

Engineering Statics - 01 Math Review Handout (Instructor Copy)

1) Solve $a^2 = b^2 + c^2$ for b .

$$b^2 = a^2 - c^2$$

$$b = \pm \sqrt{a^2 - c^2}$$

2) Solve $V = \frac{4}{3}\pi r^3$ for r .

$$r^3 = \frac{3V}{4\pi}$$

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

3) Solve $c^2 = a^2 + b^2 - 2bc \cos C$ for $\cos C$.

$$2bc \cos C = a^2 + b^2 - c^2$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2bc}$$

4) Solve $b^2 = a^2 + c^2 - 2ac \cos B$ for B .

$$2ac \cos B = a^2 + c^2 - b^2$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2bc}$$

$$B = \cos^{-1} \left(\frac{a^2 + c^2 - b^2}{2bc} \right)$$

5) Solve the equation for h_L , then evaluate h_L using the values $Q = 135$, $C = 120$, $D = 202.7$ and $L = 1200$

$$Q = \frac{CD^{2.63} \left(\frac{h_L}{L} \right)^{0.54}}{279000}$$

$$\Rightarrow \left(\frac{h_L}{L} \right)^{0.54} = \frac{279000Q}{CD^{2.63}}$$

$$\Rightarrow \frac{h_L}{L} = \left(\frac{279000Q}{CD^{2.63}} \right)^{1/0.54}$$

$$\Rightarrow h_L = L \left(\frac{279000Q}{CD^{2.63}} \right)^{1.85}$$

$$h_L = 1200 \left(\frac{279000 \times 135}{120 \times 202.7^{2.63}} \right)^{1.85}$$

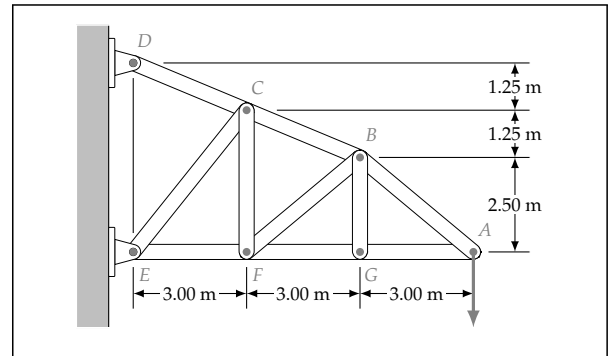
$$= 105.73$$

$$h_L = 106$$

6) Use the Pythagorean Theorem to determine the lengths of CE and CB

$$\begin{aligned} CE^2 &= EF^2 + CF^2 \\ &= (3.00 \text{ m})^2 + 3.75^2 \\ CE &= 4.8023 \\ &= \mathbf{4.80 \text{ m}} \end{aligned}$$

$$\begin{aligned} CB^2 &= 1.25^2 + 3.00^2 \\ CB &= 3.2500 \\ &= \mathbf{3.25 \text{ m}} \end{aligned}$$



7) Use the tangent function to calculate $\angle CEF$

$$\angle CEF = \tan^{-1} \left(\frac{CF}{EF} \right) = \tan^{-1} \left(\frac{3.75 \text{ m}}{3.00 \text{ m}} \right) = 51.340^\circ = \mathbf{51.3^\circ}$$

8) Use $\angle CEF$ just found and the sine function to verify the length of CE found above.

$$\begin{aligned} \frac{CD}{\sin 90^\circ} &= \frac{CF}{\sin \angle CEF} \\ \Rightarrow CE &= \frac{(3.75 \text{ m}) \sin 90^\circ}{\sin 51.340^\circ} \\ &= 4.8024 \\ \mathbf{CE} &= \mathbf{4.80 \text{ m}} \end{aligned}$$

9) Use the cosine function and the length of BC found earlier to calculate the angle, θ , between BC and the horizontal.

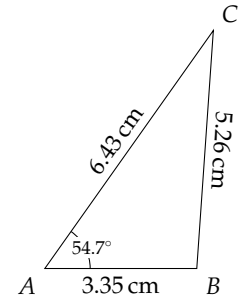
$$\begin{aligned} (1.25 \text{ m})^2 &= (3.00 \text{ m})^2 + (3.25 \text{ m})^2 - 2(3.00 \text{ m})(3.25 \text{ m}) \cos \theta \\ \Rightarrow \cos \theta &= \frac{(3.00 \text{ m})^2 + (3.25 \text{ m})^2 - (1.25 \text{ m})^2}{2(3.00 \text{ m})(3.25 \text{ m})} \\ &= 0.92308 \\ \Rightarrow \theta &= 22.619^\circ \\ \mathbf{\theta} &= \mathbf{22.6^\circ} \end{aligned}$$

10) Use the tangent function to verify the previous result.

$$\theta = \tan^{-1} \left(\frac{1.25 \text{ m}}{3.00 \text{ m}} \right) = 22.620 = \mathbf{22.6^\circ}$$

11) Using the sine rule, find $\angle ACB$.

$$\begin{aligned}\frac{\angle ACB}{3.35 \text{ cm}} &= \frac{\sin 54.7^\circ}{5.26 \text{ cm}} \\ \Rightarrow \angle ACB &= \sin^{-1} \left(\frac{(3.35 \text{ cm}) \sin 54.7^\circ}{5.26 \text{ cm}} \right) \\ &= 31.318^\circ \\ &= \mathbf{31.3^\circ}\end{aligned}$$



12) Using the sine rule, find $\angle ABC$.

$$\begin{aligned}\frac{\angle ABC}{6.43 \text{ cm}} &= \frac{\sin 54.7^\circ}{5.26 \text{ cm}} \\ \Rightarrow \angle ABC &= \sin^{-1} \left(\frac{(6.43 \text{ cm}) \sin 54.7^\circ}{5.26 \text{ cm}} \right) \\ &= 86.091^\circ \\ \angle ABC &= \mathbf{86.1^\circ} \quad (\text{or } 93.9(09)^\circ \dots)\end{aligned}$$

13) Sum the interior angles of the triangle.

$$= 54.7^\circ + 33.318^\circ + 86.091^\circ = \mathbf{172.11^\circ}$$

(Explain it's due to choice of $\angle ABC = 86.1^\circ$ from the inverse sin above. Or wait until $\angle ABC$ is found from the cos rule in the next exercise.)

14) Using the cosine rule, determine $\angle ABC$

15) Compare with the earlier value calculated for $\angle ABC$

$$\begin{aligned}(6.43 \text{ cm})^2 &= (3.35 \text{ cm})^2 + (5.26 \text{ cm})^2 - 2(3.35 \text{ cm})(5.26 \text{ cm}) \cos \angle ABC \\ \Rightarrow \angle ABC &= \cos^{-1} \left[\frac{(3.35 \text{ cm})^2 + (5.26 \text{ cm})^2 - (6.43 \text{ cm})^2}{2(3.35 \text{ cm})(5.26 \text{ cm})} \right] \\ &= 93.994^\circ \\ \angle ABC &= \mathbf{94.0^\circ}\end{aligned}$$

(Slightly different since from 93.9° since 54.7° is accurate to 3 significant digits and not a more precise given value.)

When horizontal force P is applied at A , $ABCD$ rotates about C and A deflects 2.45 mm horizontally rightwards.

Assume that BF remains horizontal and that DE remains vertical.

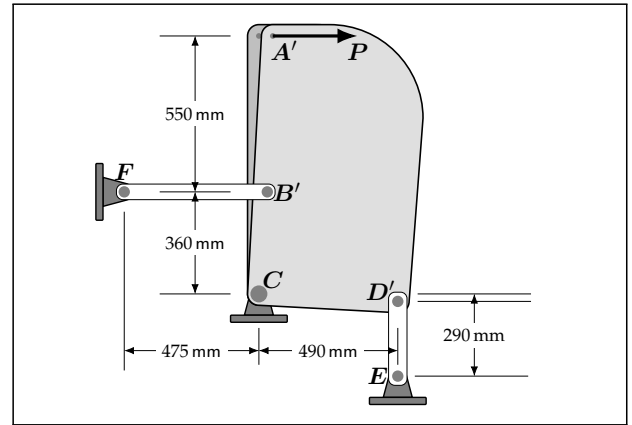
16) Determine δ_{BF} , the change in length of BF .

$\triangle CAA'$, $\triangle CBB'$ and $\triangle CDD'$ are all similar.

$$\frac{\delta_{BF}}{2.45 \text{ mm}} = \frac{360 \text{ mm}}{360 \text{ mm} + 550 \text{ mm}}$$

$$\Rightarrow \delta_{BF} = 0.96923 \text{ mm}$$

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17) Determine δ_{DE} , the change in length of DE .

$$\frac{\delta_{DE}}{2.45 \text{ mm}} = \frac{490 \text{ mm}}{360 \text{ mm} + 550 \text{ mm}}$$

$$\Rightarrow \delta_{BF} = 1.3192$$

$$= 1.32 \text{ mm}$$

18) Show that right triangles $\triangle ABC$, $\triangle ABD$ and $\triangle ACD$ all have the same angles (i.e. they are all similar).

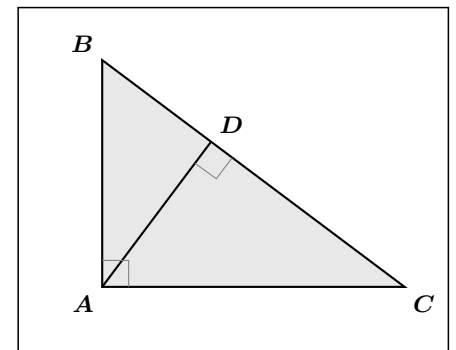
$$\text{Let } \angle DCA = \theta$$

$$\text{Then } \angle DAC = 90^\circ - \theta$$

$$\Rightarrow \angle DAB = \theta$$

$$\Rightarrow \angle DBA = 90^\circ - \theta$$

Each of the three triangles has angles θ , $90^\circ - \theta$ and 90° , so they are similar.



19) Given that $AC = 100 \text{ mm}$ and $AD = 65 \text{ mm}$, determine $\angle ACD$ and $\angle ABD$.

$$\angle ACD = \sin^{-1} \left(\frac{65 \text{ mm}}{100 \text{ mm}} \right) = 40.542^\circ = 40.5^\circ.$$

$$\angle ABD = 90^\circ - 40.542^\circ = 49.458^\circ = 49.5^\circ.$$

20) Find the remaining lengths: AB , BD and CD .

$$\frac{AB}{100 \text{ mm}} = \tan 40.542^\circ \Rightarrow AB = 85.263 \text{ mm} = \mathbf{85.2 \text{ mm}}$$

$$\frac{BD}{65 \text{ mm}} = \tan 40.542^\circ \Rightarrow BD = 55.598 \text{ mm} = \mathbf{55.6 \text{ mm}}$$

$$\frac{DC}{65 \text{ mm}} = \tan (90^\circ - 40.542^\circ) \Rightarrow DC = 75.993 \text{ mm} = \mathbf{76.0 \text{ mm}}$$

21) Verify the lengths found above by using the Pythagorean Theorem on $\triangle ABC$

$$AC^2 + AB^2 = (100 \text{ mm})^2 + (85.263 \text{ mm})^2 = 17270 \text{ mm}^2 = BC^2$$

$$\Rightarrow BC = 131.42 \text{ mm}$$

$$BD + CD = 55.598 \text{ mm} + 75.993 \text{ mm} = BC \Rightarrow BC = 131.59 \text{ mm}$$

An accumulation of rounding errors can start to cause differences when there are so many calculations and so many rounded intermediate values to use.

22) Find θ_{AC} .

23) Find θ_{BC} .

$$AB^2 = OA^2 + OB^2$$

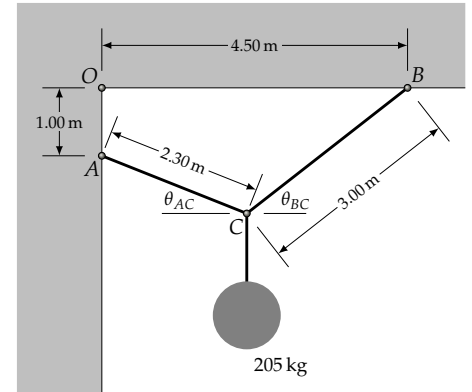
$$= (1.00 \text{ m})^2 + (4.50 \text{ m})^2$$

$$\Rightarrow AB = 4.6098 \text{ m}$$

$$\cos \angle ACB = \frac{AB^2 + BC^2 - AC^2}{2(AB)(BC)}$$

$$= \frac{(2.30 \text{ m})^2 + (3.00 \text{ m})^2 - (4.6098 \text{ m})^2}{2(2.30 \text{ m})(3.00 \text{ m})}$$

$$\Rightarrow \angle ACB = 120.29^\circ$$



$$\angle OBA = \tan^{-1} \left(\frac{1.00 \text{ m}}{4.50 \text{ m}} \right) = 12.529^\circ$$

$$\frac{\sin \angle ABC}{AC} = \frac{\sin \angle ACB}{AB} \Rightarrow \frac{\sin \angle ABC}{2.30 \text{ m}} = \frac{\sin 120.29^\circ}{4.6098 \text{ m}} \Rightarrow \angle ABC = 25.520^\circ$$

$$\theta_{BC} = \angle OBA + \angle ABC = 12.529^\circ + 25.520^\circ = 38.049^\circ \quad (\text{Alternate angles.})$$

$$\theta_{AC} = 180^\circ - \angle ACB - \theta_{BC} = 180^\circ - 120.29^\circ - 38.049^\circ = 21.661^\circ$$

$$\theta_{AC} = 21.7^\circ, \quad \theta_{BC} = 38.0^\circ$$

24) and 25) Find the values of x and y in the system shown.

$$0.36911x + 0.61633y = 2011.1 \quad (1)$$

$$0.78748y - 0.92938x = 0 \quad (2)$$

From (2),

$$y = \frac{0.92938}{0.78748}x = 1.1802x \quad (3)$$

Substitute in (1) to find x

$$0.36911x + 0.61633(1.1802x) = 2011.1$$

$$\Rightarrow (0.36911 + 0.61633(1.1802))x = 2011.1$$

$$\Rightarrow x = \frac{2011.1}{(0.36911 + 0.61633(1.1802))}$$

$$= 1834.1$$

$$x = 1830$$

Substitute back in (3) for y

$$0.78748y - 0.92938(1834.1) = 0$$

$$y = 2164.6$$

$$y = 2160$$

26) and 27) Determine F_{AC} and F_{BC}

$$F_{BC} \sin 15^\circ + F_{AC} \cos 35^\circ + 1030.1 = 0 \quad (1)$$

$$F_{BC} \cos 15^\circ + F_{AC} \sin 35^\circ = 0 \quad (2)$$

From (2)

$$F_{BC} = -\frac{\sin 35^\circ}{\cos 15^\circ}F_{AC} = -0.59381F_{AC} \quad (3)$$

Substitute into (1)

$$(-0.59381F_{AC}) \sin 15^\circ + F_{AC} \cos 35^\circ = -1030.1$$

$$-0.15369F_{AC} + 0.81915F_{AC} = -1030.1$$

$$\Rightarrow F_{AC} = \frac{-1030.1}{-0.15369 + 0.81915}$$

$$= -1548.0$$

$$= -1550$$

Substitute back into (3)

$$F_{BC} = -0.59381(-1548.0) = 919.19 = 919$$