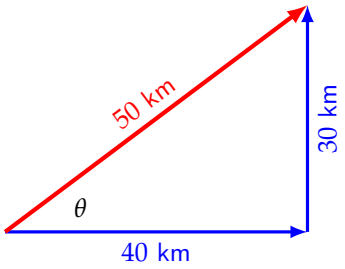
02 Force Vectors Handout - Instructor Copy

Example 1

A truck drives due east on a straight road for 40 km, then drives north on a straight road for 30 km before stopping.

What is the resultant displacement of the truck?



$$R^2 = (40.0\text{ km})^2 + (30.0\text{ km})^2$$
$$\Rightarrow R = 50.0\text{ km}$$

$$\theta = \tan^{-1}\left(\frac{30\text{ km}}{40\text{ km}}\right)$$
$$= 36.870^\circ$$

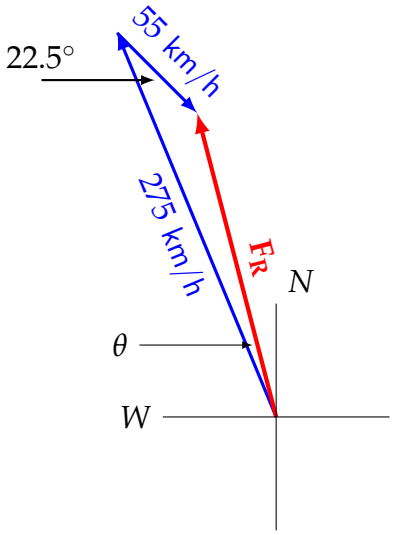
The displacement of the truck is 50.0 km, 36.9° north of east (i.e., counterclockwise from the positive x axis).

Example 2

A plane flies NNW (i.e., 22.5° west of north) with a velocity of 275 km/h. There is a wind blowing at 55 km/h from the NW (i.e., 45° west of north).

Determine the resultant velocity of the plane relative to the ground.

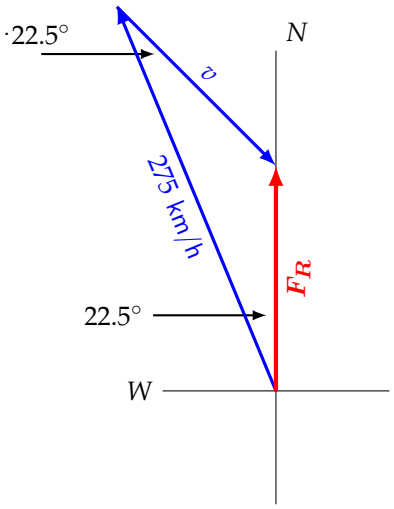
Determine the wind speed that would cause the plane to fly due north. What is the ground speed in this case?



$$|F_R|^2 = 275^2 + 55^2 - 2(275)(55)\cos 22.5^\circ$$
$$\Rightarrow |F_R| = 225.17\text{ km/h}$$

$$\frac{\sin\theta}{55} = \frac{\sin 22.5^\circ}{275}$$
$$\Rightarrow \theta = 4.3895^\circ$$

$$\mathbf{F_R = 225 km/h\text{ at }117^\circ}$$
(ccw from pos x axis)
(or 17.1° W of N)



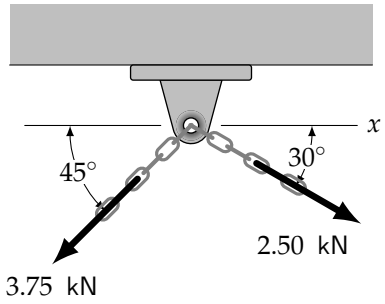
$$\frac{v}{\sin 22.5^\circ} = \frac{275 \text{ km/h}}{\sin 135^\circ}$$

$$\Rightarrow v = 148.83 \text{ km/h}$$
$$\Rightarrow \boldsymbol{F_R} = 148.83 \text{ km/h}$$

Because of the isosceles triangle, wind speed equals ground speed and is **149 km/h**.

Example 3

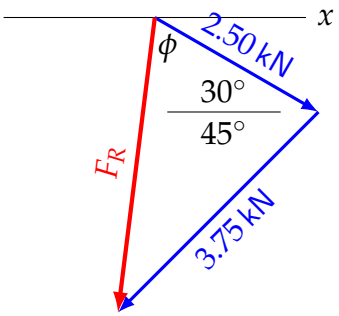
Determine the magnitude and the direction (measured clockwise from the the positive x-axis) of the resultant of the two forces.



$$\begin{aligned}(R)^2 &= (2.50\,\text{kN})^2 + (3.75\,\text{kN})^2 - 2(2.50\,\text{kN})(3.75\,\text{kN})\cos 75^\circ \\ &= (15.460\,\text{kN})^2 \\ \Rightarrow |R| &= 3.9319\,\text{kN}\end{aligned}$$

$$\frac{\sin\phi}{3.75\,\text{kN}} = \frac{\sin 75^\circ}{R} = \frac{\sin 75^\circ}{3.9319\,\text{kN}}$$
$$\Rightarrow \phi = \sin^{-1}\left[\frac{3.75\,\text{kN}\cdot\sin 75^\circ}{3.9319\,\text{kN}}\right]$$
$$= 67.108^\circ$$

$$R_\theta = \phi + 30^\circ$$
$$= 97.108^\circ$$

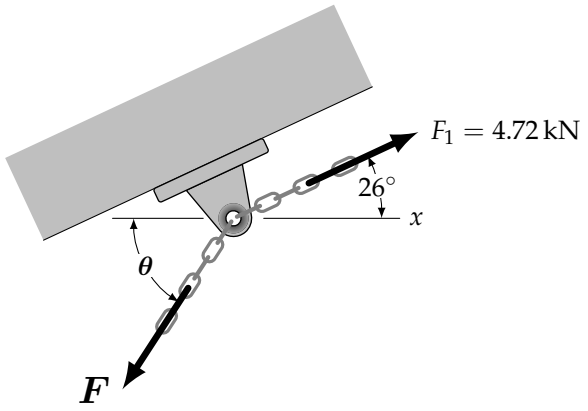


The resultant of the two forces is 3.93 kN at 97.1° measured clockwise from the positive x-axis.

Exercise 1

The resultant of the forces F and F_1 is 3.14 kN at 37° clockwise from the positive x axis.

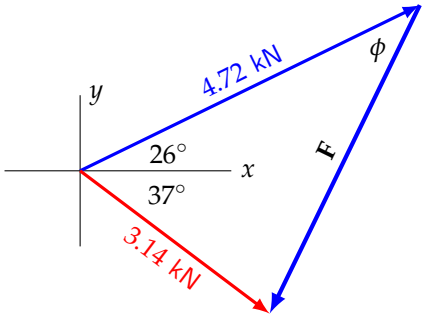
Determine F and θ.



$$|F|^2 = (4.72\text{ kN})^2 + (3.14\text{ kN})^2 - 2(4.72\text{ kN})(3.14\text{ kN})\cos 63^\circ$$
$$\Rightarrow |F| = 4.3222\text{ kN}$$

$$\frac{\sin\phi}{3.14\text{ kN}} = \frac{\sin 63^\circ}{4.3222\text{ kN}}$$
$$\Rightarrow \phi = 40.338^\circ$$
$$\Rightarrow \theta = 40.338^\circ + 26^\circ$$

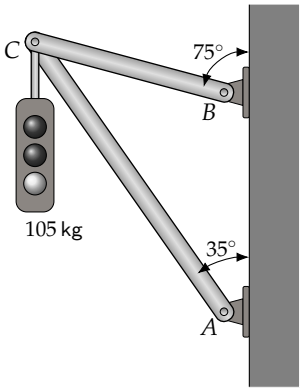
$$\boldsymbol{F = 4.32\,\text{kN}} \text{ and } \boldsymbol{\theta = 66.3^\circ}$$



Example 4

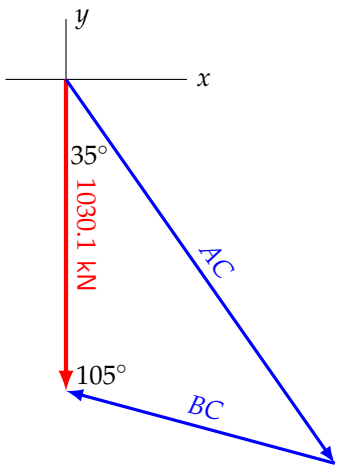
The weight, W, of the traffic lights (with mass 105 kg) acts vertically downward.

Find the value of W and use it to determine the magnitudes of its two components directed along the axes of AC and BC.



$$\frac{AC}{\sin 105^\circ} = \frac{1030.1\text{ kN}}{\sin 40^\circ} \Rightarrow AC = 1547.9\text{ kN} = \boldsymbol{1550\text{ kN}}$$

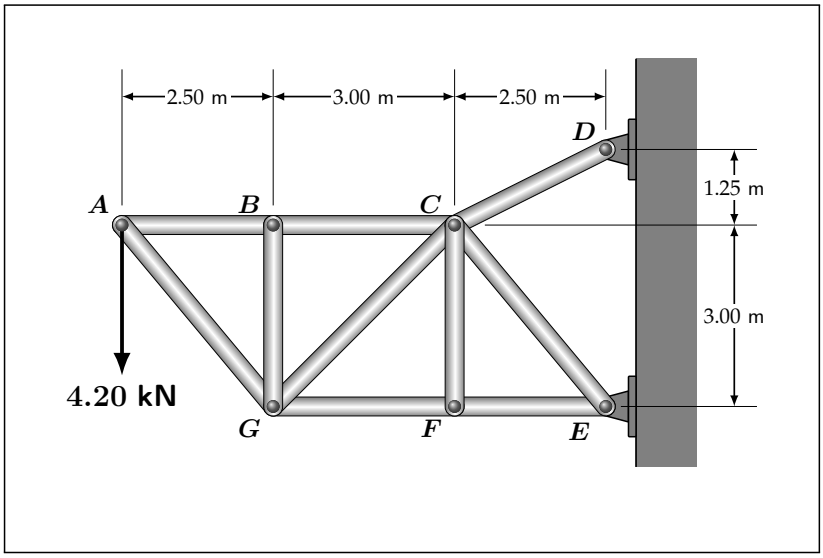
$$\frac{BC}{\sin 35^\circ} = \frac{1030.1\text{ kN}}{\sin 40^\circ} \Rightarrow AC = 919.19\text{ kN} = \boldsymbol{919\text{ kN}}$$

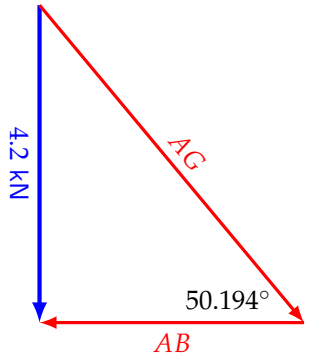


Exercise 2

Resolve the 4.20 kN load suspended from A into components parallel to the truss members AB and AG.

Give the magnitude of the components and their direction measured counter-clockwise from the positive x axis.





$$\angle BAG = \tan^{-1}\frac{BG}{AB} = \tan^{-1}\frac{3.00\,\text{m}}{2.50\,\text{m}} = 50.194^\circ$$

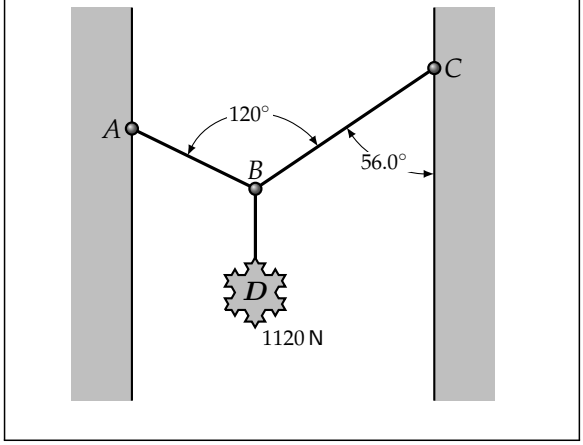
$$AG = \frac{4.2\ \text{kN}}{\sin 50.194^\circ} = 5.4672\ \text{kN} = \mathbf{5.47\ kN\ at\ 310^\circ}$$

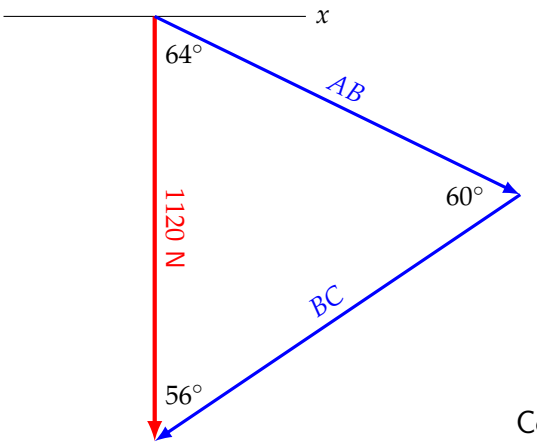
$$AB = (5.4672\ \text{kN})\cos 50.194 = 3.5000\ \text{kN} = \mathbf{3.50\,kN\ at\ 180^\circ}$$

Exercise 3

The decoration suspended at D weighs 1120 N.

Determine the magnitudes of the two force components of the weight of D, in the direction of AB and BC.





$$\frac{BC}{\sin 64^\circ} = \frac{1120\ \text{N}}{\sin 60^\circ} \Rightarrow BC = 1162.4\ \text{N}$$

$$\frac{AB}{\sin 56^\circ} = \frac{1120\ \text{N}}{\sin 60^\circ} \Rightarrow AB = 1072.2\ \text{N}$$

Components are 1070 N along AB, 1160 N along BC

Example 5

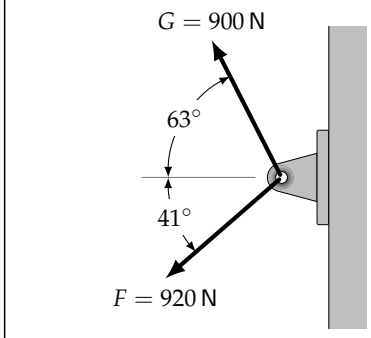
a) Determine the resultant $\boldsymbol{R}$ of the two vectors $\boldsymbol{F}$ and $\boldsymbol{G}$.

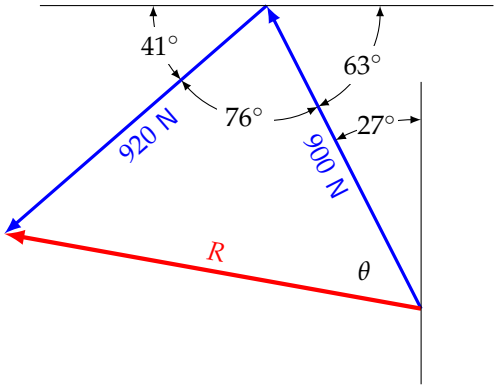
b) Determine the x-component of $\boldsymbol{R}$ (i.e., the horizontal component).

c) Determine the x-component of $\boldsymbol{F}$.

d) Determine the x-component of $\boldsymbol{G}$.

e) Add the two previous results.





a)

$$|\mathbf{R}|^2 = (920\,\text{N})^2 + (900\,\text{N})^2 - 2\,(920\,\text{N})\,(900\,\text{N})\cos 76^\circ$$

$$|\mathbf{R}| = 1120.6\,\text{N}$$

$$\frac{\sin\theta}{920\,\text{N}} = \frac{\sin 76^\circ}{1120.6} \Rightarrow \theta = 52.807^\circ$$

$$\begin{aligned}\mathbf{R} &= 1121\,\text{N at } (90^\circ + 27^\circ + 52.807^\circ)\\ &= 1121\,\text{N at } 169.81^\circ\\ &= 1121\,\text{N at } 170^\circ \text{ (measured c-wise from the pos } x\text{-axis.)}\end{aligned}$$

b) $\mathrm{R}_x = -(1120.6\,\text{N})\cos 10.190^\circ = \mathbf{-1102.9\,N}$

c) $\mathrm{F}_x = -(920\,\text{N})\cos 41^\circ = -694.33\,\text{N}$

d) $\mathrm{G}_x = -(900\,\text{N})\cos 63^\circ = -408.59\,\text{N}$

e) $\mathrm{F}_x + \mathrm{G}_x = -694.33\,\text{N} - 408.59\,\text{N} = \mathbf{-1102.9\,N}$

Exercise 4 Using R from Example 5:

a) Determine the magnitude of the component of $\mathbf{R}$ along the y axis (i.e., the vertical component).

b) Determine the magnitude of the component of $\mathbf{F}$ along the y axis.

c) Determine the magnitude of the component of $\mathbf{G}$ along the y axis.

d) Add the two previous results.

Using R from previous example:

a) $\mathrm{R}_y = (1120.6\,\text{N})\sin 10.190^\circ = 198.25\text{N}$

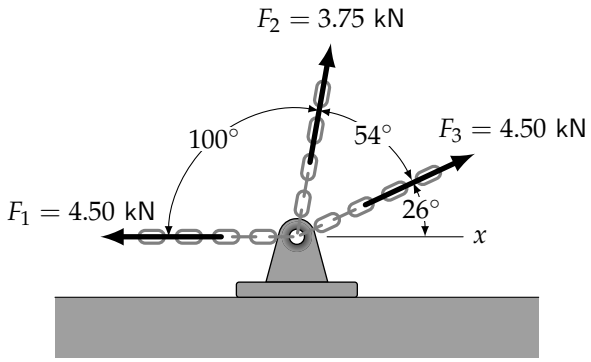
b) $\mathrm{F}_y = -(920\,\text{N})\sin 41^\circ = -603.57\text{N}$

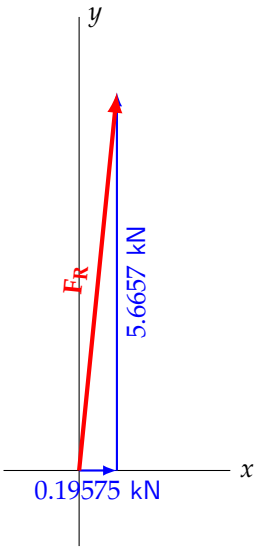
c) $\mathrm{G}_y = -(900\,N)\sin 63^\circ = 801.91\text{N}$

d) $\mathrm{F}_y + \mathrm{G}_y = 198.34\,\text{N}$

Example 9

Determine the resultant (magnitude and direction counterclockwise from the positive x axis) of the three forces F_1, F_2 and F_3 acting at a single point.





$$R_x = (4.50\text{ kN})\cos 26^\circ + (3.75\text{ kN})\cos 80^\circ - 4.50\text{ kN} = 0.19575\text{ kN}$$

$$R_y = (4.50\text{ kN})\sin 26^\circ + (3.75\text{ kN})\sin 80^\circ = 5.6657\text{ kN}$$

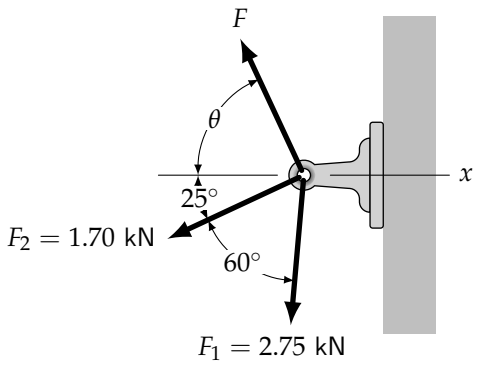
$$|R| = \sqrt{(0.19575\text{ kN})^2 + (5.6657\text{ kN})^2} = 5.6691\text{ kN} = 5.67\,\text{kN}$$

$$\theta = \tan^{-1}\left(\frac{5.6657}{0.19575}\right) = 88.021^\circ = 88.0^\circ$$

R is 5.67 kN at 88.0°, measured counter-clockwise from the positive x-axis.

Example 10

The resultant of the forces F, F_1 and F_2 acting upon the eye-bolt is 3.07 kN at 197° measured counter-clockwise from the positive x axis. Determine F and θ.



$$\mathrm{R}_x = -(3.07\ \text{kN})\cos 17^\circ = -2.9359\ \text{kN}$$
$$\text{and } \mathrm{R}_y = -(3.07\ \text{kN})\sin 17^\circ = -0.89758\ \text{kN}$$

$$\text{Then: } \Sigma F_x = \mathrm{R}_x$$
$$= -F\cos\theta - (1.70\ \text{kN})\cos 25^\circ - (2.75\ \text{kN})\cos 85^\circ$$
$$= -F\cos\theta - 1.7804\ \text{kN}$$
$$= -2.9359\ \text{kN}$$
$$-2.9359\ \text{kN} = -F\cos\theta - 1.7804\ \text{kN}$$
$$\Rightarrow F\cos\theta = 1.1555\ \text{kN}$$

$$\text{And: } \Sigma F_y = \mathrm{R}_y$$
$$= F\sin\theta - (1.70\ \text{kN})\sin 25^\circ - (2.75\ \text{kN})\sin 85^\circ$$
$$= F\sin\theta - 3.4580\ \text{kN}$$
$$= -0.89758\ \text{kN}$$
$$-0.89758\ \text{kN} = F\sin\theta - 3.4580\ \text{kN}$$
$$\Rightarrow F\sin\theta = 2.5604\ \text{kN}$$
$$\Rightarrow \frac{F\sin\theta}{F\cos\theta} = \frac{2.5604\ \text{kN}}{1.1555\ \text{kN}}$$
$$\Rightarrow \tan\theta = 2.2185$$
$$\Rightarrow \theta = 65.710^\circ$$
$$\Rightarrow F\sin 65.710^\circ = 2.5604\ \text{kN}$$
$$\Rightarrow F = 2.8091\ \text{kN}$$

$\underline{\boldsymbol{F = 2.81\,\text{kN}}\ \text{and}\ \boldsymbol{\theta = 65.7^\circ}.}$