Engineering Statics - 01 Math Review Handout (Instructor Copy)

1) Solve $a^2 = b^2 + c^2$ for b.

$$b^2 = a^2 - c^2$$
$$b = \pm\sqrt{\boldsymbol{a^2 - c^2}}$$

2) Solve $V = \frac{4}{3}\pi r^3$ for r.

$$r^3 = \frac{3V}{4\pi}$$
$$r = \sqrt[3]{\boldsymbol{\frac{3V}{4\pi}}}$$

3) Solve $c^2 = a^2 + b^2 - 2bc\cos C$ for $\cos C$.

$$2bc\cos C = a^2 + b^2 - c^2$$
$$\boldsymbol{\cos C = \frac{a^2 + b^2 - c^2}{2bc}}$$

4) Solve $b^2 = a^2 + c^2 - 2ac\cos B$ for B.

$$2ac\cos B = a^2 + c^2 - b^2$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2bc}$$
$$\boldsymbol{B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2bc}\right)}$$

5) Solve the equation for h_L, then evaluate h_L using the values $Q = 135$, $C = 120$, $D = 202.7$ and $L = 1200$

$$Q = \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}$$

$$\Rightarrow \left(\frac{h_L}{L}\right)^{0.54} = \frac{279000Q}{CD^{2.63}}$$

$$\Rightarrow \frac{h_L}{L} = \left(\frac{279000Q}{CD^{2.63}}\right)^{1/0.54}$$

$$\Rightarrow \boldsymbol{h_L = L\left(\frac{279000Q}{CD^{2.63}}\right)^{1.85}}$$

$$h_L = 1200\left(\frac{279000 \times 135}{120 \times 202.7^{2.63}}\right)^{1.85}$$

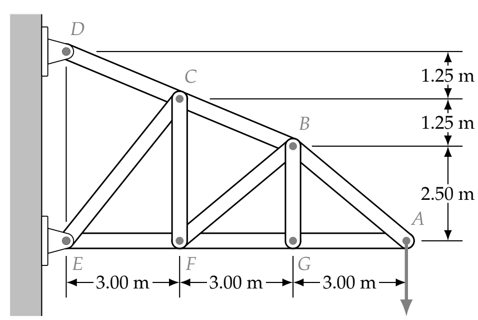
$$= 105.73$$

$$\boldsymbol{h_L = 106}$$

6) Use the Pythagorean Theorem to determine the lengths of CE and CB

$$\begin{aligned} CE^2 &= EF^2 + CF^2 \\ &= (3.00\,\mathrm{m})^2 + 3.75^2 \\ CE &= 4.8023 \\ &= \mathbf{4.80\,m} \end{aligned}$$

$$\begin{aligned} CB^2 &= 1.25^2 + 3.00^2 \\ CB &= 3.2500 \\ &= \mathbf{3.25\,m} \end{aligned}$$



7) Use the tangent function to calculate $\angle CEF$

$$\angle CEF = \tan^{-1}\left(\frac{CF}{EF}\right) = \tan^{-1}\left(\frac{3.75\,\mathrm{m}}{3.00\,\mathrm{m}}\right) = 51.340^\circ = \mathbf{51.3^\circ}$$

8) Use $\angle CEF$ just found and the sine function to verify the length of CE found above.

$$\begin{aligned}
\frac{CD}{\sin 90^\circ} &= \frac{CF}{\sin \angle CEF} \\
\Rightarrow CE &= \frac{(3.75\,\mathrm{m})\sin 90^\circ}{\sin 51.340^\circ} \\
&= 4.8024 \\
\boldsymbol{CE} &= \mathbf{4.80\,m}
\end{aligned}$$

9) Use the cosine function and the length of BC found earlier to calculate the angle, θ, between BC and the horizontal.

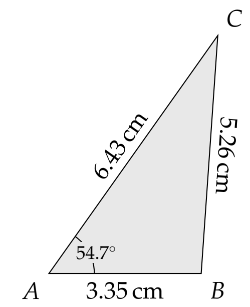
$$\begin{aligned}
(1.25\,\mathrm{m})^2 &= (3.00\,\mathrm{m})^2 + (3.25\,\mathrm{m})^2 - 2(3.00\,\mathrm{m})(3.25\,\mathrm{m})\cos\theta \\
\Rightarrow \cos\theta &= \frac{(3.00\,\mathrm{m})^2 + (3.25\,\mathrm{m})^2 - (1.25\,\mathrm{m})^2}{2(3.00\,\mathrm{m})(3.25\,\mathrm{m})} \\
&= 0.92308 \\
\Rightarrow \theta &= 22.619^\circ \\
\boldsymbol{\theta} &= \mathbf{22.6^\circ}
\end{aligned}$$

10) Use the tangent function to verify the previous result.

$$\theta = \tan^{-1}\left(\frac{1.25\,\mathrm{m}}{3.00\,\mathrm{m}}\right) = 22.620 = \mathbf{22.6^\circ}$$

11) Using the sine rule, find $\angle ACB$.

$$\frac{\angle ACB}{3.35\,\text{cm}} = \frac{\sin 54.7^\circ}{5.26\,\text{cm}}$$
$$\Rightarrow \angle ACB = \sin^{-1}\left(\frac{(3.35\,\text{cm})\sin 54.7^\circ}{5.26\,\text{cm}}\right)$$
$$= 31.318^\circ$$
$$= \mathbf{31.3^\circ}$$



12) Using the sine rule, find $\angle ABC$.

$$\frac{\angle ABC}{6.43\,\text{cm}} = \frac{\sin 54.7^\circ}{5.26\,\text{cm}}$$

$$\Rightarrow \angle ABC = \sin^{-1}\left(\frac{(6.43\,\text{cm})\sin 54.7^\circ}{5.26\,\text{cm}}\right)$$
$$= 86.091^\circ$$

$$\boldsymbol{\angle ABC = 86.1^\circ} \qquad (\text{or } 93.9(09)^\circ\ldots)$$

13) Sum the interior angles of the triangle.

$$= 54.7^\circ + 33.318^\circ + 86.091^\circ = \mathbf{172.11^\circ}$$

(Explain it's due to choice of $\angle ABC = 86.1^\circ$ from the inverse sin above. Or wait until $\angle ABC$ is found from the cos rule in the next exercise.)

14) Using the cosine rule, determine $\angle ABC$

15) Compare with the earlier value calculated for $\angle ABC$

$$(6.43\,\text{cm})^2 = (3.35\,\text{cm})^2 + (5.26\,\text{cm})^2 - 2(3.35\,\text{cm})(5.26\,\text{cm})\cos\angle ABC$$
$$\Rightarrow \angle ABC = \cos^{-1}\left[\frac{(3.35\,\text{cm}^2) + (5.26\,\text{cm})^2 - (6.43\,\text{cm})^2}{2(3.35\,\text{cm})(5.26\,\text{cm})}\right]$$
$$= 93.994^\circ$$
$$\boldsymbol{\angle ABC = 94.0^\circ}$$

(Slightly different since from 93.9° since 54.7° is accurate to 3 significant digits and not a more precise given value.)

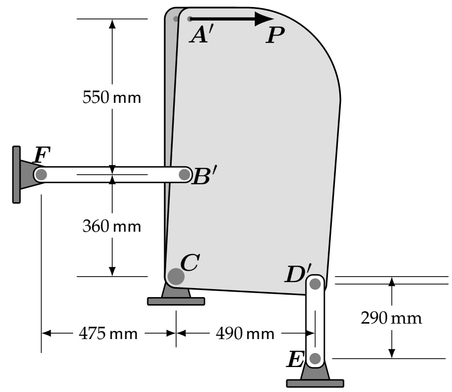
When horizontal force P is applied at A, $ABCD$ rotates about C and A deflects 2.45 mm horizontally rightwards.

Assume that BF remains horizontal and that DE remains vertical.

16) Determine δ_{BF}, the change in length of BF.

$\triangle CAA'$, $\triangle CBB'$ and $\triangle CDD'$ are all similar.

$$\frac{\delta_{BF}}{2.45\,\text{mm}} = \frac{360\,\text{mm}}{360\,\text{mm} + 550\,\text{mm}}$$
$$\Rightarrow \delta_{BF} = 0.96923\,\text{mm}$$
$$\boldsymbol{\delta_{BF} = 0.969\ \text{mm}}$$



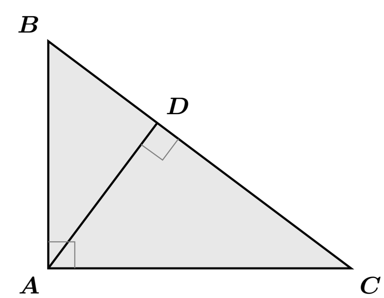
17) Determine δ_{DE}, the change in length of DE.

$$\frac{\delta_{DE}}{2.45\,\text{mm}} = \frac{490\,\text{mm}}{360\,\text{mm} + 550\,\text{mm}}$$
$$\Rightarrow \delta_{BF} = 1.3192$$
$$= \mathbf{1.32\ mm}$$

18) Show that right triangles $\triangle ABC$, $\triangle ABD$ and $\triangle ACD$ all have the same angles (i.e. they are all similar).

$$\text{Let } \angle DCA = \theta$$
$$\text{Then } \angle DAC = 90^\circ - \theta$$
$$\Rightarrow \angle DAB = \theta$$
$$\Rightarrow \angle DBA = 90^\circ - \theta$$

Each of the three triangles has angles θ, $90^\circ - \theta$ and 90°, so they are similar.



19) Given that $AC = 100$ mm and $AD = 65$ mm, determine $\angle ACD$ and $\angle ABD$.

$$\angle ACD = \sin^{-1}\left(\frac{65\,\text{mm}}{100\,\text{mm}}\right) = 40.542^\circ = \mathbf{40.5^\circ}.$$
$$\angle ABD = 90^\circ - 40.542^\circ = 49.458^\circ = \mathbf{49.5^\circ}.$$

20) Find the remaining lengths: AB, BD and CD.

$$\frac{AB}{100\,\text{mm}} = \tan 40.542^\circ \Rightarrow AB = 85.263\,\text{mm} = \mathbf{85.2\,mm}$$

$$\frac{BD}{65\,\text{mm}} = \tan 40.542^\circ \Rightarrow BD = 55.598\,\text{mm} = \mathbf{55.6\,mm}$$

$$\frac{DC}{65\,\text{mm}} = \tan\left(90^\circ - 40.542^\circ\right) \Rightarrow DC = 75.993\,\text{mm} = \mathbf{76.0\,mm}$$

21) Verify the lengths found above by using the Pythagorean Theorem on $\triangle ABC$

$$AC^2 + AB^2 = (100\,\text{mm})^2 + (85.263\,\text{mm})^2 = 17270\,\text{mm}^2 = BC^2$$

$$\Rightarrow BC = 131.42\,\text{mm}$$

$$BD + CD = 55.598\,\text{mm} + 75.993\,\text{mm} = BC \Rightarrow BC = 131.59\,\text{mm}$$

An accumulation of rounding errors can start to cause differences when there are so many calculations and so many rounded intermediate values to use.

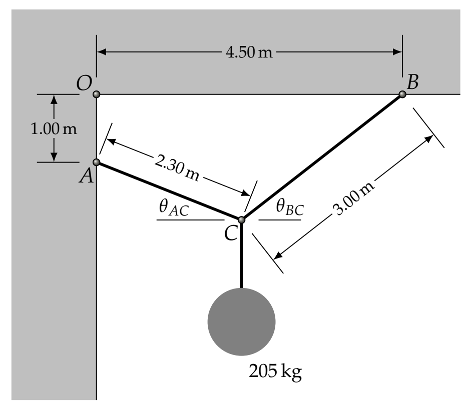
22) Find θ_{AC}. **23)** Find θ_{BC}.

$$\begin{aligned} AB^2 &= OA^2 + OB^2 \\ &= (1.00\,\text{m})^2 + (4.50\,\text{m})^2 \\ \Rightarrow AB &= 4.6098\,\text{m} \end{aligned}$$

$$\cos\angle ACB = \frac{AB^2 + BC^2 - AB^2}{2(AB)(BC)}$$

$$= \frac{(2.30\,\text{m})^2 + (3.00\,\text{m})^2 - (4.6098\,\text{m})^2}{2(2.30\,\text{mm})(3.00\,\text{m}^2)}$$

$$\Rightarrow \angle ACB = 120.29^\circ$$



$$\angle OBA = \tan^{-1}\left(\frac{1.00\,\text{m}}{4.50\,\text{m}}\right) = 12.529^\circ$$

$$\frac{\sin ABC}{AC} = \frac{\sin ACB}{AB} \Rightarrow \frac{\sin ABC}{2.30\,\text{m}} = \frac{\sin 120.29^\circ}{4.6098\,\text{m}} \Rightarrow \angle ABC = 25.520^\circ$$

$$\theta_{BC} = \angle OBA + \angle ABC = 12.259^\circ + 25.520^\circ = 38.049^\circ \qquad \text{(Alternate angles.)}$$

$$\theta_{AC} = 180^\circ - \angle ACB - \theta_{BC} = 180^\circ - 120.29^\circ - 38.049^\circ = 21.661^\circ$$

$$\boldsymbol{\theta_{AC} = 21.7^\circ, \quad \theta_{BC} = 38.0^\circ}$$

24) and **25)** Find the values of $\boldsymbol{x}$ and $\boldsymbol{y}$ in the system shown.

$$0.36911x + 0.61633y = 2011.1 \quad (1)$$
$$0.78748y - 0.92938x = 0 \quad (2)$$

From (2),

$$y = \frac{0.92938}{0.78748}x = 1.1802x \quad (3)$$

Substitute in (1) to find x

$$\begin{aligned} 0.36911x + 0.61633(1.1802x) &= 2011.1 \\ \Rightarrow (0.36911 + 0.61633(1.1802))\, x &= 2011.1 \\ \Rightarrow x &= \frac{2011.1}{(0.36911 + 0.61633(1.1802))} \\ &= 1834.1 \\ \boldsymbol{x} &= \mathbf{1830} \end{aligned}$$

Substitute back in (3) for y

$$\begin{aligned} 0.78748y - 0.92938(1834.1) &= 0 \\ y &= 2164.6 \\ \boldsymbol{y} &= \mathbf{2160} \end{aligned}$$

26) and **27)** Determine F_{AC} and F_{BC}

$$F_{BC}\sin 15^\circ + F_{AC}\cos 35^\circ + 1030.1 = 0 \quad (1)$$
$$F_{BC}\cos 15^\circ + F_{AC}\sin 35^\circ = 0 \quad (2)$$

From (2)

$$F_{BC} = -\frac{\sin 35^\circ}{\cos 15^\circ}F_{AC} = -0.59381F_{AC} \quad (3)$$

Substitute into (1)

$$\begin{aligned} (-0.59381F_{AC})\sin 15^\circ + F_{AC}\cos 35^\circ &= -1030.1 \\ -0.15369F_{AC} + 0.81915F_{AC} &= -1030.1 \\ \Rightarrow F_{AC} &= \frac{-1030.1}{-0.15369 + 0.81915} \\ &= -1548.0 \\ &= \mathbf{-1550} \end{aligned}$$

Substitute back into (3)

$$F_{BC} = -0.59381(-1548.0) = 919.19 = \mathbf{919}$$