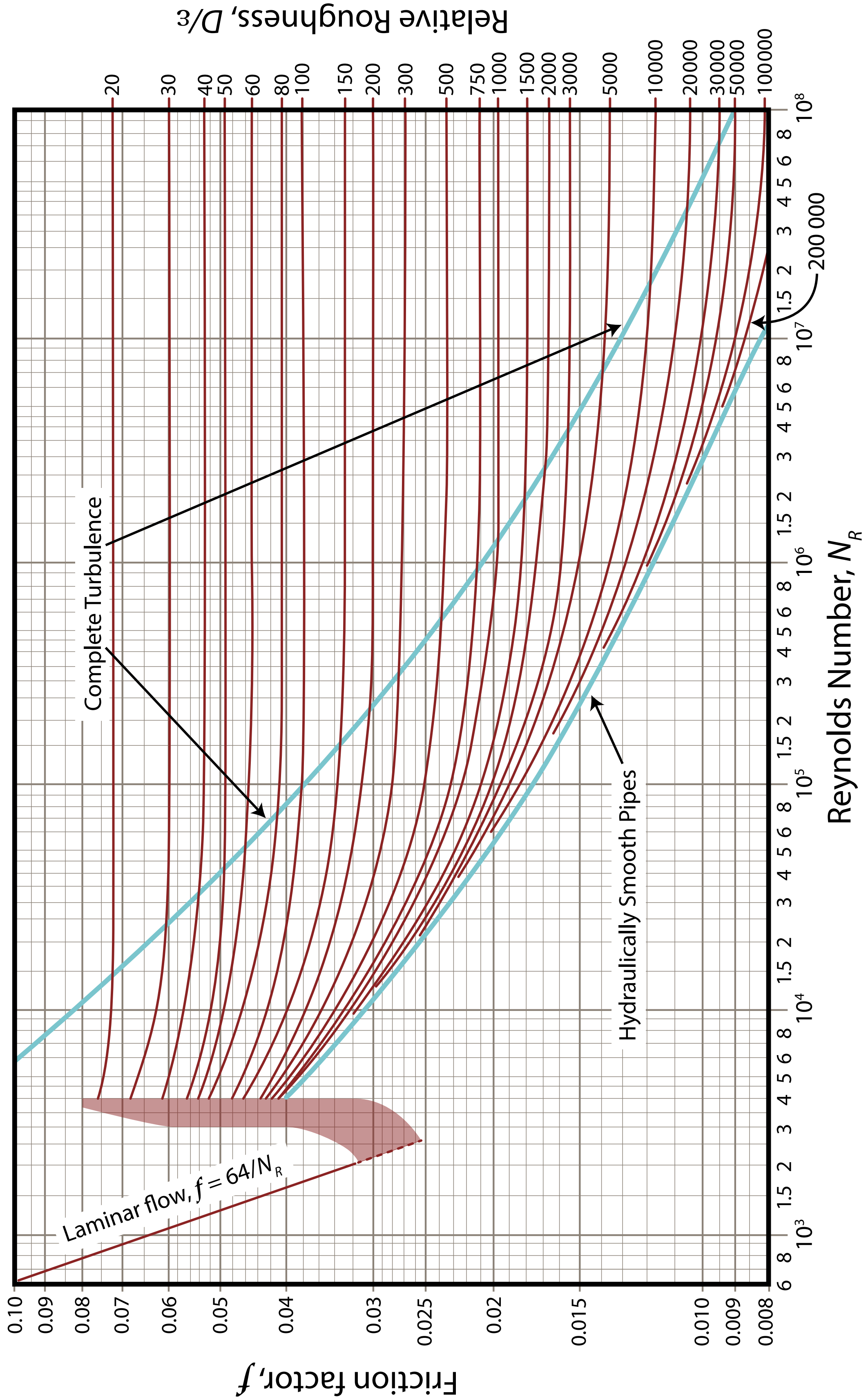
Note that, for laminar flow, $h_L \propto v$

Note also that a loss of 25.479 m of head is equivalent to an energy loss of 25.479 N · m of energy per N of oil.

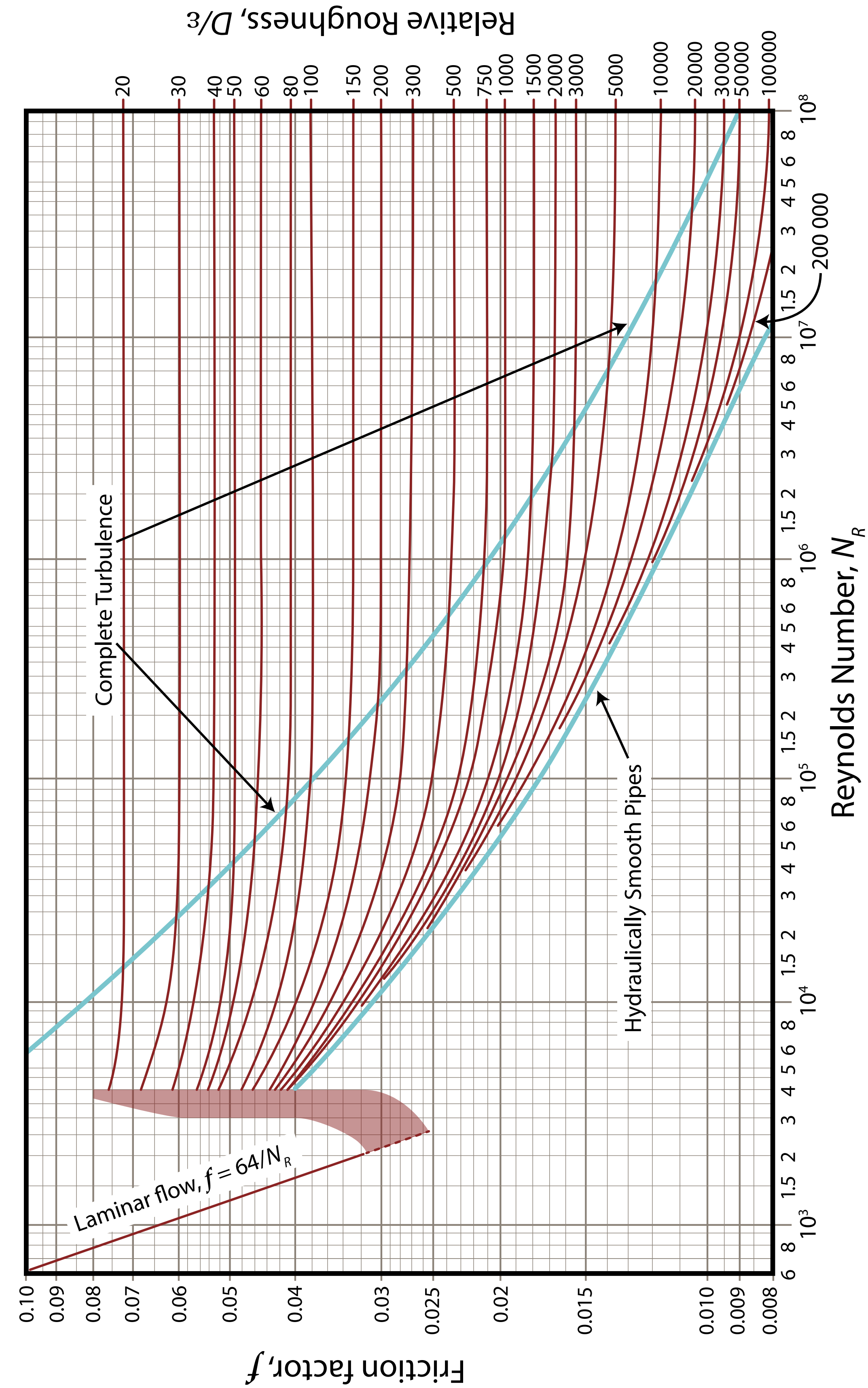
Example 3 Use the Moody diagram to determine the friction factor for flow with $N_R = 2 \times 10^6$ and a relative roughness of 1428.



Example 4 Use the Moody diagram to determine the friction factor for flow with $N_R = 1.6 \times 10^5$ and in new clean $1/2$-in copper tubing.



Example 5 A 75 m section of wooden flume is replaced with 54-in high density polyethylene (HDPE) pipe with inside diameter of 1.37 m. The pipe is smooth and transports 190×10^3 m3/day. Determine the headloss due to friction in the pipe, assuming an average temperature of 10 °C.



Example 6:

Ethyl alcohol at 25°C flows through $1\frac{1}{2}$-in Schedule 80 steel pipe at 5 L/s. Determine the pressure drop, due to friction losses, in a 125 m section of pipe.

Solution: Find average flow velocity and associated velocity head:

$$\begin{aligned} v &= \frac{0.005\ \text{m}^3/\text{s}}{\pi(0.0381\ \text{m})^2/4} \\ &= 4.3856\ \text{m/s} \\ \frac{v^2}{2g} &= 0.98031\ \text{m} \end{aligned}$$

Find the Reynolds number:

$$\begin{aligned} N_R &= \frac{vD\rho}{\eta} \\ &= \frac{4.3856 \times 0.0381 \times 787}{1.00 \times 10^{-3}} \\ &= 131500 \end{aligned}$$

Find the relative roughness:

$$\begin{aligned} \frac{D}{\epsilon} &= \frac{0.0381}{4.6 \times 10^{-5}} \\ &= 828.26 \end{aligned}$$

From the Moody diagram, $f = 0.0225$.
Determine the head loss due to friction:

$$\begin{aligned} h_L &= f \times \frac{L}{D} \times v^2 2g \\ &= 0.0225 \times \frac{125}{0.0381} \times 0.98031 \\ &= 72.365\ \text{m} \end{aligned}$$

Find the pressure drop:

$$\begin{aligned} \frac{P_A}{\gamma} + \cancel{z_A} + \cancel{\frac{v_A^2}{2g}} - h_L &= \frac{P_B}{\gamma} + \cancel{z_B} + \cancel{\frac{v_B^2}{2g}} \\ \frac{P_A}{\gamma} - h_L &= \frac{P_B}{\gamma} \\ P_A - P_B &= \gamma h_L \\ &= (7.72\ \text{kN/m}^3)(72.365\ \text{m}) \\ &= 558.66\ \text{kPa} \end{aligned}$$

Example 7:

Ethyl alcohol at 25°C flows through 3-in Schedule 80 steel pipe at 5 L/s. Determine the pressure drop, due to friction losses, in a 125 m section of pipe.

Solution:

Find average flow velocity and associated velocity head:

$$\begin{aligned} v &= \frac{0.005\ \mathrm{m^3/s}}{\pi(0.0737\ \mathrm{m})^2/4} \\ &= 1.1720\ \mathrm{m/s} \\ \frac{v^2}{2g} &= 0.070015\ \mathrm{m} \end{aligned}$$

Find the Reynolds number:

$$\begin{aligned} N_R &= \frac{vD\rho}{\eta} \\ &= \frac{1.1720 \times 0.0737 \times 787}{1.00 \times 10^{-3}} \\ &= 67978 \end{aligned}$$

Find the relative roughness:

$$\begin{aligned} \frac{D}{\epsilon} &= \frac{0.0737}{4.6 \times 10^{-5}} \\ &= 1602.2 \end{aligned}$$

From the Swamee-Jain, $f = 0.022000$

(From the Moody diagram, $f = 0.022$)

Determine the head loss due to friction:

$$\begin{aligned} h_L &= f \times \frac{L}{D} \times \frac{v^2}{2g} \\ &= 0.022 \times \frac{125}{0.0737} \times 0.070015 \\ &= 2.6125\ \mathrm{m} \end{aligned}$$

Find the pressure drop:

$$\begin{aligned} \frac{P_A}{\gamma} + \cancel{z_A} + \cancel{\frac{v_A^2}{2g}} - h_L &= \frac{P_B}{\gamma} + \cancel{z_B} + \cancel{\frac{v_B^2}{2g}} \\ \frac{P_A}{\gamma} - h_L &= \frac{P_B}{\gamma} \\ P_A - P_B &= \gamma h_L \\ &= (7.72\ \mathrm{kN/m^3})(2.6125\ \mathrm{m}) \\ &= 20.169\ \mathrm{kPa} \end{aligned}$$

Example 8:

A horizontal 12-in Schedule 80 steel pipe transports oil (sg $= 0.85$, $\eta = 3.0 \times 10^{-3}$ Pa $\cdot$ s) at 185 L/s. The pipe has pumping stations spaced at 6.0 km intervals. Determine the power required by each pumpto maintain the same pressure at each pump outlet if all losses are due to friction.

Solution:

Velocity and velocity head:

$$
\begin{aligned}
v &= \frac{0.185 \text{ m}^3/\text{s}}{\pi(0.289 \text{ m})^2/4} \\
&= 2.8202 \text{ m/s} \\
\frac{v^2}{2g} &= 0.40539 \text{ m}
\end{aligned}
$$

Reynolds number:

$$
\begin{aligned}
N_R &= \frac{(2.8202 \text{ m/s})(0.289 \text{ m})(850 \text{ kg/m}^3)}{3.0 \times 10^{-3} \text{ Pa} \cdot \text{s}} \\
&= 230930
\end{aligned}
$$

Relative roughness:

$$
\begin{aligned}
\frac{D}{\epsilon} &= \frac{0.289 \text{ m}}{4.6 \times 10^{-5} \text{ m}} \\
&= 6282.6
\end{aligned}
$$

Friction factor: $f = 0.0166$ (Moody)
$f = 0.016511$ (Swamee-Jain)

Head loss:

$$
\begin{aligned}
h_L &= 0.0166 \times \frac{6000}{0.289} \times 0.40539 \\
&= 139.71 \text{ m}
\end{aligned}
$$

This is the head lost between the outlet of one pump and the inlet of the next. The job of the pump is to replace that lost head, i.e. $h_A = 139.71$ m

The power added by each pump must be:

$$
\begin{aligned}
P_{added} &= h_A \gamma Q \\
&= (139.71 \text{ m})(0.85 \times 9.81 \text{ kN/m}^3)(0.185 \text{ kN/m}^3) \\
&= 215.52 \text{ kW}
\end{aligned}
$$