Module 1: The Nature of Fluids/Pressure Measurement (CIVL 318)

Some useful results:

Pressure:	P	$=$	$\frac{F}{A}$	$\left(\frac{\text{Force}}{\text{Area}}\right)$
Pascal's Laws:	Pressure acts uniformly in all directions on a small volume of liquid. Pressure acts perpendicularly to the solid boundaries of a fluid.			
Density:	ρ (rho)	$=$	$\frac{m}{V}$	$\left(\frac{\text{Mass}}{\text{Volume}}\right)$
Specific Weight:	γ (gamma)	$=$	$\frac{w}{V}$	$\left(\frac{\text{Weight}}{\text{Volume}}\right)$
Specific Gravity:	sg	$=$	$\frac{\rho_s}{\rho_w @ 4^\circ\text{C}}$	$= \frac{\gamma_s}{\gamma_w @ 4^\circ\text{C}}$
Density & Specific Weight:	γ	$=$	ρg	
Pressure Relationship:	p_{abs}	$=$	$p_{atm} + p_{gauge}$	
Pressure-Elevation Relationship:	Δp	$=$	γh	

Table A: Properties of Water

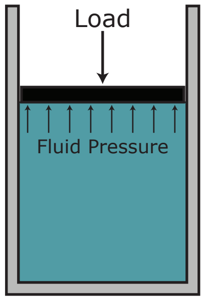
Temperature (°C)	Specific Weight γ (kN/m^3)	Density ρ (kg/m^3)	Dynamic Viscosity η ($Pa \cdot s$)
0	9.81	1000	1.75×10^{-3}
5	9.81	1000	1.52×10^{-3}
10	9.81	1000	1.30×10^{-3}
15	9.81	1000	1.15×10^{-3}
20	9.79	998	1.02×10^{-3}
25	9.78	997	8.91×10^{-4}
30	9.77	996	9.00×10^{-4}
35	9.75	994	7.18×10^{-4}
40	9.73	992	6.51×10^{-4}
45	9.71	990	5.94×10^{-4}
50	9.69	988	5.41×10^{-4}
55	9.67	986	4.98×10^{-4}
60	9.65	984	4.60×10^{-4}
65	9.62	981	4.31×10^{-4}
70	9.59	978	4.02×10^{-4}
75	9.56	975	3.73×10^{-4}
80	9.53	971	3.50×10^{-4}
85	9.50	968	3.30×10^{-4}
90	9.47	965	3.11×10^{-4}
95	9.44	962	2.92×10^{-4}
100	9.40	958	2.82×10^{-4}

Table B: Properties of Common Liquids

(at 101 kPa and 25°C)

Liquid	Specific Gravity	Specific Weight γ (kN/m^3)	Density ρ (kg/m^3)	Dynamic Viscosity η ($Pa \cdot s$)
Acetone	0.787	7.72	787	3.16×10^{-4}
Alcohol, Ethyl	0.787	7.72	787	1.00×10^{-3}
Alcohol, Methyl	0.789	7.74	789	5.60×10^{-4}
Alcohol, Propyl	0.802	7.87	802	1.92×10^{-3}
Benzene	0.876	8.59	876	6.03×10^{-4}
Carbon Tetrachloride	1.590	15.60	1590	9.10×10^{-4}
Castor Oil	0.960	9.42	960	6.51×10^{-1}
Ethylene Glycol	1.100	10.79	1100	1.62×10^{-2}
Gasoline	0.68	6.67	680	2.87×10^{-4}
Glycerine	1.258	12.34	1258	9.60×10^{-1}
Kerosene	0.823	8.07	823	1.64×10^{-3}
Linseed Oil	0.930	9.12	930	3.31×10^{-2}
Mercury	13.54	132.8	13540	1.53×10^{-3}
Propane	0.495	4.86	495	1.10×10^{-4}
Seawater	1.030	10.10	1030	1.03×10^{-3}
Turpentine	0.870	8.53	870	1.37×10^{-3}
Fuel Oil, medium	0.852	8.36	852	2.99×10^{-3}
Fuel Oil, heavy	0.906	8.89	906	1.07×10^{-1}

Example 1:



A piston confines oil in a closed circular cylinder. The maximum operating pressure for the piston is 17.8 MPa. The piston has a diameter of 62.5 mm. What is the maximum load that the piston can support?

Solution:

$$
\begin{aligned}
F &= P \times A \\
&= 17.8 \times 10^6 \text{ N/m}^2 \times \frac{\pi(0.0625 \text{ m})^2}{4} \\
&= 54610 \text{ N} \\
&\approx 54.6 \text{ kN} \qquad\qquad \text{(3 sig digs)}
\end{aligned}
$$

Alternative units:

$$
\begin{aligned}
F &= P \times A \\
&= 17.8 \text{ N/mm}^2 \times \frac{\pi(6.25 \text{ mm})^2}{4} \\
&= 54610 \text{ N} \\
&\approx 54.6 \text{ kN}
\end{aligned}
$$

Exercise 1:
A press used to produce coins requires a force of 8.20 kN.
The hydraulic cylinder has a diameter of 63.5 mm.

What is the oil pressure needed to generate this force?

Solution:

$$
\begin{aligned}
P &= \frac{F}{A} \\
&= \frac{8.20 \text{ kN}}{\pi(0.0635 \text{ m})^2/4} \\
&= 2\,589.3 \text{ kN/m}^2 \\
&\approx 2.59 \text{ MPa}
\end{aligned}
$$

Alternative units:

$$
\begin{aligned}
P &= \frac{F}{A} \\
&= \frac{8200 \text{ N}}{\pi(6.35 \text{ mm})^2/4} \\
&= 2.5893 \text{ N/mm}^2 \\
&\approx 2.59 \text{ MPa}
\end{aligned}
$$

Example 3:
An empty barrel with an inside diameter of 900 mm weighs 205 N.

What does the barrel weigh when it is filled to a depth of 750 mm with water at 25°C?

Solution:

The volume of water is the volume of a cylinder with diameter 900 mm and height 750 mm:

$$\begin{aligned} v &= \frac{\pi d^2}{4} \cdot h \\ &= \frac{\pi (0.900 \text{ m})^2}{4} \cdot (0.75 \text{ m}) \\ &= 0.47713 \text{ m}^3 \end{aligned}$$

(Use 5 significant digits for interim calculations and 3 significant digits for solutions.)

The specific weight of water at 25°C is 9.78 kN/m^3 (Table A.1) so

$$\begin{aligned} w &= \gamma v \\ &= 9.78 \text{ kN/m}^3 \times 0.47713 \text{ m}^3 \\ &= 4.6663 \text{ kN} \end{aligned}$$

The combined weight of the barrel and the water is given by:

$$0.205 \text{ kN} + 4.6663 \text{ kN} \approx 4.87 \text{ kN}$$

Example 4:
Calculate the density and the specific weight of benzene if it has a specific gravity of 0.876.

Solution:

$$\begin{aligned} 0.876 &= \frac{\rho_b}{\rho_{water@4^\circ C}} \\ \rho_b &= 0.876 \times 1000 \text{ kg/m}^3 \\ &= 876 \text{ kg/m}^3 \\ 0.876 &= \frac{\gamma_b}{\gamma_{water@4^\circ C}} \\ \gamma_b &= 0.876 \times 9.81 \text{ kN/m}^3 \\ &= 8.59 \text{ kN/m}^3 \end{aligned}$$

Example 5:
An open cylindrical tank with diameter 5.75 m and depth 3.30 m is filled to the top with water at 10°C. The water is heated to 55°C. Assuming that the tank dimensions remain constant and there are no losses due to evaporation, calculate the mass of water that overflows.

Solution:

Volume of tank:

$$\begin{aligned} v_{tank} &= \frac{\pi(5.75\ \text{m})^2}{4} \times 3.30\ \text{m} \\ &= 85.692\ \text{m}^3 \end{aligned}$$

Mass of the water in the tank at 10°C:

$$\begin{aligned} m &= \rho_{10°\text{C}} \times v_{10°\text{C}} \\ &= 1000\ \text{kg/m}^3 \times 85.692\ \text{m}^3 \\ &= 85\,692\ \text{kg} \end{aligned}$$

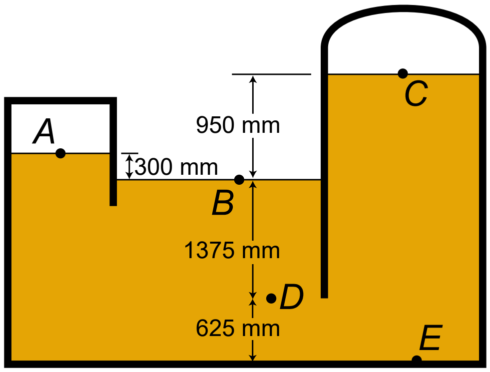
Mass of the water in the tank at 70°C:

$$\begin{aligned} m &= \rho_{55°\text{C}} \times v_{55°\text{C}} \\ &= 986\ \text{kg/m}^3 \times 85.692\ \text{m}^3 \\ &= 84\,492\ \text{kg} \end{aligned}$$

Mass of water that overflows:

$$\begin{aligned} m_{overflow} &= 85\,692\ \text{kg} - 84\,492\ \text{kg} \\ &= 1\,200\ \text{kg} \end{aligned}$$

Example 6:



A tank, open to the atmosphere in the centre, contains medium fuel oil. Atmospheric pressure is 102.1 kPa. Calculate the gauge pressure and the absolute pressure for locations *A*, *B*, *C*, *D* and *E*.

Solution:

Pressure at B: B is open to the atmosphere so

$P_B = 0$ and $P_{B(abs)} = P_{(atm)} = 102.1$ kPa

Note that pressure is assumed to be gauge pressure unless otherwise specified.

Also, atmospheric pressure is generally specified to four significant digits; there is a distinct difference in pressure between 100.5 kPa and 101.4 kPa.

Pressure at A:

$$\begin{aligned}
P_A &= P_B - \Delta p \\
&= 0 - \gamma h \\
&= -(8.36 \text{ kN/m}^3)(0.30 \text{ m}) \\
&= -2.5080 \text{ kPa} \\
&\approx -2.51 \text{ kPa}
\end{aligned}$$

$$\begin{aligned}
P_{A(abs)} &= P_{atm} + P_{A(gauge)} \\
&= 102.1 \text{ kPa} - 2.5080 \text{ kPa} \\
&= 99.592 \text{ kPa} \\
&\approx 99.6 \text{ kPa}
\end{aligned}$$

Pressure at C:

$$\begin{aligned}
P_C &= P_C - \Delta p \\
&= 0 - \gamma h \\
&= -(8.36 \text{ kN/m}^3)(0.950 \text{ m}) \\
&= -7.9420 \text{ kN/m}^2 \\
&\approx -7.94 \text{ kPa}
\end{aligned}$$

$$\begin{aligned}
P_{C(abs)} &= P_{atm} + P_{C(gauge)} \\
&= 102.1 \text{ kPa} - 7.9420 \text{ kPa} \\
&= 94.158 \text{ kPa} \\
&\approx 94.2 \text{ kPa}
\end{aligned}$$

Pressure at D:

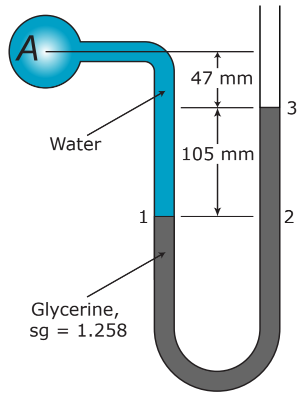
$$\begin{aligned}
P_D &= P_B + \Delta p \\
&= 0 + \gamma h \\
&= (8.36 \text{ kN/m}^3)(1.375 \text{ m}) \\
&= 11.495 \text{ kPa} \\
&\approx 11.50 \text{ kPa} \\
P_{D(abs)} &= P_{atm} + P_{D(gauge)} \\
&= 102.1 \text{ kPa} + 11.495 \text{ kPa} \\
&= 113.60 \text{ kPa} \\
&= 113.6 \text{ kPa}
\end{aligned}$$

Pressure at E:

$$\begin{aligned}
P_E &= P_B + \Delta p \\
&= 0 + \gamma h \\
&= (8.36 \text{ kN/m}^3)(2.0 \text{ m}) \\
&= 16.720 \text{ kPa}
\end{aligned}$$

$$\begin{aligned}
P_{E(abs)} &= P_{atm} + P_{D(gauge)} \\
&= 102.1 \text{ kPa} + 16.72 \text{ kPa} \\
&= 118.82 \text{ kPa} \\
&\approx 118.8 \text{ kPa}
\end{aligned}$$

Example 7:
Determine the pressure at A given that the temperature of the water is 25°C.

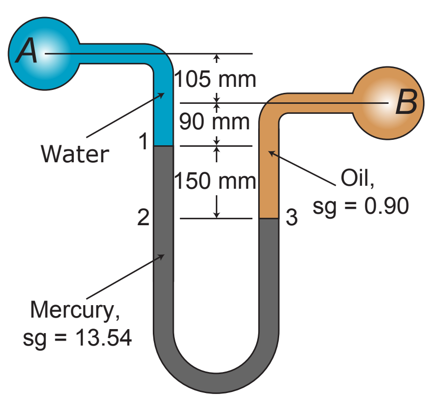


Solution:

$$\begin{aligned}
P_3 &= 0 \\
P_2 &= P_3 + \gamma h \\
&= 0 + (1.258)(9.81\ \text{kN/m}^3)(0.105\ \text{m}) \\
&= 1.2958\ \text{kPa} \\
P_1 &= 1.2958\ \text{kPa} \\
P_A &= P_1 - \gamma h \\
&= 1.2958\ \text{kPa} - (9.78)(0.152)\ \text{kPa} \\
&= -0.1907\ \text{kPa}
\end{aligned}$$

Note: There is not much difference in pressure for a difference in levels of 0.105 m. For this reason, a gauge fluid with a higher specific gravity, such as mercury, is usually used to measure larger pressure differences.

Example 8:
Find the pressure difference between A and B



Solution:

$$\begin{aligned}
P_1 &= P_A + \gamma h \\
&= P_A + (9.81\ \text{kN/m}^3)(0.195\ \text{m}) \\
&= P_A + 1.913\ \text{kPa} \\
P_2 &= P_1 + (13.54)(9.81\ \text{kN/m}^3)(0.15\ \text{m}) \\
&= P_A + (1.913 + 19.924)\ \text{kPa} \\
&= P_A + 21.837\ \text{kPa} \\
P_3 &= P_A + 21.837\ \text{kPa} \\
P_B &= P_3 - (0.90)(9.81\ \text{kN/m}^3)(0.240\ \text{m}) \\
&= P_A + 21.837\ \text{kPa} - 2.119\ \text{kPa} \\
\Delta p &= 19.72\ \text{kPa}
\end{aligned}$$