01 Math Review

Engineering Statics

Updated on: August 29, 2025

Statics and Math

- Statics is all math! All but the most trivial statics problems require algebra and/or trigonometry and/or geometry to solve.
- The good news is that the math is not very difficult. You won't need anything more advanced than high-school math.
- We will do a quick review here that should cover all the math you'll need for this course.

We frequently need to solve an equation for a particular variable (i.e., rearrange an equation to isolate a given variable)

Example

Solve $\delta = \dfrac{F \cdot L}{A \cdot E}$ for E

Solution:

Multiply both sides of the equation by E/δ. Then:

$$\cancel{\delta} \cdot \frac{E}{\cancel{\delta}} = \frac{F \cdot L}{A \cdot \cancel{E}} \cdot \frac{\cancel{E}}{\delta}$$

$$E = \frac{F \cdot L}{A \cdot \delta}$$

Algebraic Manipulation - Exercises

1. Solve $a^2 = b^2 + c^2$ for b.
2. Solve $V = \frac{4}{3}\pi r^3$ for r.
3. Solve $c^2 = a^2 + b^2 - 2bc\cos C$ for $\cos C$.
4. Solve $b^2 = a^2 + c^2 - 2ac\cos B$ for B.
5. One representation of the Hazen-Williams Equation for flow of water in a pipe is:

$$Q = \frac{CD^{2.63}\left(\frac{h_L}{L}\right)^{0.54}}{279000}$$

   Solve the equation for h_L, then evaluate h_L using the values $Q = 135$, $C = 120$, $D = 202.7$ and $L = 1200$.

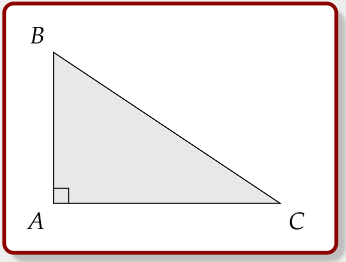
Triangles are a strong, stable shape and often used in engineering.

Triangles help avoid issues like this:

Triangles mean we need trigonometry.

Right Triangle

A **right triangle** is a triangle having one 90° angle.



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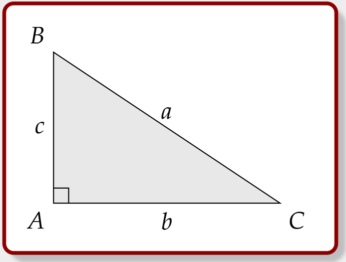

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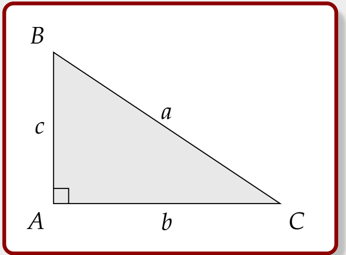
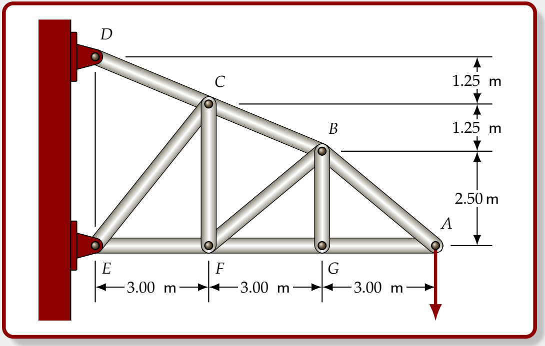


Label the three sides a, b and c. The side a, opposite the right angle, is called the **hypotenuse**.

If we know the lengths of any two sides, we can calculate the length of the third side using the **Pythagorean Theorem**:

$$a^2 = b^2 + c^2$$

Right Triangle Exercises (1)



6. Use the Pythagorean Theorem to determine the lengths of CE and CB

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- Significant digits are a measure of the **accuracy** of a number.

  It is **extremely important** to recognize that we can get no more accuracy out of a calculation than we put in. If the inputs to a problem have three significant digits, we cannot expect any higher accuracy than three significant digits in our result — even if the calculator does give ten digits.

Significant Digits

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Non-zero digits **are** significant:

- 1234 has 4 significant digits.
- 12.34 has 4 significant digits.

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Zeros between non-zero digits are significant

- 12034 has 5 significant digits.
- 12.0034 has 6 significant digits.

Significant Digits

Leading zeros are **not** *significant*

- 0.1234 has 4 significant digits.
- 0.0001234 has 4 significant digits.

Significant Digits

*Leading zeros are **not** significant*

- 0.1234 has 4 significant digits.
- 0.0001234 has 4 significant digits.

*Trailing zeros (after a decimal point) **are** significant*

- 1234.0 has 5 significant digits.
- 1.23400 has 6 significant digits.

Significant Digits

Trailing zeros (on whole numbers, i.e. integers) are more **complicated**

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  - Now consider 12.30 m. This value has 4 significant digits. But it is still equal to 12300 mm, so in this case the 12300 has 4 significant digits.

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  - What if 12300 mm refers to 12.300 m? Then it has 5 significant digits.
- Usually, the trailing zeros are placeholders for the magnitude of a value and we don't need to worry unduly.
- If we want to emphasize that 12300 has 4 significant digits, we can write $1.230 \times \left(10^3\right)$.

Calculations for Exercises

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- In practice, it is often difficult to measure objects more accurately than to three significant digits so **input values for exercises are generally given to 3 significant digits**.
- We cannot expect to get more accuracy in our result at the end of a calculation than from our given input values at the beginning of the calculation so **solutions should be correct to 3 significant digits, not more than the accuracy of the calculation inputs!**
- Intermediate calculations will accumulate rounding errors quickly if we use only three significant digits and these can affect the final result. **For intermediate calculations, use 5 or more significant digits.**

  (When I write solutions down, I use 5 significant digits for intermediate calculations. You may use more if it is more convenient for you, e.g., if you are storing intermediate results in your calculator.)

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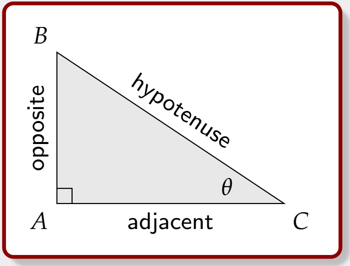
- When the first discarded digit is a 5 (or higher), round up the digit before the 5 (or higher)
- There are various rules (such as the odd-even rule) which take a more complicated approach to rounding 5 but, for our purposes, **5 rounds up**!

More About Right Triangle

The sine, cosine and tangent trigonometrical functions relate an acute angle (θ, in this example) in a right triangle to two of the sides of the triangle.



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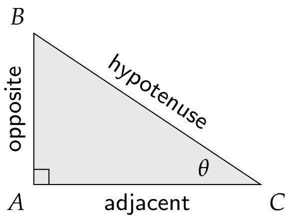
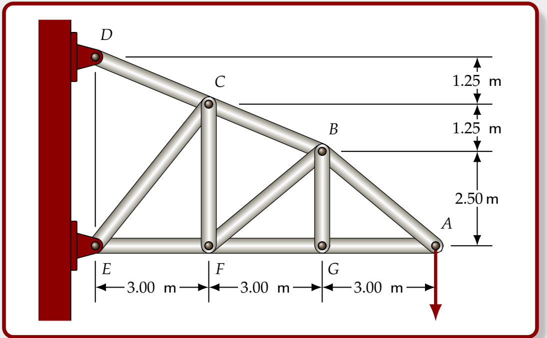


Right Triangle Trigonometry Formulæ

$$\sin\theta = \frac{\textbf{o}\text{pposite}}{\textbf{h}\text{ypotenuse}}, \quad \cos\theta = \frac{\textbf{a}\text{djacent}}{\textbf{h}\text{ypotenuse}}, \quad \tan\theta = \frac{\textbf{o}\text{pposite}}{\textbf{a}\text{djacent}}$$

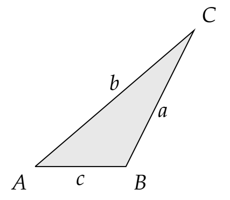
Remember: **SOHCAHTOA**



7. Use the **tangent** function to calculate $\angle CEF$.
8. From $\angle CEF$ just found (**use the intermediate, 5 or more significant digit, form!**) and the **sine** rule to verify the length of CE found earlier.
9. Use the **cosine** function and the length of CB found earlier to calculate the angle between BC and the horizontal.
10. Use the **tangent** function to verify the previous result.

Triangles - Sine Rule

Not all triangles contain a right angle. To solve for these triangles (finding the lengths of the side and the triangle angles), we have to employ some different tools: the **sine rule** and (later) the **cosine rule**



Sine Rule

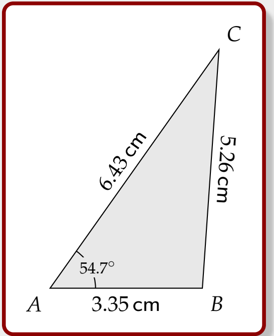
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

11. Using the sine rule, find $\angle ACB$.
12. Using the sine rule, find $\angle ABC$.
13. Sum the interior angles of the triangle.



Trig Identities

A couple of trig identities that will come in useful:

Identities

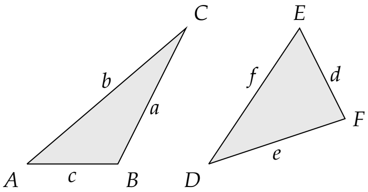
$$\sin(180^\circ - \theta) = \sin\theta$$
$$\cos(-\theta) = \cos\theta$$

Note:

$$\begin{aligned}\sin(140^\circ) &= \sin(40^\circ) &= 0.64279\\ \cos(42^\circ) &= \cos(-42^\circ) &= 0.74314\end{aligned}$$

Thus, we have to be careful when using inverse trigonometric functions:

$$\begin{aligned}\sin^{-1}(0.64279) &= 40^\circ \text{ or } 140^\circ\\ \cos^{-1} 0.74314) &= -42^\circ \text{ or } 42^\circ\end{aligned}$$

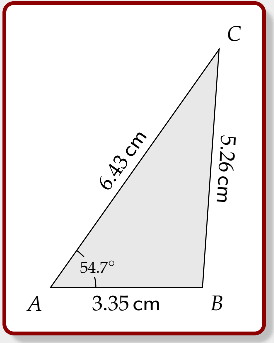


Cosine Rule

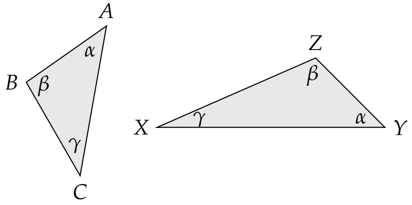
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

The cosine rule is useful when you have all the sides of a triangle and want to find the angles.

14. Determine $\angle ABC$, using the value for AB found earlier
15. Compare the value for $\angle ABC$ with the value calculated earlier. Is it the same? It should be!



Similar Triangles



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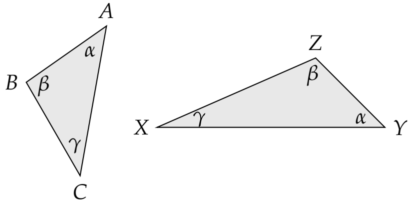
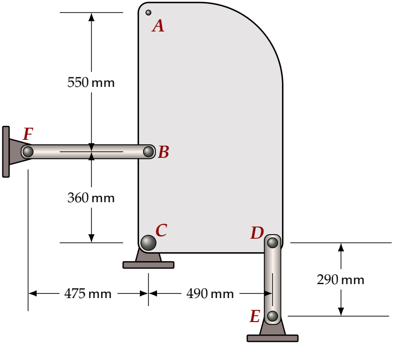


If triangles ABC and XYZ have the same angles, they are said to be **similar triangles**.

The ratios of the lengths of corresponding sides of similar triangles are equal:

$$\frac{AB}{XY} = \frac{BC}{XZ} = \frac{AC}{YZ}$$

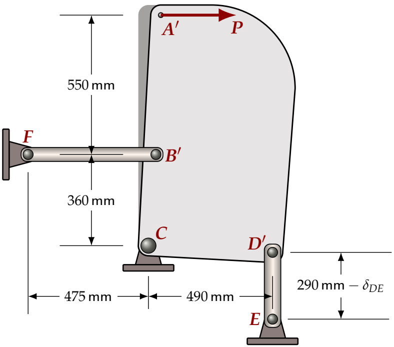
$ABCD$ is a rigid (i.e., it does not deform) plate, pinned at C.



$ABCD$ is a rigid (i.e., it does not deform) plate, pinned at C.

When horizontal force P is applied at A, $ABCD$ rotates about C and A deflects 2.45 mm horizontally rightwards.

Assume that BF remains horizontal and that DE remains vertical.



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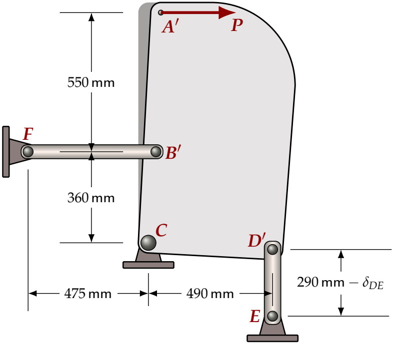
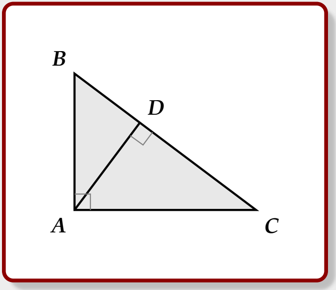


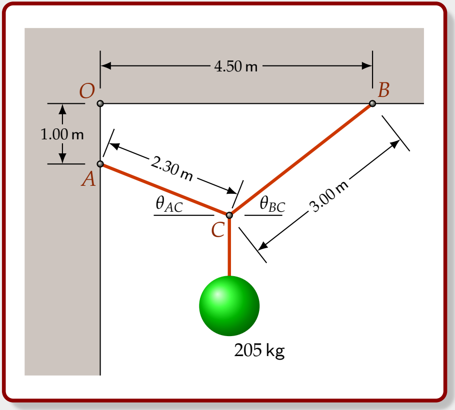
20. Determine δ_{BF}, the change in length of BF.
21. Determine δ_{DE}, the change in length of DE.

22. Show that right triangles $\triangle ABC$, $\triangle ABD$ and $\triangle ACD$ all have the same angles (i.e., they are all similar).
23. Given that $AC = 100$ mm and $AD = 65$ mm, determine $\angle ACD$ and $\angle ABD$.
24. Find the remaining lengths: AB, BD and CD.
25. Verify your lengths found above by using the Pythagorean Theorem on $\triangle ABC$

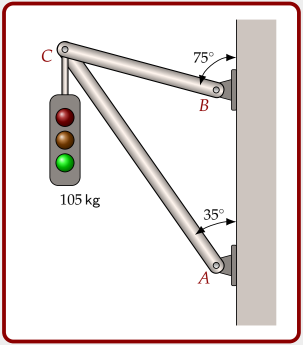


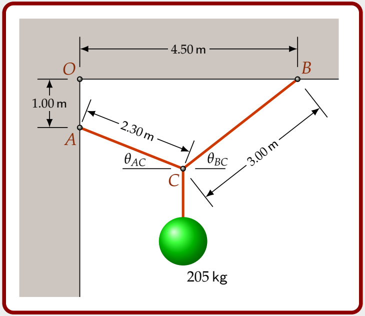
This is a standard type of statics problem to determine the forces in cables AC and BC. But first we have to find the angles θ_{AC} and θ_{BC}. This involves the use of the Pythagorean Theorem, the sine and cosine rules, and one of the trigonometric functions.

26. Find θ_{AC}.
27. Find θ_{BC}.



Simultaneous Equations





Calculating the forces in AC and BC in each of the examples shown involves solving two equations in two unknowns (also known as solving a system of simultaneous equations).

We'll review how to do this.

Solving Simultaneous Equations

This is a simple system of simultaneous equations:

$$2x + 3y = 7 \tag{1}$$

$$6x - y = 1 \tag{2}$$

Our objective is to find the value of x and y that satisfies both equations.

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Equation (1) is a straight line, with slope $-2/3$, that intersects the x axis at $x = 3.5$ and intersects the y axis at $y = 7/3$.

Equation (2) is a straight line, with slope 6, that intersects the x axis at $x = 1/6$ and intersects the y axis at $y = -1$.

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These lines have different slopes, so they must intersect somewhere. At the point (x, y) where they intersect, both equations are satisfied. This is the solution we're looking for.

Solving Simultaneous Equations (cont'd)

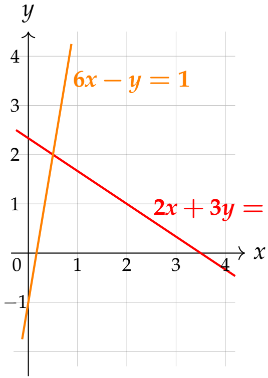
Graphically, the lines look like this.

It looks like the point at which they intersect is in the region of $(0.5, 2))$

We can check whether this is the correct solution be substituting the values of $x = 0.5$ and $y = 2$ to see whether they satisfy both equations.

Generally, we can solve algebraically using a procedure called the Method of Subsitution. Or use the **system-solver** on your calculator.

(The system-solver is allowed for quizzes and examinations; it saves time — and improves accuracy when rushed. Make sure you know how your calculator works. Trying to figure it out in an exam is not recommended!)



Solving Simultaneous Equations using the Method of Subsitution

$$2x + 3y = 7 \quad (1)$$
$$6x - y = 1 \quad (2)$$

The process:

(a) Choose an equation and solve for one of the variables. Here I choose equation (2) and solve for the variable y.

$$y = 6x - 1 \quad (3)$$

(b) Use equation (3) to substitute $6x - 1$ wherever y occurs in the other equation:

$$\begin{aligned} 2x + 3(6x - 1) &= 7 \\ 2x + 18x - 3 &= 7 \\ 20x &= 10 \\ x &= 0.5 \end{aligned}$$

(c) Substitute this value for x in either of equation (1) or (2):

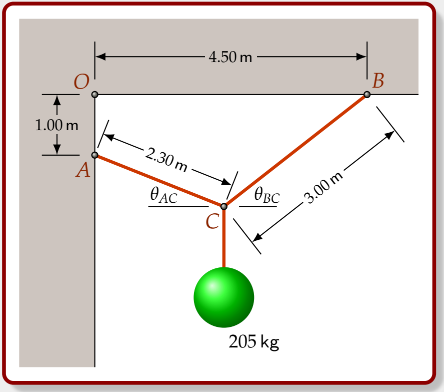
$$\begin{aligned} 6x - y &= 1 \\ 3d - y &= 1 \\ y &= 2 \end{aligned}$$

(d) We have our solution: $(0.5, 2)$.

Typical Statics System of Equations

In an earlier exercise, we calculated that $\theta_{AC} = 21.661^\circ$ and $\theta_{BC} = 38.049^\circ$.

(Note that we are using 5 significant digits here because we will be using these values for calculations.)



The system shown yields the following two equations:

$$F_{AC} \sin(21.661^\circ) + F_{BC} \sin(38.049^\circ) = 2011.1\ \mathrm{N}$$
$$F_{BC} \cos(38.049^\circ) - F_{AC} \cos(21.661^\circ) = 0$$

System of Equations Exercise (1)

This looks more difficult than our previous example.

$$F_{AC} \sin(21.661^\circ) + F_{BC} \sin(38.049^\circ) = 2011.1 \qquad (1)$$

$$F_{BC} \cos(38.049^\circ) - F_{AC} \cos(21.661^\circ) = 0 \qquad (2)$$

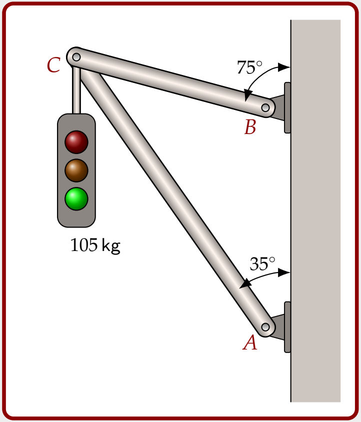
Fortunately, it only **looks** harder. F_{AC} and F_{BC} are variables, just like x and y in the earlier example. And the sines and cosines are just numbers. We get:

$$0.36911x + 0.61633y = 2011.1 \qquad (3)$$

$$0.78748y - 0.92938x = 0 \qquad (4)$$

This is the same system, with x replacing F_{AC}, y replacing F_{BC} and the trigonemetric functions evaluated.

28. Solve for x (F_{AC} in the original system)
29. Solve for y (F_{BC} in the original system)



The system shown yields the following two equations in the two unknowns F_{AC} and F_{BC}:

$$F_{BC}\sin 15^\circ + F_{AC}\cos 35^\circ + 1030.1 = 0$$
$$F_{BC}\cos 15^\circ + F_{AC}\sin 35^\circ = 0$$

30. Determine F_{AC}
31. Determine F_{BC}