Module 12: Open Channel Flow (CIVL 318)

Hydraulic Radius: $R = A/WP$

Circular Channel:

$$A = \frac{(\theta - \sin\theta)\,D^2}{8}$$

$$WP = \theta D/2$$

$$R = \left(\frac{\theta - \sin\theta}{\theta}\right)\frac{D}{4}$$

Manning's Equation:

$$v = \frac{1}{n}R^{2/3}S^{1/2}$$

$$Q = \frac{1}{n}AR^{2/3}S^{1/2}$$

Open Channel Energy Equation:

$$y_1 + \frac{v_1^2}{2g} + z_1 - h_L = y_2 + \frac{v_2^2}{2g} + z_2 - h_L$$

Specific Energy:

$$E_S = y + \frac{v^2}{2g}$$

Froude Number:

$$N_F = \frac{v}{\sqrt{g(A/T)}}$$

$N_F > 1 \Rightarrow$ Super-critical flow

$N_F = 1 \Rightarrow$ Critical flow

$N_F < 1 \Rightarrow$ Sub-critical flow

Hydraulic Jump:

$$y_2 = \frac{y_1}{2}\left[\sqrt{1 + 8N_{F_1}^2} - 1\right]$$

$$\Delta E = (y_2 - y_1)^3/4y_1y_2$$

Critical Depth, Velocity and Energy

Channel Shape	Critical Depth	Critical Velocity
Rectangular	$y_c = \frac{2}{3}E_{min}$	$v_c = \sqrt{gy_c}$
Triangular	$y_c = \frac{4}{5}E_{min}$	$v_c = \sqrt{\frac{gy_c}{2}}$
Trapezoidal	$y_c = \frac{4zE_{min} - 3b + \sqrt{16z^2E_{min}^2 + 16zE_{min}b + 9b^2}}{10z}$	$v_c = \sqrt{\frac{gy_c\,(b + zy_c)}{b + 2zy_c}}$

Manning's Equation - Resistance Factors

Channel Description	n-value
Plastic - continuous	0.009
Plastic - pipe with joints	0.011
Smooth metal	0.010
Concrete - steel trowelled or slip-formed	0.012
Concrete - pipe with joints	0.013
Concrete - shotcreted	0.017
Riveted Steel	0.018
Corrugated Steel	0.022
Earth - firm clay	0.020
Earth - alluvial silt and clay	0.021
Earth - medium sand	0.023
Earth - fine gravel	0.024
Earth - coarse gravel	0.028
Earth - cobbles Rock cuts - smooth	0.030 0.030
Rock cuts - jagged	0.040
Highway Ditches - cut grass	0.040
Highway Ditches - uncut grass	0.090
Highway Ditches - uncut grass and small bushes	0.125
Natural Streams - straight, uniform cross-section, no growth	0.025
Natural Streams - fairly straight, moderate changes in cross-section, some growth	0.033
Natural Streams - curving, moderate changes in cross-section, some growth	0.040
Flood Plains - with high grass and bushes, some developed channels	0.070
Flood Plains - with bushes, trees and pools	0.100
Mountain Creeks - with pools and waterfalls	0.130

Soil Type	ParticleSize	Maximum Velocity
Fine sand, no clay	$62 - 250\,\mu$m	0.45 m/s
Medium sand, no clay	$250 - 500\,\mu$m	0.50 m/s
Coarse sand, no clay	$500 - 2000\,\mu$m	0.60 m/s
Fine gravel	$4 - 8$ mm	0.74 m/s
Coarse gravel	$8 - 64$ mm	1.25 m/s
Cobbles	$64 - 256$ mm	1.55 m/s
Firm clay		1.15 m/s
Alluvial silt (high clay)		1.15 m/s
Alluvial silt (low clay)		0.60 m/s

Example 1

The Pontcysyllte Aqueduct is 3.4 m wide, 1.60 m deep and is constructed of cast iron with a resistance factor of $n = 0.018$.

If the flow velocity is 0.1 m/s, determine the slope.

Solution:

$$
\begin{aligned}
A &= 3.40\,\mathrm{m} \times 1.60\,\mathrm{m} \\
&= 5.44\,\mathrm{m}^2 \\
WP &= 1.6\,\mathrm{m} + 3.4\,\mathrm{m} + 1.6\,\mathrm{m} \\
&= 6.60\,\mathrm{m} \\
R &= A/WP \\
&= \frac{5.44\,\mathrm{m}^2}{6.60\,\mathrm{m}} \\
&= 0.82424\,\mathrm{m}
\end{aligned}
$$

Find the slope:

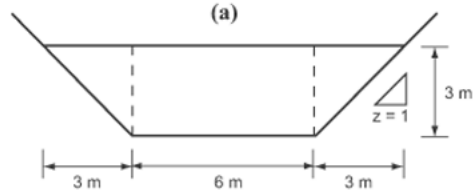
$$
\begin{aligned}
v &= \frac{1}{n} R^{2/3} S^{0.5} \\
\implies S &= \left[\frac{nv}{R^{2/3}}\right]^2 \\
&= \left[\frac{0.018 \times 0.1}{0.82424^{2/3}}\right]^2 \\
&= \mathbf{0.000419\%}
\end{aligned}
$$

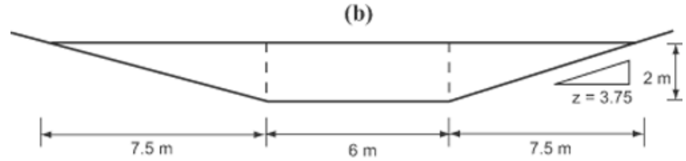
Example 2

A circular culvert of diameter $2.0\,\mathrm{m}$ is made from corrugated metal. It has a slope of 1 in 500. Determine the normal discharge, and the velocity when:

(a) the culvert runs half full
(b) the culvert runs full
(c) the culvert runs at a depth of $1.8\,\mathrm{m}$

Example 3:





Determine the normal discharge for the two channels shown. Both are in fine grained soil with an estimated n-value of 0.020. The channel slope is 0.1%.

Then, calculate the flow rate when the more efficient channel is lined with concrete ($n = 0.013$).

Example 4:

A rectangular channel is constructed out of concrete with $n = 0.013$. The channel has a slope of 0.0005 and a base width of $b = 4.0\,\mathrm{m}$

Find the depth of flow, y, if $Q = 6.25\,\mathrm{m}^3/\mathrm{s}$.

Example 5:

A trapezoidal channel constructed from coarse sand without clay ($n = 0.025$) has a base width of 7.0 m and a flow depth of 1.7 m.

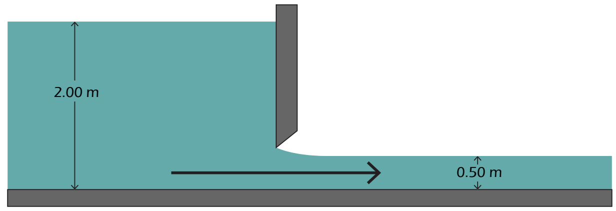
If the sidewall have a z-value of 3, find the average flow rate and the average velocity. Check this against allowable values.

(Slope is 0.015% and the maximum allowable velocity for this type of soil is 0.60 m/s.)

Example 6:

Determine the specific energy in a rectangular channel with a base of 6.0 m and a flow depth of $y = 1.8$ m if the volume flow is $20\,\text{m}^3/\text{s}$

Example 7:

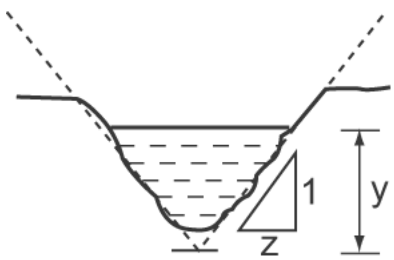


Water flows at a depth of 2.0 m in a rectangular channel with width 5.0 m. The flow passes under a sluice gate so that the downstream flow has a depth of 0.5 m. Determine the flow in the channel. (There is little energy loss due to the sluice gate.)

Example 8:

A channel with a triangular section is cut through rock and lined with un-trowelled shotcrete. It has a slope of 0.1201 %. The water depth, $y = 3.5$ m and the side slope $z = 1.0$. The channel has an estimated $n = 0.017$. Find:

a) the flow rate
b) the velocity
c) the critical velocity
d) whether the slope is sub-critical or super-critical.
e) the slope at which flow becomes critical

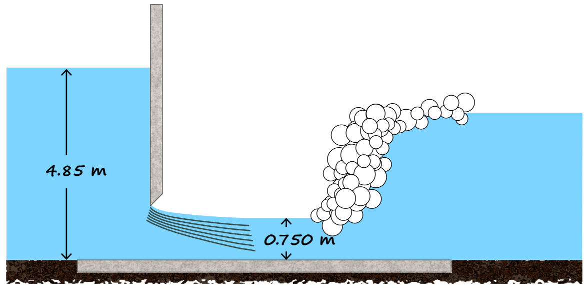


Solution:

$$\begin{aligned} A &= \frac{1}{2}(7.0\text{m})(3.5\text{ m}) \\ &= 12.25\text{ m}^2 \\ WP &= 2\sqrt{2(3.5\text{ m})^2} \\ &= 9.8995\text{ m} \\ R &= \frac{A}{WP} \\ &= \frac{12.250\text{ m}^2}{9.8995\text{ m}} \\ &= 1.2374\text{ m} \end{aligned}$$

$$\begin{aligned} Q &= \frac{1}{n}AR^{2/3}S^{1/2} \\ &= \frac{1}{0.017}(12.250)(1.2374)^{2/3}(0.0012)^{1/2} \\ &= 28.771\text{ m}^3/\text{s} \end{aligned}$$

Example 9: Flow over a rectangular dam spillway is approximately 1150 m^3/s. If the width of the spillway is 32.0 m and the depth of the supercritical flow before the hydraulic jump is 1.4 m, determine the velocities of the flow before and after the jump, the depth after the jump and the energy dissipated.



Example 10: Flow from a reservoir (i.e. it has no initial velocity) into a rectangular channel is controlled by a sluice gate as shown. Verify that the flow after the gate is supercritical and determine the depth of flow downstream from the jump.

Example 11: Water flows from a reservoir with a depth of 3.60 m down a trapezoidal concrete-lined channel ($n = 0.017$) with a base width of 4.0 m and a side slope value of $z = 2.5$.

Determine:

(1) The flow under critical conditions (when specific energy is at a minimum)
(2) The slope required to give critical flow