02 Force Vectors

Engineering Statics

Updated on: August 22, 2025

Scalars versus Vectors

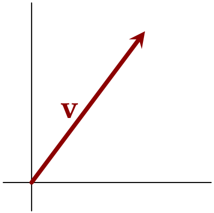
- Physical quantities in this course are measured using either **scalars** or **vectors**.
- A scalar quantity can be fully specified by its **magnitude** (or size) and units alone.

  Examples are temperature, speed, mass, time, length, volume, density and energy.
- A vector quantity requires both magnitude **and direction** - in addition to units - to be fully specified.

  Examples are displacement, velocity, force and momentum.
- 110 km/h is a speed. 110 km/h in a north-easterly direction is a vector.
- The vector quantity that is of most interest to us is **force**.

Graphical Vector Representation

- To represent a vector on a diagram, we draw a directed line segment – a line with an arrow tip.
- The length of the line segment is proportional to the magnitude of the vector.
- The direction of the line segment shows the direction of the vector.
- The arrow head gives the sense of that direction (up and rightwards in this case).



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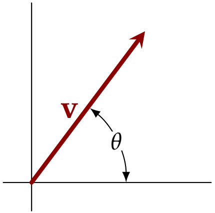
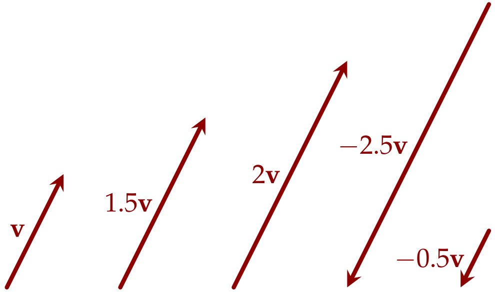


- θ indicates the direction of the line of action of the vector $\mathbf{v}$ relative to some reference.
  (I.e., the horizontal axis in this case.)

Multiplication of a vector by a scalar

Multiplication of a vector by a scalar affects the magnitude and, if the scalar is negative, the sense of the direction of the vector.



Addition of Vectors

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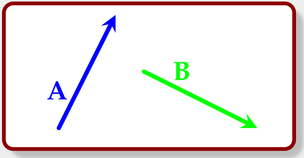

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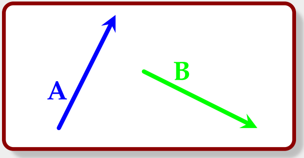
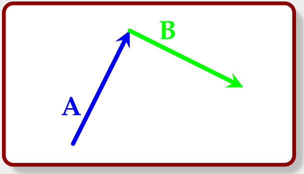


To add vectors $\mathbf{A}$ and $\mathbf{B}$, written $\mathbf{A} + \mathbf{B}$, place the tail of $\mathbf{B}$ at the tip of $\mathbf{A}$.



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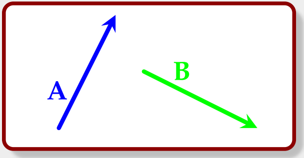

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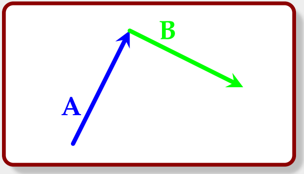
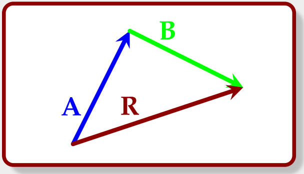


The sum, $A+B$, is obtained by drawing a vector R from the tail of $\mathbf{A}$ to the tip of $\mathbf{B}$.

$$\mathbf{A}+\mathbf{B}=\mathbf{R}$$



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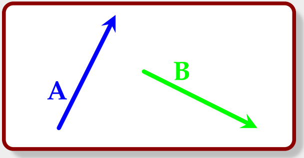

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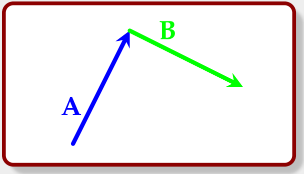

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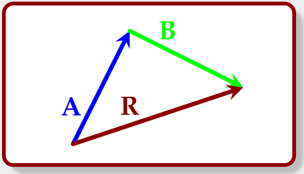

Note that the sum of two vectors is itself a vector.

Addition of Vectors :: It's commutative!

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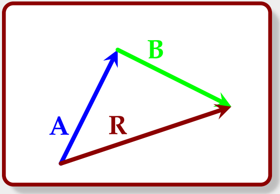

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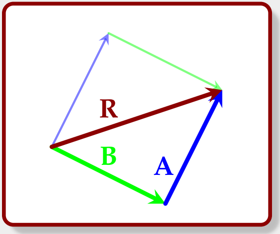

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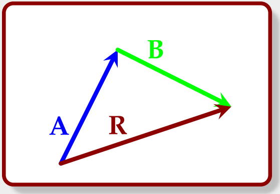

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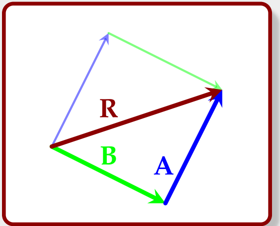


$$\boxed{\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}}$$

Example 1: Displacement Vectors

A displacement is a change in position. It has a magnitude (the distance moved) and a direction, so displacement is a vector quantity.

A truck drives due east on a straight road for 40 km, then drives north on a straight road for 30 km before stopping.

What is the resultant displacement of the truck?

Example 2: Velocity Vectors

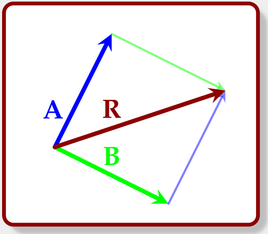
A plane flies NNW (i.e., 22.5° west of north) with a velocity of 275 km/h. There is a wind blowing at 55 km/h from the NW (i.e., 45° west of north).

Determine the resultant velocity of the plane relative to the ground.

Determine the wind speed that would cause the plane to fly due north. What is the ground speed in this case?

The Parallelogram Law:

1. Draw the vectors with their tails at the same point.
2. Form a parallelogram
3. The diagonal of the parallelogram, starting from the tails of the two vectors, is the resultant.



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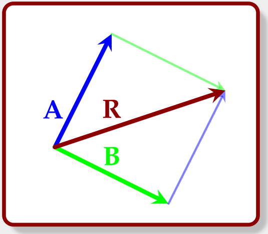
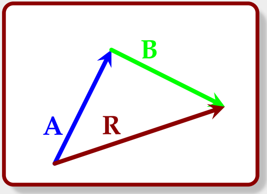


The Triangle Law:

1. This is what we have been doing
2. To do the calculations on the parallelogram above, we end up working with the triangle(s) anyway
3. **Use the triangle law**.



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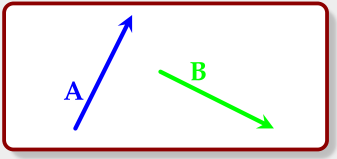

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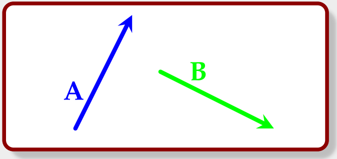
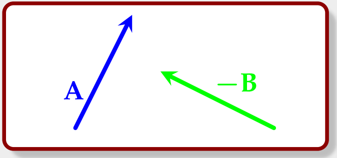


- Now consider $-\mathbf{B}$, which is obtained by reversing the sense of $\mathbf{B}$.



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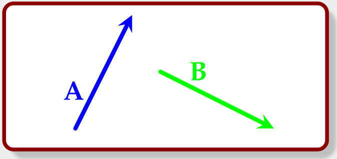

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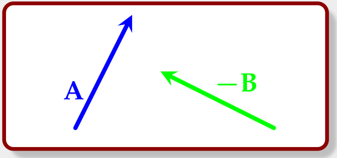
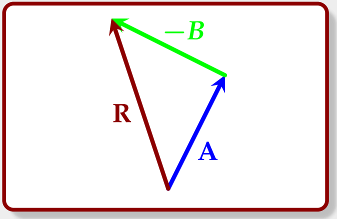


- Then, add $\mathbf{A}$ to $-\mathbf{B}$:

$$\begin{aligned} A - B &= \mathbf{A} + (-\mathbf{B}) \\ &= \mathbf{R} \end{aligned}$$



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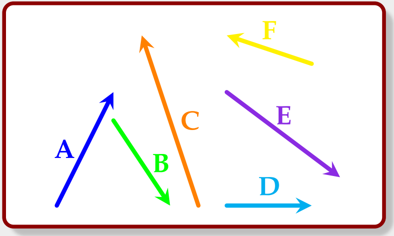

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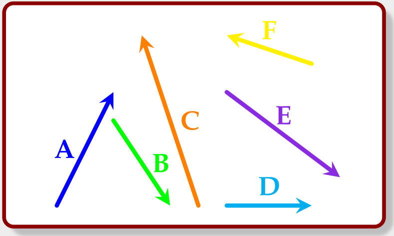

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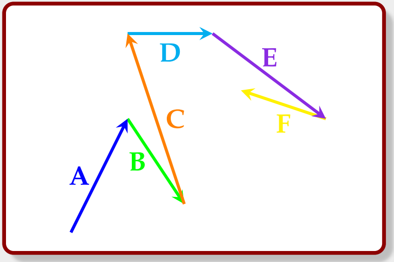

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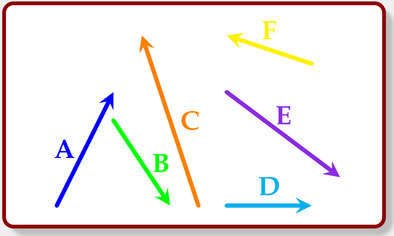

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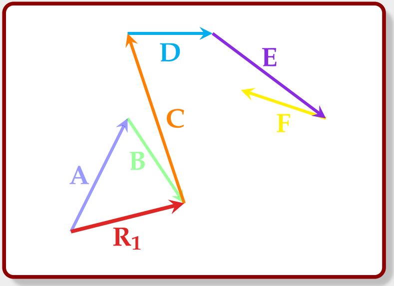

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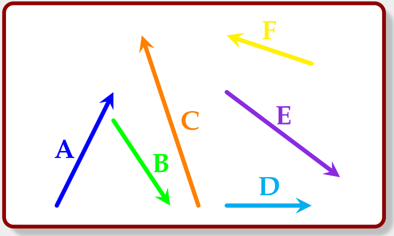

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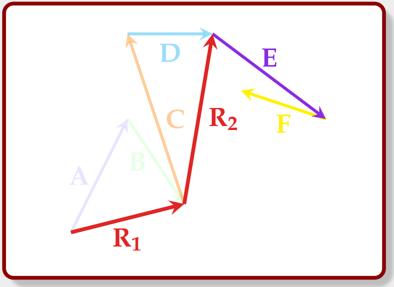

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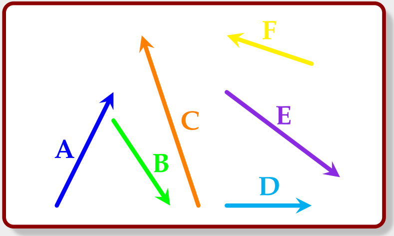

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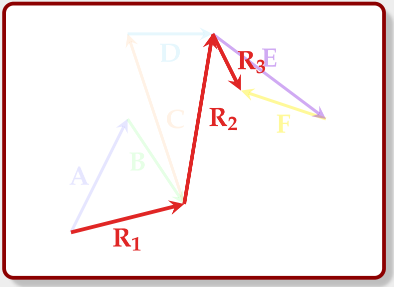

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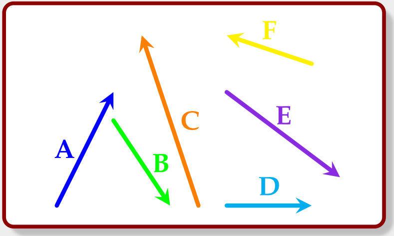

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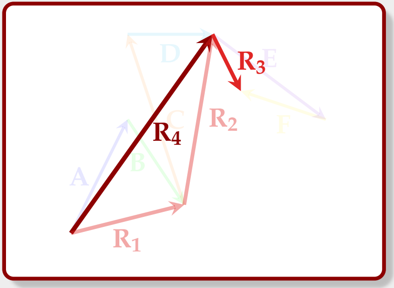

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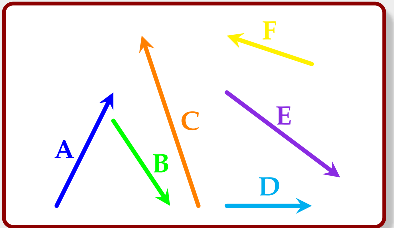

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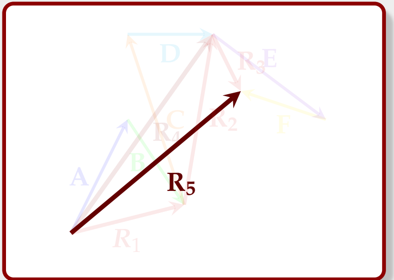

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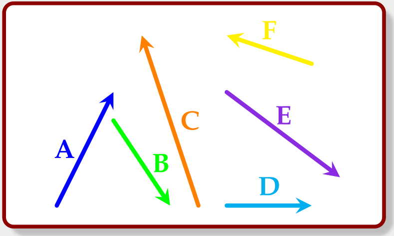

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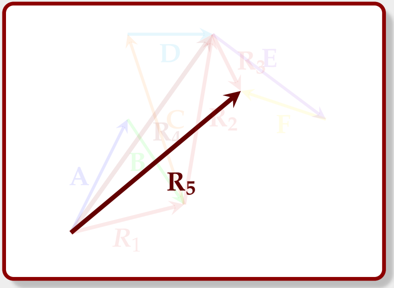
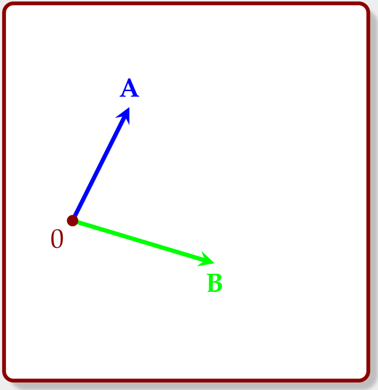
- **This is too much work!** We will find a more efficient way soon :)



Vector Addition of Forces

- **Force** is a vector quantity. It has a magnitude and a direction.
- Consider your weight. It is a force; it has a magnitude (newtons or pounds). It has a direction (along a line of action that passes between you and the centre of the earth). And it has a sense: down, towards the centre of the earth. If you step off a diving board, you will accelerate downwards in a predictable fashion.
- We have seen that we can add vectors together so we can do the same for forces.
- Multiple forces acting on an object through a point have a **resultant** force (the net force, which is the combined result of the summing together of the multiple forces).
- The forces which, when combined, sum to this resultant are known as **component** forces.

- Consider forces $\mathbf{A}$ and $\mathbf{B}$ acting at point 0, as shown.



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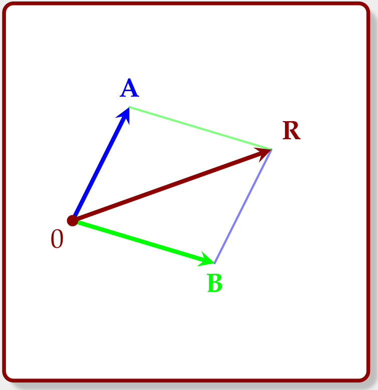

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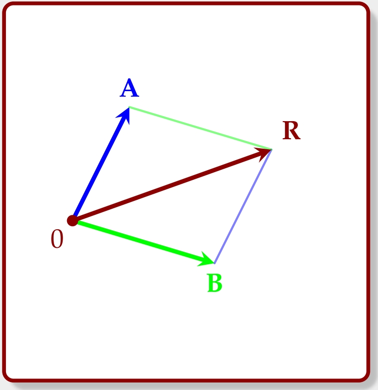

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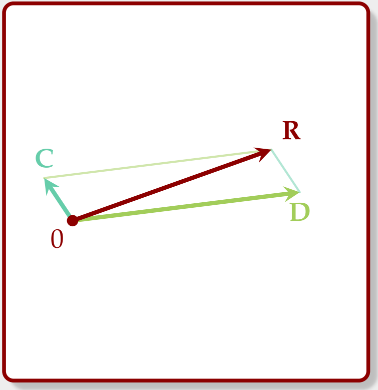

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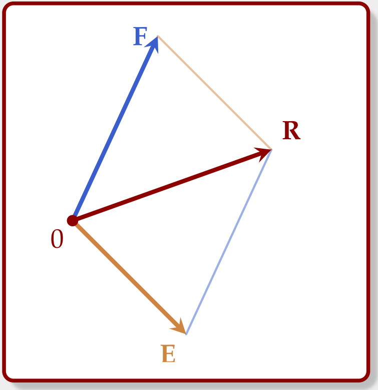

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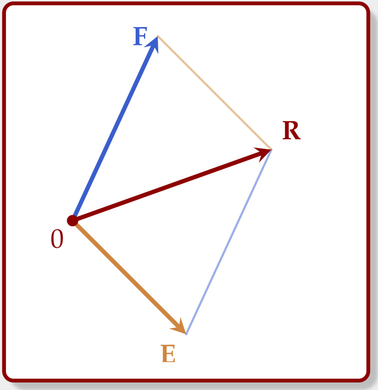
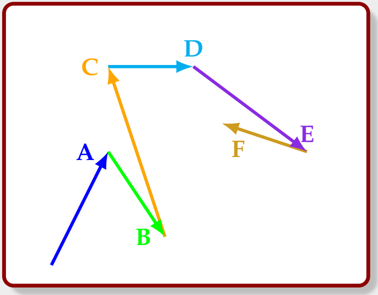
- There are infinitely many possible component forces for each force $\mathbf{R}$



Finding Resultants and Components

- In each of the examples just given, there were only two components but there can be many components for each resultant.



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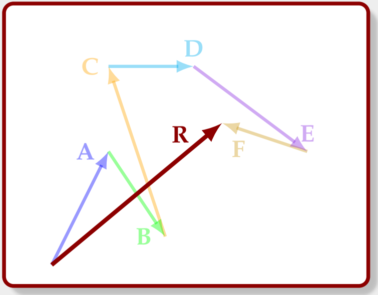

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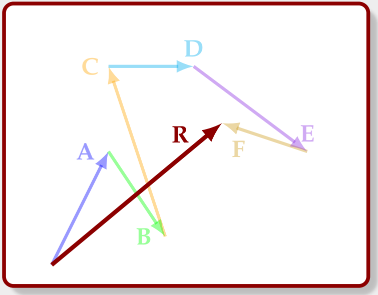
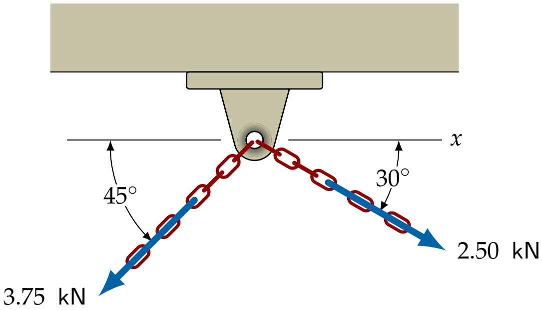
- In this course, for each force $\boldsymbol{R}$ we shall generally need only two components.



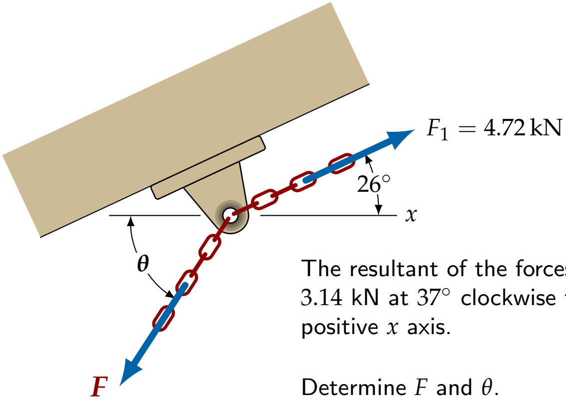
Example 3

Determine the magnitude and the direction (measured clockwise from the the positive x-axis) of the resultant of the two forces.

Note: One kilonewton is one thousand newtons, i.e., $1\,\text{kN} = 1000\,\text{N}$



Exercise 1



The resultant of the forces F and F_1 is 3.14 kN at 37° clockwise from the positive x axis.

Determine F and θ.

- From Newton's Second Law of Motion, $\boldsymbol{F} = \boldsymbol{ma}$ (force = mass x acceleration)
- The weight of an object is a force. It is the gravitational attractive force between the object and the earth.
- If we denote the acceleration due to gravity by $\boldsymbol{g}$ ($g = 9.81\ \text{m/s}^2 = 32.2\ \text{ft/s}^2$) and denote the mass of the object by $\boldsymbol{m}$, then the weight of the object, $\boldsymbol{W}$, is given by:

$$\boxed{W = mg}$$

Metric (SI) Units of Measurement

- There are four basic quantities involved in $F = ma$: force, mass, distance and time.

- According to Newton, a net force of one newton (**N**) causes a mass of one kilogram (**kg**) to accelerate by 1 metre/sec/sec (**m/s^2**).

- A rock dropped from a bridge is subject to a gravitational attractive force and will have an acceleration of 9.81 m/s^2. If the rock has a mass of 2.75 kg, this force is given by:

$$\begin{aligned} F &= ma \\ &= 2.75 \text{ kg} \times 9.81 \text{ m/s}^2 \\ &= 27.0 \text{ N} \end{aligned}$$

  This force F is the weight, W, of the rock. So, $W = mg$

To summarize:

1. $1\,\mathrm{N} = 1\,\mathrm{kg} \times 1\,\mathrm{m/s^2} = 1\,\mathrm{kg} \cdot \mathrm{m/s^2}$

2. To calculate the weight of an object from its mass, multiply the mass in kg by $g = 9.81\,\mathrm{m/s^2}$. The weight is in newtons.

3. To calculate the mass of an object from its weight, divide the weight in newtons by $g = 9.81\,\mathrm{m/s^2}$. The mass is in kilograms.

US Customary (or Imperial) Units of Measurement

- Acceleration due to gravity in feet and seconds is

$$g = 32.2\,\mathrm{ft/s^2}$$

- The unit for force is the pound, or **lb**.
(It may also be described as pound-force, or **lbf**.)

- The unit for mass is the **slug**.

- $1\mathrm{lb} = 1\,\mathrm{slug} \times 1\,\mathrm{ft/s^2} \Rightarrow 1\,\mathrm{slug}$ has the units $\frac{\mathrm{lb \cdot sec}}{\mathrm{ft^2}}$

- To calculate the mass of an object from its weight, divide the weight in pounds by $g = 32.2\,\mathrm{ft/s^2}$. The mass is in slugs.

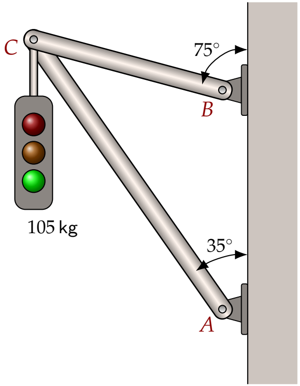
- To calculate the weight of an object from its mass, multiply the mass in slugs by $g = 32.2\,\mathrm{ft/s^2}$. The weight is in pounds.

Component Forces

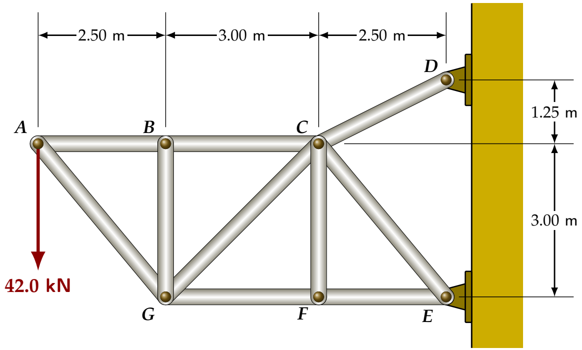
Example 4

The weight, W, of the traffic lights (with mass 105 kg) acts vertically downwards.

Find the value of W and use it to determine the magnitudes of its two components directed along the axes of AC and BC.



Exercise 2



Resolve the 42.0 kN load suspended from A into components parallel to the truss members AB and AG. Give the magnitude of the components and their direction measured counter-clockwise from the positive x axis.

Component Forces

Exercise 3

The decoration suspended at D weighs 1124 N.

Determine the magnitudes of the two force components of the weight of D, in the direction of AB and BC.