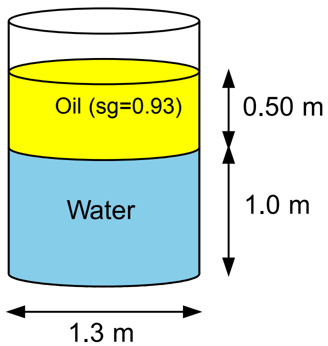
Module 2: Forces Due To Static Fluids (CIVL 318)

Pressure and forces on plane areas:	$P_{avg} = \gamma h_C$ $F_R = \gamma h_C A$
Centre of pressure for plane areas:	$L_p - L_c = \dfrac{I_c}{L_c A}$

Example 1:



Determine the force exerted by the oil and water upon the bottom plane surface of the barrel

Solution:

Pressure at oil-water boundary:

$$\begin{aligned} P_{O-W} &= \gamma h = (0.93)\left(9.81\ \text{kN/m}^3\right)(0.50\ \text{m}) \\ &= 4.5617\ \text{kPa} \end{aligned}$$

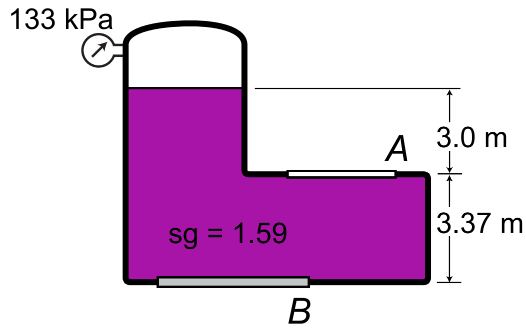
Pressure at bottom of the barrel:

$$\begin{aligned} P_{BB} &= P_{O-W} + \left(9.81\ \text{kN/m}^3\right)(1.0\ \text{m}) \\ &= 4.5617\ \text{kPa} + 9.81\ \text{kPa} \\ &= 14.372\ \text{kPa} \end{aligned}$$

Force on the bottom of the barrel:

$$\begin{aligned} P_{BB} &= 14.372\ \text{kPa} \\ F &= pA = (14.372\ \text{kN/m}^2) \cdot \frac{\pi(1.3\ \text{m})^2}{4} \\ &= 19.076\ \text{kN} \\ F &\approx 19.08\text{kN} \end{aligned}$$

Example 2:



A pressurized tank contains liquid with sg= 1.59. There are rectangular inspection hatches at A (400 mm $\times$ 250 mm) and at B (500 mm $\times$ 750 mm).

Determine the force exerted by the fluid on the hatch at A.

Solution:

$$\begin{aligned} p_A &= 133 \text{ kPa} \\ &\quad + (1.59)(9.81 \text{ kN/m}^3)(3.0 \text{ m}) \\ &= 133 \text{ kPa} + 46.794 \text{ kPa} \\ &= 179.79 \text{ kPa} \end{aligned}$$

$$\begin{aligned} F_A &= p_A \cdot A_A \\ &= (179.79 \text{ kN/m}^2)(0.4 \text{ m} \times 0.25 \text{ m}) \\ &= 17.979 \text{ kN} \end{aligned}$$

$$F_A \approx 17.98 \text{ kN}$$

Exercise 1:

Determine the force exerted by the fluid on the hatch at B.

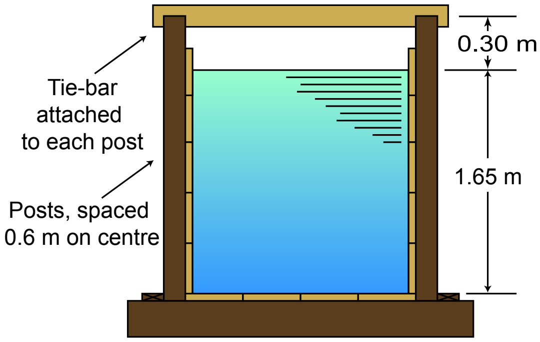
Solution:

$$\begin{aligned} p_B &= 133 \text{ kPa} \\ &\quad + (1.59)(9.81 \text{ kN/m}^3)(6.37 \text{ m}) \\ &= 133 \text{ kPa} + 99.358 \text{ kPa} \\ &= 232.36 \text{ kPa} \end{aligned}$$

$$\begin{aligned} F_B &= p_B \cdot A_B \\ &= (232.36 \text{ kN/m}^2)(0.5 \text{ m} \times 0.75 \text{ m}) \\ &= 87.134 \text{ kN} \end{aligned}$$

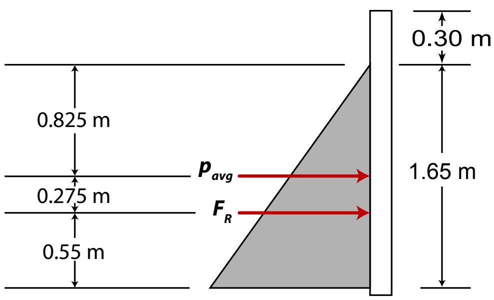
$$F_B \approx 87.1 \text{ kN}$$

Example 3:



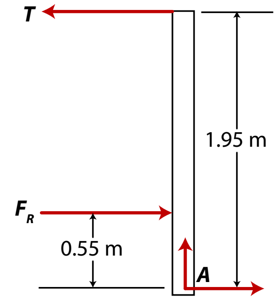
The tie-bars have cross-sectional dimension of 90mm $\times$ 90mm. Determine the normal stress in the tie-bars, and the bearing stress if the vertical posts are cut halfway into each tie-bar. (Assume a pinned connection at the bottom of the sidewalls and treat pressure as though the fluid is static.)

Solution:



$$
\begin{aligned}
p_{avg} &= \gamma \frac{h}{2} \\
&= \left(9.81\,\text{kN/m}^3\right)\left(\frac{1.65}{2}\,\text{m}\right) \\
&= 8.0933\,\text{kPa}
\end{aligned}
$$

$$
\begin{aligned}
F_R &= p_{avg} A \\
&= (8.0933\,\text{kPa})\,(0.6\,\text{m} \times 1.65\,\text{m}) \\
&= 8.0123\,\text{kN}
\end{aligned}
$$



Draw a free body diagram of the forces acting on one wall and take moments about the pinned connection at the bottom of the wall

$$\Sigma M_A = (1.95\,\text{m})T - (8.0123\,\text{kN})(0.55\,\text{m}) = 0$$

$$
\begin{aligned}
T &= \frac{8.0123 \times 0.55}{1.95}\,\text{kN} \\
&= 2.2599\,\text{kN}
\end{aligned}
$$

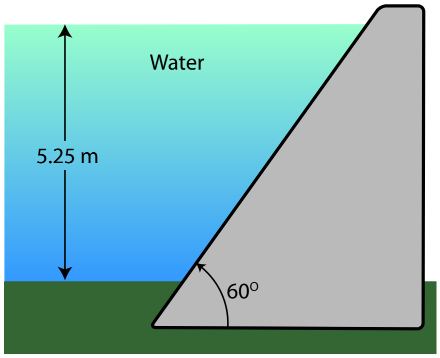
The normal stress is given by $\sigma = F/A$:

$$
\begin{aligned}
\sigma &= \frac{2.2599\,\text{kN}}{(0.09\,\text{m} \times 0.09\,\text{m})} \\
&= 279.0\,\text{kPa}
\end{aligned}
$$

The bearing stress is given by $\sigma_b = F/A_b$. Here, the bearing area is half the cross-section of the tie-bar:

$$
\begin{aligned}
\sigma_b &= \frac{2.2599\,\text{kN}}{(0.09\,\text{m} \times 0.045\,\text{m})} \\
&= 558.0\,\text{kPa}
\end{aligned}
$$

Example 4:



The wall has a rectangular plane area in contact with the water, is inclined at 60° to the horizontal and is 17 m long.

Determine the force exerted on the dam plane area by the water.

Solution:

The average pressure for the area in contact with the water is at h_C, i.e. at half-depth:

$$p_{avg} = \gamma h_C$$

$$= \left(9.81\,\mathrm{kN/m^3}\right)\left(\frac{5.25\,\mathrm{m}}{2}\right) = 25.751\,\mathrm{kN/m^2}$$

The area A of the dam wall is:

$$A = \left(\frac{5.25}{\sin 60^\circ}\,\mathrm{m}\right)(17\,\mathrm{m})$$

$$= 103.06\,\mathrm{m^2}$$

The resultant force, F_R, is:

$$F_R = 25.751\,\mathrm{kN/m^2} \times 103.06\,\mathrm{m^2}$$

$$= 2653.9\,\mathrm{kN}$$

$$\approx 2650\,\mathrm{kN}$$

Example 5:

A vertical retaining wall supports water to a depth of 4.75 m. There is a rectangular hatch in the wall. The top of the hatch is at a depth of 1.25 m; the hatch is 2.25 m wide × 1.5 m high.

What is the magnitude of the force exerted upon the hatch by the water?

Solution

The average pressure on the hatch is at centre-height of the hatch which is at:

$$h = 1.25\,\mathrm{m} + \frac{1.5\,\mathrm{m}}{2} = 2.0\,\mathrm{m}$$

$$\begin{aligned} p_{avg} &= \gamma h \\ &= \left(9.81\,\mathrm{kN/m^3}\right)(2.0\,\mathrm{m}) \\ &= 19.62\,\mathrm{kN/m^2} \end{aligned}$$

$$\begin{aligned} F_R &= p_{avg} A \\ &= \left(19.62\,\mathrm{kN/m^2}\right)(2.25\,\mathrm{m} \times 1.50\,\mathrm{m}) \\ &= 66.218\,\mathrm{kN} \\ &\approx 66.2\,\mathrm{kN} \end{aligned}$$

Exercise 2:

A vertical retaining wall supports water to a depth of 6.25 m. There is a rectangular hatch in the wall. The hatch has dimensions $3.75\,\mathrm{m}$ wide $\times$ $1.6\,\mathrm{m}$ height.

At what depth below the surface can the top of this hatch be placed if the maximum allowable force on the hatch is $128\,\mathrm{kN}$?

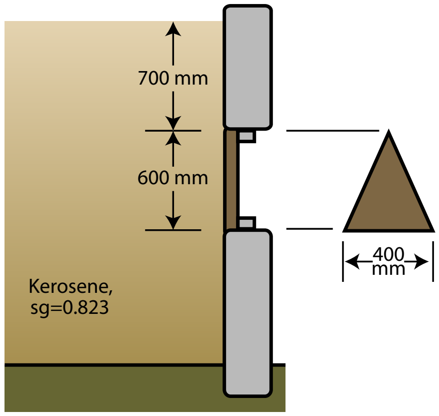
Solution

The average pressure on the hatch is at $h = d + 0.8\,\mathrm{m}$, where d is the depth of the top of the hatch. The force on the hatch is:

$$\begin{aligned} F &= p_{avg} A \\ 128\,\mathrm{kN} &= \left(9.81\,\mathrm{kN/m^3}\right)(d + 0.8\,\mathrm{m})(1.6\,\mathrm{m} \times 3.75\,\mathrm{m}) \\ d + 0.8\,\mathrm{m} &= \frac{128\,\mathrm{kN}}{\left(9.81\,\mathrm{kN/m^3}\right)(3.75\,\mathrm{m} \times 1.6\,\mathrm{m})} \\ &= 2.1724\,\mathrm{m} \end{aligned}$$

$$d \approx 1.372\,\mathrm{m}$$

Example 6:



A tank containing kerosene (sg=0.823) has a triangular inspection hatch in a vertical sidewall. The hatch has a base of 400 mm and a height of 600 mm. The top of the hatch is located at a depth of 700 mm.

Determine the force exerted on the hatch by the kerosene.

Solution:

The height of the triangular hatch is 600 mm so the location of the centroidal x-axis is 200 mm above the base

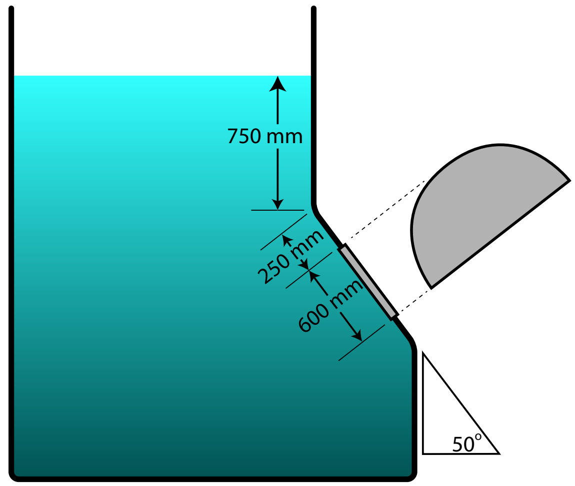
Thus the average pressure on the hatch is the pressure at a depth of $700 + (600 - 200) = 1100$ mm below the surface.

Then

$$
\begin{aligned}
p_{avg} &= \gamma h_C \\
&= (0.823)\left(9.81\,\mathrm{kN/m^3}\right)(1.10\,\mathrm{m}) \\
&= 8.8810\,\mathrm{kPa}
\end{aligned}
$$

$$
\begin{aligned}
F_R &= p_{avg} A \\
&= \left(8.8810\,\mathrm{kN/m^2}\right)(0.5 \times 0.4\,\mathrm{m} \times 0.6\,\mathrm{m}) \\
&\approx 1.066\,\mathrm{kN}
\end{aligned}
$$

Example 7:



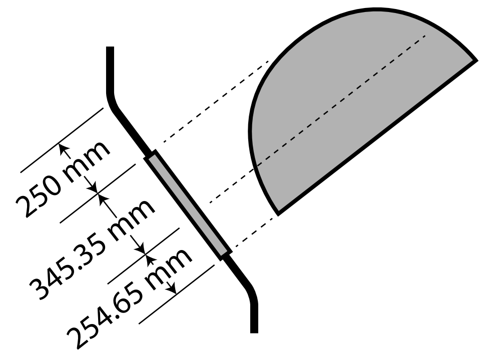
A water tank has a semi-circular inspection hatch, as illustrated.

Determine the force exerted on the hatch by the water.

(For a semi-circle, $\bar{y} = \frac{4r}{3\pi}$.)

Solution:

$$\boxed{\bar{y} = \frac{4(600\,\text{mm})}{3\pi} = 254.65\,\text{mm}}$$



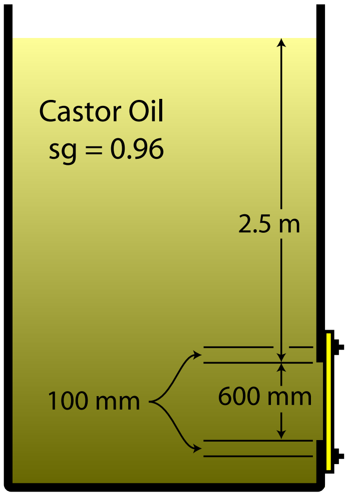
$$\begin{aligned} h_c &= (345.35\,\text{mm} + 250\,\text{mm})\cos 40^\circ + 750\,\text{mm} \\ &= 1206.1\,\text{mm} \end{aligned}$$

$$\begin{aligned} p_{avg} &= \gamma h_c = \left(9.81\,\text{kN/m}^3\right) \times 1.2061\,\text{m} \\ &= 11.832\,\text{kPa} \end{aligned}$$

$$A = \pi(0.6\,\text{m})^2/2 = 0.56549\,\text{m}^2$$

$$\begin{aligned} F_R &= p_{avg} A \\ &= \left(11.832\,\text{kN/m}^2\right) \times (0.56549\,\text{m}^2) \\ &= 6.6908\,\text{kN} \\ &= 6.69\,\text{kN} \end{aligned}$$

Example 8:



A tank containing castor oil has a 1.0 m wide × 600 mm high rectangular inspection hatch.

The top of the hatch is 2.5 m below the surface of the castor oil. The hatch cover is attached to the tank by 8 bolts, four at the top of the hatch and four at the bottom.

The bolts are offset the from the hatch opening by 100 mm, as shown.

Calculate the tension in each of the top and in each of the bottom bolts.

(Assume that all the top bolts have the same tension and that all the bottom bolts have the same tension.)

Solution:

From the hatch calculations:

$$A = 1.0\,\text{m} \times 0.6\,\text{m} = 0.6\,\text{m}^2$$

$$I_c = \frac{bh^3}{12} = \frac{(1.0\,\text{m}) \times (0.6\,\text{m})^3}{12} = 0.018\,\text{m}^4$$

$$L_c = 2.5\,\text{m} + \frac{0.6\,\text{m}}{2} = 2.8\,\text{m}$$

Then

$$\begin{aligned} L_p - L_c &= \frac{I_c}{L_c \cdot A} \\ &= \frac{0.018\,\text{m}^4}{(2.8\,\text{m}) \times (0.6\,\text{m}^2)} \\ &= 0.010714\,\text{m} \end{aligned}$$

The centre of pressure is 10.714 mm below the centroid of the hatch.

The average pressure on the hatch:

$$\begin{aligned} p_{avg} &= \gamma L_c \\ &= (0.96)\left(9.81\,\text{kN/m}^3\right)(2.8\,\text{m}) \\ &= 26.369\,\text{kN/m}^2 \end{aligned}$$

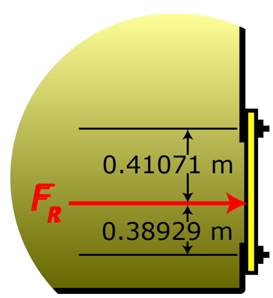
Then the magnitude of the resultant force on the hatch is:

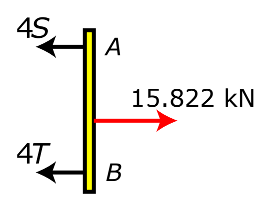
$$\begin{aligned} F_R &= p_{avg} A \\ &= \left(26.369\,\text{kN/m}^2\right)(1.0\,\text{m} \times 0.6\,\text{m}) \\ &= 15.822\,\text{kN} \end{aligned}$$

The upper row of bolts is at a depth of 2.4 m and the lower row is at 3.2 m.

The upper row of bolts is 0.41071 m above the centre of pressure

The lower row of bolts is 0.38929 m below the centre of pressure



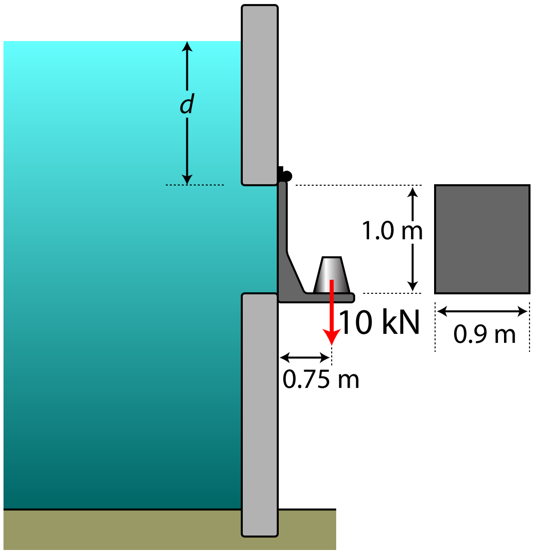


Let S and T be the tensions in each upper and each lower bolt respectively. Then

$$\Sigma M_B = 4S \times (0.8\,\text{m}) - (0.38929\,\text{m}) \times (15.822\,\text{kN}) = 0$$
$$S = 1.925\,\text{kN}$$

$$\Sigma M_A = -4T \times (0.8\,\text{m}) - (0.4107\,\text{m}) \times (15.822\,\text{kN}) = 0$$
$$S = 2.03\,\text{kN}$$

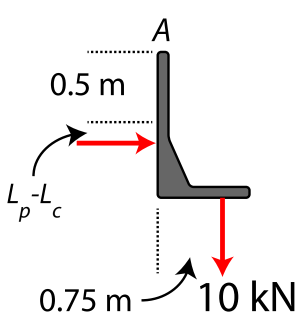
Example 9:



This is an example of a "self-levelling" gate. It is hinged along its top edge

When the water exceeds a certain height, the hydrostatic force on the gate is sufficient to open the gate. Water drains until the level allows the gate to close again.

Find the value d for which the gate opens.



Solution:

We don't know L_c but we can calculate a few results in terms of L_c

$$A = (0.9\,\text{m}) \times (1.0\,\text{m}) = 0.9\,\text{m}^2$$

$$I_c = \frac{bh^3}{12} = \frac{(0.9\,\text{m})\,(1.0\,\text{m})^3}{12} = 0.075\,\text{m}^4$$

$$L_p - L_c = \frac{I_c}{L_c A} = \frac{0.075\,\text{m}^4}{L_c(0.9\,\text{m})^2} = \frac{0.083333}{L_c}\,\text{m}$$

$$p_{avg} = \gamma h = \left(9.81\,\text{kN/m}^3\right)(L_c\,\text{m}) = 9.81 L_c\,\text{kN/m}^2$$

$$F_R = \left(9.81 L_c\,\text{kN/m}^2\right)(0.9\,\text{m}) = 8.8290 L_c\,\text{kN}$$

When the gate just begins to open, the reaction at B is 0. Then, if we take moments about A, there are only two moments to consider: the moment due to the resultant force, F_R, of the water pressure and the moment due to the 10 kN weight:

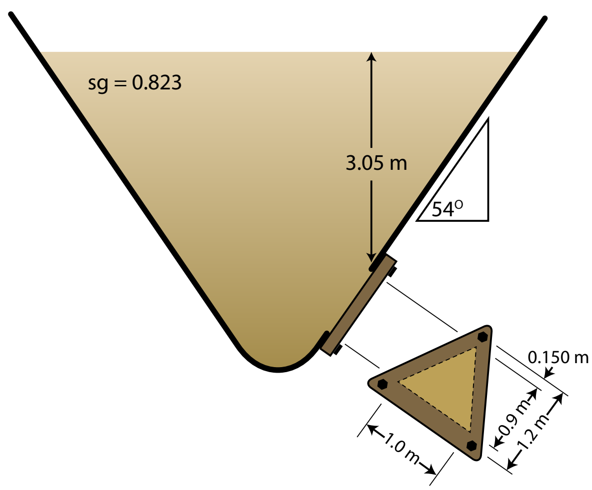
$$\begin{aligned}\Sigma M_A &= F_R\left(0.5 + (L_p - L_c)\right) - 10(0.75)\\ &= 8.8290 L_c\left(0.5 + \frac{0.083333}{L_c}\right) - 7.5\\ &= 4.4145 L_c + 0.73572 - 7.5\\ &= 0\end{aligned}$$

$$L_c = 1.5322\,\text{m}$$

The centroid of the gate is $1.5322 L_c$ m from the surface when the gate opens.
Therefore,

$$d \approx 1.032\,\text{m}$$

Example 10:



Determine the tension T in the upper bolt and tension S in each of the lower bolts.

Solution:

$$
\begin{aligned}
L_c &= \frac{3.05\,\text{m}}{\sin 54^\circ} + 0.6\,\text{m} \\
&= 4.3700\,\text{m}
\end{aligned}
$$

$$
\begin{aligned}
h_c &= L_c \sin 54^\circ \\
&= 3.5354\,\text{m}
\end{aligned}
$$

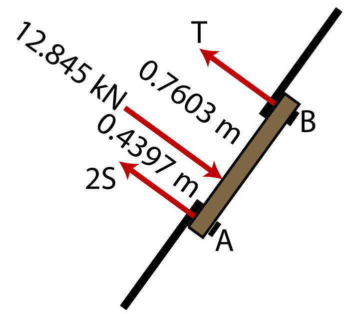
$$
\begin{aligned}
p_{avg} &= \gamma h_c \\
&= 0.823\left(9.81\,\text{kN/m}^3\right)(3.5354\,\text{m}) \\
&= 28.544\,\text{kPa}
\end{aligned}
$$

$$
\begin{aligned}
A &= \frac{(0.9\,\text{m}) \times (1.0\,\text{m})}{2} \\
&= 0.45\,\text{m}^2
\end{aligned}
$$

$$
\begin{aligned}
F_R &= \left(28.544\,\text{kN/m}^2\right)(0.45\,\text{m}^2) \\
&= 12.845\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
I_c &= \frac{bh^3}{36} \\
&= \frac{(1.0\,\text{m})\,(0.9\,\text{m})^3}{36} \\
&= 0.020250\,\text{m}^4
\end{aligned}
$$

$$
\begin{aligned}
L_p - L_c &= \frac{I_c}{L_c A} \\
&= \frac{0.020250\,\text{m}^4}{(4.3700\,\text{m})\,(0.45\,\text{m}^2)} \\
&= 0.010297\,\text{m}
\end{aligned}
$$



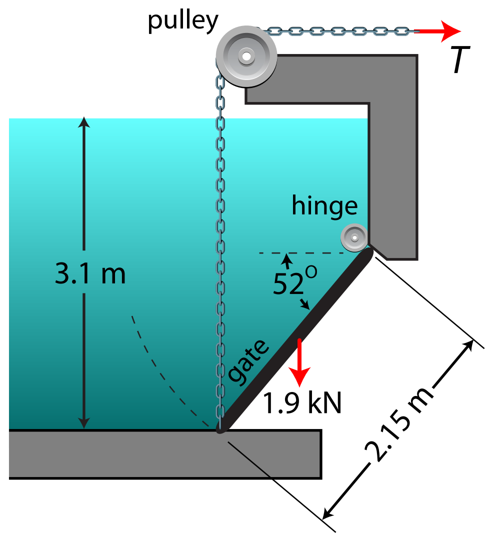
Sum the moments about the lower bolts at A:

$$
\begin{aligned}
\Sigma M_A &= (T\,\text{kN})(1.2\,\text{m}) - (12.845\,\text{kN})(0.43970\,\text{m}) \\
&= 0 \\
\Rightarrow T &= \frac{(12.845\,\text{kN})(0.43970\,\text{m})}{(1.2\,\text{m})} \\
&= 4.7066\,\text{kN} \\
&\approx 4.71\,\text{kN}
\end{aligned}
$$

Sum the moments about the upper bolt at B:

$$
\begin{aligned}
\Sigma M_B &= (12.845\,\text{kN})(0.76030\,\text{m}) - (2S\,\text{kN})(1.2\,\text{m}) \\
&= 0 \\
\Rightarrow S &= \frac{(12.845\,\text{kN})(0.76030\,\text{m})}{2(1.2\,\text{m})} \\
&= 4.0692\,\text{kN} \\
&\approx 4.07\,\text{kN}
\end{aligned}
$$

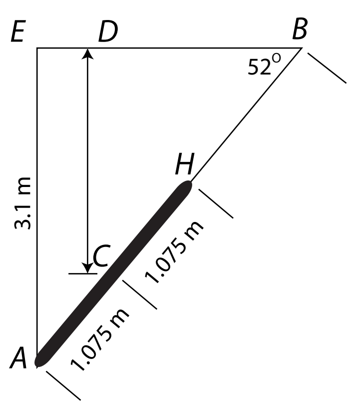
Example 11:



A rectangular steel gate (1.5 m × 2.15 m) is used to regulate the level of a water storage pond. The gate has a weight of 1.9 kN which can be thought of as acting through the centre of the gate.

Determine the force T required to begin to open the gate when the pond has a depth of 3.1 m.

Solution:



$$\frac{3.1}{AB} = \frac{\text{opp}}{\text{hyp}} = \sin 52^\circ$$

$$\Rightarrow AB = \frac{3.1}{\sin 52^\circ} = 3.9340\,\text{m}$$

$$\Rightarrow BH = 3.9340 - 2.15 = 1.7840\,\text{m}$$

$$\Rightarrow BC = 3.9340 - 1.075 = 2.8590\,\text{m}$$

$$\Rightarrow DC = 2.8590 \sin 52^\circ = 2.2529\,\text{m}$$

$$h_c = DC = 2.2529\,\text{m}$$
$$L_c = BC = 2.859\,\text{m}$$

$$\begin{aligned} p_{avg} &= \gamma h_c \\ &= \left(9.81\,\text{kN/m}^3\right)(2.2529\,\text{m}) \\ &= 22.101\,\text{kPa} \end{aligned}$$

$$A = (1.5\,\text{m} \times 2.15\,\text{m}) = 3.225\,\text{m}^2$$

$$\begin{aligned} F_R &= p_{avg} A \\ &= \left(22.101\,\text{kN/m}^2\right)\left(3.225\,\text{m}^2\right) \\ &= 71.275\,\text{kN} \end{aligned}$$

$$I_c = \frac{bh^3}{12} = \frac{(1.5\,\text{m})(2.15\,\text{m})^2}{12} = 1.2423\,\text{m}^2$$

$$\begin{aligned} \Longrightarrow\ L_p &= L_c + \frac{I_c}{L_c A} \\ &= 2.8590\,\text{m} + \frac{1.2423\,\text{m}^4}{(2.8590\,\text{m})\,(3.225\,\text{m}^2)} \\ &= 2.9937\,\text{m} \end{aligned}$$

Sum moments about the hinge, noting that the reaction at A is 0 when the gate begins to open:

$$\begin{aligned} \Sigma M_H &= (71.275\,\text{kN})(2.9937\,\text{m} - 1.7840\,\text{m}) \\ &\quad + (1.9\,\text{kN})(1.075 \cos 52^\circ\,\text{m}) \\ &\quad - (T\,\text{kN})(2.15 \cos 52^\circ\,\text{m}) \\ &= 0 \end{aligned}$$

$$\begin{aligned} T &= \frac{(71.275\,\text{kN})(1.2097\,\text{m} + (1.9\,\text{kN})(0.66184\,\text{m})}{1.3237\,\text{m}} \\ &= 66.087\,\text{kN} \\ &\approx 66.1\,\text{kN} \end{aligned}$$