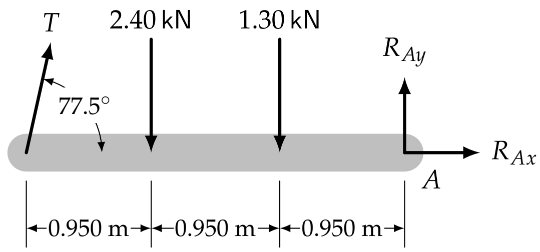
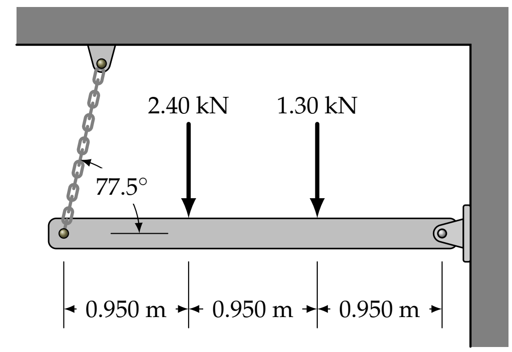
Engineering Statics - 06 Equilibrium of Rigid Bodies - Instructor Copy

<u>Example 1:</u> Determine the tension in the chain and the reaction at A.





$$\Sigma M_A = (1.30\,\text{kN})\cdot(0.950\,\text{m}) + (2.40\,\text{kN})\cdot(1.90\,\text{m}) - (T\sin 77.5^\circ)\cdot(2.85\,\text{m}) = 0$$

$$\Rightarrow T = \frac{(1.30\,\text{kN})\cdot(0.950\,\text{m}) + (2.40\,\text{kN})\cdot(1.90\,\text{m})}{\sin 77.5^\circ \times 2.85\,\text{m}}$$

$$= 2.0827\,\text{kN}$$

$$\Sigma F_x = R_{Ax} + T\cos 77.5^\circ = 0$$

$$\Rightarrow R_{Ax} = -(2.0827\,\text{kN})\cdot\cos 77.5^\circ = -0.45078\,\text{kN}$$

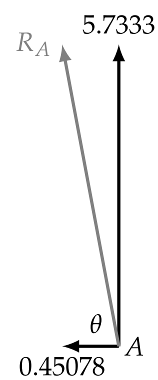
$$\Sigma F_y = R_{Ay} + T\sin 77.5^\circ - 2.40\,\text{kN} - 1.30\,\text{kN} = 0$$

$$\Rightarrow R_{Ay} = 2.40\,\text{kN} + 1.30\,\text{kN} - (2.0827\,\text{kN})\cdot\sin 77.5^\circ$$

$$= 5.7333\,\text{kN}$$

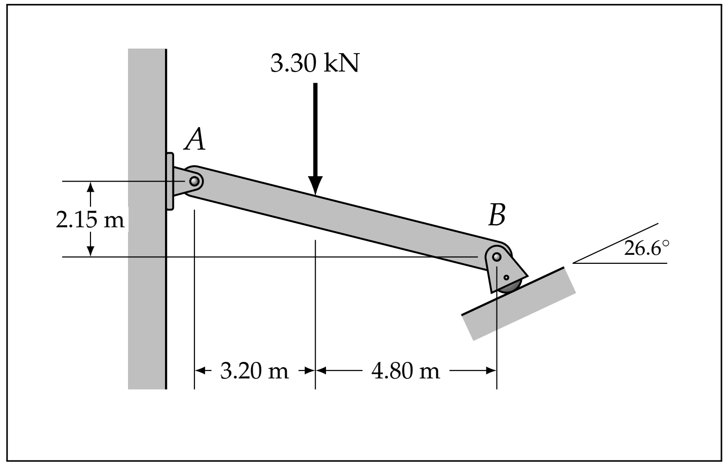
$$R_A = \sqrt{(-0.45078\,\text{kN})^2 + (5.7333\,\text{kN})^2} = 5.75103\,\text{kN}$$

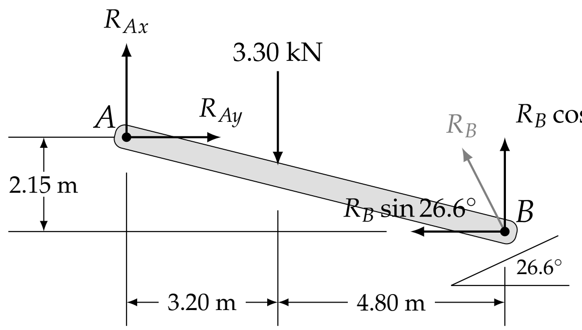
$$\theta = \tan^{-1}\left[\frac{5.7333}{0.45078}\right] = 85.504^\circ$$



The tension in the chain is **<u>2.08 kN</u>** and the reaction at A is **<u>5.75 kN</u>** at **<u>94.5°</u>** measured counter-clockwise from the positive x-axis.

Example 2: Determine the reactions at A and B.





$$
\begin{aligned}
\Sigma M_A &= (R_B \cos 26.6^\circ \text{ kN})(8.00\,\text{m}) - (3.30\,\text{kN})(3.20\,\text{m}) \\
&\quad - (R_B \sin 26.6^\circ \text{ kN})(2.15\,\text{m}) \\
&= 0 \\
\Rightarrow R_B &= \frac{(3.30\,\text{kN})(3.20\,\text{m})}{(8.00 \cos 26.6^\circ \text{ m}) - (2.15 \sin 26.6^\circ \text{ m})} \\
&= 1.7058\,\text{kN}
\end{aligned}
$$

$$
\underline{\boldsymbol{R_B = 1.71\,\text{kNat}117^\circ}}
$$

$$
\begin{aligned}
\Sigma F_y &= R_{Ay} - 3.30\,\text{kN} + 1.7058 \cos 26.6^\circ \text{ kN} = 0 \\
\Rightarrow R_{Ay} &= 1.7748\,\text{kN}
\end{aligned}
$$

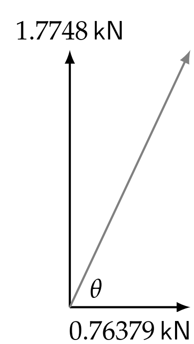
$$
\begin{aligned}
\Sigma F_x &= R_{Ax} - 1.7058 \sin 26.6^\circ \text{ kN} = 0 \\
\Rightarrow R_{Ax} &= 0.76379\,\text{kN}
\end{aligned}
$$

At this point, there is an opportunity to check our work so far. If we take moments about anywhere (except about A which we've already set to zero), they should sum to zero. Taking moments about B is the most convenient:

$$
\begin{aligned}
\Sigma M_B &= (3.30\,\text{kN})(4.80\,\text{m}) - (1.7748\,\text{m})(8.00\,\text{m}) \\
&\quad - (0.76379\,\text{m})(2.15\,\text{m}) \\
&= -0.000549 \\
&\approx 0 \quad \checkmark
\end{aligned}
$$

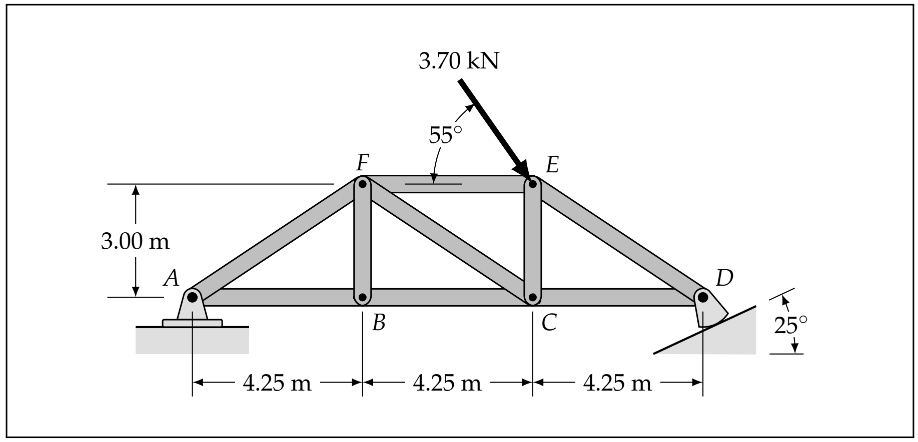
$$
\begin{aligned}
R_A &= \sqrt{(1.7748\,\text{kN})^2 + (0.76379\,\text{kN})^2} \\
&= 1.9322\,\text{kN}
\end{aligned}
$$

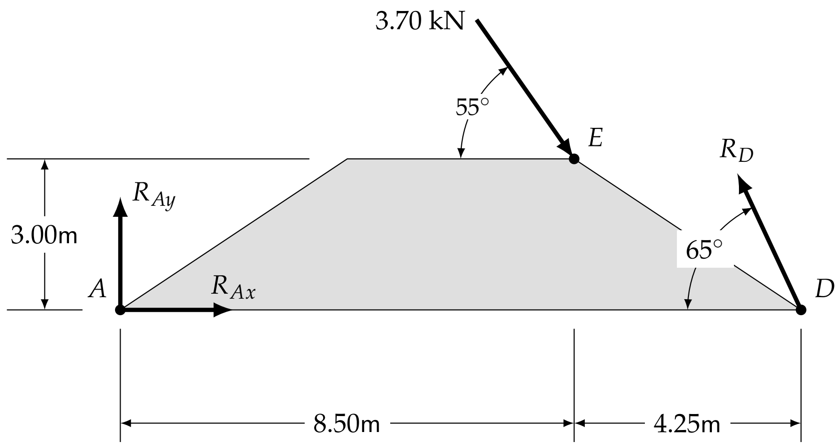
$$
\begin{aligned}
\theta &= \tan^{-1}\left[\frac{1.7748}{0.76379}\right] \\
&= 66.715^\circ
\end{aligned}
$$



$$
\underline{\boldsymbol{R_A = 1.93\,\text{kNat}66.7^\circ}}
$$

Exercise 1: Determine the reactions at A and D.





$$
\begin{aligned}
\Sigma M_A &= (R_D \sin 65^\circ)\cdot(12.75\,\mathrm{m}) - (3.70\,\mathrm{kN}\cdot\sin 55^\circ)\cdot(8.50\,\mathrm{m}) - (3.70\,\mathrm{kN}\cdot\cos 55^\circ)\cdot(3.00\,\mathrm{m}) \\
&= R_D\cdot 11.555\,\mathrm{m} - 32.129\,\mathrm{kN\cdot m} = 0 \\
\Rightarrow R_D &= \frac{32.129\,\mathrm{kN\cdot m}}{11.555\,\mathrm{m}} = 2.7805\,\mathrm{kN}
\end{aligned}
$$

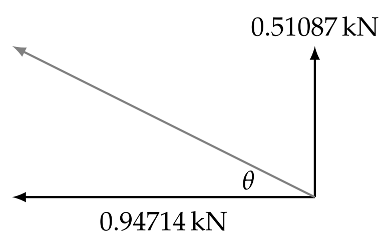
The reaction at D is **2.78 kN** at **115°** measured counter-clockwise from the positive x-axis.

$$
\begin{aligned}
\Sigma F_x &= R_{Ax} + (3.70\,\mathrm{kN})\cdot\cos 55^\circ - R_D \cos 65^\circ = 0 \\
\Rightarrow R_{Ax} &= 2.7805\,\mathrm{kN}\cdot\cos 65^\circ - (3.70\,\mathrm{kN})\cdot\cos 55^\circ \\
&= -0.94714\,\mathrm{kN}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_y &= R_{Ay} + R_D \sin 65^\circ - (3.70\,\mathrm{kN})\cdot\sin 55^\circ = 0 \\
\Rightarrow R_{Ay} &= (3.70\,\mathrm{kN})\cdot\sin 55^\circ - 2.7805\,\mathrm{kN}\cdot\sin 65^\circ \\
&= 0.51087\,\mathrm{kN}
\end{aligned}
$$

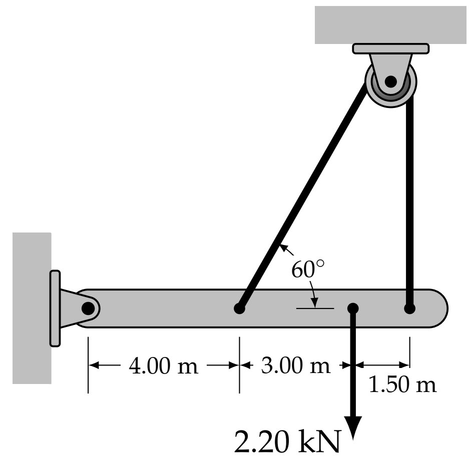
$$
R_A = \sqrt{(-0.94714\,\mathrm{kN})^2 + (-0.51087\,\mathrm{kN})^2} = 1.0761\,\mathrm{kN}
$$

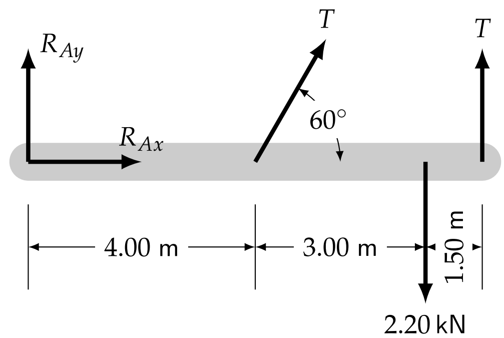
$$
\theta = \tan^{-1}\left[\frac{0.51087}{0.94714}\right] = 28.342^\circ
$$



The reaction at A is **1.08 kN** at **152°** measured counter-clockwise from the positive x-axis.

Exercise 2: Determine the reactions at the pinned connection and the tension in the cable.





$$\Sigma M_A = (T \sin 60°)\cdot(4.00\,\text{m}) + T\cdot(8.50\,\text{m}) - (2.20\,\text{kN})\cdot(7.00\,\text{m}) = 0$$

$$\Rightarrow T = \frac{(2.20\,\text{kN})\cdot(7.00\,\text{m})}{(4.00\,\text{m})\cdot(\sin 60°) + (8.50\,\text{m})} = 1.2872\,\text{kN}$$

$$\Sigma F_x = R_{Ax} + T\cos 60° = 0$$

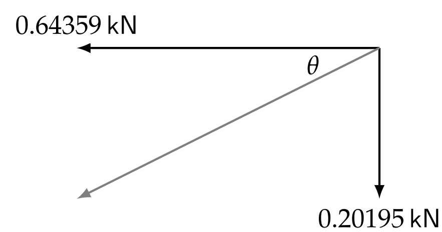
$$\Rightarrow R_{Ax} = -(1.2872\,\text{kN})\cdot\cos 60° = -0.64359\,\text{kN}$$

$$\Sigma F_y = R_{Ay} + T\sin 60° + T - 2.20\,\text{kN} = 0$$

$$\Rightarrow R_{Ay} = 2.20\,\text{kN} - (1.2872\,\text{kN})\cdot(1+\sin 60°) = -0.20195\,\text{kN}$$

$$R_A = \sqrt{(-0.64359\,\text{kN})^2 + (-0.20195\,\text{kN})^2} = 0.67453\,\text{kN}$$

$$\theta = \tan^{-1}\left[\frac{-0.20195}{-0.64359}\right] = 17.421°$$

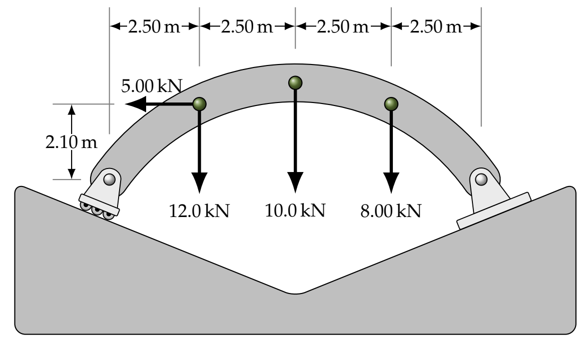


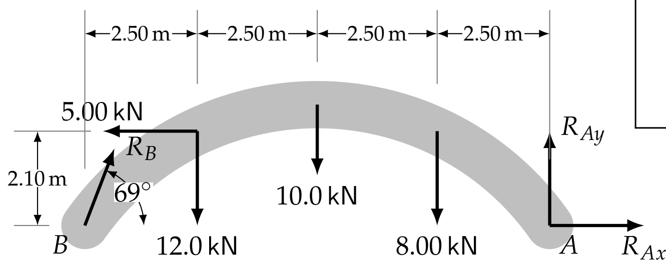
The tension in the cable is **1.29 kN**

The reaction at A is **0.675 kN** at **197°** measured counter-clockwise from the positive x-axis.

Example 3: The roller and the pinned connection are on slopes inclined at 21° to the horizontal; they are both at the same elevation.

Determine the reactions at the pinned connection and the tension in the cable.



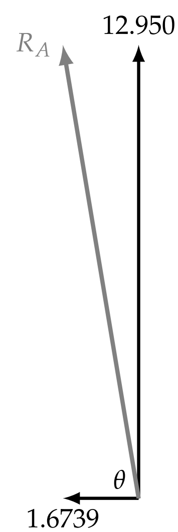


$$\begin{aligned}\Sigma M_A &= (8.00\,\text{kN})\cdot(2.50\,\text{m}) + (10.0\,\text{kN})\cdot(5.00\,\text{m}) + (12.0\,\text{kN})\cdot(7.50\,\text{m}) \\ &\qquad + (5.00\,\text{kN})\cdot(2.10\,\text{m}) - (R_B \sin 69^\circ)\cdot(10.0\,\text{m}) \\ &= 170.5\,\text{kN}\cdot\text{m} - (9.3358\,\text{m})\cdot R_B = 0 \\ \Rightarrow R_B &= \frac{170.5\,\text{kN}\cdot\text{m}}{9.3358\,\text{m}} = 18.263\,\text{kN}\end{aligned}$$

$$\begin{aligned}\Sigma F_x &= R_B\cdot\cos 69^\circ + R_{Ax} - 5.00\,\text{kN} = 0 \\ \Rightarrow R_{Ax} &= 5.00\,\text{kN} - (18.623\,\text{kN})\cdot\cos 69^\circ = -1.6739\,\text{kN}\end{aligned}$$

$$\begin{aligned}\Sigma F_y &= R_{Ay} + R_B \sin 69^\circ - (12.0\,\text{kN} + 10.0\,\text{kN} + 8.00\,\text{kN}) = 0 \\ R_{Ay} &= 30.0\,\text{kN} - (18.263\,\text{kN})\cdot\sin 69^\circ = 12.950\,\text{kN}\end{aligned}$$

$$\begin{aligned}R_A &= \sqrt{(-1.6739\,\text{kN})^2 + (12.950\,\text{kN})^2} = 13.058\,\text{kN} \\ \theta &= \tan^{-1}\left[\frac{12.950}{1.6739}\right] = 82.635^\circ\end{aligned}$$



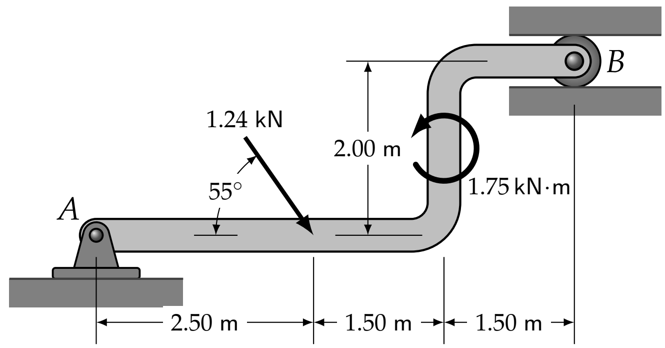
The reaction at the roller is **<u>18.3 kN</u>** at **<u>69°</u>** measured counter-clockwise from the positive x-axis.

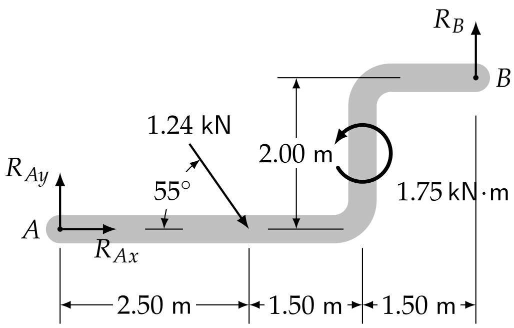
The reaction at the pinned connection is **<u>13.1 kN</u>** at **<u>97.4°</u>** measured counter-clockwise from the positive x-axis.

Example 4:

The roller at B is in a smooth slot.

Determine the reactions at A and B.





$$\Sigma M_A = R_B\cdot(5.50\,\text{m}) + 1.75\,\text{kN}\cdot\text{m} - (1.24\,\text{kN})\cdot(2.50\,\text{m}) = 0$$

$$\Rightarrow R_B = \frac{3.1\,\text{kN}\cdot\text{m} - 1.75\,\text{kN}\cdot\text{m}}{5.50\,\text{m}} = 0.24545\,\text{kN}$$

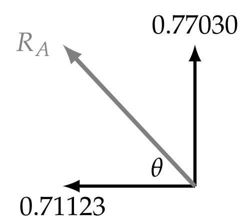
Note: We began by assuming a reaction R_B in the positive direction (that is, that the roller at B was pressing down on the slot). Our result for R_B is positive, so that initial assumption is correct. If R_B had evaluated to a negative value, then the roller would be pressing upward on the slot and the reaction would be downward.

$$\Sigma F_x = R_{Ax} + (1.24\,\text{kN})\cdot\cos 55^\circ = 0$$

$$\Rightarrow R_{Ax} = -0.71123\,\text{kN}$$

$$\Sigma F_y = R_{Ay} + R_B - (1.24\,\text{kN})\cdot\sin 55^\circ = 0$$

$$\Rightarrow R_{Ay} = (1.24\,\text{kN})\cdot\sin 55^\circ - 0.24545\,\text{kN} = 0.77030\,\text{kN}$$



$$R_A = \sqrt{(-0.71123\,\text{kN})^2 + (0.77030\,\text{kN})^2} = 1.0484\,\text{kN}$$

$$\theta = \tan^{-1}\left[\frac{0.77030}{0.71123}\right] = 47.283^\circ$$

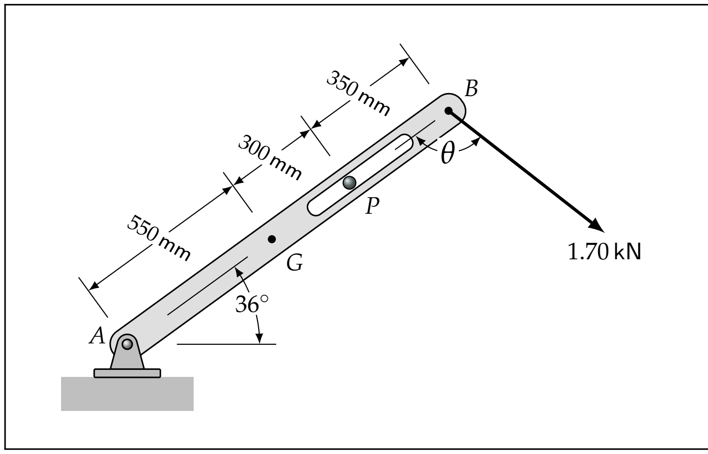
The reaction at A is **<u>1.05 kN</u>** at **<u>133°</u>**, measured counter-clockwise from the positive x-axis.

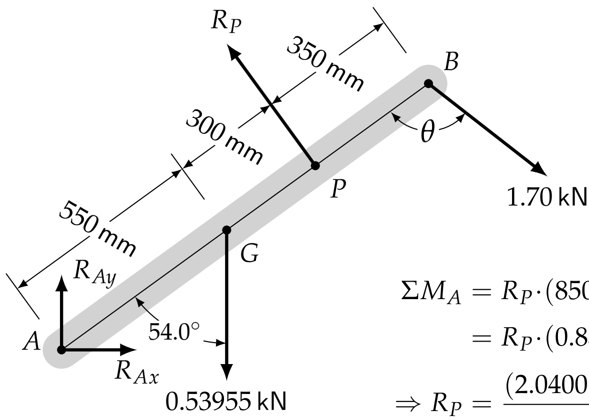
The reaction at B is **<u>0.245 kN</u>** at **<u>90°</u>**, measured counter-clockwise from the positive x-axis.

Example 5:

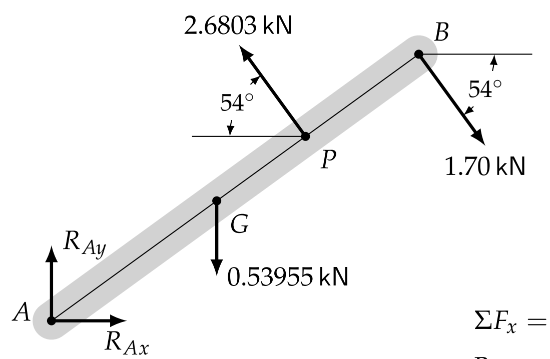
55 − kg bar AB has its centre of gravity at G. It is supported by a pinned connection at A and a smooth peg at C. A cable is attached at B and has a tensile force of 1.70 kN. The direction of the cable varies between $\theta = 60°$ and $\theta = 135°$.

What is the maximum reaction at P? Determine the reaction at A for this reaction at P.





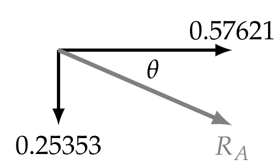
$$\Sigma M_A = R_P \cdot (850\ \text{mm}) - (1.70\ \text{kN} \cdot \sin\theta)(1200\ \text{mm}) - (0.53955\ \text{kN} \cdot \sin 54°) \cdot (550\ \text{mm})$$
$$= R_P \cdot (0.850\ \text{m}) - (2.0400\ \text{kN} \cdot \text{m}) \cdot \sin\theta - 0.23828\ \text{kN} \cdot \text{m} = 0$$
$$\Rightarrow R_P = \frac{(2.0400\ \text{kN} \cdot \text{m}) \cdot \sin\theta + 0.23828\ \text{kN} \cdot \text{m}}{0.850\ \text{m}} = 2.4000\ \text{kN} \cdot \sin\theta + 0.28033\ \text{kN}$$
$$\Rightarrow R_{P_{max}} = 2.4000\ \text{kN} \cdot \sin 90° + 0.28033\ \text{kN} = 2.6803\ \text{kN}$$



$$\Sigma F_x = R_{Ax} - (2.6803\ \text{kN}) \cdot \cos 54° + (1.70\ \text{kN}) \cdot \cos 54° = 0$$
$$\Rightarrow R_{Ax} = 0.57621\ \text{kN}$$

$$\Sigma F_y = R_{Ay} - 0.53955\ \text{kN} + (2.6803\ \text{kN}) \cdot \sin 54° - (1.70\ \text{kN}) \cdot \sin 54° = 0$$
$$\Rightarrow R_{Ay} = -0.25353\ \text{kN}$$

$$R_A = \sqrt{(0.57621\ \text{kN})^2 + (0.25353\ \text{kN})^2} = 0.62952\ \text{kN}$$
$$\theta = \tan^{-1}\left[\frac{0.25353}{0.57621}\right] = 23.749°$$

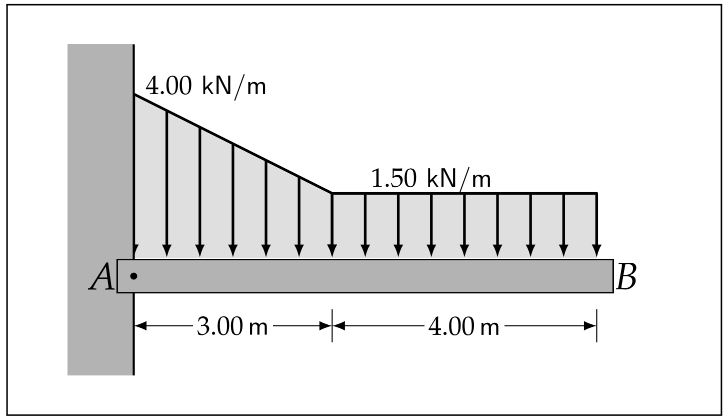


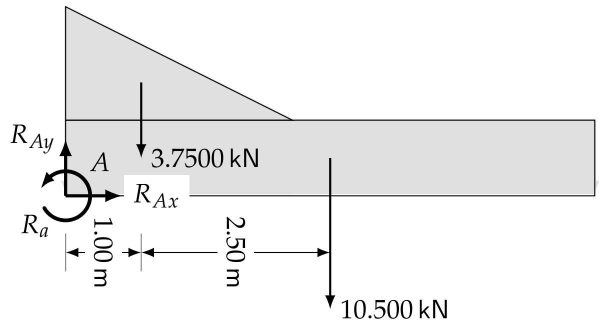
The maximum reaction at P is **2.68 kN** at **126°**, measured counter-clockwise from the positive x-axis.

The associated reaction at A is **0.630 kN** at **23.7°**, measured **clockwise** from the positive x-axis.

Example 6: Beam AB has a fixed support at A. (Fixed supports offer resistance to rotation in the form of a reacting couple at A; clearly, without this, equilibrium would not be possible.)

Determine the reaction and the reacting couple at A.





$$\Sigma M_A = R_a - (3.7500\,\mathrm{kN})\cdot(1.0000\,\mathrm{m}) - (10.500\,\mathrm{kN})\cdot(3.5000\,\mathrm{m}) = 0$$
$$\Rightarrow R_a = 40.500\,\mathrm{kN\cdot m}$$

$$\Sigma F_x = R_{Ax} = 0;$$

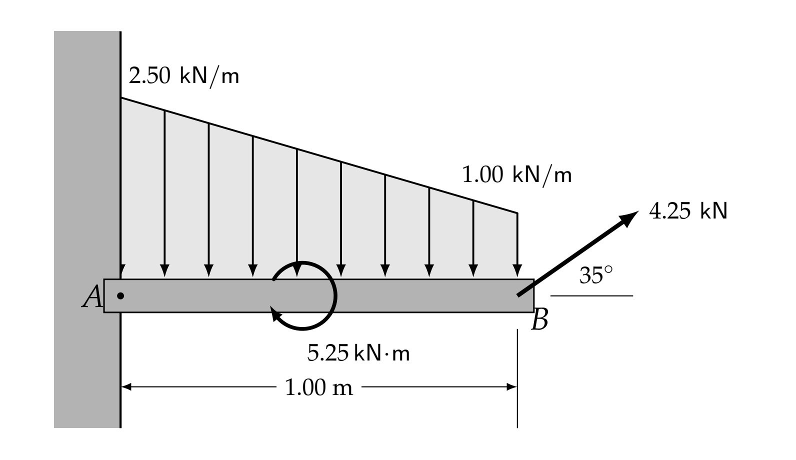
$$\Sigma F_y = R_{Ay} - 3.7500\,\mathrm{kN} - 10.500\,\mathrm{kN} = 0$$
$$\Rightarrow R_{Ay} = 14.250\,\mathrm{kN}$$

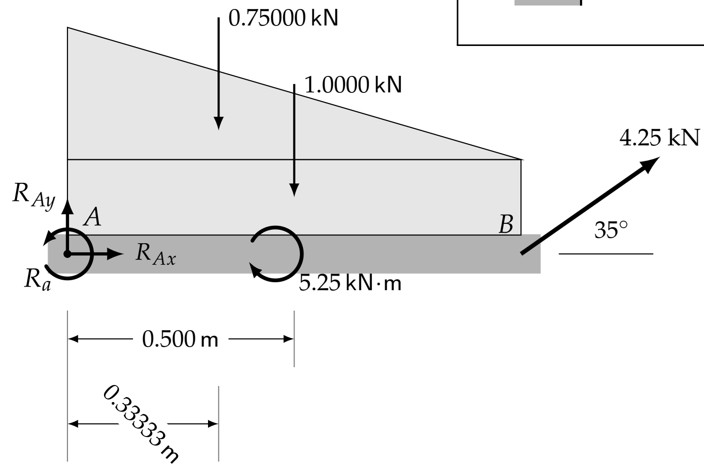
(Note that calculation for the magnitude and direction of the reaction at A is 'trivial' and can be just written down, since R_{Ax} is 0.)

The reacting moment at A is **40.5 kN·m**.

The reaction at A is **14.3 kN** at **90°**, measured counter-clockwise from the positive x-axis.

Exercise 3: Determine the reaction and the reacting moment at A.





$$\Sigma M_A = R_a - (0.75000\,\text{kN})\cdot(0.33333\,\text{m}) - (1.0000\,\text{kN})\cdot(0.50000\,\text{m})$$
$$- 5.25\,\text{kN}\cdot\text{m} + (4.25\,\text{kN}\cdot\sin 35^\circ)\cdot(1.00\,\text{m}) = 0$$
$$\Rightarrow R_a = 3.5623\,\text{kN}\cdot\text{m}$$

$$\Sigma F_x = R_{Ax} + (4.25\,\text{kN})\cdot\cos 35^\circ$$
$$\Rightarrow R_{Ax} = -3.4814\,\text{kN}$$

$$\Sigma F_y = R_{Ay} + (4.25\,\text{kN})\cdot\sin 35^\circ - 0.75000\,\text{kN} - 1.0000\,\text{kN} = 0$$
$$\Rightarrow R_{Ay} = -0.68770\,\text{kN}$$

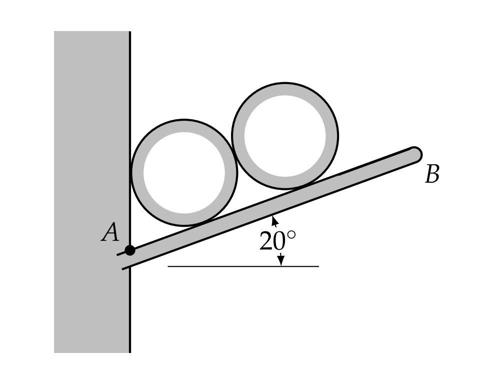
$$R_A = \sqrt{(-3.4814\,\text{kN})^2 + (-0.68770\,\text{kN})^2} = 3.5487\,\text{kN}$$
$$\theta = \tan^{-1}\left[\frac{0.68770}{3.4814}\right] = 11.174^\circ$$

The reacting moment at A is **3.56 kN·m**.

The reaction at A is **3.55 kN** at **191°**, measured counter-clockwise from the positive x-axis.

Example 5: Pipe racks (AB, and two hidden behind it) support two smooth Schedule 40 pipes, with an outside diameter of 508 mm, as shown. The pipes are 10 m in length with a mass of 78.5 kg/m. Each rack supports one-third of the weight of each pipe.

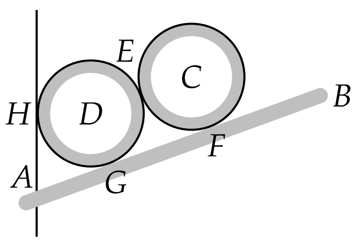
Determine the reaction at the fixed connection A.

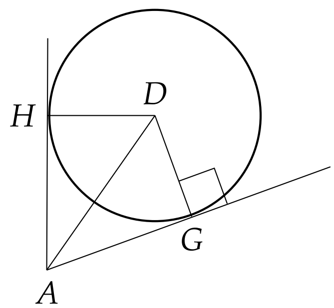


The weight of each pipe bearing on AB:

$$W = 78.5\,\mathrm{kg/m} \times 9.81\,\mathrm{m/s^2} \times 10\,\mathrm{m}/3 = 2.5670\,\mathrm{kN}$$

Add some labels, find some distances:





$$\angle HAD = \angle GAD = 35^\circ$$

$$\angle GAD = 55^\circ$$

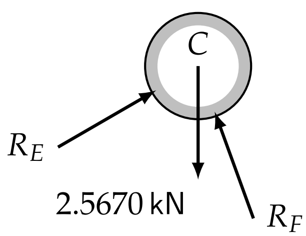
$$\frac{AG}{GD} = \tan 55^\circ$$

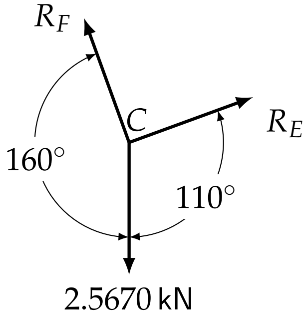
$$AG = \frac{508\,\mathrm{mm}}{2} \tan 55^\circ$$

$$= 362.75\,\mathrm{mm}$$

$$GF = CD = 508\,\mathrm{mm}$$

Forces acting upon the upper (rightmost) pipe, *C*:





This is now a simple concurrent forces problem, solved with simultaneous equations. Notice, however, that the direction of R_F is perpendicular to the direction of R_E.

If we choose axes x' and y', rotated 20° in the counter clockwise direction around C, then the direction of R_E is the x'-axis and the direction of R_F is the y'-axis. Now we can solve without simultaneous equations.

(Why bother complicating things? This will become a useful technique towards the end of the module and it's easy to introduce here.)

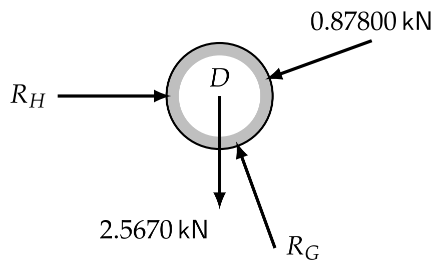
$$\Sigma F_{x'} = R_E - 2.5670\,\mathrm{kN} \cdot \cos 70^\circ = 0$$

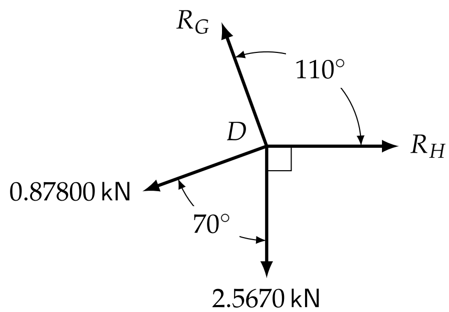
$$\Rightarrow R_E = 0.87800\,\mathrm{kN}$$

$$\Sigma F_{y'} = R_F - 2.5670\,\mathrm{kN} \cdot \cos 20^\circ = 0$$

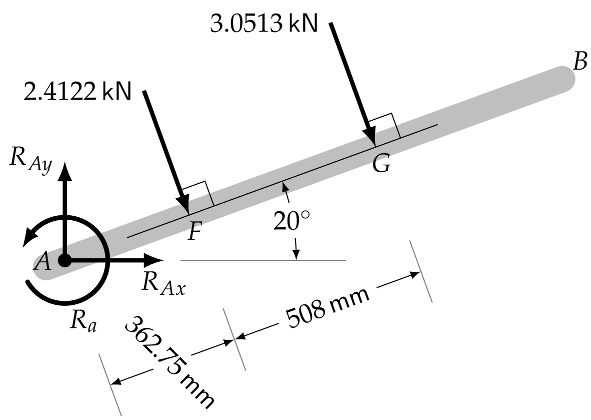
$$\Rightarrow R_F = 2.4122\,\mathrm{kN}$$

Forces acting upon the lower (leftmost) pipe, D:

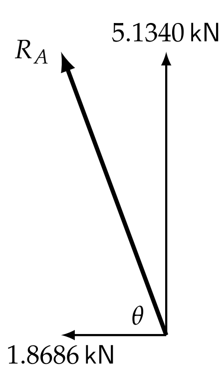




$$
\begin{aligned}
\Sigma F_y &= R_G \cdot \cos 20^\circ - 2.5670\,\text{kN} - 0.87800\,\text{kN} \cdot \cos 70^\circ \\
&= 0 \\
\Rightarrow R_G &= \frac{2.5670\,\text{kN} + 0.87800\,\text{kN} \cdot \cos 70^\circ}{\cos 20^\circ} \\
\Rightarrow R_G &= 3.0513\,\text{kN}
\end{aligned}
$$



$$
\begin{aligned}
\Sigma M_A &= R_a - 2.4122\,\text{kN} \cdot 362.75\,\text{mm} - 3.0513\,\text{kN} \cdot 870.75\,\text{mm} = 0 \Rightarrow R_a = 3531.9\,\text{kN} \cdot \text{mm} = 3.5319\,\text{kN} \cdot \text{m} \\
\Sigma F_x &= R_{Ax} + 2.4122\,\text{kN} \cdot \sin 20^\circ + 3.0513\,\text{kN} \cdot \sin 20^\circ = 0 \Rightarrow R_{Ax} = -1.8686\,\text{kN} \\
\Sigma F_x &= R_{Ay} - 2.4122\,\text{kN} \cdot \cos 20^\circ - 3.0513\,\text{kN} \cdot \cos 20^\circ = 0 \Rightarrow R_{Ay} = 5.1340\,\text{kN}
\end{aligned}
$$



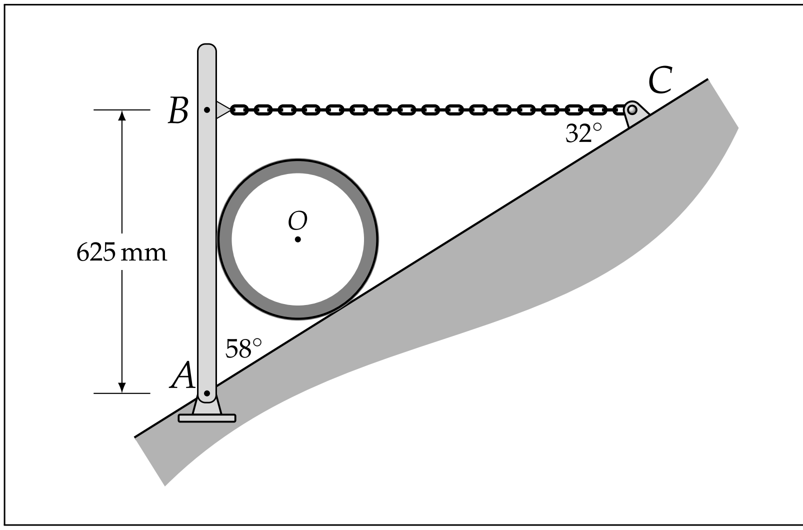
$$
R_A = \sqrt{(1.8686\,\text{kN})^2 + (5.1340\,\text{kN})^2} = 5.4635\,\text{kN}
$$

$$
\theta = \tan^{-1}\left[\frac{5.1340\,\text{kN}}{1.8686\,\text{kN}}\right] = 70^\circ
$$

The reaction at A is **5.46 kN** at **110°** counter-clockwise from the positive x-axis. The reacting moment at A is **3.53 kN·m**

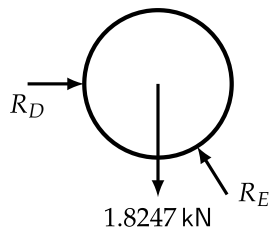
Exercise 4: A section of smooth pipe, centred at O, has a diameter of 457 mm and a mass of 186 kg. It is secured by vertical structural member AB, hinged with a pinned connection at A and held in place by chain BC.

Determine the tension in the chain, and the reaction at A.

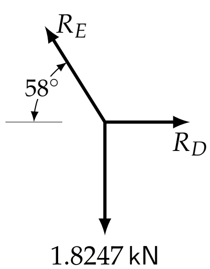


The weight of the pipe is:
$W = 186 \times 9.81\,\text{N} = 1824.7\,\text{N}$

Forces acting on the pipe:



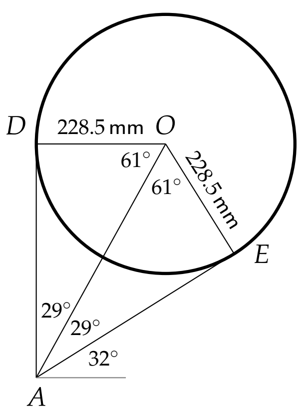
Free body diagram:



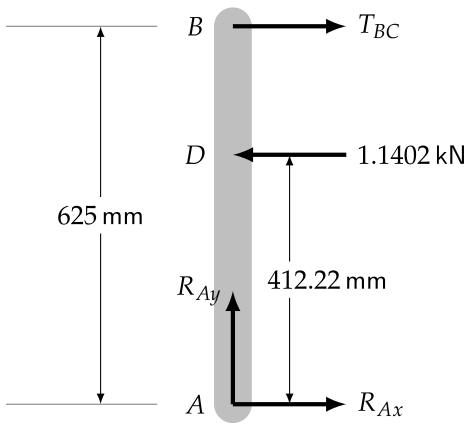
$$\Sigma F_y = R_E \cdot \sin 58^\circ - 1.8247\,\text{kN} = 0$$
$$\Rightarrow R_E = 2.1516\,\text{kN}$$

$$\Sigma F_x = R_D - (2151.6\,\text{kN}) \cdot \cos 58^\circ = 0$$
$$\Rightarrow R_D = 1.1402\,\text{kN}$$

Determine the length of the moment-arm, AD, where the pipe bears on the vertical member.



$$\frac{AD}{228.5\,\text{mm}} = \tan 61^\circ \Rightarrow AD = 412.22\,\text{mm}$$



$$\Sigma M_A = (1.1402\,\text{kN})\cdot(412.22\,\text{mm}) - (625\,\text{mm})T_{BC} = 0$$
$$\Rightarrow T_{BC} = 0.75202\,\text{kN}$$

$$\Sigma F_x = R_{Ax} - 1.1402\,\text{kN} + 0.75202\,\text{kN} = 0$$
$$\Rightarrow R_{Ax} = 0.38818\,\text{kN}$$

$$\Sigma F_y = R_{Ay} = 0$$

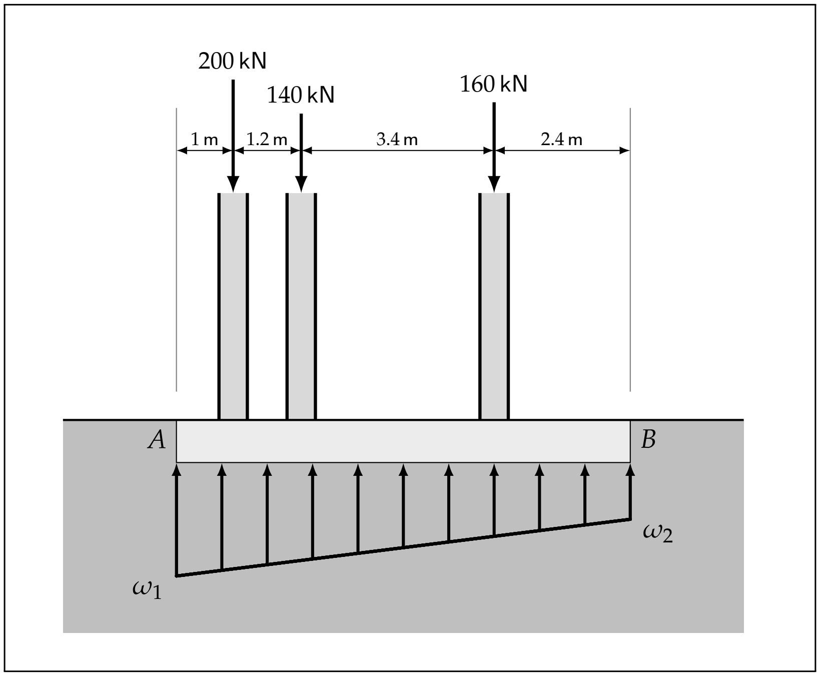
The tension in the chain between B and C is **0.752 kN**.

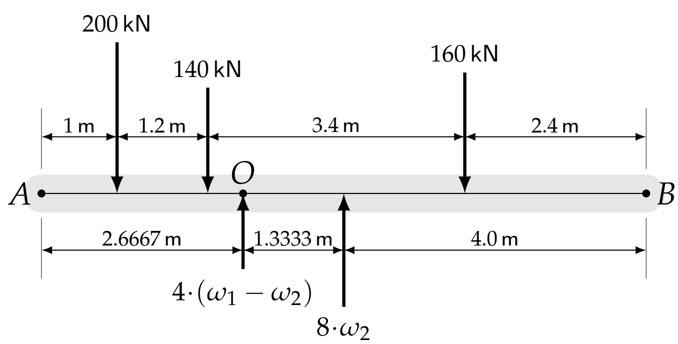
The reaction at A is **0.388 kN** in the direction of the positive x-axis

Example 8:

Soil exerts a trapezoidal distributed load on the bottom of the footing AB.

Determine the values of ω_1 and ω_2 that support the column loadings in static equilibrium.





We could start by taking moments about A to get an expression with ω_1 and ω_2. Then do the same with moments about B. And get a system of two equations in two unknowns.

Or we could take moments about O and not have to deal with simultaneous equations!

Note: The $8{\cdot}\omega_2$ is in kN where the 8 is in m and the ω_2 is in kN/m. Similarly for $4(\omega_1 - \omega_2)$.

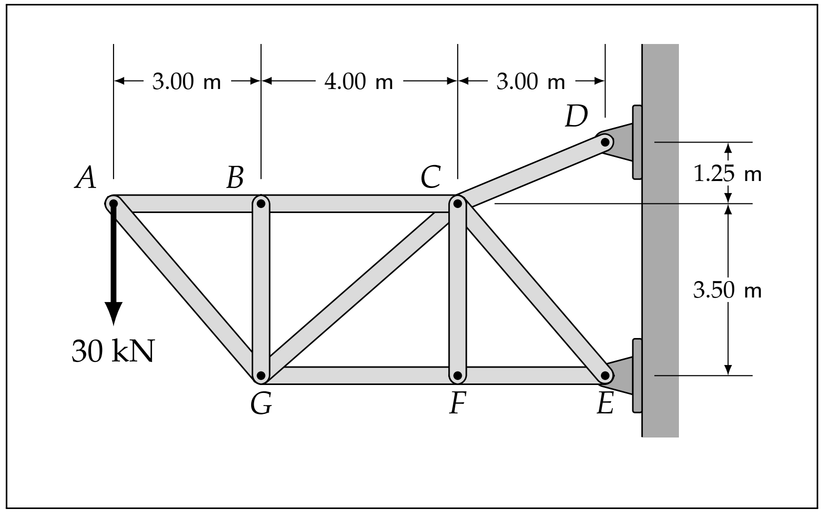
$$\begin{aligned}
\Sigma M_O &= (8\,\text{m})\omega_2{\cdot}(1.3333\,\text{m}) + (140\,\text{kN})(0.46670\,\text{m}) + (200\,\text{kN})(1.6667\,\text{m}) - (160\,\text{kN})(2.9333\,\text{m}) \\
&= (10.644\,\text{m}^2)\omega_2 - 70.650\,\text{kN}{\cdot}\text{m} = 0 \\
\Rightarrow \omega_2 &= 6.6236\,\text{kN/m}
\end{aligned}$$

$$\begin{aligned}
\Sigma F_y &= (8\,\text{m})\omega_2 + (4\,\text{m})(\omega_1 - \omega_2) - 500\,\text{kN} \\
&= (4\,\text{m})\omega_1 + (4\,\text{m})\omega_2 - 500\,\text{kN} = (4\,\text{m})\omega_1 + 26.494\,\text{kN} - 500\,\text{kN} = 0 \\
\Rightarrow \omega_1 &= \frac{473.51\,\text{kN}}{4\,\text{m}} = 118.38\,\text{kN/m}
\end{aligned}$$

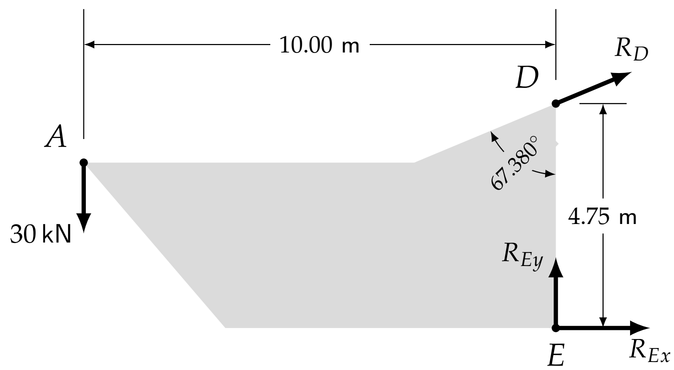
$$\begin{aligned}
\omega_1 &= \underline{118\,\text{kN/m}} \\
\omega_2 &= \underline{6.62\,\text{kN/m}}
\end{aligned}$$

Example 9:

Determine the reactions at D and E.



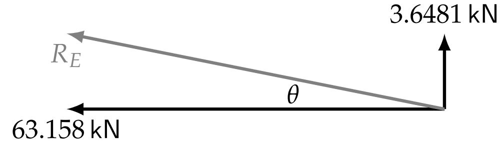
Note: We cannot usually solve a system with two pinned connections because there are two unknowns. In this case, though, CD is a two-force member so we know its direction and the only unknown at D is the magnitude of the direction. We can treat CD as we would a cable or a strut.



$$
\begin{aligned}
\Sigma M_E &= (30\,\text{kN})\cdot(10.00\,\text{m}) - R_D\cdot(4.75\,\text{m}\cdot\sin 67.380°) = 0\\
\Rightarrow R_D &= \frac{(30\,\text{kN})\cdot(10.00\,\text{m})}{4.75\,\text{m}\cdot\sin 67.380°}\\
&= 68.421\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_x &= R_{Ex} + (68.421\,\text{kN})\cdot\sin 67.380° = 0\\
\Rightarrow R_{Ex} &= -63.158\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_y &= R_{Ey} + (68.421\,\text{kN})\cdot\cos 67.380° - 30\,\text{kN} = 0\\
\Rightarrow R_{Ey} &= 3.6481\,\text{kN}
\end{aligned}
$$

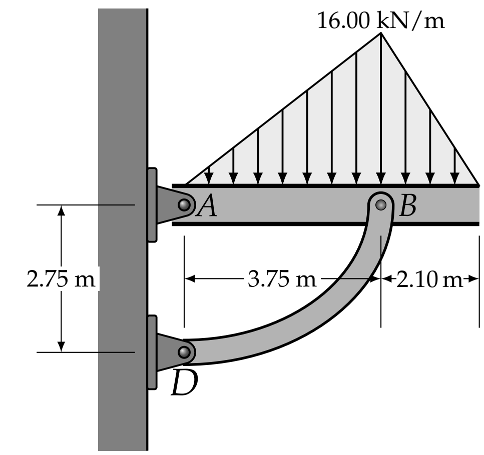


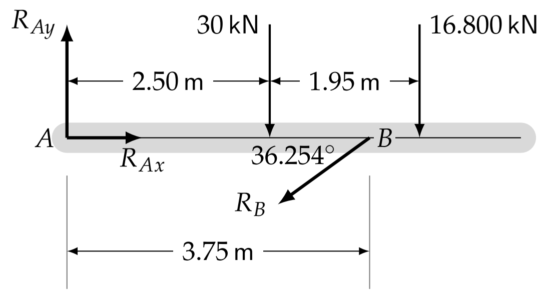
$$
\begin{aligned}
R_E &= \sqrt{(-63.158\,\text{kN})^2 + (-3.6481\,\text{kN})^2} = 63.263\,\text{kN}\\
\theta &= \tan^{-1}\left[\frac{3.6481}{63.158}\right] = 3.3058°
\end{aligned}
$$

The reaction at E is **68.4 kN** at 22.6°, measured counter-clockwise from the positive x-axis.

The reaction at D is **63.3 kN** at 177°, measured counter-clockwise from the positive x-axis.

Exercise 5:
Determine the reactions at A and D.





$$\Sigma M_A = -(30\,\text{kN})\cdot(2.50\,\text{m}) - (16.800\,\text{kN})\cdot(4.45\,\text{m}) - R_B\cdot\sin 36.254°\cdot(3.75\,\text{m}) = 0$$
$$\Rightarrow R_B = -67.532\,\text{kN}$$

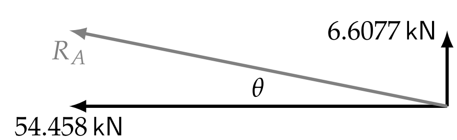
R_B is negative, so BD is in compression. We know that BD is a two-force member so the reaction at D is equal and opposite to that of the reaction at B.

$$\Sigma F_x = R_{Ax} - (-67.532\,\text{kN})\cdot\cos 36.254° = 0$$
$$\Rightarrow R_{Ax} = -54.458\,\text{kN}$$

$$\Sigma F_y = R_{Ay} - 30\,\text{kN} - 16.800\,\text{kN} - (-67.532\,\text{kN})\cdot\sin 36.254° = 0$$
$$\Rightarrow R_{Ay} = 6.6077\,\text{kN}$$

$$R_A = \sqrt{(-54.458\,\text{kN})^2 + (6.6077\,\text{kN})^2} = 54.857\,\text{kN}$$

$$\theta = \tan^{-1}\left[\frac{6.6077}{54.458}\right] = 6.9182°$$

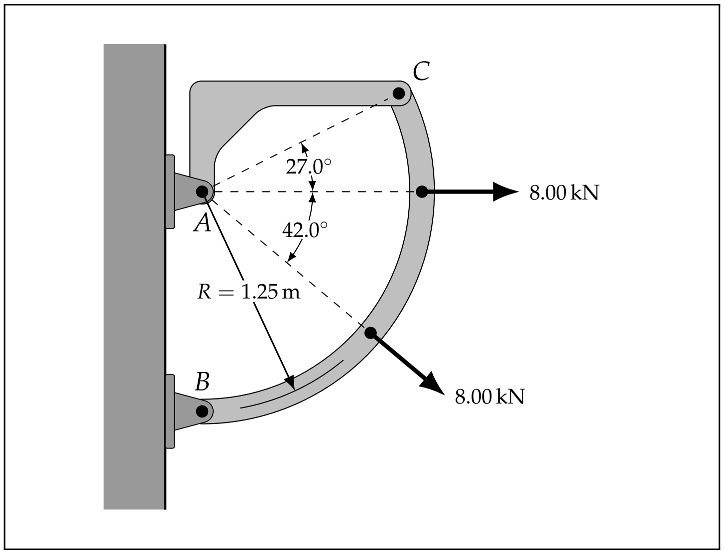


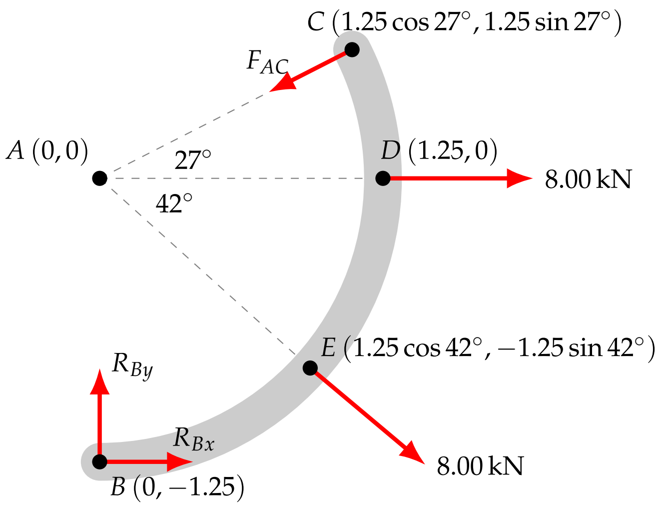
The reaction at E is **54.9 kN** at **173°**, measured counter-clockwise from the positive x-axis.

The reaction at D is **67.5 kN** at **36.3°**, measured counter-clockwise from the positive x-axis.

Exercise 6:
Member CD is a circular arc, centred at A

Determine the reactions at A and B.





$$\begin{aligned}\Sigma M_B &= F_{AC}\cdot\cos 27°\cdot(1.25\,\mathrm{m} + 1.25\,\mathrm{m}\cdot\sin 27°) - F_{AC}\cdot\sin 27°\cdot(1.25\,\mathrm{m}\cdot\cos 27°)\\ &\quad - 8.00\,\mathrm{kN}\cdot(1.25\,\mathrm{m})\\ &\quad\quad - 8.00\,\mathrm{kN}\cdot\cos 42°\cdot(1.25\,\mathrm{m} - 1.25\,\mathrm{m}\cdot\sin 42°) - 8.00\,\mathrm{kN}\cdot\sin 42°\cdot(1.25\,\mathrm{m}\cdot\cos 42°)\end{aligned}$$

$$\begin{aligned}&= F_{AC}\cdot\cos 27°\cdot(1.8175\,\mathrm{m}) - F_{AC}\cdot\sin 27°\cdot(1.1138\,\mathrm{m}) - 10.000\,\mathrm{kN\cdot m}\\ &\quad\quad - 8.00\,\mathrm{kN}\cdot\cos 42°\cdot(0.41359\,\mathrm{m}) - 8.00\,\mathrm{kN}\cdot\sin 42°\cdot(0.92893\,\mathrm{m})\end{aligned}$$

$$= F_{AC}\cdot(1.1137\,\mathrm{m}) - 17.431\,\mathrm{kN\cdot m} = 0$$

$$\Rightarrow F_{AC} = 15.651\,\mathrm{kN}$$

$$\Sigma F_x = R_{Bx} + 8.00\,\mathrm{m}\cdot\cos 42° + 8.00\,\mathrm{m} - 15.651\,\mathrm{kN}\cdot\cos 27° = 0$$
$$\Rightarrow R_{Bx} = 0$$

$$\Sigma F_y = R_{By} - 8.00\,\mathrm{kN}\cdot\sin 42° - 15.651\,\mathrm{kN}\cdot\sin 27° = 0$$
$$\Rightarrow R_{By} = 12.458\,\mathrm{kN}$$

The reaction at A is **15.7 kN** at 207°, measured counter-clockwise from the positive x-axis.

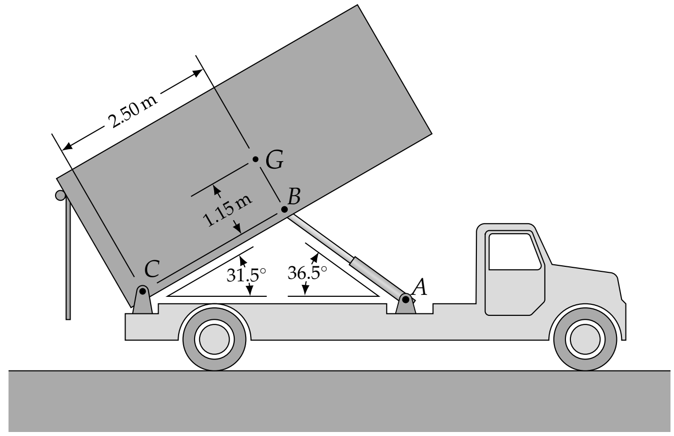
The reaction at B is **12.5 kN** at 90°, measured counter-clockwise from the positive x-axis.

Questions:

1. Why is R_{AC} away from C instead of in the direction of C, cancelling out F_{AC}?
2. Can you see that $R_{Bx} = 0$ directly from the free body diagram?

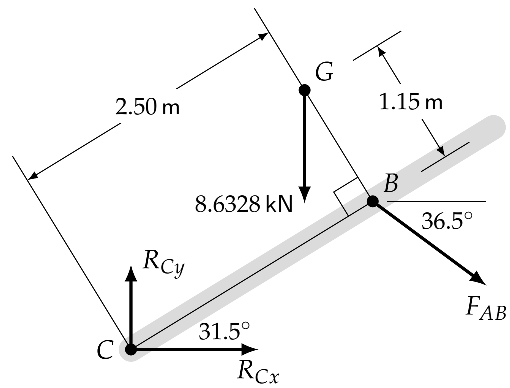
Example 10:

The bin of a dump-truck is being tipped by hydraulic lift AB. (AB can be considered a two-force member.) The bin rotates about a pin at C. Determine the force in the lift AB and find the reaction at C. The bin has a mass of 880 kg and G marks its centre of mass.



Method 1: Our usual approach.

$$M = 880\,\text{kg} \Rightarrow W = \frac{880 \times 9.81}{1000} = 8.6328\,\text{kN}$$



$$\begin{aligned}
\Sigma M_C &= -(8.6328\,\text{kN})\cdot(2.50\,\text{m}\cdot\cos 31.5^\circ - 1.15\,\text{m}\cdot\sin 31.5^\circ) \\
&\quad - F_{AB}\cdot\cos 36.5^\circ\cdot(2.50\,\text{m}\cdot\sin 31.5^\circ) \\
&\quad - F_{AB}\cdot\sin 36.5^\circ\cdot(2.50\,\text{m}\cdot\cos 31.5^\circ) \\
&= -13.214\,\text{kN}\cdot\text{m} - 2.3180\,\text{m}\cdot F_{AB} = 0 \\
\Rightarrow F_{AB} &= -5.7006\,\text{kN}
\end{aligned}$$

$$\begin{aligned}
\Sigma F_x &= R_{Cx} + (-5.7006\,\text{kN})\cdot\cos 36.5^\circ = 0 \\
\Rightarrow R_{Cx} &= 4.5825\,\text{kN}
\end{aligned}$$

$$\begin{aligned}
\Sigma F_y &= R_{Cy} - 8.6328\,\text{kN} - (-5.7006\,\text{kN})\cdot\sin 36.5^\circ = 0 \\
\Rightarrow R_{Cy} &= 5.2420\,\text{kN}
\end{aligned}$$

$$R_C = \sqrt{(4.5825\,\text{kN})^2 + (5.2420\,\text{kN})^2} = 6.9626\,\text{kN}$$

$$\theta = \tan^{-1}\left[\frac{5.2420}{4.5825}\right] = 48.840^\circ$$

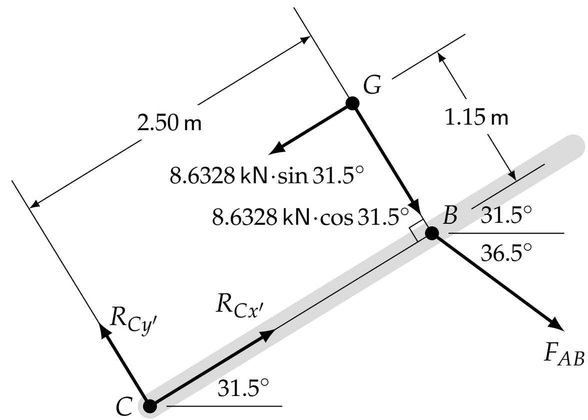
The force in AB is **5.70 kN** in **compression**.

The reaction at C is **6.96 kN** at **48.8°**, measured counter-clockwise from the positive x-axis.

Method 2: By rotation of our axes of reference.

Rotating the xy axes coordinate system, by 31.5° to a new $x'y'$ axes system, makes for (possibly) clearer calculations.

$$M = 880\,\text{kg} \Rightarrow W = \frac{880 \times 9.81}{1000} = 8.6328\,\text{kN}$$

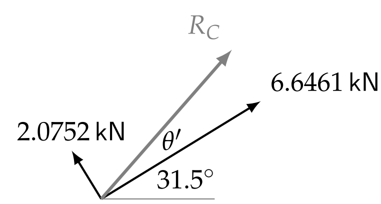


$$\begin{aligned}
\Sigma M_C &= (8.6328\,\text{kN}\cdot\sin 31.5°)\cdot(1.15\,\text{m}) - (8.6328\,\text{kN}\cdot\cos 31.5°)\cdot(2.50\,\text{m}) - F_{AB}\cdot\sin 68°\cdot 2.50\,\text{m} \\
&= -13.214\,\text{kN}\cdot\text{m} - F_{AB}\cdot 2.3180\,\text{m} = 0 \\
\Rightarrow F_{AB} &= -5.7006\,\text{kN}
\end{aligned}$$

$$\begin{aligned}
\Sigma F_{x'} &= R_{Cx'} + (-5.7006\,\text{kN})\cdot\cos 68° - 8.6328\,\text{kN}\cdot\sin 31.5° = 0 \\
\Rightarrow R_{Cx'} &= 6.6461\,\text{kN}
\end{aligned}$$

$$\begin{aligned}
\Sigma F_{y'} &= R_{Cy'} - (-5.7006\,\text{kN})\cdot\sin 68° - 8.6328\,\text{kN}\cdot\cos 31.5° = 0 \\
\Rightarrow R_{Cy'} &= 2.0752\,\text{kN}
\end{aligned}$$

$$\begin{aligned}
R_C &= \sqrt{(2.0752\,\text{kN})^2 + (6.6461\,\text{kN})^2} = 6.9625\,\text{kN} \\
\theta' &= \tan^{-1}\left[\frac{2.0752}{6.6461}\right] = 17.341° \\
\Rightarrow \theta &= \theta' + 31.5° = 48.841°
\end{aligned}$$

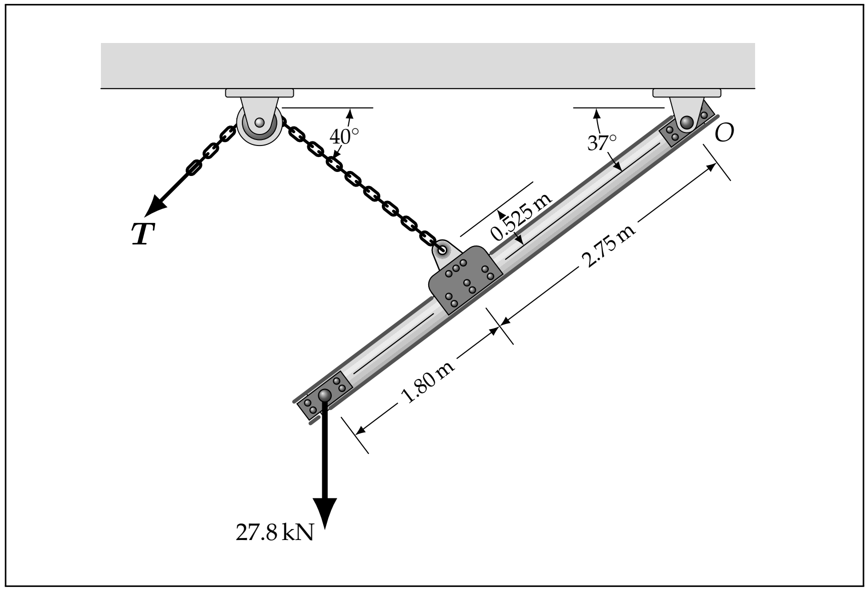


The force in AB is **5.70 kN** in **compression**.

The reaction at C is **6.96 kN** at 48.8°, measured counter-clockwise from the positive x-axis.

Exercise 7:

The pulley is frictionless. Determine the tension T and the reaction at the pinned connection O.



Method 1: