

Module 7: Series A Pipeline (CIVL 318)

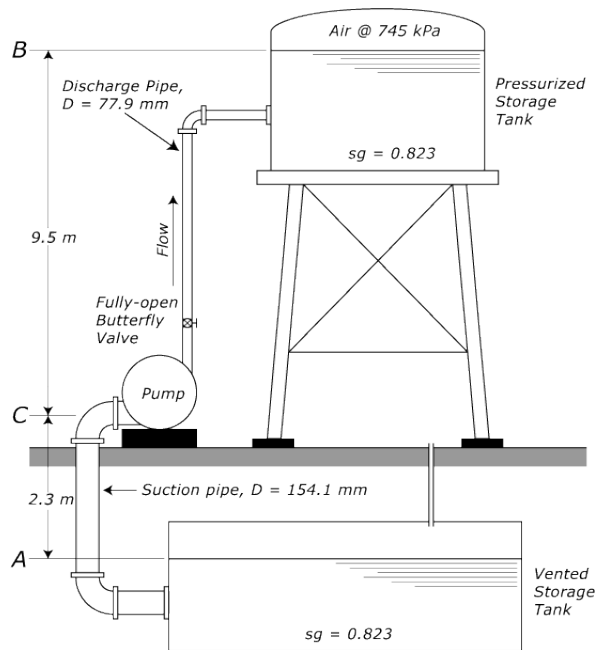
Example 1:

A pump delivers 13.5 L/s of kerosene at 25°C from an underground vented storage tank to an elevated storage tank pressurized to 745 kPa.

The suction pipe is 6-in Schedule 40 steel pipe and is 5.0 m long. It has a round-edged entrance with a radius of $r = 15$ mm.

The discharge pipe is 3-in Schedule 40 steel pipe, is 11.0 m long and includes a fully open butterfly valve with $L_e/D = 45$.

All elbows are “standard” with $L_e/D = 30$.



Solution:

Suction Pipe

$$v = \frac{0.0135 \text{ m}^3/\text{s}}{\pi(0.1540 \text{ m})^2/4} = 0.72383 \text{ m/s}$$

$$\frac{v^2}{2g} = 0.026704 \text{ m}$$

$$N_R = \frac{0.72383(0.1541)823}{1.64 \times 10^{-3}} = 55975 \approx 5.6 \times 10^4$$

Friction Losses:

$$\frac{D}{\epsilon} = \frac{0.1541}{4.6 \times 10^{-5}} = 3350$$

$$f = 0.0215 \quad (\text{Moody})$$

$$= 0.02144 \quad (\text{Swamee-Jain})$$

$$h_L = 0.0215 \left(\frac{5.0}{0.1541} \right) 0.026704$$

$$= \mathbf{0.018629 \text{ m}}$$

Minor Losses:

$$f_T = 0.015 \quad (\text{Moody})$$

$$K_{\text{entrance}} = 0.09 \quad (r/D = 0.1)$$

$$K_{\text{elbow}} = f_T \left(\frac{L_e}{D} \right) = 0.015(30) = 0.45$$

$$h_{L_{\text{minor}}} = K_{\text{entrance}} \frac{v^2}{2g} + 2K_{\text{elbow}} \frac{v^2}{2g}$$

$$= (0.09 + 2 \times 0.45) 0.026704$$

$$= \mathbf{0.026437 \text{ m}}$$

Discharge Pipe

$$v = \frac{0.0135 \text{ m}^3/\text{s}}{\pi(0.0779 \text{ m})^2/4} = 2.8325 \text{ m/s}$$

$$\frac{v^2}{2g} = 0.40892 \text{ m}$$

$$N_R = \frac{2.8325(0.0779)823}{1.64 \times 10^{-3}} = 110730 \approx 1.1 \times 10^5$$

Friction Losses:

$$\frac{D}{\epsilon} = \frac{0.0779}{4.6 \times 10^{-5}} = 1693$$

$$f = 0.0205 \quad (\text{Moody})$$

$$h_L = 0.0205 \left(\frac{11.0}{0.0779} \right) 0.40892$$

$$= \mathbf{1.1837 \text{ m}}$$

Minor Losses:

$$f_T = 0.018 \quad (\text{Table})$$

$$K_{\text{elbow}} = f_T \left(\frac{L_e}{D} \right) = 0.018(30) = 0.54$$

$$K_{\text{valve}} = f_T \left(\frac{L_e}{D} \right) = 0.018(45) = 0.81$$

$$K_{\text{exit}} = 1$$

$$h_{L_{\text{minor}}} = (0.54 + 0.81 + 1) 0.40892 \\ = \mathbf{0.96096 \text{ m}}$$

Total head losses:

$$h_L = 0.018629 + 0.026437 \\ + 1.1837 + 0.96096 \\ = \mathbf{2.1897 \text{ m}}$$

General Energy Equation:

$$\frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g} + h_A - h_L = \frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g} \\ h_A - 2.1897 = \frac{745}{0.823 \times 9.81} + 11.8 \\ h_A = 106.27 \text{ m}$$

$$P_{\text{added}} = h_A \gamma Q \\ = 106.27 (0.823 \times 9.81) 0.0135 \\ = 11.583$$

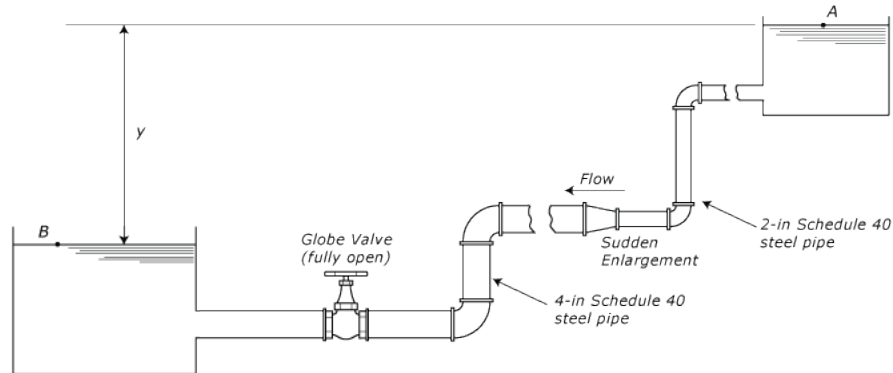
$$P_I = \frac{11.583}{0.73} \\ = \mathbf{15.87 \text{ kW}}$$

Example 2:

Gasoline at 25 °C flows under gravity from tank *A* to tank *B*; both tanks are open to the atmosphere.

The 2-in Schedule 40 steel pipe has a square entrance is 45.7 m long. The 4-in Schedule 40 steel pipe contains a fully-open globe valve and is 87.5 m long. There is a sudden enlargement between the two pipes, as shown. Both pipes are new commercial steel. All elbows are standard 90°.

Determine the difference in surface elevation between tanks *A* and *B* required to maintain a flow of 425 L/min.

2-in Pipe

Minor Losses:

$$v = \frac{0.425/60 \text{ m}^3/\text{s}}{\pi(0.0525 \text{ m})^2/4} = 3.2721 \text{ m/s}$$

$$\frac{v^2}{2g} = 0.54571 \text{ m}$$

$$N_R = \frac{3.2721(0.0525)680}{2.87 \times 10^{-4}} = 407020 = 4.07 \times 10^5$$

$N_R > 4000$ so flow is turbulent.

$$\begin{aligned} f_T &= 0.0192 \quad (\text{Moody}) \\ &= 0.019026 \quad (\text{Swamee-Jain}) \\ &= 0.019 \quad (\text{from table for steel pipe}) \end{aligned}$$

Use $f = 0.019$

$$K_{\text{entrance}} = 0.5$$

$$\begin{aligned} 2 \times K_{\text{elbow}} &= 2 \times f_T \left(\frac{L_e}{D} \right) \\ &= 2 \times 0.019(30) = 1.14 \end{aligned}$$

$$K_{\text{enlargement}} = 0.52$$

$$\Sigma K = 2.16$$

$$\begin{aligned} h_{L_{\text{minor}}} &= \Sigma K \times \frac{v^2}{2g} \\ &= (2.16)0.54571 \\ &= \mathbf{1.1787 \text{ m}} \end{aligned}$$

Friction Losses:

$$\frac{D}{\epsilon} = \frac{0.0525}{4.6 \times 10^{-5}} = 1141.3$$

$$\begin{aligned} f &= 0.0203 \quad (\text{Moody}) \\ &= 0.019945 \quad (\text{Swamee-Jain}) \end{aligned}$$

Using $f = 0.0203$

$$\begin{aligned} h_L &= 0.019 \left(\frac{45.7}{0.0525} \right) 0.54571 \\ &= \mathbf{9.6431 \text{ m}} \end{aligned}$$

4-in Pipe

$$v = \frac{0.425/60 \text{ m}^3/\text{s}}{\pi(0.1023 \text{ m})^2/4} = 0.86178 \text{ m/s}$$
$$\frac{v^2}{2g} = 0.037852 \text{ m}$$
$$N_R = \frac{0.86178(0.1023)680}{2.87 \times 10^{-4}}$$
$$= 208880 = 2.08 \times 10^5$$

$N_R > 4000$ so flow is turbulent.

Friction Losses:

$$\frac{D}{\epsilon} = \frac{0.1023}{4.6 \times 10^{-5}} = 2223.9$$
$$f = 0.0177 \quad (\text{Moody})$$
$$= 0.017661 \quad (\text{Swamee-Jain})$$

Using $f = 0.0177$

$$h_L = 0.0177 \left(\frac{87.5}{0.1023} \right) 0.037852$$
$$= \mathbf{0.57305 \text{ m}}$$

Minor Losses:

$$f_T = 0.0162 \quad (\text{Moody})$$
$$= 0.016319 \quad (\text{Swamee-Jain})$$
$$= 0.017 \quad (\text{from table for steel pipe})$$

Use $f_T = 0.017$

$$K_{\text{exit}} = 1$$

$$2 \times K_{\text{elbow}} = 2 \times f_T \left(\frac{L_e}{D} \right)$$
$$= 2 \times 0.017(30) = 1.02$$

$$K_{\text{globe}} = f_T \left(\frac{L_e}{D} \right)$$
$$= 0.017(340) = 5.78$$

$$\Sigma K = 7.8$$

$$h_{L_{\text{minor}}} = \Sigma K \times \frac{v^2}{2g}$$
$$= (7.8)0.037852$$
$$= \mathbf{0.29525 \text{ m}}$$

Total losses in system:

$$h_L = 9.6431 + 1.1787$$
$$+ 0.57305 + 0.29525$$
$$= 11.690 \text{ m}$$

Find y

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} - h_L = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$
$$0 + y + 0 - 11.690 = 0 + 0 + 0$$
$$y = 11.690 \text{ m}$$

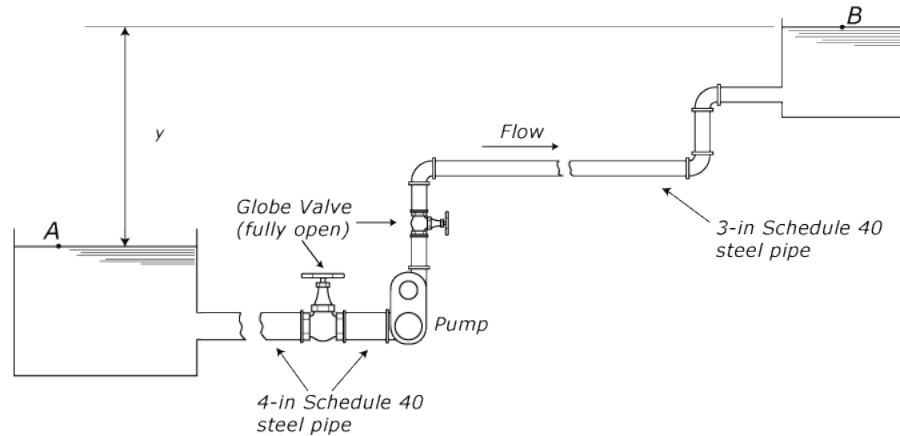
Example 3:

Water at 25 °C is pumped from tank *A* to tank *B*. Both tanks are open to the atmosphere.

The suction pipe is 4-in Schedule 40 steel pipe, has a well-rounded ($r/D > 0.15$) entrance, contains a fully open globe valve, and is 17.0 m long. The discharge pipe is 3-in Schedule 40 steel pipe, contains a fully open globe valve and three standard 90° elbows; it is 163.3 m long.

The elevation difference between *A* and *B* is 12.75 m and the volume flow rate is $Q = 900$ L/min.

If the pump is 78% efficient, determine the electrical power it uses.



Solution:

Suction Pipe

$$v = \frac{0.900/60 \text{ m}^3/\text{s}}{\pi(0.1023 \text{ m})^2/4} = 1.8249 \text{ m/s}$$

$$\frac{v^2}{2g} = 0.16974 \text{ m}$$

$$N_R = \frac{1.8249(0.1023)997}{8.91 \times 10^{-4}} = 208900$$

$$\approx 2.1 \times 10^5$$

$$\frac{D}{\epsilon} = \frac{0.1023}{4.6 \times 10^{-5}} = 2223.9$$

$$f_{(s-J)} = 0.018587$$

Head-loss (Friction)

$$h_L = f \frac{L}{D} \frac{v^2}{2g}$$

$$= 0.018587 \left(\frac{17}{0.1023} \right) 0.16974$$

$$= 0.52432 \text{ m}$$

Head-loss (Minor)

$$f_T = 0.017$$

$$k_{ent} = 0.04$$

$$k_{valve} = 0.017(340) = 5.78$$

$$h_L = (\Sigma k) \frac{v^2}{2g}$$

$$= 5.82(0.16975)$$

$$= 0.98795 \text{ m}$$

Discharge Pipe

$$v = \frac{0.900/60 \text{ m}^3/\text{s}}{\pi(0.0779 \text{ m})^2/4} = 3.1472 \text{ m/s}$$

$$\frac{v^2}{2g} = 0.50484 \text{ m}$$

$$N_R = \frac{3.1472(0.0779)997}{8.91 \times 10^{-4}} = 274330$$

$$\approx 2.74 \times 10^5$$

$$\frac{D}{\epsilon} = \frac{0.0779}{4.6 \times 10^{-5}} = 1693.5$$

$$f_{(s-J)} = 0.018941$$

Head-loss (Friction)

$$\begin{aligned}h_L &= f \frac{L}{D} \frac{v^2}{2g} \\&= 0.018941 \left(\frac{163.3}{0.0779} \right) 0.50484 \\&= 20.045 \text{ m}\end{aligned}$$

Head-loss (Minor)

$$\begin{aligned}f_T &= 0.018 \\k_{exit} &= 1 \\k_{elbows} &= 3 \times 0.018(30) \\&= 1.62 \\k_{valve} &= 0.018(340) = 6.12 \\h_L &= (\Sigma k) \frac{v^2}{2g} \\&= 8.74(0.50484) \\&= 4.4123 \text{ m}\end{aligned}$$

Total Head-loss

$$\begin{aligned}&0.52432 \\&0.98795 \\&20.045 \\&\underline{4.4123} \\h_L &= 25.970\end{aligned}$$

General Energy Equation

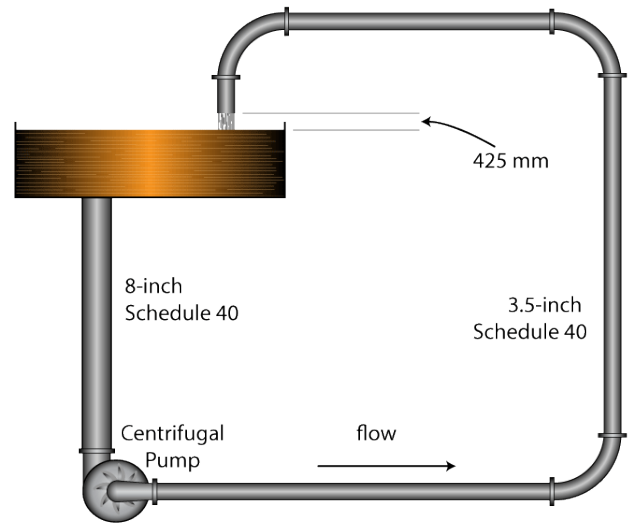
$$\begin{aligned}\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} + h_A - h_L &= \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g} \\0 + 0 + 0 + h_A - 25.970 &= 0 + 12.75 + 0 \\h_A &= 38.720 \text{ m} \\pow_{added} &= h_A \gamma Q \\&= 38.720(9.78)(0.900/60) \\&= 5.6802 \text{ kW} \\pow_{drawn} &= \frac{pow_{added}}{e_M} \\&= \frac{5.6802}{.78} \\&= 7.28 \text{ kW}\end{aligned}$$

Example 4:

Heavy machine oil ($sg=0.89$ and $\eta = 3.80 \times 10^{-2} \text{ Pa} \cdot \text{s}$) is circulated through a system repeatedly to test its stability.

The 8-inch Schedule steel pipe on the suction side of the pump has a square entrance and a length of 6.25 m and the 3.5-inch Schedule steel pipe on the discharge side of the pump has a length of 18.0 m. All elbows are long radius. The flow rate through the system is 13.5 L/s.

Determine the head added by the pump.



Suction Pipe:

$$v = \frac{0.0135 \text{ m}^3/\text{s}}{\pi(0.2027)^2/4} = 0.41835 \text{ m/s}$$
$$\frac{v^2}{2g} = 0.0089202 \text{ m}$$
$$N_R = \frac{0.41835(0.2027)890}{3.80 \times 10^{-2}} = 1986.1$$

Flow is laminar.

Friction losses:

$$f = \frac{64}{N_R} = \frac{64}{1986.1} = 0.032224$$
$$h_L = 0.032224 \left(\frac{6.25}{0.2027} \right) 0.0089202 \text{ m}$$
$$= 0.0088630 \text{ m}$$

Minor Losses:

$$k_{ent} = 0.5$$
$$h_L = 0.5 \times 0.0089202 \text{ m}$$
$$= 0.0044601 \text{ m}$$

Discharge Pipe:

$$v = \frac{0.0135 \text{ m}^3/\text{s}}{\pi(0.0901)^2/4} = 2.1174 \text{ m/s}$$
$$\frac{v^2}{2g} = 0.22850 \text{ m}$$
$$N_R = \frac{2.1174(0.0901)890}{3.80 \times 10^{-2}} = 4468.2$$

Flow is turbulent.

Friction losses:

$$\frac{D}{\epsilon} = \frac{0.0901}{4.6 \times 10^{-5}}$$
$$f = 0.039 \text{ (Moody)}$$
$$= (0.039792 \text{ from S-J})$$
$$h_L = 0.039 \left(\frac{18.0}{0.0901} \right) 0.22850 \text{ m}$$
$$= 1.7803 \text{ m}$$

Minor losses:

$$f_T = 0.0165 \text{ Moody}$$
$$3 \times k_{elb} = 3(0.0165)(20) = 0.99$$
$$h_L = 0.99 \times 0.22850 = 0.22622 \text{ m}$$

Total headloss = 2.0198 m

Head added by the pump is found by applying the GEE to the surface of the tank (A) and the outlet from the discharge pipe (B):

$$\frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g} + h_A - h_L = \frac{P_B}{\gamma} + z_B + \frac{v_B^2}{2g}$$

$$h_A - 2.0198 = 0.425 + 0.22850$$

$$h_A \approx 2.67 \text{ m}$$

Example 5:

The system illustrated is a pumped storage system. During periods of high demand for electricity, water flows down from the upper lake and drives the turbine at D . (During periods of low demand when electricity is cheap, such as at night-time, D acts as a pump and pumps water back up to the upper lake.)

At times of maximum demand, the system has a maximum volume flow rate of $420 \text{ m}^3/\text{s}$. Base your calculations on this flow. The water is at 10°C .

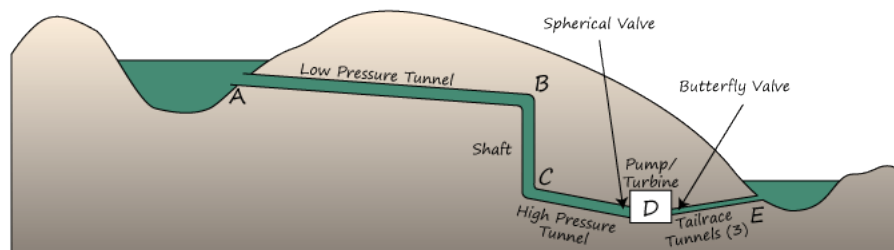
The difference in elevation between the surfaces of the two lakes is 542 m .

The low pressure tunnel from A to B is 1700 m in length, has a diameter of 10.5 m and is lined with concrete. The shaft and high pressure tunnel from B to D is 1140 m in length, has a diameter of 10.5 m and is lined with welded steel.

There are three tailrace tunnels from the turbine to the lower reservoir with the flow equally distributed between them. Each tailrace tunnel is 382 m in length, has a diameter of 8.5 m and is lined with concrete.

The entrance to the low pressure tunnel at the upper lake has an equivalent length ratio of $Le/D = 420$. The bends at B and C are in the steel pipe and each have an equivalent length ratio of 16 . A spherical valve at the inlet of the turbine that shuts off flow when the turbines are not operating is hydraulically efficient and has no losses associated with it. Each tailrace tunnel contains a butterfly valve ($Le/D = 20$).

At maximum capacity, the turbine outputs 1800 MW . Determine the efficiency of the turbine at this output.

**Solution:**
Pipe AB

Velocity and velocity head in the 10.5 m diameter tunnel:

$$v = \frac{Q}{A} = \frac{420 \text{ m}^3/\text{s}}{\pi(10.5\text{m})^2/4} = 4.8504 \text{ m/s}$$

$$\frac{v^2}{2g} = \frac{(4.8504 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} = 1.1991 \text{ m}$$

Reynolds Number, relative roughness and friction factors for tunnel AB:

$$N_R = \frac{vD\rho}{\eta} = \frac{4.8504 \text{ m/s} \times 10.5 \text{ m} \times 1000 \text{ kg/m}^3}{0.0013 \text{ Pa} \cdot \text{s}} = 39176000$$

$$\frac{D}{\epsilon_{\text{concrete}}} = \frac{10.5 \text{ m}}{1.2 \times 10^{-3} \text{ m}} = 8750$$

Using Swamee-Jain:

$$f = \frac{0.25}{\left[\log \left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_R^{0.9}} \right) \right]^2} = \frac{0.25}{\left[\log \left(\frac{1}{3.7(8750)} + \frac{5.74}{3917600^{0.9}} \right) \right]^2} = 0.012354$$

$$f_T = \frac{0.25}{\left[\log \left(\frac{1}{3.7(8750)} \right) \right]^2} = 0.012290$$

Using Moody: Readings from the Moody Diagram for f and f_T are both approximately 0.0123. This is the value I shall use.

Losses for tunnel AB:

$$\text{Entrance Losses: } h_L = k \frac{v^2}{2g} = f_T \left(\frac{L_e}{D} \right) \frac{v^2}{2g} = 0.0123 \times 420 \times 1.1991 = 6.1946$$

$$\text{Friction Losses: } h_L = f \cdot \frac{L}{D} \cdot \frac{v^2}{2g} = 0.0123 \cdot \frac{1700}{10.5} \cdot 1.1991 = 2.3879$$

$$h_{LAB} = 6.1946 + 2.3879 = 8.5825 \text{ m}$$

Tunnel BCD

Velocity, velocity head and Reynolds Number for tunnel BCD

$$v = 4.8504 \text{ m/s}, \frac{v^2}{2g} = 1.1991 \text{ m}, N_R = 39176000$$

Relative roughness and friction factors for tunnel BCD:

$$\frac{D}{\epsilon_{steel}} = \frac{10.5 \text{ m}}{4.6 \times 10^{-5} \text{ m}} = 228260$$

Using Swamee-Jain:

$$f = \frac{0.25}{\left[\log \left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_R^{0.9}} \right) \right]^2} = \frac{0.25}{\left[\log \left(\frac{1}{3.7(228260)} + \frac{5.74}{39176000^{0.9}} \right) \right]^2} = 0.0077125 \approx 0.0077$$

$$f_T = \frac{0.25}{\left[\log \left(\frac{1}{3.7(228260)} \right) \right]^2} = 0.0071174 \approx 0.0071$$

Using Moody: Readings for f and f_T lie outside the boundary of the Moody Diagram (because of the relatively smooth tunnel and high Reynolds Number) but extrapolating would seem to indicate (roughly) that $f \approx 0.0075$ and $f_T \approx 0.0070$. In this case, the Swamee-Jain is probably more accurate than my imperfect extrapolation.

Losses for tunnel BCD:

$$2 \text{ bends: } h_L = 2k \frac{v^2}{2g} = 2f_T \left(\frac{L_e}{D} \right) \frac{v^2}{2g} = 2 \times 0.0071 \times 16 \times 1.1991 = 0.27244$$

$$\text{Friction Losses: } h_L = f \cdot \frac{L}{D} \cdot \frac{v^2}{2g} = 0.0077 \cdot \frac{1140}{10.5} \cdot 1.1991 = 1.0024$$

$$h_{LBCD} = 0.27244 + 1.0024 = 1.2749 \text{ m}$$

Single Tailrace Tunnel BCD

Velocity and velocity head in the 8.5 m diameter tunnel:

$$v = \frac{Q}{A} = \frac{140 \text{ m}^3/\text{s}}{\pi(8.5\text{m})^2/4} = 2.4672 \text{ m/s}$$

$$\frac{v^2}{2g} = \frac{(2.4672 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2} = 0.31025 \text{ m}$$

Reynolds Number, relative roughness and friction factors for tunnel BCD:

$$N_R = \frac{vD\rho}{\eta} = \frac{2.4672 \text{ m/s} \times 8.5 \text{ m} \times 1000 \text{ kg/m}^3}{0.0013 \text{ Pa} \cdot \text{s}} = 16132000$$

$$\frac{D}{\epsilon_{\text{concrete}}} = \frac{8.5 \text{ m}}{1.2 \times 10^{-3} \text{ m}} = 7083$$

Using Swamee-Jain:

$$f = \frac{0.25}{\left[\log \left(\frac{1}{3.7(D/\epsilon)} + \frac{5.74}{N_R^{0.9}} \right) \right]^2} = \frac{0.25}{\left[\log \left(\frac{1}{3.7(7083)} + \frac{5.74}{16132000^{0.9}} \right) \right]^2} = 0.012927$$

$$f_T = \frac{0.25}{\left[\log \left(\frac{1}{3.7(7083)} \right) \right]^2} = 0.012806$$

Using Moody:

Readings from the Moody Diagram: $f \approx 0.0129$ and $f_T \approx 0.0127$. These are the values I shall use.

Losses for tailrace tunnel DE:

$$\text{Butterfly Valve: } h_L = k \frac{v^2}{2g} = f_T \left(\frac{L_e}{D} \right) \frac{v^2}{2g} = 0.0127 \times 20 \times 0.31025 = 0.078804$$

$$\text{Exit Losses: } h_L = \frac{v^2}{2g} = 0.31025$$

$$\text{Friction Losses: } h_L = f \cdot \frac{L}{D} \cdot \frac{v^2}{2g} = 0.0129 \cdot \frac{382}{8.5} \cdot 0.31025 = 0.17986$$

$$h_{L_{DE}} = 0.078804 + 0.31025 + 0.17986 = 0.56891 \text{ m}$$

Head Losses for System

$$\begin{aligned} h_L &= h_{L_{AB}} + h_{L_{BCD}} + 3 \times h_{L_{DE}} \\ &= 8.5825 + 1.2749 + 3 \times 0.56891 \\ &= 11.564 \text{ m} \end{aligned}$$

Applying the General Energy Equation

Apply between the surfaces of the upper and lower lakes:

$$\frac{P_U}{\gamma} + z_U + \frac{v_U^2}{2g} - h_L - h_R = \frac{P_L}{\gamma} + z_L + \frac{v_L^2}{2g}$$

$$0 + 542 + 0 - 11.564 - h_R = 0 + 0 + 0$$

$$h_R = 530.44 \text{ m}$$

Find the power removed:

$$P_R = h_R \cdot \gamma \cdot Q = 530.44 \text{ m} \cdot 9.81 \text{ kg/m}^3 \cdot 420 \text{ m}^3/\text{s} = 2185500 \text{ kW} = 2185.50 \text{ MW}$$

Find the turbine efficiency:

$$e_M = \frac{P_O}{P_R} = \frac{1800 \text{ MW}}{2185.5 \text{ MW}} = 0.82361$$

The turbine efficiency is 82.4%

Phew!

