06 Equilibrium of Rigid Bodies

Engineering Statics

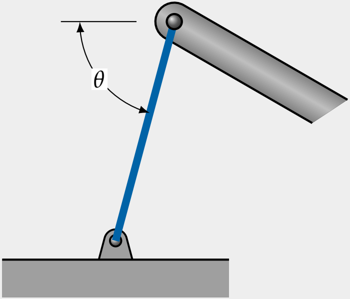
Updated on: September 30, 2025

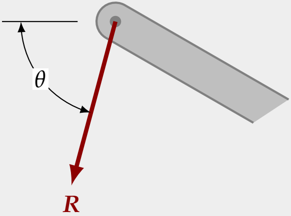
Overview

- Much of the study of statics involves the calculations of reaction forces generated between a structural body and its supports when loads are applied.

- There are various connection types used between a structural body and its supports. These connections influence the direction and the sense of the reaction. We shall examine some of these connections now.

Types of Connections: Cable



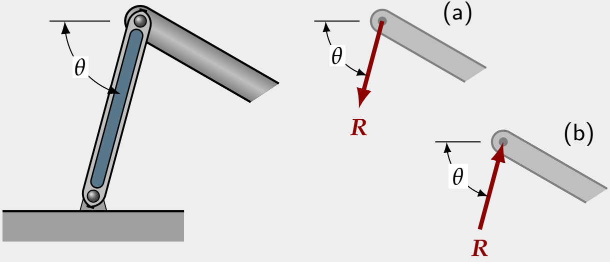


When a structural member or body is supported by a cable (or rope or chain), the cable is assumed to be weightless (and consequently straight) and the cable exerts a reaction on the structural member **in the same direction** as the cable.

A cable is in tension and can only **pull**; it cannot push.

There is only one unknown: the magnitude of the force. The direction and sense of the force are known.

Types of Connections: Strut

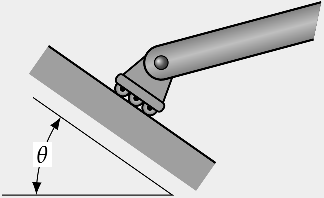


Like a cable, a straight strut (or link) exerts a reaction on a structural member in the direction of the strut. Unlike a cable, a strut can pull or push.

If we don't know whether a strut is pushing or pulling, we generally assume that the reaction is directed away from the structural member (pulling, in tension), as shown in (a). If our calculations then determine that F is negative, the direction is opposite to our assumption (i.e., pushing) (b).

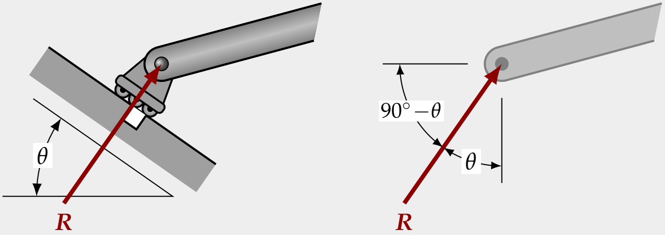
As with the cable, there is only one unknown. The sign of F determines the sense of F.

Types of Connections: Roller



A roller (assumed weightless and frictionless) can provide no resistance **along** the slope on which it is resting.

Types of Connections: Roller

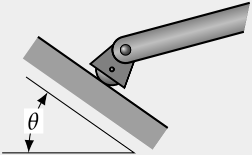


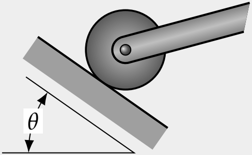
A roller (assumed weightless and frictionless) can provide no resistance **along** the slope on which it is resting. The only reaction a roller can provide is **perpendicular** to the slope.

A roller can only push. It is not fixed to the sloped surface and would lift off the surface if pulled.

As with the cable, there is only one unknown. If your math is correct, the sign of F should always be positive.

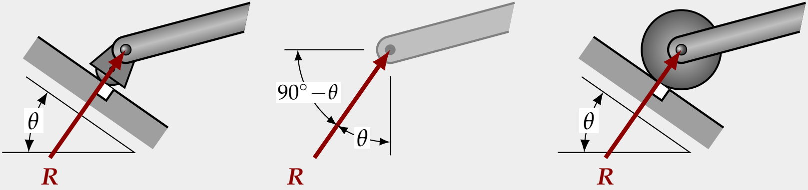
Types of Connections: More Rollers





Rollers come in different shapes. But they all react in the same way.

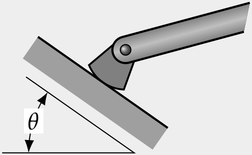
Types of Connections: More Rollers

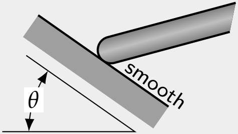


Rollers come in different shapes. But they all react in the same way.

The reaction force is perpendicular to the surface that the roller bears on.

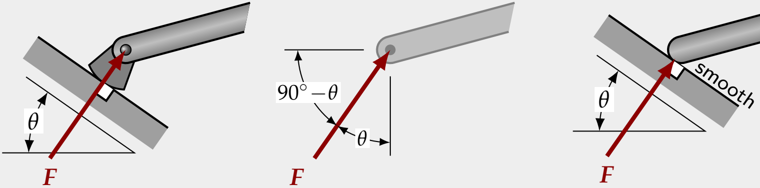
Types of Connections: Rockers and Smooth Surfaces





Two more connection types that react in the same way as rollers:

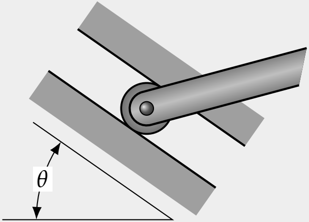
Types of Connections: Rockers and Smooth Surfaces



Two more connection types that react in the same way as rollers:

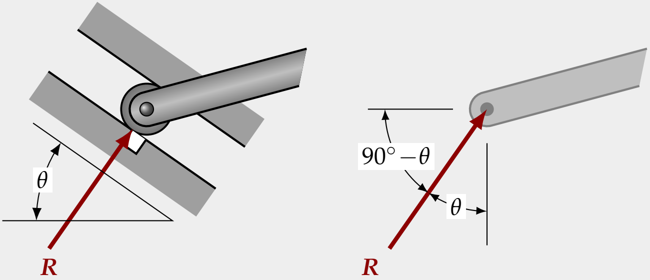
The **rocker**: the reaction force is perpendicular to the surface that the rocker bears on.

The **smooth surface** has no friction along the surface: the reaction force is perpendicular to the smooth surface that the structural member bears on.



With a roller in a smooth slot, we don't necessarily know whether the roller is:

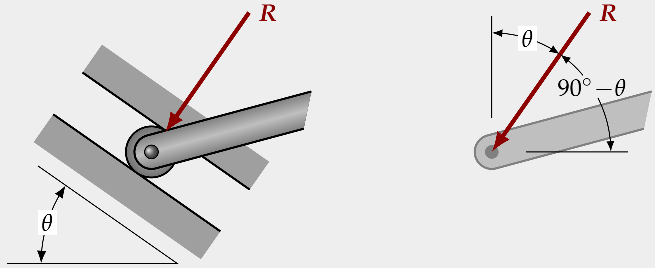
Types of Connections: Roller in Smooth Slot



With a roller in a smooth slot, we don't necessarily know whether the roller is:

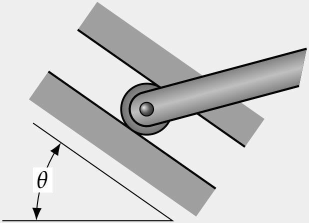
- Bearing on the lower smooth surface; in this case it is the lower surface that provides the reaction.

Types of Connections: Roller in Smooth Slot



With a roller in a smooth slot, we don't necessarily know whether the roller is:

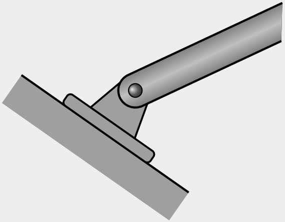
- Bearing on the lower smooth surface; in this case it is the lower surface that provides the reaction.
- Bearing on the upper smooth surface; in this case it is the upper surface that provides the reaction.



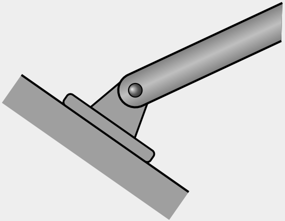
With a roller in a smooth slot, we don't necessarily know whether the roller is:

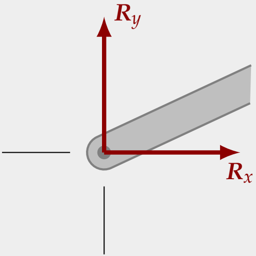
- Bearing on the lower smooth surface; in this case it is the lower surface that provides the reaction.
- Bearing on the upper smooth surface; in this case it is the upper surface that provides the reaction.
- In each case, the reaction is perpendicular to the bearing surface of the slot.

Types of Connections: Pinned Connection



With the pinned connection (also known as a hinged connection), movement is restricted in all directions. The reaction can be in any direction.

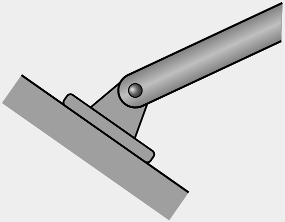


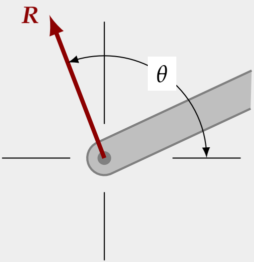


With the pinned connection (also known as a hinged connection), movement is restricted in all directions. The reaction can be in any direction.

- This gives us **two** unknowns: one generally an x-component and one generally a y-component.

Types of Connections: Pinned Connection

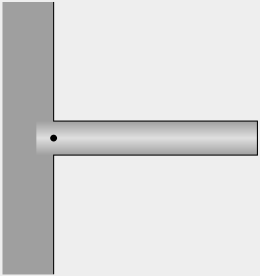




With the pinned connection (also known as a hinged connection), movement is restricted in all directions. The reaction can be in any direction.

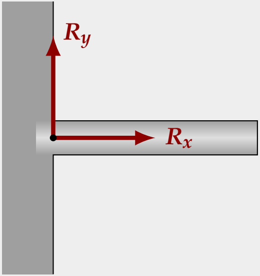
- This gives us **two** unknowns: one generally an x-component and one generally a y-component.
- Alternatively (and equivalently), the reaction can be specified by a magnitude and a direction. This is the form that we use to describe the reaction.

Types of Connections: Fixed Connection



The fixed connection has **three** unknowns.

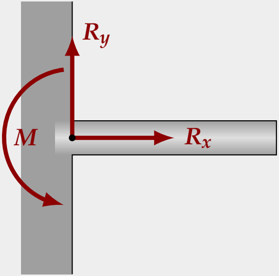
Types of Connections: Fixed Connection



The fixed connection has **three** unknowns.

- Components in the x and y directions (like the pinned connection).

Types of Connections: Fixed Connection



The fixed connection has **three** unknowns.

- Components in the x and y directions (like the pinned connection).
- A reacting moment, M, since (unlike the pinned connection) there can be no rotation about the connection.

Equilibrium: The Rules

- Here we add another major part of the puzzle: the third and final equation that, combined with our results from the equilibrium of concentric forces, allows us analyze the equilibrium of rigid bodies.
- In our discussion of force couples, we noted that although $\Sigma F_x = 0$ and $\Sigma F_y = 0$ there was still some tendency for the force couple to cause rotation.
- For a system to be in equilibrium, there must be no net moment: the sum of the moments of the forces acting upon a rigid body must be zero. There must be no tendency to rotate.

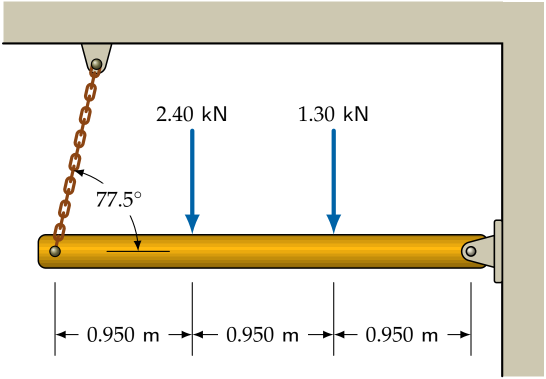
General Conditions for Equilibrium of Rigid Bodies

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M = 0$$

In Practice:

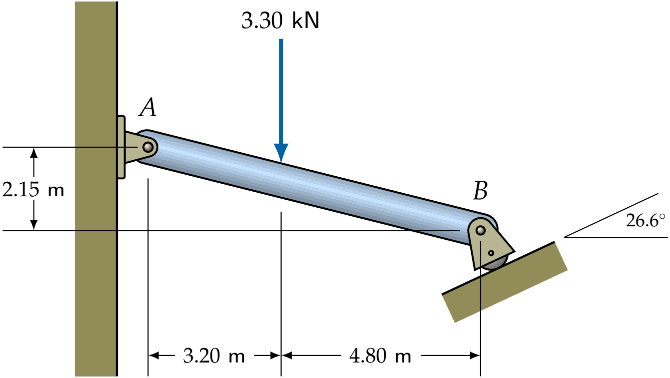
- In general, we use these equations to find the reactions at connections between a rigid body (not just a particle) and a support.
- Often, these support reactions will have three unknowns: a pinned connection with two unknown components; and a connection with a known direction (roller, cable, two-force member, etc.) but an unknown magnitude (and, possibly, unknown sense).
- Usually, our first step after drawing the FBD **(!)** is to take moments about the pinned connection to solve directly for the third unknown (the magnitude of the connection with the known direction).
- Then we sum the x-components of all the forces involved to solve for the x-component of the reaction at the pinned connection. We do the same for the y-component.
- From the components of the reaction at the pinned connection, R_x and R_y, we use the Pythagorean Theorem and the inverse tangent function to find the magnitude R and direction θ of the reaction.

Example 1



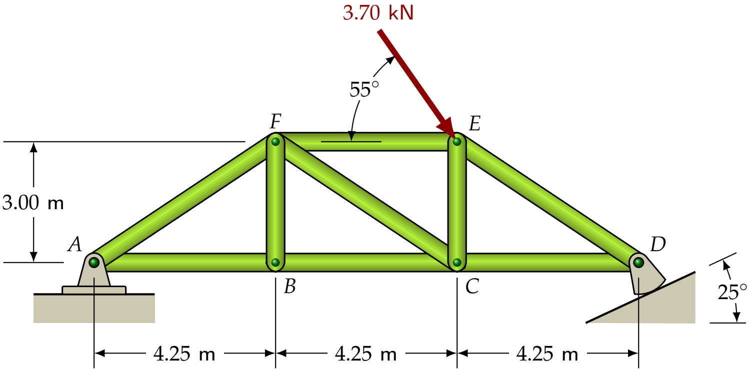
Determine the tension in the chain and the reaction at A.

Example 2



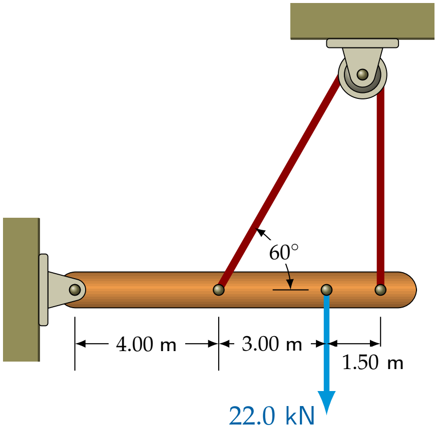
Determine the reactions at A and at B.

Exercise 1



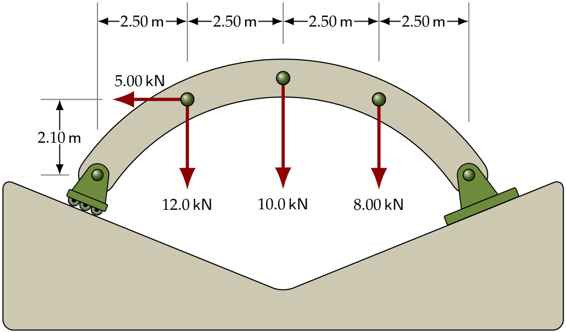
Determine the reactions at A and D.

Exercise 2



Determine the reactions at the pinned connection and the tension in the cable.

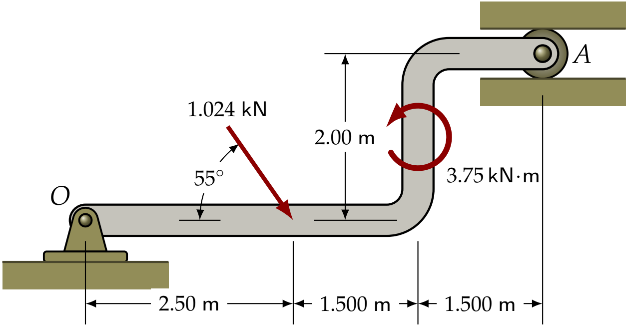
Example 3



The roller and the pinned connection are on slopes inclined at 21° to the horizontal; they are both at the same elevation.

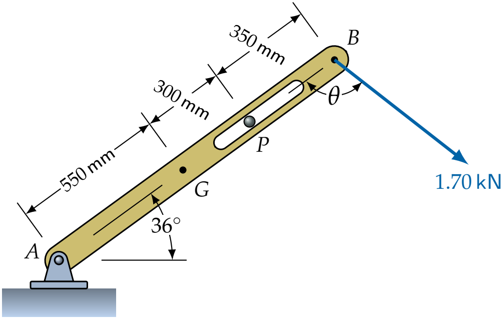
Determine the reaction at each connection. Indicate direction by measuring counter-clockwise from the positive x-axis.

Example 4



The roller at B is in a smooth slot.
Determine the reactions at A and B.

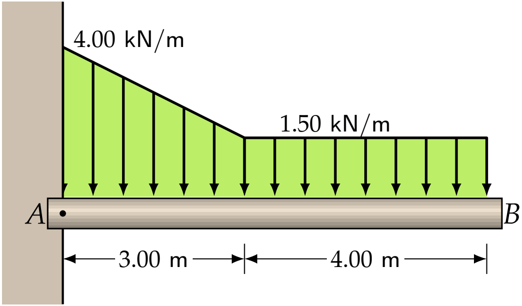
Example 5



55-kg bar AB has its centre of gravity at G. It is supported by a pinned connection at A and a smooth peg at C. A cable is attached at B and has a tensile force of 1.70 kN. The direction of the cable varies between $\theta = 60^\circ$ and $\theta = 135^\circ$.

What are the maximum reactions at A and C?

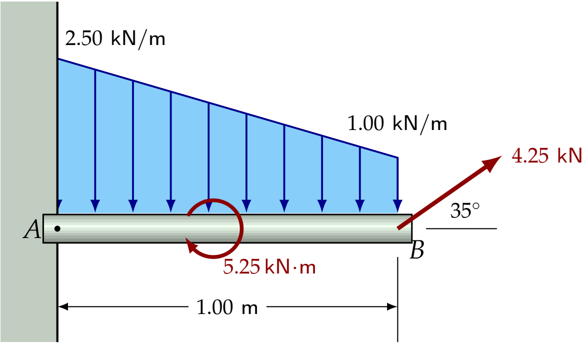
Example 6



Beam AB has a fixed support at A. (Fixed supports offer resistance to rotation in the form of a reacting couple at A; clearly, without this, equilibrium would not be possible.)

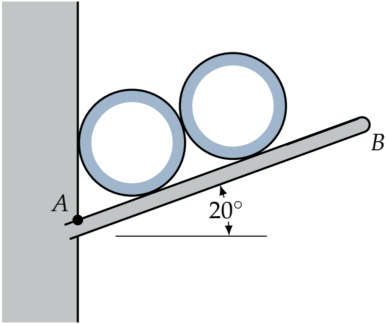
Determine the reaction and the reacting couple at A.

Exercise 3



Determine the reactions at A.

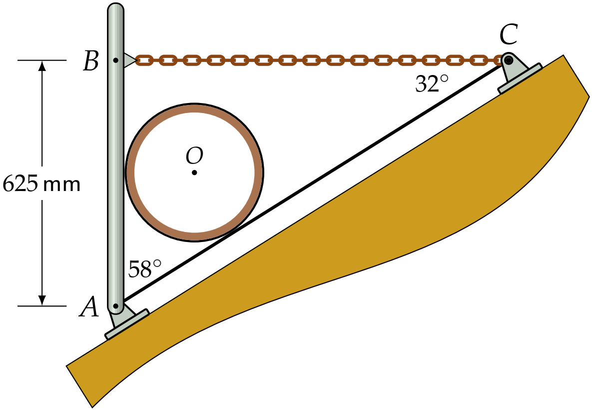
Example 7



Pipe racks (AB, and two hidden behind it) support two smooth Schedule 40 pipes, with an outside diameter of $508\,\text{mm}$, as shown. The pipes are $10\,\text{m}$ in length with a mass of $78.5\,\text{kg/m}$, Each rack supports one-third of the weight of each pipe.

Determine the reaction at the fixed connection A.

Exercise 4



A section of smooth pipe, centred at O, has a diameter of $457\,\text{mm}$ and a mass of $186\,\text{kg}$. It is secured by vertical structural member AB, hinged with a pinned connection at A and held in place by chain BC.

Determine the tension in the chain, and the reaction at A.

Notes:

1. For a body in equilibrium, $\Sigma M = 0$ for moments summed about **any** point in the plane containing the body.

2. We have used all the three equations of statics to solve for three unknowns. It would also have been possible to solve using $\Sigma M = 0$ three times by choosing appropriate points about which to take moments. There are occasions when it is more convenient to sum moments two or three times, rather than summing components. (We will see this when using the Method of Sections to analyze forces in truss members.)

3. Since there are only three equations for statics, we can only solve a system for three unknowns. Frequently, two unknowns come from a pinned connection and the third from a cable, roller or some other connection whose reaction direction is known. (Alternatively, we can solve for a fixed connection, which has three unknowns.)

4. Generally, we start by taking moments about the pinned connection - so that we don't need to consider the moments of the pinned connection unknowns. Then, the moment equation will immediately give a solution for the third unknown, e.g., for a connection with a known force direction.

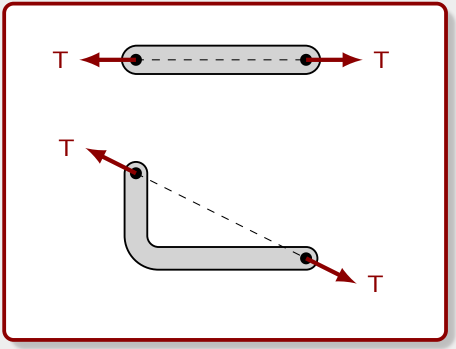
Two-Force Members

For a member with only two external forces acting upon it to be in equilibrium:

- Both forces must share the same line of action
- Both forces must have the same magnitude
- The forces must have opposite directions

Then, $\Sigma F_x = 0$, $\Sigma F_y = 0$ **and** there is no tendency for the member to rotate.

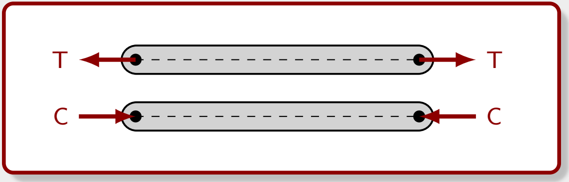
Strut connections and cables are two-force members!



Straight Two-Force Members

In straight two-force members, the lines of action of the forces are along the member.

The forces shown here are **external** forces applied to the member.



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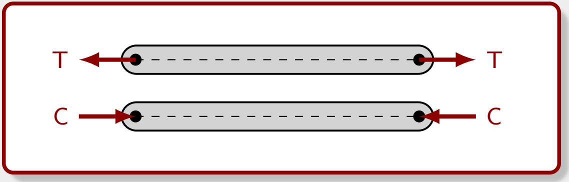
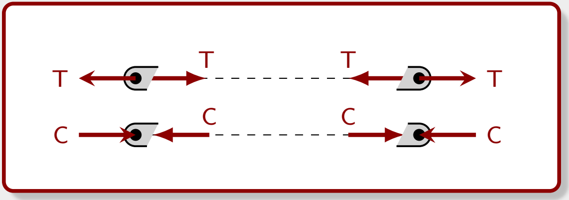


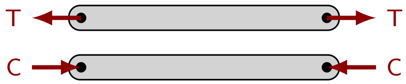
These external forces generate corresponding **internal** forces which can be examined by 'cutting' a section through the member:



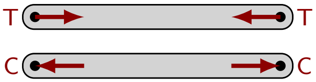
External and Internal Forces

It is important to distinguish between external and internal forces.
When a two-force member is in tension, external forces pull **away** from the member (attempting to make the member longer). The opposite is true for compression.

External Forces



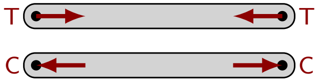
Internal Forces



External and Internal Forces

Initially, the direction of internal forces may seem counter-intuitive.

Internal Forces

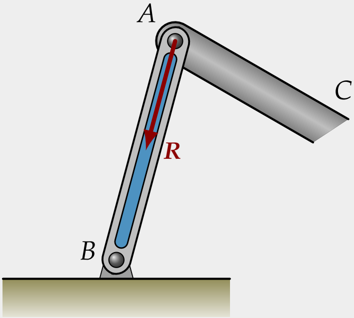


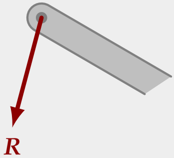
Why do internal forces pull towards each other when the member is in tension? It is because external forces are trying to 'stretch' the member and the internal forces are opposing this stretch, maintaining the length of the member.

Similarly, when a straight two-force member is in compression, internal forces in the member 'push back' against the external compression.

Furthermore, at the ends of the two-force members, external and internal forces must be equal and opposite to each other to maintain equilibrium.

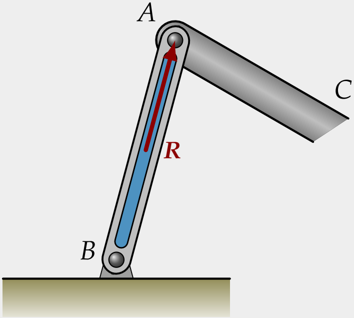
A Second Look at Strut/Link Connections

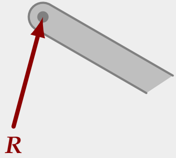




When strut AB is in **tension** (due to forces in AC), it 'pulls' back against AC to maintain equilibrium.

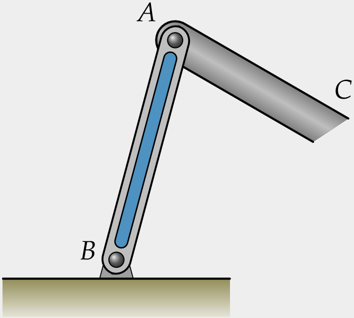
A Second Look at Strut/Link Connections

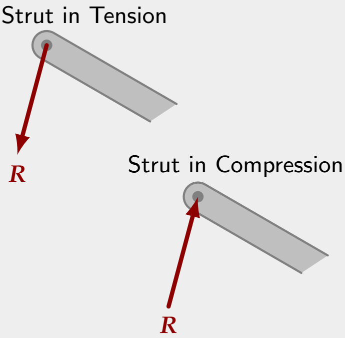




When strut AB is in **compression** (due to forces in AC), it ‘pushes’ against AC to maintain equilibrium.

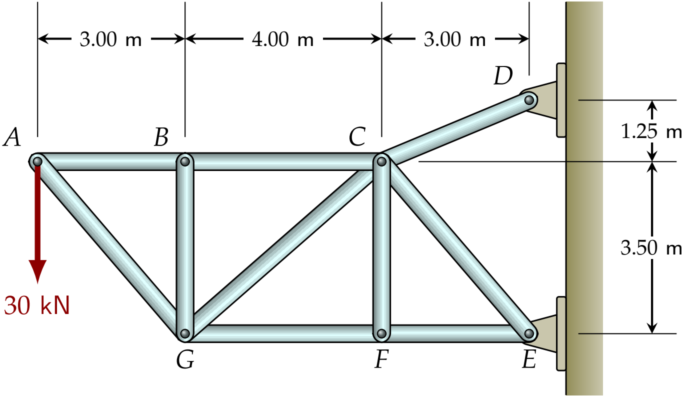
A Second Look at Strut/Link Connections





Frequently, we don't know in advance whether AB is in tension or in compression. In that case, we draw the force acting on AC in tension (i.e., arrow directed **away** from AC). If our calculations determine that R is negative, then AB is in compression.

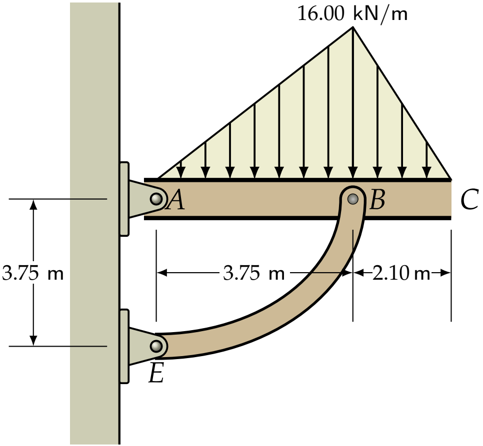
Example 8



Determine the reactions at D and E.

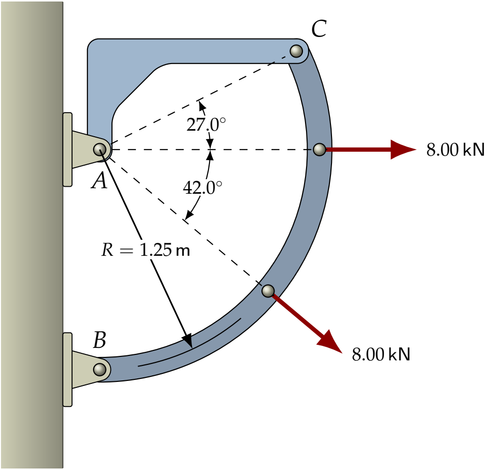
Note: There are two pinned connections, but there is a single link between C and D so CD is a **two-force member**, so the direction of the reaction at D is in the direction of CD and has only one unknown - its magnitude.

Exercise 5



Determine the reactions at A and E.

Exercise 6



Determine the reactions at A and B.