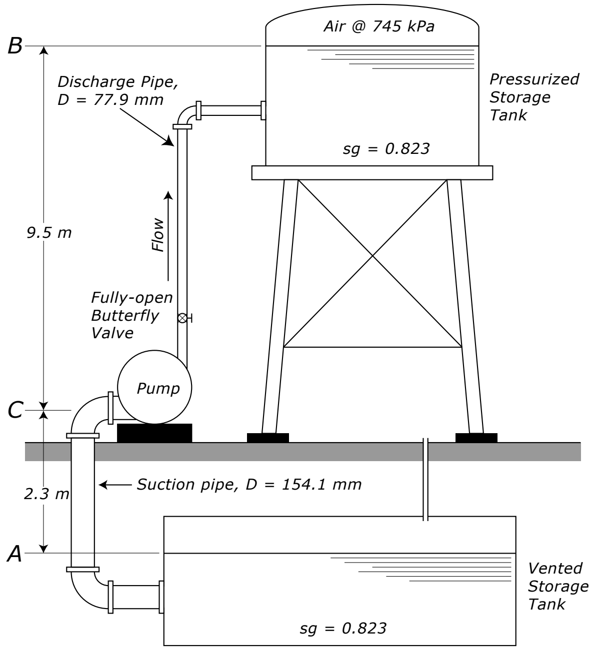
Module 7: Series A Pipeline (CIVL 318)

Example 1:



A pump delivers 13.5 L/s of kerosene at 25°C from an underground vented storage tank to an elevated storage tank pressurized to 745 kPa.

The suction pipe is 6-in Schedule 40 steel pipe and is 5.0 m long. It has a round-edged entrance with a radius of $r = 15$ mm.

The discharge pipe is 3-in Schedule 40 steel pipe, is 11.0 m long and includes a fully open butterfly valve with $L_e/D = 45$.

All elbows are "standard" with $L_e/D = 30$.

Determine the power drawn (the power in, P_I) by the pump.

Solution:

Suction Pipe

$$v = \frac{0.0135\,\text{m}^3/\text{s}}{\pi(0.1540\,\text{m})^2/4} = 0.72383\,\text{m/s}$$

$$\frac{v^2}{2g} = 0.026704\,\text{m}$$

$$N_R = \frac{0.72383(0.1541)823}{1.64\times10^{-3}} = 55975$$
$$= 5.5975\times10^4$$

$$\frac{D}{\epsilon} = \frac{0.1541}{4.6\times10^{-5}} = 3350$$

$N_R > 4000$ so flow is turbulent.

Friction Losses:

$$f = 0.0215 \quad \text{(Moody)}$$
$$= 0.02144 \quad \text{(Swamee-Jain)}$$

Using the Moody result:

$$h_L = 0.0215\left(\frac{5.0}{0.1541}\right)0.026704$$
$$= \mathbf{0.018629}\,\text{m}$$

Minor Losses:

$$f_T = 0.015 \quad \text{(Moody)}$$
$$K_{\text{entrance}} = 0.09 \quad (r/D = 0.1)$$
$$K_{\text{elbow}} = f_T\left(\frac{L_e}{D}\right) = 0.015(30) = 0.45$$
$$h_{L_{\text{minor}}} = K_{\text{entrance}}\frac{v^2}{2g} + 2K_{\text{elbow}}\frac{v^2}{2g}$$
$$= (0.09 + 2\times0.45)0.026704$$
$$= \mathbf{0.026437}\,\text{m}$$

Discharge Pipe

$$v = \frac{0.0135\,\text{m}^3/\text{s}}{\pi(0.0779\,\text{m})^2/4} = 2.8325\,\text{m/s}$$

$$\frac{v^2}{2g} = 0.40892\,\text{m}$$

$$N_R = \frac{2.8325(0.0779)823}{1.64\times10^{-3}} = 110730$$
$$= 1.1073\times10^5$$

$$\frac{D}{\epsilon} = \frac{0.0779}{4.6\times10^{-5}} = 1693$$

$N_R > 4000$ so flow is turbulent.

Friction Losses:

$$f = 0.0205 \quad \text{(Moody)}$$

$$h_L = 0.0205\left(\frac{11.0}{0.0779}\right)0.40892$$
$$= \mathbf{1.1837}\,\text{m}$$

Minor Losses:

$$
\begin{aligned}
f_T &= 0.018 \quad \text{(Table)} \\
K_{\text{elbow}} &= f_T \left(\frac{L_e}{D} \right) = 0.018(30) = 0.54 \\
K_{\text{valve}} &= f_T \left(\frac{L_e}{D} \right) = 0.018(45) = 0.81 \\
K_{\text{exit}} &= 1 \\
h_{L_{\text{minor}}} &= (0.54 + 0.81 + 1)\, 0.40892 \\
&= \mathbf{0.96096}\,\text{m}
\end{aligned}
$$

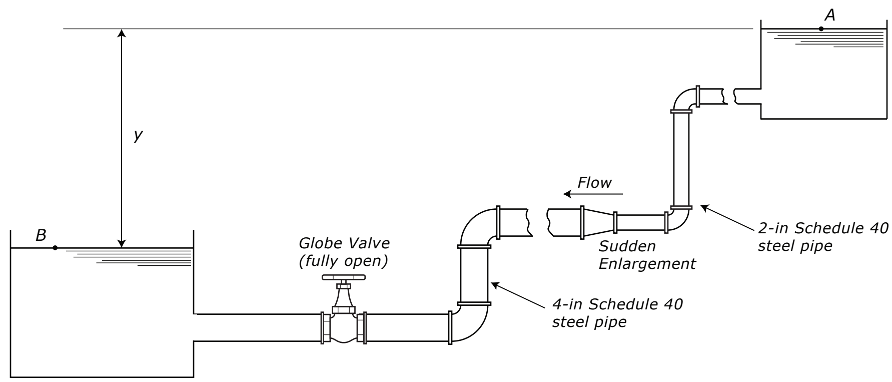
Total head losses:

$$
\begin{aligned}
h_L &= 0.018629 + 0.026437 \\
&\quad + 1.1837 + 0.96096 \\
&= \mathbf{2.1897}\,\text{m}
\end{aligned}
$$

General Energy Equation:

$$
\begin{aligned}
\cancel{\frac{P_A}{\gamma}} + \cancel{z_A} + \cancel{\frac{v_A^2}{2g}} + h_A - h_L = \frac{P_A}{\gamma} + z_A + \cancel{\frac{v_A^2}{2g}} \\
h_A - 2.1897 &= \frac{745}{0.823 \times 9.81} + 11.8 \\
h_A &= 106.27\,\text{m} \\
\\
P_{\text{added}} &= h_A \gamma Q \\
&= 106.27\,(0.823 \times 9.81)\, 0.0135 \\
&= 11.583\,\text{kW} \\
\\
P_I &= \frac{11.583}{0.73} \\
&= 15.867\,\text{kW} \\
\\
\boldsymbol{P_I} &= \mathbf{15.87\,kW}
\end{aligned}
$$

Example 2:



Gasoline at 25°C flows under gravity from tank A to tank B; both tanks are open to the atmosphere.

The 2-in Schedule 40 steel pipe has a square entrance and is 45.7 m in length. The 4-in Schedule 40 steel pipe contains a fully-open globe valve and is 87.5 m in length. There is a sudden enlargement between the two pipes, as shown. Both pipes are new commercial steel. All elbows are standard 90°.

Determine the elevation difference, y, between the surfaces of tanks A and B that is required to maintain a flow of 425 L/min.

Solution:

2-in Pipe

$$v = \frac{0.425/60\,\mathrm{m^3/s}}{\pi(0.0525\,\mathrm{m})^2/4} = 3.2721\,\mathrm{m/s}$$

$$\frac{v^2}{2g} = 0.54571\,\mathrm{m}$$

$$N_R = \frac{3.2721(0.0525)680}{2.87\times10^{-4}}$$
$$= 407020 = 4.0702\times10^5$$

$$\frac{D}{\epsilon} = \frac{0.0525}{4.6\times10^{-5}} = 1141.3$$

$N_R > 4000$ so flow is turbulent.

Friction Losses:

$$f = 0.0203 \quad \text{(Moody)}$$
$$= 0.019945 \quad \text{(Swamee-Jain)}$$

Using the Moody value:

$$h_L = 0.0203\left(\frac{45.7}{0.0525}\right)0.54571$$
$$= \mathbf{9.6431}\,\mathrm{m}$$

Minor Losses:

$$f_T = 0.0192 \quad \text{(Moody)}$$
$$= 0.019026 \quad \text{(Swamee-Jain)}$$
$$= 0.019 \quad \text{(from table for \textbf{steel} pipe)}$$

Use $f_T = 0.019$ from the steel pipe table:

$$K_{\text{entrance}} = 0.5$$

$$2\times K_{\text{elbow}} = 2\times f_T\left(\frac{L_e}{D}\right)$$
$$= 2\times0.019(30) = 1.14$$

Using $D_2/D_1 \approx 1.95$ and $v \approx 3.27\,\mathrm{m/s}$ in sudden enlargement table:

$$K_{\text{enlargement}} = 0.52$$

$$\Sigma K = 2.16$$

$$h_{L_{\text{minor}}} = \Sigma K \times \frac{v^2}{2g}$$
$$= (2.16)0.54571$$
$$= \mathbf{1.1787}\,\mathrm{m}$$

4-in Pipe

$$v = \frac{0.425/60\,\mathrm{m^3/s}}{\pi(0.1023\,\mathrm{m})^2/4} = 0.86178\,\mathrm{m/s}$$
$$\frac{v^2}{2g} = 0.037852\,\mathrm{m}$$
$$N_R = \frac{0.86178(0.1023)680}{2.87\times10^{-4}} = 208880 = 2.0888\times10^5$$
$$\frac{D}{\epsilon} = \frac{0.1023}{4.6\times10^{-5}} = 2223.9$$

$N_R > 4000$ so flow is turbulent.

Friction Losses:

$$f = 0.0177 \quad \text{(Moody)}$$
$$= 0.017661 \quad \text{(Swamee-Jain)}$$

Using $f = 0.0177$

$$h_L = 0.0177\left(\frac{87.5}{0.1023}\right)0.037852$$
$$= \mathbf{0.57305}\,\mathrm{m}$$

Minor Losses:

$$f_T = 0.0162 \quad \text{(Moody)}$$
$$= 0.016319 \quad \text{(Swamee-Jain)}$$
$$= 0.017 \quad \text{(from table for steel pipe)}$$

Use $f_T = 0.017$

$$K_{\text{exit}} = 1$$
$$2\times K_{\text{elbow}} = 2\times f_T\left(\frac{L_e}{D}\right)$$
$$= 2\times0.017(30) = 1.02$$
$$K_{\text{globe}} = f_T\left(\frac{L_e}{D}\right)$$
$$= 0.017(340) = 5.78$$
$$\Sigma K = 7.8$$
$$h_{L_{\text{minor}}} = \Sigma K\times\frac{v^2}{2g}$$
$$= (7.8)0.037852$$
$$= \mathbf{0.29525}\,\mathrm{m}$$

Total losses in system:

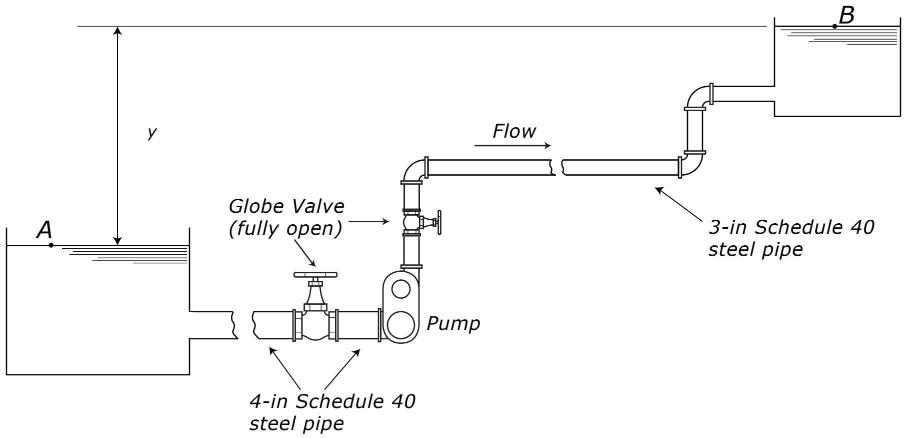
$$h_L = 9.6431 + 1.1787 + 0.57305 + 0.29525 = 11.690\,\mathrm{m}$$

Find y:

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} - h_L = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$
$$0 + y + 0 - 11.690 = 0 + 0 + 0$$
$$y = 11.690\,\mathrm{m}$$

$$\boldsymbol{y = 11.69\,\mathrm{m}}$$

Example 3:



Water at 25 °C is pumped from tank A to tank B. Both tanks are open to the atmosphere.

The suction pipe is 4-in Schedule 40 steel pipe, has a well-rounded ($r/D > 0.15$) entrance, contains a fully open globe valve, and is 17.0 m long.

The discharge pipe is 3-in Schedule 40 steel pipe, contains a fully open globe valve and three standard 90° elbows; it is 163.3 m long.

The elevation difference between A and B is $y = 12.75$ m and the volume flow rate is $Q = 900$ L/min.

If the pump is 78% efficient, determine the electrical power it uses.

Solution:

Suction Pipe

$$v = \frac{0.900/60\,\mathrm{m}^3/\mathrm{s}}{\pi(0.1023\,\mathrm{m})^2/4} = 1.8249\,\mathrm{m/s}$$

$$\frac{v^2}{2g} = 0.16974\,\mathrm{m}$$

$$N_R = \frac{1.8249(0.1023)997}{8.91 \times 10^{-4}} = 208900$$

$$= 2.0890 \times 10^5$$

$$\frac{D}{\epsilon} = \frac{0.1023}{4.6 \times 10^{-5}} = 2223.9$$

$N_R > 4000$ so flow is turbulent

Friction Losses

$$f = 0.018587 \quad \text{(Swamee-Jain)}$$

$$h_L = f\frac{L}{D}\frac{v^2}{2g}$$

$$= 0.018587\left(\frac{17}{0.1023}\right)0.16974$$

$$= \mathbf{0.52432}\ \mathrm{m}$$

Minor Losses

$$f_T = 0.017$$

$$k_{ent} = 0.04$$

$$k_{valve} = 0.017(340) = 5.78$$

$$h_L = (\Sigma k)\frac{v^2}{2g}$$

$$= 5.82(0.16975)$$

$$= \mathbf{0.98795}\ \mathrm{m}$$

Discharge Pipe

$$v = \frac{0.900/60\,\text{m}^3/\text{s}}{\pi(0.0779\,\text{m})^2/4} = 3.1472\,\text{m/s}$$
$$\frac{v^2}{2g} = 0.50484\,\text{m}$$
$$N_R = \frac{3.1472(0.0779)997}{8.91 \times 10^{-4}} = 274330$$
$$= 2.7433 \times 10^5$$
$$\frac{D}{\epsilon} = \frac{0.0779}{4.6 \times 10^{-5}} = 1693.5$$

Friction Losses

$$f = 0.018941 \quad \text{(Swamee-Jain)}$$
$$h_L = f\frac{L}{D}\frac{v^2}{2g}$$
$$= 0.018941\left(\frac{163.3}{0.0779}\right)0.50484$$
$$= \mathbf{20.045}\,\text{m}$$

Minor Losses

$$f_T = 0.018$$
$$k_{exit} = 1$$
$$k_{elbows} = 3 \times 0.018(30)$$
$$= 1.62$$
$$k_{valve} = 0.018(340) = 6.12$$

$$h_L = (\Sigma k)\frac{v^2}{2g}$$
$$= 8.74(0.50484)$$
$$= \mathbf{4.4123}\,\text{m}$$

Total Head-loss

$$h_L = 0.52432 + 0.98795$$
$$+ 20.045 + 4.4123$$
$$h_L = 25.970\,\text{m}$$

General Energy Equation

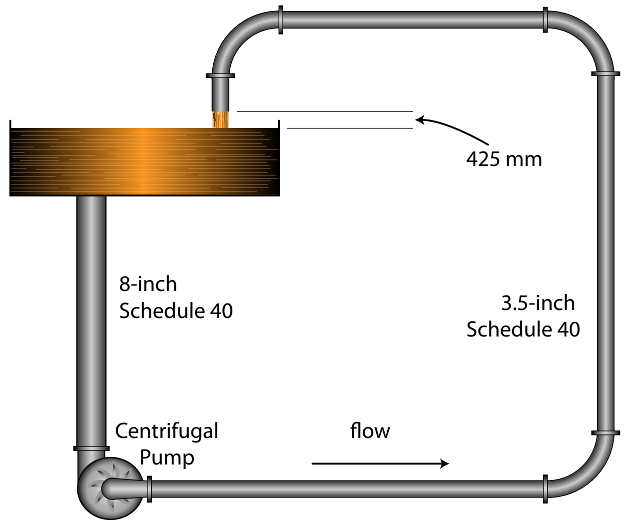
$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_A - h_L = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$
$$0 + 0 + 0 + h_A - 25.970 = 0 + 12.75 + 0$$
$$h_A = 38.720\,\text{m}$$

$$P_{\text{added}} = h_A \gamma Q$$
$$= 38.720(9.78)(0.900/60)$$
$$= 5.6802 \text{ kW}$$

$$P_{\text{in}} = \frac{P_{\text{added}}}{e_M}$$
$$= \frac{5.6802}{.78}$$
$$= 7.2823 \text{ kW}$$

$$\boldsymbol{P_{\text{in}} = 7.28\,\text{kW}}$$

Example 4:



Heavy machine oil (sg=0.89, $\eta = 3.80 \times 10^{-2}$ Pa·s) is circulated through a system repeatedly to test its stability.

The 8-inch Schedule steel pipe on the suction side of the pump has a square entrance and a length of 6.25 m and the 3.5-inch Schedule steel pipe on the discharge side of the pump has a length of 18.0 m.

(Note that the 3.5-inch discharges into the atmosphere above the tank so there is no exit loss in this question!)

All elbows are long radius. The flow rate through the system is 13.5 L/s.

Determine the head added by the pump.

Solution:

Suction Pipe

$$\begin{aligned} v &= \frac{0.0135\ \text{m}^3/\text{s}}{\pi(0.2027\,\text{m})^2/4} = 0.41835\,\text{m/s} \\ \frac{v^2}{2g} &= 0.0089202\,\text{m} \\ N_R &= \frac{0.41835(0.2027)890}{3.80 \times 10^{-2}} \\ &= 1986.1 \end{aligned}$$

$N_R < 2000$ so flow is **laminar** (and D/ϵ is not required)

Friction losses:

$$\begin{aligned} f &= \frac{64}{N_R} = \frac{64}{1986.1} = 0.032224 \\ h_L &= 0.032224\left(\frac{6.25}{0.2027}\right)0.0089202\,\text{m} \\ &= \mathbf{0.0088630}\,\text{m} \end{aligned}$$

Minor Losses:

$$\begin{aligned} k_{ent} &= 0.5 \\ h_L &= 0.5 \times 0.0089202\,\text{m} \\ &= \mathbf{0.0044601}\,\text{m} \end{aligned}$$

Discharge Pipe

$$\begin{aligned} v &= \frac{0.0135\ \text{m}^3/\text{s}}{\pi(0.0901)^2/4} = 2.1174\ \text{m/s} \\ \frac{v^2}{2g} &= 0.22850\,\text{m} \\ N_R &= \frac{2.1174(0.0901)890}{3.80 \times 10^{-2}} = 4468.2 \\ \frac{D}{\epsilon} &= \frac{0.0901}{4.6 \times 10^{-5}} \end{aligned}$$

$N_R > 4000$ so flow is turbulent.

Friction losses:

$$\begin{aligned} f &= 0.039\ (\text{Moody}) \\ &= (0.039792\ \text{from S-J}) \\ h_L &= 0.039\left(\frac{18.0}{0.0901}\right)0.22850\,\text{m} \\ &= \mathbf{1.7803}\,\text{m} \end{aligned}$$

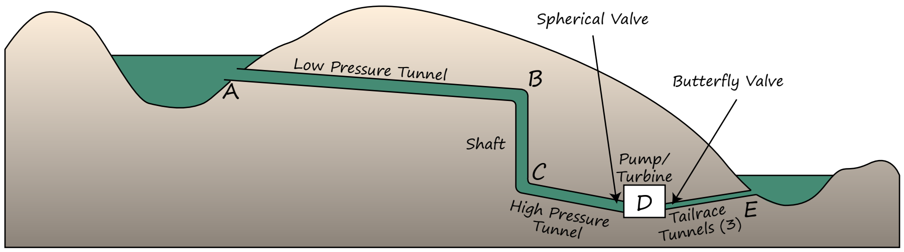
Minor losses:

$$\begin{aligned} f_T &= 0.0165 \quad (\text{Moody}) \\ 3 \times k_{elb} &= 3(0.0165)(20) = 0.99 \\ h_L &= 0.99 \times 0.22850 \\ &= \mathbf{0.22622}\,\text{m} \end{aligned}$$

Total headloss = 2.0198 m

$$\begin{aligned} \cancel{\frac{P_A}{\gamma}} + \cancel{z_A} + \cancel{\frac{v_A^2}{2g}} + h_A - h_L &= \cancel{\frac{P_B}{\gamma}} + z_B + \frac{v_B^2}{2g} \\ h_A - 2.0198 &= 0.425 + 0.22850 \\ &= 2.6733\,\text{m} \\ \boldsymbol{h_A} &= \mathbf{2.67}\,\text{m} \end{aligned}$$

Example 5:



The system illustrated is a pumped storage system. During periods of high demand for electricity, water flows down from the upper lake and drives the turbine at D. (During periods of low demand when electricity is cheap, such as at night-time, D acts as a pump and pumps water back up to the upper lake.)

At times of maximum demand, the system has a maximum volume flow rate of 420 m^3/s. Base your calculations on this flow. The water is at 10°C.

The difference in elevation between the surfaces of the two lakes is 542 m.

The low pressure tunnel from A to B is 1700 m in length, has a diameter of 10.5 m and is lined with concrete. The shaft and high pressure tunnel from B to D is 1140 m in length, has a diameter of 10.5 m and is lined with welded steel.

There are three tailrace tunnels from the turbine to the lower reservoir with the flow equally distributed between them. Each tailrace tunnel is 382 m in length, has a diameter of 8.5 m and is lined with concrete.

The entrance to the low pressure tunnel at the upper lake has an equivalent length ratio of $Le/D = 420$. The bends at B (considered to be part of tunnel BCD) and C are in the steel pipe and each have at equivalent length ratio of 16. A spherical valve at the inlet of the turbine that shuts off flow when the turbines are not operating is hydraulically efficient and has no losses associated with it. Each tailrace tunnel contains a butterfly valve ($Le/D = 20$).

At maximum capacity, the turbine outputs 1800 MW. Determine the efficiency of the turbine at this output.

Solution:

Tunnel AB

$$v = \frac{420\ \text{m}^3/\text{s}}{\pi(10.5\text{m})^2/4} = 4.8504\,\text{m/s}$$

$$\frac{v^2}{2g} = \frac{(4.8504\ \text{m/s})^2}{2 \times 9.81\ \text{m/s}^2} = 1.1991\,\text{m}$$

$$N_R = \frac{4.8504(10.5)1000}{0.0013} = 39176000 = 3.9176 \times 10^7$$

$$\frac{D}{\epsilon_{\text{concrete}}} = \frac{10.5\ \text{m}}{1.2 \times 10^{-3}\ \text{m}} = 8750$$

$N_R > 4000$ so flow is turbulent.

Friction Losses

$$f = 0.0123 \quad \text{(Moody)}$$

$$h_L = 0.0123\left(\frac{1700}{10.5}\right)1.1991 = \mathbf{2.3879}\,\text{m}$$

Minor Losses

$$f_T = 0.0123 \quad \text{(Moody)}$$

$$k_{\text{entrance}} = 0.0123(420) = 5.1660$$

$$h_L = 5.1660(1.1991) = \mathbf{6.1946}\,\text{m}$$

Tunnel BCD

$$
\begin{aligned}
v &= 4.8504\,\mathrm{m/s} \\
\frac{v^2}{2g} &= 1.1991\,\mathrm{m} \\
N_R &= 3.9176 \times 10^7 \\
\frac{D}{\epsilon_{steel}} &= \frac{10.5\ \mathrm{m}}{4.6 \times 10^{-5}\ \mathrm{m}} = 228260
\end{aligned}
$$

$N_R > 4000$ so flow is turbulent.

Friction Losses

Note: Readings for f and f_T lie outside the boundary of the Moody Diagram (because of the high relative roughness and high Reynolds Number) so the Swamee-Jain is probably more accurate than attempting extrapolation from the Moody Diagram.

$$
\begin{aligned}
f &= 0.0077125 \\
h_L &= 0.0077125 \cdot \frac{1140}{10.5} \cdot 1.1991 \\
&= \mathbf{1.0041}\,\mathrm{m}
\end{aligned}
$$

Minor Losses

$$
\begin{aligned}
f_T &= 0.0071174 \\
k_{\text{bend at B}} &= (0.0071174)16 = 0.11388 \\
k_{\text{bend at C}} &= (0.0071174)16 = 0.11388 \\
h_L &= 2(0.11388)1.1991 \\
&= \mathbf{0.27310}\,\mathrm{m}
\end{aligned}
$$

A single tailrace tunnel *DE*

$$
\begin{aligned}
v &= \frac{140\,\mathrm{m}^3/\mathrm{s}}{\pi(8.5\,\mathrm{m})^2/4} = 2.4672\,\mathrm{m/s} \\
\frac{v^2}{2g} &= 0.31025\,\mathrm{m} \\
N_R &= \frac{2.4672(8.5)1000}{0.0013} = 16132000 \\
&= 1.6132 \times 10^7
\end{aligned}
$$

$$
\frac{D}{\epsilon_{concrete}} = \frac{8.5\ \mathrm{m}}{1.2 \times 10^{-3}\ \mathrm{m}} = 7083
$$

$N_R > 4000$ so flow is turbulent.