01 Math Review

Engineering Statics, STCS 200

Updated on: August 14, 2025

Statics and Math

- Statics is all math! All but the most trivial statics problems require algebra and/or trigonometry and/or geometry to solve.
- The good news is that the math is not very difficult. You won't need anything more advanced than high-school math.
- We will do a quick review here that should cover all the math you'll need for STCS 200.

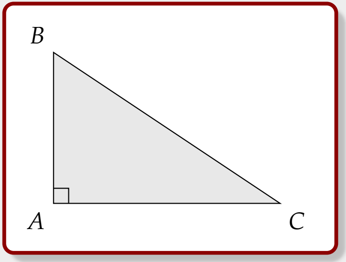
Triangles are a strong, stable shape and often used in engineering.

Triangles help avoid issues like this:

Triangles mean we need trigonometry.

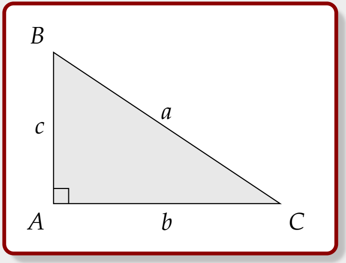
Right Triangle

A **right triangle** is a triangle having one 90° angle.



Right Triangle

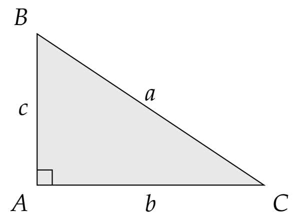
A **right triangle** is a triangle having one 90° angle.



Label the three sides a, b and c. The side a, opposite the right angle, is called the **hypotenuse**.

Right Triangle

A **right triangle** is a triangle having one 90° angle.



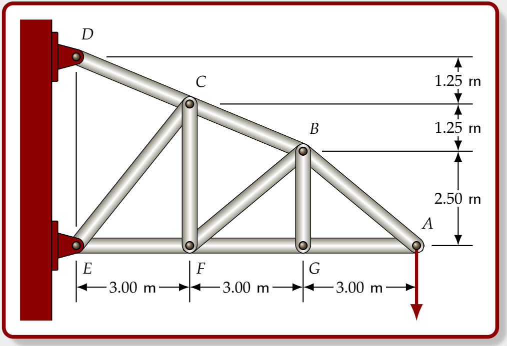
Label the three sides a, b and c. The side a, opposite the right angle, is called the **hypotenuse**.

If we know the lengths of any two sides, we can calculate the length of the third side using the **Pythagorean Theorem**:

Pythagorean Theorem

$$a^2 = b^2 + c^2$$

Right Triangle Exercises (1)



1. Use the Pythagorean Theorem to determine the lengths of CE and CB

Significant Digits

- Significant **digits** and significant **figures** are the same thing.

Significant Digits

- Significant **digits** and significant **figures** are the same thing.
- Significant digits are a measure of the **accuracy** of a number.

Significant Digits

- Significant **digits** and significant **figures** are the same thing.
- Significant digits are a measure of the **accuracy** of a number.

  It is **extremely important** to recognize that we can get no more accuracy out of a calculation than we put in. If the inputs to a problem have three significant digits, we cannot expect any higher accuracy than three significant digits in our result — even if the calculator does give ten digits.

Significant Digits

Non-zero digits

Non-zero digits **are** significant:

- 1234 has 4 significant digits.
- 12.34 has 4 significant digits.

Significant Digits

Non-zero digits

Non-zero digits **are** significant:

- 1234 has 4 significant digits.
- 12.34 has 4 significant digits.

Zeros between non-zero digits are significant

- 12034 has 5 significant digits.
- 12.0034 has 6 significant digits.

Significant Digits

Leading zeros are not significant

- 0.1234 has 4 significant digits.
- 0.0001234 has 4 significant digits.

Significant Digits

Leading zeros are not significant

- 0.1234 has 4 significant digits.
- 0.0001234 has 4 significant digits.

Trailing zeros (after a decimal point) are significant

- 1234.0 has 5 significant digits.
- 1.23400 has 6 significant digits.

Trailing zeros (on whole numbers, i.e. integers) are more complicated

- 12300 can have 3, 4 or 5 significant digits!

Significant Digits

Trailing zeros (on whole numbers, i.e. integers) are more complicated

- 12300 can have 3, 4 or 5 significant digits!
    - Consider $12.3\,\mathrm{m}$. This value has 3 significant digits. It is equal to $12300\,\mathrm{mm}$, so in this case the 12300 also has 3 significant digits.

Significant Digits

Trailing zeros (on whole numbers, i.e. integers) are more complicated

- 12300 can have 3, 4 or 5 significant digits!
  - Consider $12.3\,\mathrm{m}$. This value has 3 significant digits. It is equal to $12300\,\mathrm{mm}$, so in this case the 12300 also has 3 significant digits.
  - Now consider $12.30\,\mathrm{m}$. This value has 4 significant digits. But it is still equal to $12300\,\mathrm{mm}$, so in this case the 12300 has 4 significant digits.

Significant Digits

Trailing zeros (on whole numbers, i.e. integers) are more complicated

- 12300 can have 3, 4 or 5 significant digits!
  - Consider $12.3\,\mathrm{m}$. This value has 3 significant digits. It is equal to $12300\,\mathrm{mm}$, so in this case the 12300 also has 3 significant digits.
  - Now consider $12.30\,\mathrm{m}$. This value has 4 significant digits. But it is still equal to $12300\,\mathrm{mm}$, so in this case the 12300 has 4 significant digits.
  - What if $12300\,\mathrm{mm}$ refers to $12.300\,\mathrm{m}$? Then it has 5 significant digits.

Trailing zeros (on whole numbers, i.e. integers) are more **complicated**

- 12300 can have 3, 4 or 5 significant digits!
  - Consider $12.3\,\mathrm{m}$. This value has 3 significant digits. It is equal to $12300\,\mathrm{mm}$, so in this case the 12300 also has 3 significant digits.
  - Now consider $12.30\,\mathrm{m}$. This value has 4 significant digits. But it is still equal to $12300\,\mathrm{mm}$, so in this case the 12300 has 4 significant digits.
  - What if $12300\,\mathrm{mm}$ refers to $12.300\,\mathrm{m}$? Then it has 5 significant digits.
- Usually, the trailing zeros are placeholders for the magnitude of a value and we don't need to worry unduly.

Significant Digits

Trailing zeros (on whole numbers, i.e. integers) are more **complicated**

- 12300 can have 3, 4 or 5 significant digits!
  - Consider $12.3\,\mathrm{m}$. This value has 3 significant digits. It is equal to $12300\,\mathrm{mm}$, so in this case the 12300 also has 3 significant digits.
  - Now consider $12.30\,\mathrm{m}$. This value has 4 significant digits. But it is still equal to $12300\,\mathrm{mm}$, so in this case the 12300 has 4 significant digits.
  - What if $12300\,\mathrm{mm}$ refers to $12.300\,\mathrm{m}$? Then it has 5 significant digits.
- Usually, the trailing zeros are placeholders for the magnitude of a value and we don't need to worry unduly.
- If we want to emphasize that 12300 has 4 significant digits, we can write $1.230 \times \left(10^3\right)$.

Calculations for Exercises

- In practice, it is often difficult to measure objects more accurately than to three significant digits so **input values for exercises are generally given to 3 significant digits**. (Or sometimes 4 significant digits when the leading significant digit is a 1)

Calculations for Exercises

- In practice, it is often difficult to measure objects more accurately than to three significant digits so **input values for exercises are generally given to 3 significant digits**. (Or sometimes 4 significant digits when the leading significant digit is a 1)

- We cannot expect to get more accuracy in our result at the end of a calculation than from our given input values at the beginning of the calculation so **solutions should be correct to 3 significant digits, not more than the accuracy of the calculation inputs!**

Calculations for Exercises

- In practice, it is often difficult to measure objects more accurately than to three significant digits so **input values for exercises are generally given to 3 significant digits**. (Or sometimes 4 significant digits when the leading significant digit is a 1)

- We cannot expect to get more accuracy in our result at the end of a calculation than from our given input values at the beginning of the calculation so **solutions should be correct to 3 significant digits, not more than the accuracy of the calculation inputs!**

- Intermediate calculations will accumulate rounding errors quickly if we use only three significant digits and these can affect the final result. **For intermediate calculations, use 5 or more significant digits.**

  (When I write solutions down, I use 5 significant digits for intermediate calculations. You may use more if it is more convenient for you, e.g., if you are storing intermediate results in your calculator.)

- When using five significant digits for intermediate calculations, it is necessary to convert to three (or four) significant digits when providing the final answer to an exercise. Then, **rounding** is often involved:

Significant Digits and Rounding

- When using five significant digits for intermediate calculations, it is necessary to convert to three (or four) significant digits when providing the final answer to an exercise. Then, **rounding** is often involved:
  - 2.3456 becomes 2.35 because the first non-significant digit (the 5 in this case) is ≥ 5 and so the 4 rounds up.

Significant Digits and Rounding

- When using five significant digits for intermediate calculations, it is necessary to convert to three (or four) significant digits when providing the final answer to an exercise. Then, **rounding** is often involved:
  - 2.3456 becomes 2.35 because the first non-significant digit (the 5 in this case) is ≥ 5 and so the 4 rounds up.
  - 2.3446 becomes 2.34 because the first non-significant digit (the 4 in this case) is < 5 and the last significant digit, 4, remains unchanged.

Significant Digits and Rounding

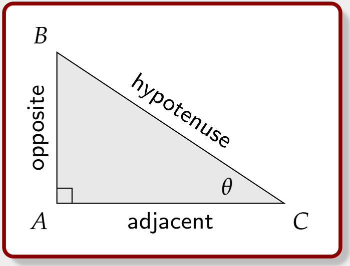
- When using five significant digits for intermediate calculations, it is necessary to convert to three (or four) significant digits when providing the final answer to an exercise. Then, **rounding** is often involved:
  - 2.3456 becomes 2.35 because the first non-significant digit (the 5 in this case) is ≥ 5 and so the 4 rounds up.
  - 2.3446 becomes 2.34 because the first non-significant digit (the 4 in this case) is < 5 and the last significant digit, 4, remains unchanged.
- When the first discarded digit is a 5 (or higher), round up the digit before the 5 (or higher)

Significant Digits and Rounding

- When using five significant digits for intermediate calculations, it is necessary to convert to three (or four) significant digits when providing the final answer to an exercise. Then, **rounding** is often involved:
  - 2.3456 becomes 2.35 because the first non-significant digit (the 5 in this case) is ≥ 5 and so the 4 rounds up.
  - 2.3446 becomes 2.34 because the first non-significant digit (the 4 in this case) is < 5 and the last significant digit, 4, remains unchanged.
- When the first discarded digit is a 5 (or higher), round up the digit before the 5 (or higher)
- There are various rules (such as the odd-even rule) which take a more complicated approach to rounding 5 but, for our purposes, **5 rounds up**!

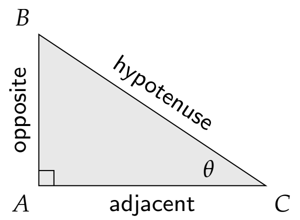
More About Right Triangle

The sine, cosine and tangent trigonometrical functions relate an acute angle (θ, in this example) in a right triangle to two of the sides of the triangle.



More About Right Triangle

The sine, cosine and tangent trigonometrical functions relate an acute angle (θ, in this example) in a right triangle to two of the sides of the triangle.

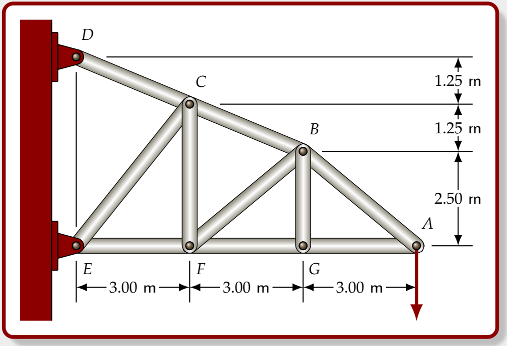


Right Triangle Trigonometry Formulæ

$$\sin\theta = \frac{\textbf{o}\text{pposite}}{\textbf{h}\text{ypotenuse}}, \quad \cos\theta = \frac{\textbf{a}\text{djacent}}{\textbf{h}\text{ypotenuse}}, \quad \tan\theta = \frac{\textbf{o}\text{pposite}}{\textbf{a}\text{djacent}}$$

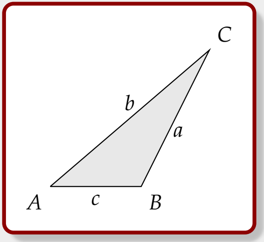
Remember: **SOHCAHTOA**

Right Triangle Exercises (2)



2. Use the **tangent** function to calculate $\angle CEF$.
3. From $\angle CEF$ just found (**use the intermediate, 5 or more significant digit, form!**) and the **sine** rule to verify the length of CE found earlier.
4. Use the **cosine** function and the length of CB found earlier to calculate the angle between BC and the horizontal.
5. Use the **tangent** function to verify the previous result.

Not all triangles contain a right angle. To solve for these triangles (finding the lengths of the side and the triangle angles), we have to employ some different tools: the **sine rule** and (later) the **cosine rule**



Sine Rule

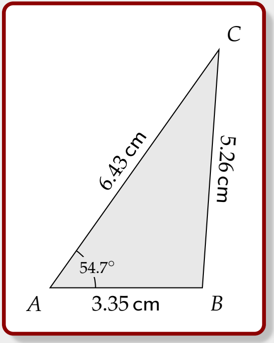
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

6. Using the sine rule, find $\angle ACB$.
7. Using the sine rule, find $\angle ABC$.
8. Sum the interior angles of the triangle.



A couple of trig identities that will come in useful:

Identities

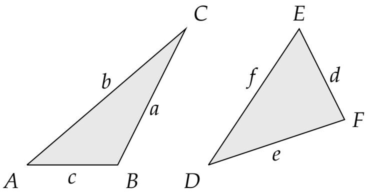
$$\sin(180^\circ - \theta) = \sin\theta$$
$$\cos(-\theta) = \cos\theta$$

Note:

$$\begin{aligned} \sin(140^\circ) &= \sin(40^\circ) &= 0.64279 \\ \cos(42^\circ) &= \cos(-42^\circ) &= 0.74314 \end{aligned}$$

Thus, we have to be careful when using inverse trigonometric functions:

$$\begin{aligned} \sin^{-1}(0.64279) &= 40^\circ \text{ or } 140^\circ \\ \cos^{-1} 0.74314) &= -42^\circ \text{ or } 42^\circ \end{aligned}$$

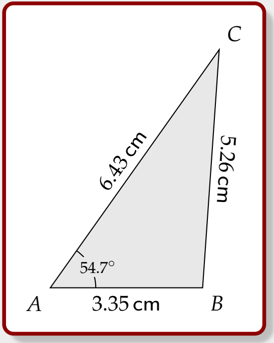


Cosine Rule

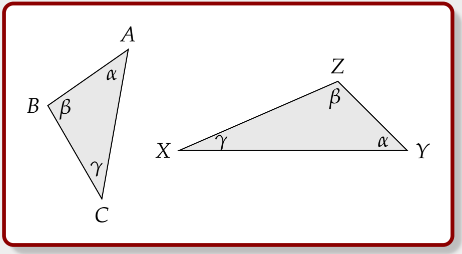
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

The cosine rule is useful when you have all the sides of a triangle and want to find the angles.

9. Determine $\angle ABC$, using the value for AB found earlier
10. Compare the value for $\angle ABC$ with the value calculated earlier.
    Is it the same? It should be!

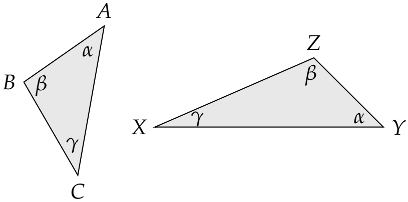


Similar Triangles



If triangles ABC and XYZ have the same angles, they are said to be **similar triangles**.

Similar Triangles



If triangles ABC and XYZ have the same angles, they are said to be **similar triangles**.

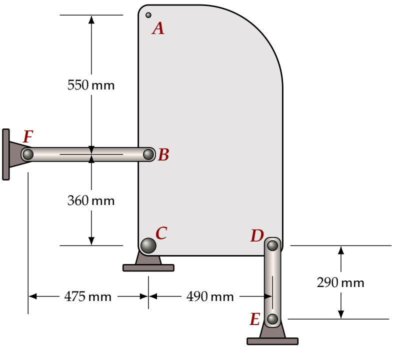
The ratios of the lengths of corresponding sides of similar triangles are equal:

$$\frac{AB}{XY} = \frac{BC}{XZ} = \frac{AC}{YZ}$$

ABCD is a rigid (i.e., it does not deform) plate, pinned at C.

When horizontal force P is applied at A, $ABCD$ rotates about C and A deflects 2.45 mm horizontally rightwards.

Assume that BF remains horizontal and that DE remains vertical.

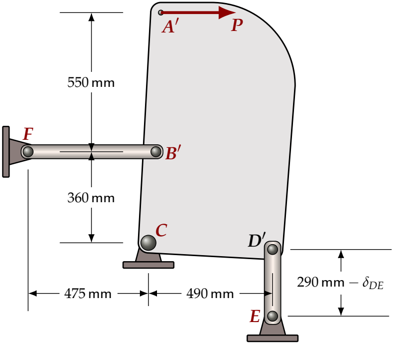


11. Determine δ_{BF}, the change in length of BF.
12. Determine δ_{DE}, the change in length of DE.

$ABCD$ is a rigid (i.e., it does not deform) plate, pinned at C.

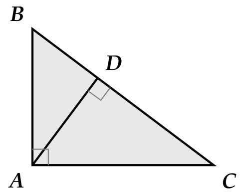
When horizontal force P is applied at A, $ABCD$ rotates about C and A deflects 2.45 mm horizontally rightwards.

Assume that BF remains horizontal and that DE remains vertical.



11. Determine δ_{BF}, the change in length of BF.
12. Determine δ_{DE}, the change in length of DE.

13. Show that right triangles $\triangle ABC$, $\triangle ABD$ and $\triangle ACD$ all have the same angles (i.e. they are all similar).
14. Given that $AC = 100$ mm and $AD = 65$ mm, determine $\angle ACD$ and $\angle ABD$.
15. Find the remaining lengths: AB, BD and CD.
16. Verify your lengths found above by using the Pythagorean Theorem on $\triangle ABC$



Triangles and Trig Exercise

This is a standard type of statics problem to determine the forces in cables AC and BC. But first we have to find the angles θ_{AC} and θ_{BC}. This involves the use of the Pythagorean Theorem, the sine and cosine rules, and one of the trigonometric functions.

17. Find θ_{AC}.
18. Find θ_{BC}.