06 Equilibrium of Rigid Bodies

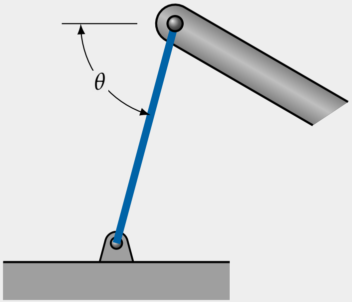
Engineering Statics

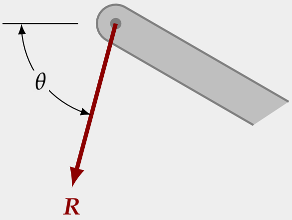
Updated on: September 26, 2025

Overview

- Much of the study of statics involves the calculations of reaction forces generated between a structural body and its supports when loads are applied.
- There are various connection types used between a structural body and its supports. These connections influence the direction and the sense of the reaction. We shall examine some of these connections now.

Types of Connections: Cable



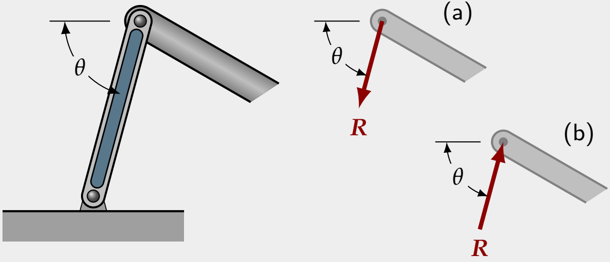


When a structural member or body is supported by a cable (or rope or chain), the cable is assumed to be weightless (and consequently straight) and the cable exerts a reaction on the structural member **in the same direction** as the cable.

A cable is in tension and can only **pull**; it cannot push.

There is only one unknown: the magnitude of the force. The direction and sense of the force are known.

Types of Connections: Strut

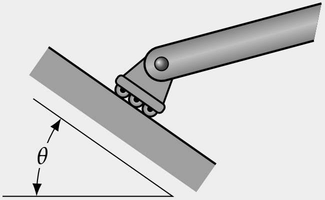


Like a cable, a straight strut (or link) exerts a reaction on a structural member in the direction of the strut. Unlike a cable, a strut can pull or push.

If we don't know whether a strut is pushing or pulling, we generally assume that the reaction is directed away from the structural member (pulling, in tension), as shown in (a). If our calculations then determine that F is negative, the direction is opposite to our assumption (i.e., pushing) (b).

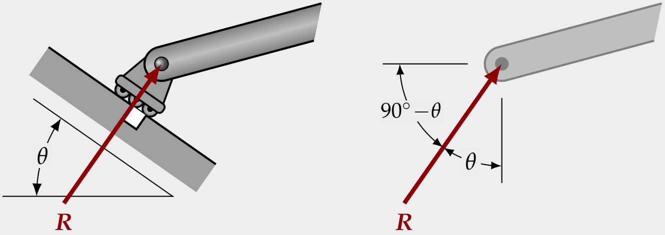
As with the cable, there is only one unknown. The sign of F determines the sense of F.

Types of Connections: Roller



A roller (assumed weightless and frictionless) can provide no resistance **along** the slope on which it is resting.

Types of Connections: Roller

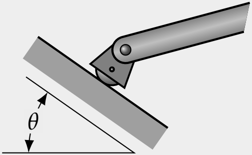


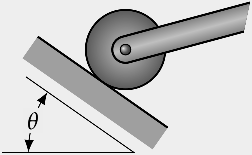
A roller (assumed weightless and frictionless) can provide no resistance **along** the slope on which it is resting. The only reaction a roller can provide is **perpendicular** to the slope.

A roller can only push. It is not fixed to the sloped surface and would lift off the surface if pulled.

As with the cable, there is only one unknown. If your math is correct, the sign of F should always be positive.

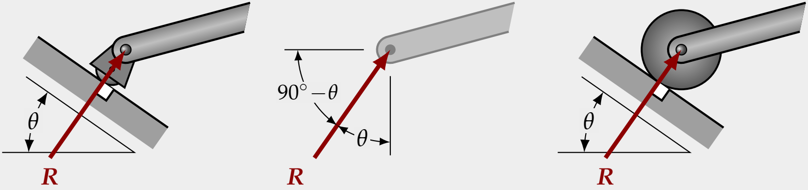
Types of Connections: More Rollers





Rollers come in different shapes. But they all react in the same way.

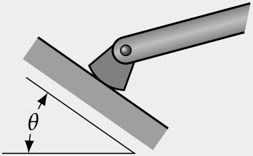
Types of Connections: More Rollers

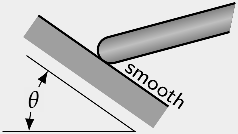


Rollers come in different shapes. But they all react in the same way.

The reaction force is perpendicular to the surface that the roller bears on.

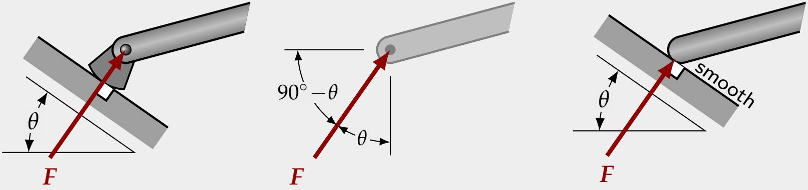
Types of Connections: Rockers and Smooth Surfaces





Two more connection types that react in the same way as rollers:

Types of Connections: Rockers and Smooth Surfaces

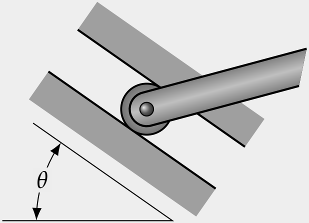


Two more connection types that react in the same way as rollers:

The **rocker**: the reaction force is perpendicular to the surface that the rocker bears on.

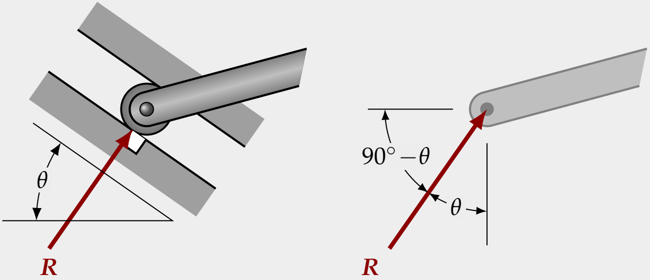
The **smooth surface** has no friction along the surface: the reaction force is perpendicular to the smooth surface that the structural member bears on.

Types of Connections: Roller in Smooth Slot



With a roller in a smooth slot, we don't necessarily know whether the roller is:

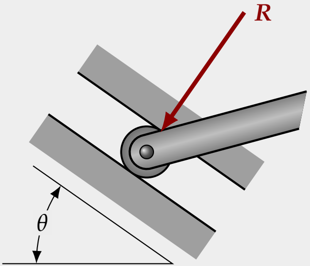
Types of Connections: Roller in Smooth Slot

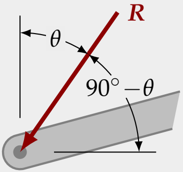


With a roller in a smooth slot, we don't necessarily know whether the roller is:

- Bearing on the lower smooth surface; in this case it is the lower surface that provides the reaction.

Types of Connections: Roller in Smooth Slot





With a roller in a smooth slot, we don't necessarily know whether the roller is:

- Bearing on the lower smooth surface; in this case it is the lower surface that provides the reaction.
- Bearing on the upper smooth surface; in this case it is the upper surface that provides the reaction.

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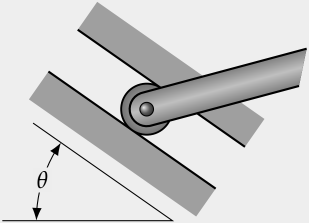
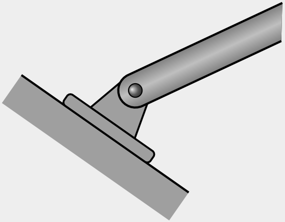


With a roller in a smooth slot, we don't necessarily know whether the roller is:

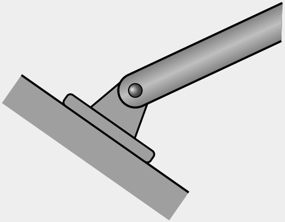
- Bearing on the lower smooth surface; in this case it is the lower surface that provides the reaction.
- Bearing on the upper smooth surface; in this case it is the upper surface that provides the reaction.
- In each case, the reaction is perpendicular to the bearing surface of the slot.

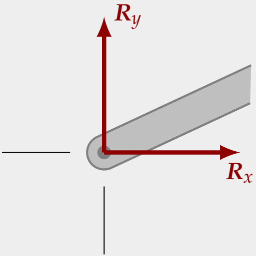
Types of Connections: Pinned Connection



With the pinned connection (also known as a hinged connection), movement is restricted in all directions. The reaction can be in any direction.

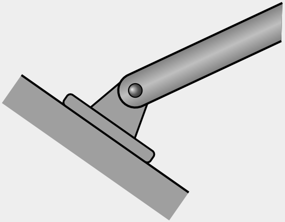
Types of Connections: Pinned Connection

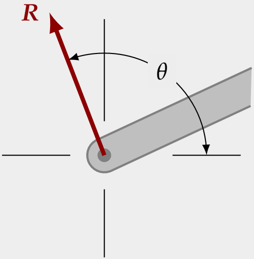




With the pinned connection (also known as a hinged connection), movement is restricted in all directions. The reaction can be in any direction.

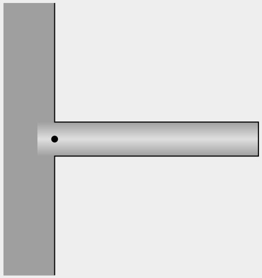
- This gives us **two** unknowns: one generally an x-component and one generally a y-component.





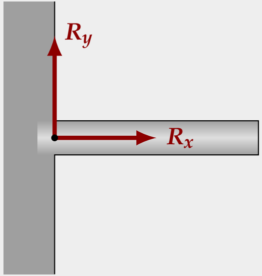
With the pinned connection (also known as a hinged connection), movement is restricted in all directions. The reaction can be in any direction.

- This gives us **two** unknowns: one generally an x-component and one generally a y-component.
- Alternatively (and equivalently), the reaction can be specified by a magnitude and a direction. This is the form that we use to describe the reaction.



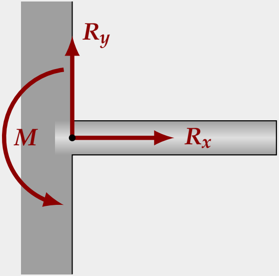
The fixed connection has **three** unknowns.

Types of Connections: Fixed Connection



The fixed connection has **three** unknowns.

- Components in the x and y directions (like the pinned connection).



The fixed connection has **three** unknowns.

- Components in the x and y directions (like the pinned connection).
- A reacting moment, M, since (unlike the pinned connection) there can be no rotation about the connection.

Equilibrium: The Rules

- Here we add another major part of the puzzle: the third and final equation that, combined with our results from the equilibrium of concentric forces, allows us analyze the equilibrium of rigid bodies.
- In our discussion of force couples, we noted that although $\Sigma F_x = 0$ and $\Sigma F_y = 0$ there was still some tendency for the force couple to cause rotation.
- For a system to be in equilibrium, there must be no net moment: the sum of the moments of the forces acting upon a rigid body must be zero. There must be no tendency to rotate.

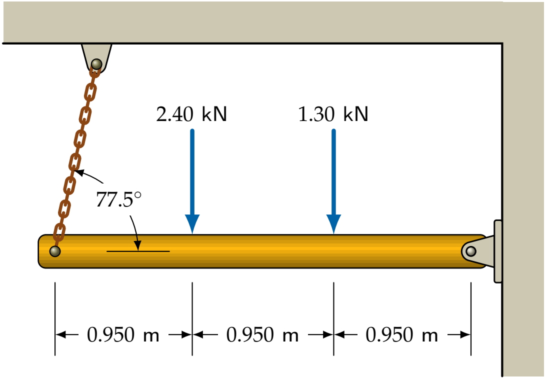
General Conditions for Equilibrium of Rigid Bodies

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M = 0$$

In Practice:

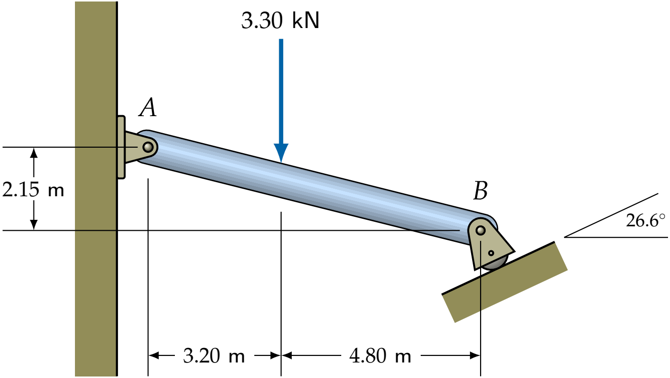
- In general, we use these equations to find the reactions at connections between a rigid body (not just a particle) and a support.
- Often, these support reactions will have three unknowns: a pinned connection with two unknown components; and a connection with a known direction (roller, cable, two-force member, etc.) but an unknown magnitude (and, possibly, unknown sense).
- Usually, our first step after drawing the FBD **(!)** is to take moments about the pinned connection to solve directly for the third unknown (the magnitude of the connection with the known direction).
- Then we sum the x-components of all the forces involved to solve for the x-component of the reaction at the pinned connection. We do the same for the y-component.
- From the components of the reaction at the pinned connection, R_x and R_y, we use the Pythagorean Theorem and the inverse tangent function to find the magnitude R and direction θ of the reaction.

Example 1



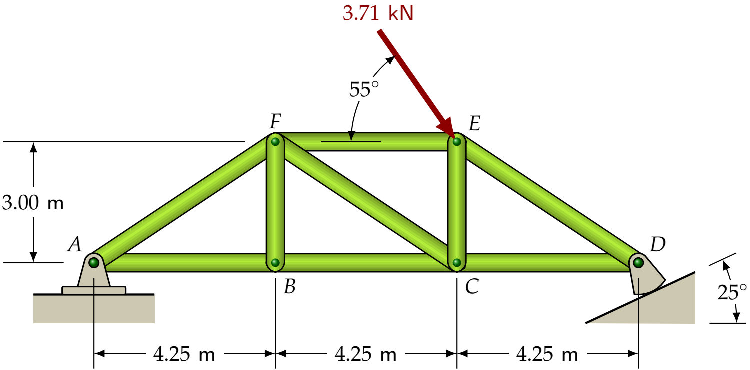
Determine the tension in the chain and the reaction at A.

Example 2



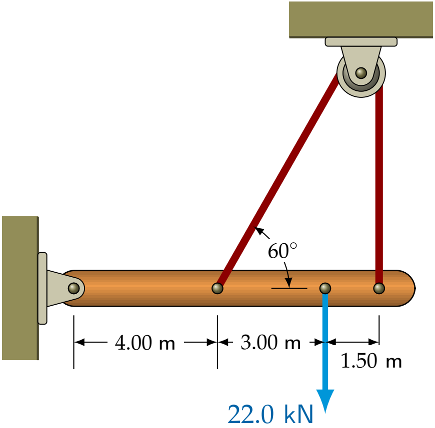
Determine the reactions at A and at B.

Exercise 1



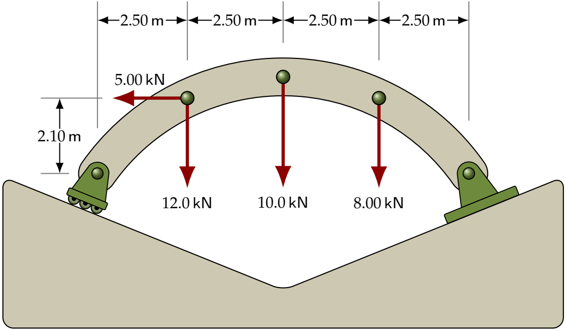
Determine the reactions at A and D.

Exercise 2



Determine the reactions at the pinned connection and the tension in the cable.

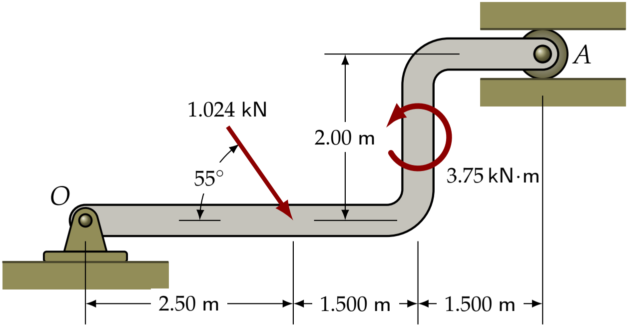
Example 3



The roller and the pinned connection are on slopes inclined at 21° to the horizontal; they are both at the same elevation.

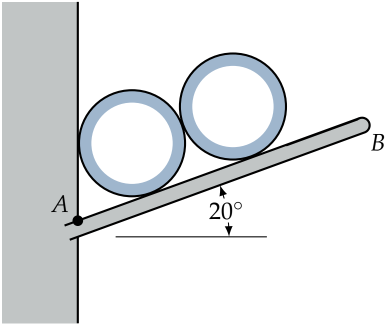
Determine the reaction at each connection. Indicate direction by measuring counter-clockwise from the positive x-axis.

Example 4



The roller at B is in a smooth slot.
Determine the reactions at A and B.

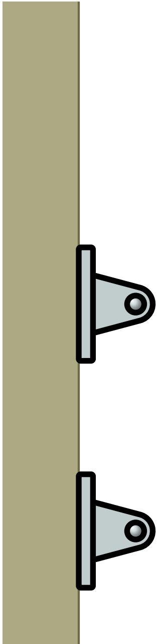
Exercise 3



Pipe racks (AB, and two hidden behind it) support two smooth Schedule 40 pipes, with an outside diameter of $508\,\text{mm}$, as shown. The pipes are $10\,\text{m}$ in length with a mass of $78.5\,\text{kg/m}$, Each rack supports one-third of the weight of each pipe.

Determine the reaction at the fixed connection A.

Example 5



Notes:

1. For a body in equilibrium, $\Sigma M = 0$ for moments summed about **any** point in the plane containing the body.

2. We used all the three equations of statics to solve for the three unknowns. It would also have been possible to solve using $\Sigma M = 0$ three times by choosing appropriate points about which to take moments. There are occasions when it is more convenient to sum moments two or three times, rather than summing components. (We will see this when using the Method of Sections to analyze forces in truss members.)

3. Since there are only three equations for statics, we can only solve a system for three unknowns. Typically, two unknowns come from a pinned connection and the third from a cable, roller or some other connection whose reaction direction is known.

4. We start the solution by taking moments about the pinned connection - this is so that we don't need to consider the moments of the pinned connection unknowns (e.g., R_x and R_y) and the moment equation will immediately give a solution for the third unknown.