

# *01 Math Review*

## *Engineering Statics, STCS 200*

Updated on: August 16, 2025

- ▶ Statics is all math! All but the most trivial statics problems require algebra and/or trigonometry and/or geometry to solve.
- ▶ The good news is that the math is not very difficult. You won't need anything more advanced than high-school math.
- ▶ We will do a quick review here that should cover all the math you'll need for STCS 200.

Triangles are a strong, stable shape and often used in engineering.

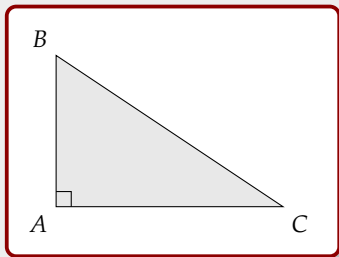
Triangles help avoid issues like this:



Triangles mean we need trigonometry.

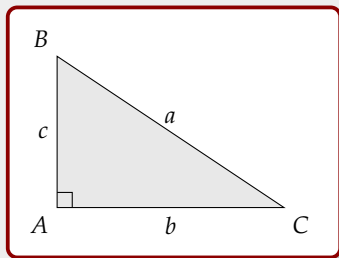
## Right Triangle

A **right triangle** is a triangle having one  $90^\circ$  angle.



## Right Triangle

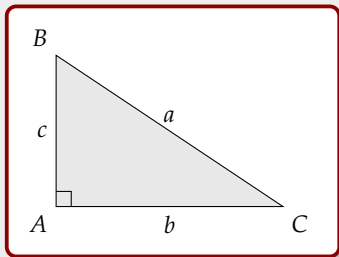
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Label the three sides  $a$ ,  $b$  and  $c$ . The side  $a$ , opposite the right angle, is called the **hypotenuse**.

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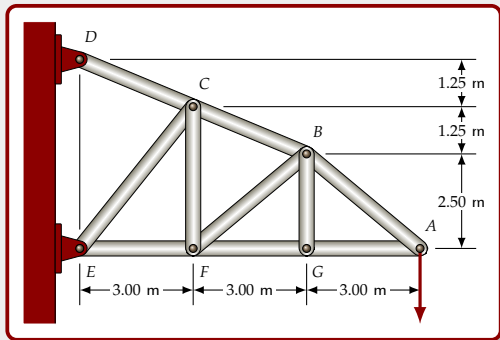


Label the three sides  $a$ ,  $b$  and  $c$ . The side  $a$ , opposite the right angle, is called the **hypotenuse**.

If we know the lengths of any two sides, we can calculate the length of the third side using the **Pythagorean Theorem**:

$$a^2 = b^2 + c^2$$

## Right Triangle Exercises (1)



1. Use the Pythagorean Theorem to determine the lengths of  $CE$  and  $CB$

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It is **extremely important** to recognize that we can get no more accuracy out of a calculation than we put in. If the inputs to a problem have three significant digits, we cannot expect any higher accuracy than three significant digits in our result — even if the calculator does give ten digits.

## *Non-zero digits*

Non-zero digits **are** significant:

- ▶ 1234 has 4 significant digits.
- ▶ 12.34 has 4 significant digits.

# Significant Digits

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## *Zeros between non-zero digits are significant*

- ▶ 12034 has 5 significant digits.
- ▶ 12.0034 has 6 significant digits.

*Leading zeros are **not** significant*

- ▶ 0.1234 has 4 significant digits.
- ▶ 0.0001234 has 4 significant digits.

*Leading zeros are **not** significant*

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*Trailing zeros (after a decimal point) **are** significant*

- ▶ 1234.0 has 5 significant digits.
- ▶ 1.23400 has 6 significant digits.

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- ▶ Usually, the trailing zeros are placeholders for the magnitude of a value and we don't need to worry unduly.
- ▶ If we want to emphasize that 12300 has 4 significant digits, we can write  $1.230 \times (10^3)$ .

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- ▶ We cannot expect to get more accuracy in our result at the end of a calculation than from our given input values at the beginning of the calculation so **solutions should be correct to 3 significant digits, not more than the accuracy of the calculation inputs!**
- ▶ Intermediate calculations will accumulate rounding errors quickly if we use only three significant digits and these can affect the final result. **For intermediate calculations, use 5 or more significant digits.**

(When I write solutions down, I use 5 significant digits for intermediate calculations. You may use more if it is more convenient for you, e.g., if you are storing intermediate results in your calculator.)

## *Significant Digits and Rounding*

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## Significant Digits and Rounding

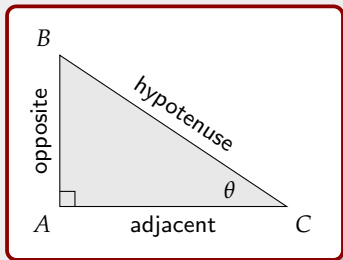
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- ▶ When the first discarded digit is a 5 (or higher), round up the digit before the 5 (or higher)
- ▶ There are various rules (such as the odd-even rule) which take a more complicated approach to rounding 5 but, for our purposes, **5 rounds up!**

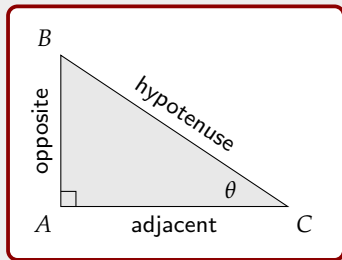
## More About Right Triangle

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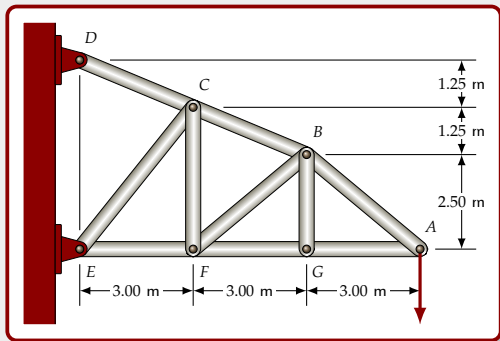


### Right Triangle Trigonometry Formulae

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Remember: **SOHCAHTOA**

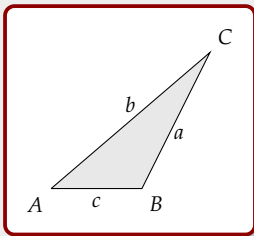
## Right Triangle Exercises (2)



2. Use the **tangent** function to calculate  $\angle CEF$ .
3. From  $\angle CEF$  just found (**use the intermediate, 5 or more significant digit, form!**) and the **sine** rule to verify the length of  $CE$  found earlier.
4. Use the **cosine** function and the length of  $CB$  found earlier to calculate the angle between  $BC$  and the horizontal.
5. Use the **tangent** function to verify the previous result.

## Triangles - Sine Rule

Not all triangles contain a right angle. To solve for these triangles (finding the lengths of the side and the triangle angles), we have to employ some different tools: the **sine rule** and (later) the **cosine rule**



*Sine Rule*

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

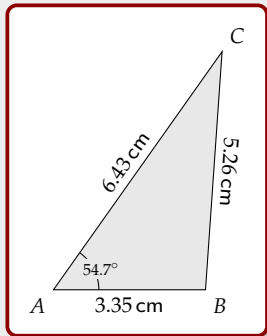
*Sine Rule*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



## Triangles - Sine Rule Exercises

6. Using the sine rule, find  $\angle ACB$ .
7. Using the sine rule, find  $\angle ABC$ .
8. Sum the interior angles of the triangle.



A couple of trig identities that will come in useful:

## *Identities*

$$\sin(180^\circ - \theta) = \sin \theta$$

$$\cos(-\theta) = \cos \theta$$

## *Note:*

$$\sin(140^\circ) = \sin(40^\circ) = 0.64279$$

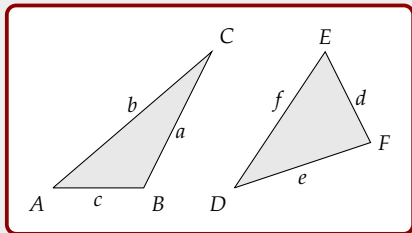
$$\cos(42^\circ) = \cos(-42^\circ) = 0.74314$$

Thus, we have to be careful when using inverse trigonometric functions:

$$\sin^{-1}(0.64279) = 40^\circ \text{ or } 140^\circ$$

$$\cos^{-1} 0.74314 = -42^\circ \text{ or } 42^\circ$$

## Triangles - Cosine Rule



### Cosine Rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

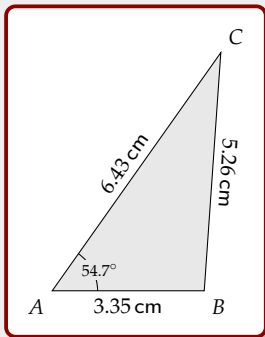
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

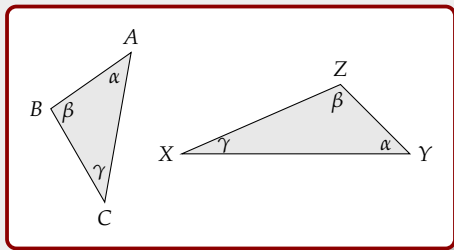
The cosine rule is useful when you have all the sides of a triangle and want to find the angles.

## Triangles - Cosine Rule Exercises

9. Determine  $\angle ABC$ , using the value for  $AB$  found earlier
10. Compare the value for  $\angle ABC$  with the value calculated earlier.  
Is it the same? It should be!

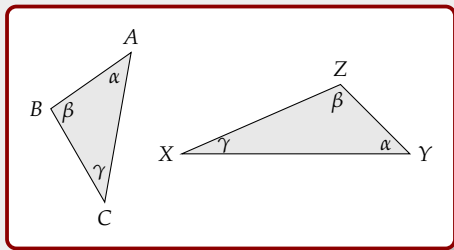


## Similar Triangles



If triangles  $ABC$  and  $XYZ$  have the same angles, they are said to be **similar triangles**.

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The ratios of the lengths of corresponding sides of similar triangles are equal:

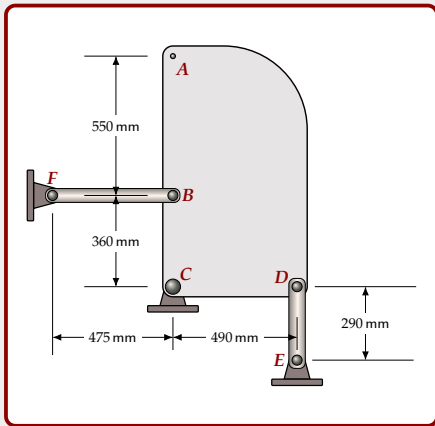
$$\frac{AB}{XY} = \frac{BC}{XZ} = \frac{AC}{YZ}$$

## Similar Triangles - Exercises

$ABCD$  is a rigid (i.e., it does not deform) plate, pinned at  $C$ .

When horizontal force  $P$  is applied at  $A$ ,  $ABCD$  rotates about  $C$  and  $A$  deflects 2.45 mm horizontally rightwards.

Assume that  $BF$  remains horizontal and that  $DE$  remains vertical.



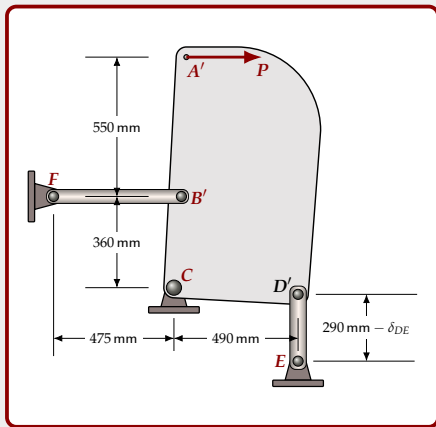
11. Determine  $\delta_{BF}$ , the change in length of  $BF$ .
12. Determine  $\delta_{DE}$ , the change in length of  $DE$ .

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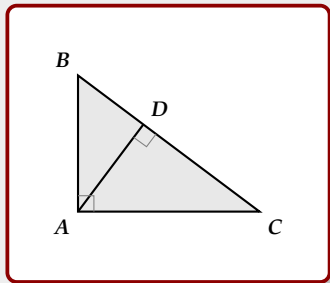


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## Right Triangles and Trigonometric Functions - Exercises

13. Show that right triangles  $\triangle ABC$ ,  $\triangle ABD$  and  $\triangle ACD$  all have the same angles (i.e. they are all similar).
14. Given that  $AC = 100$  mm and  $AD = 65$  mm, determine  $\angle ACD$  and  $\angle ABD$ .
15. Find the remaining lengths:  $AB$ ,  $BD$  and  $CD$ .
16. Verify your lengths found above by using the Pythagorean Theorem on  $\triangle ABC$

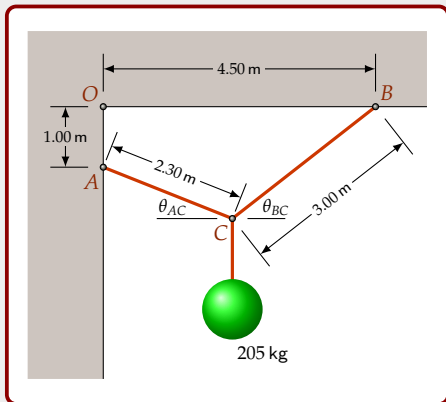


## Triangles and Trig Exercise

This is a standard type of statics problem to determine the forces in cables  $AC$  and  $BC$ . But first we have to find the angles  $\theta_{AC}$  and  $\theta_{BC}$ . This involves the use of the Pythagorean Theorem, the sine and cosine rules, and one of the trigonometric functions.

17. Find  $\theta_{AC}$ .

18. Find  $\theta_{BC}$ .



We frequently need to solve an equation for a particular variable (i.e., rearrange an equation to isolate a given variable)

For example:

### Example

Solve  $\delta = \frac{F \cdot L}{A \cdot E}$  for  $E$

Solution:

Multiply both sides of the equation by  $E/\delta$ . Then:

$$\cancel{\delta} \cdot \frac{E}{\cancel{\delta}} = \frac{F \cdot L}{A \cdot \cancel{E}} \cdot \frac{\cancel{E}}{\delta}$$

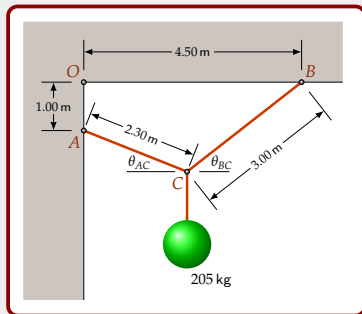
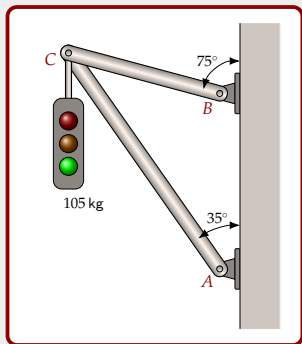
$$E = \frac{F \cdot L}{A \cdot \delta}$$

17. Solve  $a^2 = b^2 + c^2$  for  $b$ .
18. Solve  $V = \frac{4}{3}\pi r^3$  for  $r$ .
19. Solve  $c^2 = a^2 + b^2 - 2bc \cos C$  for  $\cos C$ .
20. Solve  $b^2 = a^2 + c^2 - 2ac \cos B$  for  $B$ .
21. One representation of the Hazen-Williams Equation for flow of water in a pipe is:

$$Q = \frac{CD^{2.63} \left( \frac{h_L}{L} \right)^{0.54}}{279000}$$

Solve the equation for  $h_L$ , then evaluate  $h_L$  using the values  $Q = 135$ ,  $C = 120$ ,  $D = 202.7$  and  $L = 1200$ .

## Simultaneous Equations



Calculating the forces in  $AC$  and  $BC$  in each of the examples shown involves solving two equations in two unknowns (also known as solving a system of simultaneous equations).

We'll review how to do this.

## *Solving Simultaneous Equations*

This is a simple system of simultaneous equations:

$$2x + 3y = 7 \quad (1)$$

$$6x - y = 1 \quad (2)$$

Our objective is to find the value of  $x$  and  $y$  that satisfies both equations.

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Equation (1) is a straight line, with slope  $-2/3$ , that intersects the  $x$  axis at  $x = 3.5$  and intersects the  $y$  axis at  $y = 7/3$ .

Equation (2) is a straight line, with slope  $6$ , that intersects the  $x$  axis at  $x = 1/6$  and intersects the  $y$  axis at  $y = -1$ .

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Equation (2) is a straight line, with slope 6, that intersects the  $x$  axis at  $x = 1/6$  and intersects the  $y$  axis at  $y = -1$ .

These lines have different slopes, so they must intersect somewhere. At the point  $(x, y)$  where they intersect, both equations are satisfied. This is the solution we're looking for.



## Solving Simultaneous Equations (cont'd)

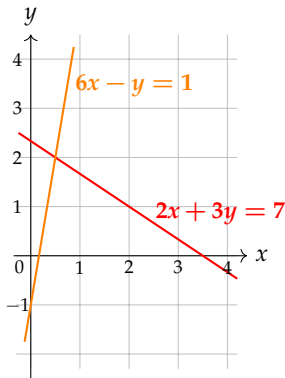
Graphically, the lines look like this.

It looks like the point at which they intersect is in the region of  $(0.5, 2)$

We can check whether this is the correct solution by substituting the values of  $x = 0.5$  and  $y = 2$  to see whether they satisfy both equations.

Generally, we can solve algebraically using a procedure called the Method of Substitution. Or use the **system-solver** on your calculator.

(The system-solver is allowed for quizzes and examinations; it saves time — and improves accuracy when rushed. Make sure you know how your calculator works. Trying to figure it out in an exam is not recommended!)



## *Solving Simultaneous Equations using the Method of Substitution*

$$2x + 3y = 7 \quad (1)$$

$$6x - y = 1 \quad (2)$$

The process:

- (a) Choose an equation and solve for one of the variables. Here I choose equation (2) and solve for the variable  $y$ .

$$y = 6x - 1 \quad (3)$$

- (b) Use equation (3) to substitute  $6x - 1$  wherever  $y$  occurs in the other equation:

$$2x + 3(6x - 1) = 7$$

$$2x + 18x - 3 = 7$$

$$20x = 10$$

$$x = 0.5$$

- (c) Substitute this value for  $x$  in either of equation (1) or (2):

$$6x - y = 1$$

$$3d - y = 1$$

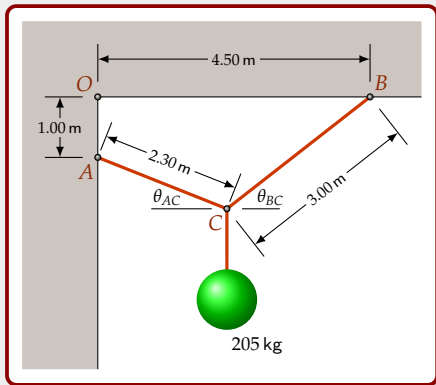
$$y = 2$$

- (d) We have our solution:  $(0.5, 2)$ .

## Typical Statics System of Equations

In an earlier exercise, we calculated that  $\theta_{AC} = 21.661^\circ$  and  $\theta_{BC} = 38.049^\circ$ .

(Note that we are using 5 significant digits here because we will be using these values for calculations.)



The system shown yields the following two equations:

$$F_{AC} \sin (21.661^\circ) + F_{BC} \sin (38.049^\circ) = 2011.1 \text{ N}$$

$$F_{BC} \cos (38.049^\circ) - F_{AC} \cos (21.661^\circ) = 0$$

## System of Equations Exercise (1)

This looks more difficult than our previous example.

$$F_{AC} \sin(21.661^\circ) + F_{BC} \sin(38.049^\circ) = 2011.1 \quad (1)$$

$$F_{BC} \cos(38.049^\circ) - F_{AC} \cos(21.661^\circ) = 0 \quad (2)$$

Fortunately, it only **looks** harder.  $F_{AC}$  and  $F_{BC}$  are variables, just like  $x$  and  $y$  in the earlier example. And the sines and cosines are just numbers. We get:

$$0.36911x + 0.61633y = 2011.1 \quad (3)$$

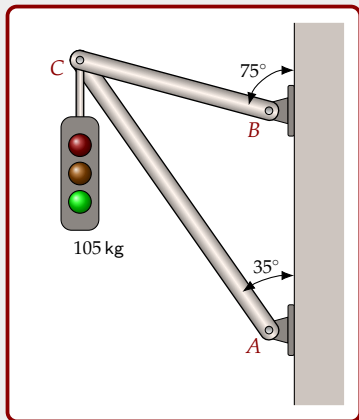
$$0.78748y - 0.92938x = 0 \quad (4)$$

This is the same system, with  $x$  replacing  $F_{AC}$ ,  $y$  replacing  $F_{BC}$  and the trigonometric functions evaluated.

22. Solve for  $x$  ( $F_{AC}$  in the original system)

23. Solve for  $y$  ( $F_{BC}$  in the original system)

## System of Equations Exercise (2)



The system shown yields the following two equations in the two unknowns  $F_{AC}$  and  $F_{BC}$ :

$$F_{BC} \sin 15^\circ + F_{AC} \cos 35^\circ + 1030.1 = 0$$

$$F_{BC} \cos 15^\circ + F_{AC} \sin 35^\circ = 0$$

24. Determine  $F_{AC}$

25. Determine  $F_{BC}$