

## Module 8: Hazen Williams Equation and Equivalent Pipes (CIVL 318)

### Hazen-Williams Equations

$$Q = \frac{C D^{2.63} \left(\frac{h_L}{L}\right)^{0.54}}{279000}, \quad h_L = L \left(\frac{279000 Q}{C D^{2.63}}\right)^{1.852}, \quad D = \left(\frac{279000 Q}{C \left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802}$$

### Equivalent-Length Ratios for Fittings

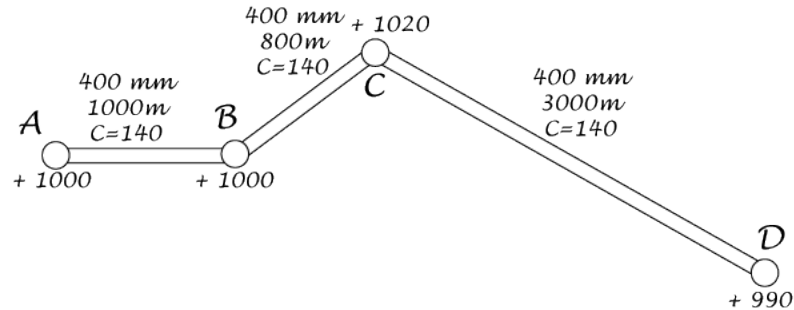
Type	$L_e/D$
Globe valve — fully open	340
Angle valve — fully open	150
Gate valve — fully open	8
— 3/4 open	35
— 1/2 open	160
— 1/4 open	900
Check valve — swing type	100
Check valve — ball type	150
Butterfly valve — fully open — 2-8"	45
— 10-14"	35
— 16-24"	25
Foot valve — poppet disc type	420
Foot valve — hinged disc type	75
90° standard elbow	30
90° long radius elbow	20
90° street elbow	50
45° standard elbow	16
45° street elbow	26
Close return bend	50
Standard tee — flow through run	20
Standard tee — flow through branch	60
Gradual enlargement — 15° cone angle	8
Gradual enlargement — 20° cone angle	15
Gradual enlargement — 30° cone angle	23
Gradual reduction — 15° to 40° cone angle	2
Pipe entrance — inward projecting	50
Pipe entrance — square	25
Pipe entrance — rounded	10
Venturi meter	100

### Example 1

For the pipeline shown, calculate the pressure at  $B$ , given that the pressure at  $A$  is 700 kPa.

The pipes are cement-lined Hyprescon with a diameter of 400 mm and a roughness coefficient of  $C = 140$ . Flow through the system is 200 L/s.

Elevations are as indicated.



### Solution:

First, apply the Hazen-Williams:

$$\begin{aligned} h_{L_{AB}} &= L \left( \frac{279000 Q}{C D^{2.63}} \right)^{1.852} \\ &= 1000 \left( \frac{279000 \times 200}{140 \times 400^{2.63}} \right)^{1.852} \\ &= 4.9903 \text{ m} \end{aligned}$$

Now, apply the GEE:

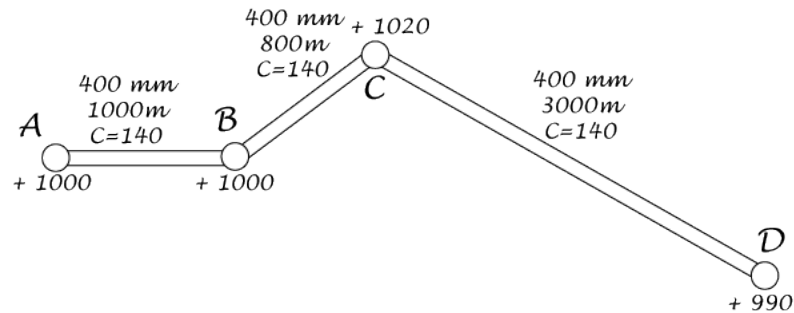
$$\begin{aligned} \frac{P_A}{\gamma} + \cancel{z_A} + \cancel{\frac{v_A^2}{2g}} - h_L &= \frac{P_B}{\gamma} + \cancel{z_B} + \cancel{\frac{v_B^2}{2g}} \\ \frac{700}{9.81} - 4.9903 &= \frac{P_B}{9.81} \\ P_B &= 651.05 \text{ kPa} \\ P_B &= \mathbf{651 \text{ kPa}} \end{aligned}$$

### Exercise 1

For the pipeline shown, calculate the pressure at  $C$  and  $D$ , given that the pressure at  $A$  is 700 kPa.

The pipes are cement-lined Hyprescon with a diameter of 400 mm and a roughness coefficient of  $C = 140$ . Flow through the system is 200 L/s.

Elevations are as indicated.



### Solution:

First, apply the Hazen-Williams:

$$\begin{aligned} h_{L_{BC}} &= L \left( \frac{279000 Q}{C D^{2.63}} \right)^{1.852} \\ &= 800 \left( \frac{279000 \times 200}{140 \times 400^{2.63}} \right)^{1.852} \\ &= 3.9922 \text{ m} \end{aligned}$$

$$\begin{aligned} h_{L_{CD}} &= L \left( \frac{279000 Q}{C D^{2.63}} \right)^{1.852} \\ &= 3000 \left( \frac{279000 \times 200}{140 \times 400^{2.63}} \right)^{1.852} \\ &= 14.971 \text{ m} \end{aligned}$$

Now, apply the GEE:

$$\begin{aligned} \frac{P_B}{\gamma} + z_B + \frac{v_B^2}{2g} - h_L &= \frac{P_C}{\gamma} + z_C + \frac{v_C^2}{2g} \\ \frac{651.05}{9.81} - 3.9922 &= \frac{P_C}{9.81} + 20 \end{aligned}$$

$$P_C = 415.69 \text{ kPa}$$

$$P_C = \mathbf{416 \text{ kPa}}$$

$$\begin{aligned} \frac{P_C}{\gamma} + z_C + \frac{v_C^2}{2g} - h_L &= \frac{P_D}{\gamma} + z_D + \frac{v_D^2}{2g} \\ \frac{415.69}{9.81} + 30 - 14.971 &= \frac{P_D}{9.81} \end{aligned}$$

$$P_D = 563.12 \text{ kPa}$$

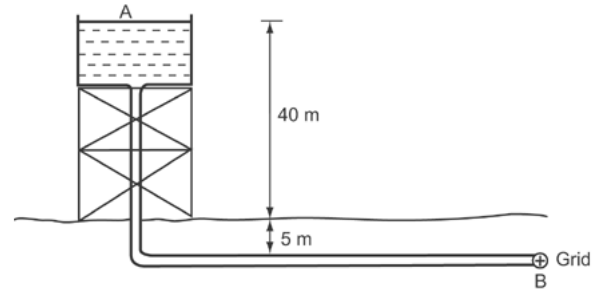
$$P_D = \mathbf{563 \text{ kPa}}$$

## Example 2

Water flows from a storage tank through a welded steel pipe that is 1200 m long and 350 mm in diameter, entering a distribution grid at point 'B'. Assume  $C=100$ . Determine:

- (1) The pressure at 'B' when the flow is 150 L/s
- (2) The maximum flow rate into the grid when the minimum allowable pressure at 'B' is 400 kPa.

Minor losses are negligible compared to friction losses.



### Solution (1):

$$\begin{aligned}
 h_L &= L \left( \frac{279000 Q}{C D^{2.63}} \right)^{1.852} \\
 &= 1200 \left( \frac{279000 \times 150}{100 \times 350^{2.63}} \right)^{1.852} \\
 &= 12.561 \text{ m} \\
 v &= \frac{Q}{A} \\
 &= \frac{0.150}{\pi(0.350)^2/4} \\
 &= 1.5591 \text{ m/s} \\
 \frac{v^2}{2g} &= 0.12389 \text{ m}
 \end{aligned}$$

GEE:

$$\begin{aligned}
 \frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g} - h_L &= \frac{P_B}{\gamma} + z_B + \frac{v_B^2}{2g} \\
 45 - 12.561 &= \frac{P_B}{9.81} + 0.12389 \\
 P_B &= 317.01 \text{ kPa} \\
 P_B &= \mathbf{317 \text{ kPa}}
 \end{aligned}$$

Notice that if we recalculated the pressure at B omitting the velocity head, then  $P_B = 318.2 \text{ kPa}$  (which is not very different from including it).

### Solution (2):

What flow/headloss will give a pressure of 400 kPa at B?

$$\begin{aligned}
 \frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g} - h_L &= \frac{P_B}{\gamma} + z_B + \frac{v_B^2}{2g} \\
 45 - h_L &= \frac{400}{9.81} + \frac{v_B^2}{2g}
 \end{aligned}$$

One equation and two unknowns! We could solve it iteratively, guessing at a flow and seeing what  $P_B$  is for this flow, then trying another flow until we converge on a pressure of 400 kPa at B.

But the velocity head had an effect of about 0.3% in part (1); it will be less here as we need less velocity/headloss to keep the pressure higher. So, in problems of this type, we **simply ignore the velocity head term...**

$$\begin{aligned}
 45 - h_L &= \frac{400}{9.81} + \frac{v_B^2}{2g} \\
 h_L &= 4.2253 \text{ m}
 \end{aligned}$$

What flow will give this headloss?

$$\begin{aligned}
 Q &= \frac{CD^{2.63} \left( \frac{h_L}{L} \right)^{0.54}}{279000} \\
 &= \frac{100 \times 350^{2.63} \left( \frac{4.2253}{1200} \right)^{0.54}}{279000} \\
 &= 83.272 \text{ L/s} \\
 Q &= \mathbf{83.3 \text{ L/s}}
 \end{aligned}$$

Let's look at the value of the velocity head we discarded...

$$\begin{aligned}v &= \frac{Q}{A} \\&= \frac{0.083272}{\pi(0.350)^2/4} \\&= 0.86551 \text{ m/s}\end{aligned}$$

$$\frac{v^2}{2g} = 0.038181 \text{ m}$$

The velocity head is small enough that we can disregard it.  
Any error from not omitting the headloss is negligible  
compared with error in estimating the  $C$ -value.

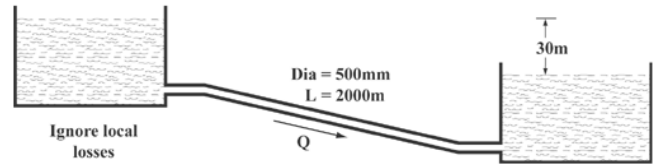
## Exercise 2

Water flows from one reservoir down to another, through a 500 mm diameter pipe that is 2000 m in length. The difference in elevation between the surfaces of the two reservoirs is 30 m.

Determine:

- (1) The flow with high density polyethylene pipe (HDPE) with  $C = 140$
- (2) The flow with welded steel with  $C = 100$
- (3) The diameter of HDPE pipe required for a flow of 1200 L/s

Disregard minor losses.



**Solution (1):** For HDPE.

At the surfaces of both reservoirs, pressure and velocity head are 0 so the GEE reduces to  $30 - h_L = 0$

$$\begin{aligned}
 Q &= \frac{CD^{2.63} \left(\frac{h_L}{L}\right)^{0.54}}{279000} \\
 &= \frac{140(500)^{2.63} \left(\frac{30}{2000}\right)^{0.54}}{279000} \\
 &= 651.48 \text{ L/s} \\
 Q &= \mathbf{651 \text{ L/s}}
 \end{aligned}$$

**Solution (3):** Diameter for a flow of 1200 L/s with HDPE,

$$\begin{aligned}
 D &= \left( \frac{279000 Q}{C \left(\frac{h_L}{L}\right)^{0.54}} \right)^{0.3802} \\
 &= \left( \frac{279000 \times 1200}{140 \left(\frac{30}{2000}\right)^{0.54}} \right)^{0.3802} \\
 &= 630.42 \text{ mm} \\
 D &= \mathbf{630 \text{ mm}}
 \end{aligned}$$

**Solution (2):** For welded steel,

$$\begin{aligned}
 Q &= \frac{CD^{2.63} \left(\frac{h_L}{L}\right)^{0.54}}{279000} \\
 &= \frac{100(500)^{2.63} \left(\frac{30}{2000}\right)^{0.54}}{279000} \\
 &= 465.35 \text{ L/s} \\
 Q &= \mathbf{465 \text{ L/s}}
 \end{aligned}$$

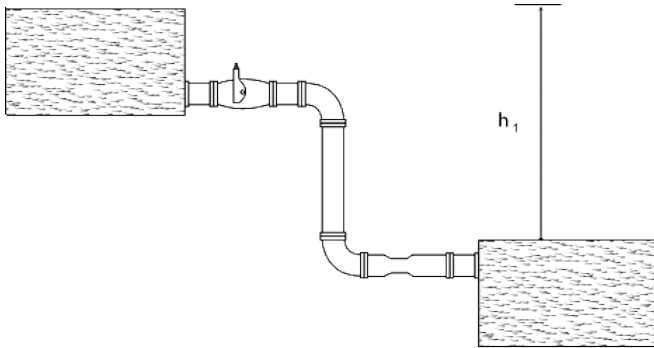
### Example 3

In a water treatment plant, water flows from a filter down to a clear well through the pipe system shown. The pipe is welded steel with a diameter of 300 mm and roughness coefficient  $C = 130$ . The total length of pipe is 50 m. Elevation difference  $h_1$  between the tanks is 5 m.

Equivalent length ratios,  $L_e/D$ , are:

Entrance and exit losses:	50	Butterfly valve:	35
Large radius elbows:	25	Venturi meter:	100

Determine the flow through the system.



#### Solution:

Effective length of the pipe: (length and diameter in metres!)

$$\begin{aligned}
 L_{\text{eff}} &= \text{Actual pipe length} + D \left( \frac{L_e}{D} \right) \\
 &= 50 + 0.3(50 + 35 + 25 + 25 + 100 + 50) \\
 &= 50 + 85.5 \\
 &= 135.5 \text{ m}
 \end{aligned}$$

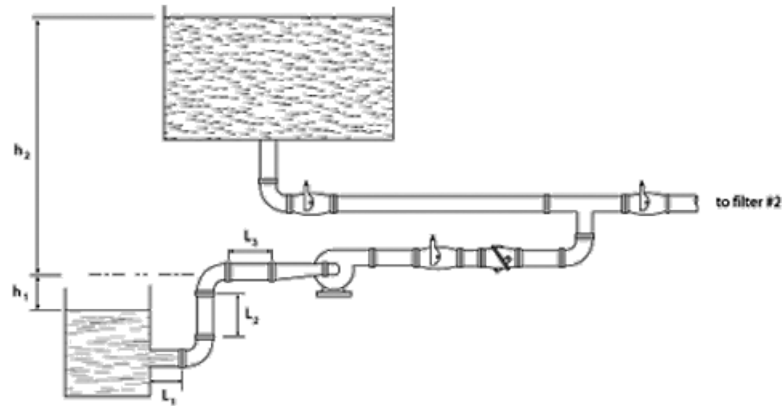
As earlier, headloss between the two surfaces is just the elevation difference:

$$h_L = 5 \text{ m}$$

Find the flow:

$$\begin{aligned}
 Q &= \frac{CD^{2.63} \left( \frac{h_L}{L} \right)^{0.54}}{279000} \\
 &= \frac{130(300)^{2.63} \left( \frac{5}{135.5} \right)^{0.54}}{279000} \\
 &= 256.66 \text{ L/s} \\
 \mathbf{Q} &= \mathbf{257 \text{ L/s}}
 \end{aligned}$$

#### Example 4



In a water treatment plant, backwash water is pumped from the clear well through the pipe system shown to the filter. The required backwash flow is 10 L/s per square meter of filter area (the filter dimensions are 10 m by 15 m). The inlet pipe is made of welded steel ( $C = 130$ ), has a diameter of 1000 mm and a total length ( $L_1 + L_2 + L_3$ ) of 10 m. The outlet pipe, from the pump to the filter, is also welded steel, has a diameter of 700 mm and a length of 70 m.

The two elevation differences are  $h_1 = 2$  m and  $h_2 = 10$  m.

Equivalent length ratios,  $L_e/D$ , are:

Entrance:	10	Elbow (inlet):	25
Eccentric Reducer:	2	Butterfly Valve:	40
Check Valve:	120	Elbow (outlet):	35
Tee Connection:	60		

Neglect exit losses into the filter.

Determine:

- (1) The head losses on the inlet side (clear well to pump)
- (2) The head losses on the outlet side (pump to filter)

#### Solution:

$Q$  required for backwash in the filter:

$$Q = 10 \text{ m} \times 15 \text{ m} \times 0.01 \text{ m}^3/\text{s} = 1.5000 \text{ m}^3/\text{s}$$

(1)

$$L_{eff} = 10 + 1(10 + 25 + 25 + 2) = 72.000 \text{ m}$$

$$h_L = 72 \left( \frac{279000 \times 1500}{130(1000)^{2.63}} \right)^{1.852}$$

$$= 0.19834 \text{ m}$$

$$h_{L(in)} = \mathbf{0.1983 \text{ m}}$$

(2)

$$L_{eff} = 70 + 0.7(40 + 120 + 35 + 60 + 40 + 35)$$

$$= 301.00 \text{ m}$$

$$h_L = 301 \left( \frac{279000 \times 1500}{130(700)^{2.63}} \right)^{1.852}$$

$$= 4.7112 \text{ m}$$

$$h_{L(out)} = \mathbf{4.71 \text{ m}}$$



### Exercise 3

This exercise is a continuation of the previous example. Determine:

- (3) The head added by the pump
- (4) The pressure at the pump outlet

**Solution:**

**(3)**

Apply the GEE between the surface of the clear well ( $W$ ) and the surface of the filter ( $F$ ):

$$\begin{aligned}\frac{P_W}{\gamma} + z_W + \frac{v_W^2}{2g} + h_A - h_L &= \frac{P_F}{\gamma} + z_F + \frac{v_F^2}{2g} \\ h_A - (0.19834 + 4.7112) &= 12 \\ h_A &= 16.910 \text{ m} \\ \mathbf{h_A = 16.91 \text{ m}}\end{aligned}$$

**(4)**

Apply the GEE between the pump ( $P$ ) and the surface of the filter ( $F$ ):

$$\begin{aligned}\frac{P_P}{\gamma} + z_P + \frac{v_P^2}{2g} - h_L &= \frac{P_F}{\gamma} + z_F + \frac{v_F^2}{2g} \\ \frac{P_P}{9.81} + 0 + 0.77430 - 4.7112 &= 0 + 10 + 0 \\ \Rightarrow P_P &= 136.72 \text{ kPa} \\ \mathbf{P_P = 136.7 \text{ kPa}}\end{aligned}$$

### Example 5

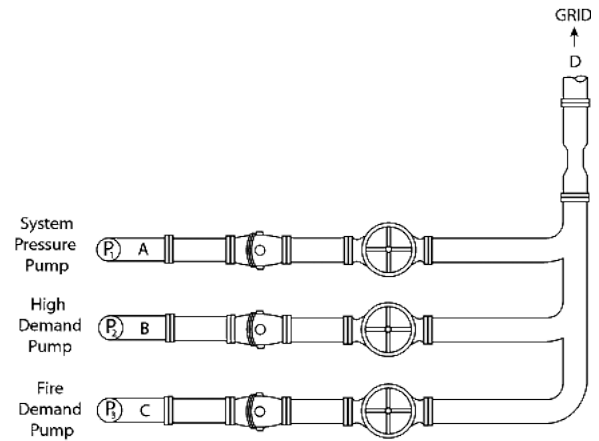
The pumps and piping system are used to supply a municipal grid. Pump  $P_1$  runs continuously and maintains the basic pressure in the distribution grid beyond point  $D$ . There is no flow from pumps  $P_2$  and  $P_3$ . (Pump  $P_2$  is, in addition to  $P_1$ , used during periods of high demand and all pumps are used during fire flow demands.)

The elevations are the same at the pump and the discharge point  $D$ . The outlet pipe, from the pump to point  $D$ , is welded steel ( $C = 130$ ) with a diameter of 200 mm and a total length between fittings of 10 m.

The minimum pressure required at  $D$  is 500 kPa for a design flow of 150 L/s.

Equivalent length ratios,  $L_e/D$ , are:

Check Valve:	120	Gate Valve:	15
Tee Connection:	60	Venturi Meter:	100



Determine:

- (1) the head losses between  $A$  and  $D$
- (2) the pressure at  $A$  required for the required pressure and flow at  $D$

**Solution:**

(1)

$$L_{\text{eff}} = 10 + 0.2(120 + 15 + 60 + 100) = 69.0 \text{ m}$$

$$h_L = 69.0 \left( \frac{279000 \times 150}{130(200)^{2.63}} \right)^{1.852} = 6.7833 \text{ m}$$

$$h_L = \mathbf{6.78 \text{ m}}$$

(2)

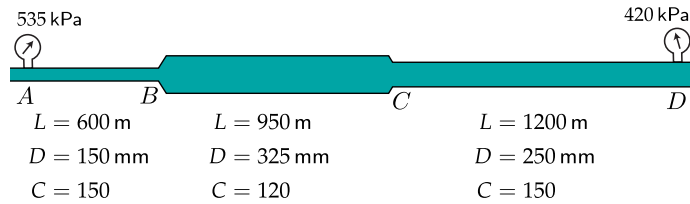
$$\frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g} - h_L = \frac{P_D}{\gamma} + z_D + \frac{v_D^2}{2g}$$

$$\frac{P_A}{9.81} - 6.7833 = \frac{500}{9.81}$$

$$\Rightarrow P_A = 566.54$$

$$P_A = \mathbf{567 \text{ kPa}}$$

### Example 6



Determine  $Q$ , the volume flow rate from  $A$  to  $D$ , through the system shown. Ignore minor losses and assume that  $A$  and  $D$  are at the same elevation.

#### Solution:

First, determine the head loss in each of the three pipes, in terms of  $Q$ :

$$\begin{aligned}
 h_{L_{AB}} &= L \left( \frac{279000Q}{CD^{2.63}} \right)^{1.852} \\
 &= L \left( \frac{279000}{CD^{2.63}} \right)^{1.852} \cdot Q^{1.852} \\
 &= 600 \left( \frac{279000}{150(150)^{2.63}} \right)^{1.852} \cdot Q^{1.852} \\
 &= 0.017142 Q^{1.852} \\
 h_{L_{BC}} &= 950 \left( \frac{279000}{120(325)^{2.63}} \right)^{1.852} \cdot Q^{1.852} \\
 &= 0.00094964 Q^{1.852} \\
 h_{L_{CD}} &= 1200 \left( \frac{279000}{150(250)^{2.63}} \right)^{1.852} \cdot Q^{1.852} \\
 &= 0.0028479 Q^{1.852}
 \end{aligned}$$

Sum these headlosses to get the headloss between  $A$  and  $D$ :

$$\begin{aligned}
 h_{L_{AD}} &= h_{L_{AB}} + h_{L_{BC}} + h_{L_{CD}} \\
 &= (0.017142 + 0.00094964 + 0.0028479) Q^{1.852} \\
 &= 0.020940 Q^{1.852} \text{ m}
 \end{aligned}$$

Use the GEE to approximate a numerical value for  $h_{L_{AD}}$ :

$$\begin{aligned}
 \frac{P_A}{\gamma} + \cancel{z_A} + \frac{v_A^2}{2g} - h_L &= \frac{P_D}{\gamma} + \cancel{z_D} + \frac{v_D^2}{2g} \\
 \Rightarrow h_L &= \frac{P_A - P_D}{2g} + \frac{v_A^2 - v_D^2}{2g}
 \end{aligned}$$

One equation and two unknowns ( $h_L$  and  $Q$ , since  $v_A$  and  $v_D$  can be expressed in terms of the single variable  $Q$ ). Here, we have the difference between the two velocity heads which is smaller than the velocity head at  $A$ . As before, we can ignore it (for now!)

$$\begin{aligned}
 h_L &\approx \frac{P_A - P_D}{2g} \\
 &= \frac{535 - 420}{19.62} \\
 &= 5.8614 \text{ m}
 \end{aligned}$$

Now, find  $Q$ :

$$\begin{aligned}
 h_{L_{AD}} &= 5.8614 = 0.020940 Q^{1.852} \\
 \Rightarrow Q &= \left( \frac{5.8614}{0.020940} \right)^{\frac{1}{1.852}} \\
 &= 20.955 \\
 Q &= \mathbf{21.0 \text{ L/s}}
 \end{aligned}$$

How large was the term we discarded?

$$\begin{aligned}
 \frac{v_A^2 - v_D^2}{2g} &= \frac{\left( \frac{20.955/1000}{\pi(0.150)^2/4} \right)^2 - \left( \frac{20.955/1000}{\pi(0.250)^2/4} \right)^2}{19.62} \\
 &= 0.062381 \text{ m}
 \end{aligned}$$

This is approximately 0.15% of the pressure head at  $D$ .

**Example 7**

- a) Determine the diameter of a pipe with length  $L = 1000$  m and resistance coefficient  $C = 100$  that is equivalent to 785 m of new Schedule 40 12-in steel pipe ( $D = 303.2$  mm,  $C = 130$ ).
- b) Verify that this equivalent pipe has the same headloss as the 12-in steel pipe for two arbitrary flows (choose a couple of flows at random, different from the flow used in part a).

- a) Assume a flow of 100 L/s through the 12-in steel pipe. Calculate the headloss for this flow:

$$h_L = 785 \left( \frac{279000(100)}{130(303.2)^{2.63}} \right)^{1.852} = 4.7995 \text{ m}$$

Now, find the diameter of the equivalent pipe that has a headloss of 4.7995 m for a flow of 100 L/s:

$$D = \left( \frac{279000(100)}{100 \left( \frac{4.7995}{1000} \right)^{0.54}} \right)^{0.3802} = 351.96 \text{ mm}$$

**$D = 352 \text{ mm}$**

- b) 128 L/s and 42 L/s are two flows chosen at random. Compare the headloss in each pipe for both of these flows

First, using  $Q = 128$  L/s:

$$h_{L(12\text{-in})} = 785 \left( \frac{279000(128)}{130(303.2)^{2.63}} \right)^{1.852} = 7.5814 \text{ m}$$

$$h_{L(\text{equiv})} = 1000 \left( \frac{279000(128)}{100(351.96)^{2.63}} \right)^{1.852} = 7.5937 \text{ m}$$

These results are the same except for rounding errors. The errors are noticeable because, in the derivation of the Hazen-Williams solution for diameter, 0.3802 was chosen at the inverse of 2.63 which is not exact. If the exponent  $1/2.63$  is used instead of 0.3802, the headloss values are closer (7.5814 and 7.5780).

For the same reasons, the results would be even closer if headloss were calculated with  $1/0.54$  instead of 1.852; these values are only approximately equal.

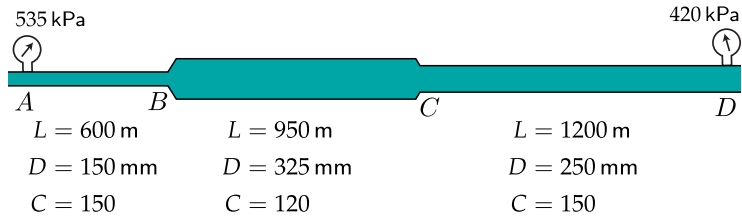
Now, using  $Q = 42$  L/s:

$$h_{L(12\text{-in})} = 785 \left( \frac{279000(42)}{130(303.2)^{2.63}} \right)^{1.852} = 0.96262 \text{ m}$$

$$h_{L(\text{equiv})} = 1000 \left( \frac{279000(42)}{100(351.96)^{2.63}} \right)^{1.852} = 0.96418 \text{ m}$$

Again, these are not exactly the same. You may need to 'cheat' with the last digit in a Qwizm assignment but the result shouldn't be out by more than one digit.

### Example 8



Use the equivalent pipe technique to determine  $Q$ , the volume flow rate from  $A$  to  $D$ , through the system shown. Ignore minor losses and assume that  $A$  and  $D$  are at the same elevation.

#### Solution:

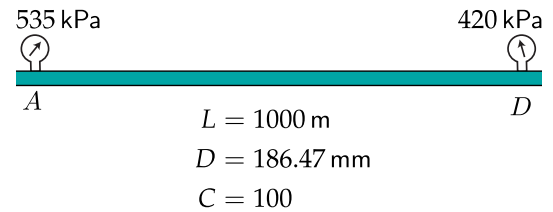
Assume a flow of  $Q = 100 \text{ L/s}$  and determine the headloss between  $A$  and  $D$ :

$$\begin{aligned}
 h_{LAB} &= 600 \left( \frac{279000 \times 100}{150(150)^{2.63}} \right)^{1.852} \\
 &= 86.710 \text{ m} \\
 h_{LBC} &= 950 \left( \frac{279000 \times 100}{120(325)^{2.63}} \right)^{1.852} \\
 &= 4.8035 \text{ m} \\
 h_{LCD} &= 1200 \left( \frac{279000 \times 100}{150(250)^{2.63}} \right)^{1.852} \\
 &= 14.406 \text{ m} \\
 h_{LAD} &= 86.710 + 4.8035 + 14.406 \\
 &= 105.92 \text{ m}
 \end{aligned}$$

Determine the diameter of the pipe, with length 1000 m and resistance coefficient  $C = 100$ , that has a headloss of 105.92 m for flow of 100 L/s.

$$\begin{aligned}
 d_{ADEquiv} &= \left( \frac{279000 \times 100}{100 \left( \frac{105.92}{1000} \right)^{0.54}} \right)^{0.3802} \\
 &= 186.47 \text{ mm}
 \end{aligned}$$

Our problem has now reduced to finding the flow through the single pipe,  $AD_{equiv}$ , shown below.



We apply the GEE to this pipe to find  $h_{LAD_{equiv}}$ , noting that  $A$  and  $D$  are at the same elevation and have the same diameter and, therefore, the same velocity head.

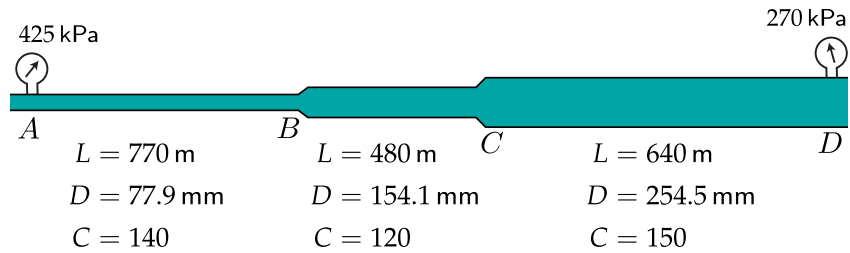
$$\begin{aligned}
 \frac{535}{9.81} - h_{LAD_{equiv}} &= \frac{420}{9.81} \\
 \Rightarrow h_{LAD_{equiv}} &= 5.8614 \text{ m}
 \end{aligned}$$

Note that this is the same loss as we found in Example 6, so ignoring the difference in velocity heads had no numerical effect (to 5 significant digits).

Find  $Q$ :

$$\begin{aligned}
 Q &= \frac{100(186.47)^{2.63} \left( \frac{5.8614}{1000} \right)^{0.54}}{279000} \\
 &= 20.932 \\
 Q &= 20.9 \text{ L/s}
 \end{aligned}$$

### Exercise 4



Use the equivalent pipe technique to determine  $Q$ , the volume flow rate from A to D, through the system shown. Ignore minor losses and assume that A and D are at the same elevation.

#### Solution:

Assume a flow of  $Q = 100 \text{ L/s}$  and determine the headloss between A and D:

$$h_{LAB} = 770 \left( \frac{279000 \times 100}{140(77.9)^{2.63}} \right)^{1.852}$$

$$= 3075.4 \text{ m}$$

$$h_{LBC} = 480 \left( \frac{279000 \times 100}{120(154.1)^{2.63}} \right)^{1.852}$$

$$= 6540.3 \text{ m}$$

$$h_{LCD} = 640 \left( \frac{279000 \times 100}{150(254.5)^{2.63}} \right)^{1.852}$$

$$= 7.0435 \text{ m}$$

$$h_{LAD} = 3075.4 + 6540.3 + 7.0435$$

$$= 9622.7 \text{ m}$$

Determine the diameter of the pipe, with length 1000 m and resistance coefficient  $C = 100$ , that has a headloss of 9622.7 m for flow of 100 L/s.

$$d_{ADEquiv} = \left( \frac{279000 \times 100}{100 \left( \frac{9622.7}{1000} \right)^{0.54}} \right)^{0.3802}$$

$$= 73.882 \text{ mm}$$

Our problem has now reduced to finding the flow through the single pipe,  $AD_{equiv}$

We apply the GEE to this pipe to find  $h_{LAD_{equiv}}$ , noting that A and D are at the same elevation and have the same diameter and, therefore, the same velocity head.

$$\frac{425}{9.81} - h_{LAD_{equiv}} = \frac{270}{9.81}$$

$$\Rightarrow h_{LAD_{equiv}} = 7.9001 \text{ m}$$

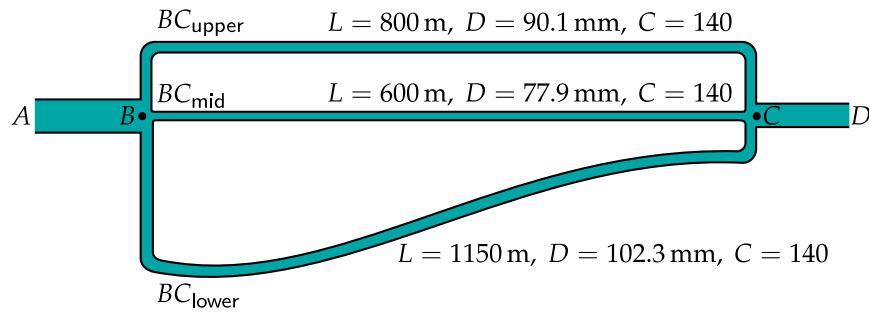
Find  $Q$ :

$$Q = \frac{100(73.882)^{2.63} \left( \frac{7.9001}{1000} \right)^{0.54}}{279000}$$

$$= 2.1561$$

$$Q = 2.16 \text{ L/s}$$

### Example 9



Given a flow of 18 L/s and ignoring minor losses:

- Determine the volume flow rate through each of the parallel pipes between B and C.
- Determine the headloss due to friction between B and C.

**Solution:**

$$h_{L_{BC_{upper}}} = h_{L_{BC_{mid}}} = h_{L_{BC_{lower}}}$$

$$800 \left( \frac{279000 Q_{upper}}{140(90.1)^{2.63}} \right)^{1.852} = 600 \left( \frac{279000 Q_{mid}}{140(77.9)^{2.63}} \right)^{1.852} = 1150 \left( \frac{279000 Q_{lower}}{140(102.3)^{2.63}} \right)^{1.852}$$

$$800 \left( \frac{Q_{upper}}{(90.1)^{2.63}} \right)^{1.852} = 600 \left( \frac{Q_{mid}}{(77.9)^{2.63}} \right)^{1.852} = 1150 \left( \frac{Q_{lower}}{(102.3)^{2.63}} \right)^{1.852}$$

Raise each term to the power 1/1.852:

$$800^{\frac{1}{1.852}} \cdot \frac{Q_{upper}}{(90.1)^{2.63}} = 600^{\frac{1}{1.852}} \cdot \frac{Q_{mid}}{(77.9)^{2.63}} = 1150^{\frac{1}{1.852}} \cdot \frac{Q_{lower}}{(102.3)^{2.63}}$$

$$\frac{Q_{upper}}{12982} = \frac{Q_{mid}}{10342} = \frac{Q_{lower}}{14903}$$

This has established a relationship between the flows in each of the three parallel pipes.

$$Q_{\text{mid}} = \frac{10342}{12982} Q_{\text{upper}} = 0.79664 Q_{\text{upper}}$$

$$Q_{\text{lower}} = \frac{14903}{12982} Q_{\text{upper}} = 1.1480 Q_{\text{upper}}$$

Find the proportion of the flow through the upper pipe and then the flows through each pipe:

$$\frac{Q_{\text{upper}}}{Q_{\text{upper}} + Q_{\text{mid}} + Q_{\text{lower}}} = \frac{Q_{\text{upper}}}{(1 + 0.79664 + 1.1480)Q_{\text{upper}}} = .33960$$

$$Q_{\text{upper}} = 0.33960 \times 18 = 6.1128$$

$$Q_{\text{mid}} = 0.79664 Q_{\text{upper}} = 4.8697$$

$$Q_{\text{lower}} = 1.1480 Q_{\text{upper}} = 7.0175$$

$$Q_{\text{upper}} = 6.11 \text{ L/s}, \quad Q_{\text{mid}} = 4.87 \text{ L/s}, \quad Q_{\text{lower}} = 7.02 \text{ L/s}$$

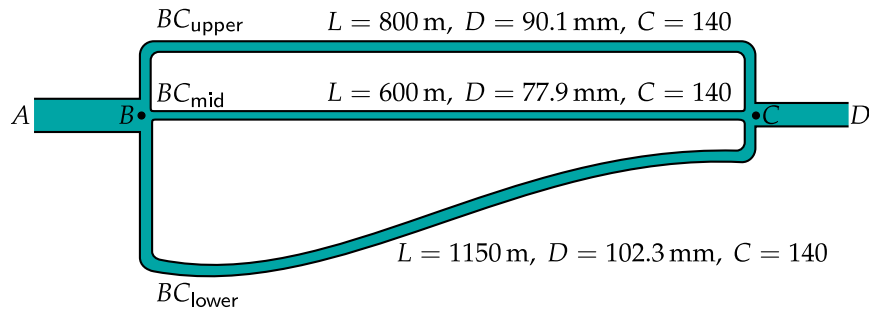
Determine the headloss between *B* and *C*: We can use any of the three parallel pipes to calculate this since they all have the same loss.

$$h_{L_{AB}} = h_{L_{AB_{\text{upper}}}} = 800 \left( \frac{279000 \times 6.1128}{140(90.1)^{2.63}} \right)^{1.852} = 8.8888 \text{ m}$$

$$h_{L_{AB}} = 8.89 \text{ m}$$



### Example 10



Given a flow of 18 L/s and ignoring minor losses:

- Determine the percentage of the flow that goes through each parallel pipe by choosing a convenient headloss between B and C.
- Determine the volume flow rate through each of the parallel pipes.

### Solution:

Assume a head loss of 10 m between B and C. Calculate the flow through each pipe, then sum the flows to get total flow from B to C that causes a headloss of 10 m.

$$Q_{upper} = \frac{140(90.1)^{2.63} \left( \frac{10}{800} \right)^{0.54}}{279000} = 6.5130 \text{ L/s}$$

$$Q_{mid} = \frac{140(77.9)^{2.63} \left( \frac{10}{600} \right)^{0.54}}{279000} = 5.1888 \text{ L/s}$$

$$Q_{lower} = \frac{140(102.3)^{2.63} \left( \frac{10}{1150} \right)^{0.54}}{279000} = 7.4769 \text{ L/s}$$

$$Q_{BC} = 6.5130 + 5.1888 + 7.4769 = 19.178 \text{ L/s}$$

- Percentages of flow through each pipe:

$$BC_{upper} = 34.0\%$$

$$BC_{mid} = 27.1\%$$

$$BC_{lower} = 39.0\%$$

- Flow rate through each pipe:

$$Q_{upper} = 0.33961 \times 18 = 6.1130 = 6.11 \text{ L/s}$$

$$Q_{upper} = 0.27056 \times 18 = 4.8701 = 4.87 \text{ L/s}$$

$$Q_{upper} = 0.38987 \times 18 = 7.0177 = 7.02 \text{ L/s}$$

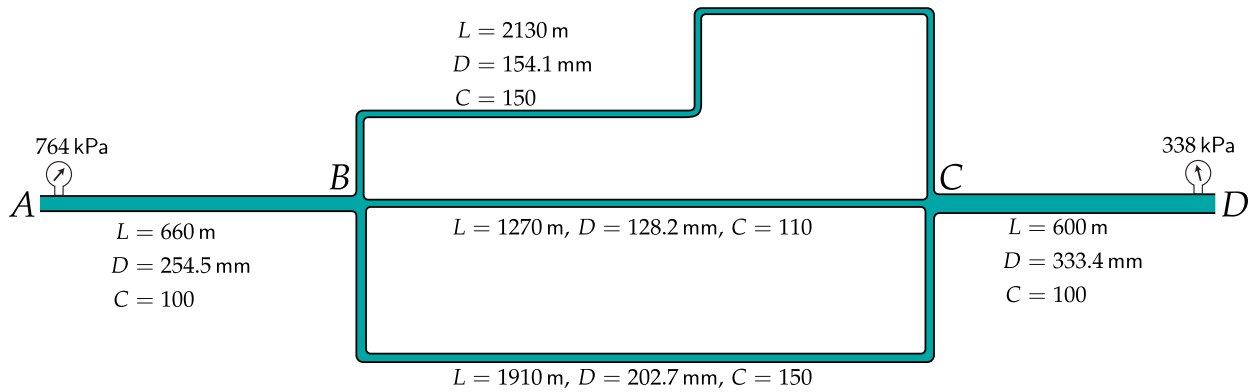
Determine the percentages of the flow that go through each pipe:

$$\frac{Q_{upper}}{Q_{BC}} = \frac{6.5130}{19.178} = 33.961\%$$

$$\frac{Q_{mid}}{Q_{BC}} = \frac{5.1888}{19.178} = 27.056\%$$

$$\frac{Q_{lower}}{Q_{BC}} = \frac{7.4769}{19.178} = 38.987\%$$

### Example 11



A, B, C and D are at the same elevation. Determine the flow through the system from A to D. (Ignore minor losses.)

### Solution:

Assume a head loss of 10 m between B and C. Calculate the flow through each pipe, then sum the flows to get total flow from B to C that causes a headloss of 10 m.

$$Q_{BCupper} = \frac{150(154.1)^{2.63} \left( \frac{10}{2130} \right)^{0.54}}{279000} = 16.869 \text{ L/s}$$

$$Q_{BCmid} = \frac{110(128.2)^{2.63} \left( \frac{10}{1270} \right)^{0.54}}{279000} = 10.080 \text{ L/s}$$

$$Q_{BClower} = \frac{150(202.7)^{2.63} \left( \frac{10}{1910} \right)^{0.54}}{279000} = 36.792 \text{ L/s}$$

$$Q_{BCequiv} = 16.869 + 10.080 + 36.792 = 63.741 \text{ L/s}$$

Determine the diameter of the equivalent pipe, with length of 1000 m and resistance coefficient of 100, that has a flow of 63.741 L/s for a headloss of 10 m:

$$D = \left( \frac{279000 \times 63.741}{100 \left( \frac{10}{1000} \right)^{0.54}} \right)^{0.3802} = 255.08 \text{ mm}$$

Now there are three pipes in series: AB, BCEquiv and CD. For pipes in series, we assume a flow of 100 L/s and find the total headloss between A and D:

$$h_{LAB} = 660 \left( \frac{279000 \times 100}{100(254.5)^{2.63}} \right)^{1.852} = 15.391$$

$$h_{LBCEquiv} = 100 \left( \frac{279000 \times 100}{100(255.08)^{2.63}} \right)^{1.852} = 23.063$$

$$h_{LCD} = 600 \left( \frac{279000 \times 100}{100(333.4)^{2.63}} \right)^{1.852} = 3.7553$$

$$h_{LAD} = 15.391 + 23.063 + 3.7553 = 42.209 \text{ m}$$

Determine the diameter of the equivalent pipe, with length of 1000 m and resistance coefficient of 100, that has a flow of 100 L/s for a headloss of 42.209 m:

$$D = \left( \frac{279000 \times 100}{100 \left( \frac{42.209}{1000} \right)^{0.54}} \right)^{0.3802} = 225.23 \text{ mm}$$

The headloss between A and D is:

$$h_{LAD} = \frac{764 - 338}{9.81} = 43.425 \text{ m}$$

The flow through the system,  $Q_{AB}$ , is:

$$Q_{AB} = \frac{100(225.23)^{2.63} \left( \frac{43.425}{1000} \right)^{0.54}}{279000} = 101.43 \text{ L/s}$$

$$Q_{AB} = 101.4 \text{ L/s}$$