

05 — Friction Losses in Pipes

Water Resources, CIVL318

Last revision on May 28, 2018

Types of Flow

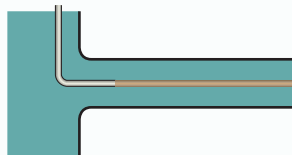
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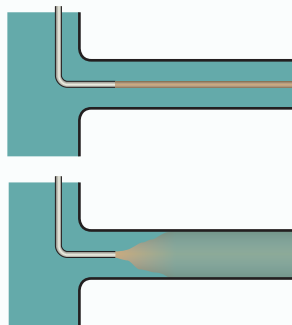
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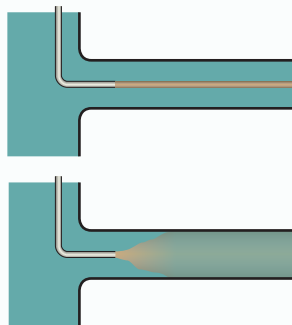
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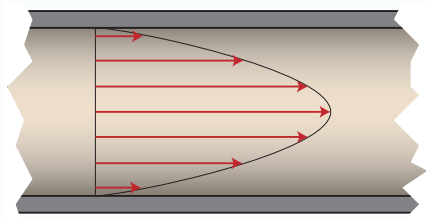
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- ▶ Between these two conditions is a range of velocities where flow is transitional



- ▶ Not all fluid particles have the same velocity within a pipe

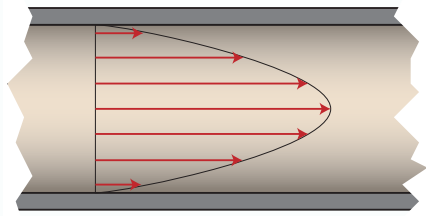
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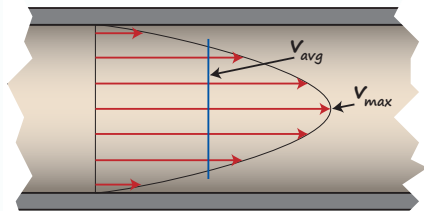
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Maximum velocity is at the centre of the pipe and is about twice the average velocity of the flow



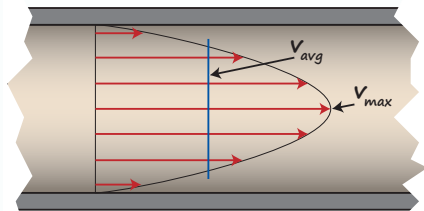
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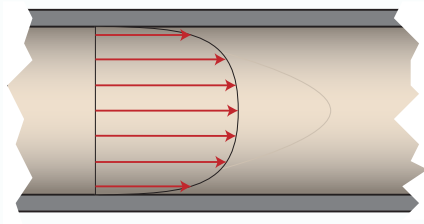
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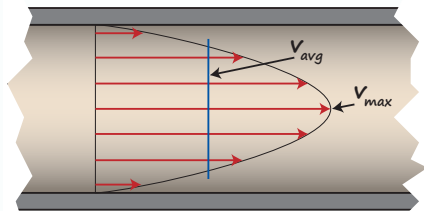
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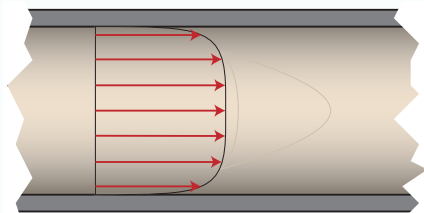
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- ▶ Turbulent flow has a flatter velocity distribution.
Especially in case of smooth pipe.



- ▶ The viscosity of a fluid is a measure of how easily it pours.
- ▶ Heating a viscous fluid, such as cold oil, lowers its viscosity and allows it to flow more easily.
- ▶ The flow of high viscosity fluids is more likely to be laminar.
- ▶ The flow of low viscosity fluids (such as water) is more likely to be turbulent.

- ▶ Energy losses within flow in pipes is dependent upon the type of flow.
- ▶ Type of flow (in circular pipes) is dependent upon the density, viscosity and velocity of the fluid, and upon the inside diameter of the pipe.
- ▶ The Reynolds Number is used to predict flow type:

Reynolds Number

$$N_R = \frac{vD\rho}{\eta}$$

where:

v is the average flow velocity (m/s)

D is the pipe inside diameter (m)

ρ is the density of the fluid (kg/m^3)

η is the dynamic viscosity of the fluid ($\text{Pa} \cdot \text{s}$)

The Reynolds Number is dimensionless:

$$\begin{aligned}\frac{\text{m/s} \times \text{m} \times \text{kg/m}^3}{\text{Pa} \cdot \text{s}} &= \frac{\text{m/s} \times \text{m} \times \text{kg/m}^3}{\text{N/m}^2 \cdot \text{s}} \\ &= \frac{\text{m/s} \times \text{m} \times \text{kg/m}^3}{(\text{kg} \cdot \text{m/s}^2) / \text{m}^2 \cdot \text{s}} \\ &= 1\end{aligned}$$

- ▶ Flows with high velocities and/or low viscosities tend to have turbulent flow.

Such flows have large Reynolds numbers.

- ▶ Flows with low velocities and/or high viscosities tend to exhibit laminar flow.

Such flows have low Reynolds numbers.

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Such flows have low Reynolds numbers.

Reynolds Number

$N_R < 2000$, flow is laminar

$2000 < N_R < 4000$, flow is in the 'critical region'

$N_R > 4000$, flow can be assumed to be turbulent

Exercise 1

Flow is said to be in the **critical region**, with neither fully laminar or fully turbulent flow, if the Reynolds number for the flow is between 2000 and 4000.

Determine the range of velocities and volume flow rates for which flow is in the critical region for:

- 1 water at 5°C flowing in 1/2-in copper tubing
- 2 water at 95°C flowing in 1/2-in copper tubing
- 3 fuel oil at 10°C ($sg = 0.94$, $\eta = 2.4 \text{ Pa} \cdot \text{s}$), flowing in 12-in Schedule 40 steel pipe

Darcy's Equation

Darcy's equation (or Darcy-Weisbach equation) is used to calculate the head loss due to friction in long, straight sections of circular pipe:

Darcy's Equation

$$h_L = f \times \frac{L}{D} \times \frac{v^2}{2g}$$

where:

h_L is energy loss due to friction (m)

f is the friction factor (dimensionless)

L is the length of the pipe (m)

D is the diameter of the pipe (m)

$\frac{v^2}{2g}$ is the velocity head of the flow based on the average flow velocity in the pipe (m)

- ▶ Darcy's Equation may be used for both laminar and turbulent flow.
- ▶ Calculation of f , the friction factor, depends upon the type of flow.
- ▶ For laminar flow,

$$f = \frac{64}{N_R}$$

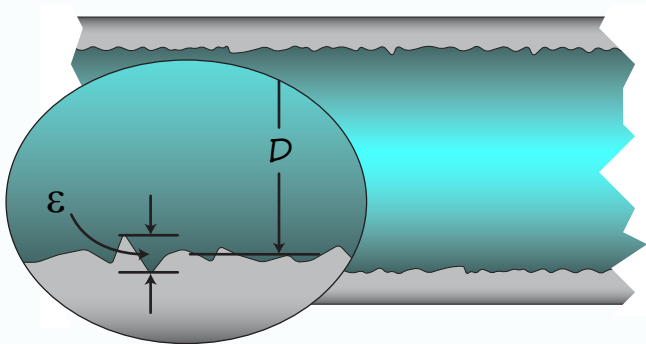
- ▶ Losses are independent of the pipe wall surface (velocity is zero at the pipe wall).
- ▶ Losses come from overcoming the frictional (shear) forces between adjacent layers of liquid moving at different velocities.

Example 1

Determine the headloss due to friction in fuel oil at 10°C flowing through 125 m of 12-in Schedule 40 steel pipe with an average flow velocity of 4.5 m/s. ($sg = 0.94$, $\eta = 2.4 \text{ Pa} \cdot \text{s}$)

Friction Loss in Turbulent Flow

- ▶ Turbulent flow is chaotic and varying, and the value of f has been determined experimentally for many flow situations
- ▶ Experiments have shown that f depends upon the Reynolds number for the flow and the **relative roughness**, the ratio D/ϵ of pipe diameter D to the average wall roughness ϵ .

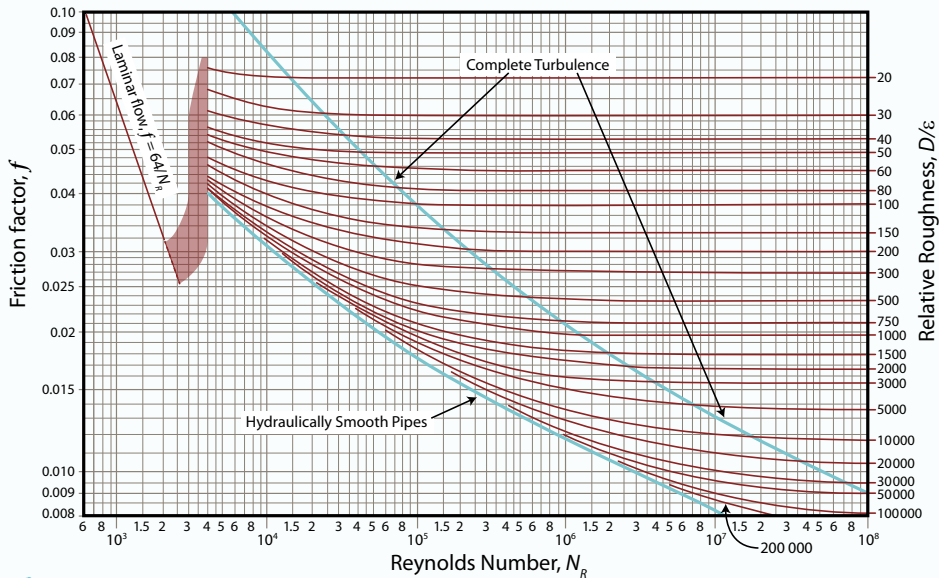


- ▶ Values for f can be read from the Moody Diagram

Roughness, ϵ :

Material (new, clean)	ϵ (m)
Glass	Smooth
Plastic	3.0×10^{-7}
Copper, brass, lead (tubing)	1.5×10^{-6}
Commercial steel, welded steel	4.6×10^{-5}
Wrought iron	4.6×10^{-5}
Ductile Iron - coated	1.2×10^{-4}
Ductile Iron - uncoated	2.4×10^{-4}
Concrete	1.2×10^{-4}
Riveted steel	1.8×10^{-3}

The Moody Diagram



"It must be recognized that any high degree of accuracy in determining f is not to be expected. With smooth tubing, it is true, good degrees of accuracy are obtainable; a probable variation in f within about $\pm 5\%$ and for commercial steel and wrought-iron piping, a variation within about $\pm 10\%$. But, in the transition and rough-pipe regions, we lack the primary and obvious essential, a technique for measuring the roughness of a pipe mechanically. . .

. . . however, fairly reasonable estimates of friction loss can be made, and, fortunately, engineering problems rarely require more than this. . .

The charts apply only to new and clean piping, since the rapidity of deterioration with age, dependent upon the quality of water or fluid and that of the pipe material, can only be guessed in most cases; and in addition to the variation in roughness there may be, in old piping, an appreciable reduction in effective diameter. . ."

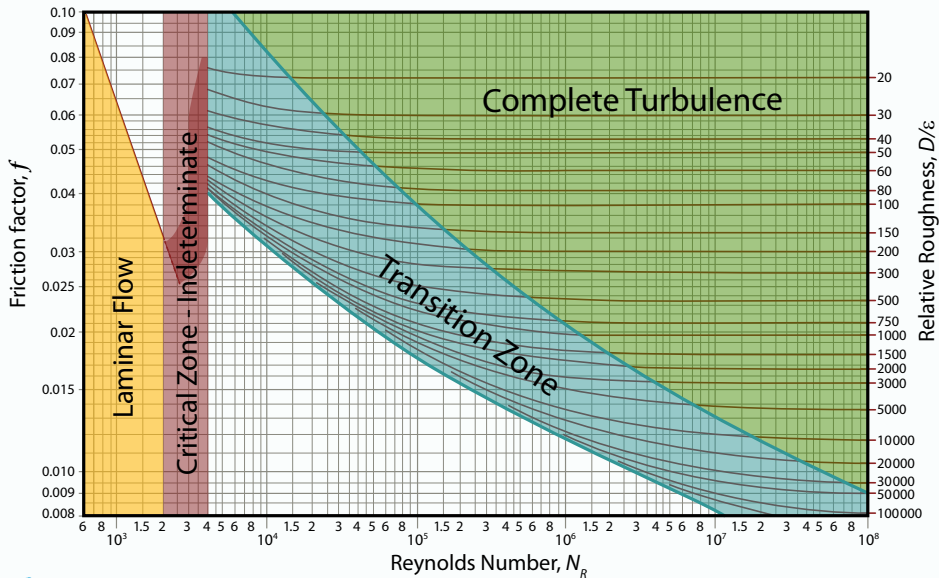
Friction Factors for Pipe Flow, Lewis F. Moody, Princeton, N.J. (1944)

Regions of Flow Characteristics

- ▶ Up to a Reynolds number of 2000, flow is laminar
- ▶ For a Reynolds number between 2000 and about 4000, conditions depend upon a number of factors (such as the shape of the pipe entrance, changes in section size, pressure waves, ...). This is the **critical region** indicated on the Moody diagram by the shaded area, where f cannot be calculated. ^a
- ▶ Above a Reynolds number of 4000, there are two regions:
 - ▶ First there is a **transition zone** of incomplete turbulence (the extent of this depends upon the relative roughness of the pipe)
 - ▶ The region of **complete turbulence**

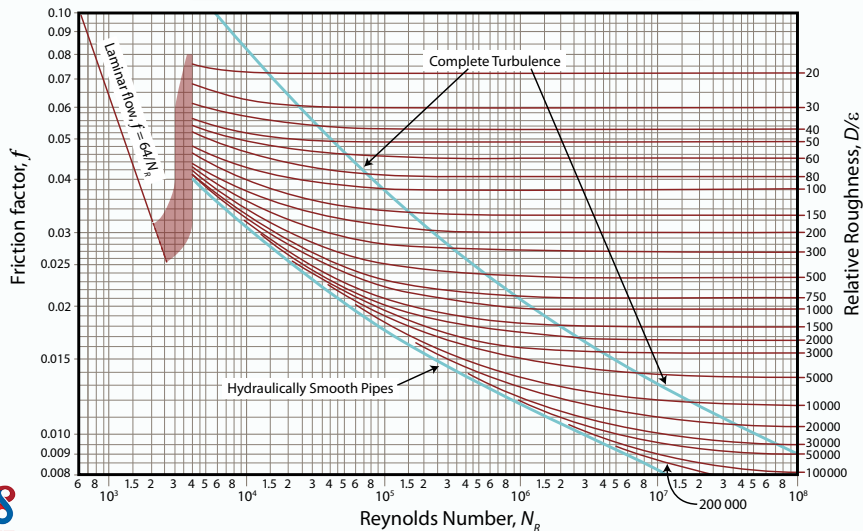
^aMore recent experimental data suggest that for smooth pipe, flow is laminar up to around $N_R = 2700$ and completely turbulent for $N_R > 3000$. We shall use the more recognized ranges where flow can not be determined for $2000 < N_r < 4000$.

Regions of Flow Characteristics



Example 2

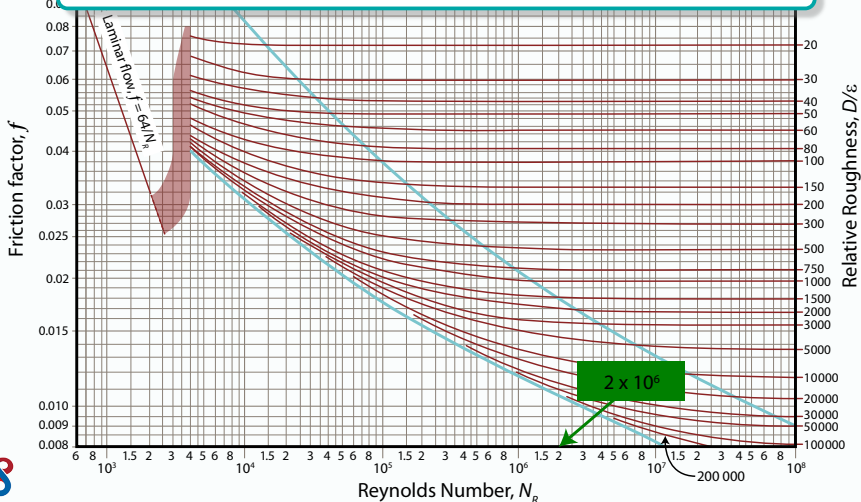
Use the Moody diagram to determine the friction factor for flow with $N_R = 2 \times 10^6$ and a relative roughness of 1428.



Example 2

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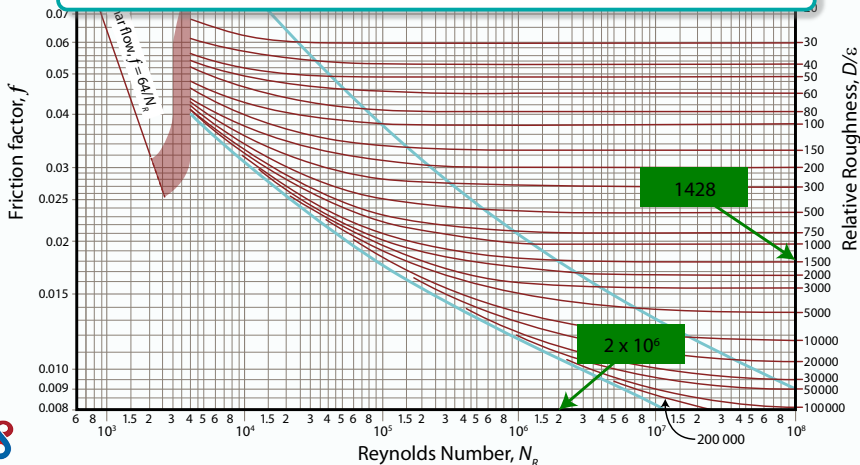
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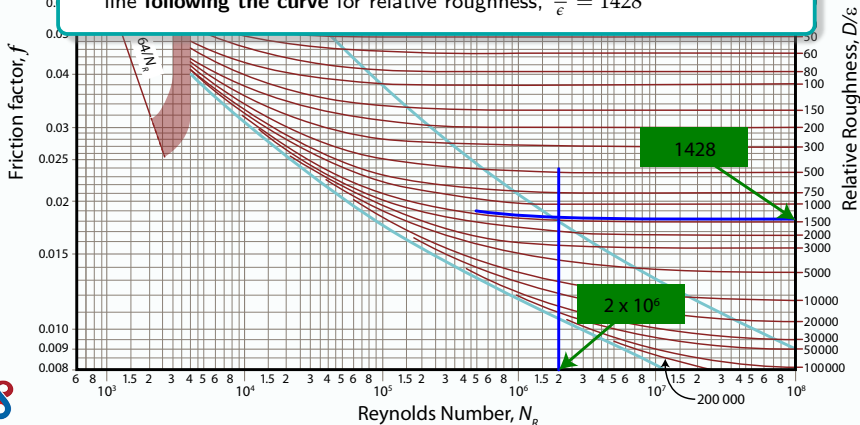
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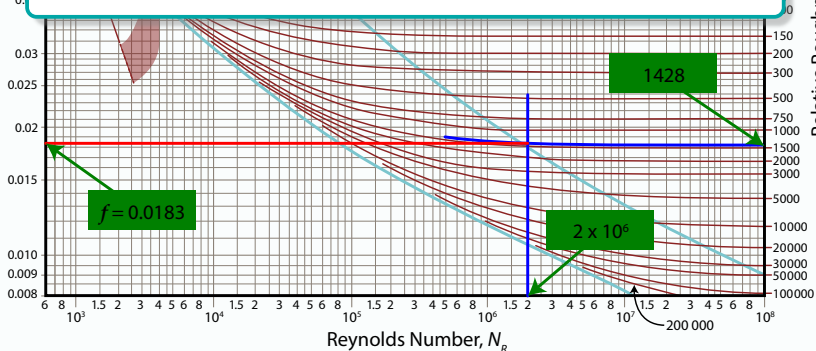


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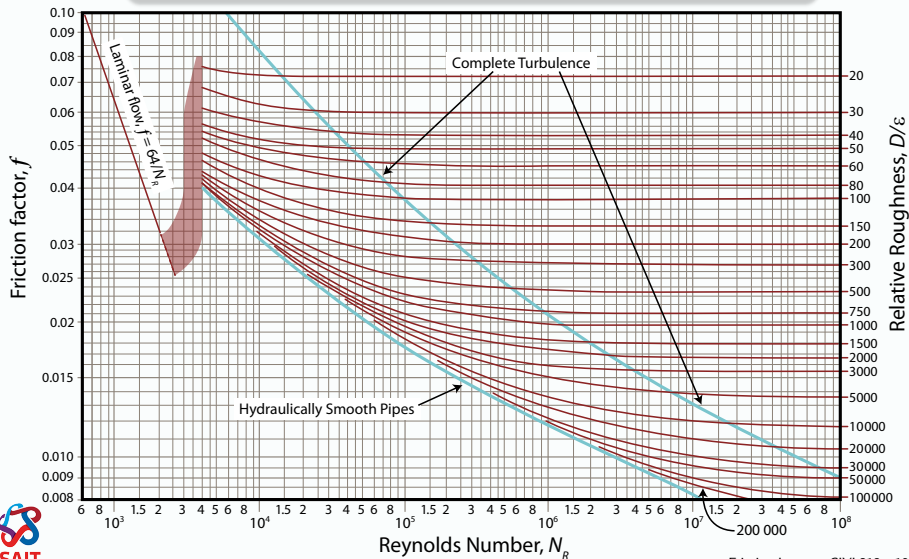
Friction factor, f



Relative Roughness, D/ϵ

Example 3

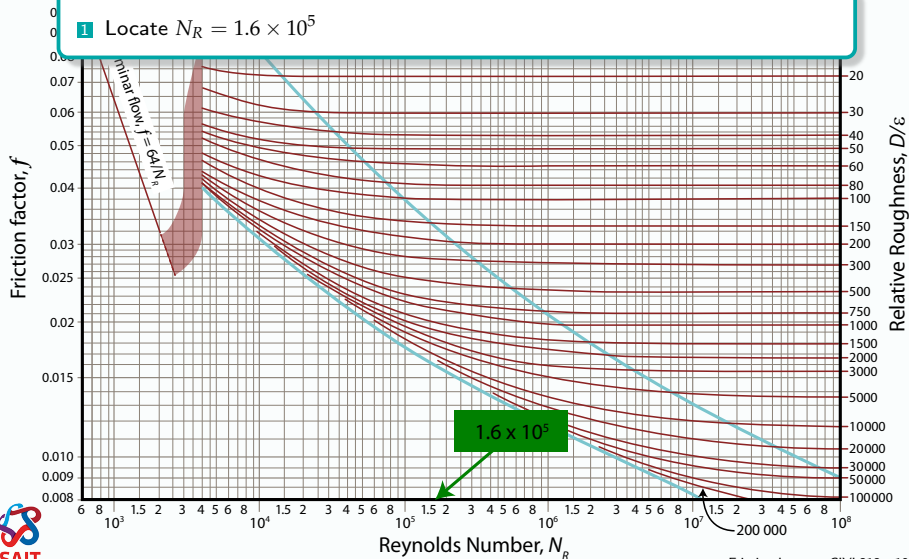
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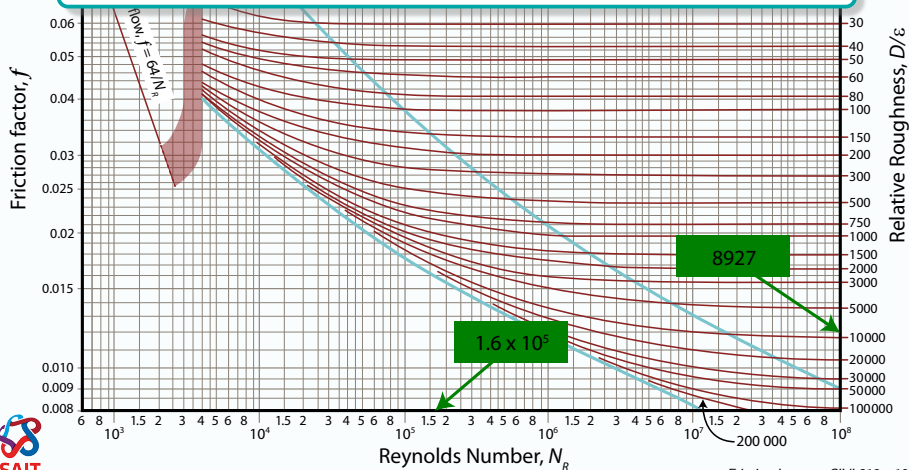


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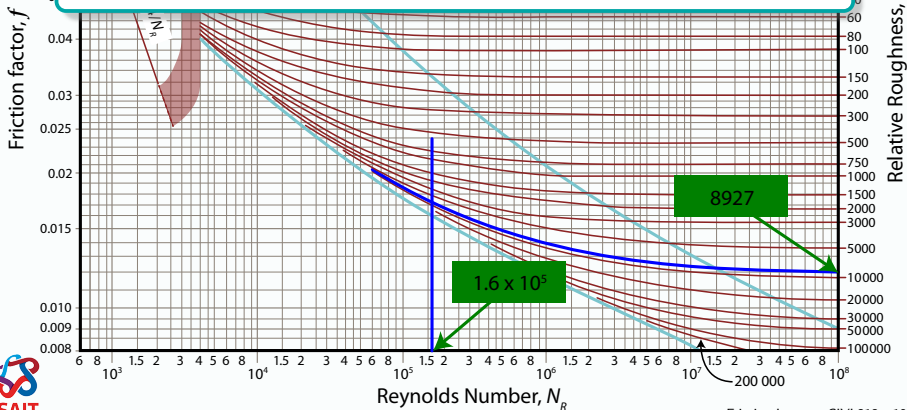
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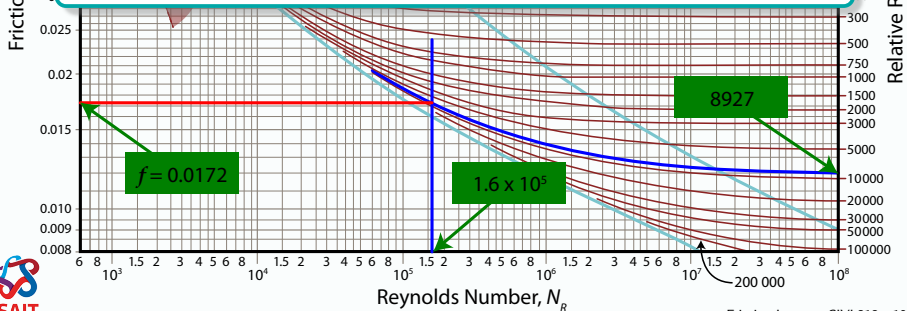


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- 4 From this intersection point, draw a horizontal line leftwards to the left hand scale to read the friction factor,

$$f = 0.0172$$



Example 4

A 75 m section of wooden flume is replaced with 54-in high density polyethylene (HDPE) pipe with inside diameter of 1.37 m. The pipe is smooth and transports $190 \times 10^3 \text{ m}^3/\text{day}$. Determine the headloss due to friction in the pipe, assuming an average temperature of 10°C .

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► Flow velocity:

$$v = \frac{Q}{A} = \frac{190 \times 10^3 / 24 / 60 / 60 \text{ m}^3/\text{s}}{\pi(1.37 \text{ m})^2 / 4} = 1.4918 \text{ m/s}$$

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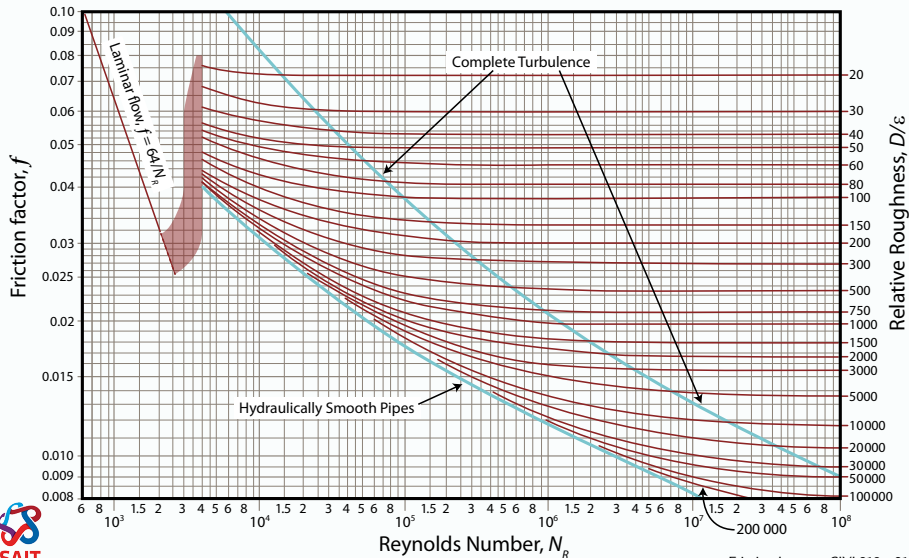
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- Relative roughness: Smooth pipe
- Find the friction factor from the Moody diagram...

Cont'd...

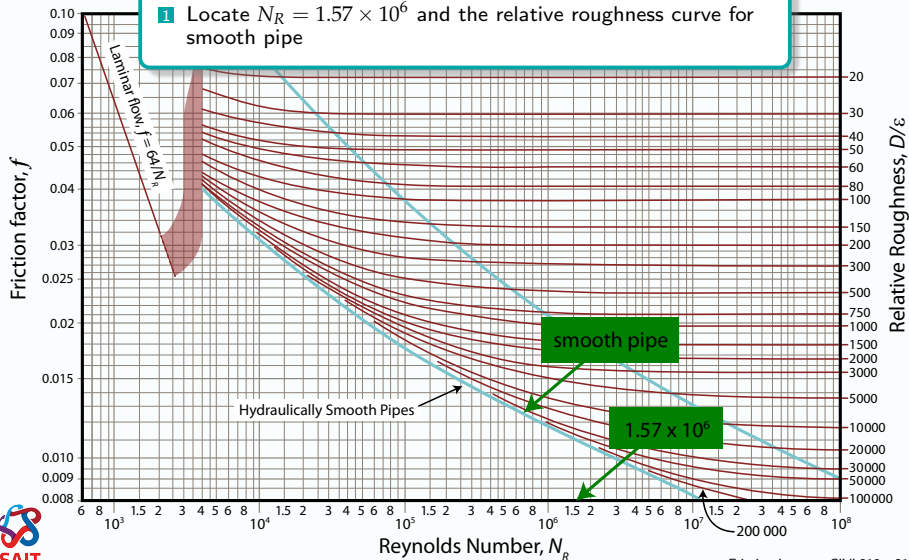
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Cont'd...

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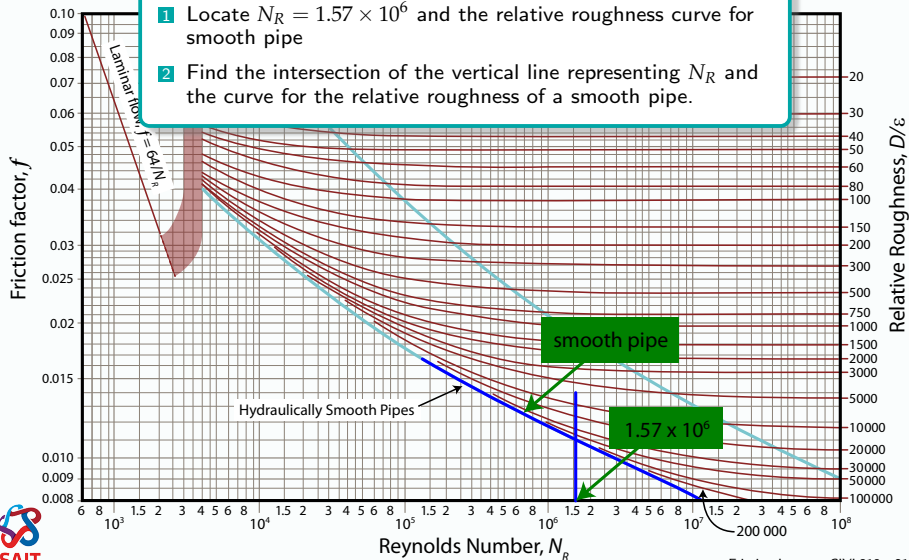
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Cont'd. . .

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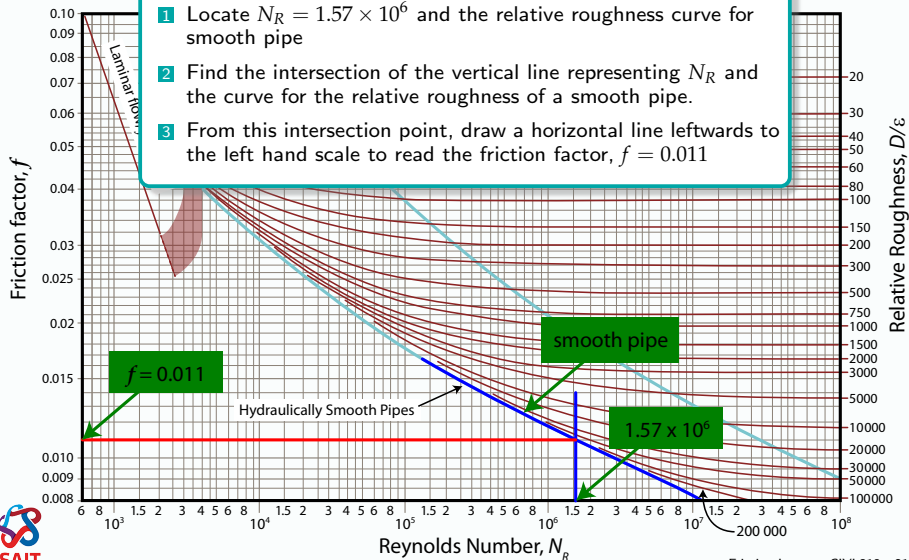
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Cont'd...

$N_R = 1.5721 \times 10^6$, smooth pipe

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- 2 Find the intersection of the vertical line representing N_R and the curve for the relative roughness of a smooth pipe.
- 3 From this intersection point, draw a horizontal line leftwards to the left hand scale to read the friction factor, $f = 0.011$



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- 2 Find the intersection of the vertical line representing N_R and the curve for the relative roughness of a smooth pipe.
- 3 From this intersection point, draw a horizontal line leftwards to the left hand scale to read the friction factor, $f = 0.011$
- 4 Now, determine the head loss due to friction. . .
- $$h_L = f \times \frac{L}{D} \times \frac{v^2}{2g}$$
- $$= 0.011 \times \frac{75 \text{ m}}{1.37 \text{ m}} \times \frac{(1.4918)^2}{2 \times 9.81} \text{ m}$$
- $$= 0.068305 \text{ m}$$

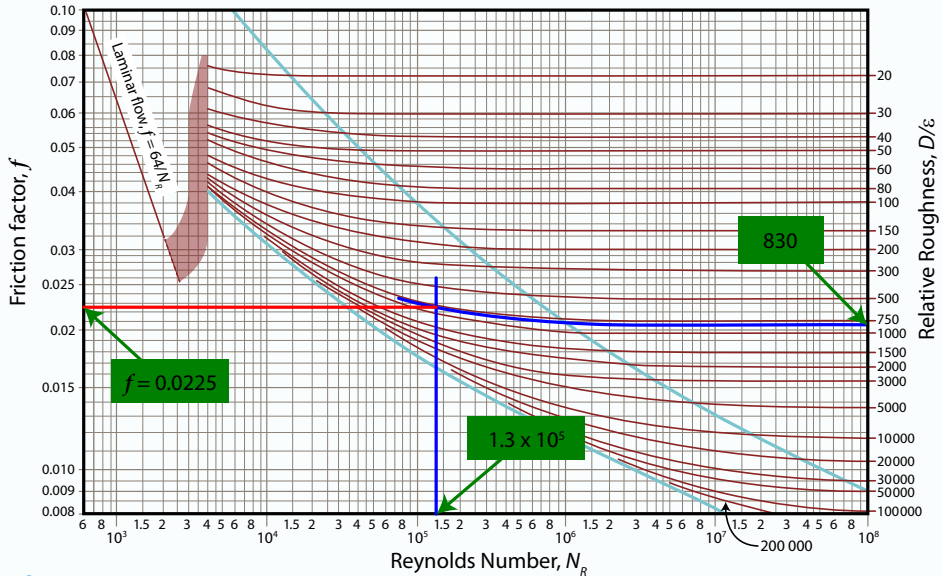
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Exercise 2

Ethyl alcohol at 25°C flows through $1\frac{1}{2}$ -in Schedule 80 steel pipe at 5 L/s.

Determine the pressure drop, due to friction losses, in a 125 m section of pipe.

What result do you get for f from the Moody Diagram...



Swamee-Jain Formula for Turbulent Flow

$$f = \frac{0.25}{\left[\log \left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_R^{0.9}} \right) \right]^2}$$

The Swamee-Jain formula is quite accurate, yielding values for f that are within $\pm 1\%$ of the Moody Diagram value.

The Swamee-Jain approximates the Moody Diagram. Results for f are no more accurate than those provided by the Moody Diagram.

Exercise 3

Ethyl alcohol at 25°C flows through 3-in Schedule 80 steel pipe at 5 L/s.

Determine the pressure drop, due to friction losses, in a 125 m section of pipe.

Two Pipes Compared:

	$1\frac{1}{2}$ -in	3-in	Ratio:
Diameter	38.1 mm	73.7 mm	≈ 2
Velocity	4.3851 m/s	1.1720 m/s	$\approx \frac{1}{4}$
Velocity Head	0.98031 m	0.070015 m	$\approx \frac{1}{14}$ $\left(\frac{1}{16} \text{ if double}\right)$
Head Loss	72.365 m	2.6125 m	$\approx \frac{1}{28}$

By (approximately) doubling the diameter, the velocity is reduced to one-quarter which, in turn, reduces the velocity head to 1/14th and losses to 1/28th.

Example 5

A horizontal 12-in Schedule 80 steel pipe transports oil ($sg = 0.85$, $\eta = 3.0 \times 10^{-3} \text{ Pa} \cdot \text{s}$) at 185 L/s. The pipe has pumping stations spaced at 6.0 km intervals.

Determine the power required by each pump to maintain the same pressure at each pump outlet if all losses are due to friction.

