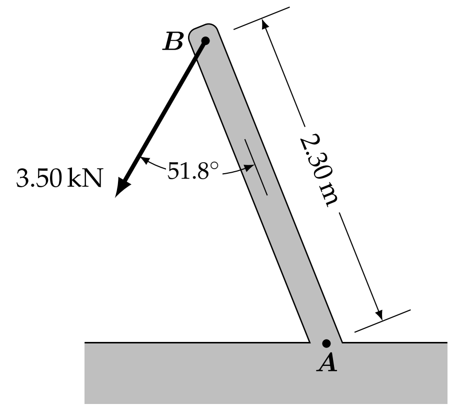
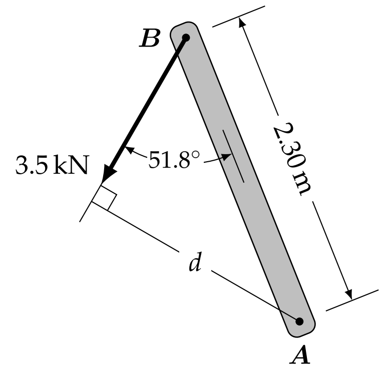
Engineering Statics - 05 Moments - Instructor Copy

Example 1: Determine the moment, M_A, of the 3.5 kN force applied at B, about the point A.

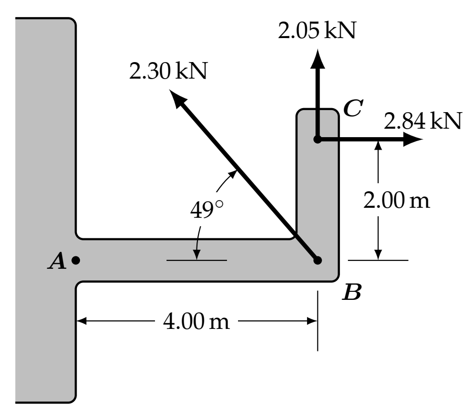


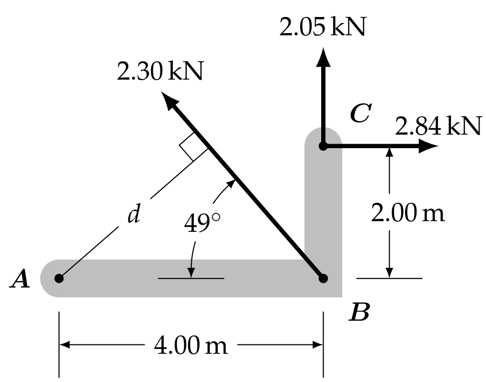


$$
\begin{aligned}
M_A &= F \cdot d \\
&= (3.5\,\text{kN}) \cdot (2.30\,\text{m})(\sin 51.8^\circ) \\
&= 6.3261\,\text{kN} \cdot \text{m} \\
&\approx 6.33\,\text{kN} \cdot \text{m}
\end{aligned}
$$

Example 2: Determine the sum of the moments of the forces, acting at B and C, about the point A.

Also, sum the moments of the forces about the point B.



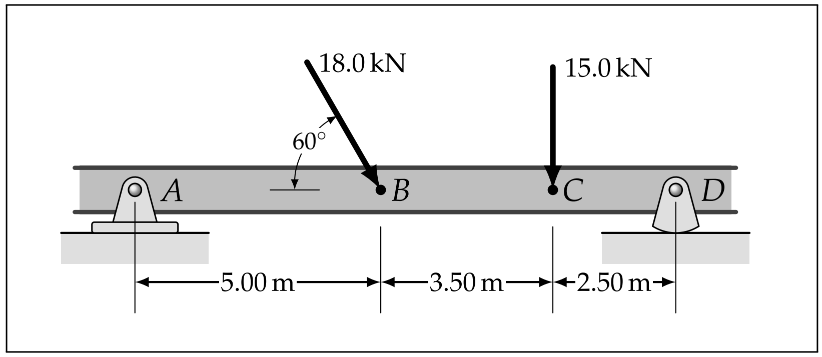


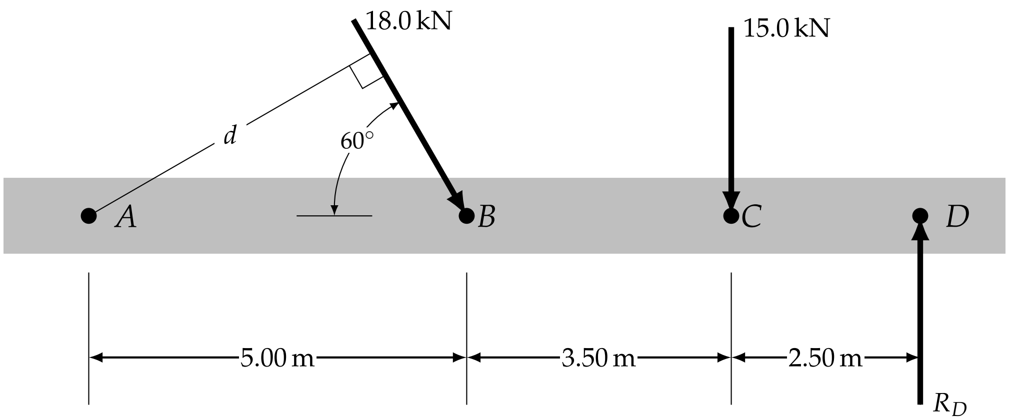
$$
\begin{aligned}
\Sigma M_A &= \Sigma F \cdot d \\
&= (2.30\,\text{kN}) \cdot (4.00\,\text{m})(\sin 49^\circ) \\
&\quad + (2.05\,\text{kN}) \cdot (4.00\,\text{m}) - (2.84\,\text{kN}) \cdot (2.00\,\text{m}) \\
&= 9.4633\,\text{kN} \cdot \text{m} \approx 9.46\,\text{kN} \cdot \text{m}
\end{aligned}
$$

$$
\Sigma M_B = 0 + 0 - (2.84\,\text{kN}) \cdot (2.00\,\text{m}) = -5.6800\,\text{kN} \cdot \text{m} = -5.68\,\text{kN} \cdot \text{m}
$$

Exercise 1: Rigid beam $ABCD$ is supported at A and at D, and is subjected to the two forces shown at B and C.

Determine the value of the reaction at D, R_D, if the sum of the moments about A, ΣM_A, is zero and the reaction at D is vertically upwards.





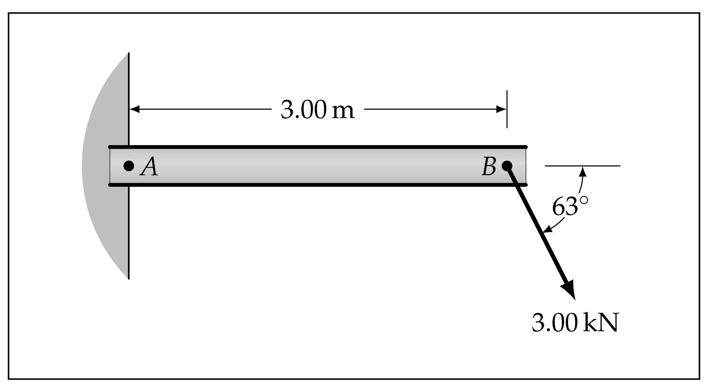
$$\sum M_A = -(18.0\,\text{kN})(5.00\,\text{m})(\sin 60^\circ) - (15.0\,\text{kN})(8.50\,\text{m}) + R_D(11.00\,\text{m}) = 0$$

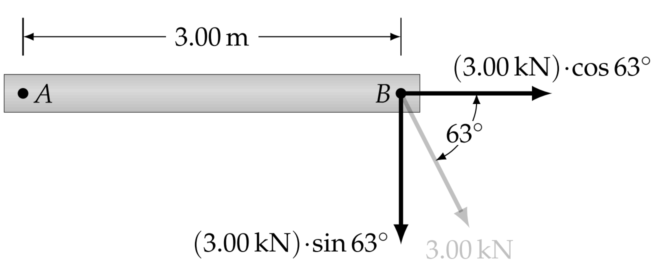
$$\Rightarrow R_D = \frac{(18.0\,\text{kN})(5.00\,\text{m})(\sin 60^\circ) + (15.0\,\text{kN})(8.50\,\text{m})}{11.00\,\text{m}}$$

$$= 18.677\,\text{kN} \approx 18.7\,\text{kN}$$

Example 3:

Determine the moment about A of the force applied to B by resolving the force at B into horizontal and vertical components.

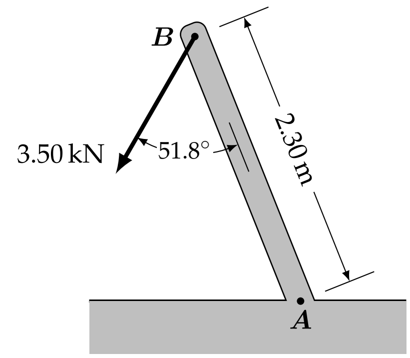


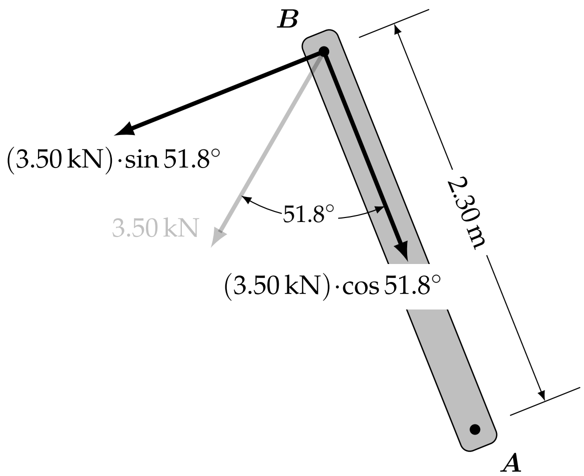


$$\Sigma M_A = (3.00\,\text{kN})\cdot\cos 63^\circ\cdot 0 + (3.00\,\text{kN})\cdot\sin 63^\circ\cdot 3.00\,\text{m} = 8.0191\,\text{kN}\cdot\text{m} \approx 8.02\,\text{kN}\cdot\text{m}$$

Example 4:

Determine the moment about A of the force at B by resolving the force into components parallel to and perpendicular to AB.

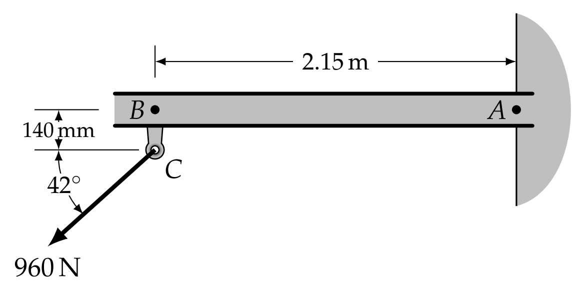


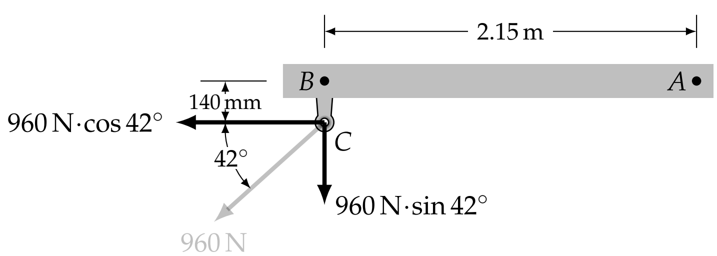


$$\begin{aligned}\Sigma M_A &= (3.50\,\mathrm{kN})\cdot\cos 51.8^\circ\cdot 0\\ &\quad + (3.50\,\mathrm{kN})\cdot\sin 51.8^\circ\cdot 2.30\,\mathrm{m}\\ &= 6.3261\,\mathrm{kN} \approx 6.33\,\mathrm{kN}\end{aligned}$$

Exercise 2:

Determine the moment about A of the force acting on D. Then determine the moment about B for the same force.



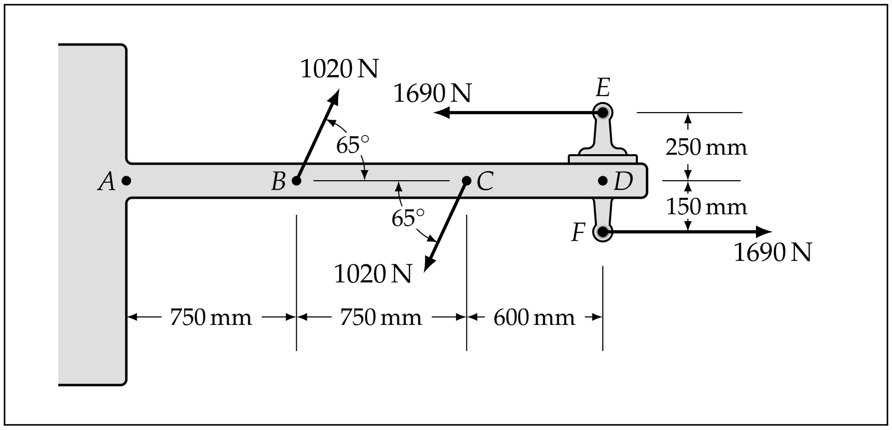


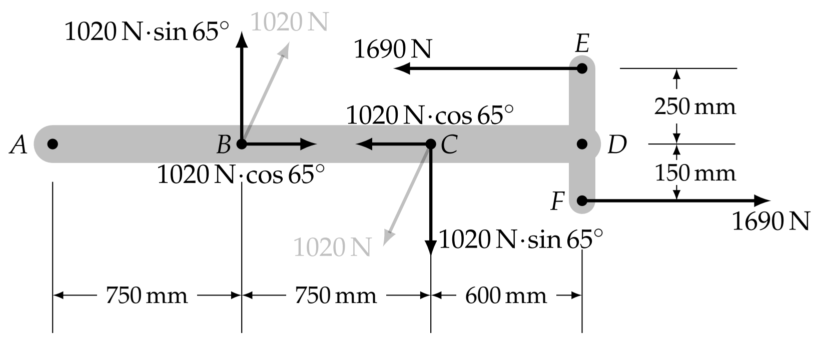
$$\Sigma M_A = 960\,\mathrm{N}\cdot\sin 42^\circ\cdot(2.15\,\mathrm{m}) - 960\,\mathrm{N}\cdot\cos 42^\circ\cdot(0.14\,\mathrm{m}) = 1281.2\,\mathrm{N\cdot m} \approx 1280\,\mathrm{N\cdot m}$$

$$\Sigma M_B = -960\,\mathrm{N}\cdot\cos 42^\circ\cdot(0.14\,\mathrm{m}) = 99.879\,\mathrm{N\cdot m} \approx 99.9\,\mathrm{N\cdot m}$$

Exercise 3:

Determine the sum of the moments about A, about C and about F of the forces shown.





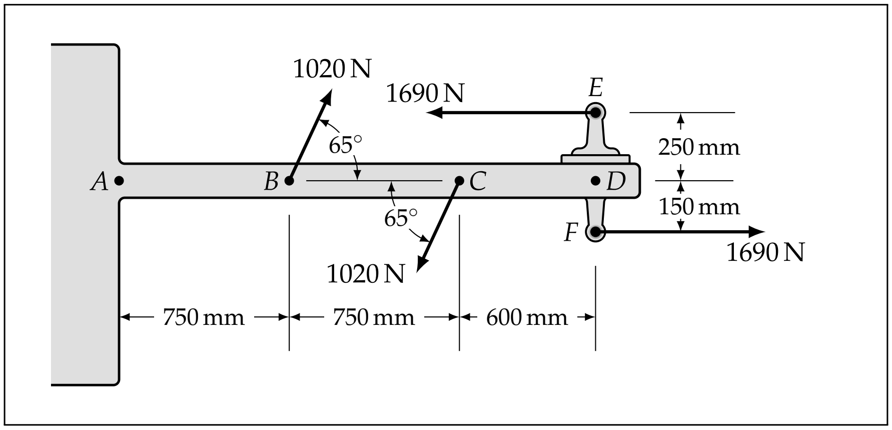
$$
\begin{aligned}
\Sigma M_A &= (1020\,\text{N}\cdot\sin 65^\circ)(750\,\text{mm}) - (1020\,\text{N}\cdot\sin 65^\circ)(1500\,\text{mm}) \\
&\qquad + (1690\,\text{N})(250\,\text{mm}) + (1690\,\text{N})(150\,\text{mm}) \\
&= -(1020\,\text{N}\cdot\sin 65^\circ)(750\,\text{mm}) + (1690\,\text{N})(400\,\text{mm}) \\
&= -17325\,\text{N}\cdot\text{mm} \approx -17.3\,\text{N}\cdot\text{m}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma M_C &= -(1020\,\text{N}\cdot\sin 65^\circ)(750\,\text{mm}) + (1690\,\text{N})(250\,\text{mm}) + (1690\,\text{N})(150\,\text{mm}) \\
&= -17325\,\text{N}\cdot\text{mm} \approx -17.3\,\text{N}\cdot\text{m}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma M_F &= (1020\,\text{N}\cdot\sin 65^\circ)(600\,\text{mm}) - (1020\,\text{N}\cdot\sin 65^\circ)(1350\,\text{mm}) + (1690\,\text{N})(400\,\text{mm}) \\
&= -17325\,\text{N}\cdot\text{mm} \approx -17.3\,\text{N}\cdot\text{m}
\end{aligned}
$$

Example 5:

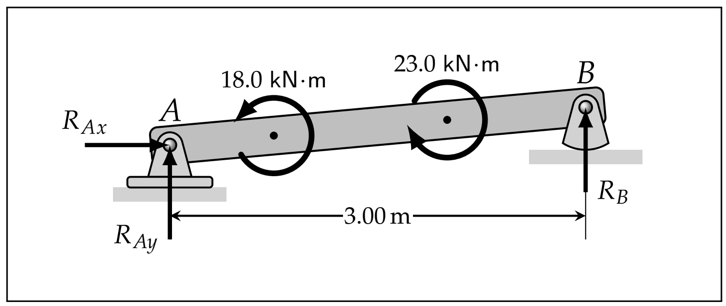
Determine ΣM, the sum of the moments, of the couples shown.

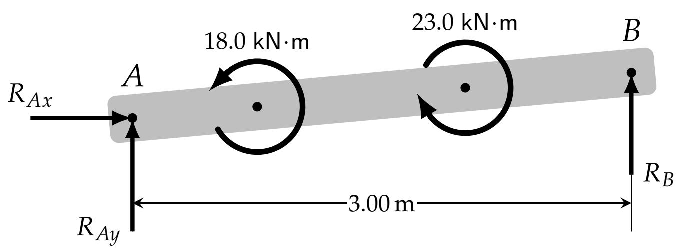


$$\Sigma M = (-1020\,\text{N})(750\,\text{mm}) + (1690\,\text{N})(400\,\text{mm}) = -17325\,\text{N}\cdot\text{mm} \approx -17.3\,\text{N}\cdot\text{m}$$

Example 6: Considering beam AB:

1. Determine the magnitude of R_D if $\Sigma M_A = 0$.
2. Determine the magnitude of R_{Ay} if $\Sigma F_y = 0$.
3. Determine the magnitude of R_{Ax} if $\Sigma F_x = 0$.





1.

$$\Sigma M_A = 18.0\,\text{kN}\cdot\text{m} - 23.0\,\text{kN}\cdot\text{m} + R_B(3.00\,\text{m}) = 0 \Rightarrow R_B = 1.6667\,\text{kN} \Rightarrow R_B \approx 1.67\,\text{kN}$$

2.

$$\Sigma F_y = R_{Ay} + R_B = 0 \Rightarrow R_{Ay} = -1.6667\,\text{kN} \Rightarrow R_{Ay} \approx -1.67\,\text{kN}$$

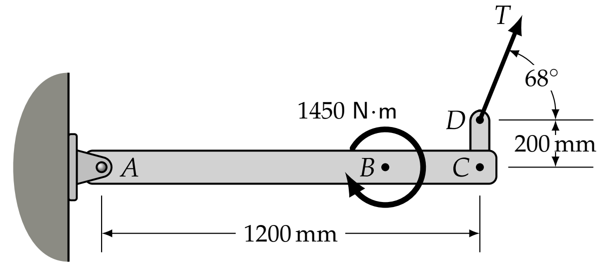
3.

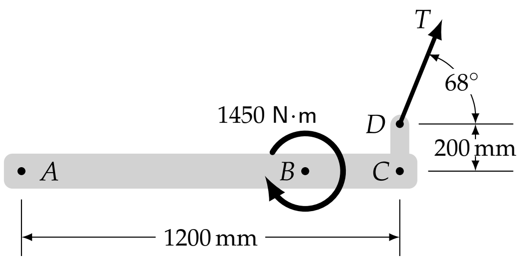
$$\Sigma F_x = R_{Ax} = 0$$

Exercise 4:

$ABCD$ has a negative couple applied at B and is supported by a cable at D.

1. Determine the magnitude of T if $\Sigma M_A = 0$.
2. Determine the reaction at A if $\Sigma F_x = \Sigma F_y = 0$.





1.

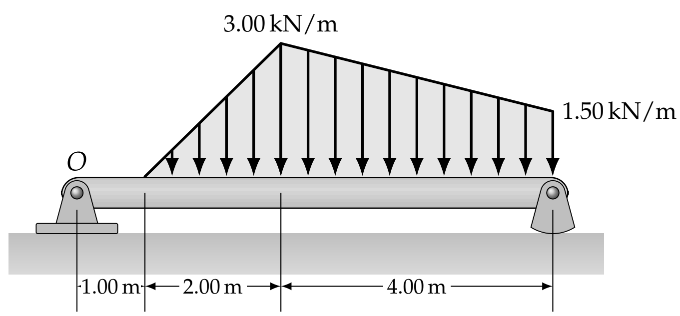
$$\Sigma M_A = -1450\,\text{N·m} - (T\cos 68^\circ)(200\,\text{mm}) + (T\sin 68^\circ)(1200\,\text{mm}) = 0$$

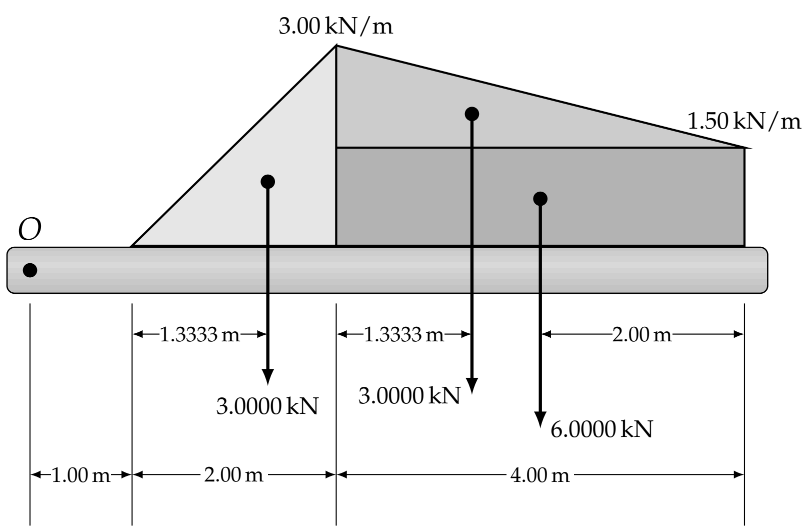
$$\Rightarrow T = \frac{1450\,\text{N·m}}{(1.200\,\text{m})\sin 68^\circ - (0.200\,\text{m})\cos 68^\circ} = 1397.3\,\text{N} = 1400\,\text{N}$$

2. $R_A = 1400\,\text{N}$ at 248° measured ccw from the positive x-axis. Why?

Example 7:

Determine the moment of the distributed load about O.

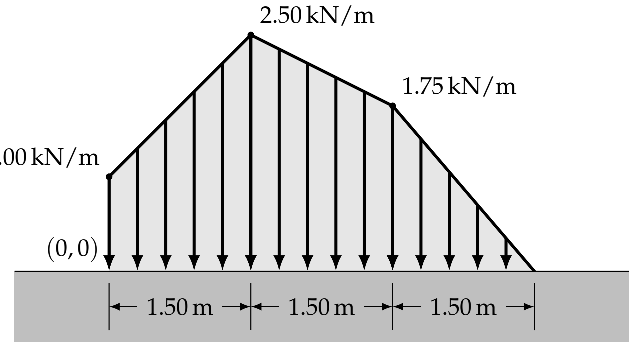


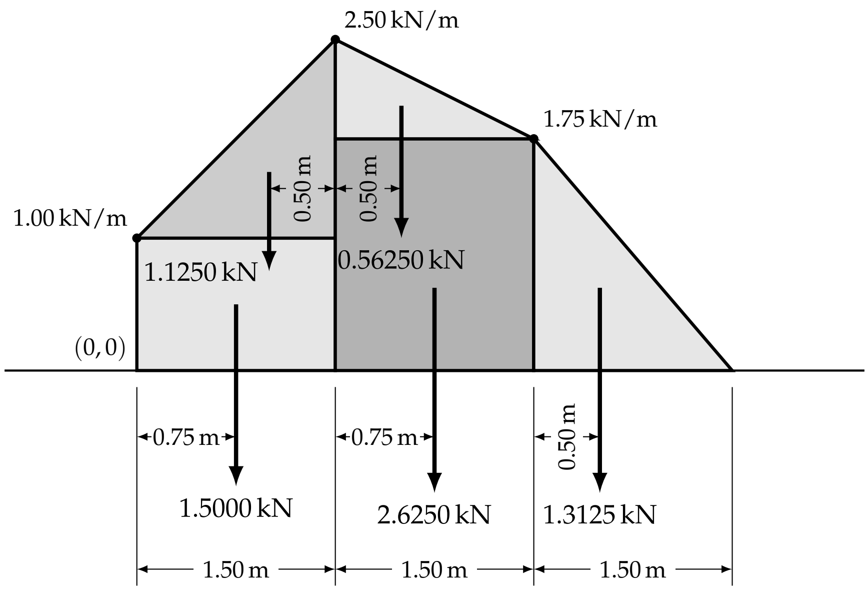


$$\Sigma M_O = -(3.00\,\text{kN})(2.3333\,\text{m}) - (3.0000\,\text{kN})(4.3333\,\text{m}) - (6.0000\,\text{kN})(5.00\,\text{m}) = -50.0\,\text{kN·m}$$

Exercise 5:

Determine the moment of the distributed load about $(0, 0)$.





$$\begin{aligned}\Sigma M_{(0,0)} &= -(1.5000\,\text{kN})(0.750\,\text{m}) - (1.1250\,\text{kN})(1.00\,\text{m}) \\ &\quad - (0.56250\,\text{kN})(2.00\,\text{m}) - (2.6250\,\text{kN})(2.25\,\text{m}) - (1.3125\,\text{kN})(3.50\,\text{m}) \\ &= -13.875\,\text{kN}\cdot\text{m} \approx -13.9\,\text{kN}\cdot\text{m}\end{aligned}$$