Module 8: Hazen Williams Equation and Equivalent Pipes (CIVL 318)

Hazen-Williams Equations

$$Q = \frac{C\,D^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}, \qquad h_L = L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852}, \qquad D = \left(\frac{279000\,Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802}$$

Equivalent-Length Ratios for Fittings

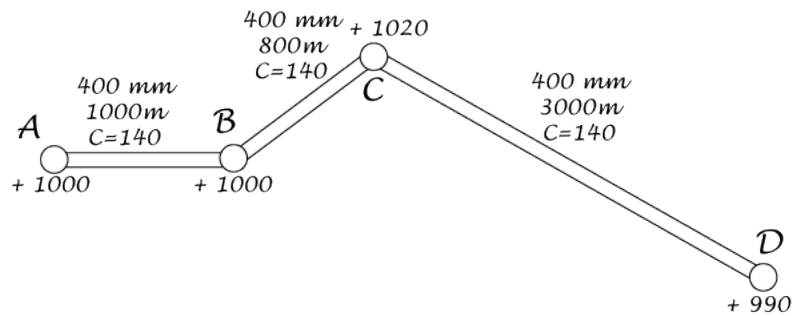
Type	L_e/D
Globe valve — fully open	340
Angle valve — fully open	150
Gate valve — fully open	8
— 3/4 open	35
— 1/2 open	160
— 1/4 open	900
Check valve — swing type	100
Check valve — ball type	150
Butterfly valve — fully open — 2-8"	45
— 10-14"	35
— 16-24"	25
Foot valve — poppet disc type	420
Foot valve — hinged disc type	75
90° standard elbow	30
90° long radius elbow	20
90° street elbow	50
45° standard elbow	16
45° street elbow	26
Close return bend	50
Standard tee — flow through run	20
Standard tee — flow through branch	60
Gradual enlargement — 15° cone angle	8
Gradual enlargement — 20° cone angle	15
Gradual enlargement — 30° cone angle	23
Gradual reduction — 15° to 40° cone angle	2
Pipe entrance — inward projecting	50
Pipe entrance — square	25
Pipe entrance — rounded	10
Venturi meter	100

Example 1

For the pipeline shown, calculate the pressure at B, given that the pressure at A is 700 kPa.

The pipes are cement-lined Hyprescon with a diameter of 400 mm and a roughness coefficient of $C = 140$. Flow through the system is 200 L/s.

Elevations are as indicated.



Solution:

First, apply the Hazen-Williams:

$$
\begin{aligned}
h_{L_{AB}} &= L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852} \\
&= 1000\left(\frac{279000 \times 200}{140 \times 400^{2.63}}\right)^{1.852} \\
&= 4.9903\,\mathrm{m}
\end{aligned}
$$

Now, apply the GEE:

$$
\frac{P_A}{\gamma} + \cancel{z_A} + \cancel{\frac{v_A^2}{2g}} - h_L = \frac{P_B}{\gamma} + \cancel{z_B} + \cancel{\frac{v_B^2}{2g}}
$$

$$
\frac{700}{9.81} - 4.9903 = \frac{P_B}{9.81}
$$

$$
P_B = 651.05\,\mathrm{kPa}
$$

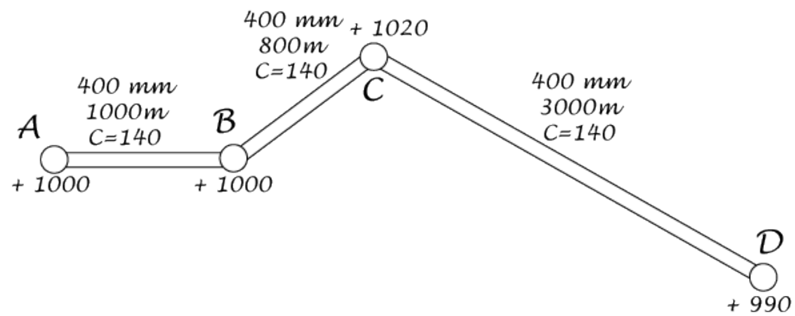
$$
\boldsymbol{P_B = 651\,\mathrm{kPa}}
$$

Exercise 1

For the pipeline shown, calculate the pressure at C and D, given that the pressure at A is 700 kPa.

The pipes are cement-lined Hyprescon with a diameter of 400 mm and a roughness coefficient of $C = 140$. Flow through the system is 200 L/s.

Elevations are as indicated.



Solution:

First, apply the Hazen-Williams:

$$
\begin{aligned}
h_{L_{BC}} &= L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852} \\
&= 800\left(\frac{279000 \times 200}{140 \times 400^{2.63}}\right)^{1.852} \\
&= 3.9922\,\text{m}
\end{aligned}
$$

$$
\begin{aligned}
h_{L_{CD}} &= L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852} \\
&= 3000\left(\frac{279000 \times 200}{140 \times 400^{2.63}}\right)^{1.852} \\
&= 14.971\,\text{m}
\end{aligned}
$$

Now, apply the GEE:

$$
\begin{aligned}
\frac{P_B}{\gamma} + z_B + \cancel{\frac{v_B^2}{2g}} - h_L &= \frac{P_C}{\gamma} + z_C + \cancel{\frac{v_C^2}{2g}} \\
\frac{651.05}{9.81} - 3.9922 &= \frac{P_C}{9.81} + 20 \\
P_C &= 415.69\,\text{kPa} \\
\boldsymbol{P_C} &= \mathbf{416\,kPa}
\end{aligned}
$$

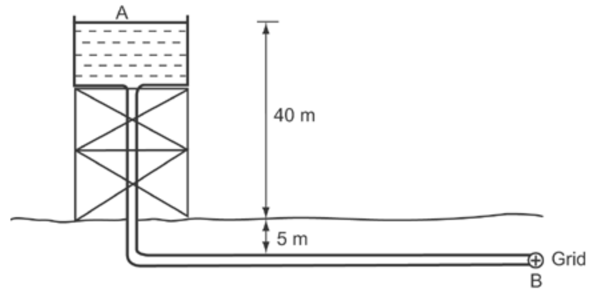
$$
\begin{aligned}
\frac{P_C}{\gamma} + z_C + \cancel{\frac{v_C^2}{2g}} - h_L &= \frac{P_D}{\gamma} + z_D + \cancel{\frac{v_D^2}{2g}} \\
\frac{415.69}{9.81} + 30 - 14.971 &= \frac{P_D}{9.81} \\
P_D &= 563.12\,\text{kPa} \\
\boldsymbol{P_D} &= \mathbf{563\,kPa}
\end{aligned}
$$

Example 2

Water flows from a storage tank through a welded steel pipe that is 1200 m long and 350 mm in diameter, entering a distribution grid at point 'B'. Assume C=100. Determine:

(1) The pressure at 'B' when the flow is 150 L/s
(2) The maximum flow rate into the grid when the minimum allowable pressure at 'B' is 400 kPa.

Minor losses are negligible compared to friction losses.



Solution (1):

$$
\begin{aligned}
h_L &= L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852} \\
&= 1200\left(\frac{279000\times 150}{100\times 350^{2.63}}\right)^{1.852} \\
&= 12.561\,\text{m} \\
v &= \frac{Q}{A} \\
&= \frac{0.150}{\pi(0.350)^2/4} \\
&= 1.5591\,\text{m/s} \\
\frac{v^2}{2g} &= 0.12389\,\text{m}
\end{aligned}
$$

GEE:

$$
\begin{aligned}
\cancelto{0}{\frac{P_A}{\gamma}} + z_A + \cancelto{0}{\frac{v_A^2}{2g}} - h_L &= \frac{P_B}{\gamma} + \cancelto{0}{z_B} + \frac{v_B^2}{2g} \\
45 - 12.561 &= \frac{P_B}{9.81} + 0.12389 \\
P_B &= 317.01\,\text{kPa} \\
\boldsymbol{P_B} &= \mathbf{317\,kPa}
\end{aligned}
$$

Notice that if we recalculated the pressure at B omitting the velocity head, then $P_B = 318.2\,\text{kPa}$ (which is not very different from including it).

Solution (2):

What flow/headloss will give a pressure of 400 kPa at B?

$$
\begin{aligned}
\cancelto{0}{\frac{P_A}{\gamma}} + z_A + \cancelto{0}{\frac{v_A^2}{2g}} - h_L &= \frac{P_B}{\gamma} + \cancelto{0}{z_B} + \frac{v_B^2}{2g} \\
45 - h_L &= \frac{400}{9.81} + \frac{v_B^2}{2g}
\end{aligned}
$$

One equation and two unknowns! We could solve it iteratively, guessing at a flow and seeing what P_B is for this flow, then trying another flow until we converge on a pressure of 400 kPa at B.

But the velocity head had an effect of about 0.3% in part (1); it will be less here as we need less velocity/headloss to keep the pressure higher. So, in problems of this type, we **simply ignore the velocity head term...**

$$
\begin{aligned}
45 - h_L &= \frac{400}{9.81} + \cancelto{0}{\frac{v_B^2}{2g}} \\
h_L &= 4.2253\,\text{m}
\end{aligned}
$$

What flow will give this headloss?

$$
\begin{aligned}
Q &= \frac{C D^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000} \\
&= \frac{100\times 350^{2.63}\left(\frac{4.2253}{1200}\right)^{0.54}}{279000} \\
&= 83.272\,\text{L/s} \\
\boldsymbol{Q} &= \mathbf{83.3\,L/s}
\end{aligned}
$$

Let's look at the value of the velocity head we discarded...

$$
\begin{aligned}
v &= \frac{Q}{A} \\
&= \frac{0.083272}{\pi(0.350)^2/4} \\
&= 0.86551\,\text{m/s}
\end{aligned}
$$

$$
\frac{v^2}{2g} = 0.038181\,\text{m}
$$

The velocity head is small enough that we can disregard it. Any error from not omitting the headloss is negligible compared with error in estimating the C-value.

Exercise 2

Water flows from one reservoir down to another, through a 500 mm diameter pipe that is 2000 m in length. The difference in elevation between the surfaces of the two reservoirs is 30 m.

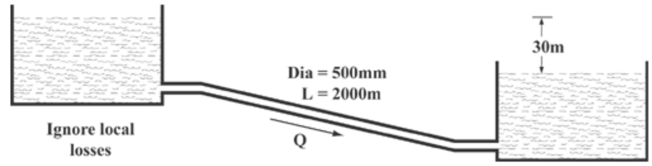
Determine:

(1) The flow with high density polyethylene pipe (HDPE) with $C = 140$

(2) The flow with welded steel with $C = 100$

(3) The diameter of HDPE pipe required for a flow of 1200 L/s

Disregard minor losses.



Solution (1): For HDPE.

At the surfaces of both reservoirs, pressure and velocity head are 0 so the GEE reduces to $\mathbf{30 - h_L = 0}$

$$
\begin{aligned}
Q &= \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000} \\
&= \frac{140(500)^{2.63}\left(\frac{30}{2000}\right)^{0.54}}{279000} \\
&= 651.48\,\mathrm{L/s} \\
\boldsymbol{Q} &= \mathbf{651\,L/s}
\end{aligned}
$$

Solution (2): For welded steel,

$$
\begin{aligned}
Q &= \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000} \\
&= \frac{100(500)^{2.63}\left(\frac{30}{2000}\right)^{0.54}}{279000} \\
&= 465.35\,\mathrm{L/s} \\
\boldsymbol{Q} &= \mathbf{465\,L/s}
\end{aligned}
$$

Solution (3): Diameter for a flow of 1200 L/s with HDPE,

$$
\begin{aligned}
D &= \left(\frac{279000\,Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802} \\
&= \left(\frac{279000 \times 1200}{140\left(\frac{30}{2000}\right)^{0.54}}\right)^{0.3802} \\
&= 630.42\,\mathrm{mm} \\
\boldsymbol{D} &= \mathbf{630\,mm}
\end{aligned}
$$

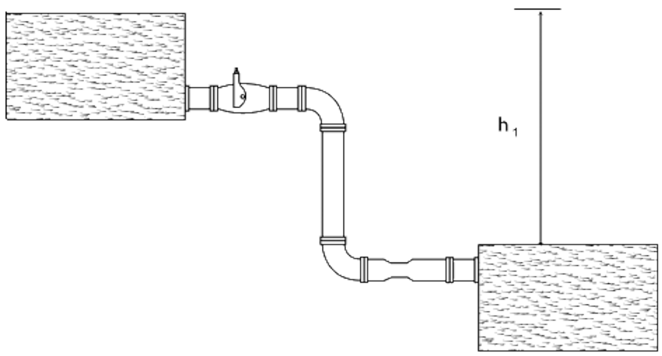
Example 3

In a water treatment plant, water flows from a filter down to a clear well through the pipe system shown. The pipe is welded steel with a diameter of 300 mm and roughness coefficient $C = 130$. The total length of pipe is 50 m. Elevation difference h_1 between the tanks is 5 m.

Equivalent length ratios, L_e/D, are:

Entrance and exit losses:	50	Butterfly valve:	35
Large radius elbows:	25	Venturi meter:	100

Determine the flow through the system.



Solution:

Effective length of the pipe: (length and diameter in metres!)

$$\begin{aligned} L_{\text{eff}} &= \text{Actual pipe length} + D\left(\frac{L_e}{D}\right) \\ &= 50 + 0.3(50 + 35 + 25 + 25 + 100 + 50) \\ &= 50 + 85.50 \\ &= 135.50\,\text{m} \end{aligned}$$

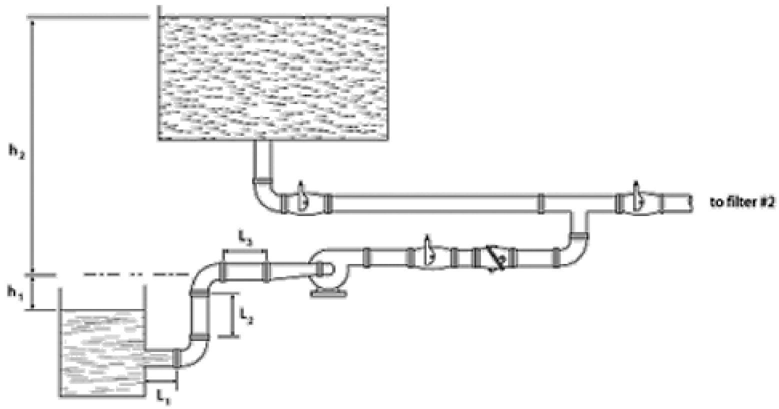
As earlier, headloss between the two surfaces is just the elevation difference:

$$h_L = 5\,\text{m}$$

Find the flow:

$$\begin{aligned} Q &= \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000} \\ &= \frac{130(300)^{2.63}\left(\frac{5}{135.50}\right)^{0.54}}{279000} \\ &= 256.66\,\text{L/s} \\ \boldsymbol{Q} &= \mathbf{257\,L/s} \end{aligned}$$

Example 4



In a water treatment plant, backwash water is pumped from the clear well through the pipe system shown to the filter. The required backwash flow is 10 L/s per square meter of filter area (the filter dimensions are 10 m by 15 m. The inlet pipe is made of welded steel $(C = 130)$, has a diameter of 1000 mm and a total length $(L_1 + L_2 + L_3)$ of 10 m. The outlet pipe, from the pump to the filter, is also welded steel, has a diameter of 700 mm and a length of 70 m.

The two elevation differences are $h_1 = 2$ m and $h_2 = 10$ m.

Equivalent length ratios, L_e/D, are:

Entrance:	10	Elbow (inlet):	25
Eccentric Reducer:	2	Butterfly Valve:	40
Check Valve:	120	Elbow (outlet):	35
Tee Connection:	60		

Neglect exit losses into the filter.

Determine:

(1) The head losses on the inlet side (clear well to pump)

(2) The head losses on the outlet side (pump to filter)

Solution:

Q required for backwash in the filter:

$$Q = 10\,\text{m} \times 15\,\text{m} \times 0.01\,\text{m}^3/\text{s} = 1.5000\,\text{m}^3/\text{s}$$

(1)

$$\begin{aligned}
L_{eff} &= 10 + 1(10 + 25 + 25 + 2) = 72.000\,\text{m} \\
h_L &= 72\left(\frac{279000 \times 1500}{130(1000)^{2.63}}\right)^{1.852} \\
&= 0.19834\,\text{m} \\
\boldsymbol{h_{L_{(in)}}} &= \mathbf{0.1983\,m}
\end{aligned}$$

(2)

$$\begin{aligned}
L_{eff} &= 70 + 0.7(40 + 120 + 35 + 60 + 40 + 35) \\
&= 301.00\,\text{m} \\
h_L &= 301\left(\frac{279000 \times 1500}{130(700)^{2.63}}\right)^{1.852} \\
&= 4.7112\,\text{m} \\
\boldsymbol{h_{L_{(out)}}} &= \mathbf{4.71\,m}
\end{aligned}$$

Exercise 3

This exercise is a continuation of the previous example. Determine:

(3) The head added by the pump

(4) The pressure at the pump outlet

Solution:

(3)

Apply the GEE between the surface of the clear well (W) and the surface of the filter (F):

$$\begin{aligned}
\frac{P_W}{\gamma} + z_W + \frac{v_W^2}{2g} + h_A - h_L &= \frac{P_F}{\gamma} + z_F + \frac{v_F^2}{2g} \\
h_A - (0.19834 + 4.7112) &= 12 \\
h_A &= 16.910\,\text{m} \\
\boldsymbol{h_A} &= \mathbf{16.91\,m}
\end{aligned}$$

(4)

Apply the GEE between the pump (P) and the surface of the filter (F):

$$\begin{aligned}
\frac{P_P}{\gamma} + z_P + \frac{v_P^2}{2g} - h_L &= \frac{P_F}{\gamma} + z_F + \frac{v_F^2}{2g} \\
\frac{P_P}{9.81} + 0 + 0.77430 - 4.7112 &= 0 + 10 + 0 \\
\Rightarrow P_P &= 136.72\,\text{kPa} \\
\boldsymbol{P_P} &= \mathbf{136.7\,kPa}
\end{aligned}$$

Example 5

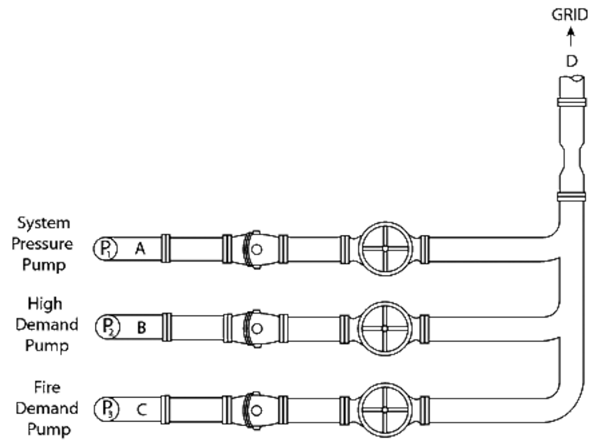
The pumps and piping system are used to supply a municipal grid. Pump P_1 runs continuously and maintains the basic pressure in the distribution grid beyond point D. There is no flow from pumps P_2 and P_3. (Pump P_2 is, in addition to P_1, used during periods of high demand and all pumps are used during fire flow demands.)

The elevations are the same at the pump and the discharge point D. The outlet pipe, from the pump to point D, is welded steel ($C = 130$) with a diameter of 200 mm and a total length between fittings of 10 m.

The minimum pressure required at D is 500 kPa for a design flow of 150 L/s.

Equivalent length ratios, L_e/D, are:

Check Valve:	120	Gate Valve:	15
Tee Connection:	60	Venturi Meter:	100



Determine:

(1) the head losses between A and D
(2) the pressure at A required for the required pressure and flow at D

Solution:

(1)

$$L_{\text{eff}} = 10 + 0.2(120 + 15 + 60 + 100)$$
$$= 69.0\,\text{m}$$

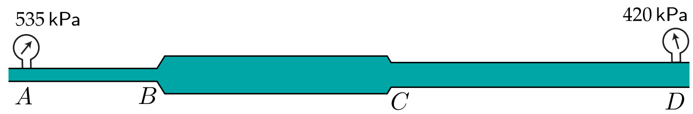
$$h_L = 69.0\left(\frac{279000 \times 150}{130(200)^{2.63}}\right)^{1.852}$$
$$= 6.7833\,\text{m}$$
$$\boldsymbol{h_L = 6.78\,\text{m}}$$

(2)

$$\frac{P_A}{\gamma} + \cancel{z_A} + \cancel{\frac{v_A^2}{2g}} - h_L = \frac{P_D}{\gamma} + \cancel{z_D} + \cancel{\frac{v_D^2}{2g}}$$
$$\frac{P_A}{9.81} - 6.7833 = \frac{500}{9.81}$$
$$\Rightarrow P_A = 566.54$$

$$\boldsymbol{P_A = 567\,\text{kPa}}$$

Example 6



$L = 600\,\text{m}$ $D = 150\,\text{mm}$ $C = 150$

$L = 950\,\text{m}$ $D = 325\,\text{mm}$ $C = 120$

$L = 1200\,\text{m}$ $D = 250\,\text{mm}$ $C = 150$

Determine Q, the volume flow rate from A to D, through the system shown. Ignore minor losses and assume that A and D are at the same elevation.

Solution:

First, determine the head loss in each of the three pipes, in terms of Q:

$$\begin{aligned} h_{L_{AB}} &= L\left(\frac{279000Q}{CD^{2.63}}\right)^{1.852} \\ &= L\left(\frac{279000}{CD^{2.63}}\right)^{1.852} \cdot Q^{1.852} \\ &= 600\left(\frac{279000}{150(150)^{2.63}}\right)^{1.852} \cdot Q^{1.852} \\ &= 0.017142\, Q^{1.852} \end{aligned}$$

$$\begin{aligned} h_{L_{BC}} &= 950\left(\frac{279000}{120(325)^{2.63}}\right)^{1.852} \cdot Q^{1.852} \\ &= 0.00094964\, Q^{1.852} \end{aligned}$$

$$\begin{aligned} h_{L_{CD}} &= 1200\left(\frac{279000}{150(250)^{2.63}}\right)^{1.852} \cdot Q^{1.852} \\ &= 0.0028479\, Q^{1.852} \end{aligned}$$

Sum these headlosses to get the headloss between A and D:

$$\begin{aligned} h_{L_{AD}} &= h_{L_{AB}} + h_{L_{BC}} + h_{L_{CD}} \\ &= (0.017142 + 0.00094964 + 0.0028479)\, Q^{1.852} \\ &= 0.020940 Q^{1.852}\ \text{m} \end{aligned}$$

Use the GEE to approximate a numerical value for $h_{L_{AD}}$:

$$\frac{P_A}{\gamma} + \cancel{z_A} + \frac{v_A^2}{2g} - h_L = \frac{P_D}{\gamma} + \cancel{z_D} + \frac{v_D^2}{2g}$$

$$\Rightarrow h_L = \frac{P_A - P_D}{2g} + \frac{v_A^2 - v_D^2}{2g}$$

One equation and two unknowns (h_L and Q, since v_A and v_B can be expressed in terms of the single variable Q). Here, we have the difference between the two velocity heads which is smaller than the velocity head at A. As before, we can ignore it (for now!)

$$\begin{aligned} h_L &\approx \frac{P_A - P_D}{2g} \\ &= \frac{535 - 420}{19.62} \\ &= 5.8614\,\text{m} \end{aligned}$$

Now, find Q:

$$\begin{aligned} h_{L_{AD}} &= 5.8614 = 0.020940 Q^{1.852} \\ \Rightarrow Q &= \left(\frac{5.1852}{0.020940}\right)^{\frac{1}{1.852}} \\ &= 20.955 \\ \boldsymbol{Q} &= \mathbf{21.0\,L/s} \end{aligned}$$

How large was the term we discarded?

$$\begin{aligned} \frac{v_A^2 - v_D^2}{2g} &= \frac{\left(\frac{20.955/1000}{\pi(0.150)^2/4}\right)^2 - \left(\frac{20.955/1000}{\pi(0.250)^2/4}\right)^2}{19.62} \\ &= 0.062381\,\text{m} \end{aligned}$$

This is approximately 0.15% (1/667 th) of the pressure head at D.

Example 7

a) Determine the diameter of a pipe with length $L = 1000\,\text{m}$ and resistance coefficient $C = 100$ that is equivalent to $785\,\text{m}$ of new Schedule 40 12-in steel pipe ($D = 303.2\,\text{mm}, C = 130$).

b) Verify that this equivalent pipe has the same headloss as the 12-in steel pipe for two arbitrary flows (choose a couple of flows at random, different from the flow used in part a).

a) Assume a flow of $100\,\text{L/s}$ through the 12-in steel pipe. Calculate the headloss for this flow:

$$\begin{aligned} h_L &= 785\left(\frac{279000(100)}{130(303.2)^{2.63}}\right)^{1.852} \\ &= 4.7995\,\text{m} \end{aligned}$$

Now, find the diameter of the equivalent pipe that has a headloss of $4.7995\,\text{m}$ for a flow of $100\,\text{L/s}$:

$$\begin{aligned} D &= \left(\frac{279000(100)}{100\left(\frac{4.7995}{1000}\right)^{0.54}}\right)^{0.3802} \\ &= 351.96\,\text{mm} \end{aligned}$$

$$\boldsymbol{D = 352\,\text{mm}}$$

b) $128\,\text{L/s}$ and $42\,\text{L/s}$ are two flows chosen at random. Compare the headloss in each pipe for both of these flows

First, using $Q = 128\,\text{L/s}$:

$$\begin{aligned} h_{L(\text{12-in})} &= 785\left(\frac{279000(128)}{130(303.2)^{2.63}}\right)^{1.852} \\ &= 7.5814\,\text{m} \end{aligned}$$

$$\begin{aligned} h_{L(\text{equiv})} &= 1000\left(\frac{279000(128)}{100(351.96^{2.63}}\right)^{1.852} \\ &= 7.5937\,\text{m} \end{aligned}$$

These results are the same except for rounding errors. The errors are noticeable because, in the derivation of the Hazen-Williams solution for diameter, 0.3802 was chosen at the inverse of 2.63 which is not exact. If the exponent $1/2.63$ is used instead of 0.3802, the headloss values are closer (7.5814 and 7.5780).

For the same reasons, the results would be even closer if headloss were calculated with $1/0.54$ instead of 1.852; these values are only approximately equal.

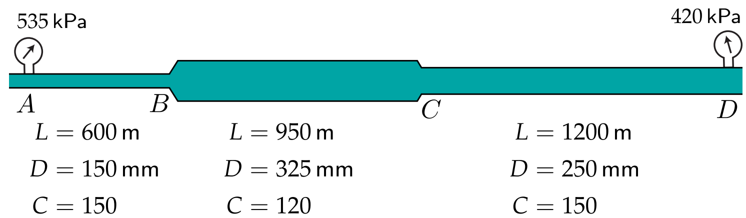
Now, using $Q = 42\,\text{L/s}$:

$$\begin{aligned} h_{L(\text{12-in})} &= 785\left(\frac{279000(42)}{130(303.2)^{2.63}}\right)^{1.852} \\ &= 0.96262\,\text{m} \end{aligned}$$

$$\begin{aligned} h_{L(\text{equiv})} &= 1000\left(\frac{279000(42)}{100(351.96^{2.63}}\right)^{1.852} \\ &= 0.96418\,\text{m} \end{aligned}$$

Again, these are not exactly the same. You may need to 'cheat' with the last digit in a Qwizm assignment but the result shouldn't be out by more than one digit.

Example 8



Use the equivalent pipe technique to determine Q, the volume flow rate from A to D, through the system shown. Ignore minor losses and assume that A and D are at the same elevation.

Solution:

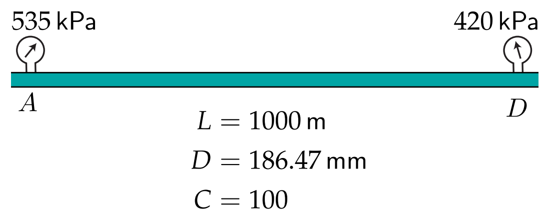
Assume a flow of $Q = 100\,\text{L/s}$ and determine the headloss between A and D:

$$\begin{aligned}
h_{L_{AB}} &= 600\left(\frac{279000 \times 100}{150(150)^{2.63}}\right)^{1.852} \\
&= 86.710\,\text{m} \\
h_{L_{BC}} &= 950\left(\frac{279000 \times 100}{120(325)^{2.63}}\right)^{1.852} \\
&= 4.8035\,\text{m} \\
h_{L_{CD}} &= 1200\left(\frac{279000 \times 100}{150(250)^{2.63}}\right)^{1.852} \\
&= 14.406\,\text{m} \\
h_{L_{AD}} &= 86.710 + 4.8035 + 14.406 \\
&= 105.92\,\text{m}
\end{aligned}$$

Determine the diameter of the pipe, with length $1000\,\text{m}$ and resistance coefficient $C = 100$, that has a headloss of $105.92\,\text{m}$ for flow of $100\,\text{L/s}$.

$$\begin{aligned}
d_{AD\text{equiv}} &= \left(\frac{279000 \times 100}{100\left(\frac{105.92}{1000}\right)^{0.54}}\right)^{0.3802} \\
&= 186.47\,\text{mm}
\end{aligned}$$

Our problem has now reduced to finding the flow through the single pipe, ADequiv, shown below.



We apply the GEE to this pipe to find $h_{L_{AD\text{equiv}}}$, noting that A and D are at the same elevation and have the same diameter and, therefore, the same velocity head.

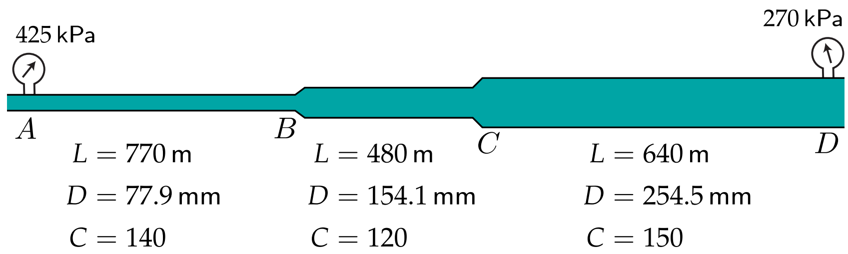
$$\begin{aligned}
\frac{535}{9.81} - h_{L_{AD\text{equiv}}} &= \frac{420}{9.81} \\
\Rightarrow h_{L_{AD\text{equiv}}} &= 5.8614\,\text{m}
\end{aligned}$$

Note that this is the same loss as we found in Example 6, so ignoring the difference in velocity heads had no numerical effect (to 5 significant digits).

Find Q:

$$\begin{aligned}
Q &= \frac{100(186.47)^{2.63}\left(\frac{5.8614}{1000}\right)^{0.54}}{279000} \\
&= 20.932 \\
\mathbf{Q} &= \mathbf{20.9}\,\text{L/s}
\end{aligned}$$

Exercise 4



Use the equivalent pipe technique to determine Q, the volume flow rate from A to D, through the system shown. Ignore minor losses and assume that A and D are at the same elevation.

Solution:

Assume a flow of $Q = 100\,\text{L/s}$ and determine the headloss between A and D:

$$\begin{aligned}
h_{L_{AB}} &= 770\left(\frac{279000\times 100}{140(77.9)^{2.63}}\right)^{1.852}\\
&= 3075.4\,\text{m}\\
h_{L_{BC}} &= 480\left(\frac{279000\times 100}{120(154.1)^{2.63}}\right)^{1.852}\\
&= 6540.3\,\text{m}\\
h_{L_{CD}} &= 640\left(\frac{279000\times 100}{150(254.5)^{2.63}}\right)^{1.852}\\
&= 7.0435\,\text{m}\\
h_{L_{AD}} &= 3075.4 + 6540.3 + 7.0435\\
&= 9622.7\,\text{m}
\end{aligned}$$

Determine the diameter of the pipe, with length $1000\,\text{m}$ and resistance coefficient $C = 100$, that has a headloss of $9622.7\,\text{m}$ for flow of $100\,\text{L/s}$.

$$\begin{aligned}
d_{AD\text{equiv}} &= \left(\frac{279000\times 100}{100\left(\frac{9622.7}{1000}\right)^{0.54}}\right)^{0.3802}\\
&= 73.882\,\text{mm}
\end{aligned}$$

Our problem has now reduced to finding the flow through the single pipe, ADequiv

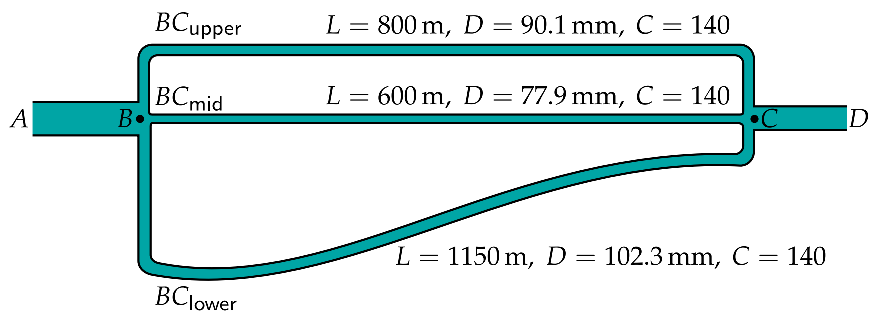
We apply the GEE to this pipe to find $h_{L_{AD\text{equiv}}}$, noting that A and D are at the same elevation and have the same diameter and, therefore, the same velocity head.

$$\begin{aligned}
\frac{425}{9.81} - h_{L_{AD\text{equiv}}} &= \frac{270}{9.81}\\
\Rightarrow h_{L_{AD\text{equiv}}} &= 7.9001\,\text{m}
\end{aligned}$$

Find Q:

$$\begin{aligned}
Q &= \frac{100(73.882)^{2.63}\left(\frac{7.9001}{1000}\right)^{0.54}}{279000}\\
&= 2.1561\\
\boldsymbol{Q} &= \mathbf{2.16}\,\text{L/s}
\end{aligned}$$

Example 9



Given a flow of 18 L/s and ignoring minor losses:

a) Determine the volume flow rate through each of the parallel pipes between B and C.
b) Determine the headloss due to friction between B and C.

Solution:

$$h_{L_{BC_{\text{upper}}}} = h_{L_{BC_{\text{mid}}}} = h_{L_{BC_{\text{lower}}}}$$

$$800\left(\frac{\cancel{279000}\,Q_{\text{upper}}}{\cancel{140}(90.1)^{2.63}}\right)^{1.852} = 600\left(\frac{\cancel{279000}\,Q_{\text{mid}}}{\cancel{140}(77.9)^{2.63}}\right)^{1.852} = 1150\left(\frac{\cancel{279000}\,Q_{\text{lower}}}{\cancel{140}(102.3)^{2.63}}\right)^{1.852}$$

$$80\cancel{0}\left(\frac{Q_{\text{upper}}}{(90.1)^{2.63}}\right)^{1.852} = 60\cancel{0}\left(\frac{Q_{\text{mid}}}{(77.9)^{2.63}}\right)^{1.852} = 115\cancel{0}\left(\frac{Q_{\text{lower}}}{(102.3)^{2.63}}\right)^{1.852}$$

Raise each term to the power $1/1.852$:

$$80^{\frac{1}{1.852}} \cdot \frac{Q_{\text{upper}}}{(90.1)^{2.63}} = 60^{\frac{1}{1.852}} \cdot \frac{Q_{\text{mid}}}{(77.9)^{2.63}} = 115^{\frac{1}{1.852}} \cdot \frac{Q_{\text{lower}}}{(102.3)^{2.63}}$$

$$\frac{Q_{\text{upper}}}{12982} = \frac{Q_{\text{mid}}}{10342} = \frac{Q_{\text{lower}}}{14903}$$

This has established a relationship between the flows in each of the three parallel pipes.

$$Q_{\text{mid}} = \frac{10342}{12982}\, Q_{\text{upper}} = 0.79664\, Q_{\text{upper}}$$

$$Q_{\text{lower}} = \frac{14903}{12982}\, Q_{\text{upper}} = 1.1480\, Q_{\text{upper}}$$

Find the proportion of the flow through the upper pipe and then the flows through each pipe:

$$\begin{aligned}\frac{Q_{\text{upper}}}{Q_{\text{upper}} + Q_{\text{mid}} + Q_{\text{lower}}} &= \frac{\cancel{Q_{\text{upper}}}}{(1 + 0.79664 + 1.1480)\cancel{Q_{\text{upper}}}} \\ &= .33960\end{aligned}$$

$$\begin{aligned}Q_{\text{upper}} &= 0.33960 \times 18 = 6.1128 \\ Q_{\text{mid}} &= 0.79664\, Q_{\text{upper}} = 4.8697 \\ Q_{\text{lower}} &= 1.1480\, Q_{\text{upper}} = 7.0175\end{aligned}$$

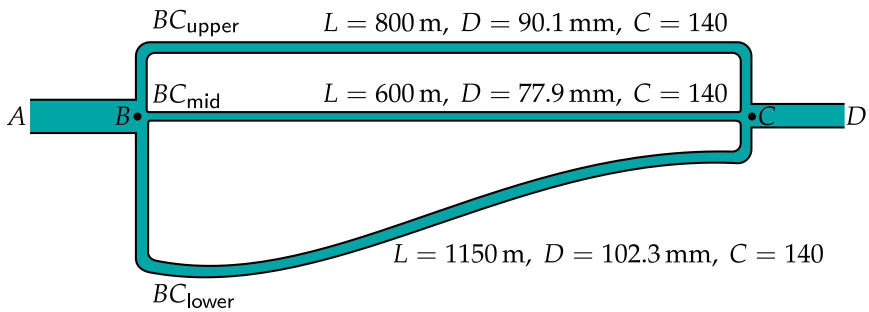
$$\boldsymbol{Q_{\text{upper}} = 6.11\,\text{L/s}, \quad Q_{\text{mid}} = 4.87\,\text{L/s}, \quad Q_{\text{lower}} = 7.02\,\text{L/s}}$$

Determine the headloss between B and C: We can use any of the three parallel pipes to calculate this since they all have the same loss.

$$h_{L_{AB}} = h_{L_{AB\text{upper}}} = 800 \left(\frac{279000 \times 6.1128}{140 (90.1)^{2.63}} \right)^{1.852} = 8.8888\,\text{m}$$

$$\boldsymbol{h_{L_{AB}} = 8.89\,\text{m}}$$

Example 10



Given a flow of 18 L/s and ignoring minor losses:

a) Determine the percentage of the flow that goes through each parallel pipe by choosing a convenient headloss between B and C.

b) Determine the volume flow rate through each of the parallel pipes.

Solution:

Assume a head loss of 10 m between B and C. Calculate the flow through each pipe, then sum the flows to get total flow from B to C that causes a headloss of 10 m.

$$Q_{\text{upper}} = \frac{140(90.1)^{2.63}\left(\frac{10}{800}\right)^{0.54}}{279000} = 6.5130\,\text{L/s}$$

$$Q_{\text{mid}} = \frac{140(77.9)^{2.63}\left(\frac{10}{600}\right)^{0.54}}{279000} = 5.1888\,\text{L/s}$$

$$Q_{\text{lower}} = \frac{140(102.3)^{2.63}\left(\frac{10}{1150}\right)^{0.54}}{279000} = 7.4769\,\text{L/s}$$

$$Q_{BC} = 6.5130 + 5.1888 + 7.4769 = 19.178\,\text{L/s}$$

Determine the percentages of the flow that go through each pipe:

$$\frac{Q_{\text{upper}}}{Q_{BC}} = \frac{6.5130}{19.178} = 33.961\%$$

$$\frac{Q_{\text{mid}}}{Q_{BC}} = \frac{5.1888}{19.178} = 27.056\%$$

$$\frac{Q_{\text{lower}}}{Q_{BC}} = \frac{7.4769}{19.178} = 38.987\%$$

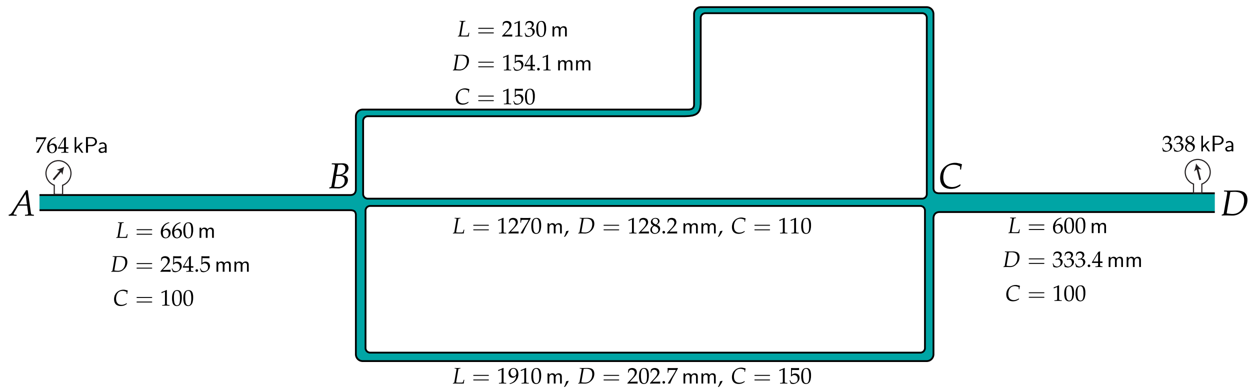
a) Percentages of flow through each pipe:

$$\begin{aligned} \boldsymbol{BC_{\text{upper}}} &= \mathbf{34.0\%} \\ \boldsymbol{BC_{\text{mid}}} &= \mathbf{27.1\%} \\ \boldsymbol{BC_{\text{lower}}} &= \mathbf{39.0\%} \end{aligned}$$

b) Flow rate through each pipe:

$$\begin{aligned} \boldsymbol{Q_{\text{upper}}} &= 0.33961 \times 18 = 6.1130 = \mathbf{6.11\,L/s} \\ \boldsymbol{Q_{\text{mid}}} &= 0.27056 \times 18 = 4.8701 = \mathbf{4.87\,L/s} \\ \boldsymbol{Q_{\text{lower}}} &= 0.38987 \times 18 = 7.0177 = \mathbf{7.02\,L/s} \end{aligned}$$

Example 11



A, B, C and D are at the same elevation. Determine the flow through the system from A to D. (Ignore minor losses.)

Solution:

Assume a head loss of 10 m between B and C. Calculate the flow through each pipe, then sum the flows to get total flow from B to C that causes a headloss of 10 m.

$$Q_{B\text{Cupper}} = \frac{150(154.1)^{2.63}\left(\frac{10}{2130}\right)^{0.54}}{279000} = 16.869\,\text{L/s}$$

$$Q_{B\text{Cmid}} = \frac{110(128.2)^{2.63}\left(\frac{10}{1270}\right)^{0.54}}{279000} = 10.080\,\text{L/s}$$

$$Q_{B\text{Clower}} = \frac{150(202.7)^{2.63}\left(\frac{10}{1910}\right)^{0.54}}{279000} = 36.792\,\text{L/s}$$

$$Q_{B\text{Cequiv}} = 16.869 + 10.080 + 36.792 = 63.741\,\text{L/s}$$

Determine the diameter of the equivalent pipe, with length of 1000 m and resistance coefficient of 100, that has a flow of 63.741 L/s for a headloss of 10 m:

$$D = \left(\frac{279000 \times 63.741}{100\left(\frac{10}{1000}\right)^{0.54}}\right)^{0.3802} = 255.08\,\text{mm}$$

Now there are three pipes in series: AB, BCequiv and CD. For pipes in series, we assume a flow of 100 L/s and find the total headloss between A and D:

$$h_{L_{AB}} = 660\left(\frac{279000 \times 100}{100(254.5)^{2.63}}\right)^{1.852} = 15.391$$

$$h_{L_{B\text{Cequiv}}} = 100\left(\frac{279000 \times 100}{100(255.08)^{2.63}}\right)^{1.852} = 23.063$$

$$h_{L_{CD}} = 600\left(\frac{279000 \times 100}{100(333.4)^{2.63}}\right)^{1.852} = 3.7553$$

$$h_{L_{AD}} = 15.391 + 23.063 + 3.7553 = 42.209\,\text{m}$$

Determine the diameter of the equivalent pipe, with length of 1000 m and resistance coefficient of 100, that has a flow of 100 L/s for a headloss of 42.209 m:

$$D = \left(\frac{279000 \times 100}{100\left(\frac{42.209}{1000}\right)^{0.54}}\right)^{0.3802} = 225.23\,\text{mm}$$

The headloss between A and D is:

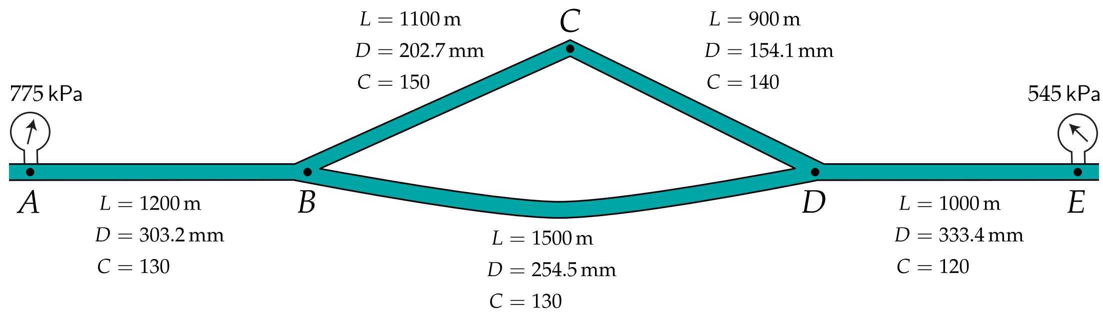
$$h_{L_{AD}} = \frac{764 - 338}{9.81} = 43.425\,\text{m}$$

The flow through the system, Q_{AB}, is:

$$Q_{AB} = \frac{100(225.23)^{2.63}\left(\frac{43.425}{1000}\right)^{0.54}}{279000} = 101.43\,\text{L/s}$$

$$\mathbf{Q_{AB} = 101.4\,L/s}$$

Exercise 5



Determine Q. All pipes have the same elevation. Disregard minor losses.

Solution:

BC and CD are in series:

Assume a flow of 100 L/s through BC and CD. Find the headloss between B and D, $h_{L_{BCD}}$, and determine the diameter of the equivalent pipe BCD_{equiv}.

$$
\begin{aligned}
h_{L_{BC}} &= 1100\left(\frac{279000 \times 100}{150(202.7)^{2.63}}\right)^{1.852} = 36.677\,\text{m} \\
h_{L_{CD}} &= 900\left(\frac{279000 \times 100}{140(154.1)^{2.63}}\right)^{1.852} = 129.60\,\text{m} \\
h_{L_{BCD}} &= 36.677 + 129.60 = 166.28\,\text{m}
\end{aligned}
$$

Determine the diameter of the equivalent pipe, with length of 1000 m and resistance coefficient of 100, that has a headloss of 166.28 m for a flow of 100 L/s

$$
D_{BCD\text{equiv}} = \left(\frac{279000 \times 100}{100\left(\frac{166.28}{1000}\right)^{0.54}}\right)^{0.3802}
$$

$$
\mathbf{D_{BCD\text{equiv}} = 169.98\,mm}
$$

BCDequiv and BD are pipes in parallel

Assume a headloss of 10 m between B and D. Find the flow from B to D for this headloss. Then determine the diameter of the equivalent pipe BD_{equiv}.

$$
\begin{aligned}
Q_{BCD_{\text{equiv}}} &= \frac{100(166.98)^{2.63}\left(\frac{10}{1000}\right)^{0.54}}{279000} \\
&= 21.895\,\text{L/s}
\end{aligned}
$$

$$
\begin{aligned}
Q_{BD} &= \frac{130(254.5)^{2.63}\left(\frac{10}{1500}\right)^{0.54}}{279000} \\
&= 66.101\,\text{L/s}
\end{aligned}
$$

$$
\begin{aligned}
Q_{BD_{\text{equiv}}} &= 21.895 + 66.101\,\text{L/s} \\
&= 87.997\,\text{L/s}
\end{aligned}
$$

Determine the diameter of the equivalent pipe, with length of 1000 m and resistance coefficient of 100, that has a flow of 87.997 L/s for a headloss of 10 m

$$
D_{BC\text{equiv}} = \left(\frac{279000 \times 87.997}{100\left(\frac{10}{1000}\right)^{0.54}}\right)^{0.3802}
$$

$$
\mathbf{D_{BC\text{equiv}} = 288.35\,mm}
$$

AB, BD_{equiv} and CD are in series:

Assume a flow of 100 L/s through AB, BD_{equiv} and DE. Find the headloss between A and E, $h_{L_{AE}}$, for this flow. Then determine the diameter of the equivalent pipe AE_{equiv}.

$$\begin{aligned}
h_{L_{AB}} &= 1200\left(\frac{279000\times 100}{130(303.2)^{2.63}}\right)^{1.852} = 7.3369\,\text{m}\\
h_{L_{BD_{\text{equiv}}}} &= 1000\left(\frac{279000\times 100}{100(288.35)^{2.63}}\right)^{1.852} = 12.693\,\text{m}\\
h_{L_{DE}} &= 1000\left(\frac{279000\times 100}{120(333.4)^{2.63}}\right)^{1.852} = 4.4653\,\text{m}\\
h_{L_{AE}} &= 7.3369 + 12.693 + 4.4653 = 24.495\,\text{m}
\end{aligned}$$

Determine the diameter of the equivalent pipe, with length of 1000 m and resistance coefficient of 100, that has a headloss of 24.495 m for a flow of 100 L/s

$$D_{AE\text{equiv}} = \left(\frac{279000\times 100}{100\left(\frac{24.495}{1000}\right)^{0.54}}\right)^{0.3802}$$

$$\boldsymbol{D_{AE\text{equiv}} = 251.86\,\text{mm}}$$

We now have a pipe, AE_{equiv}, that is equivalent to the pipe system between A and E. We need the flow through AE_{equiv} for the pressure drop from A to E. Find the headloss between A and E:

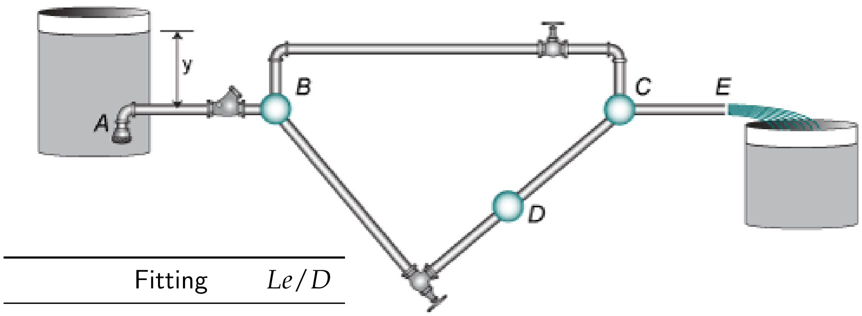
$$\begin{aligned}
\frac{P_A}{\gamma} + \cancel{z_A} + \cancel{\frac{v_A^2}{2g}} - h_L &= \frac{P_E}{\gamma} + \cancel{z_E} + \cancel{\frac{v_E^2}{2g}}\\
\Rightarrow h_L &= \frac{P_A - P_E}{\gamma}\\
&= \frac{775 - 545}{9.81}\\
&= 23.445\,\text{m}
\end{aligned}$$

The flow through the system:

$$\begin{aligned}
Q_{AE_{\text{equiv}}} &= \frac{100(251.86)^{2.63}\left(\frac{23.445}{1000}\right)^{0.54}}{279000}\\
&= 97.560\,\text{L/s}
\end{aligned}$$

$$\boldsymbol{Q_{AE} = 97.6\,\text{L/s}}$$

Exercise 6



Fitting	Le/D
Angle Valve	150
Check Valve	100
Elbow	50
Foot Valve	75
Gate Valve	35

Pipe	Length (m)	diam (mm)	C
AB	10	500	125
BC	2000	275	150
BD	1500	250	100
DC	1000	300	100
CE	10	500	125

Given that $y = 6.7\,\mathrm{m}$, determine the flow through the system.

(Nodes B and E are at the same elevation. Disregard exit losses.)

Solution:

Find the effective length of the pipes that have valves or fittings:

$$L_{AB_{\text{eff}}} = 10 + 0.5(75 + 50 + 100) = 122.50\,\mathrm{m}$$
$$L_{BC_{\text{eff}}} = 2000 + 0.275(50 + 35 + 50) = 2037.1\,\mathrm{m}$$
$$L_{BD_{\text{eff}}} = 1500 + 0.250(150) = 1537.5\,\mathrm{m}$$

<u>BD and DC are in series</u>:

Assume a flow of $100\,\mathrm{L/s}$ through BD and DC. Find the headloss between B and C (through D), and determine the diameter of the equivalent pipe BDC_{equiv}.

$$h_{L_{BD}} = 1537.5\left(\frac{279000 \times 100}{100(250)^{2.63}}\right)^{1.852} = 39.110\,\mathrm{m}$$
$$h_{L_{DC}} = 1000\left(\frac{279000 \times 100}{100(300)^{2.63}}\right)^{1.852} = 10.466\,\mathrm{m}$$
$$h_{L_{BDC}} = 39.110 + 10.466 = 49.576\,\mathrm{m}$$

Determine the diameter of the equivalent pipe, with length of $1000\,\mathrm{m}$ and resistance coefficient of 100, that has a headloss of $49.576\,\mathrm{m}$ for a flow of $100\,\mathrm{L/s}$.

$$D_{BDC\text{equiv}} = \left(\frac{279000 \times 100}{100\left(\frac{49.576}{1000}\right)^{0.54}}\right)^{0.3802}$$

$$\boldsymbol{D_{BDC\text{equiv}} = 217.92\,\mathrm{mm}}$$

<u>Pipes BC and BDC_{equiv} are in parallel</u>:

Assuming a headloss of $10\,\mathrm{m}$ between B and C, find the flow from B to C.

$$Q_{BC_{\text{upper}}} = \frac{150(275)^{2.63}\left(\frac{10}{2037.1}\right)^{0.54}}{279000}$$
$$= 79.262\,\text{L/s}$$

$$Q_{BDC_{\text{equiv}}} = \frac{100(217.92)^{2.63}\left(\frac{10}{1000}\right)^{0.54}}{279000}$$
$$= 42.084\,\text{L/s}$$

$$Q_{BC_{\text{equiv}}} = 79.262 + 42.084\,\text{L/s}$$
$$= 121.35\,\text{L/s}$$

Determine the diameter of the equivalent pipe, with length of 1000 m and resistance coefficient of 100, that has a headloss of 121.35 m for a flow of 100 L/s.

$$D_{BC\text{equiv}} = \left(\frac{279000 \times 100}{100\left(\frac{121.35}{1000}\right)^{0.54}}\right)^{0.3802}$$

$$\boldsymbol{D_{BC\text{equiv}} = 181.33\,\text{mm}}$$

AB, $D_{BC\text{equiv}}$ and CE are in series:

Assume a flow of 100 L/s between A and E. Find the headloss between A and E, then determine the diameter of the equivalent pipe AE_{equiv}.

$$h_{L_{AB}} = 122.5\left(\frac{279000 \times 100}{125(500)^{2.63}}\right)^{1.852}$$
$$= 0.070450\,\text{m}$$

$$h_{L_{BC\text{equiv}}} = 1000\left(\frac{279000 \times 100}{100(181.33)^{2.63}}\right)^{1.852}$$
$$= 121.56\,\text{m}$$

$$h_{L_{CE\text{equiv}}} = 1000\left(\frac{279000 \times 100}{100(500)^{2.63}}\right)^{1.852}$$
$$= 0.0057510\,\text{m}$$

$$h_{L_{AE}} = 0.070450 + 121.56 + 0.0057510$$
$$= 121.64\,\text{m}$$

Find the equivalent pipe that has a headloss of 121.64 m for a flow of 100 L/s:

$$D_{AE\text{equiv}} = \left(\frac{279000 \times 100}{100\left(\frac{121.64}{1000}\right)^{0.54}}\right)^{0.3802}$$
$$= 181.24\,\text{mm}$$

$D_{AE\text{equiv}}$ is a single pipe equivalent to the whole system. The headloss, applying the GEE at the surface of the 'upstream' tank and at E, ignoring the exit loss, is 6.7 m. The flow through $D_{AE\text{equiv}}$, and through the real system, is:

$$Q_{AE\text{equiv}} = \frac{100(181.24)^{2.63}\left(\frac{6.7}{1000}\right)^{0.54}}{279000}$$
$$= 20.878\,\text{L/s}$$

$$\boldsymbol{Q_{AE} = 20.9\,\text{L/s}}$$