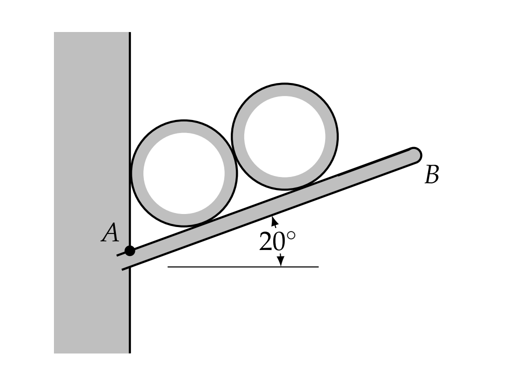
Engineering Statics - 06 Equilibrium of Rigid Bodies - Instructor Copy

Example 5: Pipe racks (AB, and two hidden behind it) support two smooth Schedule 40 pipes, with an outside diameter of 508 mm, as shown. The pipes are 10 m in length with a mass of 78.5 kg/m. Each rack supports one-third of the weight of each pipe.

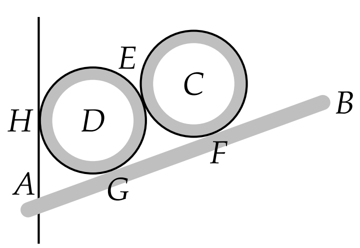
Determine the reaction at the fixed connection A.

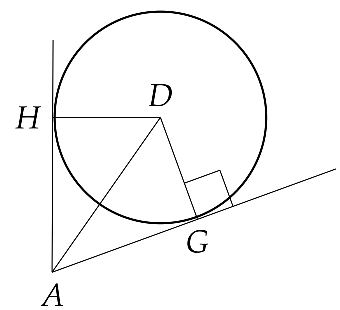


The weight of each pipe bearing on AB:

$$W = 78.5\,\mathrm{kg/m} \times 9.81\,\mathrm{m/s^2} \times 10\,\mathrm{m}/3 = 2.5670\,\mathrm{kN}$$

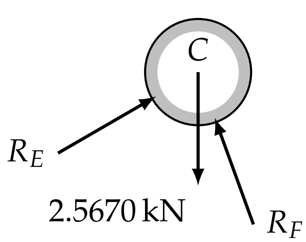
Add some labels, find some distances:

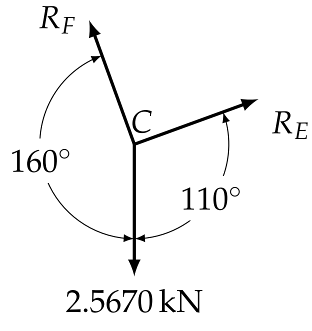




$$\angle HAD = \angle GAD = 35^\circ$$
$$\angle GAD = 55^\circ$$
$$\frac{AG}{GD} = \tan 55^\circ$$
$$AG = \frac{508\,\mathrm{mm}}{2}\tan 55^\circ$$
$$= 362.75\,\mathrm{mm}$$
$$GF = CD = 508\,\mathrm{mm}$$

Forces acting upon the upper (rightmost) pipe, C:





This is now a simple concurrent forces problem, solved with simultaneous equations. Notice, however, that the direction of R_F is perpendicular to the direction of R_E.

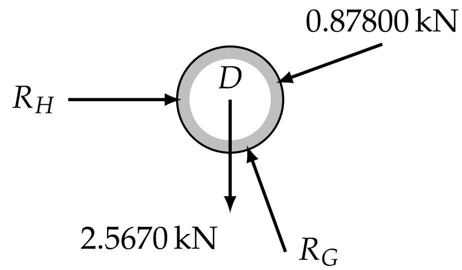
If we choose axes x' and y', rotated 20° in the counter clockwise direction around C, then the direction of R_E is the x'-axis and the direction of R_F is the y'-axis. Now we can solve without simultaneous equations.

(Why bother complicating things? This will become a useful technique towards the end of the module and it's easy to introduce here.)

$$\Sigma F_{x'} = R_E - 2.5670\,\mathrm{kN}\cdot\cos 70^\circ = 0$$
$$\Rightarrow R_E = 0.87800\,\mathrm{kN}$$

$$\Sigma F_{y'} = R_F - 2.5670\,\mathrm{kN}\cdot\cos 20^\circ = 0$$
$$\Rightarrow R_F = 2.4122\,\mathrm{kN}$$

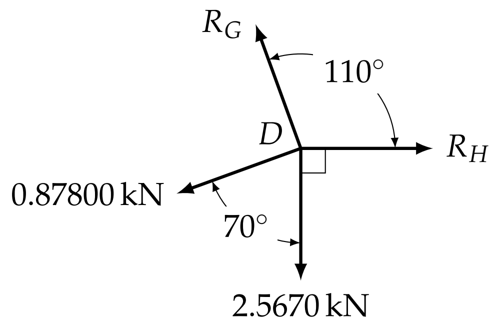
Forces acting upon the lower (leftmost) pipe, D:

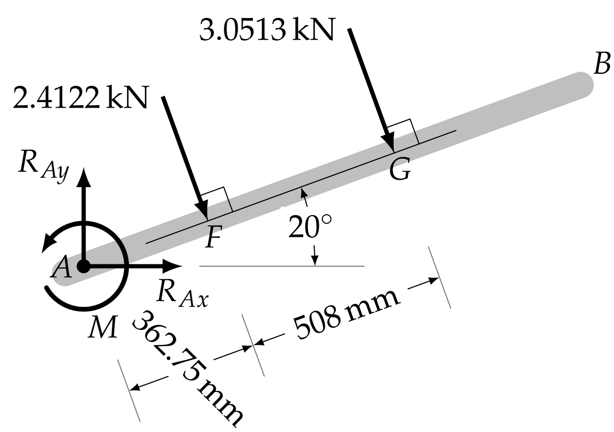


$$\Sigma F_y = R_G \cdot \cos 20^\circ - 2.5670\,\text{kN} - 0.87800\,\text{kN} \cdot \cos 70^\circ = 0$$

$$\Rightarrow R_G = \frac{2.5670\,\text{kN} + 0.87800\,\text{kN} \cdot \cos 70^\circ}{\cos 20^\circ}$$

$$\Rightarrow R_G = 3.0513\,\text{kN}$$

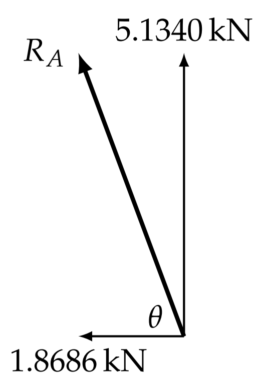




$$\Sigma M_A = M - 2.4122\,\text{kN} \cdot 362.75\,\text{mm} - 3.0513\,\text{kN} \cdot 870.75\,\text{mm} = 0 \Rightarrow M = 3531.9\,\text{kN} \cdot \text{mm} = 3.5319\,\text{kN} \cdot \text{m}$$

$$\Sigma F_x = R_{Ax} + 2.4122\,\text{kN} \cdot \sin 20^\circ + 3.0513\,\text{kN} \cdot \sin 20^\circ = 0 \Rightarrow R_{Ax} = -1.8686\,\text{kN}$$

$$\Sigma F_x = R_{Ay} - 2.4122\,\text{kN} \cdot \cos 20^\circ - 3.0513\,\text{kN} \cdot \cos 20^\circ = 0 \Rightarrow R_{Ay} = 5.1340\,\text{kN}$$



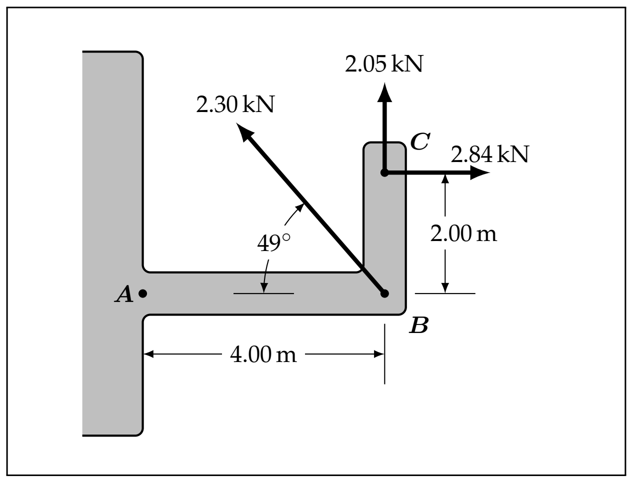
$$R_A = \sqrt{(1.8686\,\text{kN})^2 + (5.1340\,\text{kN})^2} = 5.4635\,\text{kN}$$

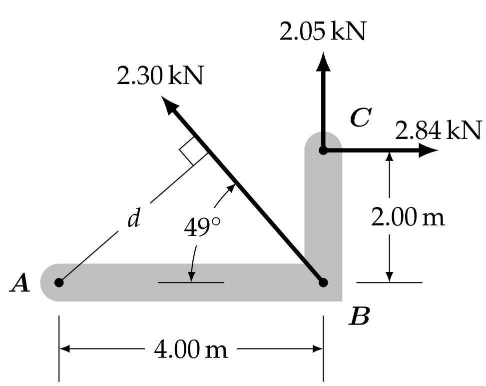
$$\theta = \tan^{-1}\left[\frac{5.1340\,\text{kN}}{1.8686\,\text{kN}}\right] = 70^\circ$$

The reaction at A is 5.46 kN at 110° counter-clockwise from the positive x-axis. The reacting moment at A is 3.53 kN·m

Example 2: Determine the sum of the moments of the forces, acting at B and C, about the point A.

Also, sum the moments of the forces about the point B.





$$\begin{aligned}
\Sigma M_A &= \Sigma F \cdot d \\
&= (2.30\,\text{kN}) \cdot (4.00\,\text{m})(\sin 49^\circ) \\
&\qquad + (2.05\,\text{kN}) \cdot (4.00\,\text{m}) - (2.84\,\text{kN}) \cdot (2.00\,\text{m}) \\
&= 9.4633\,\mathsf{kN \cdot m} \approx 9.46\,\mathsf{kN \cdot m}
\end{aligned}$$

$$\Sigma M_B = 0 + 0 - (2.84\,\text{kN}) \cdot (2.00\,\text{m}) = -5.6800\,\mathsf{kN \cdot m} = -5.68\,\mathsf{kN \cdot m}$$