Engineering Statics - 01 Math Review Handout (Instructor Copy)

1) Solve
$$a^2 = b^2 + c^2$$
 for *b*.

$$b^2 = a^2 - c^2$$
$$b = \pm \sqrt{a^2 - c^2}$$

2) Solve
$$V = \frac{4}{3}\pi r^3$$
 for *r*.

$$r^3 = \frac{3V}{4\pi}$$

$$r=\sqrt[3]{rac{3V}{4\pi}}$$

3) Solve
$$c^2 = a^2 + b^2 - 2bc \cos C$$
 for $\cos C$.

$$2bc \cos C = a^2 + b^2 - c^2$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2bc}$$

4) Solve
$$b^2 = a^2 + c^2 - 2ac \cos B$$
 for B.

$$2ac\cos B = a^2 + c^2 - b^2$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2bc}$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2bc}\right)$$

5) Solve the equation for h_L , then evaluate h_L using the values Q=135, C=120, D=202.7 and L=1200

$$Q = \frac{CD^{2.63} \left(\frac{h_L}{L}\right)^{0.54}}{279000}$$

$$\Rightarrow \left(\frac{h_L}{L}\right)^{0.54} = \frac{279000Q}{CD^{2.63}}$$

$$\Rightarrow \frac{h_L}{L} = \left(\frac{279000Q}{CD^{2.63}}\right)^{1/0.54}$$

$$\Rightarrow h_L = L \left(\frac{279000Q}{CD^{2.63}}\right)^{1.85}$$

$$h_L = 1200 \left(\frac{279000 \times 135}{120 \times 202.7^{2.63}}\right)^{1.85}$$

$$= 105.73$$

$$h_L = 106$$

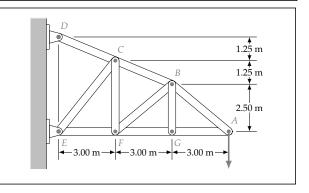
6) Use the Pythagorean Theorem to determine the lengths of *CE* and *CB*

$$CE^2 = EF^2 + CF^2$$

= $(3.00 \text{ m})^2 + 3.75^2$
 $CE = 4.8023$
= 4.80 m

$$CB^2 = 1.25^2 + 3.00^2$$

 $CB = 3.2500$
= 3.25 m



7) Use the tangent function to calculate ∠*CEF*

$$\angle CEF = \tan^{-1}\left(\frac{CF}{EF}\right) = \tan^{-1}\left(\frac{3.75 \text{ m}}{3.00 \text{ m}}\right) = 51.340^{\circ} = 51.3^{\circ}$$

8) Use $\angle CEF$ just found and the sine function to verify the length of CE found above.

$$\frac{CD}{\sin 90^{\circ}} = \frac{CF}{\sin \angle CEF}$$

$$\Rightarrow CE = \frac{(3.75 \text{ m}) \sin 90^{\circ}}{\sin 51.340^{\circ}}$$

$$= 4.8024$$

$$CE = 4.80 \text{ m}$$

9) Use the cosine function and the length of BC found earlier to calculate the angle, θ , between BC and the horizontal.

$$(1.25 \,\mathrm{m})^2 = (3.00 \,\mathrm{m})^2 + (3.25 \,\mathrm{m})^2 - 2(3.00 \,\mathrm{m})(3.25 \,\mathrm{m}) \cos \theta$$

$$\Rightarrow \cos \theta = \frac{(3.00 \,\mathrm{m})^2 + (3.25 \,\mathrm{m})^2 - (1.25 \,\mathrm{m})^2}{2(3.00 \,\mathrm{m})(3.25 \,\mathrm{m})}$$

$$= 0.92308$$

$$\Rightarrow \theta = 22.619^{\circ}$$

$$\theta = 22.6^{\circ}$$

10) Use the tangent function to verify the previous result.

$$\theta = \tan^{-1} \left(\frac{1.25 \,\mathrm{m}}{3.00 \,\mathrm{m}} \right) = 22.620 = 22.6^{\circ}$$

3

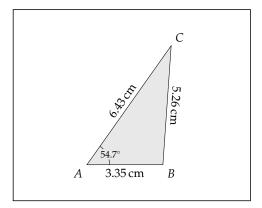
11) Using the sine rule, find $\angle ACB$.

$$\frac{\angle ACB}{3.35 \text{ cm}} = \frac{\sin 54.7^{\circ}}{5.26 \text{ cm}}$$

$$\Rightarrow \angle ACB = \sin^{-1} \left(\frac{(3.35 \text{ cm}) \sin 54.7^{\circ}}{5.26 \text{ cm}} \right)$$

$$= 31.318^{\circ}$$

$$= 31.3^{\circ}$$



12) Using the sine rule, find $\angle ABC$.

$$\frac{\angle ABC}{6.43 \text{ cm}} = \frac{\sin 54.7^{\circ}}{5.26 \text{ cm}}$$

$$\Rightarrow \angle ABC = \sin^{-1} \left(\frac{(6.43 \text{ cm}) \sin 54.7^{\circ}}{5.26 \text{ cm}} \right)$$

$$= 86.091^{\circ}$$

$$\angle ABC = 86.1^{\circ} \qquad (\text{or } 93.9(09)^{\circ}...)$$

13) Sum the interior angles of the triangle.

$$=54.7^{\circ} + 33.318^{\circ} + 86.091^{\circ} = 172.11^{\circ}$$

(Explain it's due to choice of $\angle ABC = 86.1^{\circ}$ from the inverse sin above. Or wait until $\angle ABC$ is found from the cos rule in the next exercise.)

- **14)** Using the cosine rule, determine $\angle ABC$
- **15)** Compare with the earlier value calculated for $\angle ABC$

$$(6.43 \,\mathrm{cm})^2 = (3.35 \,\mathrm{cm})^2 + (5.26 \,\mathrm{cm})^2 - 2(3.35 \,\mathrm{cm})(5.26 \,\mathrm{cm}) \cos \angle ABC$$

$$\Rightarrow \angle ABC = \cos^{-1} \left[\frac{(3.35 \,\mathrm{cm}^2) + (5.26 \,\mathrm{cm})^2 - (6.43 \,\mathrm{cm})^2}{2(3.35 \,\mathrm{cm})(5.26 \,\mathrm{cm})} \right]$$

$$= 93.994^{\circ}$$

$$\angle ABC = 94.0^{\circ}$$

(Slightly different since from 93.9° since 54.7° is accurate to 3 significant digits and not a more precise given value.)

4

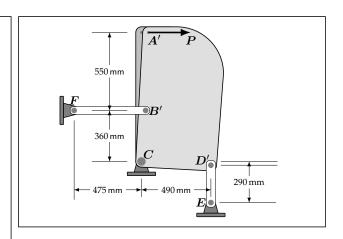
When horizontal force *P* is applied at *A*, *ABCD* rotates about *C* and *A* deflects 2.45 mm horizontally rightwards.

Assume that *BF* remains horizontal and that *DE* remains vertical.

16) Determine δ_{BF} , the change in length of BF.

 $\triangle CAA'$, $\triangle CBB'$ and $\triangle CDD'$ are all similar.

$$rac{\delta_{BF}}{2.45\,\mathrm{mm}} = rac{360\,\mathrm{mm}}{360\,\mathrm{mm} + 550\,\mathrm{mm}}$$
 $\Rightarrow \delta_{BF} = 0.96923\,\mathrm{mm}$
 $egin{equation} \delta_{BF} = 0.969\,\mathrm{mm} \end{gathered}$



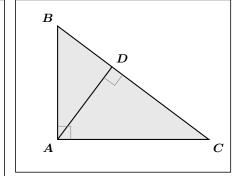
17) Determine δ_{DE} , the change in length of DE.

$$\begin{split} \frac{\delta_{DE}}{2.45\,\mathrm{mm}} &= \frac{490\,\mathrm{mm}}{360\,\mathrm{mm} + 550\,\mathrm{mm}} \\ \Rightarrow \delta_{BF} &= 1.3192 \\ &= \mathbf{1.32}\,\mathrm{mm} \end{split}$$

18) Show that right triangles $\triangle ABC$, $\triangle ABD$ and $\triangle ACD$ all have the same angles (i.e. they are all similar).

Let
$$\angle DCA = \theta$$

Then $\angle DAC = 90^{\circ} - \theta$
 $\Rightarrow \angle DAB = \theta$
 $\Rightarrow \angle DBA = 90^{\circ} - \theta$



Each of the three triangles has angles θ , $90^{\circ} - \theta$ and 90° , so they are similar.

19) Given that AC = 100 mm and AD = 65 mm, determine $\angle ACD$ and $\angle ABD$.

5

$$\angle ACD = \sin^{-1}\left(\frac{65 \text{ mm}}{100 \text{ mm}}\right) = 40.542^{\circ} = 40.5^{\circ}.$$

 $\angle ABD = 90^{\circ} - 40.542^{\circ} = 49.458^{\circ} = 49.5^{\circ}.$

20) Find the remaining lengths: *AB*, *BD* and *CD*.

$$\frac{AB}{100\,\mathrm{mm}} = \tan 40.542^\circ \Rightarrow AB = 85.263\,\mathrm{mm} = 85.2\,\mathrm{mm}$$

$$\frac{BD}{65\,\mathrm{mm}} = \tan 40.542^\circ \Rightarrow BD = 55.598\,\mathrm{mm} = 55.6\,\mathrm{mm}$$

$$\frac{DC}{65\,\mathrm{mm}} = \tan \left(90^\circ - 40.542^\circ\right) \Rightarrow DC = 75.993\,\mathrm{mm} = 76.0\,\mathrm{mm}$$

21) Verify the lengths found above by using the Pythagorean Theorem on $\triangle ABC$

$$AC^2 + AB^2 = (100 \text{ mm})^2 + (85.263 \text{ mm})^2 = 17270 \text{ mm}^2 = BC^2$$

 $\Rightarrow BC = 131.42 \text{ mm}$
 $BD + CD = 55.598 \text{ mm} + 75.993 \text{ mm} = BC \Rightarrow BC = 131.59 \text{ mm}$

An accumulation of rounding errors can start to cause differences when there are so many calculations and so many rounded intermediate values to use.

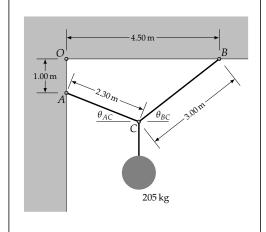
22) Find θ_{AC} . **23)** Find θ_{BC} .

$$AB^2 = OA^2 + OB^2$$

= $(1.00 \text{ m})^2 + (4.50 \text{ m})^2$
 $\Rightarrow AB = 4.6098 \text{ m}$

$$\cos \angle ACB = \frac{AB^2 + BC^2 - AB^2}{2(AB)(BC)}$$
$$= \frac{(2.30 \text{ m})^2 + (3.00 \text{ m})^2 - (4.6098 \text{ m})^2}{2(2.30 \text{ mm})(3.00 \text{ m}^2)}$$

$$\Rightarrow \angle ACB = 120.29^{\circ}$$



$$\angle OBA = \tan^{-1}\left(\frac{1.00 \text{ m}}{4.50 \text{ m}}\right) = 12.529^{\circ}$$

$$\frac{\sin ABC}{AC} = \frac{\sin ACB}{AB} \Rightarrow \frac{\sin ABC}{2.30 \text{ m}} = \frac{\sin 120.29^{\circ}}{4.6098 \text{ m}} \Rightarrow \angle ABC = 25.520^{\circ}$$

$$\theta_{BC} = \angle OBA + \angle ABC = 12.259^{\circ} + 25.520^{\circ} = 38.049^{\circ} \qquad \text{(Alternate angles.)}$$

$$\theta_{AC} = 180^{\circ} - \angle ACB - \theta_{BC} = 180^{\circ} - 120.29^{\circ} - 38.049^{\circ} = 21.661^{\circ}$$

$$\theta_{AC} = 21.7^{\circ}, \quad \theta_{BC} = 38.0^{\circ}$$

24) and **25)** Find the values of
$$x$$
 and y in the system shown.

$$0.36911x + 0.61633y = 2011.1$$

(1)

(2)

$$0.78748y - 0.92938x = 0$$

From (2),

$$y = \frac{0.92938}{0.78748}x = 1.1802x\tag{3}$$

Substitute in (1) to find x

$$0.36911x + 0.61633(1.1802x) = 2011.1$$

$$\Rightarrow (0.36911 + 0.61633(1.1802)) x = 2011.1$$

$$\Rightarrow x = \frac{2011.1}{(0.36911 + 0.61633(1.1802))}$$

$$= 1834.1$$

$$x = 1830$$

Substitute back in (3) for *y*

$$0.78748y - 0.92938(1834.1) = 0$$

 $y = 2164.6$
 $y = 2160$

$$F_{BC}\sin 15^{\circ} + F_{AC}\cos 35^{\circ} + 1030.1 = 0$$
 (1)

26) and **27)** Determine F_{AC} and F_{BC}

$$F_{BC}\cos 15^{\circ} + F_{AC}\sin 35^{\circ} = 0 \quad (2)$$

From (2)

$$F_{BC} = -\frac{\sin 35^{\circ}}{\cos 15^{\circ}} F_{AC} = -0.59381 F_{AC} \tag{3}$$

Substitute into (1)

$$(-0.59381F_{AC}) \sin 15^{\circ} + F_{AC} \cos 35^{\circ} = -1030.1$$

$$-0.15369F_{AC} + 0.81915F_{AC} = -1030.1$$

$$\Rightarrow F_{AC} = \frac{-1030.1}{-0.15369 + 0.81915}$$

$$= -1548.0$$

$$= -1550$$

Substitute back into (3)

$$F_{BC} = -0.59381(-1548.0) = 919.19 = \mathbf{919}$$