

The 'gold' area is given by

$$A = \frac{(\theta - \sin \theta)D^2}{8}$$

where θ is the angle subtended at O by BB', in radians $(\pi/3)$ and D is the diameter (12).

$$A_{Gold} = \frac{\left(\frac{\pi}{3} - \sin 60^{\circ}\right) (24)^{2}}{8}$$
$$= 13.044$$

(Yes, I know I'm mixing units here but $\sin 60^\circ = \sin \frac{\pi}{3}$ radians and I like to keep my calculator in degree mode – because I have a habit of forgetting to return to degree mode...)

Similarly, the 'gold and green' areas are given by:

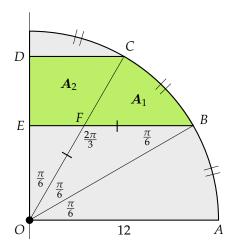
$$A_{Green+Gold} = \frac{\left(\frac{2\pi}{3} - \sin 120^{\circ}\right) (24)^{2}}{8}$$

= 88.443

So, the 'green' area (AA'B'B) is simply the difference between the two areas:

$$A_{Green} = 88.443 - 13.044 = 75.399$$

But the original question only wants half of this so the answer is 37.7 units.



$$m{A_1} = ext{Area} \ OBC - ext{Area} \ OBF$$
 Area $OBC = rac{\pi/6}{2\pi} \cdot \pi (12)^2 = 12\pi$

For $\triangle OBF$:

$$\frac{BF}{\sin \pi/6} = \frac{12}{\sin 2\pi/3}$$

$$\Rightarrow \frac{BF}{1/2} = \frac{12}{\sqrt{3}/2}$$

$$\Rightarrow BF = \frac{12}{\sqrt{3}}$$

$$OE = 12\cos \pi/3 = 6$$

$$\Rightarrow \text{Area } \triangle OBF = \frac{1}{2} \cdot \left(\frac{12}{\sqrt{3}}\right) \cdot 6 = \frac{36}{\sqrt{3}}$$

Thus:

$$A_1 = 12\pi - \frac{36}{\sqrt{3}}$$

Now, for A_2

$$OD = 12\cos \pi/6 = 12 \cdot \frac{\sqrt{3}}{2} = 6\sqrt{3}$$

$$\Rightarrow DE = OD - OE = 6\sqrt{3} - 6$$

$$CD = 12\sin \pi/6 = 6$$

$$BE = 12\cos \pi/6 = 6\sqrt{3}$$

$$\Rightarrow \text{Area } \triangle OCD = \frac{1}{2} \cdot CD \cdot OD = \frac{1}{2} \cdot 6 \cdot 6\sqrt{3} = 18\sqrt{3}$$
and area $\triangle OEF = \frac{1}{2} \cdot EF \cdot OE = \frac{1}{2} \cdot \left(6\sqrt{3} - \frac{12}{\sqrt{3}}\right) \cdot 6$

$$= 18\sqrt{3} - \frac{36}{\sqrt{3}}$$

$$\Rightarrow A_2 = 18\sqrt{3} - 18\sqrt{3} + \frac{36}{\sqrt{3}} = \frac{36}{\sqrt{3}}$$
Finally, $A_1 + A_2 = 12\pi - \frac{36}{\sqrt{3}} + \frac{36}{\sqrt{3}} = 12\pi$