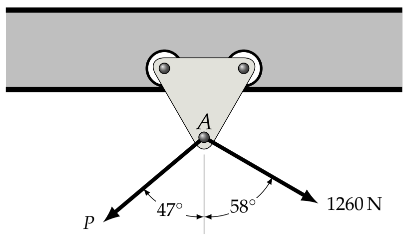
Engineering Statics - 03 Equilibrium of a Particle / Concurrent Forces Handout - Instructor Copy

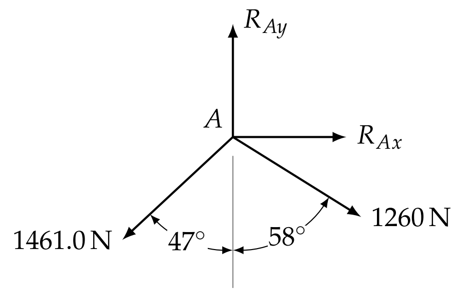
Exercise 1

The trolley can move freely along the horizontal beam on frictionless rollers. Currently, it is in equilibrium. Determine the reaction at A..



A is in equilibrium so:

$$\begin{aligned} \sum F_x &= 0 \\ \Rightarrow P \sin 47^\circ &= 1260\,\text{N} \cdot \sin 58^\circ \\ \Rightarrow P &= \frac{1260\,\text{N} \cdot \sin 58^\circ}{\sin 47^\circ} \\ &= 1461.0\,\text{N} \end{aligned}$$



The reaction at A:

$$\begin{aligned} \sum F_x &= R_{Ax} + (1260\,\text{N}) \sin 58^\circ - (1461.0\,\text{N}) \sin 47^\circ \\ &= 0 \\ \Rightarrow R_{Ax} &= (1461.0\,\text{N}) \sin 47^\circ - (1260\,\text{N}) \sin 58^\circ \\ &= -0.032543\,\text{N} \quad \text{(Rounding errors; should be 0)} \\ &\approx 0 \end{aligned}$$

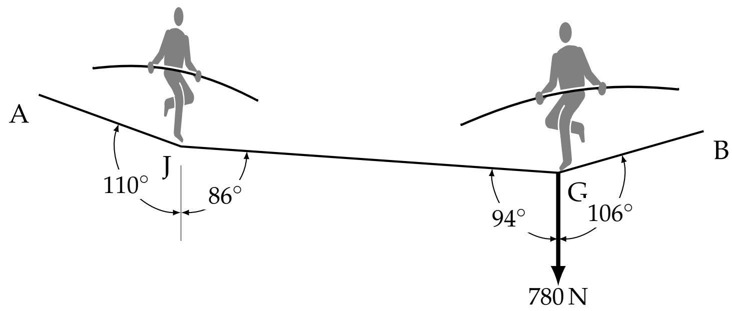
Why? Because if the two horizontal components do not sum to 0, the frictionless trolley will move.

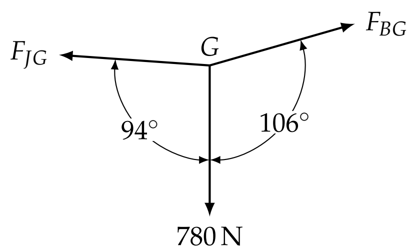
$$\begin{aligned} \sum F_y &= R_{Ay} - (1260\,\text{N}) \cos 58^\circ - (1461.0\,\text{N}) \cos 47^\circ \\ &= 0 \\ \Rightarrow R_{Ay} &= (1461.0\,\text{N}) \cos 47^\circ + (1260\,\text{N}) \cos 58^\circ \\ &= 1664.1\,\text{N} \end{aligned}$$

$R_A = 1660\,\text{N}$ at 90°, measured ccw from the +ve x-axis

Exercise 2

Jacques and Gilles are high-wire artistes. Gille weighs 780 N. How much does Jacques weigh?





$$\sum F_x = F_{BG} \cos 16^\circ - F_{JG} \cos 4^\circ = 0$$

$$\Rightarrow F_{BG} = \frac{F_{JG} \cos 4^\circ}{\cos 16^\circ}$$

$$\Rightarrow F_{BG} = 1.0378 F_{JG}$$

$$\sum F_y = F_{BG} \sin 16^\circ + F_{JG} \sin 4^\circ - 780\,\text{N} = 0$$

$$\Rightarrow 780\,\text{N} = 1.0378 F_{JG} \sin 16^\circ + F_{JG} \sin 4^\circ$$

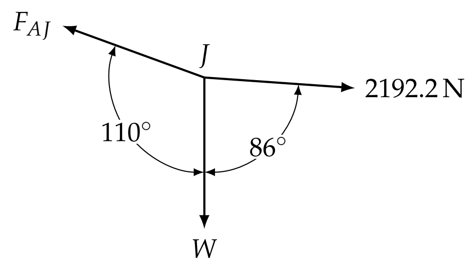
$$\Rightarrow F_{JG} = \frac{780\,\text{N}}{1.0378 \sin 16^\circ + \sin 4^\circ}$$

$$= 2192.2\,\text{N}$$

$$\Rightarrow F_{BG} = 1.0378(2192.2\,\text{N})$$

$$= 2275.0\,\text{N}$$

We don't need F_{BG} for the next part.



$$\sum F_x = (2192\,\text{N}) \cos 4^\circ - F_{AJ} \cos 20^\circ = 0$$

$$\Rightarrow F_{AJ} = \frac{(2192\,\text{N}) \cos 4^\circ}{\cos 20^\circ}$$

$$= 2327.0\,\text{N}$$

$$\sum F_y = F_{AJ} \sin 20^\circ - 2192.2 \sin 4^\circ - W = 0$$

$$\Rightarrow W = (2327.0\,\text{N}) \sin 20^\circ - (2192.2\,\text{N}) \sin 4^\circ$$

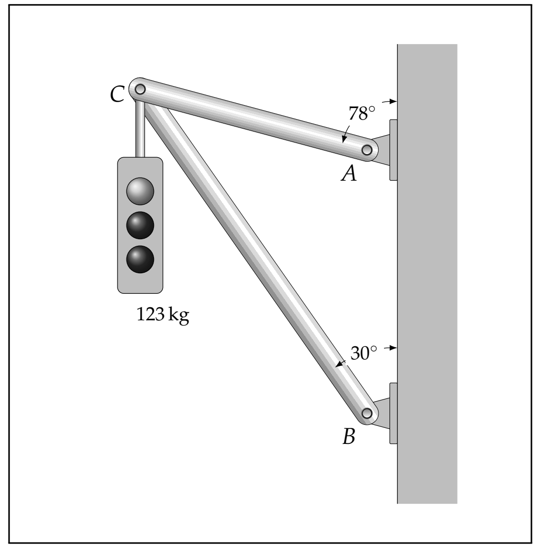
$$= 642.96\,\text{N}$$

Jacques weighs 643 N

(Or use the system-solver to avoid simultaneous equations!)

Example 3

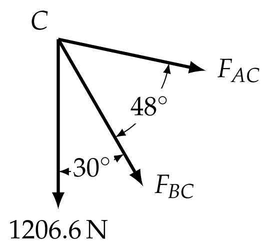
Determine the internal forces in members AC and BC. Specify whether they are in tension or compression.



Convert the mass of the traffic lights into a force:

$$W = 123\,\text{kg} \times 9.81\,\text{m/s}^2 = 1206.6\,\text{N}$$

Note: By default, draw F_{AC} and F_{BC} in tension.



$$\sum F_x = F_{AC}\cos 12^\circ + F_{BC}\cos 60^\circ = 0$$

$$\Rightarrow F_{BC} = -\frac{F_{AC}\cos 12^\circ}{\cos 60^\circ}$$

$$= -1.9563 F_{AC}$$

$$\sum F_y = F_{AC}\sin 12^\circ + F_{BC}\sin 60^\circ + 1206.6\,\text{N} = 0$$

$$\Rightarrow 0 = F_{AC}\sin 12^\circ + F_{BC}\sin 60^\circ + 1206.6\,\text{N}$$

$$\Rightarrow 0 = F_{AC}\sin 12^\circ - 1.9563 F_{AC}\cdot\sin 60^\circ + 1206.6\,\text{N}$$

$$\Rightarrow 0 = F_{AC}\,(\sin 12^\circ - 1.9563\cdot\sin 60^\circ) + 1206.6\,\text{N}$$

$$\Rightarrow 0 = -1.4863 F_{AC} + 1206.6\,\text{N}$$

$$\Rightarrow F_{AC} = 811.81\,\text{N}$$

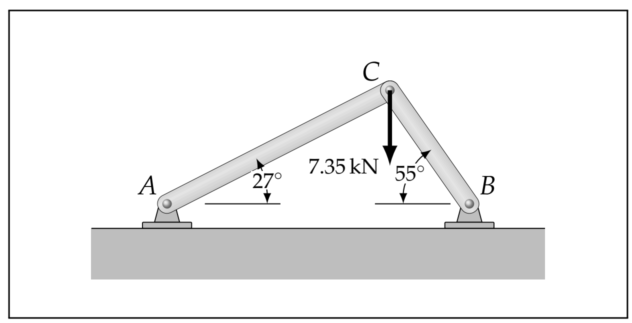
$$\Rightarrow F_{BC} = -1.9563(811.81\,\text{N})$$

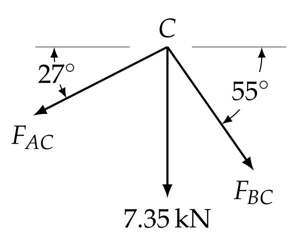
$$\Rightarrow F_{BC} = -1588.2\,\text{N}$$

The force in $\boldsymbol{AC} = \mathbf{811}\,\text{N}$ (Tension). The force in $\boldsymbol{BC} = \mathbf{1590}\,\text{N}$ (Compression).

Exercise 3

Determine the internal forces in members AC and BC. Specify whether they are in tension or compression.





$$\sum F_x = F_{BC}\cos 55^\circ - F_{AC}\cos 27^\circ = 0$$

$$\Rightarrow F_{AC} = \frac{F_{BC}\cos 55^\circ}{\cos 27^\circ} = 0.64374 F_{BC}$$

$$\sum F_y = F_{BC}\sin 55^\circ + F_{AC}\sin 27^\circ + 7.35\,\text{N} = 0$$

$$\Rightarrow F_{BC}\sin 55^\circ + (0.64374 F_{BC})\sin 27^\circ + 7.35\,\text{N} = 0$$

$$\Rightarrow F_{BC} = \frac{-7.35\,\text{N}}{\sin 55^\circ + 0.64374\sin 27^\circ} = -6.6133\,\text{N}$$

$$\Rightarrow F_{AC} = 0.64374(-6.6133\,\text{N}) = -4.2572\,\text{N}$$

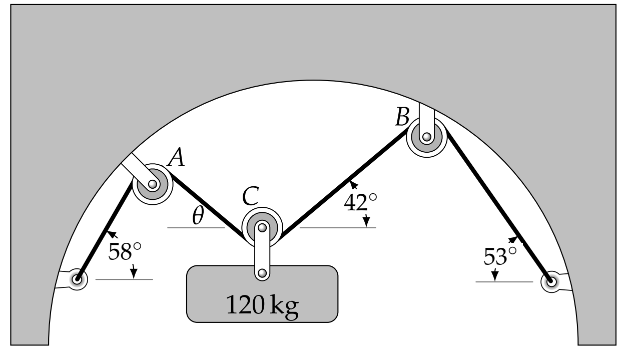
Or, setting up for the system-solver:

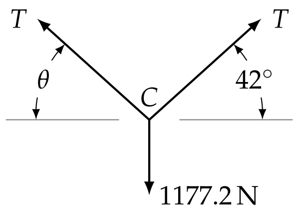
$$\begin{aligned} -\cos 27^\circ\cdot x + \cos 55^\circ\cdot y &= 0 \\ \sin 27^\circ\cdot x + \sin 55^\circ\cdot y &= -7.35 \end{aligned}$$

The force in member AC is 4.26 N in compression and the force in member BC is 6.61 N in compression.

Example 4

Determine θ. Then find the tension in the rope and the pulley reaction at B due to the suspended mass.

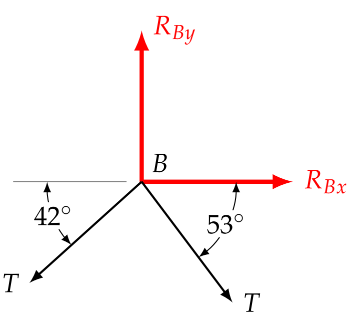




$$\sum F_x = T\cos 42^\circ - T\cos\theta = 0$$
$$\Rightarrow \cos 42^\circ = \cos\theta$$
$$\Rightarrow \theta = 42^\circ$$

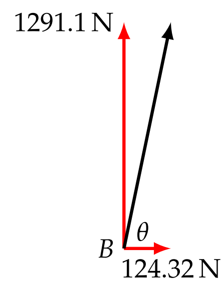
Note that pulley C finds equilibrium when the angles of the cable on either side are equal.

$$\sum F_y = 2T\sin 42^\circ - 1177.2\,\text{N} = 0$$
$$\Rightarrow T = \frac{1177.2\,\text{N}}{2\sin 42^\circ} = 879.65\,\text{N}$$



$$\sum F_x = R_{Bx} + T\cos 53^\circ - T\cos 42^\circ = 0$$
$$\Rightarrow R_{Bx} = T(\cos 42^\circ - \cos 53^\circ)$$
$$= 124.32\,\text{N}$$

$$\sum F_y = R_{By} - T\sin 53^\circ - T\sin 42^\circ = 0$$
$$\Rightarrow R_{By} = T(\sin 42^\circ + \sin 53^\circ)$$
$$= 1291.1\,\text{N}$$



$$R_B = \sqrt{(124.32\,\text{N})^2 + (1291.1\,\text{N})^2}$$
$$= 1297.1\,\text{N}$$

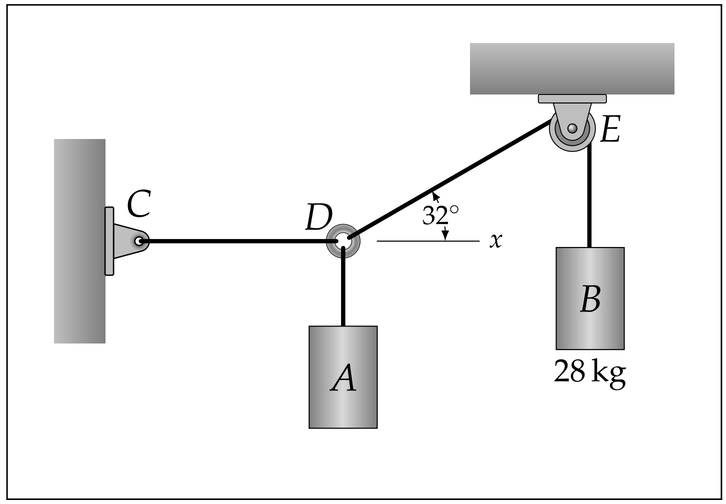
$$R_\theta = \tan^{-1}\left[\frac{1291.1}{124.32}\right]$$
$$= 84.500^\circ$$

The tension in the cable is 880 N

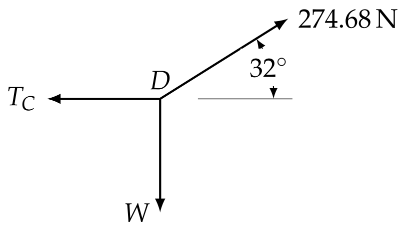
The reaction at B is 1297.1 N at 84.5°, measured counter-clockwise from the positive x-axis.

Exercise 4

Cylinder B has a mass of 28 kg. The system is in equilibrium. Determine the mass of A and the reactions at C and E.



$$T_{DEB} = 28\,\text{kg} \times 9.81\,\text{m/s}^2 = 274.68\,\text{N}$$

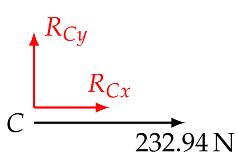


$$\sum F_x = (274.68\,\text{N})\cos 32^\circ - T_{CD} = 0$$
$$\Rightarrow T_{CD} = 232.94\,\text{N}$$

$$\sum F_y = (274.68\,\text{N})\sin 32^\circ - W = 0$$
$$\Rightarrow W = 145.56\,\text{N}$$

Cylinder A has a weight of 145.56 N so it has mass:

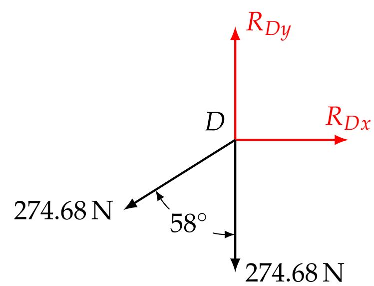
$$A = \frac{145.56\,\text{N}}{9.81\,\text{m/s}^2} = 14.838\,\text{kg}$$



$$\sum F_x = 232.94\,\text{N} + R_{Cx} = 0$$
$$\Rightarrow R_{Cx} = -232.94\,\text{N}$$

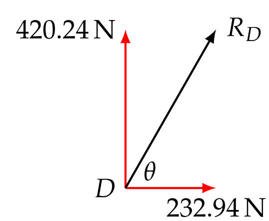
$$\sum F_y = R_{Cy} = 0$$

The reaction at C is in the direction of the negative x-axis (180° counter-clockwise from the positive x-axis.



$$\sum F_x = R_{Dx} - (274.68\,\text{N})\sin 58^\circ = 0$$
$$\Rightarrow R_{Dx} = 232.94\,\text{N}$$

$$\sum F_y = R_{Dy} - (274.68\,\text{N})\cos 58^\circ - (274.68\,\text{N}) = 0$$
$$\Rightarrow R_{Dy} = 420.24\,\text{N}$$



$$R_D = \sqrt{(420.24\,\text{N})^2 + (232.94\,\text{N})^2}$$
$$= 480.48\,\text{N}$$

$$\theta = \tan^{-1}\left[\frac{420.24\,\text{N}}{232.94\,\text{N}}\right]$$
$$= 61.000^\circ$$

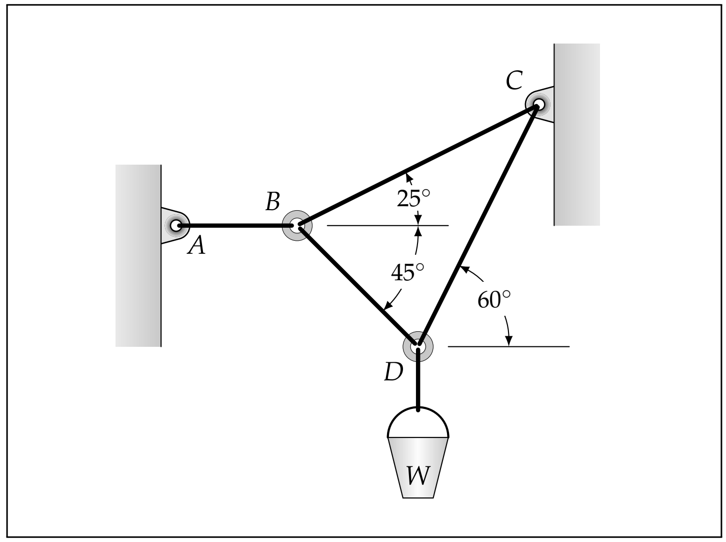
The mass of A is 14.8 kg

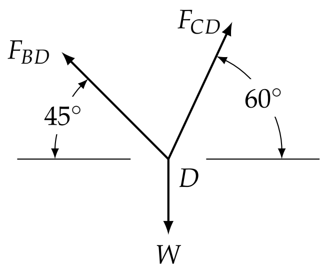
The reaction at C is 233 N at 180°, counter-clockwise from the positive x-axis.

The reaction D is 480 N at 61°, counter-clockwise from the positive x-axis.

Example 5

Determine the maximum weight W of the bucket that the system can support given that no single wire may support more than 450 N. Determine R_C, the reaction at C, for this value of W.





$$\sum F_x = F_{CD}\cos 60^\circ - F_{BD}\cos 45^\circ = 0$$

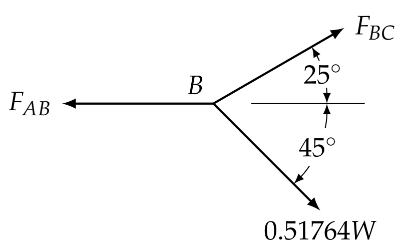
$$\Rightarrow F_{CD} = \frac{F_{BD}\cos 45^\circ}{\cos 60^\circ} = 1.4142 F_{BD}$$

$$\sum F_y = F_{CD}\sin 60^\circ + F_{BD}\sin 45^\circ - W = 0$$

$$\begin{aligned}\Rightarrow W &= F_{CD}\sin 60^\circ + F_{BD}\sin 45^\circ\\ &= 1.4142 F_{BD}\sin 60^\circ + F_{BD}\sin 45^\circ\\ &= 1.9318 F_{BD}\end{aligned}$$

$$\Rightarrow F_{BD} = 0.51764W$$

$$\Rightarrow F_{CD} = 0.73205W$$



$$\sum F_y = F_{BC}\sin 25^\circ - 0.51764W \cdot \sin 45^\circ = 0$$

$$\Rightarrow F_{BC} = \frac{0.51764W \cdot \sin 45^\circ}{\sin 25^\circ} = 0.86609W$$

$$\sum F_x = (0.86609W)\cos 25^\circ + 0.51764W \cdot \cos 45^\circ - F_{AB} = 0$$

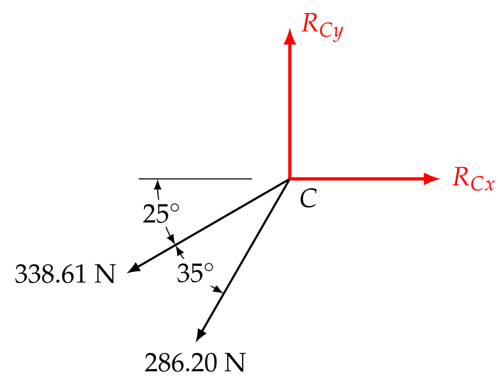
$$\Rightarrow F_{AB} = 1.1510W$$

Comparing the tension values for all four cables, then highest tension is in cable AB. Set this tension to 450 N.

$$1.1510W = 450\,\text{N}$$

$$\Rightarrow W = \frac{450\,\text{N}}{1.1510} = 390.96\,\text{N}$$

Then, $F_{BC} = 0.86609 \times 390.96 = 338.61$ N
and $F_{CD} = 0.73205 \times 390.96 = 286.20$ N



$$\sum F_x = R_{Cx} - (338.61\,\text{N})\cos 25^\circ - (286.20\,\text{N})\cos 60^\circ = 0$$

$$\Rightarrow R_{Cx} = 449.98\,\text{N}$$

$$\sum F_y = R_{Cy} - (338.61\,\text{N})\sin 25^\circ - (286.20\,\text{N})\sin 60^\circ = 0$$

$$\Rightarrow R_{Cy} = 390.96\,\text{N}$$

$$\Rightarrow R_C = \sqrt{449.98^2 + 390.96^2} = 596.10\,\text{N}$$

$$\theta = \tan^{-1}\left[\frac{390.96}{449.98}\right] = 40.985^\circ$$

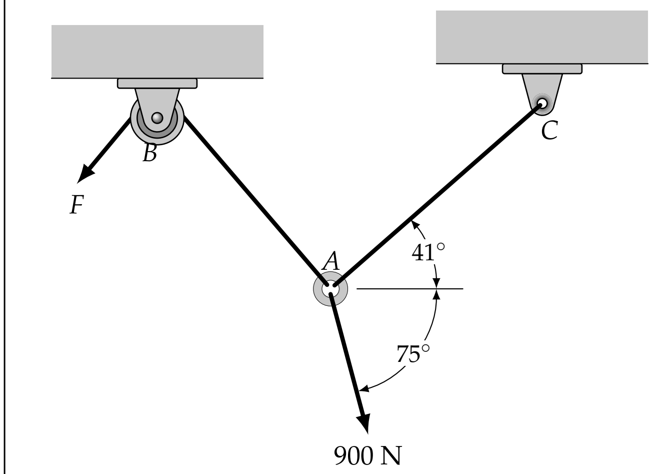
Maximum value for W is 390 N

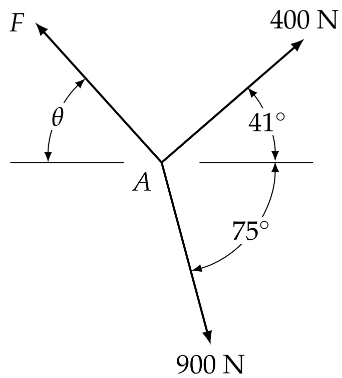
Reaction at C is 596 N at 41.0° ccw from the +ve x axis.

(Discussion: Should we round 390.96 up to 391 as we would normally do? Or not, because this pushes the tension in AB marginally over 450 N?)

Exercise 5

The tension in cable AC is 400 N. Determine the force F necessary to hold the ring A in the position shown..





$$\Sigma F_x = 400\,\text{N}\cdot\cos 41^\circ + 900\,\text{N}\cdot\cos 75^\circ - F\cos\theta = 0$$
$$\Rightarrow F\cos\theta = 534.82\text{ N}$$

$$\Sigma F_y = 400\,\text{N}\cdot\sin 41^\circ - 900\,\text{N}\cdot\sin 75^\circ + F\sin\theta = 0$$
$$\Rightarrow F\sin\theta = 606.91\text{ N}$$

$$\theta = \tan^{-1}\left[\frac{F\sin\theta}{F\cos\theta}\right] = \tan^{-1}\left[\frac{606.91}{534.82}\right] = 48.613^\circ$$

$$F = \frac{606.91\text{ N}}{\sin 48.613^\circ} = 808.93\text{ N}$$

$$F = 809\text{ N and } \theta = 48.6^\circ$$