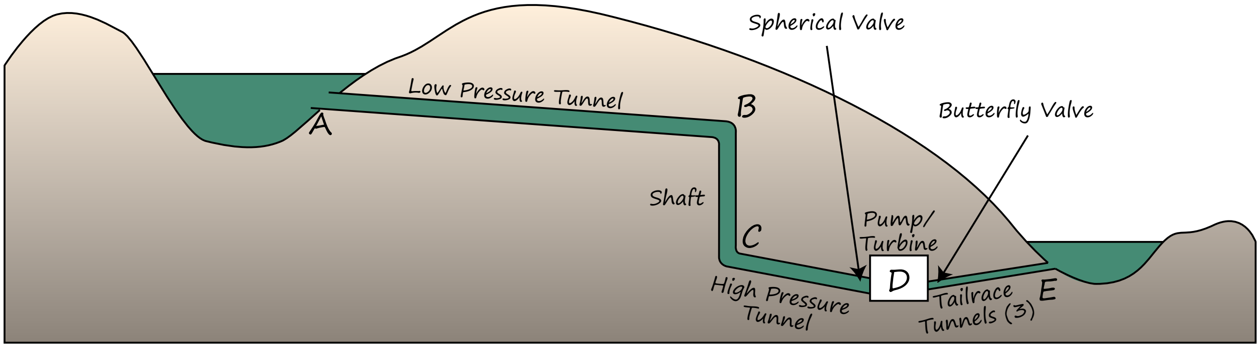
Pumped Storage Example (Series Pipeline)



The system illustrated is a pumped storage system. During periods of high demand for electricity, water flows from the upper lake and drives the turbine at D. (During periods of low demand when electricity is cheap, such as at night-time, D acts as a pump and pumps water back up to the upper lake.)

At times of maximum demand, the system has a maximum volume flow rate of 420 m^3/s. Base your calculations on this flow. The water is at 10°C, with a specific gravity of 1.0 and a viscosity of 0.0013 Pa $\cdot$ s.

The difference in elevation between the surfaces of the two lakes is 542 m.

The low pressure tunnel from A to B is 1700 m in length, has a diameter of 10.5 m and is lined with concrete. ($\epsilon = 1.2 \times 10^{-3}$ m.)

The shaft and high pressure tunnel from B to D is 1140 m in length, has a diameter of 10.5 m and is lined with welded steel ($\epsilon = 4.6 \times 10^{-5}$ m.)

(Notice that tunnel AB and tunnel BD are of different materials so their losses must be calculated separately!)

There are **three** tailrace tunnels from the turbine to the lower reservoir with the flow equally distributed between them (that is, each tailrace tunnel has a flow of 140 m^3/s). Each tailrace tunnel is 382 m in length, has a diameter of 8.5 m and is lined with concrete. ($\epsilon = 1.2 \times 10^{-3}$ m.)

(Calculate the headlosses for one tailrace tunnel and simply multiply that loss by three to get the total tailrace tunnel losses.)

The entrance to the low pressure tunnel at the upper lake has an equivalent length ratio of $Le/D = 420$. The bends at B and C are in the steel pipe and each have at equivalent length ratio of 16. A spherical valve at the inlet of the turbine that shuts off flow when the turbines are not operating is hydraulically efficient and has no losses associated with it. Each tailrace tunnel contains a butterfly valve ($Le/D = 20$).

At maximum capacity, the turbine outputs 1800 MW. Determine the efficiency of the turbine at this output.

Solution: (Text in red is explanatory and is not required in your solution.)

Series pipelines problems require finding the head losses, both due to friction and minor losses, to find the total head loss in the system. Then the General Energy Equation is applied, using the combined head losses already calculated.

Pipe AB

Velocity and velocity head in the 10.5 *m diameter tunnel*:

$$v = \frac{Q}{A} = \frac{420 \text{ m}^3/\text{s}}{\pi(10.5\text{m})^2/4} = 4.8504 \text{ m/s}$$

$$\frac{v^2}{2g} = \frac{(4.8504 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} = 1.1991 \text{ m}$$

Reynolds Number, relative roughness and friction factors for tunnel AB:

$$N_R = \frac{vD\rho}{\eta} = \frac{4.8504 \text{ m/s } \times 10.5 \text{ m } \times 1000 \text{ kg/m}^3}{0.0013 \text{ Pa}\cdot\text{s}} = 39176000$$

$$\frac{D}{\epsilon_{concrete}} = \frac{10.5 \text{ m}}{1.2 \times 10^{-3} \text{ m}} = 8750$$

Using Swamee-Jain:

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_R^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{1}{3.7(8750)} + \frac{5.74}{39176000^{0.9}}\right)\right]^2} = 0.012354$$

$$f_T = \frac{0.25}{\left[\log\left(\frac{1}{3.7(8750)}\right)\right]^2} = 0.012290$$

f_T can be found in a number of ways: from the Moody Diagram using a Reynolds Number of 10^8 (that is, the right hand edge of the diagram), from the Swamee-Jain using a Reynolds Number of 10^8, or from the Swamee-Jain omitting the $5.74/N_R^{0.9}$ term altogether since this term is very small. The Swamee-Jain value for f_T in this problem is 0.012318 with $N_R = 10^8$ or 0.012290 omitting the $5.74/N_R^{0.9}$ term. Given that the Swamee-Jain formula is an approximation of the Moody Diagram and that we can't distinguish between 0.012318 and 0.012290 on the Moody Diagram, the difference may be ignored.

Using Moody: Readings from the Moody Diagram for f and f_T are both approximately 0.0123. This is the value I shall use.

Losses for tunnel AB:

Entrance Losses: $$h_L = k\frac{v^2}{2g} = f_T\left(\frac{Le}{D}\right)\frac{v^2}{2g} = 0.0123 \times 420 \times 1.1991 = 6.1946$$

Friction Losses: $$h_L = f \cdot \frac{L}{D} \cdot \frac{v^2}{2g} = 0.0123 \cdot \frac{1700}{10.5} \cdot 1.1991 = 2.3879$$

$$h_{L_{AB}} = 6.1946 + 2.3879 = 8.5825 \text{ m}$$

Tunnel BCD

Velocity, velocity head and Reynolds Number for tunnel BCD (unchanged from tunnel AB calculations above since the dimensions are unchanged):

$$v = 4.8504 \text{ m/s}, \ \frac{v^2}{2g} = 1.1991 \text{ m}, \ N_R = 39176000$$

Relative roughness and friction factors for tunnel BCD:

$$\frac{D}{\epsilon_{steel}} = \frac{10.5 \text{ m}}{4.6 \times 10^{-5} \text{ m}} = 228260$$

Using Swamee-Jain:

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_R^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{1}{3.7(228260)} + \frac{5.74}{39176000^{0.9}}\right)\right]^2} = 0.0077125 \approx 0.0077$$

$$f_T = \frac{0.25}{\left[\log\left(\frac{1}{3.7(228260)}\right)\right]^2} = 0.0071174 \approx 0.0071$$

Using Moody: Readings for f and f_T lie outside the boundary of the Moody Diagram (because of the relatively smooth tunnel and high Reynolds Number) but extrapolating would seem to indicate (roughly) that $f \approx 0.0075$ and $f_T \approx 0.0070$. In this case, the Swamee-Jain is probably more accurate than my imperfect extrapolation.

Losses for tunnel BCD:

2 bends: $$h_L = 2k\frac{v^2}{2g} = 2f_T\left(\frac{Le}{D}\right)\frac{v^2}{2g} = 2 \times 0.0071 \times 16 \times 1.1991 = 0.27244$$

Friction Losses: $$h_L = f \cdot \frac{L}{D} \cdot \frac{v^2}{2g} = 0.0077 \cdot \frac{1140}{10.5} \cdot 1.1991 = 1.0024$$

$$h_{L_{BCD}} = 0.27244 + 1.0024 = 1.2749 \text{ m}$$

Single Tailrace Tunnel BCD

Velocity and velocity head in the 8.5 m diameter tunnel:

$$v = \frac{Q}{A} = \frac{140 \text{ m}^3/\text{s}}{\pi(8.5\text{m})^2/4} = 2.4672 \text{ m/s}$$

$$\frac{v^2}{2g} = \frac{(2.4672 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} = 0.31025 \text{ m}$$

Reynolds Number, relative roughness and friction factors for tunnel BCD:

$$N_R = \frac{vD\rho}{\eta} = \frac{2.4672 \text{ m/s} \times 8.5 \text{ m} \times 1000 \text{ kg/m}^3}{0.0013 \text{ Pa}\cdot\text{s}} = 16132000$$

$$\frac{D}{\epsilon_{concrete}} = \frac{8.5 \text{ m}}{1.2 \times 10^{-3} \text{ m}} = 7083$$

Using Swamee-Jain:

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_R^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{1}{3.7(7083)} + \frac{5.74}{16132000^{0.9}}\right)\right]^2} = 0.012927$$

$$f_T = \frac{0.25}{\left[\log\left(\frac{1}{3.7(7083)}\right)\right]^2} = 0.012806$$

Using Moody:

Readings from the Moody Diagram: $f \approx 0.0129$ and $f_T \approx 0.0127$. These are the values I shall use.

Losses for tailrace tunnel DE:

Butterfly Valve: $$h_L = k\frac{v^2}{2g} = f_T\left(\frac{Le}{D}\right)\frac{v^2}{2g} = 0.0127 \times 20 \times 0.31025 = 0.078804$$

Exit Losses: $$h_L = \frac{v^2}{2g} = 0.31025$$

Friction Losses: $$h_L = f \cdot \frac{L}{D} \cdot \frac{v^2}{2g} = 0.0129 \cdot \frac{382}{8.5} \cdot 0.31025 = 0.17986$$

$$h_{L_{DE}} = 0.078804 + 0.31025 + 0.17986 = 0.56891 \text{ m}$$

Head Losses for System

$$\begin{aligned} h_L &= h_{L_{AB}} + h_{L_{BCD}} + 3 \times h_{L_{DE}} \\ &= 8.5825 + 1.2749 + 3 \times 0.56891 \\ &= 11.564 \text{ m} \end{aligned}$$

Applying the General Energy Equation

Apply between the surfaces of the upper and lower lakes:

$$\begin{aligned} \frac{P_U}{\gamma} + z_U + \frac{v_U^2}{2g} - h_L - h_R &= \frac{P_L}{\gamma} + z_L + \frac{v_L^2}{2g} \\ 0 + 542 + 0 - 11.564 - h_R &= 0 + 0 + 0 \\ h_R &= 530.44 \text{ m} \end{aligned}$$

Find the power removed:

$$P_R = h_R \cdot \gamma \cdot Q = 530.44 \text{ m} \cdot 9.81 \text{ kg/m}^3 \cdot 420 \text{ m}^3\text{/s} = 2185500 \text{ kW} = 2185.50 \text{ MW}$$

Find the turbine efficiency:

$$e_M = \frac{P_O}{P_R} = \frac{1800 \text{ MW}}{2185.5 \text{ MW}} = 0.82361$$

The turbine efficiency is 82.4%

Phew!