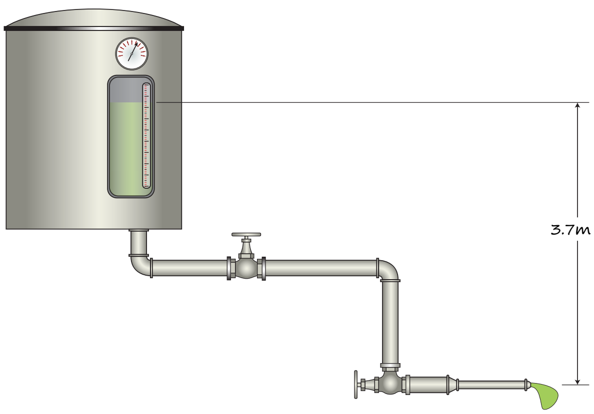
Module 4: The General Energy Equation (CIVL 318)

GEE:	$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} + h_A - h_R - h_L = \frac{p_B}{\gamma} + z_B + \frac{v_B^2}{2g}$
Power added by a pump:	$P_A = h_A \gamma Q$
Power removed by a turbine:	$P_R = h_R \gamma Q$
Efficiency of a pump:	$e_M = \frac{\text{power delivered to fluid}}{\text{power input to pump}} = \frac{P_A}{P_I}$
Efficiency of a turbine:	$e_M = \frac{\text{power output from turbine}}{\text{power removed from fluid}} = \frac{P_O}{P_R}$

Example 1:



Liquid with a specific gravity of 0.9 flows from a tank, pressurized to 57 kPa, through the pipe system shown, before entering the atmosphere through a nozzle with diameter 125 mm.

If the volume flow rate is $Q = 89$ L/s, determine h_L, the head loss due to friction and fittings.

Solution:

Find the average flow velocity at the nozzle:

$$v_N = \frac{Q}{A_N} = \frac{0.089}{\pi(0.125)^2/4} = 7.2524 \text{ m/s}$$

$$\frac{v_N^2}{2g} = 2.6808 \text{ m}$$

There is no head added or head removed in this problem so those terms may be omitted from the GEE. Apply the GEE to the liquid surface in the tank and to the nozzle:

$$\frac{P_S}{\gamma} + z_S + \frac{v_S^2}{2g} - h_L = \frac{P_N}{\gamma} + z_N + \frac{v_N^2}{2g}$$

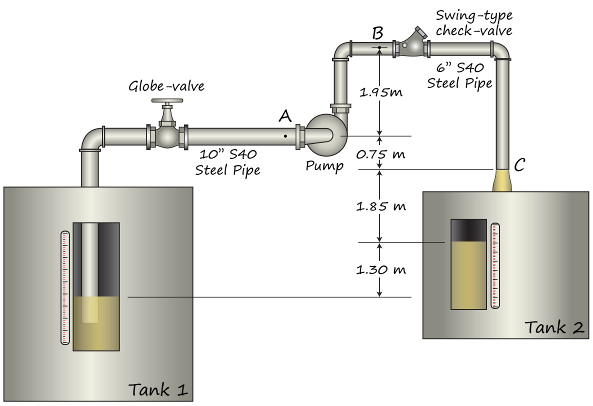
$$\frac{57}{0.9 \times 9.81} + 3.7 + 0 - h_L = 0 + 0 + \frac{v_N^2}{2g}$$

$$h_L = \frac{57}{0.9 \times 9.81} + 3.7 - 2.6808$$

$$= 7.4752$$

$$\boldsymbol{h_L = 7.48\,\mathrm{m}}$$

Example 2:



Liquid with a specific gravity of 0.87 is pumped from Tank 1; the liquid exits the pipe at C before dropping into Tank 2 at 180 L/s.

Determine the head added by the pump and the pressure at A.

(Assume that friction losses are not significant.)

Solution:

The diameter of the 10″ S40 Steel Pipe is 254.5 mm and the diameter of the 6″ S40 Steel Pipe is 154.1 mm.
Then, the average flow velocity at the pipe outlet, C, above Tank 2 is:

$$v_C = \frac{Q}{A_C} = \frac{0.180}{\pi(0.1541)^2/4} = 9.6511 \text{ m/s}$$

The velocity head at C is:

$$\frac{v_C^2}{2g} = 4.7474 \text{ m}$$

Apply the GEE to the surface, S, of the liquid in Tank 1 and to the outlet, N, above above Tank 2:

$$\frac{P_S}{\gamma} + z_S + \frac{v_S^2}{2g} + h_A = \frac{P_C}{\gamma} + z_C + \frac{v_C^2}{2g}$$

$$0 + 0 + 0 + h_A = 0 + (1.30 + 1.85) + 4.7474$$

$$h_A = 7.8974 \text{ m}$$

$$\boldsymbol{h_A = 7.90\,\text{m}}$$

The average flow velocity at A is:

$$v_A = \frac{Q}{A_A} = \frac{0.180}{\pi(0.2545)^2/4} = 3.5384 \text{ m/s}$$

The velocity head at A is:

$$\frac{v_A^2}{2g} = 0.63814 \text{ m}$$

Now, apply the GEE to the surface, S, of the liquid in Tank 1 and to A. Note that A is before the pump so no head has been added by the time fluid reaches A.

$$\frac{P_S}{\gamma} + z_S + \frac{v_S^2}{2g} = \frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g}$$

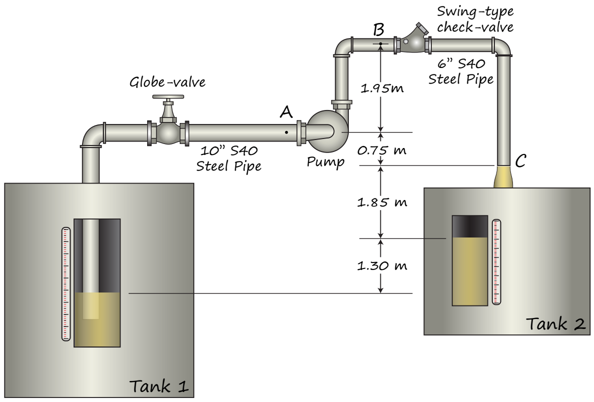
$$0 + 0 + 0 = \frac{P_A}{0.87 \times 9.81} + (1.30 + 1.85 + 0.75) + 0.63814$$

$$P_A = -38.732 \text{ kPa}$$

$$\boldsymbol{P_A = -38.7\,\text{kPa}}$$

Note that if the GEE had been applied between A and C, then the 7.8974 m of head added would have had to be included in the GEE since that head is added to the flow between A and C.

Exercise 1:



Liquid with a specific gravity of 0.87 is pumped from Tank 1; the liquid exits the pipe at C before dropping into Tank 2 at 180 L/s. (Neglect any head losses due to friction and valves.)

Determine the pressure at B:

(1) First, by applying the GEE between the surface of Tank 1 and B;
(2) Second, by applying the GEE between A and B;
(3) Finally, by applying the GEE between B and C.

Solution:

Note that the velocities at B and at C are the same.

(1)

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_A = \frac{P_B}{\gamma} + z_B + \frac{v_B^2}{2g}$$
$$0 + 0 + 0 + 7.8974 = \frac{P_B}{0.87 \times 9.81} + 5.85 + 4.7474$$
$$\Longrightarrow P_B = -23.044\,\text{kPa}$$
$$\Longrightarrow \boldsymbol{P_B = -23.0\,\textbf{kPa}}$$

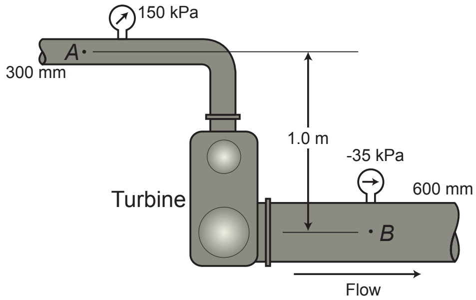
(2)

$$\frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g} + h_A = \frac{P_B}{\gamma} + z_B + \frac{v_B^2}{2g}$$
$$\frac{-38.732}{0.87 \times 9.81} + 0 + 0.63814 + 7.8974 = \frac{P_B}{0.87 \times 9.81} + 1.95 + 4.7474$$
$$\Longrightarrow P_B = -23.044\text{ kPa}$$
$$\Longrightarrow \boldsymbol{P_B = -23.0\,\textbf{kPa}}$$

(3)

$$\frac{P_B}{\gamma} + z_B + \frac{v_B^2}{2g} = \frac{P_C}{\gamma} + z_C + \frac{v_C^2}{2g}$$
$$\frac{P_B}{0.87 \times 9.81} + 2.7 + \cancel{\frac{v_B^2}{2g}} = 0 + 0 + \cancel{\frac{v_C^2}{2g}}$$
$$\Longrightarrow P_B = -23.044\,\text{kPa}$$
$$\Longrightarrow \boldsymbol{P_B = -23.0\,\textbf{kPa}}$$

Example 3:



Water flows from A to B at the rate of 120 L/s
Determine the head removed by the turbine.

Solution:

The velocity heads at A and B are found as follows:

$$\begin{aligned} v_A &= \frac{0.120\,\mathrm{m}^3/s}{\pi(0.300\ \mathrm{m})^2/4} \\ &= 1.6977\ \mathrm{m/s} \\ \frac{v_A^2}{2g} &= 0.14689\ \mathrm{m} \end{aligned}$$

$$\begin{aligned} v_B &= \frac{0.120\,\mathrm{m}^3/s}{\pi(0.600\ \mathrm{m})^2/4} \\ &= 0.42441\,\mathrm{m/s} \\ \frac{v_B^2}{2g} &= 0.0091808\,\mathrm{m} \end{aligned}$$

Apply the GEE between A and B:

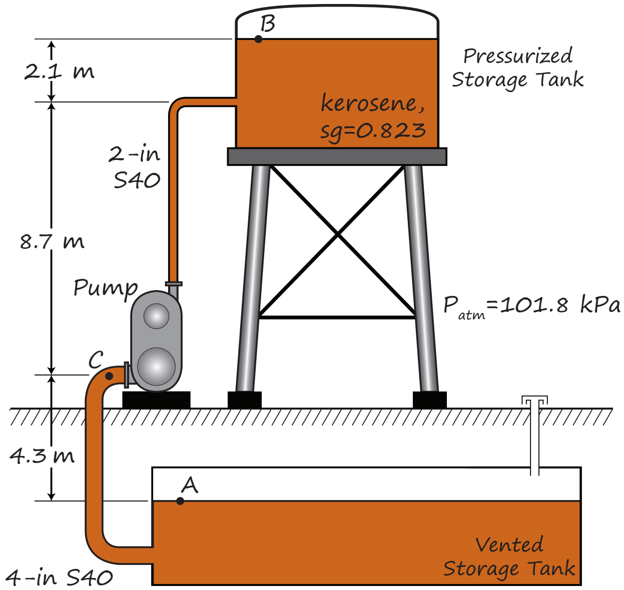
$$\frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g} - h_R = \frac{P_B}{\gamma} + \cancelto{0}{z_B} + \frac{v_B^2}{2g}$$

$$\frac{150}{9.81} + 1.0 + 0.14689 - h_R = \frac{-35}{9.81} + 0.0091808$$

$$h_R = 19.996\ \mathrm{m}$$

$$\boldsymbol{h_R = 20.0\,\mathrm{m}}$$

Example 4:



A pump produces a flow of 1024 L/min of kerosene with a specific gravity of 0.823 from vented underground storage to an elevated tank pressurized to 512 kPa. Energy loss between the underground storage and the pump is 0.95 m and energy loss between the pump and the elevated tank is 4.9 m.

(a) Determine the power added to the fluid by the pump.
(b) If the pump has an efficiency of 73%, determine the (electrical) power drawn by the pump.
(c) Determine the gauge and the absolute pressure at the pump inlet.

Solution:

Find the power added by the pump:

$$Q = \frac{1024\,\text{L/min}}{60\,\text{s/min} \times 1000\,\text{L/m}^3} = 0.017067\ \text{m}^3/\text{s}$$

$$\frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g} + h_A - h_L = \frac{P_B}{\gamma} + z_B + \frac{v_B^2}{2g}$$

$$0 + 0 + 0 + h_A - 5.85 = \frac{512}{0.823 \times 9.81} + 15.1 + 0$$

$$h_A = 84.366\,\text{m}$$

$$P_A = h_A \gamma Q$$

$$= (84.366\ \text{m})\left(0.823 \times 9.81\ \text{kN/m}^3\right)\left(0.017067\ \text{m}^3/\text{s}\right)$$

$$= 11.625\ \text{kN} \cdot \text{m/s}$$

The power added by the pump, $\boldsymbol{P_A = 11.63\,\text{kW}}$

Find the power drawn by the pump:

$$P_I = \frac{P_A}{e_M} = \frac{11.652}{73\%} = 15.924\,\text{kW}$$

$$\boldsymbol{P_I = 15.92\,\text{kW}}$$

Find the velocity head at C, the pump inlet:

$$v_C = Q/A_C = \frac{0.017067\ \text{m}^3/\text{s}}{\pi(0.1023\ \text{m})^2/4} = 2.0764\ \text{m/s}$$

$$\frac{v_C^2}{2g} = 0.21975\ \text{m}$$

Using the GEE, find the pressure at C:

$$\frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g} + h_A - h_L = \frac{P_C}{\gamma} + z_C + \frac{v_C^2}{2g}$$

$$0 + 0 + 0 - 0.95 = \frac{P_C}{0.823 \times 9.81} + 4.3 + 0.21975$$

$$P_C = -44.161\,\text{kPa}$$

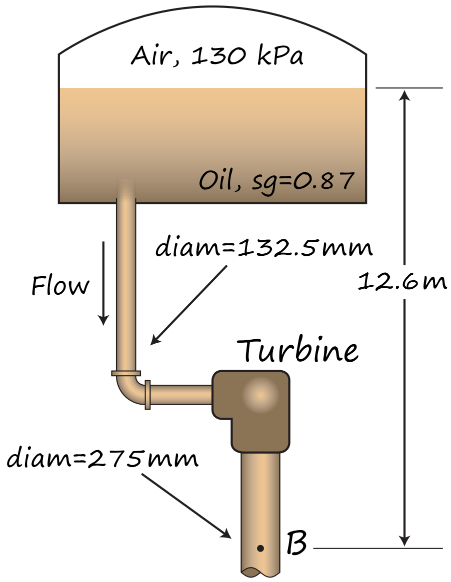
$$\boldsymbol{P_{C(\text{gauge})} = -44.2\,\text{kPa}}$$

Note that this is the **gauge** pressure.

$$p_{\text{abs}} = p_{\text{atm}} + p_{\text{gauge}} = 101.8 - 44.2\ \text{kPa} = 57.639\ \text{kPa}$$

$$\boldsymbol{P_{C(\text{abs})} = 57.6\,\text{kPa}}$$

Exercise 2:



Oil, with sg $= 0.87$, flows from a tank pressurized at 130 kPa at a rate of 72 L/s and powers a fluid motor as shown. Energy losses due to friction and fittings between the tank and B are estimated to be 1.81 m.

If the pressure at B is found to be -56 kPa and the motor has an efficiency of 78%, determine the power output from the motor.

Solution:

Find the velocity head at B:

$$\begin{aligned} v_B &= \frac{Q}{A_B} \\ &= \frac{0.072 \text{ m}^3/\text{s}}{\pi(0.275\,\text{m})^2/4} \\ &= 1.2122\,\text{m/s} \\ \frac{v_B^2}{2g} &= 0.074895\,\text{m} \end{aligned}$$

Apply the GEE between the oil surface in the tank and B:

$$\frac{P_A}{\gamma} + z_A + \cancelto{0}{\frac{v_A^2}{2g}} - h_L - h_R = \frac{P_b}{\gamma} + \cancelto{0}{z_B} + \frac{v_B^2}{2g}$$

$$\frac{130}{0.87 \times 9.81} + 12.6 - 1.81 - h_R = \frac{-56}{0.87 \times 9.81} + 0.074895$$

$$\frac{130 + 56}{0.87 \times 9.81} + 12.6 - 1.81 - 0.0749 = h_R$$

$$h_R = 32.508 \text{ m}$$

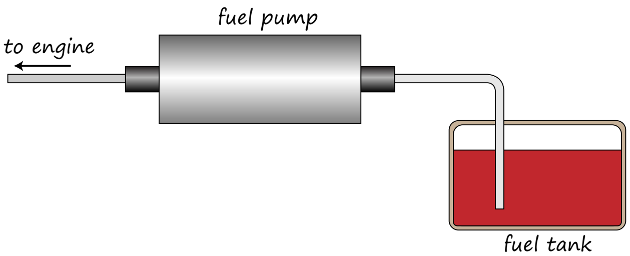
The power removed is:

$$\begin{aligned} P_R &= h_R \gamma Q \\ &= (32.508\,\text{m})\left(0.87 \times 9.81\,\text{kN/m}^3\right)\left(0.072\,\text{m}^3/\text{s}\right) \\ &= 19.976\,\text{kN·m/s} \\ &= 19.976 \text{ kW} \end{aligned}$$

The power output by the motor is:

$$\begin{aligned} P_O &= P_R \times e_M \\ &= 19.976 \times 78\% \\ &= 15.581 \text{ kW} \\ \boldsymbol{P_O} &= \mathbf{15.58\,kW} \end{aligned}$$

Example 5:



A car fuel pump pumps 1 L of gasoline every 45 s when is has a suction pressure of 155 mm of mercury vacuum and a discharge pressure of 32 kPa. Both the suction and the discharge lines have the same diameter.

If the pump efficiency is 68%, determine the power drawn from the engine.

Solution:

First, calculate the input pressure:
"155 mm of mercury vacuum" means a pressure that is below atmospheric by an amount due to a height of 155 mm of mercury.

$$\begin{aligned}\Delta p &= \gamma h \\ &= (13.54 \times 9.81 \text{ kN/m}^3)(0.155 \text{ m}) \\ &= 20.588 \text{ kPa}\end{aligned}$$

Since this pressure is below atmospheric, it is negative.
Thus the inlet pressure is -20.588 kPa.

Find the head added by the pump. The lines are of the same diameter so velocity heads cancel out. Similarly, the lines are at the same elevation so elevation head cancels out too.

$$\begin{aligned}h_A &= \frac{P_{\text{dischage}} - P_{\text{suction}}}{\gamma} \\ &= \frac{32 - (-20.588)}{0.68 \times 9.81} \\ &= 7.8833 \text{ m}\end{aligned}$$

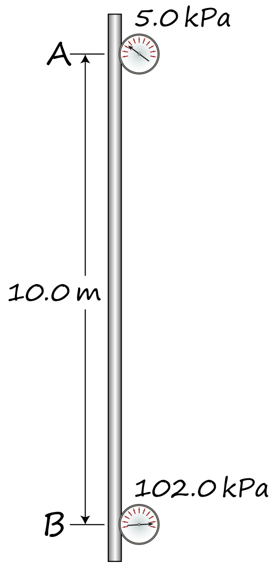
Then, the power added by the pump is:

$$\begin{aligned}p_A &= h_A \gamma Q \\ &= (7.8833 \text{ m})(0.68 \times 9.81 \text{ kN/m}^3)\left(\frac{0.001 \text{ m}^3}{45 \text{ s}}\right) \\ &= 0.0011686 \text{ kW} \\ &= 1.1686 \text{ W}\end{aligned}$$

$$\begin{aligned}P_I &= \frac{1.1686 \text{ W}}{0.68} \\ &= 1.7186 \text{ W}\end{aligned}$$

$$\boldsymbol{P_I = 1.72\ \mathbf{W}}$$

Example 6:



Water flows at a steady rate in a vertical pipe. Two pressure gauges are set $10\,\text{m}$ apart, as shown. There are no pumps or turbines and the pipe is of constant diameter.

Determine which of the following is true:

(a) flow is upward
(b) flow is downward
(c) there is no flow

Solution:

The difference in pressure between the two gauges is

$$102.0 - 5.0 = 97.0 \text{ kPa}$$

If there is no flow, then the difference in pressures should be given by

$$\Delta p = \gamma h = 9.81 \text{ kN/m}^3 \times 10 \text{ m} = 98.1 \text{ kPa}$$

If the flow is upward, apply the GEE from B to A:

$$\frac{P_B}{\gamma} + \cancelto{0}{z_B} + \cancel{\frac{v_B^2}{2g}} - h_L = \frac{P_A}{\gamma} + \cancelto{10}{z_A} + \cancel{\frac{v_A^2}{2g}}$$

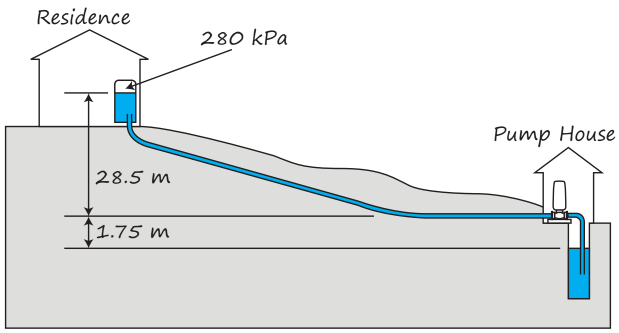
$$\begin{aligned} h_L &= \frac{P_B - P_A}{\gamma} - 10 \\ &= \frac{102.0 - 5.0}{9.81} - 10 \\ &= -0.11213 \text{ m} \end{aligned}$$

A negative headloss is not possible!

Applying the GEE from A to B (i.e., downward flow) yields a headloss of $+0.11213\,\text{m}$.

Flow is downward!

Exercise 3:



A rural house relies upon a shallow well for its water supply. The pump at the well is required to supply 210 L/min of water. The water tank at the house maintains a pressure of 280 kPa. Friction losses in the pipe amount to 4.35 m.

If the pump is 72% efficient, determine the power delivered to the pump by the electrical supply and the power added to the water by the pump.

Solution:

Find the head added by the pump:

$$\frac{P_P}{\gamma} + z_P + \frac{v_P^2}{2g} + h_A - h_L = \frac{P_R}{\gamma} + z_R + \frac{v_R^2}{2g}$$

$$0 + 0 + 0 + h_A - 4.35 = \frac{280}{9.81} + 30.25 + 0$$

$$h_A = 63.142 \text{ m}$$

Find the volume flow rate:

$$\begin{aligned} Q &= 210\,\text{L/min} \\ &= \frac{210}{1000 \times 60}\ \text{m}^3/\text{s} \\ &= 0.0035\,\text{m}^3/\text{s} \end{aligned}$$

Now, the power added to the pump:

$$\begin{aligned} P_{\text{added}} &= h_A \gamma Q \\ &= (63.142\,\text{m})(9.81\,\text{kN/m}^3)(0.0035\,\text{m}^3/\text{s}) \\ &= 2.1680\,\text{kN}\cdot\text{m/s} \\ \boldsymbol{P_{\text{added}}} &= \mathbf{2.17\,kW} \end{aligned}$$

And the power delivered to the pump:

$$\begin{aligned} P_I &= \frac{P_{\text{added}}}{e_M} \\ &= \frac{P_{\text{added}}}{e_M} \\ &= \frac{2.1680\,\text{kW}}{0.72} \\ &= 3.0111\,\text{kW} \\ \boldsymbol{P_I} &= \mathbf{3.01\,kW} \end{aligned}$$