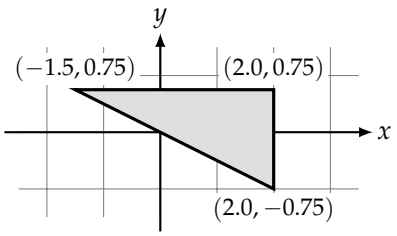
Engineering Statics - 04 Centroids - Instructor Copy

Exercise 1: Determine the coordinates of the centroid of the triangle shown.

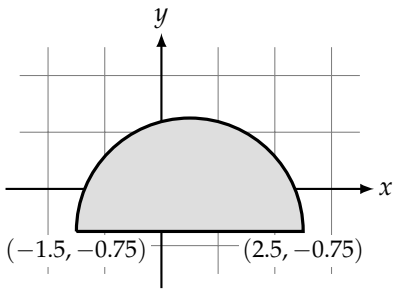


$$\overline{x} = (2.0) - \frac{2.0 - (-1.5)}{3} = 0.83333$$

$$\overline{y} = (0.75) - \frac{0.75 - (-0.75)}{3} = 0.25000$$

$$\boldsymbol{(\overline{x}, \overline{y}) = (0.833, 0.250)}$$

Exercise 2: Determine the coordinates of the centroid of the semi-circle shown.



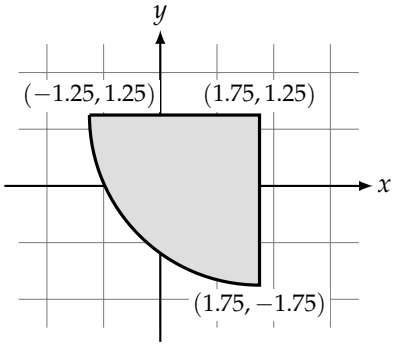
$$r = 2.0$$

$$\overline{x} = \frac{(-1.5) + (2.5)}{2} = 0.50$$

$$\overline{y} = -0.75 + \frac{4 \times 2.0}{3\pi} = 0.098826$$

$$\boldsymbol{(\overline{x}, \overline{y}) = (0.500, 0.988)}$$

Exercise 3: Determine the location of the centroid of the quarter-circle shown.



$$r = 3.00$$

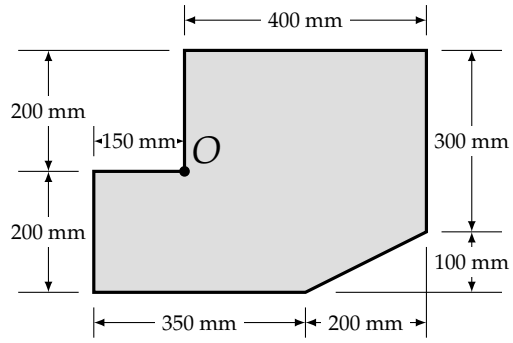
$$\overline{x} = 1.75 - \frac{4 \times 3.00}{3\pi} = 0.47676$$

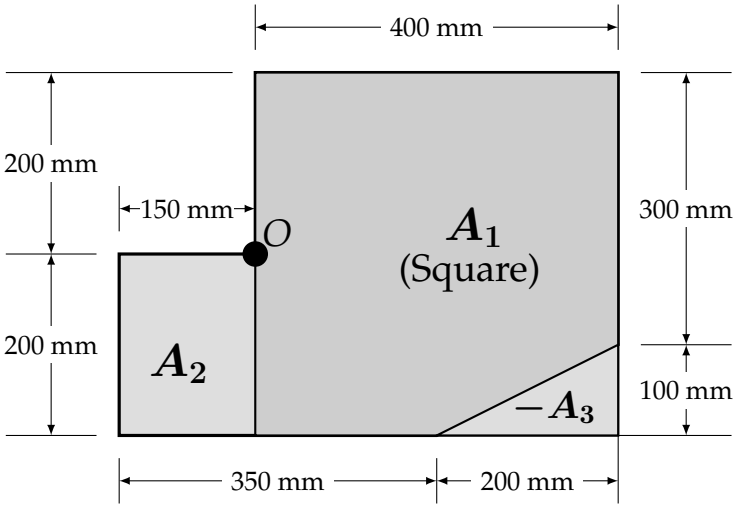
$$\overline{y} = 1.25 - \frac{4 \times 3.00}{3\pi} = -0.023240$$

$$\boldsymbol{(\overline{x}, \overline{y}) = (0.477, -0.0232)}$$

Exercise 4

Find the location of the centroid, C, relative to the point O.





Shape	Area (m^2)	x_i (m)	y_i (m)	$A_i x_i$ (m^3)	$A_i y_i$ (m^3)
A_1	0.4×0.4	0.2	0	0.032000	0
A_2	0.15×0.2	-0.075	-0.1	-0.0022500	-0.0030000
A_3	$-(0.2 \times 0.1)/2$	0.33333	-0.16667	-0.0033333	0.0016667
Σ	0.18			0.026417	-0.0013333

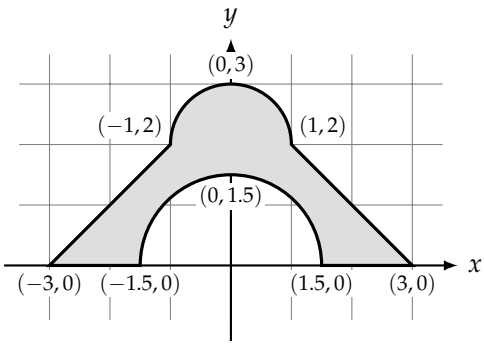
$$\overline{x} = \frac{\sum A_i x_1}{\sum A_1} = \frac{0.026417}{0.18} = 0.14676\,\mathrm{m}$$

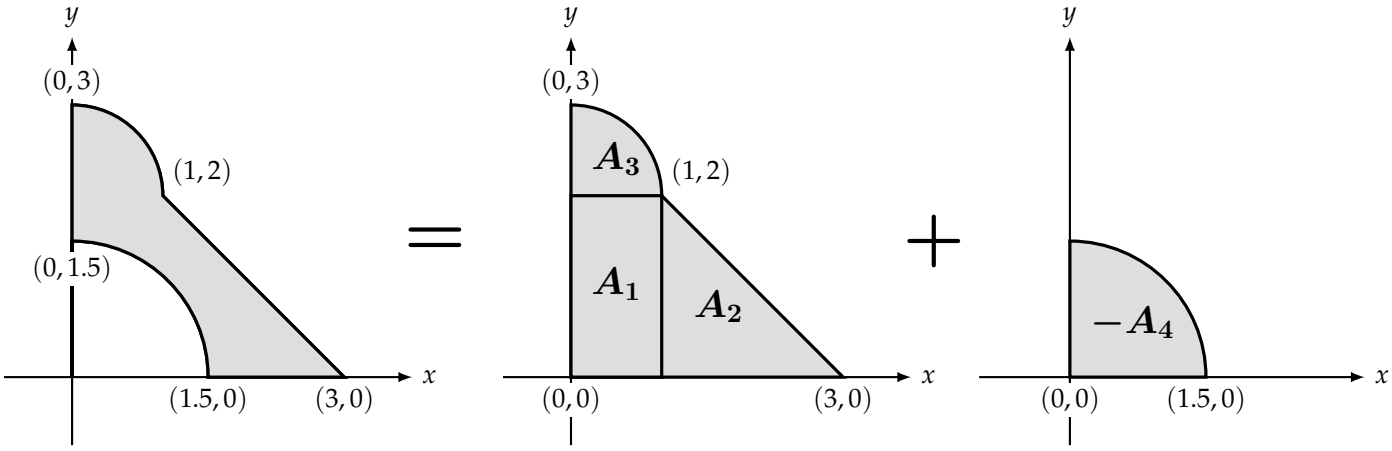
$$\overline{y} = \frac{\sum A_i y_1}{\sum A_1} = \frac{-0.0013333}{0.18} = -0.0074072\,\mathrm{m}$$

$$\boldsymbol{(\overline{x}, \overline{y}) = (147\,\mathrm{mm}, -7.41\,\mathrm{mm})}$$

Exercise 5

Find the location of the centroid, C, relative to the coordinate origin.





Shape	Area	y_i	$A_i y_i$
A_1	1×2	1	2
A_2	$(2\times 2)/2$	0.66667	1.3333
A_3	$\pi/4$	$2+\frac{4}{3\pi}$	1.9041
$-A_4$	$-\frac{\pi(1.5)^2}{4}$	$\frac{4\times 1.5}{3\pi}$	-1.125
Σ	3.0183		4.1124

$$\overline{y} = \frac{\sum A_i y_1}{\sum A_1} = \frac{4.1124}{3.0183} = 1.3625$$

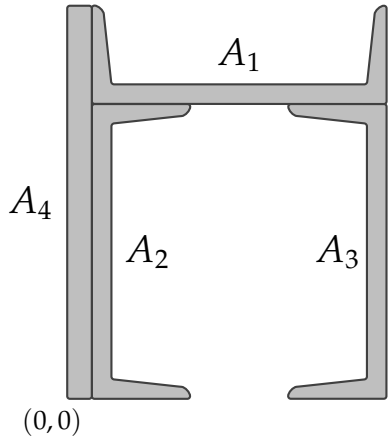
$$\mathbf{(\overline{x}, \overline{y}) = (0, 1.36)}$$

Exercise 6

Three C130X13 channels and a steel plate (15mm$\times$174mm) are welded together.

Determine the location of the centroid, relative to the bottom left hand corner of the composite area.

Properties for C130X13:
Area $= 1700\,\text{mm}^2$
Depth $= 127\,\text{mm}$
$\overline{x} = 12\,\text{mm}$



Shape	Area(mm^2)	$x_i\,(\text{mm})$	$y_i\,(\text{mm})$	$A_i x_i\,(\text{mm}^3)$	$A_i y_i\,(\text{mm}^3)$
A_1	1700	$15 + \frac{127}{2}$	$127 + 12$	133450	236300
A_2	1700	$15 + 12$	$\frac{127}{2}$	45900	107950
A_3	1700	$15 + 127 - 12$	$\frac{127}{2}$	221000	107950
A_4	2610	7.5	87	19575	227070
$\sum$	7710			419925	679270

$$\overline{x} = \frac{\sum A_i x_1}{\sum A_1} = \frac{419925}{7710} = 54.465$$

$$\overline{y} = \frac{\sum A_i y_1}{\sum A_1} = \frac{679270}{7710} = 88.102$$

$$\mathbf{(\overline{x}, \overline{y}) = (54.5\,mm, 88.1\,mm)}$$