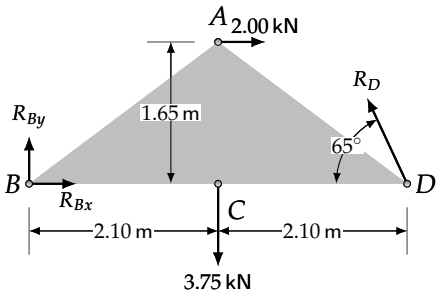
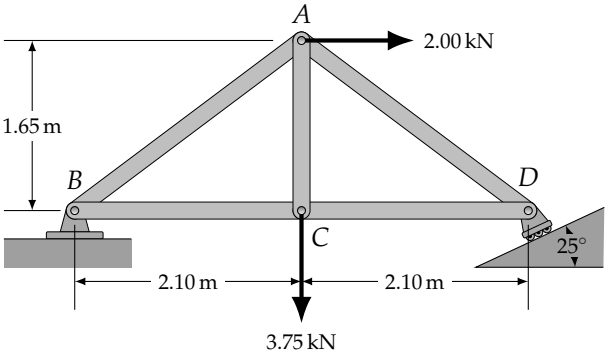
Engineering Statics - 07 Method of Joints - Instructor Copy

Example 1: Determine the force in each truss member.





Reactions

$$\Sigma M_B = R_D \sin 65^\circ \cdot 4.20\,\text{m} - 2.00\,\text{kN} \cdot 1.65\,\text{m} - 3.75\,\text{kN} \cdot 2.10\,\text{m} = 0$$

$$\Rightarrow R_D = \frac{2.00\,\text{kN} \cdot 1.65\,\text{m} + 3.75\,\text{kN} \cdot 2.10\,\text{m}}{\sin 65^\circ \cdot 4.20\,\text{m}} = 2.9358\,\text{kN}$$

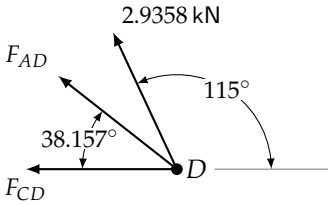
$$\Sigma F_x = R_{Bx} + 2.00\,\text{kN} - 2.9358\,\text{kN} \cdot \cos 65^\circ = 0$$

$$\Rightarrow R_{Bx} = -0.75928\,\text{kN}$$

$$\Sigma F_y = R_{By} - 3.75\,\text{kN} + 2.9358\,\text{kN} \cdot \sin 65^\circ = 0$$

$$\Rightarrow R_{By} = 1.0893\,\text{kN}$$

Joint D



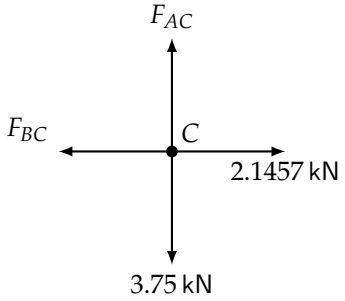
$$\Sigma F_y = 2.9358\,\text{kN} \cdot \cos 25^\circ + F_{AD} \sin 38.157^\circ = 0$$

$$\Rightarrow F_{AD} = -4.3067\,\text{kN}$$

$$\Sigma F_x = -F_{CD} - (-4.3067\,\text{kN}) \cdot \cos 38.157^\circ - 2.9358\,\text{kN} \cos 65^\circ = 0$$

$$\Rightarrow F_{CD} = 2.1457\,\text{kN}$$

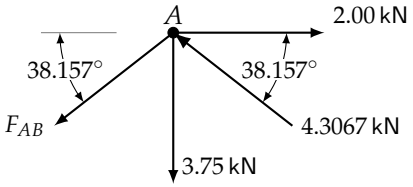
Joint C



$$F_{BC} = 2.1457\,\text{kN}$$

$$F_{AC} = 3.75\,\text{kN}$$

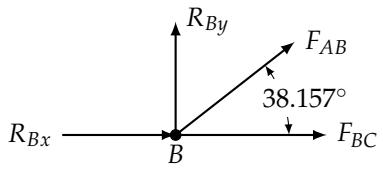
Joint A



$$\Sigma F_x = 2.00\,\text{kN} - 4.3067\,\text{kN} \cdot \cos 38.157^\circ - F_{AB} \cdot \cos 38.157^\circ = 0$$

$$\Rightarrow F_{AB} = \frac{2.00\,\text{kN} - 4.3067\,\text{kN} \cdot \cos 38.157^\circ}{\cos 38.157^\circ} = -1.7632\,\text{kN}$$

Check at B



$$\Sigma F_y = R_{By} + F_{AB} \cdot \sin 38.517^\circ$$

$$= 1.0893\,\text{kN} + (-1.7632\,\text{kN}) \cdot \sin 38.517^\circ$$

$$= -0.0087272\,\text{kN} \approx 0 \qquad \checkmark$$

$$\Sigma F_y = R_{Bx} + F_{AB} \cdot \cos 38.517^\circ + F_{BC}$$

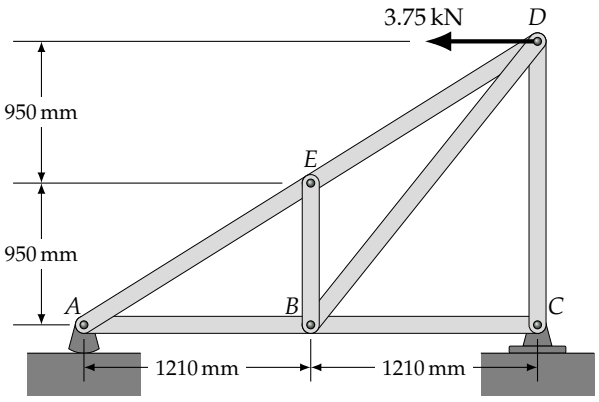
$$= (-0.75928\,\text{kN}) + (-1.7632\,\text{kN}) \cdot \cos 38.517^\circ + 2.1457\,\text{kN}$$

$$= -0.000020803\,\text{kN} \approx 0 \qquad \checkmark$$

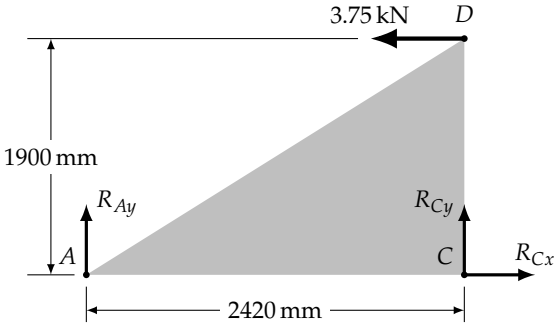
Answers

$AB = 1.76\,\text{kN}$	(Compression)
$BC = 2.15\,\text{kN}$	(Tension)
$CD = 2.15\,\text{kN}$	(Tension)
$AC = 3.75\,\text{kN}$	(Tension)
$AD = 4.13\,\text{kN}$	(Compression)

Example 3: Determine the force in each truss member.



FBD and external forces



$$\Sigma M_C = (3.75\,\text{kN})\cdot(1.900\,\text{m}) - R_{Ay}\cdot(2.420\,\text{m}) = 0$$
$$\Rightarrow R_{Ay} = 2.9442\,\text{kN}\cdot\text{m}$$

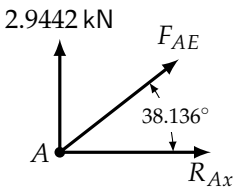
$$\Sigma F_x = R_{Cx} - 3.75\,\text{kN} = 0$$
$$\Rightarrow R_{Cx} = 3.75\,\text{kN}$$

$$\Sigma F_y = R_{Cy} + 2.9442\,\text{kN} = 0$$
$$\Rightarrow R_{Cy} = -2.9442\,\text{kN}$$

Note:

1. BE is a zero-force member
2. Since, BE is a zero force member, so is BD
3. Thus, $F_{AE} = F_{ED}$ and $F_{AB} = F_{BC}$

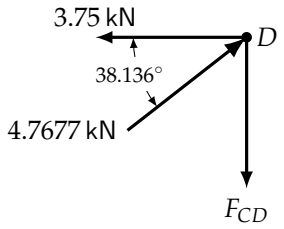
Joint A:



$$\Sigma F_y = F_{AE}\cdot\sin 38.136^\circ + 2.9442\,\text{kN} = 0$$
$$\Rightarrow F_{AE} = -4.7677\,\text{kN}$$

$$\Sigma F_x = (-4.7677\,\text{kN})\cdot\cos 38.136^\circ + R_{Ax} = 0$$
$$\Rightarrow F_{AB} = 3.7500\,\text{kN}$$

Joint D:

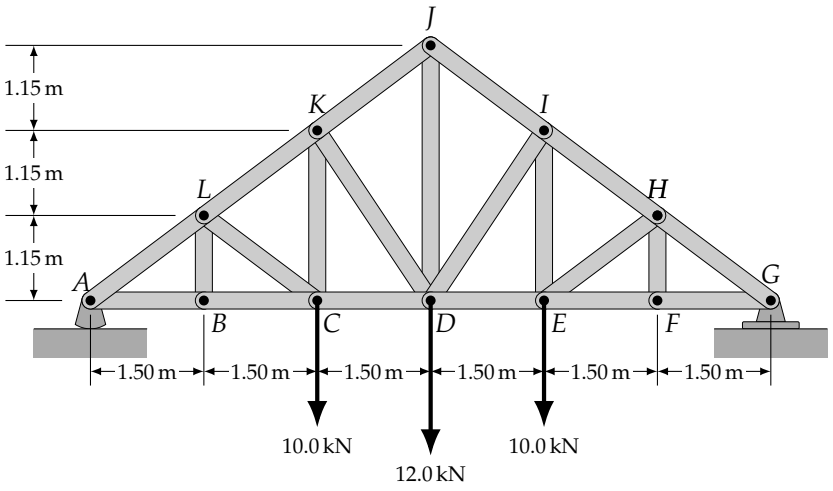


$$\Sigma F_y = (4.7677\,\text{kN})\cdot\sin 38.136^\circ - F_{CD} = 0$$
$$F_{CD} = 2.9442\,\text{kN}$$

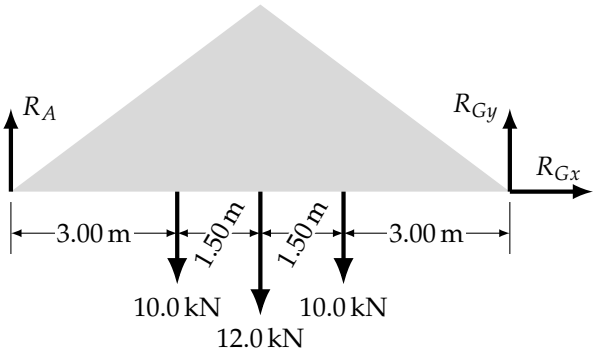
Answers

$AB = 3.75\,\text{kN}$	(Tension)
$AE = 4.77\,\text{kN}$	(Compress)
$BC = 3.75\,\text{kN}$	(Tension)
$BD = 0$	
$BE = 0$	
$CD = 2.94\,\text{kN}$	(Tension)
$DE = 4.77\,\text{kN}$	(Compression)

Example 4: Determine the force in each truss member.



FBD and reactions



$$\Sigma M_G = (10.0\,\text{kN})\cdot(3.00\,\text{m}) + (12.0\,\text{kN})\cdot(4.50\,\text{m}) + (10.0\,\text{kN})\cdot(6.00\,\text{m}) + R_A\cdot(9.00\,\text{m}) = 0$$

$$\Rightarrow R_A = 16.000\,\text{m}$$

$$\Sigma F_x = R_{Gx} = 0$$

$$\Rightarrow R_{Gx} = 0$$

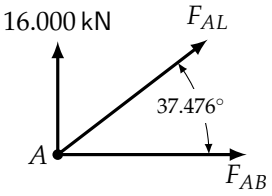
$$\Sigma F_y = R_{Gy} + 16.000\,\text{kN} - 10.0\,\text{kN} - 12.0\,\text{kN} - 10.0\,\text{kN} = 0$$

$$\Rightarrow R_{Gx} = 16.000\,\text{kN}$$

Note:

1. The system is symmetrical about member DJ so we only have to solve half of it - either the right half or the left half.
2. Member BF is a zero-force member and, because of this, CL is also a zero-force member. (Similarly, so are FH and EH.)

Joint A:



$$\Sigma F_y = F_{AL}\cdot\sin 37.476^\circ + 16.000\,\text{kN} = 0$$

$$F_{AL} = -26.297\,\text{kN}$$

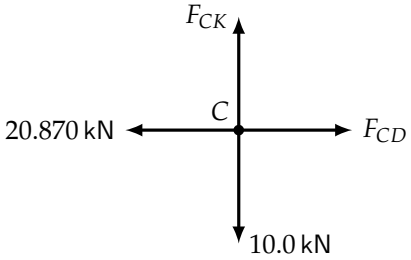
$$\Sigma F_x = F_{AB} + (-26.297\,\text{kN})\cdot\cos 37.476^\circ = 0$$

$$F_{AB} = 20.870\,\text{kN}$$

Note:

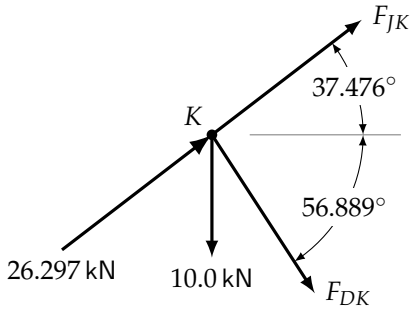
Since BL and CL are zero-force members, we can ignore them. That means that $F_{AB} = F_{BC}$ and $F_{AL} = F_{KL}$.

Joint C:



$$F_{CD} = 20.870\,\text{kN},\ F_{CK} = 10.0\,\text{kN}$$

Joint K:



$$\Sigma F_x = F_{JK}\cdot\cos 37.476^\circ + F_{DK}\cdot\cos 56.889^\circ + (26.297\,\text{kN})\cdot\cos 37.476^\circ$$
$$= 0.79361F_{JK} + 0.54626F_{DK} + 20.870\,\text{kN} = 0$$

$$\Sigma F_y = F_{JK}\cdot\sin 37.476^\circ - F_{DK}\cdot\sin 56.889^\circ + (26.297\,\text{kN})\cdot\sin 37.476^\circ - 10.0\,\text{kN}$$
$$= 0.60843F_{JK} - 0.83761F_{DK} + 5.9999\,\text{kN} = 0$$

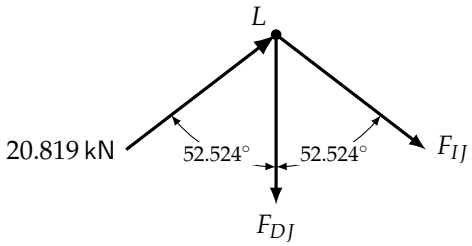
Solve the system:

$$0.79361F_{JK} + 0.54626F_{DK} = -20.870\,\text{kN} \quad (1)$$
$$0.60843F_{JK} - 0.83761F_{DK} = -5.9999\,\text{kN} \quad (2)$$

From the system-solver:

$$F_{DK} = -7.9595\,\text{kN},\ F_{JK} = 20.819\,\text{kN}$$

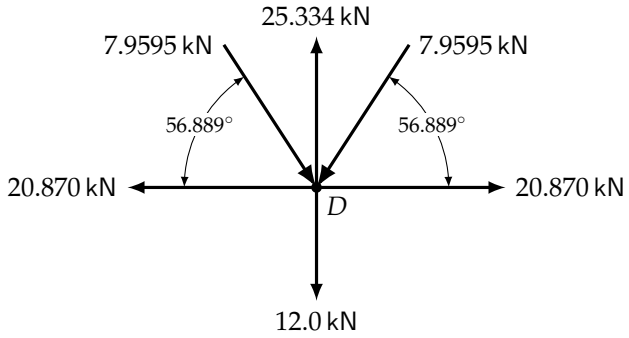
Joint L:



$$\Sigma F_x = F_{IJ}\cdot\sin 52.524^\circ + 20.819\,\text{kN}\cdot\sin 52.524^\circ = 0$$
$$\Rightarrow F_{IJ} = -20.819\,\text{kN}$$

$$\Sigma F_y = 20.819\,\text{kN}\cdot\cos 52.524^\circ - (-20.819\,\text{kN})\cdot\cos 52.524^\circ - F_{DJ} = 0$$
$$\Rightarrow F_{DJ} = 25.334\,\text{kN}$$

Check at Joint D (assuming symmetry):

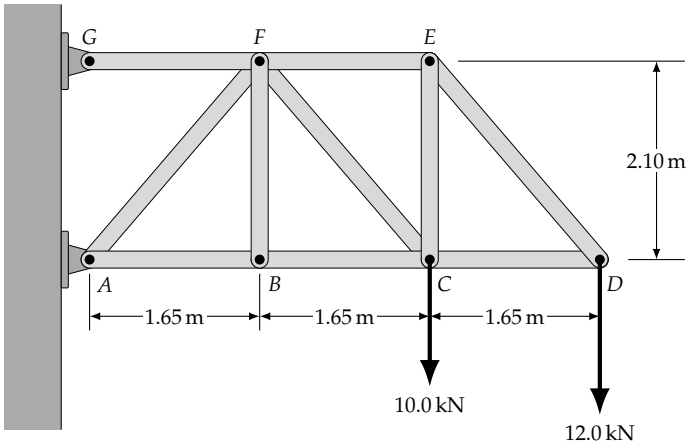


$$\Sigma F_y = 25.334\,\text{kN} - 12.0\,\text{kN} - 2(7.9595\,\text{kN})\cdot\sin 56.889^\circ$$
$$= 0.00002\,\text{kN} \approx 0 \quad \checkmark$$

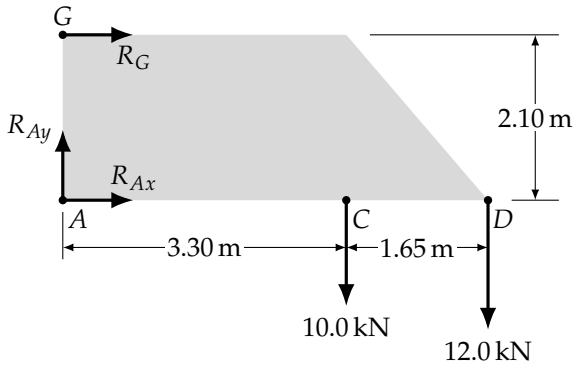
Answers

$AB = 20.9\,\text{kN}$	(Tension)
$AL = 26.3\,\text{kN}$	(Compression)
$BC = 20.9\,\text{kN}$	(Tension)
$BL = 0$	
$CD = 20.9\,\text{kN}$	(Tension)
$CK = 10.0\,\text{kN}$	(Tension)
$CL = 0$	
$DE = 20.9\,\text{kN}$	(Tension)
$DI = 7.96\,\text{kN}$	(Compression)
$DJ = 25.3\,\text{kN}$	(Tension)
$DK = 7.96\,\text{kN}$	(Compression)
$EF = 20.9\,\text{kN}$	(Tension)
$EH = 0$	
$EI = 10.0\,\text{kN}$	(Tension)
$FG = 20.9\,\text{kN}$	(Tension)
$FH = 0$	
$GH = 26.3\,\text{kN}$	(Compression)
$HI = 26.3\,\text{kN}$	(Compression)
$IJ = 20.8\,\text{kN}$	(Compression)
$JK = 20.8\,\text{kN}$	(Compression)
$KL = 26.3\,\text{kN}$	(Compression)

Exercise 1: Determine the force in each truss member.



FBD and reactions:

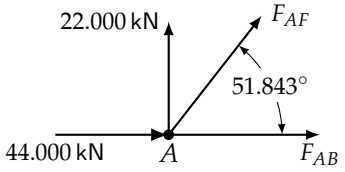


$$\Sigma M_A = -(10.0\,\text{kN})\cdot(3.30\,\text{m}) - (12.0\,\text{kN})\cdot(4.95\,\text{m}) - R_G\cdot(2.10\,\text{m}) = 0$$
$$\Rightarrow R_G = -44.000\,\text{kN}$$

$$\Sigma F_x = R_{Ax} - 44.000\,\text{kN} = 0$$
$$\Rightarrow R_{Ax} = 44.000\,\text{kN}$$

$$\Sigma F_y = R_{Ay} - 10.0\,\text{kN} - 12.0\,\text{kN} = 0$$
$$\Rightarrow R_{Ax} = 22.000\,\text{kN}$$

Joint A:



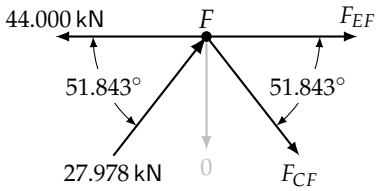
$$\Sigma F_y = F_{AF}\cdot\sin 51.843^\circ + 22.000\,\text{kN} = 0$$
$$\Rightarrow F_{AF} = -27.978\,\text{kN}$$

$$\Sigma F_x = F_{AB} + (-27.978\,\text{kN})\cdot\cos 51.843^\circ + 44.000\,\text{kN} = 0$$
$$\Rightarrow F_{AB} = -26.715\,\text{kN}$$

Note:

BF is a zero-force member and $F_{BC} = F_{AB} = 17.286\,\text{kN}$. Also, $F_{FG} = 44.000\,\text{kN}$, equal and opposite to the reaction R_G.

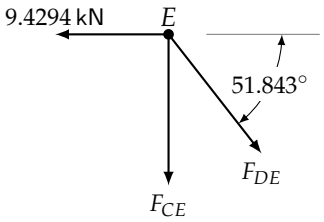
Joint F:



$$\Sigma F_y = (27.978\,\text{kN})\cdot\sin 51.843^\circ - F_{CF}\cdot\sin 51.843^\circ = 0$$
$$\Rightarrow F_{CF} = 27.978\,\text{kN}$$

$$\Sigma F_x = F_{EF} + 2(27.978\,\text{kN})\cdot\cos 51.843^\circ - 44.000\,\text{kN} = 0$$
$$\Rightarrow F_{EF} = 9.4294\,\text{kN}$$

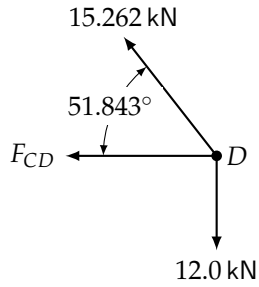
Joint E:



$$\Sigma F_x = F_{DE}\cdot\cos 51.843^\circ - 9.4294\,\text{kN} = 0$$
$$\Rightarrow F_{DE} = 15.262\,\text{kN}$$

$$\Sigma F_y = -F_{CE} - (15.262\,\text{kN})\cdot\sin 51.843^\circ = 0$$
$$\Rightarrow F_{CE} = -12.001\,\text{kN}$$

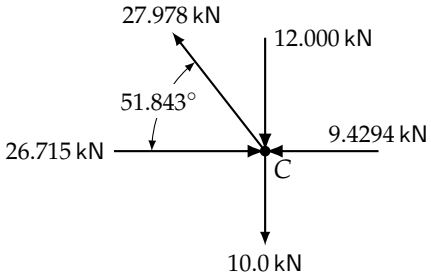
Joint D:



$$\Sigma F_x = -(15.262\,\text{kN})\cdot\cos 51.843^\circ - F_{CD} = 0$$

$$\Rightarrow F_{CD} = -9.4294\,\text{kN}$$

Do a check at joint C:



$$\Sigma F_x = 26.715\,\text{kN} - 9.4294\,\text{kN} - (27.978\,\text{kN})\cos 51.843^\circ$$

$$= 0.00027562\,\text{kN} \approx 0 \quad \checkmark$$

$$\Sigma F_y = (27.978\,\text{kN})\cdot\sin 51.843^\circ - 10.0\,\text{kN} - 12.001\,\text{kN}$$

$$= -0.0013172\,\text{kN} \approx 0 \quad \checkmark$$

Answers

$AB = 26.7\,\text{kN}$	(Compression)
$AF = 28.0\,\text{kN}$	(Compression)
$BC = 26.7\,\text{kN}$	(Compression)
$BF = 0$	
$CD = 9.43\,\text{kN}$	(Compression)
$CE = 12.0\,\text{kN}$	(Compression)
$CF = 28.0\,\text{kN}$	(Tension)
$DE = 15.3\,\text{kN}$	(Tension)
$EF = 9.43\,\text{kN}$	(Tension)
$FG = 44.0\,\text{kN}$	(Tension)