03 Equilibrium of a Particle (Concurrent Forces)

Engineering Statics

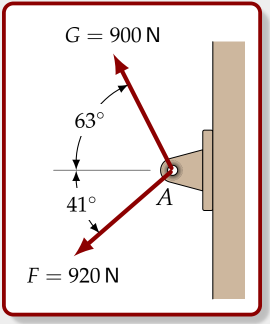
Updated on: September 4, 2025

Statics and Equilibrium

- Mechanics is a branch of physics dealing with bodies that are subject to forces.
- Statics is the branch of mechanics that deals with bodies that are in **equilibrium**.
- Bodies in equilibrium are either **static** (not moving, or at rest) or are moving with a constant velocity.
- In this course, we are interested in the forces necessary to keep a body at rest.
- We only consider forces in the plane, i.e. this course is limited to bodies described in only two dimensions (2-D), not those described in space (3-D).
- Furthermore, these bodies are assumed to be **rigid**. That is, they do not deform when loads are applied. In practice, structural members **do** deform when loaded but, if correctly designed, deformation is generally minimal and we can ignore the slight geometrical changes.
- Unless otherwise noted, structural bodies are presumed to have no weight. We do this because the weight of the body is generally small compared to the loads imposed.

Conditions for Equilibrium

- In the previous module, we investigated the resultant force $\boldsymbol{R}$ of several forces acting at a single point.
- $\boldsymbol{R}$ is the net force that results from combining these several component forces $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, $\boldsymbol{F_3}, \ldots$
- The resultants that we found have been non-zero.
- From Newton's Laws, we know that a net force acting on a particle will cause this particle to accelerate. So, should ring at $\boldsymbol{A}$ be moving? Or are other forces involved that we haven't considered?
- The ring at $\boldsymbol{A}$ resists the effects of the two forces $\boldsymbol{F}$ and $\boldsymbol{G}$, maintaining the ring in its stationary position.



- These resisting forces that maintain equilibrium are called **reactions**.

Conditions for Equilibrium

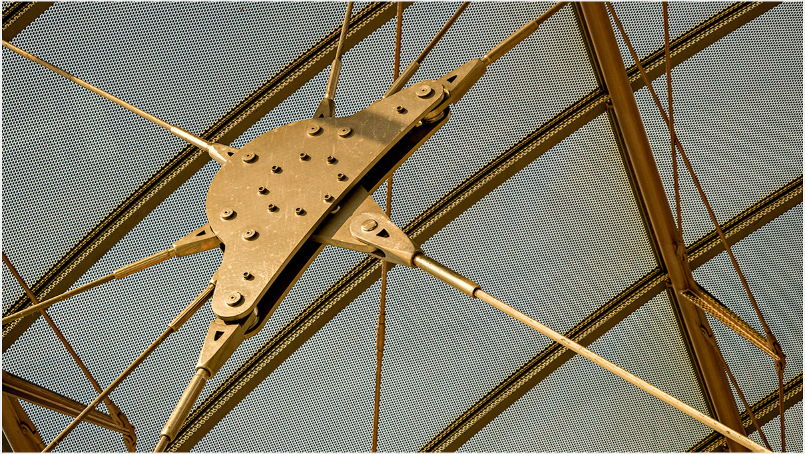
- Reactions are necessary for equilibrium to occur in a structure.
- When we walk across a room, our weight bears vertically down onto the floor through one or both of our feet. It is the reaction from the floor that stops us crashing down into the room below. And the one below that.
- The study of **statics is the study of equilibrium**, so the reactions that maintain equilibrium are an important part of statics.

Equation of Equilibrium for a Particle/Concurrent Forces

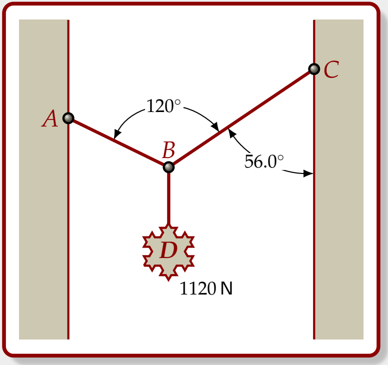
- When in equilibrium, there is no resultant: $\mathbf{\Sigma F = 0}$
- We frequently solve the two equations $\mathbf{\Sigma F}_x = \mathbf{0}$ and $\mathbf{\Sigma F}_y = \mathbf{0}$, setting the sum of the x components and the sum of the y components to 0.
- Note that with two equations, **we can only solve for two unknowns** (two forces, if the force lines of action are known, or one force and its direction). If there are three unknowns, we cannot find any of them.
- Forces whose lines of action pass through a particle, or point, are known as **concurrent forces**.

Equations of Equilibrium for Systems of Concurrent Forces

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

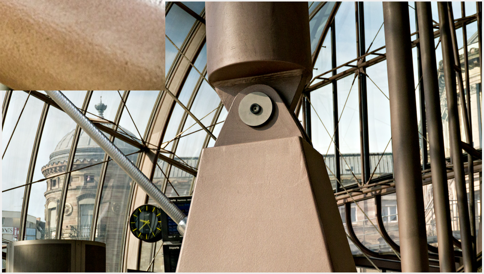


- If a system is in equilibrium, then each part of that system must be in equilibrium (otherwise that part of the system would move).



- This system is in equilibrium (it's static).
- To solve, break the system down into smaller, separate problems:
  1. Find the force in BD
  2. Analyze the forces at B to determine the forces in AB and BC. (This is the main part of the problem.)
  3. Determine the reaction at A
  4. Determine the reaction at C.

Actions & Reactions (Forces Acting on the Pin)



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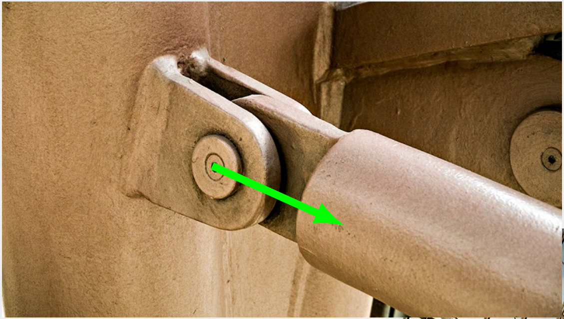

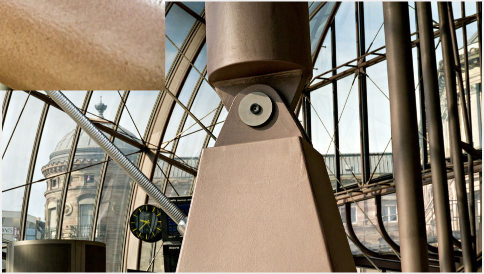

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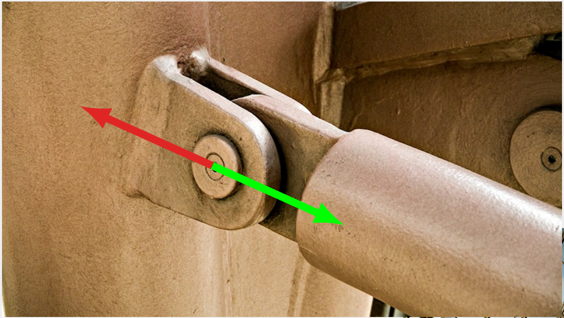

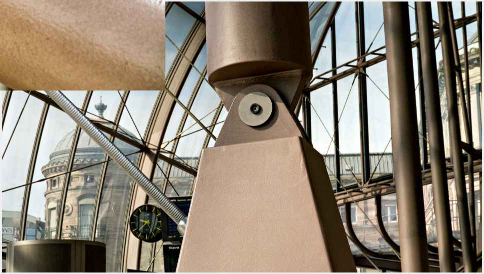

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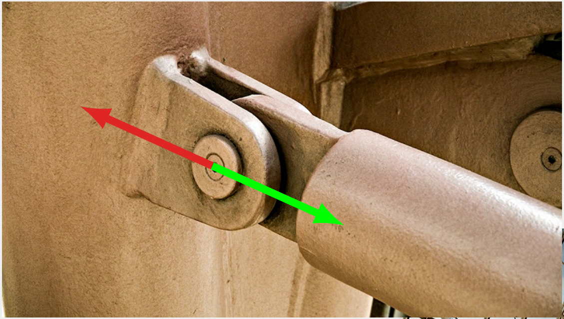

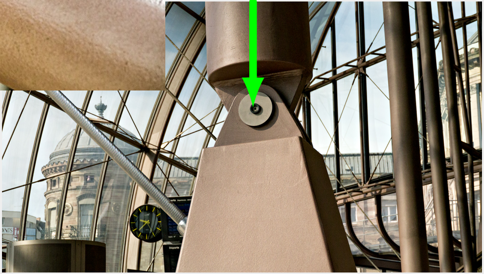

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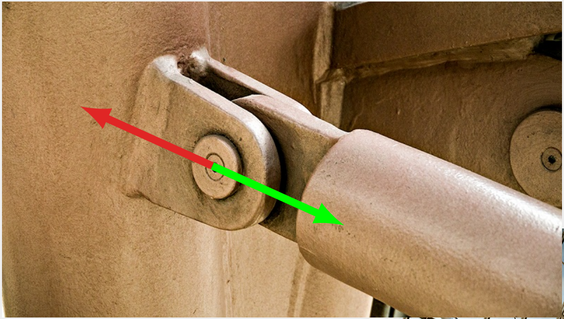
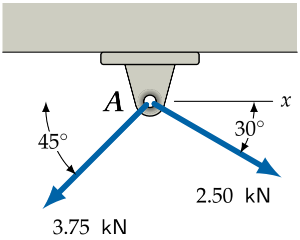


The Free-Body Diagram

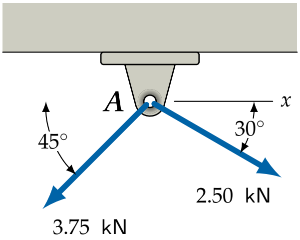
- An essential part of statics problems is the free-body diagram (*FBD*).
- We need to include all forces acting concurrently (acting on the particle) when using the equations of equilibrium. These forces are shown on the *FBD*.
  1. Isolate the particle from surrounding detail.
  2. Draw **all** forces that act on the particle: active forces, and the reactive forces that restrict the particle from moving.
  3. If the direction of an active or a reactive force is unknown, draw horizontal and vertical components where the reactive force acts. They will be solved later.
  4. Forces that are known should be drawn or labelled with their actual magnitude and direction.
- Free body diagrams are fairly straightforward for equilibrium of concurrent forces problems.

Example 1



Draw an *FBD* for the forces acting at A.
Use this to solve for the reaction at A.

Example 1



Draw the *FBD*:

1. Include **all** the information necessary to solve the problem but omit unnecessary details.

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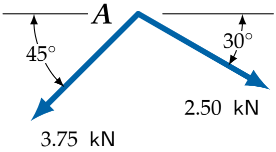

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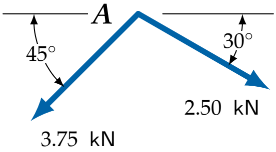

Draw the *FBD*:

1. Include **all** the information necessary to solve the problem but omit unnecessary details.
2. It is good practice to draw the reaction components in the direction of the positive axes. Then, if the answer turns out to be negative, you know that the component direction is negative also.

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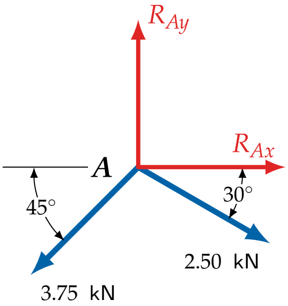

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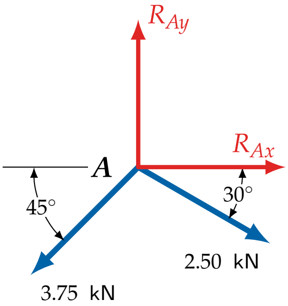

Draw the *FBD*:

1. Include **all** the information necessary to solve the problem but omit unnecessary details.
2. It is good practice to draw the reaction components in the direction of the positive axes. Then, if the answer turns out to be negative, you know that the component direction is negative also.
3. The *FBD* now has all the information necessary to solve for the reaction.

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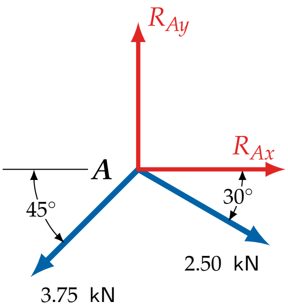
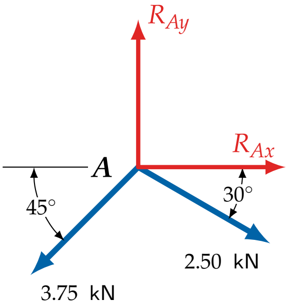


Sum all x-components and set to 0:

$$
\begin{aligned}
\Sigma F_x &= R_{Ax} + (2.50\,\text{kN})\cdot\cos 30^\circ \\
&\qquad\qquad - (3.75\,\text{kN})\cdot\cos 45^\circ \\
&= R_{Ax} - 0.48659\,\text{kN} \\
&= 0 \quad \text{(for equilibrium)} \\
\Rightarrow R_{Ax} &= 0.48659\,\text{kN}
\end{aligned}
$$

Example 1



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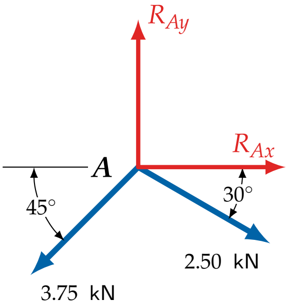
$$\begin{aligned}\Sigma F_x &= R_{Ax} + (2.50\,\text{kN})\cdot\cos 30^\circ \\ &\qquad - (3.75\,\text{kN})\cdot\cos 45^\circ \\ &= R_{Ax} - 0.48659\,\text{kN} \\ &= 0 \quad \text{(for equilibrium)} \\ \Rightarrow R_{Ax} &= 0.48659\,\text{kN}\end{aligned}$$

Sum all y-components and set to 0:

$$\begin{aligned}\Sigma F_y &= R_{Ay} - (2.50\,\text{kN})\cdot\sin 30^\circ \\ &\qquad - (3.75\,\text{kN})\cdot\sin 45^\circ \\ &= R_{Ay} - 3.9017\,\text{kN} \\ &= 0 \quad \text{(for equilibrium)} \\ \Rightarrow R_{Ay} &= 3.9017\,\text{kN}\end{aligned}$$

Example 1



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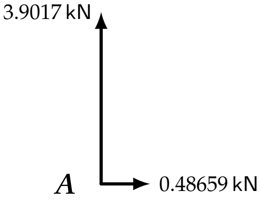
Now we can find the reaction from its components.

Example 1

$$R_{Ax} = 0.48659\,\mathrm{kN},\ R_{Ay} = 3.9017\,\mathrm{kN}$$

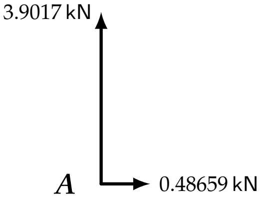


Solving a Concurrent Force System

Example 1

$$R_{Ax} = 0.48659\,\text{kN},\ R_{Ay} = 3.9017\,\text{kN}$$

Both are positive so the reaction is in the first quadrant. Draw the reaction.



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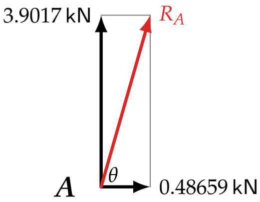

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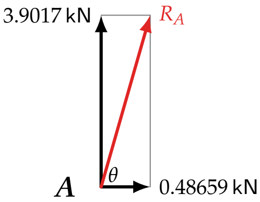

$$R_{Ax} = 0.48659\,\text{kN},\ R_{Ay} = 3.9017\,\text{kN}$$

Both are positive so the reaction is in the first quadrant. Draw the reaction.

$$\begin{aligned}|R_A| &= \sqrt{(R_{Ax})^2 + (R_{Ay})^2} \\ &= \sqrt{(0.48659\,\text{kN})^2 + (3.9017\,\text{kN})^2} \\ &= 3.9319\,\text{kN}\end{aligned}$$

Solving a Concurrent Force System

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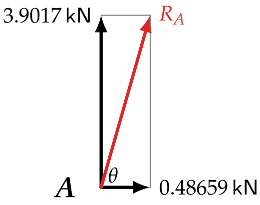

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$$\begin{aligned}\theta &= \tan^{-1}\left[\frac{R_{Ay}}{R_{Ax}}\right]\\ &= \tan^{-1}\left[\frac{3.9017\,\text{kN}}{0.48659\,\text{kN}}\right]\\ &= 82.891^\circ\end{aligned}$$

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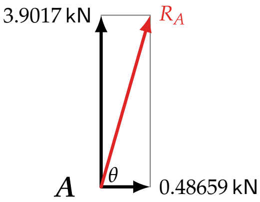

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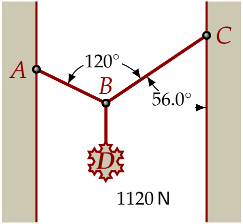
The reaction, R_A, at A is 3.93 kN at 82.9° (ccw from the positive x-axis).

Solving a Concurrent Force System

Example 2



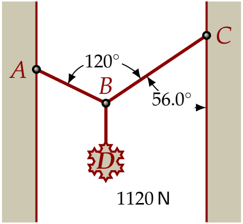
The forces in cables AB, BC and BD are concurrent; they act through the single particle/point at B.

Draw the *FBD*s for the forces acting upon particles A, B, C and D in the system shown.

Then solve for the tensions in the support cables AB, BC and BD, and the reactions at A and at C.

Solving a Concurrent Force System

Example 2



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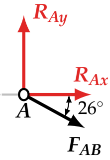

1. As mentioned before, it is good practice to draw the reaction components in the direction of the positive axes. Then, when our calculations are complete, if the result is positive (if $R_{Ay} > 0$), the reaction is in the positive direction. And, then, a negative result (if $R_{Ay} < 0$) will always indicate a reaction in the direction of the negative axis.

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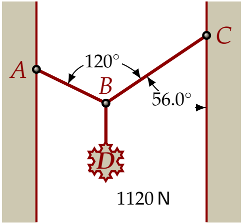

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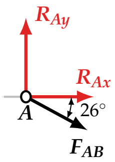

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2. F_{AB} drawn pointing away from A; the cable is 'pulling' on A and the cable is in tension. Again, it is good practice to always draw unknown forces in tension; then, when the result is positive it follows that the structural member is in tension. A negative result will indicate that member is in compression, 'pushing' on its supports. (Note that cables can only be in tension, never compression. Cables don't push!)

Solving a Concurrent Force System

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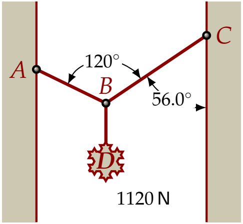

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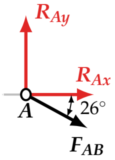

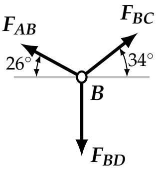

Solving a Concurrent Force System

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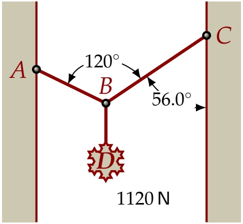

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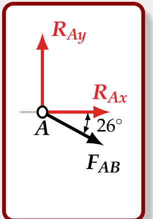

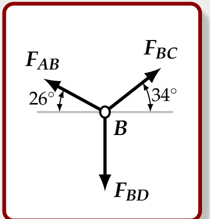

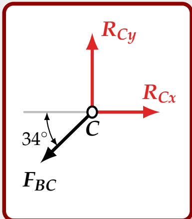

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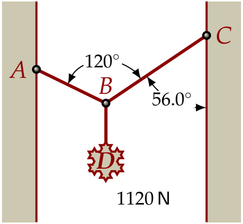

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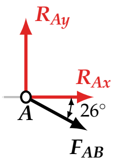
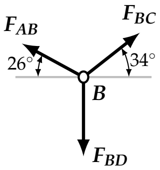
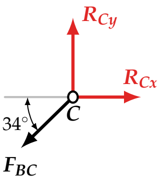
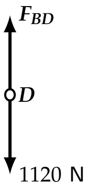


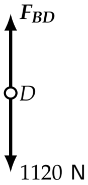






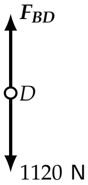
Example 2: Analyze **D**

Each of the *FBD*s for particles A, B and C has three unknowns so we cannot solve them yet. We start with D.



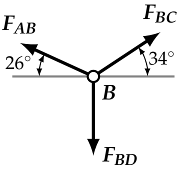
*Example 2: Analyze **D***

Each of the *FBD*s for particles A, B and C has three unknowns so we cannot solve them yet. We start with D.



$$\Sigma F_x = 0$$
$$\Sigma F_y = F_{BD} - 1120\,N = 0$$
$$\Rightarrow F_{BD} = 1120\,\text{N}$$

Example 2: Analyze ***B***

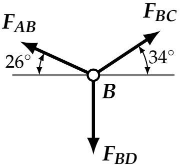


Now that we have found that $F_{BD} = 1120\,\text{N}$, there are only two unknowns at B and we can proceed. This analysis of particle B involves the solving of two simultaneous equations.

$$\Sigma F_x = F_{BC} \cdot \cos 34^\circ - F_{AB} \cdot \cos 26^\circ = 0$$

$$\Rightarrow F_{BC} = F_{AB} \cdot \frac{\cos 26^\circ}{\cos 34^\circ}$$

Example 2: Analyze *B*



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$$\Rightarrow F_{BC} = F_{AB} \cdot \frac{\cos 26^\circ}{\cos 34^\circ}$$

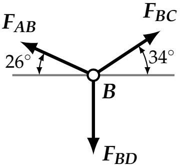
$$\Sigma F_y = F_{AB} \cdot \sin 26^\circ + F_{BC} \cdot \sin 34^\circ - 1120 \text{ N} = 0$$

$$\Rightarrow F_{AB} \cdot \sin 26^\circ + \left[F_{AB} \cdot \frac{\cos 26^\circ}{\cos 34^\circ} \right] \cdot \sin 34^\circ = 1120 \text{ N}$$

$$\Rightarrow F_{AB} \left[\sin 26^\circ + \cos 26^\circ \tan 34^\circ \right] = 1120 \text{ N}$$

$$\Rightarrow F_{AB} = 1072.2\text{N}$$

Example 2: Analyze B



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$$\Sigma F_y = F_{AB} \cdot \sin 26^\circ + F_{BC} \cdot \sin 34^\circ - 1120 \text{ N} = 0$$

$$\Rightarrow F_{AB} \cdot \sin 26^\circ + \left[F_{AB} \cdot \frac{\cos 26^\circ}{\cos 34^\circ}\right] \cdot \sin 34^\circ = 1120 \text{ N}$$

$$\Rightarrow F_{AB} \left[\sin 26^\circ + \cos 26^\circ \tan 34^\circ\right] = 1120 \text{ N}$$

$$\Rightarrow F_{AB} = 1072.2\text{N}$$

$$\Rightarrow F_{BC} = F_{AB} \cdot \frac{\cos 26^\circ}{\cos 34^\circ}$$

$$= 1072.2\text{N} \cdot \frac{\cos 26^\circ}{\cos 34^\circ}$$

$$= 1162.4\,\text{N}$$

Example 2: Analyze B

Now that we have found that $F_{BD} = 1120\,\text{N}$, there are only two unknowns at B and we can proceed. This analysis of particle B involves the solving of two simultaneous equations.

$$\Sigma F_x = F_{BC} \cdot \cos 34^\circ - F_{AB} \cdot \cos 26^\circ = 0$$

$$\Rightarrow F_{BC} = F_{AB} \cdot \frac{\cos 26^\circ}{\cos 34^\circ}$$

$$\Sigma F_y = F_{AB} \cdot \sin 26^\circ + F_{BC} \cdot \sin 34^\circ - 1120\ \text{N} = 0$$

$$\Rightarrow F_{AB} \cdot \sin 26^\circ$$

$$\Rightarrow$$

Or use the system-solver?

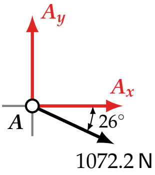
I recommend that you learn how to use the system-solver on your calculator to save time and reduce the chance of errors...

The two equations you enter are:

$$-\cos 26^\circ \cdot x + \cos 34^\circ \cdot y = 0$$
$$\sin 26^\circ \cdot x + \sin 34^\circ \cdot y = 1120\ \text{N}$$

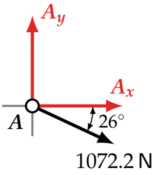
where x represents F_{AB} and y represents F_{BC}; x and are just algebraic variables used by the calculator and are not related to the x or y-axes. These equations are identical to the ones used for the simultaneous equation solution, with F_{AB} and F_{BC} relabeled x and y, and a little reordering of terms.

Example 2: Analyze A



$$
\begin{aligned}
\Sigma F_y &= A_y - F_{AB}\sin 26^\circ = 0 \\
\Rightarrow A_y &= F_{AB}\sin 26^\circ \\
&= (1072.2\,\mathrm{N})\sin 26^\circ \\
&= 470.02\,\mathrm{N}
\end{aligned}
$$

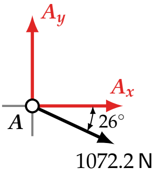
Example 2: Analyze $\boldsymbol{A}$



$$\begin{aligned} \Sigma F_y &= A_y - F_{AB}\sin 26^\circ = 0 \\ \Rightarrow A_y &= F_{AB}\sin 26^\circ \\ &= (1072.2\,\text{N})\sin 26^\circ \\ &= 470.02\,\text{N} \end{aligned}$$

$$\begin{aligned} \Sigma F_x &= A_x + F_{AB}\cos 26^\circ = 0 \\ \Rightarrow A_x &= -(1072.2\,\text{N})\cos 26^\circ \\ &= -963.69\,\text{N} \end{aligned}$$

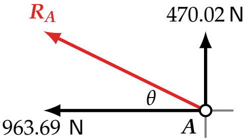
Example 2: Analyze A



$$\begin{aligned} \Sigma F_y &= A_y - F_{AB}\sin 26^\circ = 0 \\ \Rightarrow A_y &= F_{AB}\sin 26^\circ \\ &= (1072.2\,\mathrm{N})\sin 26^\circ \\ &= 470.02\,\mathrm{N} \end{aligned}$$

$$\begin{aligned} \Sigma F_x &= A_x + F_{AB}\cos 26^\circ = 0 \\ \Rightarrow A_x &= -(1072.2\,\mathrm{N})\cos 26^\circ \\ &= -963.69\,\mathrm{N} \end{aligned}$$

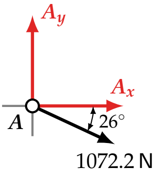
Now, find the resultant, R_A, of the two reaction components A_x and A_y:



$$\begin{aligned} R_A &= \sqrt{(470.02\,\mathrm{N})^2 + (963.69\,\mathrm{N})^2} \\ &= 1072.2\ \mathrm{N} \end{aligned}$$

$$\theta = \tan^{-1}\left(\frac{470.02}{963.69}\right) = 26.000^\circ$$

Example 2: Analyze A



$$
\begin{aligned}
\Sigma F_y &= A_y - F_{AB}\sin 26^\circ = 0 \\
\Rightarrow A_y &= F_{AB}\sin 26^\circ \\
&= (1072.2\,\mathrm{N})\sin 26^\circ \\
&= 470.02\,\mathrm{N}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_x &= A_x + F_{AB}\cos 26^\circ = 0 \\
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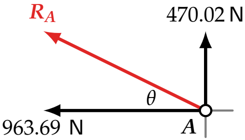
&= -963.69\,\mathrm{N}
\end{aligned}
$$

Now, find the resultant, R_A, of the two reaction components A_x and A_y:

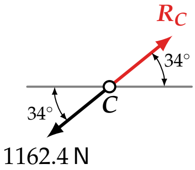


$$
\begin{aligned}
R_A &= \sqrt{(470.02\,\mathrm{N})^2 + (963.69\,\mathrm{N})^2} \\
&= 1072.2\ \mathrm{N}
\end{aligned}
$$

$$
\theta = \tan^{-1}\left(\frac{470.02}{963.69}\right) = 26.000^\circ
$$

R_A is equal and opposite to F_{AB}, which shouldn't be a surprise.

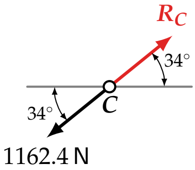
Example 2: Analyze C



Based on our result for the reaction at A, it follows that R_C is equal and opposite to F_{BC}.

R_C is 1024 N at 34°, measured counter-clockwise from the positive x-axis.

Example 2: Analyze *C*



Based on our result for the reaction at A, it follows that R_C is equal and opposite to F_{BC}.

R_C is 1024 N at 34°, measured counter-clockwise from the positive x-axis.

Example 2: The Answers

All rounded to three significant digits:

$$F_{AB} = 1070\,\text{N},\ F_{BC} = 1160\,\text{N},\ F_{BD} = 1120\,\text{N}$$

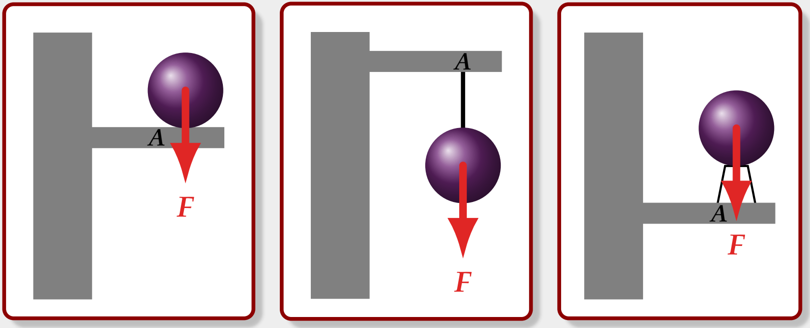
$$R_A = 1070\,\text{N at } 154^\circ \text{ ccw from the positive } x\text{-axis}$$

$$R_C = 1160\,\text{N at } 34^\circ \text{ ccw from the positive } x\text{-axis}$$

Notice that tension (and compression) do not have a direction. Is the tension in AB from A to B or from B to A? Tension and compression are scalar values.

Force Transmissability

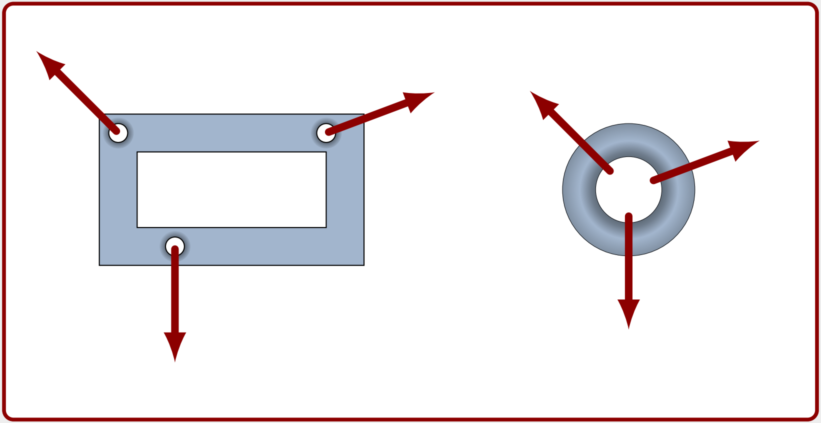
- The **line of action** of a force is the line along the force direction, extended infinitely in both directions.
- The effect of a force on a body is the same wherever the force is applied along its line of action.



- All these loadings have the same effect on the beam at A.

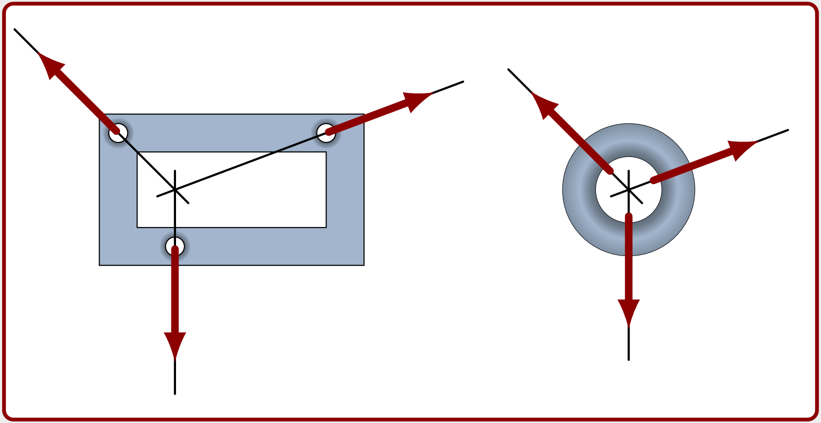
Force Transmissability and Concurrency

- If the lines of action of forces go through a single point (or particle), then the forces are concurrent.
- Each of the force systems shown is a concurrent system.



Force Transmissability and Concurrency

- If the lines of action of forces go through a single point (or particle), then the forces are concurrent.
- Each of the force systems shown is a concurrent system. Their force lines of action intersect at a single point.



Exercise 1

Determine the reaction at A.

(R = 14.9 kN at 2.91° cw from positive x axis.)