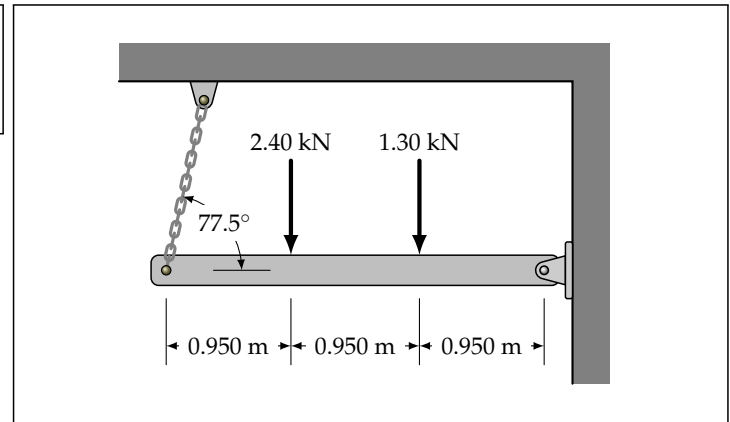
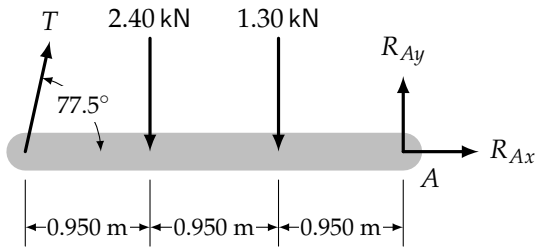


# Engineering Statics - 06 Equilibrium of Rigid Bodies - Instructor Copy

**Example 1:** Determine the tension in the chain and the reaction at A.

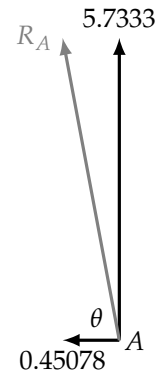


$$\begin{aligned}\Sigma M_A &= (1.30 \text{ kN}) \cdot (0.950 \text{ m}) + (2.40 \text{ kN}) \cdot (1.90 \text{ m}) - (T \sin 77.5^\circ) \cdot (2.85 \text{ m}) = 0 \\ \Rightarrow T &= \frac{(1.30 \text{ kN}) \cdot (0.950 \text{ m}) + (2.40 \text{ kN}) \cdot (1.90 \text{ m})}{\sin 77.5^\circ \times 2.85 \text{ m}} \\ &= 2.0827 \text{ kN}\end{aligned}$$

$$\begin{aligned}\Sigma F_x &= R_{Ax} + T \cos 77.5^\circ = 0 \\ \Rightarrow R_{Ax} &= -(2.0827 \text{ kN}) \cdot \cos 77.5^\circ = -0.45078 \text{ kN}\end{aligned}$$

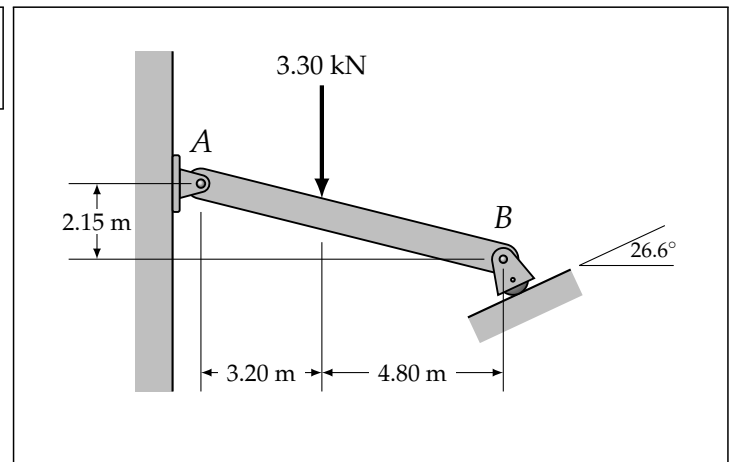
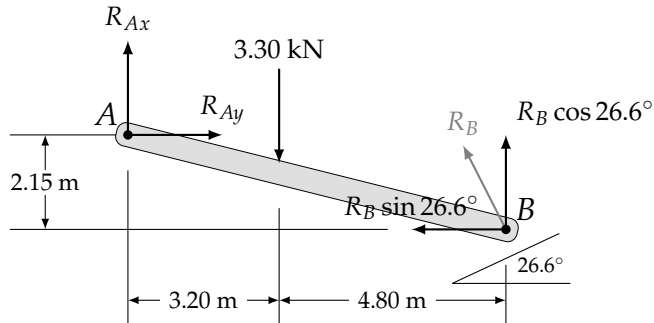
$$\begin{aligned}\Sigma F_y &= R_{Ay} + T \sin 77.5^\circ - 2.40 \text{ kN} - 1.30 \text{ kN} = 0 \\ \Rightarrow R_{Ay} &= 2.40 \text{ kN} + 1.30 \text{ kN} - (2.0827 \text{ kN}) \cdot \sin 77.5^\circ \\ &= 5.7333 \text{ kN}\end{aligned}$$

$$\begin{aligned}R_A &= \sqrt{(-0.45078 \text{ kN})^2 + (5.7333 \text{ kN})^2} = 5.75103 \text{ kN} \\ \theta &= \tan^{-1} \left[ \frac{5.7333}{0.45078} \right] = 85.504^\circ\end{aligned}$$



The tension in the chain is **2.08 kN** and the reaction at A is **5.75 kN** at **94.5°** measured counter-clockwise from the positive x-axis.

**Example 2:** Determine the reactions at A and B.



$$\begin{aligned}\Sigma M_A &= (R_B \cos 26.6^\circ \text{ kN})(8.00 \text{ m}) - (3.30 \text{ kN})(3.20 \text{ m}) \\ &\quad - (R_B \sin 26.6^\circ \text{ kN})(2.15 \text{ m}) \\ &= 0 \\ \Rightarrow R_B &= \frac{(3.30 \text{ kN})(3.20 \text{ m})}{(8.00 \cos 26.6^\circ \text{ m}) - (2.15 \sin 26.6^\circ \text{ m})} \\ &= 1.7058 \text{ kN}\end{aligned}$$

$$\underline{R_B = 1.71 \text{ kN at } 117^\circ}$$

$$\begin{aligned}\Sigma F_y &= R_{Ay} - 3.30 \text{ kN} + 1.7058 \cos 26.6^\circ \text{ kN} = 0 \\ \Rightarrow R_{Ay} &= 1.7748 \text{ kN}\end{aligned}$$

$$\begin{aligned}\Sigma F_x &= R_{Ax} - 1.7058 \sin 26.6^\circ \text{ kN} = 0 \\ \Rightarrow R_{Ax} &= 0.76379 \text{ kN}\end{aligned}$$

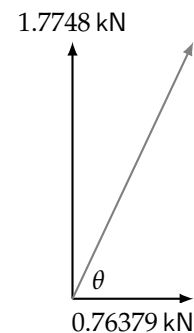
At this point, there is an opportunity to check our work so far. If we take moments about anywhere (except about A which we've already set to zero), they should sum to zero. Taking moments about B is the most convenient:

$$\begin{aligned}\Sigma M_B &= (3.30 \text{ kN})(4.80 \text{ m}) - (1.7748 \text{ kN})(8.00 \text{ m}) \\ &\quad - (0.76379 \text{ kN})(2.15 \text{ m}) \\ &= -0.000549 \\ &\approx 0 \quad \checkmark\end{aligned}$$

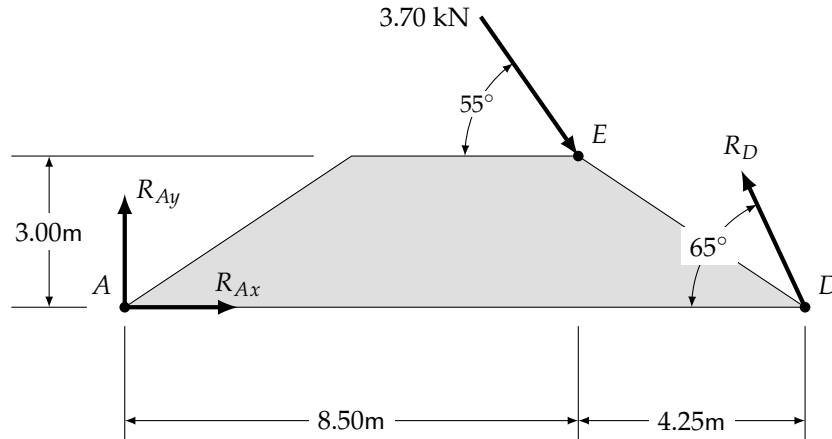
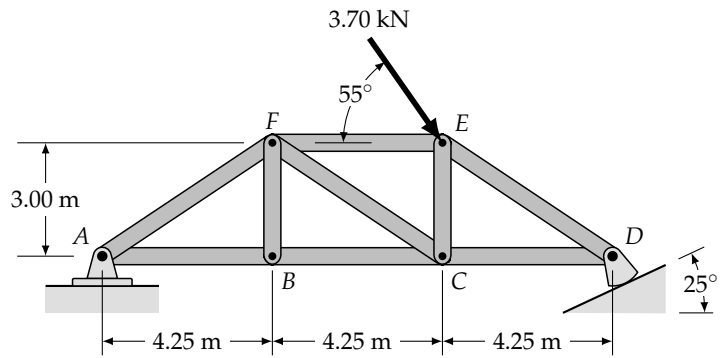
$$\begin{aligned}R_A &= \sqrt{(1.7748 \text{ kN})^2 + (0.76379 \text{ kN})^2} \\ &= 1.9322 \text{ kN}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1} \left[ \frac{1.7748}{0.76379} \right] \\ &= 66.715^\circ\end{aligned}$$

$$\underline{R_A = 1.93 \text{ kN at } 66.7^\circ}$$



**Exercise 1:** Determine the reactions at A and D.



$$\begin{aligned}\Sigma M_A &= (R_D \sin 65^\circ) \cdot (12.75 \text{ m}) - (3.70 \text{ kN} \cdot \sin 55^\circ) \cdot (8.50 \text{ m}) - (3.70 \text{ kN} \cdot \cos 55^\circ) \cdot (3.00 \text{ m}) \\ &= R_D \cdot 11.555 \text{ m} - 32.129 \text{ kN} \cdot \text{m} = 0 \\ \Rightarrow R_D &= \frac{32.129 \text{ kN} \cdot \text{m}}{11.555 \text{ m}} = 2.7805 \text{ kN}\end{aligned}$$

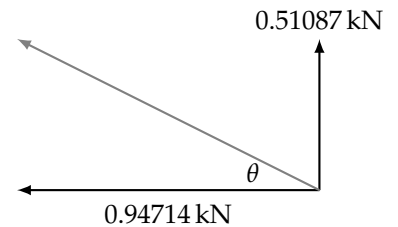
The reaction at D is **2.78 kN** at **115°** measured counter-clockwise from the positive x-axis.

$$\begin{aligned}\Sigma F_x &= R_{Ax} + (3.70 \text{ kN}) \cdot \cos 55^\circ - R_D \cos 65^\circ = 0 \\ \Rightarrow R_{Ax} &= 2.7805 \text{ kN} \cdot \cos 65^\circ - (3.70 \text{ kN}) \cdot \cos 55^\circ \\ &= -0.94714 \text{ kN}\end{aligned}$$

$$\begin{aligned}\Sigma F_y &= R_{Ay} + R_D \sin 65^\circ - (3.70 \text{ kN}) \cdot \sin 55^\circ = 0 \\ \Rightarrow R_{Ay} &= (3.70 \text{ kN}) \cdot \sin 55^\circ - 2.7805 \text{ kN} \cdot \sin 65^\circ \\ &= 0.51087 \text{ kN}\end{aligned}$$

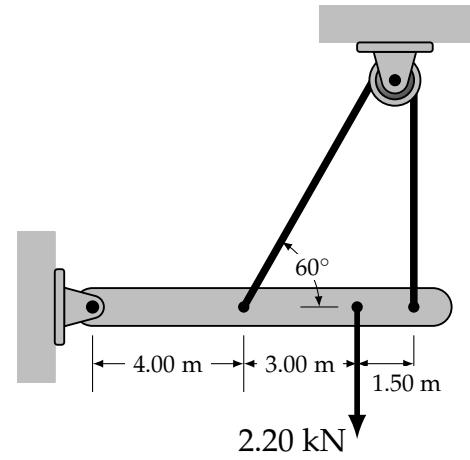
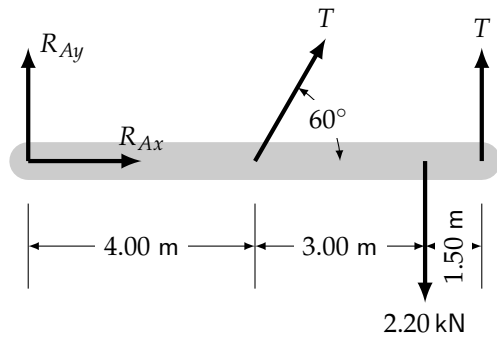
$$R_A = \sqrt{(-0.94714 \text{ kN})^2 + (0.51087 \text{ kN})^2} = 1.0761 \text{ kN}$$

$$\theta = \tan^{-1} \left[ \frac{0.51087}{0.94714} \right] = 28.342^\circ$$



The reaction at A is **1.08 kN** at **152°** measured counter-clockwise from the positive x-axis.

**Exercise 2:** Determine the reactions at the pinned connection and the tension in the cable.



$$\Sigma M_A = (T \sin 60^\circ) \cdot (4.00 \text{ m}) + T \cdot (8.50 \text{ m}) - (2.20 \text{ kN}) \cdot (7.00 \text{ m}) = 0$$

$$\Rightarrow T = \frac{(2.20 \text{ kN}) \cdot (7.00 \text{ m})}{(4.00 \text{ m}) \cdot (\sin 60^\circ) + (8.50 \text{ m})} = 1.2872 \text{ kN}$$

$$\Sigma F_x = R_{Ax} + T \cos 60^\circ = 0$$

$$\Rightarrow R_{Ax} = -(1.2872 \text{ kN}) \cdot \cos 60^\circ = -0.64359 \text{ kN}$$

$$\Sigma F_y = R_{Ay} + T \sin 60^\circ + T - 2.20 \text{ kN} = 0$$

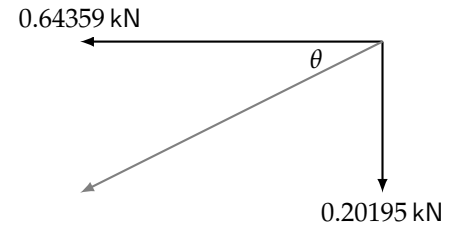
$$\Rightarrow R_{Ay} = 2.20 \text{ kN} - (1.2872 \text{ kN}) \cdot (1 + \sin 60^\circ) = -0.20195 \text{ kN}$$

$$R_A = \sqrt{(-0.64359 \text{ kN})^2 + (-0.20195 \text{ kN})^2} = 0.67453 \text{ kN}$$

$$\theta = \tan^{-1} \left[ \frac{-0.20195}{-0.64359} \right] = 17.421^\circ$$

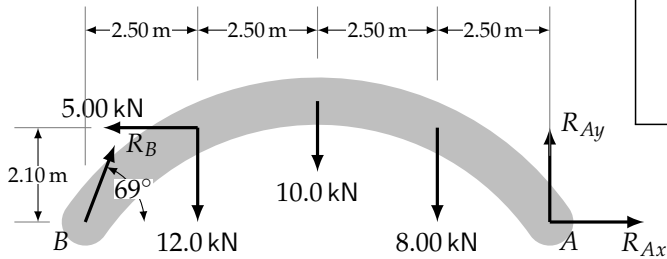
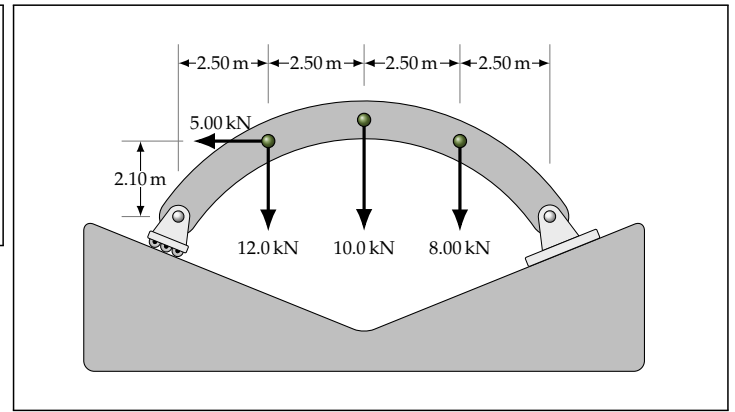
The tension in the cable is **1.29 kN**

The reaction at A is **0.675 kN** at **197°** measured counter-clockwise from the positive  $x$ -axis.



**Example 3:** The roller and the pinned connection are on slopes inclined at  $21^\circ$  to the horizontal; they are both at the same elevation.

Determine the reactions at the pinned connection and the tension in the cable.

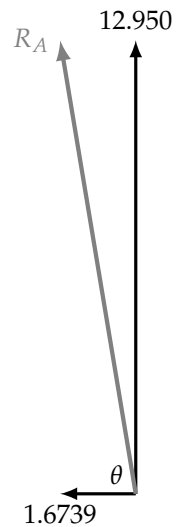


$$\begin{aligned}\Sigma M_A &= (8.00 \text{ kN}) \cdot (2.50 \text{ m}) + (10.0 \text{ kN}) \cdot (5.00 \text{ m}) + (12.0 \text{ kN}) \cdot (7.50 \text{ m}) \\ &\quad + (5.00 \text{ kN}) \cdot (2.10 \text{ m}) - (R_B \sin 69^\circ) \cdot (10.0 \text{ m}) \\ &= 170.5 \text{ kN} \cdot \text{m} - (9.3358 \text{ m}) \cdot R_B = 0 \\ \Rightarrow R_B &= \frac{170.5 \text{ kN} \cdot \text{m}}{9.3358 \text{ m}} = 18.263 \text{ kN}\end{aligned}$$

$$\begin{aligned}\Sigma F_x &= R_B \cdot \cos 69^\circ + R_{Ax} - 5.00 \text{ kN} = 0 \\ \Rightarrow R_{Ax} &= 5.00 \text{ kN} - (18.263 \text{ kN}) \cdot \cos 69^\circ = -1.6739 \text{ kN}\end{aligned}$$

$$\begin{aligned}\Sigma F_y &= R_{Ay} + R_B \sin 69^\circ - (12.0 \text{ kN} + 10.0 \text{ kN} + 8.00 \text{ kN}) = 0 \\ R_{Ay} &= 30.0 \text{ kN} - (18.263 \text{ kN}) \cdot \sin 69^\circ = 12.950 \text{ kN}\end{aligned}$$

$$\begin{aligned}R_A &= \sqrt{(-1.6739 \text{ kN})^2 + (12.950 \text{ kN})^2} = 13.058 \text{ kN} \\ \theta &= \tan^{-1} \left[ \frac{12.950}{1.6739} \right] = 82.635^\circ\end{aligned}$$



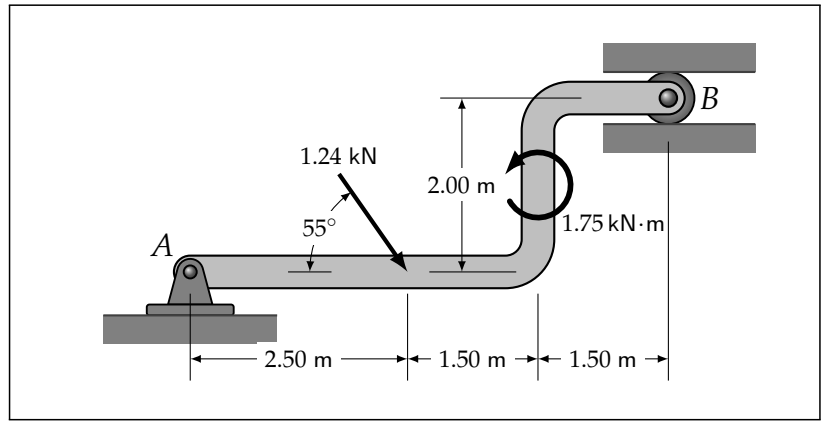
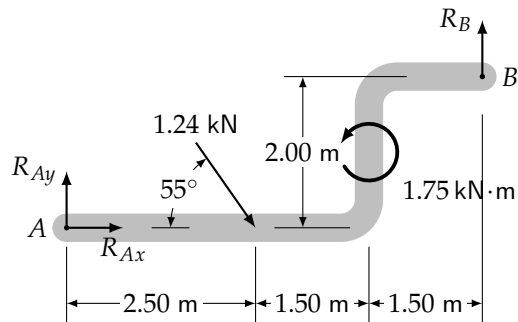
The reaction at the roller is **18.3 kN** at  **$69^\circ$**  measured counter-clockwise from the positive  $x$ -axis.

The reaction at the pinned connection is **13.1 kN** at  **$97.4^\circ$**  measured counter-clockwise from the positive  $x$ -axis.

**Example 4:**

The roller at  $B$  is in a smooth slot.

Determine the reactions at  $A$  and  $B$ .



$$\Sigma M_A = R_B \cdot (5.50 \text{ m}) + 1.75 \text{ kN} \cdot \text{m} - (1.24 \text{ kN}) \cdot (2.50 \text{ m}) = 0$$

$$\Rightarrow R_B = \frac{3.1 \text{ kN} \cdot \text{m} - 1.75 \text{ kN} \cdot \text{m}}{5.50 \text{ m}} = 0.24545 \text{ kN}$$

**Note:** We began by assuming a reaction  $R_B$  in the positive direction (that is, that the roller at  $B$  was pressing down on the slot). Our result for  $R_B$  is positive, so that initial assumption is correct. If  $R_B$  had evaluated to a negative value, then the roller would be pressing upward on the slot and the reaction would be downward.

$$\Sigma F_x = R_{Ax} + (1.24 \text{ kN}) \cdot \cos 55^\circ = 0$$

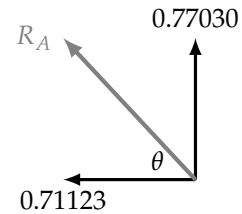
$$\Rightarrow R_{Ax} = -0.71123 \text{ kN}$$

$$\Sigma F_y = R_{Ay} + R_B - (1.24 \text{ kN}) \cdot \sin 55^\circ = 0$$

$$\Rightarrow R_{Ay} = (1.24 \text{ kN}) \cdot \sin 55^\circ - 0.24545 \text{ kN} = 0.77030 \text{ kN}$$

$$R_A = \sqrt{(-0.71123 \text{ kN})^2 + (0.77030 \text{ kN})^2} = 1.0484 \text{ kN}$$

$$\theta = \tan^{-1} \left[ \frac{0.77030}{0.71123} \right] = 47.283^\circ$$



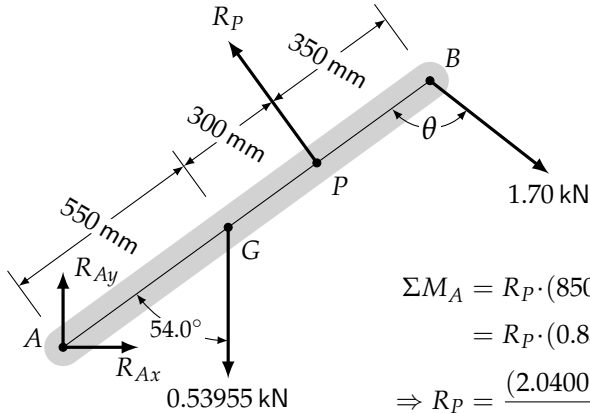
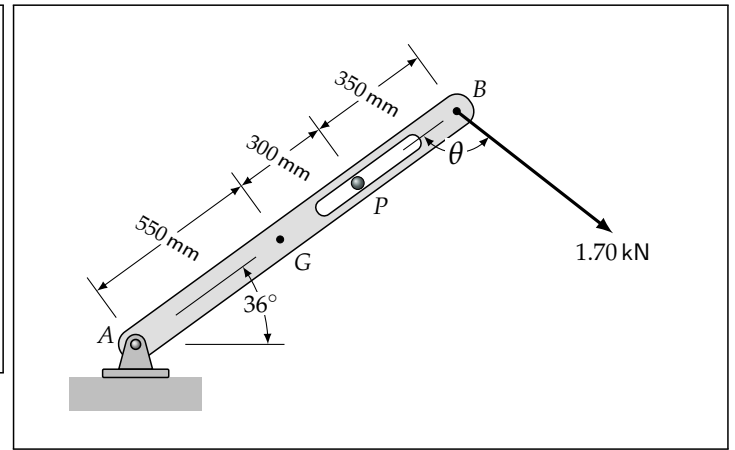
The reaction at  $A$  is **1.05 kN** at **133°**, measured counter-clockwise from the positive  $x$ -axis.

The reaction at  $B$  is **0.245 kN** at **90°**, measured counter-clockwise from the positive  $x$ -axis.

**Example 5:**

55 – kg bar  $AB$  has its centre of gravity at  $G$ . It is supported by a pinned connection at  $A$  and a smooth peg at  $C$ . A cable is attached at  $B$  and has a tensile force of 1.70 kN. The direction of the cable varies between  $\theta = 60^\circ$  and  $\theta = 135^\circ$ .

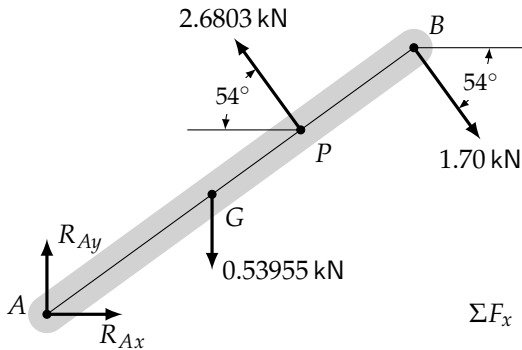
What is the maximum reaction at  $P$ ? Determine the reaction at  $A$  for this reaction at  $P$ .



$$\begin{aligned}\Sigma M_A &= R_P \cdot (850 \text{ mm}) - (1.70 \text{ kN} \cdot \sin \theta)(1200 \text{ mm}) - (0.53955 \text{ kN} \cdot \sin 54^\circ) \cdot (550 \text{ mm}) \\ &= R_P \cdot (0.850 \text{ m}) - (2.0400 \text{ kN} \cdot \text{m}) \cdot \sin \theta - 0.23828 \text{ kN} \cdot \text{m} = 0\end{aligned}$$

$$\Rightarrow R_P = \frac{(2.0400 \text{ kN} \cdot \text{m}) \cdot \sin \theta + 0.23828 \text{ kN} \cdot \text{m}}{0.850 \text{ m}} = 2.4000 \text{ kN} \cdot \sin \theta + 0.28033 \text{ kN}$$

$$\Rightarrow R_{P_{\max}} = 2.4000 \text{ kN} \cdot \sin 90^\circ + 0.28033 \text{ kN} = 2.6803 \text{ kN}$$



$$\begin{aligned}\Sigma F_x &= R_{Ax} - (2.6803 \text{ kN}) \cdot \cos 54^\circ + (1.70 \text{ kN}) \cdot \cos 54^\circ = 0 \\ \Rightarrow R_{Ax} &= 0.57621 \text{ kN}\end{aligned}$$

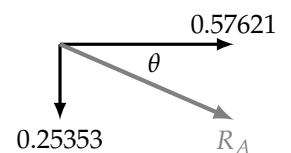
$$\begin{aligned}\Sigma F_y &= R_{Ay} - 0.53955 \text{ kN} + (2.6803 \text{ kN}) \cdot \sin 54^\circ - (1.70 \text{ kN}) \cdot \sin 54^\circ = 0 \\ \Rightarrow R_{Ay} &= -0.25353 \text{ kN}\end{aligned}$$

$$R_A = \sqrt{(0.57621 \text{ kN})^2 + (0.25353 \text{ kN})^2} = 0.62952 \text{ kN}$$

$$\theta = \tan^{-1} \left[ \frac{0.25353}{0.57621} \right] = 23.749^\circ$$

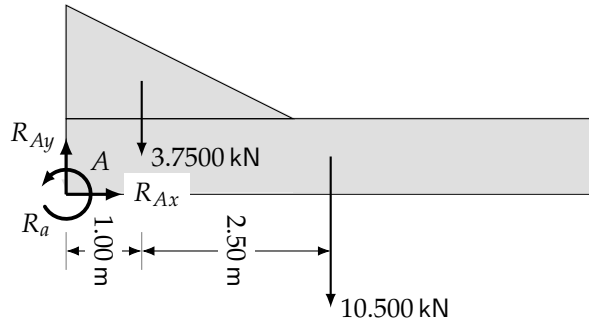
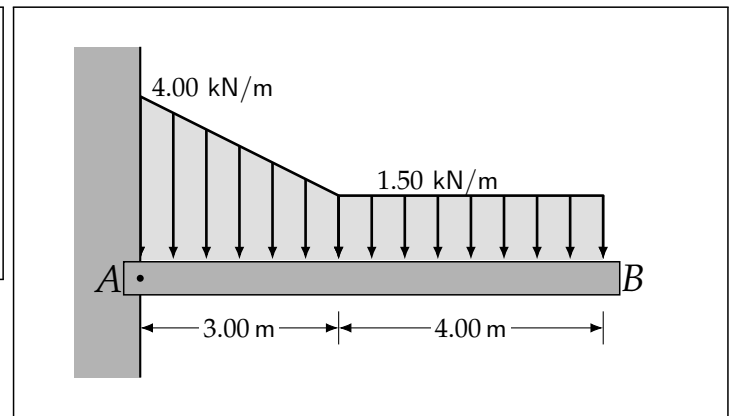
The maximum reaction at  $P$  is **2.68 kN** at **126°**, measured counter-clockwise from the positive  $x$ -axis.

The associated reaction at  $A$  is **0.630 kN** at **23.7°**, measured **clockwise** from the positive  $x$ -axis.



**Example 6:** Beam  $AB$  has a fixed support at  $A$ . (Fixed supports offer resistance to rotation in the form of a reacting couple at  $A$ ; clearly, without this, equilibrium would not be possible.)

Determine the reaction and the reacting couple at  $A$ .



$$\begin{aligned}\Sigma M_A &= R_a - (3.7500 \text{ kN}) \cdot (1.0000 \text{ m}) - (10.500 \text{ kN}) \cdot (3.5000 \text{ m}) = 0 \\ \Rightarrow R_a &= 40.500 \text{ kN} \cdot \text{m}\end{aligned}$$

$$\Sigma F_x = R_{Ax} = 0;$$

$$\begin{aligned}\Sigma F_y &= R_{Ay} - 3.7500 \text{ kN} - 10.500 \text{ kN} = 0 \\ \Rightarrow R_{Ay} &= 14.250 \text{ kN}\end{aligned}$$

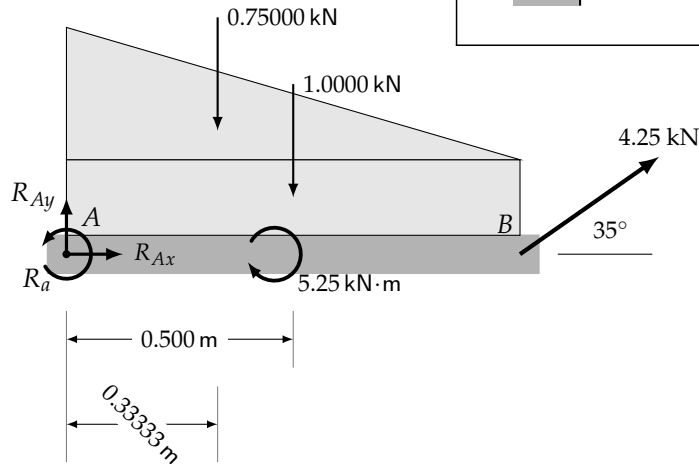
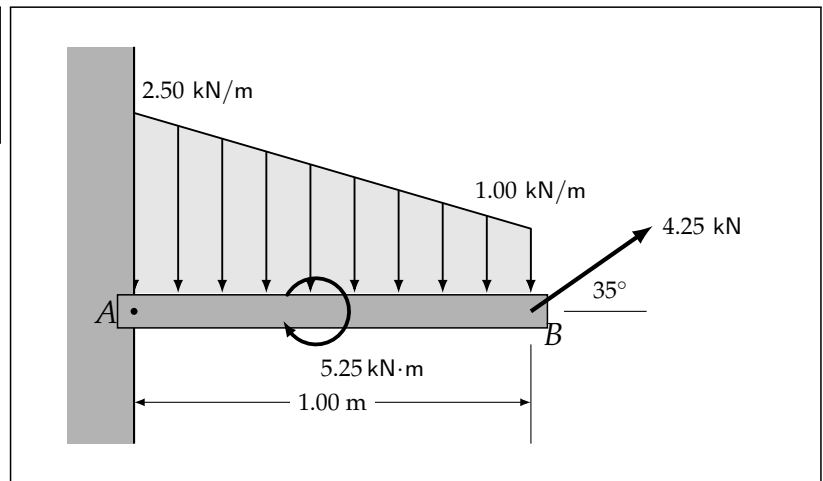
(Note that calculation for the magnitude and direction of the reaction at  $A$  is 'trivial' and can be just written down, since  $R_{Ax}$  is 0.)

The reacting moment at  $A$  is **40.5 kN·m**.

The reaction at  $A$  is **14.3 kN** at **90°**, measured counter-clockwise from the positive  $x$ -axis.



**Exercise 3:** Determine the reaction and the reacting moment at A.



$$\begin{aligned}\Sigma M_A = R_a - (0.75000 \text{ kN}) \cdot (0.33333 \text{ m}) - (1.0000 \text{ kN}) \cdot (0.50000 \text{ m}) \\ - 5.25 \text{ kN} \cdot \text{m} + (4.25 \text{ kN} \cdot \sin 35^\circ) \cdot (1.00 \text{ m}) = 0\end{aligned}$$

$$\Rightarrow R_a = 3.5623 \text{ kN} \cdot \text{m}$$

$$\begin{aligned}\Sigma F_x = R_{Ax} + (4.25 \text{ kN}) \cdot \cos 35^\circ \\ \Rightarrow R_{Ax} = -3.4814 \text{ kN}\end{aligned}$$

$$\begin{aligned}\Sigma F_y = R_{Ay} + (4.25 \text{ kN}) \cdot \sin 35^\circ - 0.75000 \text{ kN} - 1.0000 \text{ kN} = 0 \\ \Rightarrow R_{Ay} = -0.68770 \text{ kN}\end{aligned}$$

$$R_A = \sqrt{(-3.4814 \text{ kN})^2 + (-0.68770 \text{ kN})^2} = 3.5487 \text{ kN}$$

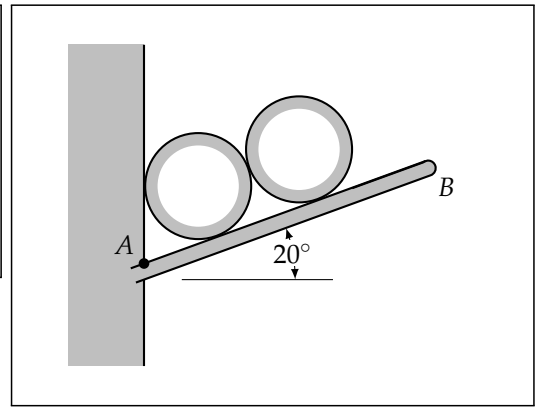
$$\theta = \tan^{-1} \left[ \frac{0.68770}{3.4814} \right] = 11.174^\circ$$

The reacting moment at A is **3.56 kN·m**.

The reaction at A is **3.55 kN** at **191°**, measured counter-clockwise from the positive  $x$ -axis.

**Example 5:** Pipe racks ( $AB$ , and two hidden behind it) support two smooth Schedule 40 pipes, with an outside diameter of 508 mm, as shown. The pipes are 10 m in length with a mass of 78.5 kg/m. Each rack supports one-third of the weight of each pipe.

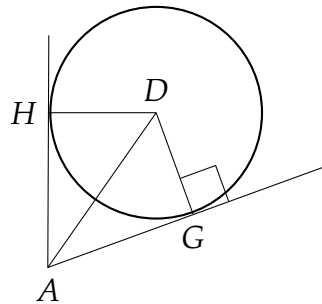
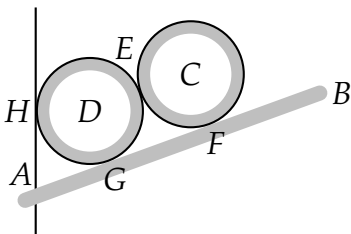
Determine the reaction at the fixed connection  $A$ .



The weight of each pipe bearing on  $AB$ :

$$W = 78.5 \text{ kg/m} \times 9.81 \text{ m/s}^2 \times 10 \text{ m} / 3 = 2.5670 \text{ kN}$$

Add some labels, find some distances:



$$\angle HAD = \angle GAD = 35^\circ$$

$$\angle GAD = 55^\circ$$

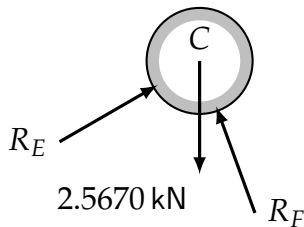
$$\frac{AG}{GD} = \tan 55^\circ$$

$$AG = \frac{508 \text{ mm}}{2} \tan 55^\circ$$

$$= 362.75 \text{ mm}$$

$$GF = CD = 508 \text{ mm}$$

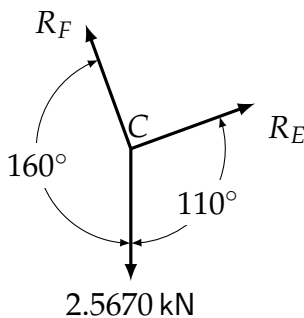
**Forces acting upon the upper (rightmost) pipe,  $C$ :**



This is now a simple concurrent forces problem, solved with simultaneous equations. Notice, however, that the direction of  $R_F$  is perpendicular to the direction of  $R_E$ .

If we choose axes  $x'$  and  $y'$ , rotated  $20^\circ$  in the counter clockwise direction around  $C$ , then the direction of  $R_E$  is the  $x'$ -axis and the direction of  $R_F$  is the  $y'$ -axis. Now we can solve without simultaneous equations.

(Why bother complicating things? This will become a useful technique towards the end of the module and it's easy to introduce here.)



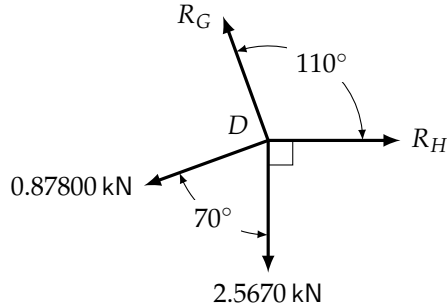
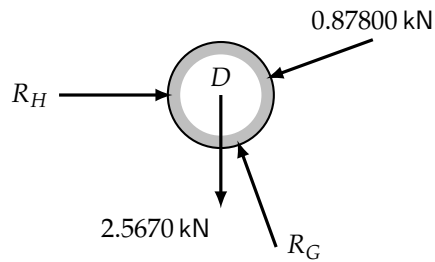
$$\Sigma F_{x'} = R_E - 2.5670 \text{ kN} \cdot \cos 70^\circ = 0$$

$$\Rightarrow R_E = 0.87800 \text{ kN}$$

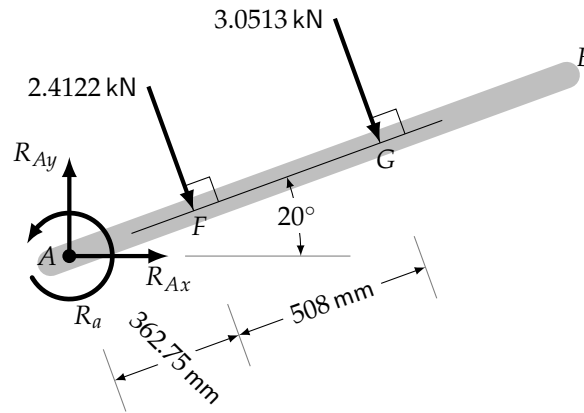
$$\Sigma F_{y'} = R_F - 2.5670 \text{ kN} \cdot \cos 20^\circ = 0$$

$$\Rightarrow R_F = 2.4122 \text{ kN}$$

**Forces acting upon the lower (leftmost) pipe, D:**



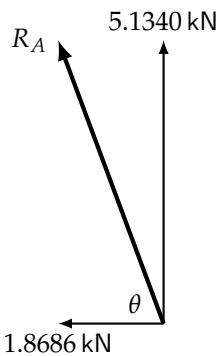
$$\begin{aligned}\Sigma F_y &= R_G \cdot \cos 20^\circ - 2.5670 \text{ kN} - 0.87800 \text{ kN} \cdot \cos 70^\circ \\ &= 0 \\ \Rightarrow R_G &= \frac{2.5670 \text{ kN} + 0.87800 \text{ kN} \cdot \cos 70^\circ}{\cos 20^\circ} \\ \Rightarrow R_G &= 3.0513 \text{ kN}\end{aligned}$$



$$\Sigma M_A = R_a - 2.4122 \text{ kN} \cdot 362.75 \text{ mm} - 3.0513 \text{ kN} \cdot 870.75 \text{ mm} = 0 \Rightarrow R_a = 3531.9 \text{ kN} \cdot \text{mm} = 3.5319 \text{ kN} \cdot \text{m}$$

$$\Sigma F_x = R_{Ax} + 2.4122 \text{ kN} \cdot \sin 20^\circ + 3.0513 \text{ kN} \cdot \sin 20^\circ = 0 \Rightarrow R_{Ax} = -1.8686 \text{ kN}$$

$$\Sigma F_y = R_{Ay} - 2.4122 \text{ kN} \cdot \cos 20^\circ - 3.0513 \text{ kN} \cdot \cos 20^\circ = 0 \Rightarrow R_{Ay} = 5.1340 \text{ kN}$$



$$R_A = \sqrt{(1.8686 \text{ kN})^2 + (5.1340 \text{ kN})^2} = 5.4635 \text{ kN}$$

$$\theta = \tan^{-1} \left[ \frac{5.1340 \text{ kN}}{1.8686 \text{ kN}} \right] = 70^\circ$$

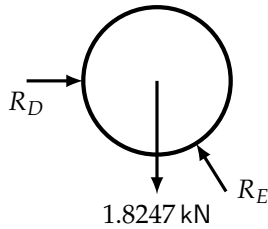
The reaction at A is **5.46 kN** at **110°** counter-clockwise from the positive  $x$ -axis. The reacting moment at A is **3.53 kN·m**

**Exercise 4:** A section of smooth pipe, centred at  $O$ , has a diameter of 457 mm and a mass of 186 kg. It is secured by vertical structural member  $AB$ , hinged with a pinned connection at  $A$  and held in place by chain  $BC$ .

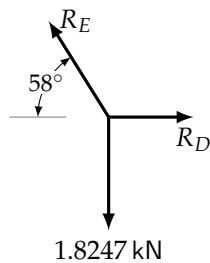
Determine the tension in the chain, and the reaction at  $A$ .

The weight of the pipe is:  
 $W = 186 \times 9.81 \text{ N} = 1824.7 \text{ N}$

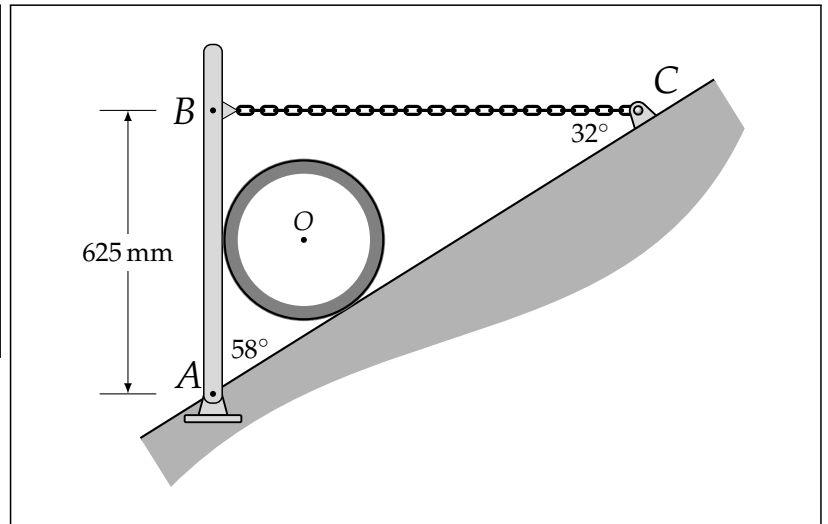
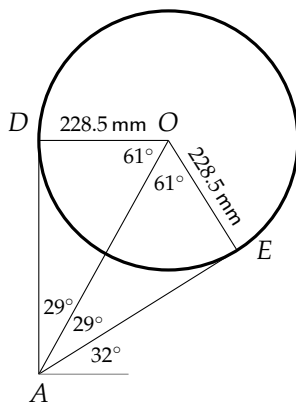
Forces acting on the pipe:



Free body diagram:



Determine the length of the moment-arm,  $AD$ , where the pipe bears on the vertical member.



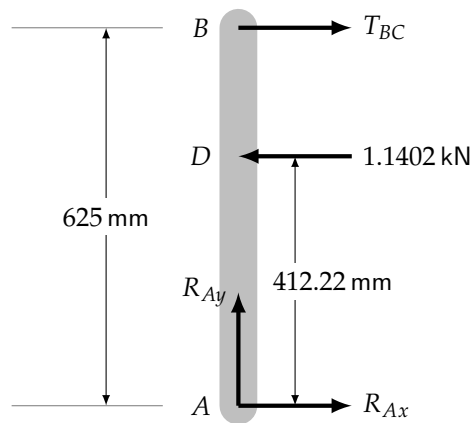
$$\Sigma F_y = R_E \cdot \sin 58^\circ - 1.8247 \text{ kN} = 0$$

$$\Rightarrow R_E = 2.1516 \text{ kN}$$

$$\Sigma F_x = R_D - (2151.6 \text{ kN}) \cdot \cos 58^\circ = 0$$

$$\Rightarrow R_D = 1.1402 \text{ kN}$$

$$\frac{AD}{228.5 \text{ mm}} = \tan 61^\circ \Rightarrow AD = 412.22 \text{ mm}$$



$$\Sigma M_A = (1.1402 \text{ kN}) \cdot (412.22 \text{ mm}) - (625 \text{ mm}) T_{BC} = 0$$

$$\Rightarrow T_{BC} = 0.75202 \text{ kN}$$

$$\Sigma F_x = R_{Ax} - 1.1402 \text{ kN} + 0.75202 \text{ kN} = 0$$

$$\Rightarrow R_{Ax} = 0.38818 \text{ kN}$$

$$\Sigma F_y = R_{Ay} = 0$$

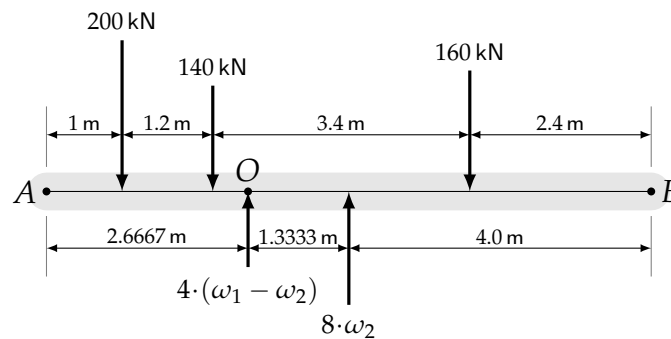
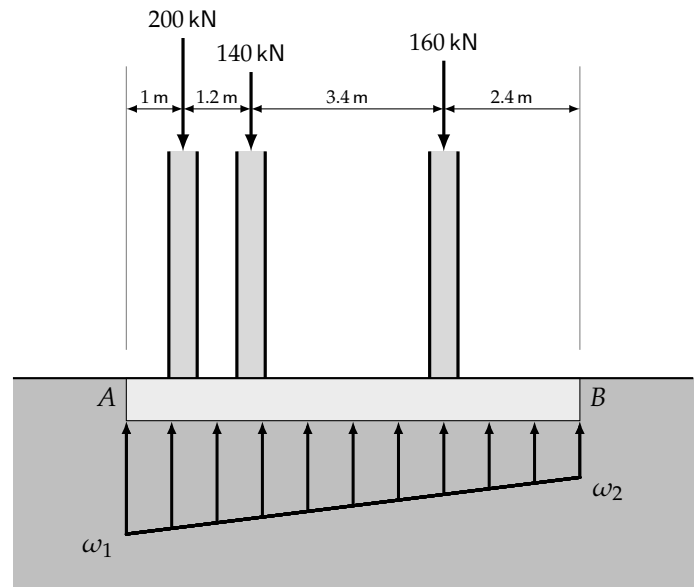
The tension in the chain between  $B$  and  $C$  is **0.752 kN**.

The reaction at  $A$  is **0.388 kN** in the direction of the positive  $x$ -axis

**Example 8:**

Soil exerts a trapezoidal distributed load on the bottom of the footing  $AB$ .

Determine the values of  $\omega_1$  and  $\omega_2$  that support the column loadings in static equilibrium.



We could start by taking moments about  $A$  to get an expression with  $\omega_1$  and  $\omega_2$ . Then do the same with moments about  $B$ . And get a system of two equations in two unknowns.

Or we could take moments about  $O$  and not have to deal with simultaneous equations!

**Note:** The  $8 \cdot \omega_2$  is in kN where the 8 is in m and the  $\omega_2$  is in kN/m. Similarly for  $4(\omega_1 - \omega_2)$ .

$$\begin{aligned}\Sigma M_O &= (8\text{ m})\omega_2 \cdot (1.3333\text{ m}) + (140\text{ kN})(0.46670\text{ m}) + (200\text{ kN})(1.6667\text{ m}) - (160\text{ kN})(2.9333\text{ m}) \\ &= (10.644\text{ m}^2)\omega_2 - 70.650\text{ kN}\cdot\text{m} = 0 \\ \Rightarrow \omega_2 &= 6.6236\text{ kN/m}\end{aligned}$$

$$\begin{aligned}\Sigma F_y &= (8\text{ m})\omega_2 + (4\text{ m})(\omega_1 - \omega_2) - 500\text{ kN} \\ &= (4\text{ m})\omega_1 + (4\text{ m})\omega_2 - 500\text{ kN} = (4\text{ m})\omega_1 + 26.494\text{ kN} - 500\text{ kN} = 0 \\ \Rightarrow \omega_1 &= \frac{473.51\text{ kN}}{4\text{ m}} = 118.38\text{ kN/m}\end{aligned}$$

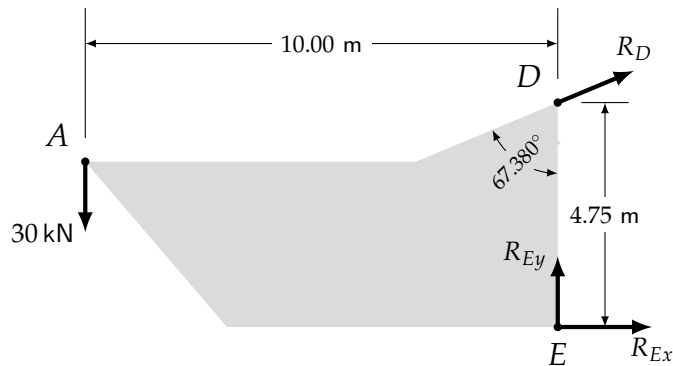
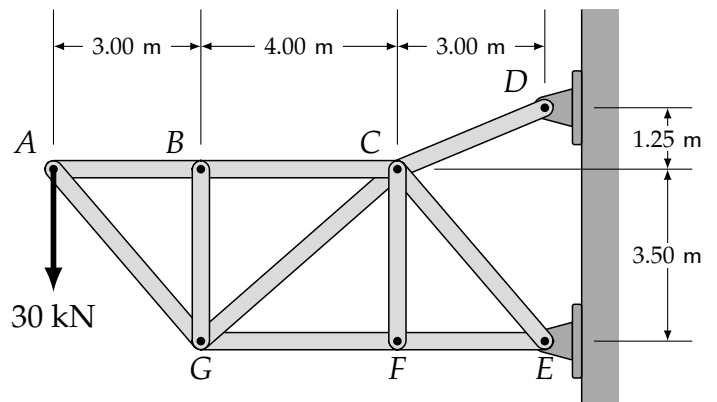
$$\omega_1 = \underline{118\text{ kN/m}}$$

$$\omega_2 = \underline{6.62\text{ kN/m}}$$

**Example 9:**

Determine the reactions at  $D$  and  $E$ .

**Note:** We cannot usually solve a system with two pinned connections because there are two unknowns. In this case, though,  $CD$  is a two-force member so we know its direction and the only unknown at  $D$  is the magnitude of the direction. We can treat  $CD$  as we would a cable or a strut.



$$\Sigma M_E = (30 \text{ kN}) \cdot (10.00 \text{ m}) - R_D \cdot (4.75 \text{ m} \cdot \sin 67.380^\circ) = 0$$

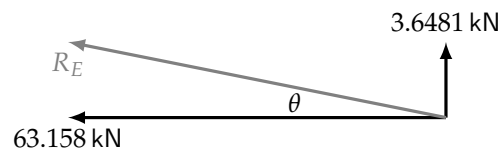
$$\Rightarrow R_D = \frac{(30 \text{ kN}) \cdot (10.00 \text{ m})}{4.75 \text{ m} \cdot \sin 67.380^\circ} = 68.421 \text{ kN}$$

$$\Sigma F_x = R_{Ex} + (68.421 \text{ kN}) \cdot \sin 67.380^\circ = 0$$

$$\Rightarrow R_{Ex} = -63.158 \text{ kN}$$

$$\Sigma F_y = R_{Ey} + (68.421 \text{ kN}) \cdot \cos 67.380^\circ - 30 \text{ kN} = 0$$

$$\Rightarrow R_{Ey} = 3.6481 \text{ kN}$$



$$R_E = \sqrt{(-63.158 \text{ kN})^2 + (3.6481 \text{ kN})^2} = 63.263 \text{ kN}$$

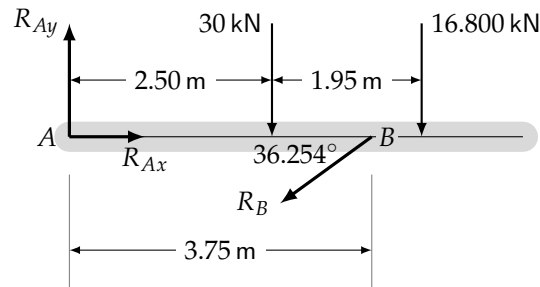
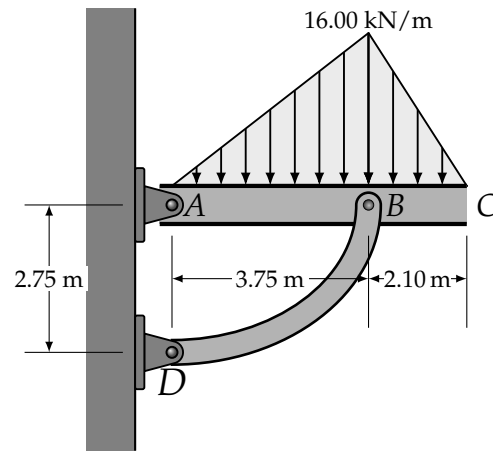
$$\theta = \tan^{-1} \left[ \frac{3.6481}{63.158} \right] = 3.3058^\circ$$

The reaction at  $E$  is **68.4 kN** at **22.6°**, measured counter-clockwise from the positive  $x$ -axis.

The reaction at  $D$  is **63.3 kN** at **177°**, measured counter-clockwise from the positive  $x$ -axis.

**Exercise 5:**

Determine the reactions at A and D.



$$\Sigma M_A = -(30 \text{ kN}) \cdot (2.50 \text{ m}) - (16.800 \text{ kN}) \cdot (4.45 \text{ m}) - R_B \cdot \sin 36.254^\circ \cdot (3.75 \text{ m}) = 0$$

$$\Rightarrow R_B = -67.532 \text{ kN}$$

$R_B$  is negative, so  $BD$  is in compression. We know that  $BD$  is a two-force member so the reaction at  $D$  is equal and opposite to that of the reaction at  $B$ .

$$\Sigma F_x = R_{Ax} - (-67.532 \text{ kN}) \cdot \cos 36.254^\circ = 0$$

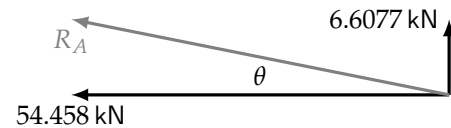
$$\Rightarrow R_{Ax} = -54.458 \text{ kN}$$

$$\Sigma F_y = R_{Ay} - 30 \text{ kN} - 16.800 \text{ kN} - (-67.532 \text{ kN}) \cdot \sin 36.254^\circ = 0$$

$$\Rightarrow R_{Ay} = 6.6077 \text{ kN}$$

$$R_A = \sqrt{(-54.458 \text{ kN})^2 + (6.6077 \text{ kN})^2} = 54.857 \text{ kN}$$

$$\theta = \tan^{-1} \left[ \frac{6.6077}{54.458} \right] = 6.9182^\circ$$



The reaction at  $E$  is **54.9 kN** at **173°**, measured counter-clockwise from the positive  $x$ -axis.

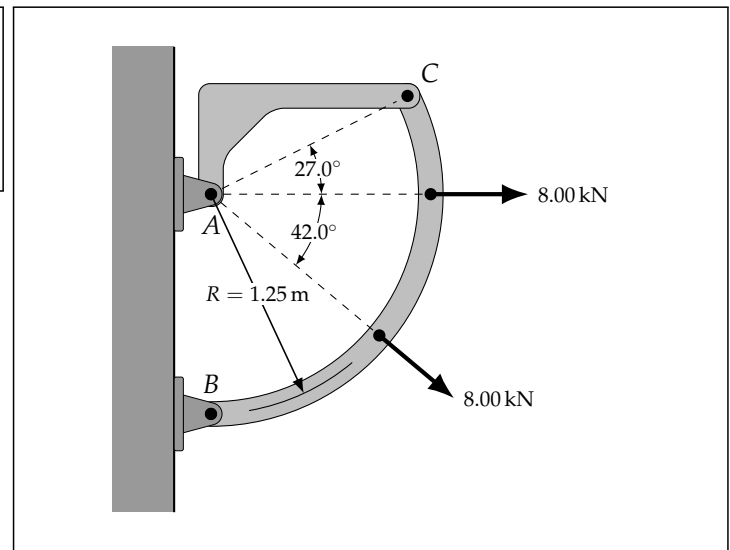
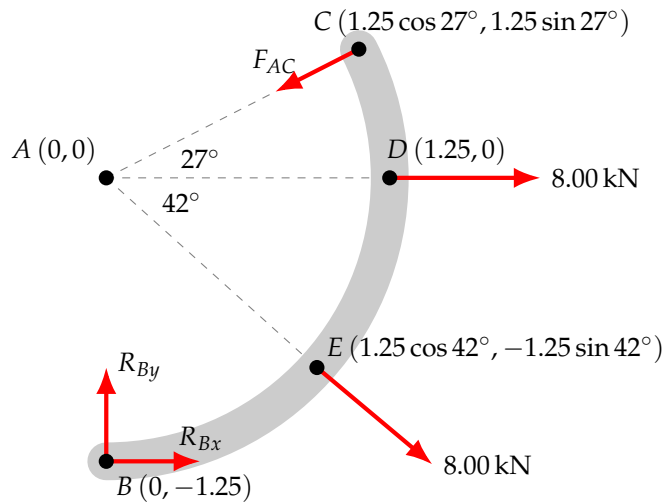
The reaction at  $D$  is **67.5 kN** at **36.3°**, measured counter-clockwise from the positive  $x$ -axis.



**Exercise 6:**

Member CD is a circular arc, centred at A

Determine the reactions at A and B.



$$\begin{aligned}
 \Sigma M_B &= F_{AC} \cdot \cos 27^\circ \cdot (1.25 \text{ m} + 1.25 \text{ m} \cdot \sin 27^\circ) - F_{AC} \cdot \sin 27^\circ \cdot (1.25 \text{ m} \cdot \cos 27^\circ) \\
 &\quad - 8.00 \text{ kN} \cdot (1.25 \text{ m}) \\
 &\quad - 8.00 \text{ kN} \cdot \cos 42^\circ \cdot (1.25 \text{ m} - 1.25 \text{ m} \cdot \sin 42^\circ) - 8.00 \text{ kN} \cdot \sin 42^\circ \cdot (1.25 \text{ m} \cdot \cos 42^\circ) \\
 &= F_{AC} \cdot \cos 27^\circ \cdot (1.8175 \text{ m}) - F_{AC} \cdot \sin 27^\circ \cdot (1.1138 \text{ m}) - 10.000 \text{ kN} \cdot \text{m} \\
 &\quad - 8.00 \text{ kN} \cdot \cos 42^\circ \cdot (0.41359 \text{ m}) - 8.00 \text{ kN} \cdot \sin 42^\circ \cdot (0.92893 \text{ m}) \\
 &= F_{AC} \cdot (1.1137 \text{ m}) - 17.431 \text{ kN} \cdot \text{m} = 0
 \end{aligned}$$

$$\Rightarrow F_{AC} = 15.651 \text{ kN}$$

$$\begin{aligned}
 \Sigma F_x &= R_{Bx} + 8.00 \text{ m} \cdot \cos 42^\circ + 8.00 \text{ m} - 15.651 \text{ kN} \cdot \cos 27^\circ = 0 \\
 \Rightarrow R_{Bx} &= 0
 \end{aligned}$$

$$\begin{aligned}
 \Sigma F_y &= R_{By} - 8.00 \text{ kN} \cdot \sin 42^\circ - 15.651 \text{ kN} \cdot \sin 27^\circ = 0 \\
 \Rightarrow R_{By} &= 12.458 \text{ kN}
 \end{aligned}$$

The reaction at A is **15.7 kN** at **207°**, measured counter-clockwise from the positive  $x$ -axis.

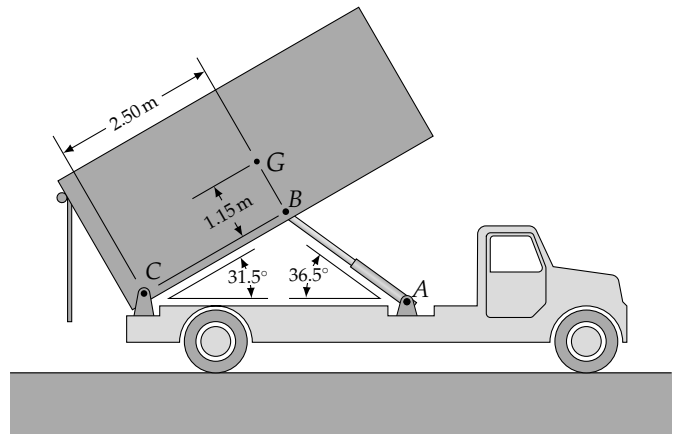
The reaction at B is **12.5 kN** at **90°**, measured counter-clockwise from the positive  $x$ -axis.

**Questions:**

1. Why is  $R_{AC}$  away from C instead of in the direction of C, cancelling out  $F_{AC}$ ?
2. Can you see that  $R_{Bx} = 0$  directly from the free body diagram?

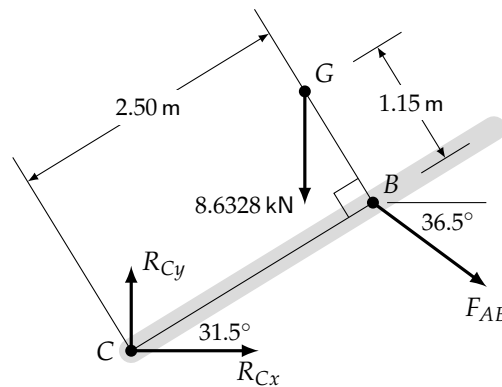
**Example 10:**

The bin of a dump-truck is being tipped by hydraulic lift  $AB$ . ( $AB$  can be considered a two-force member.) The bin rotates about a pin at  $C$ . Determine the force in the lift  $AB$  and find the reaction at  $C$ . The bin has a mass of 880 kg and  $G$  marks its centre of mass.



**Method 1:** Our usual approach.

$$M = 880 \text{ kg} \Rightarrow W = \frac{880 \times 9.81}{1000} = 8.6328 \text{ kN}$$



$$\begin{aligned} \Sigma M_C &= -(8.6328 \text{ kN}) \cdot (2.50 \text{ m} \cdot \cos 31.5^\circ - 1.15 \text{ m} \cdot \sin 31.5^\circ) \\ &\quad - F_{AB} \cdot \cos 36.5^\circ \cdot (2.50 \text{ m} \cdot \sin 31.5^\circ) \\ &\quad - F_{AB} \cdot \sin 36.5^\circ \cdot (2.50 \text{ m} \cdot \cos 31.5^\circ) \end{aligned}$$

$$= -13.214 \text{ kN} \cdot \text{m} - 2.3180 \text{ m} \cdot F_{AB} = 0$$

$$\Rightarrow F_{AB} = -5.7006 \text{ kN}$$

$$\Sigma F_x = R_{Cx} + (-5.7006 \text{ kN}) \cdot \cos 36.5^\circ = 0$$

$$\Rightarrow R_{Cx} = 4.5825 \text{ kN}$$

$$\Sigma F_y = R_{Cy} - 8.6328 \text{ kN} - (-5.7006 \text{ kN}) \cdot \sin 36.5^\circ = 0$$

$$\Rightarrow R_{Cy} = 5.2420 \text{ kN}$$

$$R_C = \sqrt{(4.5825 \text{ kN})^2 + (5.2420 \text{ kN})^2} = 6.9626 \text{ kN}$$

$$\theta = \tan^{-1} \left[ \frac{5.2420}{4.5825} \right] = 48.840^\circ$$

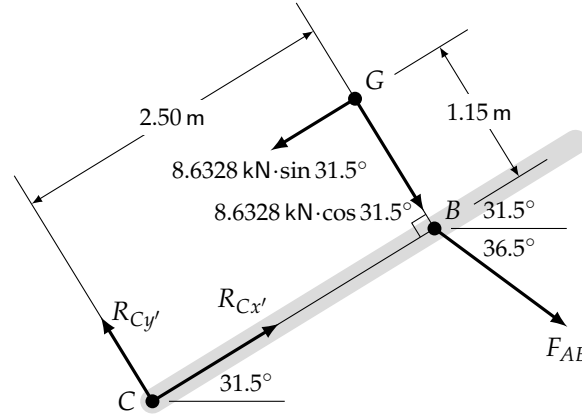
The force in  $AB$  is **5.70 kN** in **compression**.

The reaction at  $C$  is **6.96 kN** at **48.8°**, measured counter-clockwise from the positive  $x$ -axis.

**Method 2:** By rotation of our axes of reference.

Rotating the  $xy$  axes coordinate system, by  $31.5^\circ$  to a new  $x'y'$  axes system, makes for (possibly) clearer calculations.

$$M = 880 \text{ kg} \Rightarrow W = \frac{880 \times 9.81}{1000} = 8.6328 \text{ kN}$$



$$\begin{aligned} \Sigma M_C &= (8.6328 \text{ kN} \cdot \sin 31.5^\circ) \cdot (1.15 \text{ m}) - (8.6328 \text{ kN} \cdot \cos 31.5^\circ) \cdot (2.50 \text{ m}) - F_{AB} \cdot \sin 68^\circ \cdot 2.50 \text{ m} \\ &= -13.214 \text{ kN} \cdot \text{m} - F_{AB} \cdot 2.3180 \text{ m} = 0 \\ \Rightarrow F_{AB} &= -5.7006 \text{ kN} \end{aligned}$$

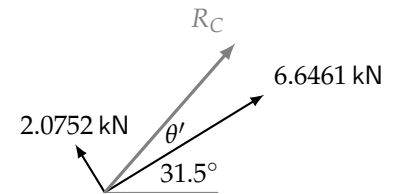
$$\begin{aligned} \Sigma F_{x'} &= R_{Cx'} + (-5.7006 \text{ kN}) \cdot \cos 68^\circ - 8.6328 \text{ kN} \cdot \sin 31.5^\circ = 0 \\ \Rightarrow R_{Cx'} &= 6.6461 \text{ kN} \end{aligned}$$

$$\begin{aligned} \Sigma F_{y'} &= R_{Cy'} - (-5.7006 \text{ kN}) \cdot \sin 68^\circ - 8.6328 \text{ kN} \cdot \cos 31.5^\circ = 0 \\ \Rightarrow R_{Cy'} &= 2.0752 \text{ kN} \end{aligned}$$

$$R_C = \sqrt{(2.0752 \text{ kN})^2 + (6.6461 \text{ kN})^2} = 6.9625 \text{ kN}$$

$$\theta' = \tan^{-1} \left[ \frac{2.0752}{6.6461} \right] = 17.341^\circ$$

$$\Rightarrow \theta = \theta' + 31.5^\circ = 48.841^\circ$$

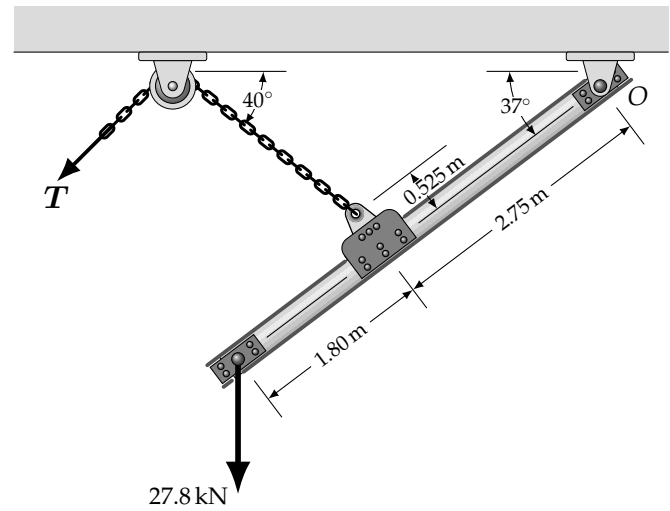


The force in  $AB$  is **5.70 kN** in **compression**.

The reaction at  $C$  is **6.96 kN** at **48.8°**, measured counter-clockwise from the positive  $x$ -axis.

**Exercise 7:**

The pulley is frictionless. Determine the tension  $T$  and the reaction at the pinned connection  $O$ .



**Method 1:**

