Module 6: Minor Losses (CIVL 318)

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_R^{0.9}}\right)\right]^2}$$

$$h_L = K \cdot \frac{v^2}{2g}$$

Flow through valves:

$$h_L = f_T \cdot \frac{L_e}{D} \cdot \frac{v^2}{2g}$$

K-values for Sudden Contraction

	Velocity, v								
D_1/D_2	0.6 m/s	1.2 m/s	1.8 m/s	2.4 m/s	3 m/s	4.5 m/s	6 m/s	9 m/s	12 m/s
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.1	0.03	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.06
1.2	0.07	0.07	0.07	0.07	0.08	0.08	0.09	0.10	0.11
1.4	0.17	0.17	0.17	0.17	0.18	0.18	0.18	0.19	0.20
1.6	0.26	0.26	0.26	0.26	0.26	0.25	0.25	0.25	0.24
1.8	0.34	0.34	0.34	0.33	0.33	0.32	0.31	0.29	0.27
2.0	0.38	0.37	0.37	0.36	0.36	0.34	0.33	0.31	0.29
2.2	0.40	0.40	0.39	0.39	0.38	0.37	0.35	0.33	0.30
2.5	0.42	0.42	0.41	0.40	0.40	0.38	0.38	0.34	0.31
3.0	0.44	0.44	0.43	0.42	0.42	0.40	0.39	0.36	0.33
4.0	0.47	0.46	0.45	0.45	0.44	0.42	0.41	0.37	0.34
5.0	0.48	0.47	0.47	0.46	0.45	0.44	0.42	0.38	0.35
10.0	0.49	0.48	0.48	0.47	0.46	0.45	0.43	0.40	0.36
∞	0.49	0.48	0.48	0.47	0.47	0.45	0.44	0.41	0.38

K-values for Gradual Contraction

$$K = \begin{cases} 0.5\sqrt{\sin\frac{\theta}{2}}\left(1 - \left(\frac{D_2}{D_1}\right)^2\right), & 45^\circ < \theta \le 180^\circ \\ 0.8\sin\frac{\theta}{2}\left(1 - \left(\frac{D_2}{D_1}\right)^2\right), & 15^\circ < \theta \le 45^\circ \end{cases}$$

where D_2 is the smaller (downstream) diameter of the pipe after the contraction and D_1 is the diameter of the larger (upstream) pipe and the head loss is based on the velocity of the smaller pipe.

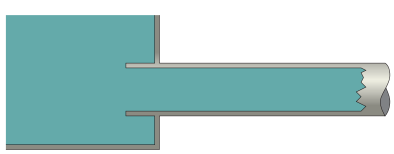
K-values for Sudden Enlargement:

	Velocity, v						
D_2/D_1	0.6 m/s	1.2 m/s	3 m/s	4.5 m/s	6 m/s	9 m/s	12 m/s
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.2	0.11	0.10	0.09	0.09	0.09	0.09	0.08
1.4	0.26	0.25	0.23	0.22	0.22	0.21	0.20
1.6	0.40	0.38	0.35	0.34	0.33	0.32	0.32
1.8	0.51	0.48	0.45	0.43	0.42	0.41	0.40
2.0	0.60	0.56	0.52	0.51	0.50	0.48	0.47
2.5	0.74	0.70	0.65	0.63	0.62	0.60	0.58
3.0	0.83	0.78	0.73	0.70	0.69	0.67	0.65
4.0	0.92	0.87	0.80	0.78	0.76	0.74	0.72
5.0	0.96	0.91	0.84	0.82	0.80	0.77	0.75
10.0	1.00	0.96	0.89	0.86	0.84	0.82	0.80
∞	1.00	0.98	0.91	0.88	0.86	0.83	0.81

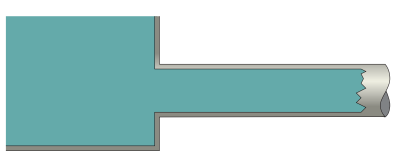
K-values for Gradual Enlargement

D_2/D_1	2°	6°	10°	15°	20°	25°	30°	35°	40°	45°	50°	60°
1.1	0.01	0.01	0.03	0.05	0.10	0.13	0.16	0.81	0.19	0.20	0.21	0.23
1.2	0.02	0.02	0.04	0.09	0.16	0.21	0.25	0.29	0.31	0.33	0.35	0.37
1.4	0.02	0.03	0.06	0.12	0.23	0.30	0.36	0.41	0.44	0.47	0.50	0.53
1.6	0.03	0.04	0.07	0.14	0.26	0.35	0.42	0.47	0.51	0.54	0.57	0.61
1.8	0.03	0.04	0.07	0.15	0.28	0.37	0.44	0.50	0.54	0.58	0.61	0.65
2.0	0.03	0.04	0.07	0.16	0.29	0.38	0.46	0.52	0.56	0.60	0.63	0.68
2.5	0.03	0.04	0.08	0.16	0.30	0.39	0.48	0.54	0.58	0.62	0.65	0.70
3	0.03	0.04	0.08	0.16	0.31	0.40	0.48	0.55	0.59	0.63	0.66	0.71
∞	0.03	0.05	0.08	0.16	0.31	0.40	0.49	0.56	0.60	0.64	0.67	0.72

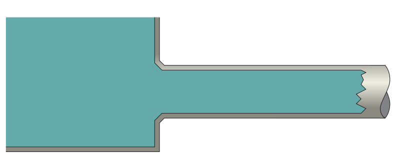
K-values for Entrance Losses



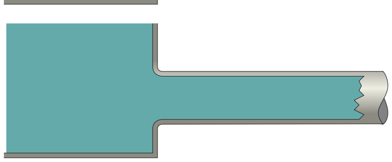
Inward-projecting: $K = 0.78 — 1.0$



Square-edged inlet: $K = 0.5$

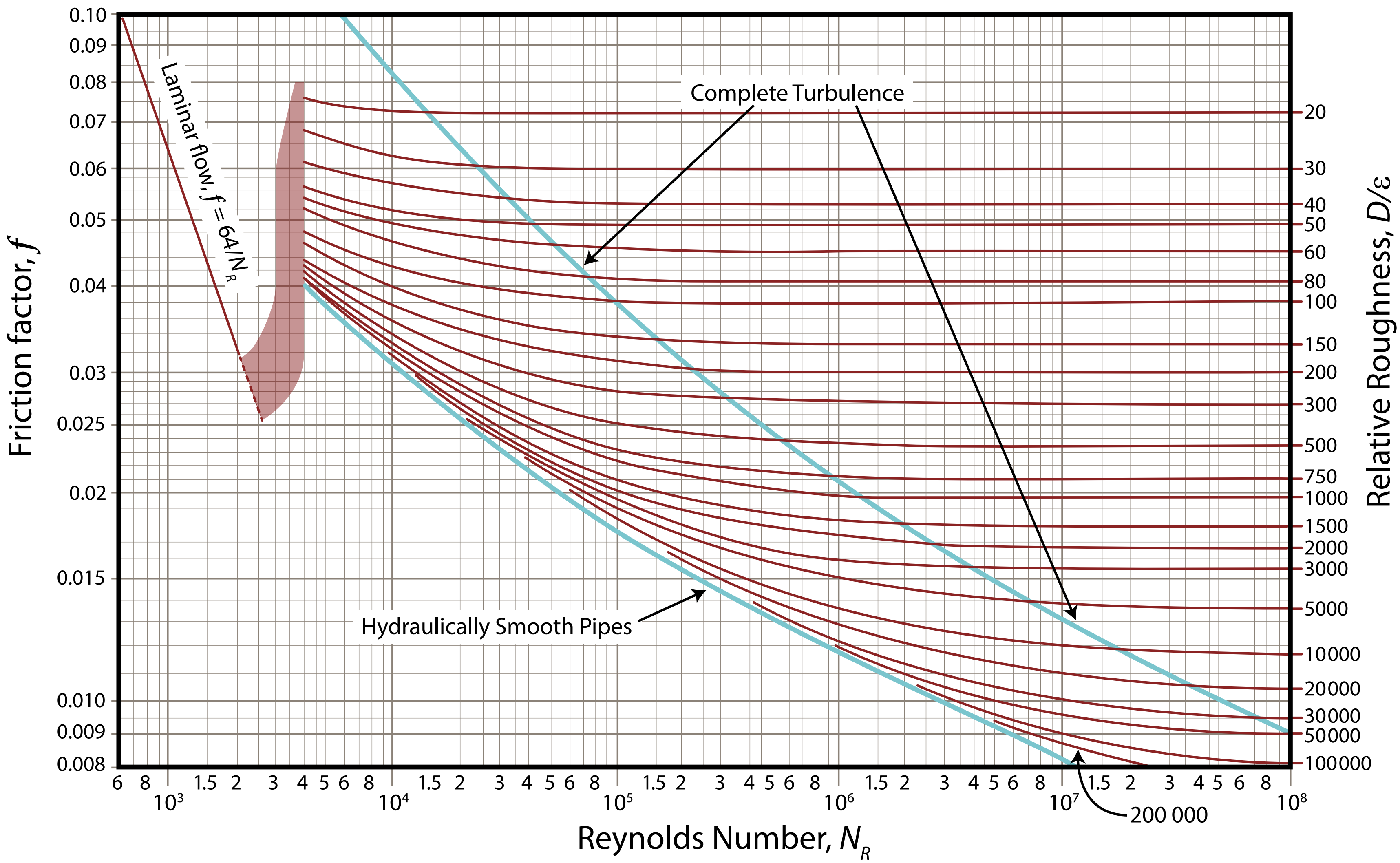


Chamfered Inlet: $K = 0.25$



Rounded inlet:

r/D	K
0	0.5
0.02	0.28
0.04	0.24
0.06	0.15
0.10	0.09
≥ 0.15	0.04


Complete Turbulence
Laminar flow, $f = 64/N_R$
Hydraulically Smooth Pipes
200 000
Friction factor, f
Relative Roughness, D/ε
Reynolds Number, N_R
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.010
0.009
0.008
20
30
40
50
60
80
100
150
200
300
500
750
1000
1500
2000
3000
5000
10000
20000
30000
50000
100000
10^3
10^4
10^5
10^6
10^7
10^8

Exit Loss (i.e. exiting a pipe into a tank where all velocity is lost):

$$K = 1 \Rightarrow h_L = \frac{v^2}{2g}$$

Friction Factor, f_T in the zone of complete turbulence for new clean commercial **steel** pipe:

Nominal Size (in)	f_T	Nominal Size (in)	f_T
$\frac{1}{2}$	0.027	$3\frac{1}{2}, 4$	0.017
$\frac{3}{4}$	0.025	5	0.016
1	0.023	6	0.015
$1\frac{1}{4}$	0.022	$8-10$	0.014
$1\frac{1}{2}$	0.021	$12-16$	0.013
2	0.019	$18-24$	0.012
$2\frac{1}{2}, 3$	0.018		

Equivalent-length ratios for valves:

Type	L_e/D
Globe valve — fully open	340
Angle valve — fully open	150
Gate valve — fully open	8
— 3/4 open	35
— 1/2 open	160
— 1/4 open	900
Check valve — swing type	100
Check valve — ball type	150
Butterfly valve — fully open	45
Foot valve — poppet disc type	420
Foot valve — hinged disc type	75

Table of Equivalent-Length Ratios for Fittings

Type	L_e/D
90° standard elbow	30
90° long radius elbow	20
90° street elbow	50
45° standard elbow	16
45° street elbow	26
Close return bend	50
Standard tee — flow through run	20
Standard tee — flow through branch	60

Example 1:

Determine the head loss that occurs when 100 L/min of fluid flows from 3-in Type K copper tube ($D = 73.84\,\text{mm}$) into 1-in Type K copper tube ($D = 25.27\,\text{mm}$) through a sudden contraction.

Solution:

From the sudden contraction K-value table, D_1 is the larger pipe diameter and D_2 the smaller pipe diameter so

$$\frac{D_1}{D_2} = \frac{73.83\,\text{mm}}{25.27\,\text{mm}} \approx 2.9$$

The velocity in the smaller pipe is

$$\begin{aligned} v_2 &= \frac{Q}{A} \\ &= \frac{0.1/60\,\text{m}^3/\text{s}}{\pi(0.02527\,\text{m})^2/4} \\ &= 3.3231\,\text{m/s} \end{aligned}$$

From the table:

$$k \approx 0.42$$

Then,

$$\begin{aligned} h_L &= k \cdot \frac{v^2}{2g} \\ &= 0.42 \cdot \frac{(3.3231\,\text{m/s})^2}{19.62\,\text{m/s}^2} \\ &= 0.23639\,\text{m} \\ &\approx 0.236\,\text{m} \end{aligned}$$

Example 2:

Determine the head loss for a gradual contraction from 4-in Schedule 80 pipe ($D = 97.2\,\text{mm}$) to a $1\frac{1}{2}$-in Schedule 80 pipe ($D = 38.1\,\text{mm}$) with a cone angle of 76°.
The flow is 450 L/min.

Solution:

For $\theta = 76^\circ$,

$$\begin{aligned} k &= 0.5\sqrt{\sin\frac{\theta}{2}}\left(1 - \left(\frac{D_2}{D_1}\right)^2\right) \\ &= 0.5\sqrt{\sin\frac{76^\circ}{2}}\left(1 - \left(\frac{38.1\,\text{mm}}{97.2\,\text{mm}}\right)^2\right) \\ &= 0.33204 \end{aligned}$$

Velocity and velocity head in the smaller downstream pipe:

$$\begin{aligned} Q &= \frac{450/60\,(m^3/s)}{\pi(0.0381\,\text{m})^2/4} \\ &= 6.5784\,\text{m/s} \end{aligned}$$

$$\frac{v^2}{2g} = 2.2057\,\text{m}$$

Then,

$$\begin{aligned} h_L &= k \cdot \frac{v^2}{2g} \\ &= 0.33204 \times 2.2057\,\text{m} \\ &\approx 0.732\,\text{m} \end{aligned}$$

Example 3:

Determine the head loss that occurs when 100 L/min flows from 1-in Type K copper tube ($D = 25.27\,\mathrm{mm}$) into 3-in Type K copper tube ($D = 73.48\,\mathrm{mm}$) through a sudden enlargement.

Solution:

Again, losses are based on the velocity head in the smaller (in this case, upstream) pipe.

$$\begin{aligned} v_1 &= \frac{0.1/60\,\mathrm{m}^3/\mathrm{s}}{\pi(0.02527\,\mathrm{m})^2)/4} \\ &= 3.3231\,\mathrm{m/s} \end{aligned}$$

$$\frac{v^2}{2g} = 0.56286\,\mathrm{m}$$

$$\begin{aligned} \frac{D_2}{D_1} &= \frac{73.84\,\mathrm{mm}}{25.27\,\mathrm{mm}} \\ &= 2.9220 \end{aligned}$$

Velocity is greater than 1.2 m/s so we use the table to find that $K \approx 0.73$

$$\begin{aligned} h_L &= K \cdot \frac{v^2}{2g} \\ &= 0.73 \times 0.56286\,\mathrm{m} \\ &\approx 0.411\,\mathrm{m} \end{aligned}$$

Example 4:

Compare the headlosses between an inward-projecting entrance and rounded entrance with a radius of 25 mm for water entering 6-in Schedule 40 steel pipe ($D = 154.1\,\mathrm{mm}$) with a flow of 75L/s. (Use $K = 0.78$.)

Solution:

$$\begin{aligned} v &= \frac{0.075\,\mathrm{m}^3/\mathrm{s}}{\pi(0.1541\,\mathrm{m})^2)/4} \\ &= 4.0213\,\mathrm{m/s} \end{aligned}$$

$$\frac{v^2}{2g} = 0.82420\,\mathrm{m}$$

For inward projecting entrance:

$$\begin{aligned} h_L &= 0.78 \times 0.82420\,\mathrm{m} \\ &= 0.64288\,\mathrm{m} \end{aligned}$$

For rounded entrance:

$$\begin{aligned} \frac{r}{D} &= \frac{25\,\mathrm{mm}}{154.1\,\mathrm{mm}} \\ &= 0.16223 \\ \Rightarrow K &= 0.04 \end{aligned}$$

Then head loss is given by:

$$\begin{aligned} h_L &= K \cdot \frac{v^2}{2g} \\ &= 0.032968\,\mathrm{m} \end{aligned}$$

Example 5:

Find the pressure drop across a fully open globe valve ($L_e/D = 340$) in 4-in Shedule 40 steel pipe ($D = 102.3\,\text{mm}$) carrying 1600 L/min.

Solution:

We have a number of options to find f_T: the Moody diagram, the Swamee-Jain formula and directly from the table for values of f_T for new commercial steel. (The table gives a value of $f_T = 0.017$.)
The relative roughness is:

$$\frac{D}{\epsilon} = \frac{0.1023\,\text{m}}{4.6 \times 10^{-5}\,\text{m}} = 2223.9$$

Using the Moody diagram, we choose a value for the Reynold's number anywhere in the zone of complete turbulence for a relative roughness of 2223.9. Reading the friction factor from the left hand side of the diagram gives $f_T \approx 0.0165$
Using the Swamee-Jain Formula with $N_R = 3.6 \times 10^6$, where flow with a relative roughness of 2223.9 becomes completely turbulent:

$$f_T = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_R^{0.9}}\right)\right]^2}$$

$$= \frac{0.25}{\left[\log\left(\frac{1}{3.7(2223.9)} + \frac{5.74}{(3.6\times10^6)^{0.9}}\right)\right]^2}$$

$$= 0.016519$$

The three results for f_T are within 3% of each other, which is more accurate than we expect from any calculations of flow so these are effectively the same result.
For new commercial steel pipe, using the appropriate table is convenient. Beware, though, that you don't use it for any other type of pipe or tubing.

Now, we can find the resistance coefficient, K, of the valve for this pipe:

$$K = f_T \cdot \frac{L_e}{D} = 0.017 \times 340 = 5.78$$

The velocity and velocity head are:

$$v = \frac{1.6/60\,\text{m}^3/\text{s}}{\pi(0.1023\,\text{m})^2/4}$$
$$= 3.2443\,\text{m/s}$$

$$\frac{v^2}{2g} = 0.53648\,\text{m}$$

Head loss across the valve is given by:

$$h_L = K \cdot \frac{v^2}{2g}$$

$$= 5.78 \times 0.53648$$

$$= 3.1008\,\text{m}$$

Use the GEE to find the pressure drop:

$$\frac{P_1}{\gamma} - h_L = \frac{P_1}{\gamma}$$

$$P_1 - P_2 = \gamma \cdot h_L$$

$$= 9.81\,\text{kN/m}^3 \times 3.1008$$

$$= 30.4\,\text{kPa}$$

Example 6:

Calculate the headloss across a ball-type check valve placed in a $1\frac{1}{4}$-in copper tubing $(D = 31.62\,\text{mm})$ if water is flowing through the tubing with a velocity of 2.35 m/s.

Solution:

The relative roughness is:

$$\frac{D}{\epsilon} = \frac{0.03162\,\text{m}}{1.5 \times 10^{-6}\,\text{m}} = 21087$$

Copper tubing so we need either the Moody diagram or the Swami-Jain. From the Moody diagram:

$$f_T = 0.0105$$

The resistance coefficient, K, of the valve for this pipe:

$$K = f_T \cdot \frac{L_e}{D} = 0.0105 \times 150 = 1.575$$

The velocity head is:

$$\frac{v^2}{2g} = \frac{(2.35\,\text{m/s})^2}{19.62\,\text{m/s}^2} = 0.28147\,\text{m}$$

Head loss across the valve is given by:

$$h_L = K \cdot \frac{v^2}{2g}$$

$$= 1.575 \times 0.28147$$

$$= 0.44332\,\text{m}$$

Example 7:

Find the pressure drop across a 90° standard elbow in a $2\frac{1}{2}$-in Schedule 40 steel pipe $(D = 62.7\,\text{mm})$ if water is flowing at the rate of 800 L/min.

Solution:

From the table of f_T values for new commercial steel pipe, $f_T = 0.018$
Then

$$\begin{aligned} v &= \frac{0.8/60\,\text{m}^3/\text{s}}{\pi(0.0627\,\text{m})^2/4} \\ &= 4.3183\,\text{m/s} \end{aligned}$$

$$\frac{v^2}{2g} = 0.95045\,\text{m}$$

$$\begin{aligned} K &= f_T \cdot \frac{L_e}{D} \\ &= 0.018 \times 30 \\ &= 0.54 \end{aligned}$$

$$\begin{aligned} h_L &= K \cdot \frac{v^2}{2g} \\ &= 0.54 \times 0.95045 \\ &= 0.51324\,\text{m} \end{aligned}$$

$$\begin{aligned} P_1 - P_2 &= \gamma \cdot h_L \\ &= 9.81\,\text{kN/m}^3 \times 0.51324 \\ &= 5.0349\,\text{kPa} \end{aligned}$$