Module 8: Hazen Williams Equation and Equivalent Pipes (CIVL 318)

Hazen-Williams Equations

$$Q = \frac{C\,D^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}, \qquad h_L = L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852}, \qquad D = \left(\frac{279000\,Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802}$$

Equivalent-Length Ratios for Fittings

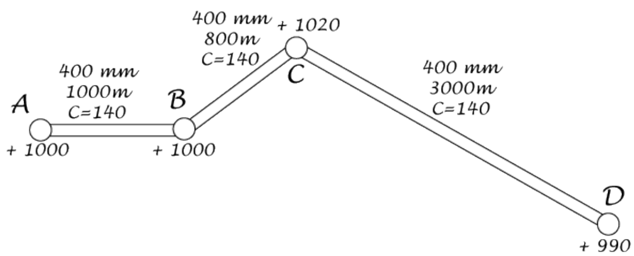
Type	L_e/D
Globe valve — fully open	340
Angle valve — fully open	150
Gate valve — fully open	8
— 3/4 open	35
— 1/2 open	160
— 1/4 open	900
Check valve — swing type	100
Check valve — ball type	150
Butterfly valve — fully open — 2-8"	45
— 10-14"	35
— 16-24"	25
Foot valve — poppet disc type	420
Foot valve — hinged disc type	75
90° standard elbow	30
90° long radius elbow	20
90° street elbow	50
45° standard elbow	16
45° street elbow	26
Close return bend	50
Standard tee — flow through run	20
Standard tee — flow through branch	60
Gradual enlargement — 15° cone angle	8
Gradual enlargement — 20° cone angle	15
Gradual enlargement — 30° cone angle	23
Gradual reduction — 15° to 40° cone angle	2
Pipe entrance — inward projecting	50
Pipe entrance — square	25
Pipe entrance — rounded	10
Venturi meter	100

Example 1:

For the pipeline shown, calculate the pressure at B, given that the pressure at A is 700 kPa.

The pipes are cement-lined Hyprescon with a diameter of 400 mm and a roughness coefficient of $C = 140$. Flow through the system is 200 L/s.

Elevations are as indicated.



Solution:

First, apply the Hazen-Williams:

$$
\begin{aligned}
h_{L_{AB}} &= L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852} \\
&= 1000\left(\frac{279000 \times 200}{140 \times 400^{2.63}}\right)^{1.852} \\
&= 4.9903\,\text{m}
\end{aligned}
$$

Now, apply the GEE:

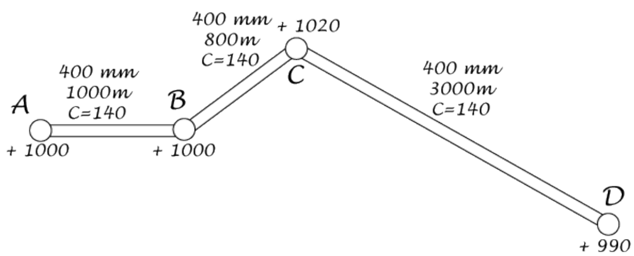
$$
\begin{aligned}
\frac{P_A}{\gamma} + \cancel{z_A} + \cancel{\frac{v_A^2}{2g}} - h_L &= \frac{P_B}{\gamma} + \cancel{z_B} + \cancel{\frac{v_B^2}{2g}} \\
\frac{700}{9.81} - 4.9903 &= \frac{P_B}{9.81} \\
P_B &= 651.05\,\text{kPa} \\
\boldsymbol{P_B} &= \mathbf{651\,kPa}
\end{aligned}
$$

Exercise 1:

For the pipeline shown, calculate the pressure at C and D, given that the pressure at A is 700 kPa.

The pipes are cement-lined Hyprescon with a diameter of 400 mm and a roughness coefficient of $C = 140$. Flow through the system is 200 L/s.

Elevations are as indicated.



Solution:

First, apply the Hazen-Williams:

$$
\begin{aligned}
h_{L_{BC}} &= L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852} \\
&= 800\left(\frac{279000 \times 200}{140 \times 400^{2.63}}\right)^{1.852} \\
&= 3.9922\,\text{m}
\end{aligned}
$$

$$
\begin{aligned}
h_{L_{CD}} &= L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852} \\
&= 3000\left(\frac{279000 \times 200}{140 \times 400^{2.63}}\right)^{1.852} \\
&= 14.971\,\text{m}
\end{aligned}
$$

Now, apply the GEE:

$$
\begin{aligned}
\frac{P_B}{\gamma} + z_B + \cancel{\frac{v_B^2}{2g}} - h_L &= \frac{P_C}{\gamma} + z_C + \cancel{\frac{v_C^2}{2g}} \\
\frac{651.05}{9.81} - 3.9922 &= \frac{P_C}{9.81} + 20
\end{aligned}
$$

$$P_C = 415.69\,\text{kPa}$$

$$\boldsymbol{P_C = 416\,\textbf{kPa}}$$

$$
\begin{aligned}
\frac{P_C}{\gamma} + z_C + \cancel{\frac{v_C^2}{2g}} - h_L &= \frac{P_D}{\gamma} + z_D + \cancel{\frac{v_D^2}{2g}} \\
\frac{415.69}{9.81} + 30 - 14.971 &= \frac{P_D}{9.81}
\end{aligned}
$$

$$P_D = 563.12\,\text{kPa}$$

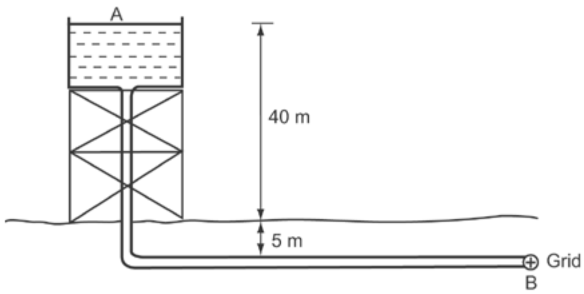
$$\boldsymbol{P_D = 563\,\textbf{kPa}}$$

Example 2:

Water flows from a storage tank through a welded steel pipe that is 1200 m long and 350 mm in diameter, entering a distribution grid at point 'B'. Assume C=100. Determine:

(1) The pressure at 'B' when the flow is 150 L/s
(2) The maximum flow rate into the grid when the minimum allowable pressure at 'B' is 400 kPa.

Minor losses are negligible compared to friction losses.



Solution (1):

$$h_L = L\left(\frac{279000\,Q}{C\,D^{2.63}}\right)^{1.852}$$
$$= 1200\left(\frac{279000\times 150}{100\times 350^{2.63}}\right)^{1.852}$$
$$= 12.561\,\text{m}$$

$$v = \frac{Q}{A}$$
$$= \frac{0.150}{\pi(0.350)^2/4}$$
$$= 1.5591\,\text{m/s}$$

$$\frac{v^2}{2g} = 0.12389\,\text{m}$$

G.E.E.:

$$\cancelto{0}{\frac{P_A}{\gamma}} + z_A + \cancelto{0}{\frac{v_A^2}{2g}} - h_L = \frac{P_B}{\gamma} + \cancelto{0}{z_B} + \frac{v_B^2}{2g}$$
$$45 - 12.561 = \frac{P_B}{9.81} + 0.12389$$

$$P_B = 317.01\,\text{kPa}$$
$$\boldsymbol{P_B = 317\,\text{kPa}}$$

Notice that if we recalculated the pressure at B omitting the velocity head, then $P_B = 318.2\,\text{kPa}$, not very different from including it.

Solution (2):

What flow/headloss will give a pressure of 400 kPa at B?

$$\cancelto{0}{\frac{P_A}{\gamma}} + z_A + \cancelto{0}{\frac{v_A^2}{2g}} - h_L = \frac{P_B}{\gamma} + \cancelto{0}{z_B} + \frac{v_B^2}{2g}$$
$$45 - h_L = \frac{400}{9.81} + \frac{v_B^2}{2g}$$

One equation and two unknowns! We could solve it iteratively, guessing at a flow and seeing what P_B is for this flow, then trying another flow until we converge on a pressure of 400 kPa at B.

But the velocity head had an effect of about 0.3% in part (1); it will be less here as we need less velocity/headloss to keep the pressure higher. So, in problems of this type, we **simply ignore the velocity head term...**

$$45 - h_L = \frac{400}{9.81} + \cancelto{0}{\frac{v_B^2}{2g}}$$
$$h_L = 4.2253\,\text{m}$$

What flow will give this headloss?

$$Q = \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}$$
$$= \frac{100\times 350^{2.63}\left(\frac{4.2253}{1200}\right)^{0.54}}{279000}$$
$$= 83.272\,\text{L/s}$$
$$\boldsymbol{Q = 83.3\,\text{L/s}}$$

Let's look at the value of the velocity head we discarded...

$$v = \frac{Q}{A}$$
$$= \frac{0.083272}{\pi(0.350)^2/4}$$
$$= 0.86551\,\text{m/s}$$

$$\frac{v^2}{2g} = 0.038181\,\text{m}$$

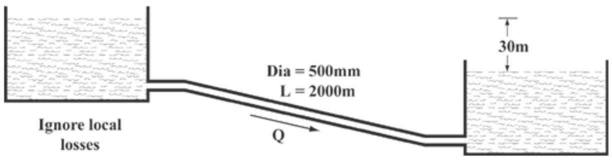
The velocity head is small enough that we can disregard it. Any error from not omitting the headloss is negligible compared with error in estimating the C-value.

Exercise 2:
Water flows from one reservoir down to another, through a 500 mm diameter pipe that is 2000 m in length. The difference in elevation between the surfaces of the two reservoirs is 30 m.

Determine:

(1) The flow with high density polyethylene pipe (HDPE) with $C = 140$
(2) The flow with welded steel with $C = 100$
(3) The diameter of HDPE pipe required for a flow of 1200 L/s

Disregard minor losses.



Note: At the surfaces of both reservoirs, pressure and velocity head are 0 so the GEE reduces to

$$30 - h_L = 0$$

Solution (1): For HDPE,

$$\begin{aligned} Q &= \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000} \\ &= \frac{140(500)^{2.63}\left(\frac{30}{2000}\right)^{0.54}}{279000} \\ &= 651.48\,\mathrm{L/s} \\ \boldsymbol{Q} &= \mathbf{651\,L/s} \end{aligned}$$

Solution (2): For welded steel,

$$\begin{aligned} Q &= \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000} \\ &= \frac{100(500)^{2.63}\left(\frac{30}{2000}\right)^{0.54}}{279000} \\ &= 465.35\,\mathrm{L/s} \\ \boldsymbol{Q} &= \mathbf{465\,L/s} \end{aligned}$$

Solution (3): Diameter for a flow of 1200 L/s with HDPE,

$$\begin{aligned} D &= \left(\frac{279000\,Q}{C\left(\frac{h_L}{L}\right)^{0.54}}\right)^{0.3802} \\ &= \left(\frac{279000 \times 1200}{140\left(\frac{30}{2000}\right)^{0.54}}\right)^{0.3802} \\ &= 630.42\,\mathrm{mm} \\ \boldsymbol{D} &= \mathbf{630\,mm} \end{aligned}$$

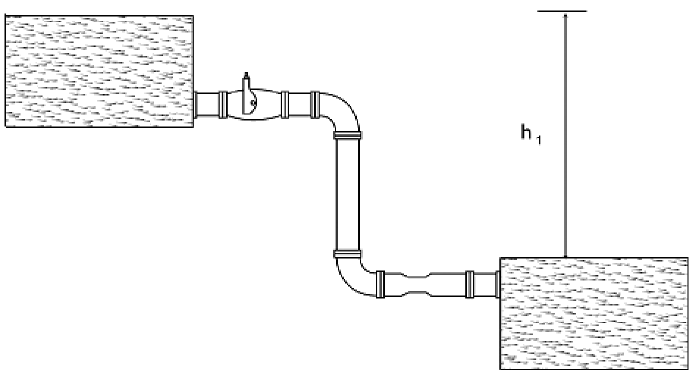
Example 3:

In a water treatment plant, water flows from a filter down to a clear well through the pipe system shown. The pipe is welded steel with a diameter of 300 mm and roughness coefficient $C = 130$. The total length of pipe is 50 m. Elevation difference h_1 between the tanks is 5 m.

Equivalent length ratios, L_e/D, are:

Entrance and exit losses:	50	Butterfly valve:	35
Large radius elbows:	25	Venturi meter:	100

Determine the flow through the system.



Solution:

Effective length of the pipe: (length and diameter in metres!)

$$\begin{aligned} L_{\text{eff}} &= \text{Actual pipe length} + D\left(\frac{L_e}{D}\right) \\ &= 50 + 0.3(50 + 35 + 25 + 25 + 100 + 50) \\ &= 50 + 85.5 \\ &= 135.5\,\text{m} \end{aligned}$$

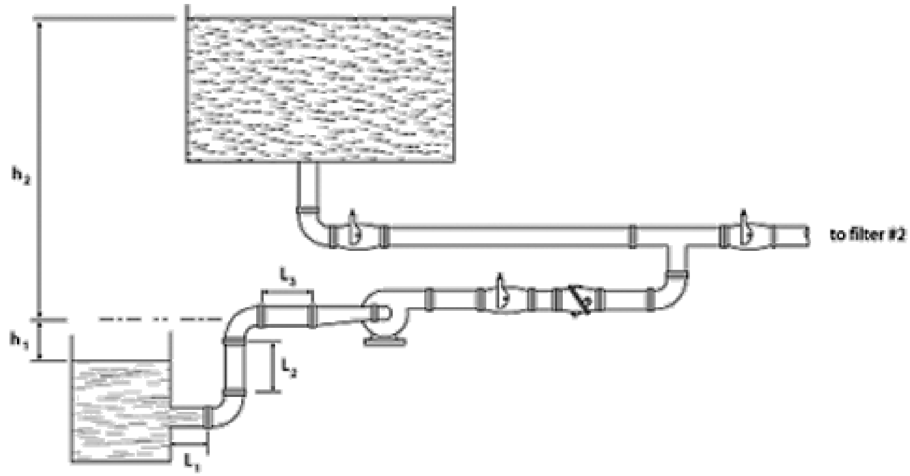
As earlier, headloss between the two surfaces is just the elevation difference:

$$h_L = 5\,\text{m}$$

Find the flow:

$$\begin{aligned} Q &= \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000} \\ &= \frac{130(300)^{2.63}\left(\frac{5}{135.5}\right)^{0.54}}{279000} \\ &= 256.66\,\text{L/s} \\ \boldsymbol{Q} &= \mathbf{257\,L/s} \end{aligned}$$

Example 4:



In a water treatment plant, backwash water is pumped from the clear well through the pipe system shown to the filter. The required backwash flow is 10 L/s per square meter of filter area (the filter dimensions are 10 m by 15 m. The inlet pipe is made of welded steel $(C = 130)$, has a diameter of 1000 mm and a total length $(L_1 + L_2 + L_3)$ of 10 m. The outlet pipe, from the pump to the filter, is also welded steel, has a diameter of 700 mm and a length of 70 m.

The two elevation differences are $h_1 = 2$ m and $h_2 = 10$ m.

Equivalent length ratios, L_e/D, are:

Entrance:	10	Elbow (inlet):	25
Eccentric Reducer:	2	Butterfly Valve:	40
Check Valve:	120	Elbow (outlet):	35
Tee Connection:	60		

Determine:

(1) The head losses on the inlet side (clear well to pump)
(2) The head losses on the outlet side (pump to filter)

Neglect exit losses into the filter.

Solution:

Q required for backwash in the filter:

$$Q = 10\,\text{m} \times 15\,\text{m} \times 0.01\,\text{m}^3/\text{s} = 1.5000\,\text{m}^3/\text{s}$$

(1)

$$\begin{aligned}
L_{eff} &= 10 + 1(10 + 25 + 25 + 2) = 72.000\,\text{m} \\
h_L &= 72\left(\frac{279000 \times 1500}{130(1000)^{2.63}}\right)^{1.852} \\
&= 0.19834\,\text{m} \\
\boldsymbol{h_{L_{(in)}}} &= \mathbf{0.1983\,m}
\end{aligned}$$

(2)

$$\begin{aligned}
L_{eff} &= 70 + 0.7(40 + 120 + 35 + 60 + 40 + 35) \\
&= 301.00\,\text{m} \\
h_L &= 301\left(\frac{279000 \times 1500}{130(700)^{2.63}}\right)^{1.852} \\
&= 4.7112\,\text{m} \\
\boldsymbol{h_{L_{(out)}}} &= \mathbf{4.71\,m}
\end{aligned}$$

Exercise 3:

This exercise is a continuation of the previous example. Determine:

(3) The head added by the pump
(4) The pressure at the pump outlet

Solution:

(3)

Apply the GEE between the surface of the clear well (W) and the surface of the filter (F):

$$\begin{aligned}\frac{P_W}{\gamma} + z_W + \frac{v_W^2}{2g} + h_A - h_L &= \frac{P_F}{\gamma} + z_F + \frac{v_F^2}{2g} \\ h_A - (0.19834 + 4.7112) &= 12 \\ h_A &= 16.910\,\mathrm{m} \\ \boldsymbol{h_A} &= \mathbf{16.91\,m}\end{aligned}$$

(4)

Apply the GEE between the pump (P) and the surface of the filter (F):

$$\begin{aligned}\frac{P_P}{\gamma} + z_P + \frac{v_P^2}{2g} - h_L &= \frac{P_F}{\gamma} + z_F + \frac{v_F^2}{2g} \\ \frac{P_P}{9.81} + 0 + 0.77430 - 4.7112 &= 0 + 10 + 0 \\ \Rightarrow P_P &= 136.72\,\mathrm{kPa} \\ \boldsymbol{P_P} &= \mathbf{136.7\,kPa}\end{aligned}$$

Example 5:

The pumps and piping system are used to supply a municipal grid.

Pump P_1 runs continuously and maintains the basic pressure in the distribution grid beyond point D.

The elevations are the same at the pump and the discharge point D.

The outlet pipe, from the pump to point D, is welded steel $(C = 130)$ with a diameter of 200 mm and a total length between fittings of 10 m.

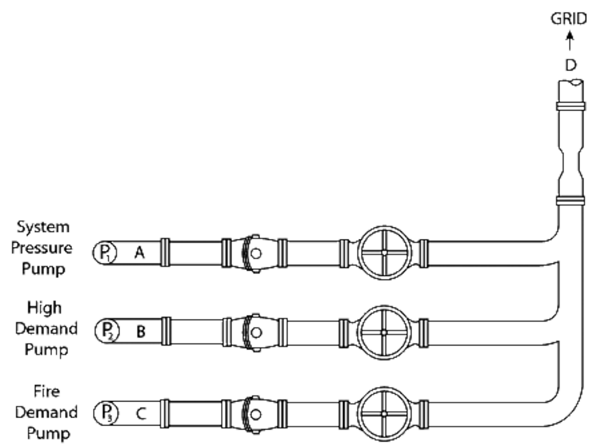
The minimum pressure required at D is 500 kPa for a design flow of 150 L/s.

Equivalent length ratios, L_e/D, are:

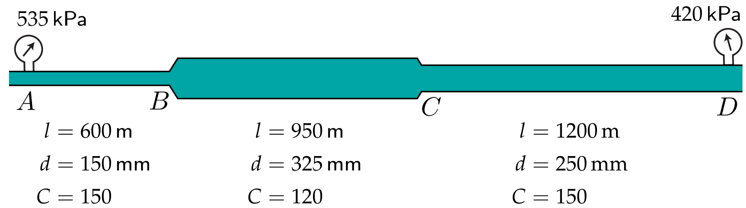
Check Valve:	120	Gate Valve:	15
Tee Connection:	60	Venturi Meter:	100

There is no flow from pumps P_2 and P_3. Determine:

(1) the head losses between A and D
(2) the pressure at A required for the required pressure and flow at D



Example 6:



Determine Q, the volume flow rate from A to D, through the system shown. Ignore minor losses and assume that A and D are at the same elevation.

Solution:

First, determine the head loss in each of the three pipes, in terms of Q:

$$\begin{aligned} h_{L_{AB}} &= L\left(\frac{279000Q}{CD^{2.63}}\right)^{1.852} \\ &= L\left(\frac{279000}{CD^{2.63}}\right)^{1.852} \cdot Q^{1.852} \\ &= 600\left(\frac{279000}{150(150)^{2.63}}\right)^{1.852} \cdot Q^{1.852} \\ &= 0.017142\, Q^{1.852} \end{aligned}$$

$$\begin{aligned} h_{L_{BC}} &= 950\left(\frac{279000}{120(325)^{2.63}}\right)^{1.852} \cdot Q^{1.852} \\ &= 0.00094964\, Q^{1.852} \end{aligned}$$

$$\begin{aligned} h_{L_{CD}} &= 1200\left(\frac{279000}{150(250)^{2.63}}\right)^{1.852} \cdot Q^{1.852} \\ &= 0.0028479\, Q^{1.852} \end{aligned}$$

Sum these headlosses to get the headloss between A and D:

$$\begin{aligned} h_{L_{AD}} &= h_{L_{AB}} + h_{L_{BC}} + h_{L_{CD}} \\ &= (0.017142 + 0.00094964 + 0.0028479)\, Q^{1.852} \\ &= 0.020940 Q^{1.852}\,\text{m} \end{aligned}$$

Use the GEE to approximate a numerical value for $h_{L_{AD}}$:

$$\begin{aligned} \frac{P_A}{\gamma} + \cancel{z_A} + \frac{v_A^2}{2g} - h_L &= \frac{P_D}{\gamma} + \cancel{z_D} + \frac{v_D^2}{2g} \\ \Rightarrow h_L &= \frac{P_A - P_D}{2g} + \frac{v_A^2 - v_D^2}{2g} \end{aligned}$$

One equation and two unknowns (h_L and Q, since v_A and v_B can be expressed in terms of the single variable Q). Here, we have the difference between the two velocity heads which is smaller than the velocity head at A. As before, we can ignore it (for now!)

$$\begin{aligned} h_L &\approx \frac{P_A - P_D}{2g} \\ &= \frac{535 - 420}{19.62} \\ &= 5.8614\,\text{m} \end{aligned}$$

Now, find Q:

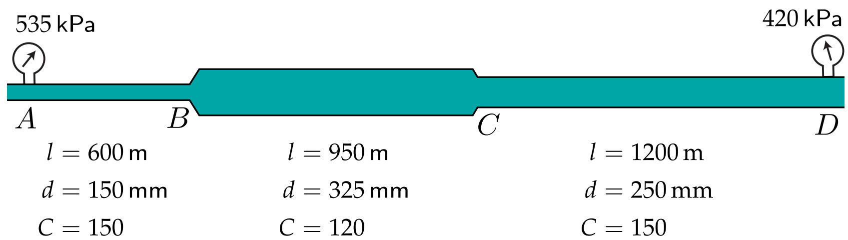
$$\begin{aligned} h_{L_{AD}} &= 5.8614 = 0.020940 Q^{1.852} \\ \Rightarrow Q &= \left(\frac{5.1852}{0.020940}\right)^{\frac{1}{1.852}} \\ &= 20.955 \\ \boldsymbol{Q} &= \mathbf{21.0\,L/s} \end{aligned}$$

How large was the term we discarded?

$$\begin{aligned} \frac{v_A^2 - v_D^2}{2g} &= \frac{\left(\frac{20.955/1000}{\pi(0.150)^2/4}\right)^2 - \left(\frac{20.955/1000}{\pi(0.250)^2/4}\right)^2}{19.62} \\ &= 0.062381\,\text{m} \end{aligned}$$

This is approximately 0.15% of the pressure head at D.

Example 7:



Use the equivialent pipe technique to determine Q, the volume flow rate from A to D, through the system shown. Ignore minor losses and assume that A and D are at the same elevation.

Solution:

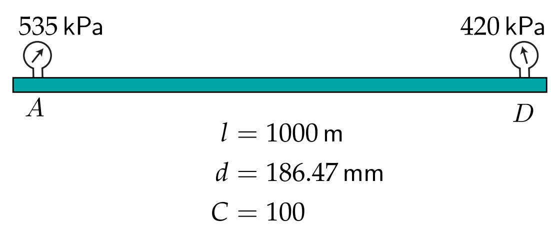
Assume a flow of $Q = 100$ L/s and determine the headloss between A and D:

$$\begin{aligned} h_{L_{AB}} &= 600\left(\frac{279000 \times 100}{150(150)^{2.63}}\right)^{1.852} \\ &= 86.710\,\text{m} \\ h_{L_{BC}} &= 950\left(\frac{279000 \times 100}{120(325)^{2.63}}\right)^{1.852} \\ &= 4.8035\,\text{m} \\ h_{L_{CD}} &= 1200\left(\frac{279000}{150(250)^{2.63}}\right)^{1.852} \\ &= 14.406\,\text{m} \\ h_{L_{AD}} &= 86.710 + 4.8035 + 14.406 \\ &= 105.92\,\text{m} \end{aligned}$$

Determine the diameter of the pipe, with length 1000 m and resistance coefficient $C = 100$, that has a headloss of 105.92 m for flow of 100 L/s.

$$\begin{aligned} d_{AD\text{equiv}} &= \left(\frac{279000 \times 100}{100\left(\frac{105.92}{1000}\right)^{0.54}}\right)^{0.3802} \\ &= 186.47\,\text{mm} \end{aligned}$$

Our problem has now reduced to finding the flow through the single pipe, ADequiv, shown below.



We apply the GEE to this pipe to find $h_{L_{AD\text{equiv}}}$, noting that A and D are at the same elevation and have the same diameter and, therefore, the same velocity head.

$$\begin{aligned} \frac{535}{9.81} - h_{L_{AD\text{equiv}}} &= \frac{420}{9.81} \\ \Rightarrow h_{L_{AD\text{equiv}}} &= 5.8614\,\text{m} \end{aligned}$$

Note that this is the same loss as we found in Example 6, so ignoring the difference in velocity heads had no numerical effect (to 5 significant digits).

Find Q:

$$\begin{aligned} Q &= \frac{100(186.47)^{2.63}\left(\frac{5.8614}{1000}\right)^{0.54}}{279000} \\ &= 20.932 \\ \boldsymbol{Q} &= \mathbf{20.9}\,\text{L/s} \end{aligned}$$