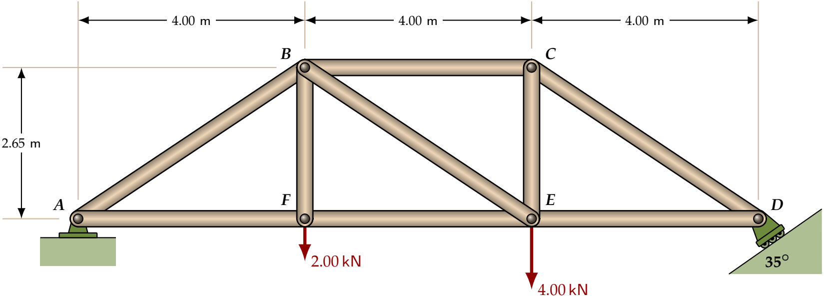
Complex Frames — Step by Step Examples

Engineering Statics

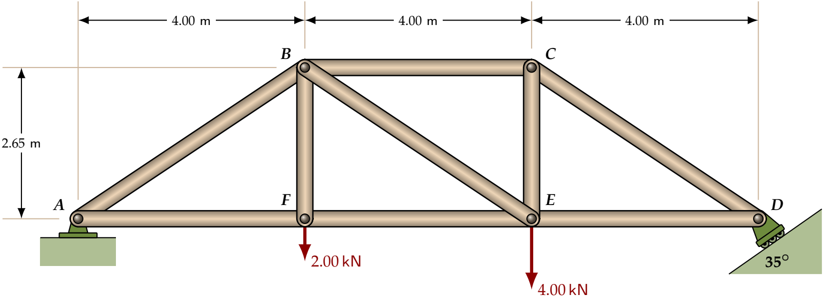
Last revision on August 11, 2025



Method of Joints: Example 1

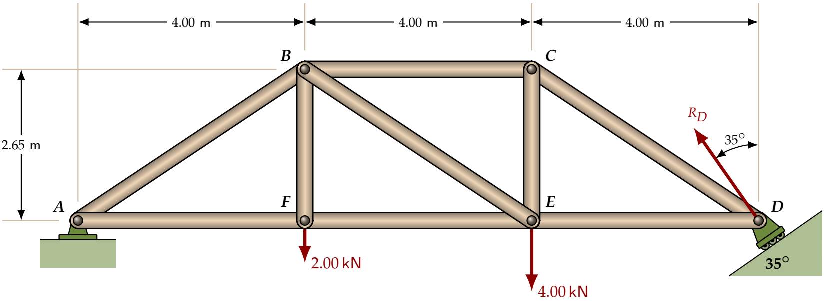
The truss is supported by a pinned connection at A and a roller, inclined at 35° to the horizontal, at D.

Determine the internal force in each truss member due to the applied loads at E and F.



Find the reaction at D

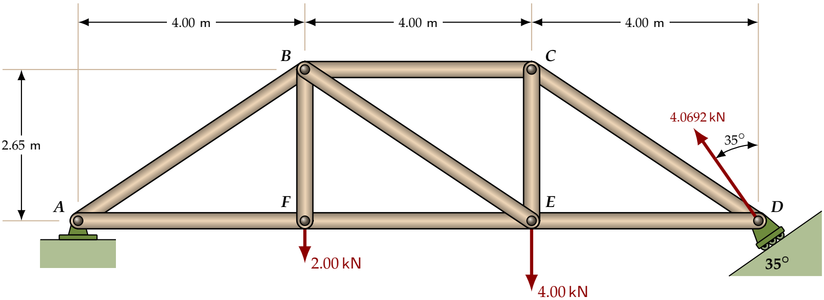
Take moments of the external forces acting on the truss, about A:



Find the reaction at D

Take moments of the external forces acting on the truss, about A:

$$\sum M_A = R_D \cos 35^\circ \times 12.0\,\mathrm{m} - 2.00\,\mathrm{kN} \times 4.00\,\mathrm{m} - 4.00\,\mathrm{kN} \times 8.00\,\mathrm{m} = 0$$

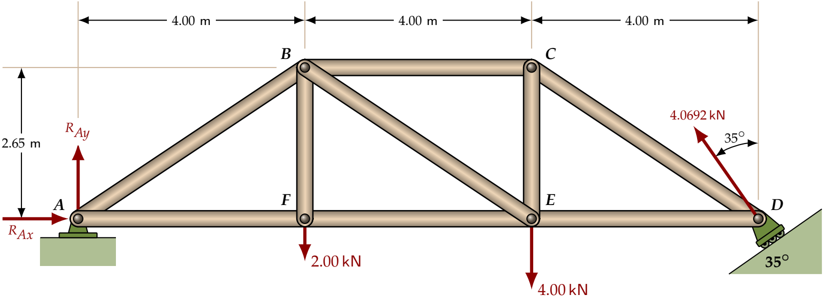


Find the reaction at D

Take moments of the external forces acting on the truss, about A:

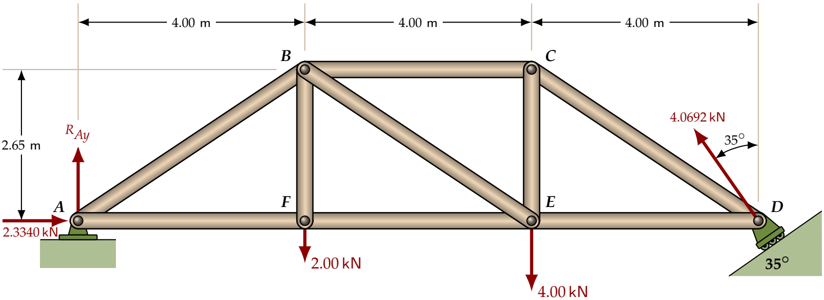
$$\sum M_A = R_D \cos 35^\circ \times 12.0\,\text{m} - 2.00\,\text{kN} \times 4.00\,\text{m} - 4.00\,\text{kN} \times 8.00\,\text{m} = 0$$

$$\Rightarrow R_D = \frac{40.0\,\text{kN}\cdot\text{m}}{12.0\,\text{m} \times \cos 35^\circ}$$
$$= 4.0692\,\text{kN}$$



Find the reaction at A

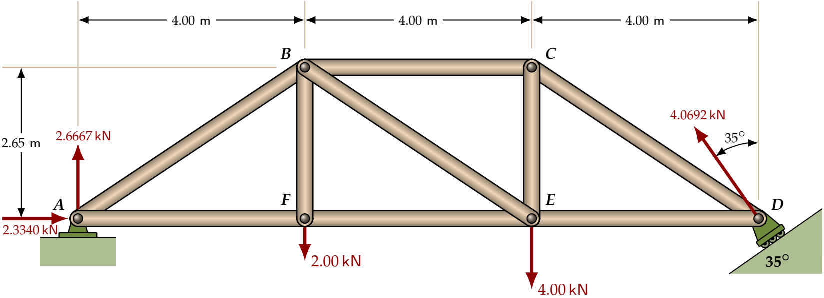
Note: We could proceed to find all the forces in the truss members, working from D back to A, **without** finding the reaction at A. But the reaction at A is useful for a check – at the end of the problem – to make sure that we haven't made any errors along the way.



Find the reaction at A

$$\sum F_x = R_{Ax} - 4.0692 \sin 35^\circ \text{ kN} = 0$$

$$\Rightarrow R_{Ax} = 2.3340 \text{ kN}$$



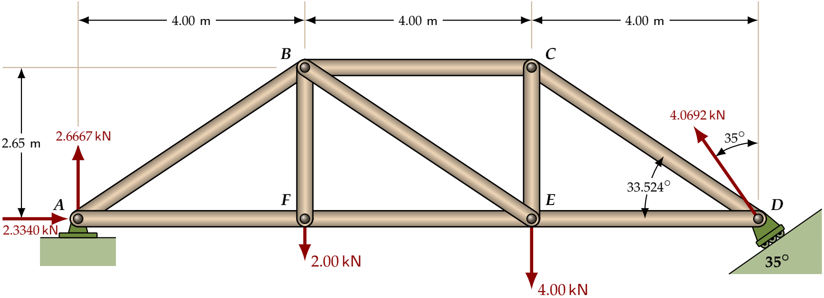
Find the reaction at A

$$\sum F_x = R_{Ax} - 4.0692 \sin 35^\circ \text{ kN} = 0$$

$$\Rightarrow R_{Ax} = 2.3340 \text{ kN}$$

$$\sum F_y = R_{Ay} + 4.0692 \cos 35^\circ \text{ kN} - 2.00 \text{ kN} - 4.00 \text{ kN} = 0$$

$$\Rightarrow R_{Ay} = 2.6667 \text{ kN}$$

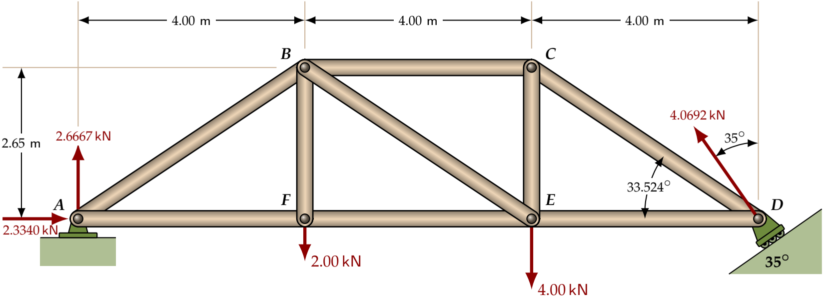


Find the truss angle

$$\angle CDE = \tan^{-1}\left[\frac{CE}{DE}\right]$$

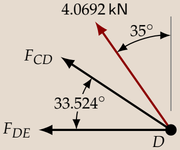
$$= \tan^{-1}\left[\frac{2.65\,\text{m}}{4.00\,\text{m}}\right]$$

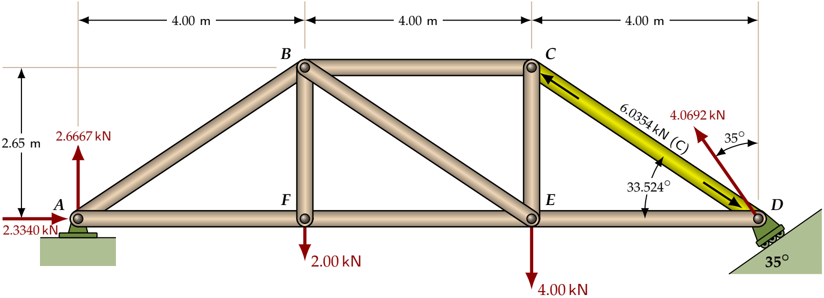
$$= 33.524^\circ$$



Joint D

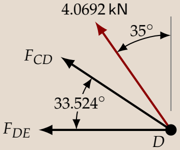
First, the free body diagram:





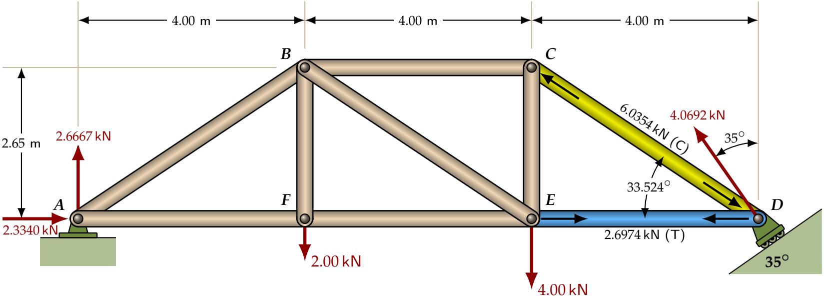
Joint D

First, the free body diagram:



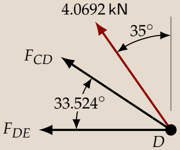
$$\sum F_y = 4.0692 \cos 35^\circ \text{ kN} + F_{CD} \sin 33.524^\circ = 0$$

$$\Rightarrow \boldsymbol{F_{CD} = -6.0354\,\text{kN}}$$



Joint D

First, the free body diagram:



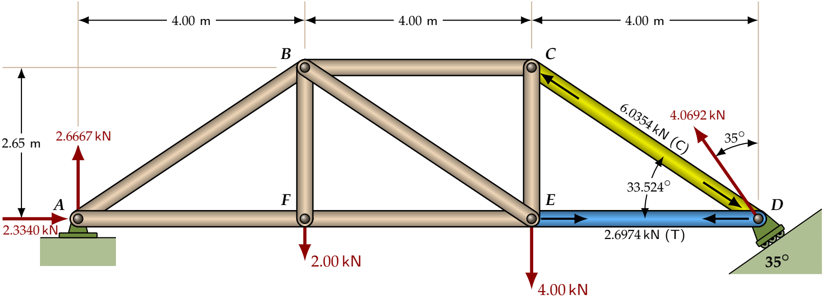
$$\sum F_y = 4.0692 \cos 35^\circ \text{ kN} + F_{CD} \sin 33.524^\circ = 0$$

$$\Rightarrow \boldsymbol{F_{CD} = -6.0354\ \text{kN}}$$

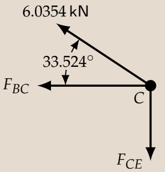
$$\sum F_x = -4.0692 \sin 35^\circ \text{ kN} - F_{CD} \cos 33.524^\circ - F_{DE} = 0$$

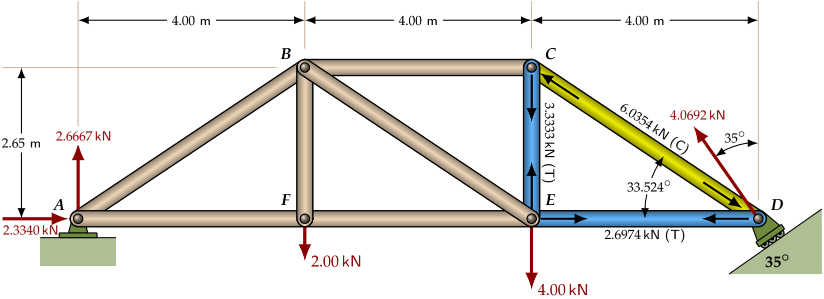
$$= -2.3340 \text{ kN} - (-6.0354 \text{ kN}) \cos 33.524^\circ - F_{DE} = 0$$

$$\Rightarrow \boldsymbol{F_{DE} = 2.6974\ \text{kN}}$$

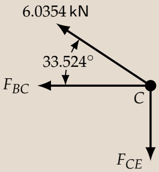

4.00 m
4.00 m
4.00 m
B
C
2.65 m
2.6667 kN
6.0354 kN (C)
4.0692 kN
35°
33.524°
A
F
E
D
2.3340 kN
2.6974 kN (T)
2.00 kN
35°
4.00 kN

Joint C


6.0354 kN
33.524°
F_{BC}
C
F_{CE}

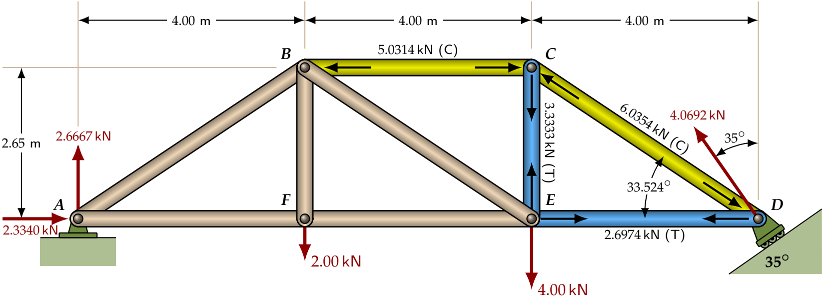


Joint C

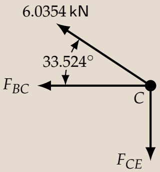


$$\sum F_y = 6.0354 \sin 33.524° \text{ kN} - F_{CE} = 0$$

$$\Rightarrow \boldsymbol{F_{CE} = 3.3333\ \mathrm{kN}}$$



Joint C

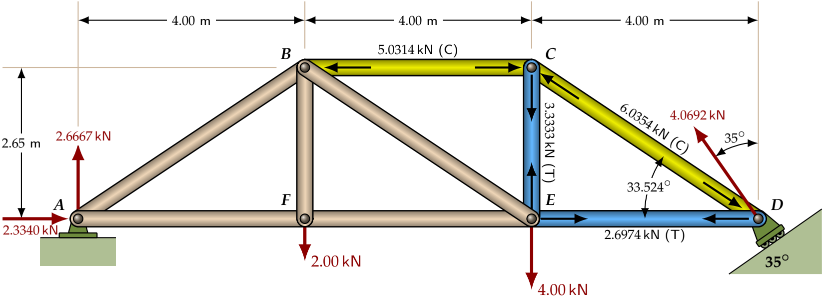


$$\sum F_y = 6.0354 \sin 33.524^\circ \text{ kN} - F_{CE} = 0$$

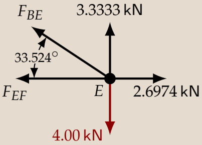
$$\Rightarrow \boldsymbol{F_{CE} = 3.3333\ \text{kN}}$$

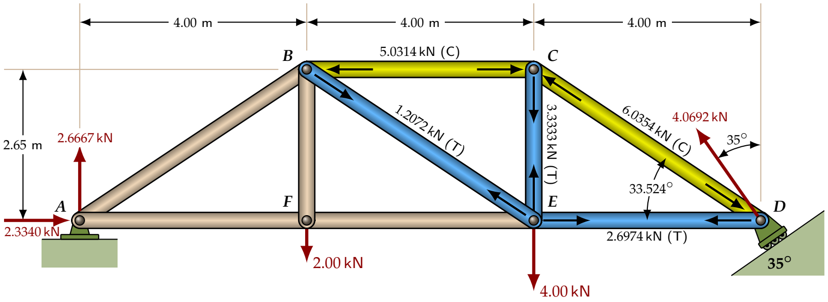
$$\sum F_x = -6.0354 \cos 33.524^\circ \text{ kN} - F_{BC} = 0$$

$$\Rightarrow \boldsymbol{F_{BC} = -5.0314}\ \text{kN}$$

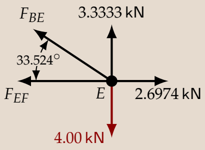

4.00 m
4.00 m
4.00 m
2.65 m
A
B
C
D
E
F
5.0314 kN (C)
6.0354 kN (C)
3.3333 kN (T)
2.6974 kN (T)
2.6667 kN
2.3340 kN
4.0692 kN
35°
33.524°
2.00 kN
4.00 kN
35°

Joint E


F_{BE}
3.3333 kN
33.524°
F_{EF}
E
2.6974 kN
4.00 kN

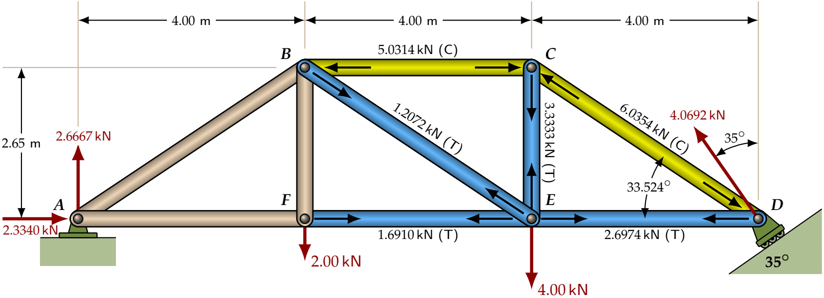


Joint E

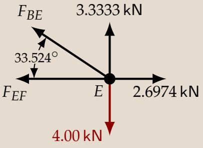


$$\sum F_y = 3.3333\,\text{kN} + F_{BE}\sin 33.524^\circ\,\text{kN} - 4.00\,\text{kN} = 0$$

$$\Rightarrow \boldsymbol{F_{BE} = 1.2072\,\text{kN}}$$



Joint E

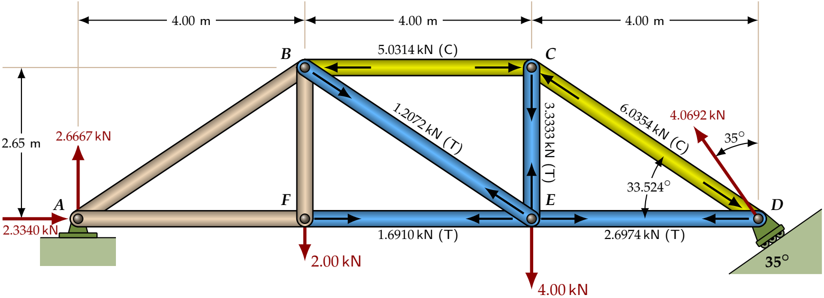


$$\sum F_y = 3.3333\,\text{kN} + F_{BE}\sin 33.524^\circ\,\text{kN} - 4.00\,\text{kN} = 0$$

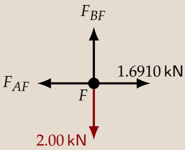
$$\Rightarrow \boldsymbol{F_{BE} = 1.2072\,\text{kN}}$$

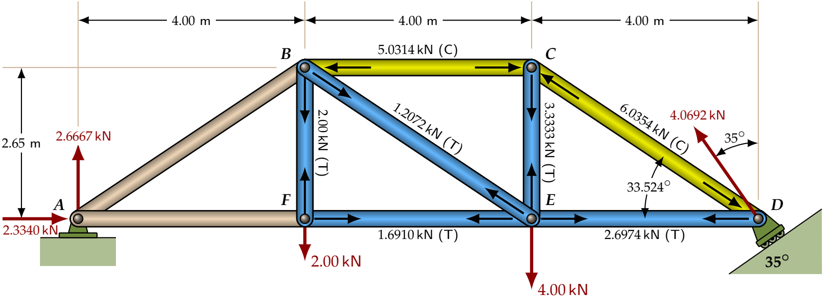
$$\sum F_x = 2.6974\,\text{kN} - 1.2072\cos 33.524^\circ\,\text{kN} - F_{EF} = 0$$

$$\Rightarrow \boldsymbol{F_{EF} = 1.6910\,\text{kN}}$$

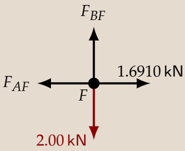

4.00 m
4.00 m
4.00 m
B
5.0314 kN (C)
C
2.65 m
2.6667 kN
1.2072 kN (T)
3.3333 kN (T)
6.0354 kN (C)
4.0692 kN
35°
33.524°
A
F
E
D
2.3340 kN
1.6910 kN (T)
2.6974 kN (T)
2.00 kN
4.00 kN
35°

Joint F


F_{BF}
F_{AF}
1.6910 kN
F
2.00 kN

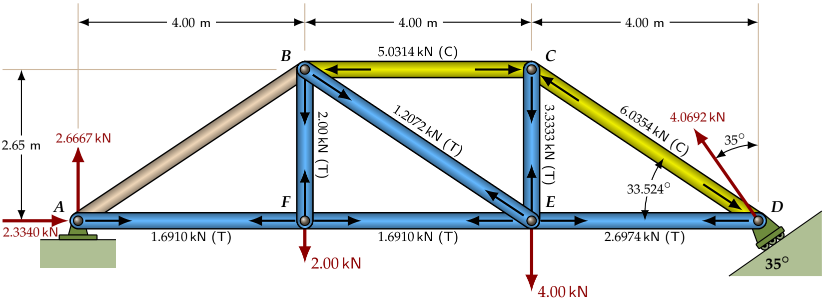


Joint F

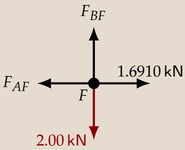


$$\sum F_y = F_{BF} - 2.00\,\text{kN}$$

$$\Rightarrow \boldsymbol{F_{BF} = 2.00\,\text{kN}}$$



Joint F

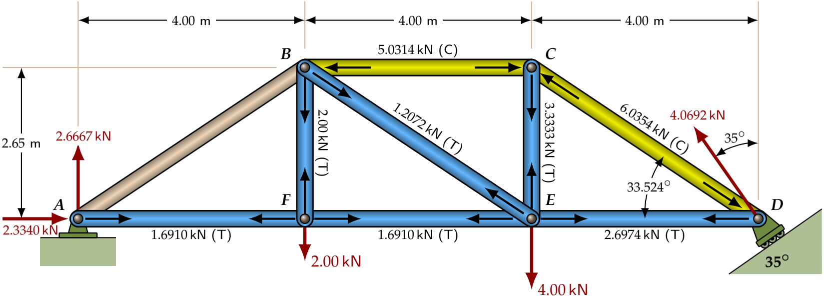


$$\sum F_y = F_{BF} - 2.00\,\text{kN}$$

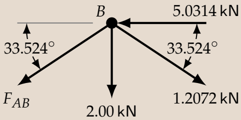
$$\Rightarrow \boldsymbol{F_{BF} = 2.00\,\text{kN}}$$

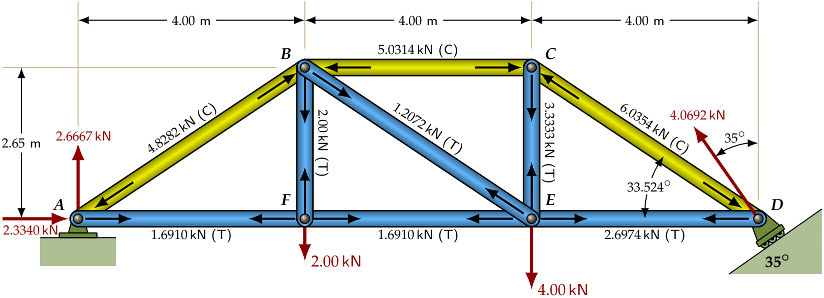
$$\sum F_x = 1.6910\,\text{kN} - F_{AF} = 0$$

$$\Rightarrow \boldsymbol{F_{AF} = 1.6910\,\text{kN}}$$

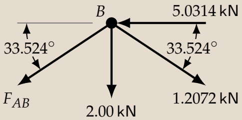

4.00 m
4.00 m
4.00 m
B
C
5.0314 kN (C)
2.65 m
2.6667 kN
1.2072 kN (T)
2.00 kN (T)
3.3333 kN (T)
6.0354 kN (C)
4.0692 kN
35°
33.524°
A
F
E
D
2.3340 kN
1.6910 kN (T)
1.6910 kN (T)
2.6974 kN (T)
2.00 kN
4.00 kN
35°

Joint B


B
5.0314 kN
33.524°
33.524°
F_{AB}
2.00 kN
1.2072 kN

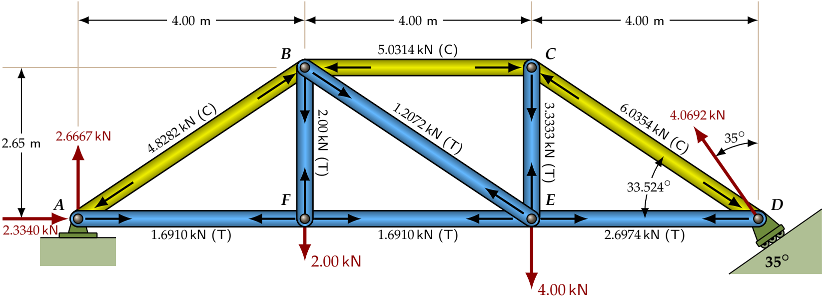


Joint B

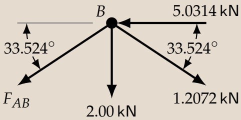


$$\sum F_x = -5.0314\,\text{kN} + 1.2072\cos 33.524^\circ\,\text{kN} - F_{AB}\cos 33.524^\circ = 0$$

$$\Rightarrow \boldsymbol{F_{AB}} = \mathbf{-4.8282}\,\text{kN}$$



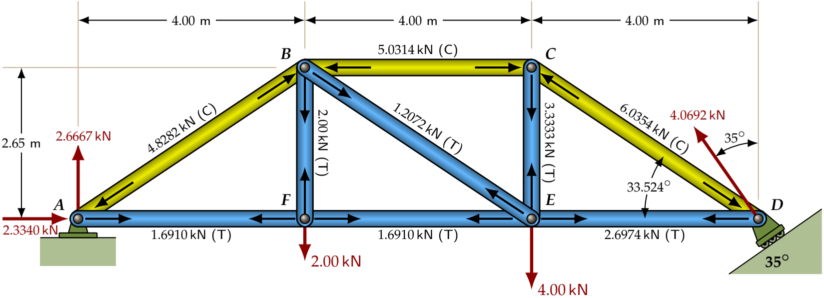
Joint B



$$\sum F_x = -5.0314\,\text{kN} + 1.2072\cos 33.524^\circ\,\text{kN} - F_{AB}\cos 33.524^\circ = 0$$

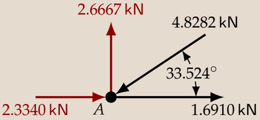
$$\Rightarrow \boldsymbol{F_{AB}} = \mathbf{-4.8282}\,\text{kN}$$

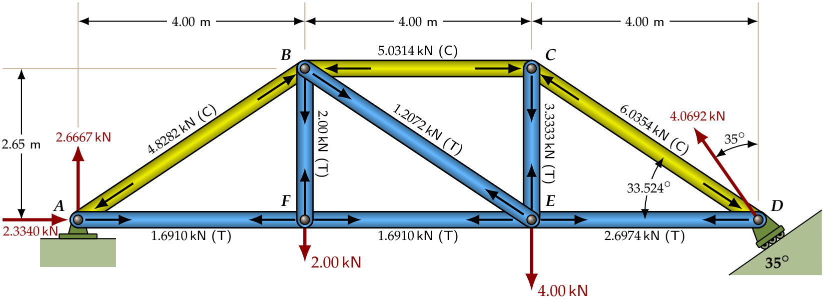
All the truss member forces are now found.



Check for equilibrium at A

This is to verify that we haven't made an error in our member force calculations.





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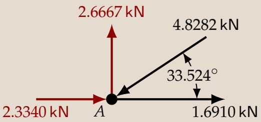
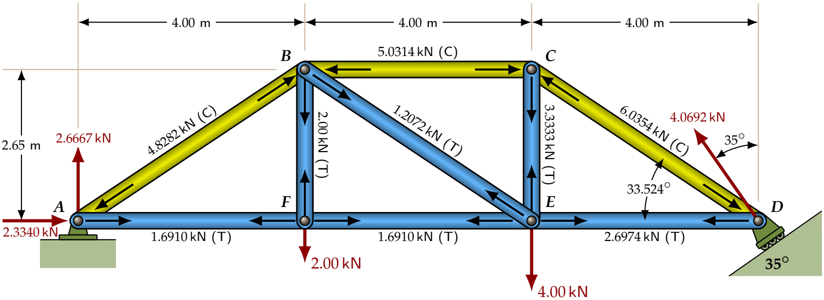
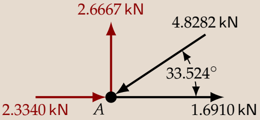


$$\sum F_x = 2.3340\,\text{kN} + 1.6910\,\text{kN} - 4.8282\cos 33.524^\circ\,\text{kN}$$
$$= -0.000050918\,\text{kN} \approx 0 \quad \checkmark$$



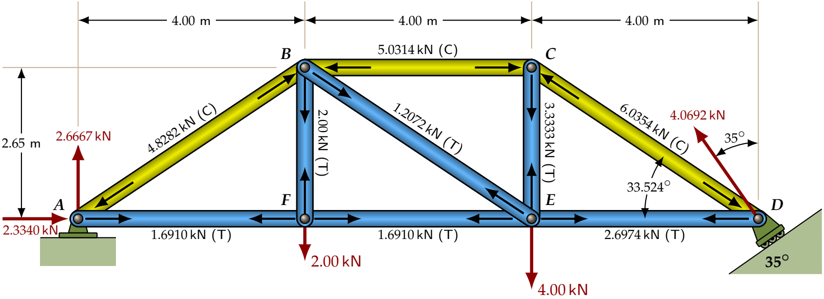
Check for equilibrium at A

This is to verify that we haven't made an error in our member force calculations.



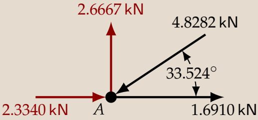
$$\sum F_x = 2.3340\,\text{kN} + 1.6910\,\text{kN} - 4.8282\cos 33.524^\circ\,\text{kN}$$
$$= -0.000050918\,\text{kN} \approx 0 \quad \checkmark$$

$$\sum F_y = 2.6667\,\text{kN} - 4.8282\sin 33.524^\circ\,\text{kN}$$
$$= 0.00015160\,\text{kN} \approx 0 \quad \checkmark$$



Check for equilibrium at A

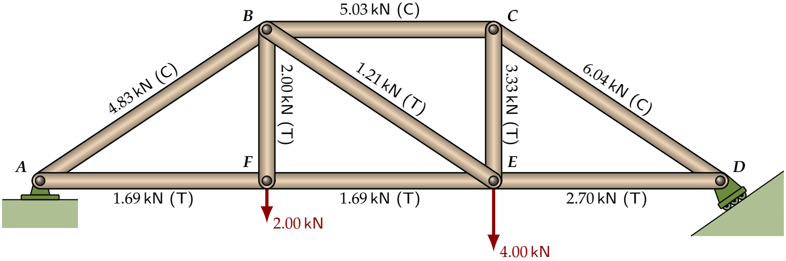
This is to verify that we haven't made an error in our member force calculations.



$$\sum F_x = 2.3340\,\text{kN} + 1.6910\,\text{kN} - 4.8282\cos 33.524^\circ\,\text{kN}$$
$$= -0.000050918\,\text{kN} \approx 0 \quad \checkmark$$

$$\sum F_y = 2.6667\,\text{kN} - 4.8282\sin 33.524^\circ\,\text{kN}$$
$$= 0.00015160\,\text{kN} \approx 0 \quad \checkmark$$

It only remains to convert the results back to the precision given by the input values.



The Results

$$
\begin{aligned}
AB &= 4.83\,\text{kN} \quad &(\text{Compression})\\
AF &= 1.69\,\text{kN} \quad &(\text{Tension})\\
BC &= 5.03\,\text{kN} \quad &(\text{Compression})\\
BE &= 1.21\,\text{kN} \quad &(\text{Tension})\\
BF &= 2.00\,\text{kN} \quad &(\text{Tension})\\
CD &= 6.04\,\text{kN} \quad &(\text{Compression})\\
CE &= 3.33\,\text{kN} \quad &(\text{Tension})\\
DE &= 2.70\,\text{kN} \quad &(\text{Tension})\\
EF &= 1.69\,\text{kN} \quad &(\text{Tension})
\end{aligned}
$$