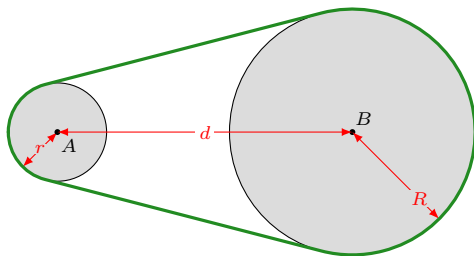


Nerdstuff

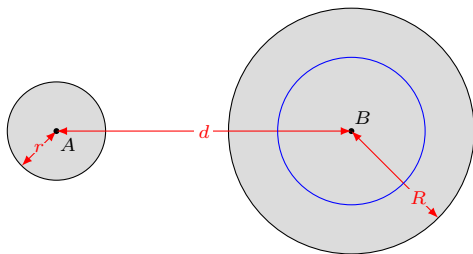
Source code at: <https://github.com/dmorgorg/LaTeX/blob/master/misc/misc.pdf>

Last updated on February 27, 2020



Two pulleys, centred at A and B , have radii r and R . The distance from A to B is d .

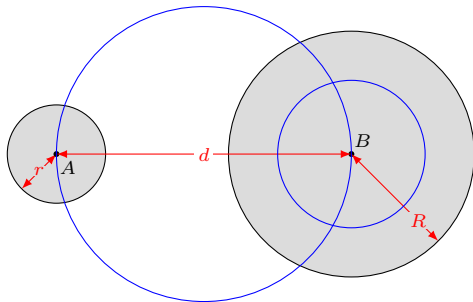
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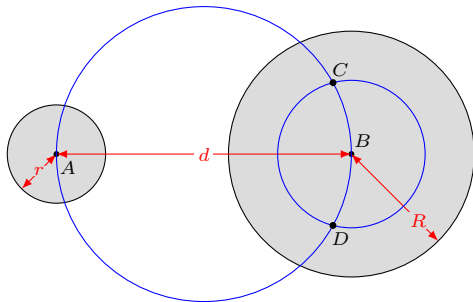
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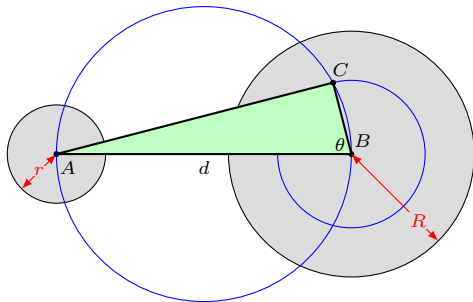


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Geometry :: Belt-Length



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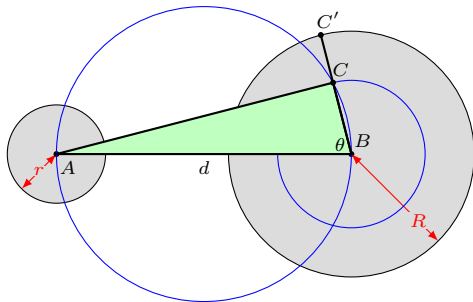
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- 4 Consider $\triangle ABC$: $\angle ACB = 90^\circ$ since it is an angle inscribed in a semicircle. Then:

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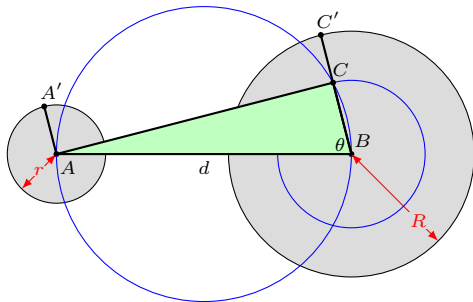
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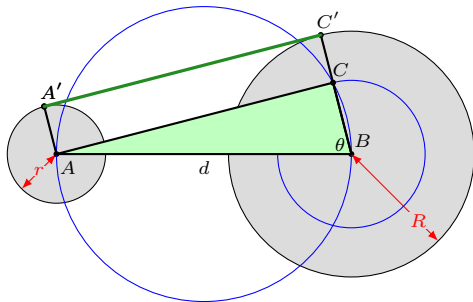
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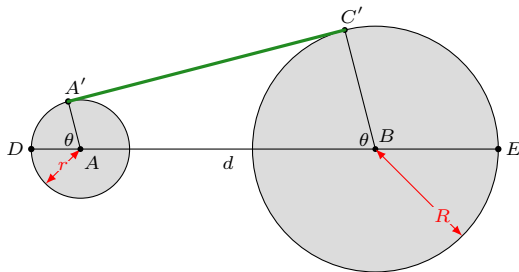
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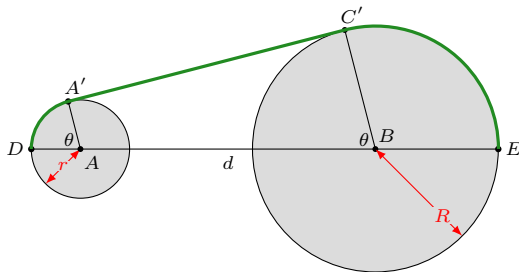
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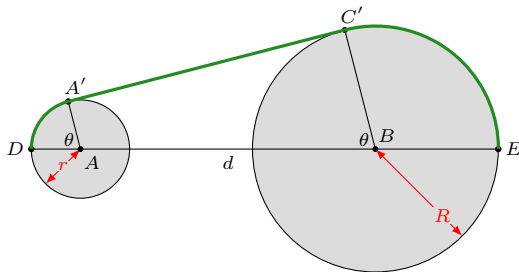
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$$DA' = r\theta \text{ and } C'E = R(\pi - \theta)$$



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10 Belt-length:

$$\begin{aligned} &= 2 (DA' + A'C' + C'E) \\ &= 2 \left(r\theta + \sqrt{d^2 - (R - r)^2} + R(\pi - \theta) \right) \end{aligned}$$

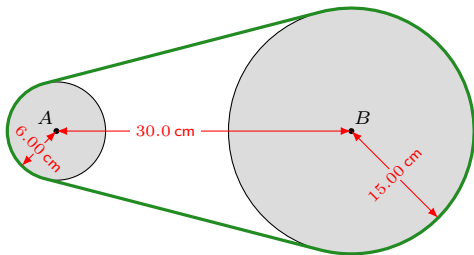
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Example:

$$\theta = \sin^{-1} \left(\frac{\sqrt{d^2 - (R - r)^2}}{d} \right) = \sin^{-1} \left(\frac{\sqrt{6.00^2 - 1.50^2}}{6.00} \right) = 1.3181 \text{ (radians)}$$

$$\text{B-L} = 2 \left(6.00 \times 1.3181 + \sqrt{6.00^2 - 1.50^2} + 15.00 \times (\pi - 1.3181) \right) = 82.141$$

The belt length is 82.1 cm.

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Still not exactly what you want? Maybe `font-size:calc(4px + 1.5vw);`?

Web :: Dynamic font sizes with CSS

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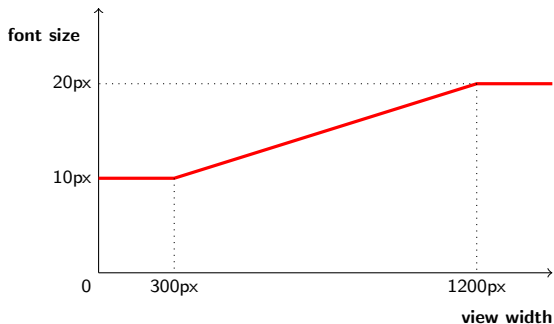
- ▶ For more precise control, without trial and error that rapidly becomes frustrating, I turned to this excellent [CSS-tricks](#) page showing examples such as

```
font-size: calc(16px + 6 * ((100vw - 320px) / 680));
```

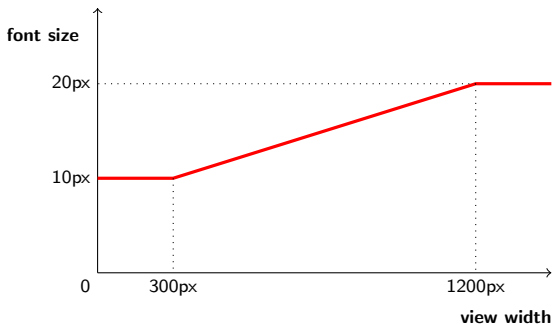
What are these magic numbers? At the moment, I can tell that, at a screen width of 320px, the font size is 16px. Font size increases smoothly until, at a screen width of 1000px, the font size of 22px. But I probably won't remember how to figure that out next week! The [CSS-tricks](#) page doesn't show how these numbers are derived...

...but it's just some (relatively) simple high-school math. All you need to recall from

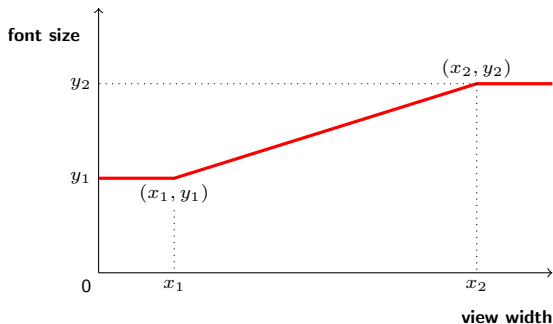
high-school is the equation of a line in the form: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$



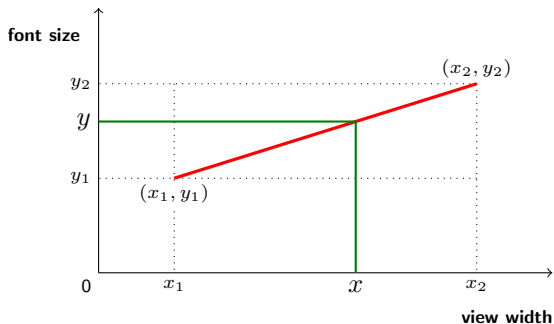
- 1 We can graph our desired font size against view width as shown. In this case: view widths less than 300px have a font size of 10px; font sizes grow uniformly from 10px at 300px view width to a font-size 20px at 1200px window width; and, for window sizes over 1200px, the font size remains 20px. **How do we achieve this with CSS?**



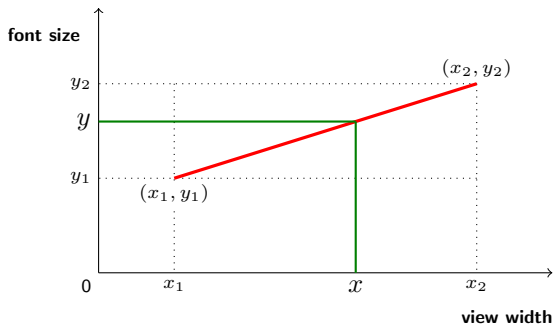
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- 2 The constant font sizes below 300px and above 1200px can be easily handled with media queries; we'll come back to them later. What is more interesting is the uniformly increasing font size calculation between view widths of 300px and 1200px.



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- 3 Instead of using the fixed numbers, we'll generalise and use variables so we can easily adjust our formula for different required values.



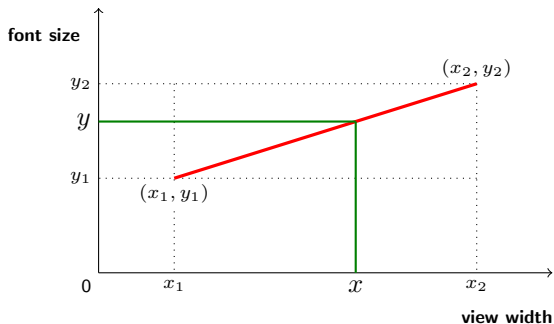
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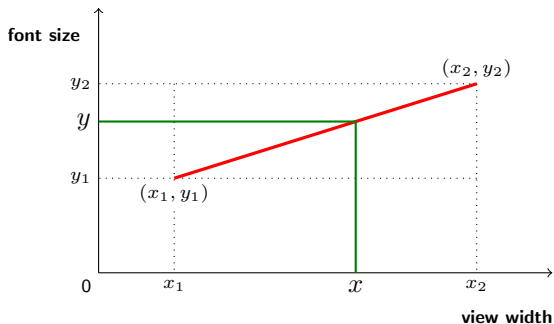
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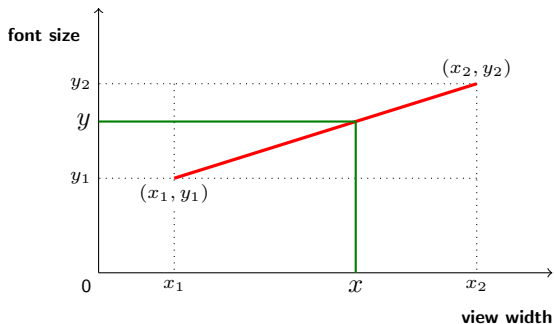
$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\Rightarrow y - y_1 = (x - x_1) \cdot \frac{y_2 - y_1}{x_2 - x_1}$$



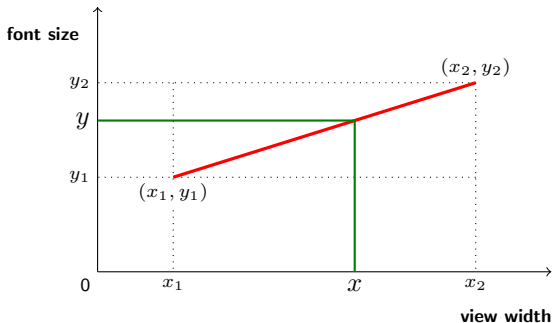
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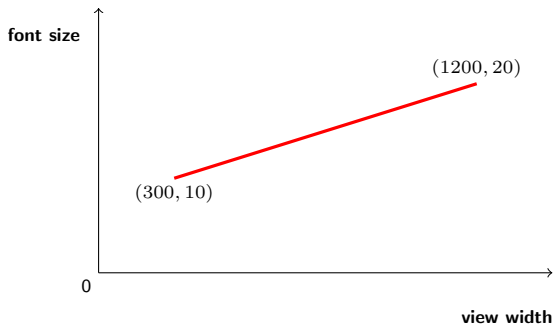
$$\begin{aligned}\frac{y - y_1}{x - x_1} &= \frac{y_2 - y_1}{x_2 - x_1} \\ \Rightarrow y - y_1 &= (x - x_1) \cdot \frac{y_2 - y_1}{x_2 - x_1} \\ \Rightarrow y &= y_1 + (x - x_1) \cdot \frac{y_2 - y_1}{x_2 - x_1}\end{aligned}$$



- 6 We have $\text{font size} = y_1 + (x - x_1) \cdot \frac{y_2 - y_1}{x_2 - x_1}$ where x_1 , y_1 , x_2 and y_2 are numbers chosen for our particular design and x is view width. Of course, CSS does not understand x but view width can be represented by 100vw .



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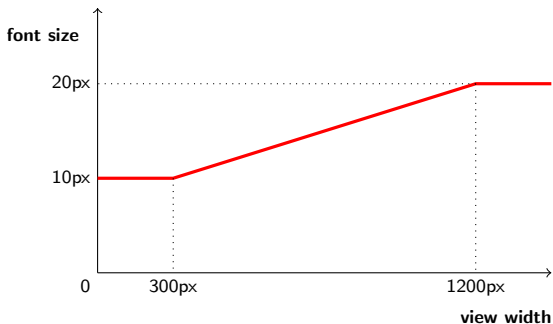
7 Thus, `font-size: calc($y_1 + (100\text{vw} - x_1) * (y_2 - y_1) / (x_2 - x_1)$);`

8 From our previous example:

```
font-size: calc(10px + (100vw - 300px) * (20px - 10px) / (1200px - 300px));
```

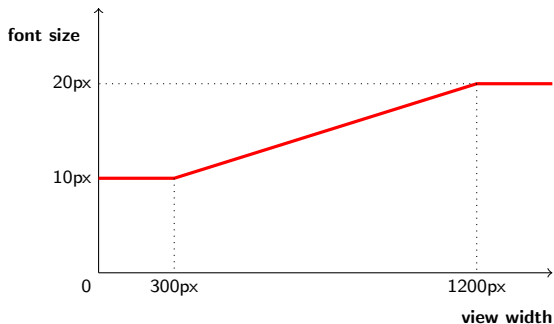
or, more concisely:

```
font-size: calc(10px + 10 * (100vw - 300px) / 900);
```



9 CSS for the complete range of view widths:

```
@media screen and (min-width:1200px) {  
  html { font-size:20px; }  
}  
  
@media screen and (max-width:1200px) {  
  html { font-size: calc(10px + (100vw - 300px)/90); }  
}  
  
@media screen and (max-width:300px) {  
  html { font-size:10px; }  
}
```



- 10 For ease of editing, using SASS variables for min-font, min-width, max-font, max-width is a better solution. Or write a mixin...