Constant Gradient 3-Magnet Design: Report 3

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Version 3.1

1. Introduction

This report outlines work done from June 23rd to July 8th to achieve a constant gradient G_z of various 3-magnet arrays, where G_z is the variation of the Z-component of the magnetic field, B_o, in the Z-direction. This follows from work done by Marble [1], Garcia [2], and Wilbur [3]; Report 1[4] and Report 2 [5] on this project; and uses the MagnetArray2020 [6] MATLAB program. Various gradients in the range of 15 Gauss/cm to 150 Gauss/cm over a cylindrical volume of about 1 cm diameter and 3-5 cm long are desired to measure how fluids flow through pipes.

2. Different Gradient Locations and Different Field Strengths

Since the results in [5] showed the locations above the magnet arrays with constant G_z to be off-centre, for some of the arrays presented in Section 3, the upper and lower bounds in Optimization2.m (see [5]) for Z were changed to be to the left side of the array, and the lower bound for Y was lowered so that the areas with constant G_z could be aligned with the array, and not above it. This is shown in fig. 1.

It was very difficult for the GA function (see [5]) to find high gradient strengths and high field strengths next to the magnets.

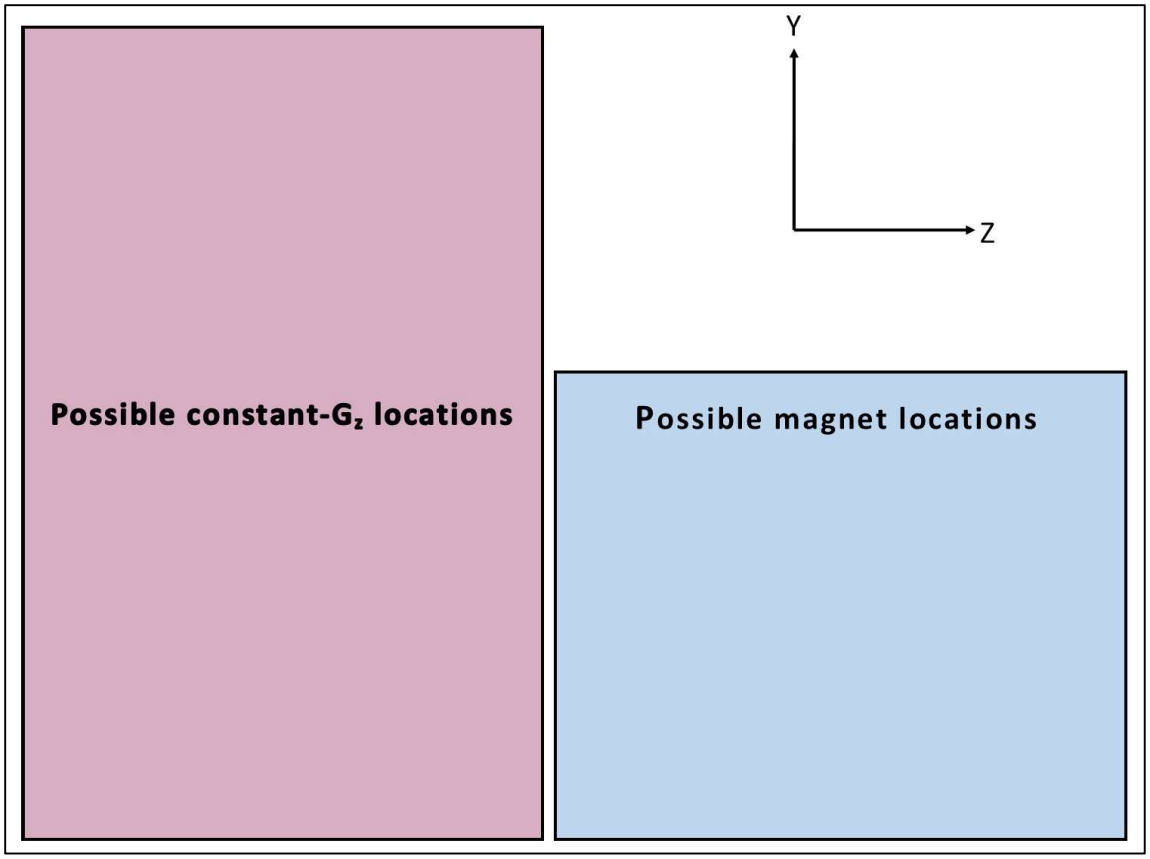

Figure 1: Area given to Optimization2.m to search for a spot with a constant G_z (red), relative to the area given to Optimization2.m to place the three magnets (blue).

In [5], all the arrays had very low field strengths. For a proton (H^1 nucleus), the gyromagnetic ration, γ_p, is $2.675 \cdot 10^8\ s^{-1}T^{-1}$ [7], or $2.675 \cdot 10^4\ s^{-1}G^{-1}$. The (absolute value of the) highest field in [5] was around 600 Gauss, in Array 7. This gives a Larmor frequency of $\omega_o = \gamma_p B_o = (2.675 \cdot 10^4\ s^{-1}G^{-1}) \cdot (600\ G) = 1.605 \cdot 10^7\ s^{-1} \approx 2$ MHz. On average, however, the Larmor frequencies were around 1 MHz for all arrays. An extra constraint was added to ensure a high field strength:

$$B_{o_{goal}} - |B_o| < 0$$

This ensured that the strength of the magnetic field found by the GA function was greater than a lower limit, B_{ogoal}. B_{ogoal} was set at 700 G. When this constraint was added, the upper and lower bounds for Z and Y were reset so the area of constant G_z would be above the array.

3. Data

Array	Magnet	Width/cm	Height/cm	Z/cm	Y/cm	Z range/cm	Y_{ga}/cm
1	Left	1	3	-4.0798	-2.8530	$[-9.76, -6.68]$	-5.2792
	Centre	3	5	0	-2.2007		
	Right	3	5	4.8801	0		
2	Left	1	3	-5	0	$[-11.96, -7.64]$	-6.53
	Centre	3	5	0	-3		
	Right	1	5	4.9842	0		
3	Left	3	5	-4.4013	-1.1456	$[-3.4, 0.04]$	2.9585
	Centre	3	5	0	-0.9432		
	Right	3	5	3.86	0		

Table 1.1: Arrays generated by the Optimization2.m program. The Z coordinate specifies the positions of the centres of the magnets. The Y coordinate specifies the positions of the tops of the magnets. 'Left', 'Centre', and 'Right' refer to the individual magnets, and subscripts 1, 2, and 3 throughout the paper refer to them respectively. Y_{ga} is the height above the array returned by the GA function that has the most constant G_z. The Z range is the range found by the Optimization2.m program over which G_z is constant.

Array	Magnet	Width/cm	Height/cm	Z/cm	Y/cm	Z range/cm	Y_{ga}/cm
4	Left	3	3	-3.8088	-1.3187	$[-2.92,0.08]$	2.0869
	Centre	3	5	0	-1.0879		
	Right	3	5	3.5344	0		
5	Left	3	5	-3.7867	-0.5476	$[-2.2,0.84]$	2.6754
	Centre	3	5	0	-0.6755		
	Right	3	5	3.6281	0		
6	Left	3	5	-3.8017	-2.0070	$[-3.36,-0.28]$	1.7158
	Centre	3	5	0	-1.3147		
	Right	3	5	3.2506	0		

Table 1.2: Arrays generated by the Optimization2.m program. The Z coordinate specifies the positions of the centres of the magnets. The Y coordinate specifies the positions of the tops of the magnets. 'Left', 'Centre', and 'Right' refer to the individual magnets, and subscripts 1, 2, and 3 throughout the paper refer to them respectively. Y_{ga} is the height above the array returned by the GA function that has the most constant G_z. The Z range is the range found by the Optimization2.m program over which G_z is constant.

4. Analysis

Array	Y/cm	Linear fit
1	-6.2792	$45.5754 \cdot Z + 721.2464$
	-5.7792	$48.3887 \cdot Z + 765.7689$
	-5.2792	$51.1276 \cdot Z + 808.4726$
	-4.7792	$55.4153 \cdot Z + 863.9289$
	-4.2792	$63.2975 \cdot Z + 950.2053$
2	-7.53	$17.6493 \cdot Z + 355.8960$
	-7.03	$20.0076 \cdot Z + 394.0781$
	-6.53	$21.9283 \cdot 427.2750$
	-6.03	$23.6662 \cdot Z + 458.2544$
	-5.53	$25.7584 \cdot 492.8160$
3	1.9585	$-53.8240 \cdot Z - 825.3892$
	2.4585	$-51.4322 \cdot Z - 782.4774$
	2.9585	$-49.8370 \cdot Z - 740.2896$
	3.4585	$-47.7957 \cdot Z - 697.7167$
	3.9585	$-45.0943 \cdot Z - 655.1023$

Table 2.1: Linear fits of B_o in Gauss as a function of Z. The slopes of the linear fits are G_z in Gauss/cm.

Array	Y/cm	Linear fit
4	1.0869	$-93.7089 \cdot Z - 934.4386$
	1.5869	$-88.9274 \cdot Z - 884.6384$
	2.0869	$-86.9346 \cdot Z - 838.8399$
	2.5869	$-83.7239 \cdot Z - 791.3514$
	3.0869	$-78.7088 - 741.8432$
5	1.6754	$-27.2117 \cdot Z - 1017$
	2.1754	$-29.1519 \cdot Z - 959.6$
	2.6754	$-30.9563 \cdot Z - 899.6$
	3.1754	$-31.2043 \cdot Z - 838.5$
	3.6754	$-30.0574 \cdot Z - 778.2$
6	0.7158	$-119.6742 \cdot Z - 1053$
	1.2158	$-112.5310 \cdot Z - 998.3$
	1.7158	$-109.0301 \cdot Z - 951.9$
	2.2158	$-104.3989 \cdot Z - 903.5381$
	2.7158	$-97.7795 \cdot Z - 851.4$

Table 2.2: Linear fits of B_o in Gauss as a function of Z. The slopes of the linear fits are G_z in Gauss/cm.

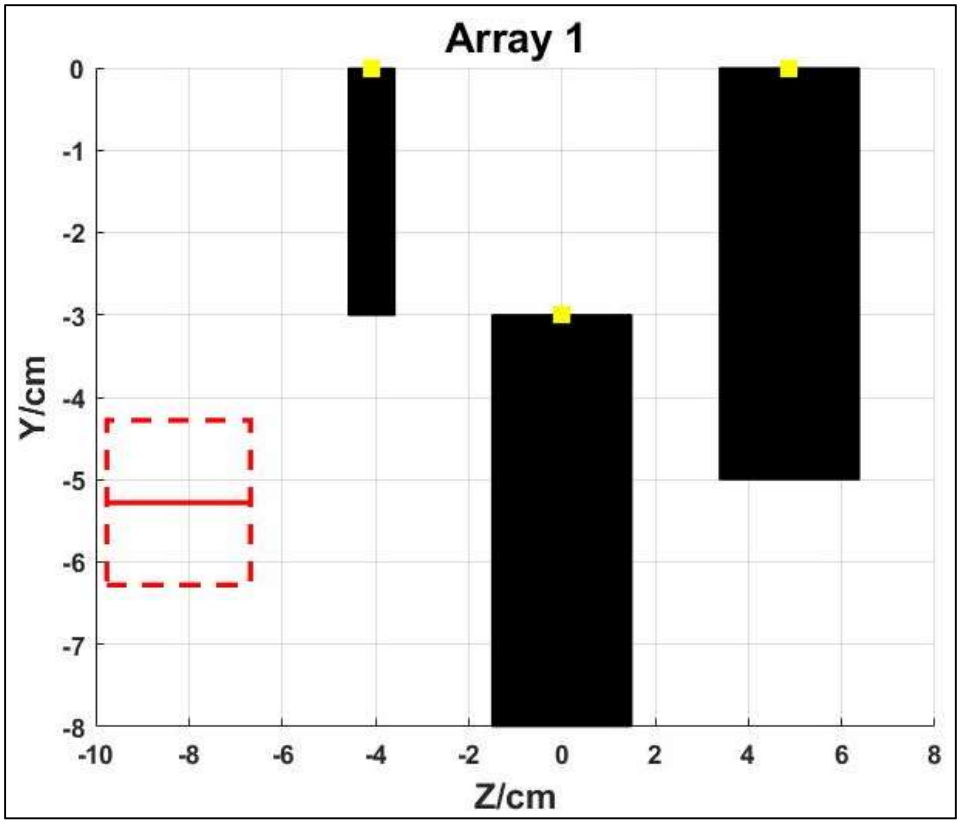

Figure 1.1: Array 1 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 1. The red dotted box is the area over which G_z is approximately constant.

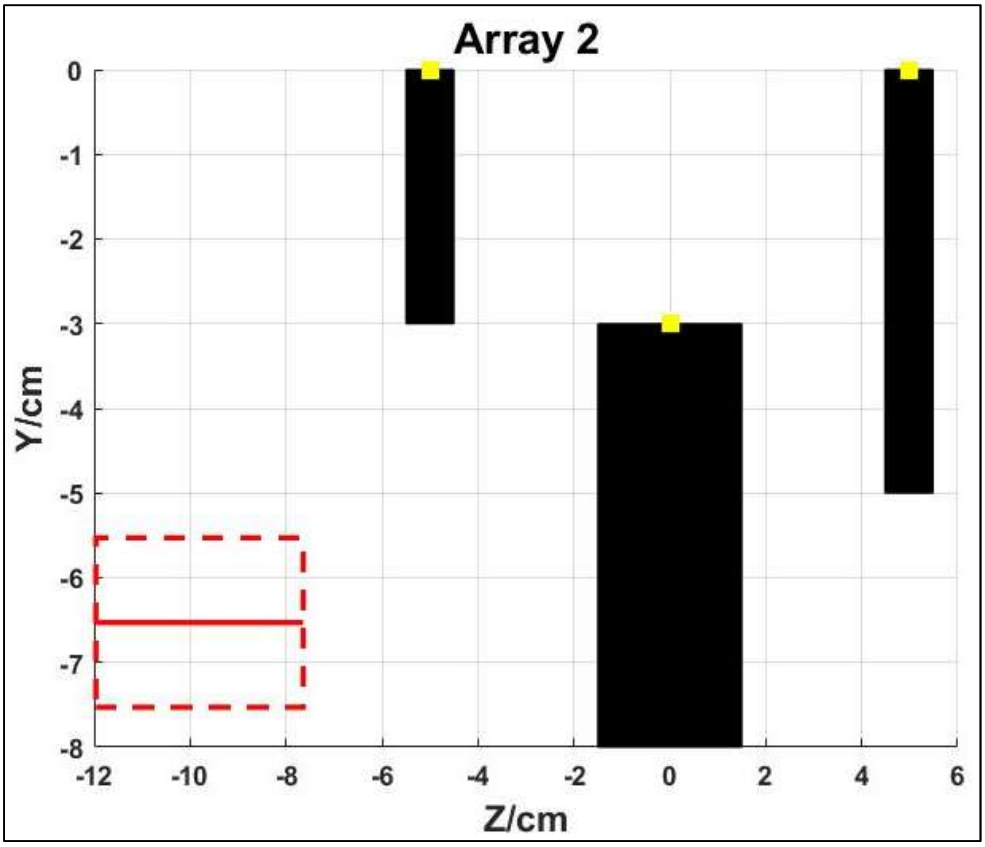

Figure 1.2: Array 2 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 1. The red dotted box is the area over which G_z is approximately constant.

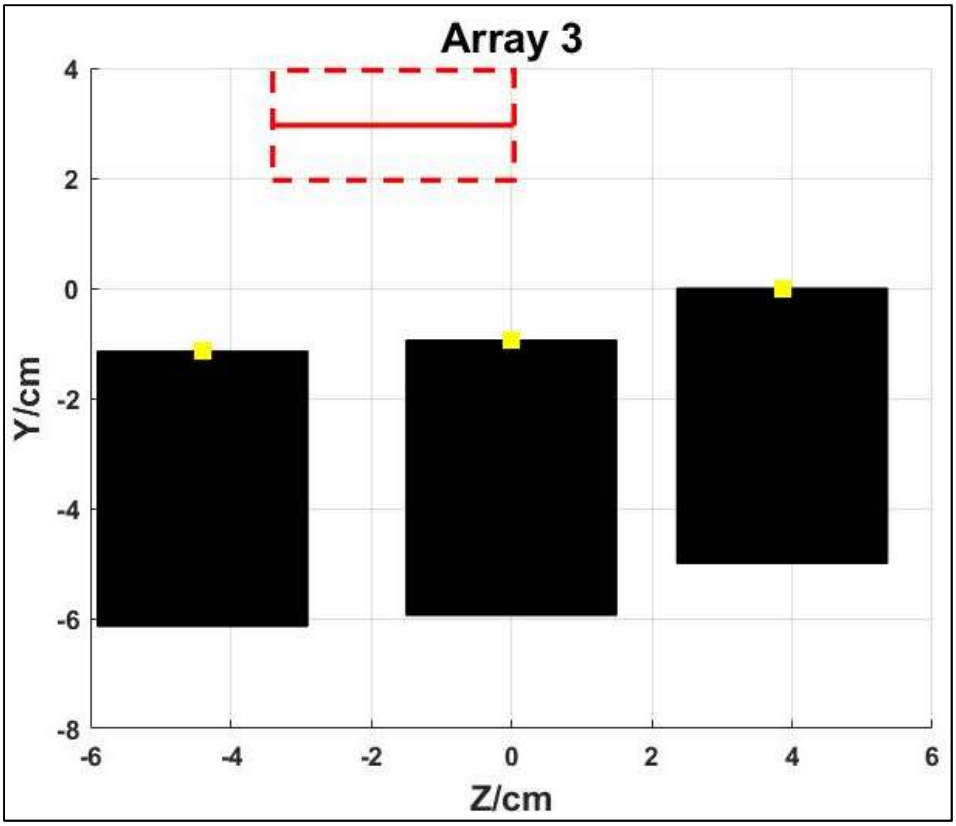

Figure 1.3: Array 3 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 1. The red dotted box is the area over which G_z is approximately constant.

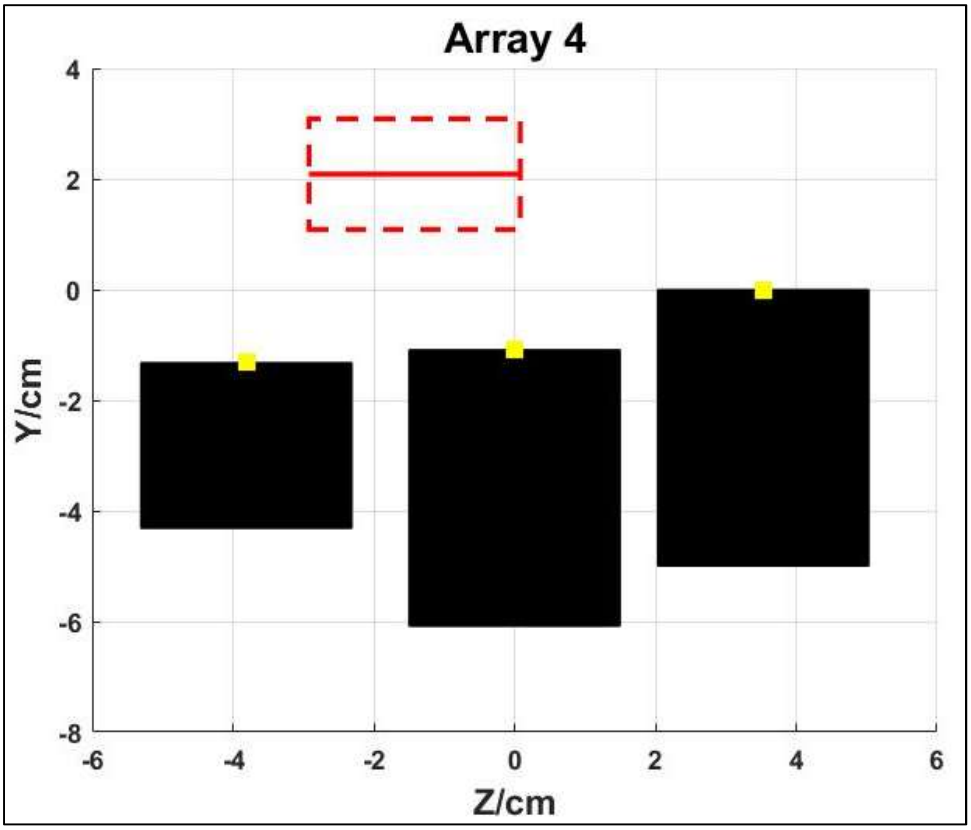

Figure 1.4: Array 4 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 1. The red dotted box is the area over which G_z is approximately constant.

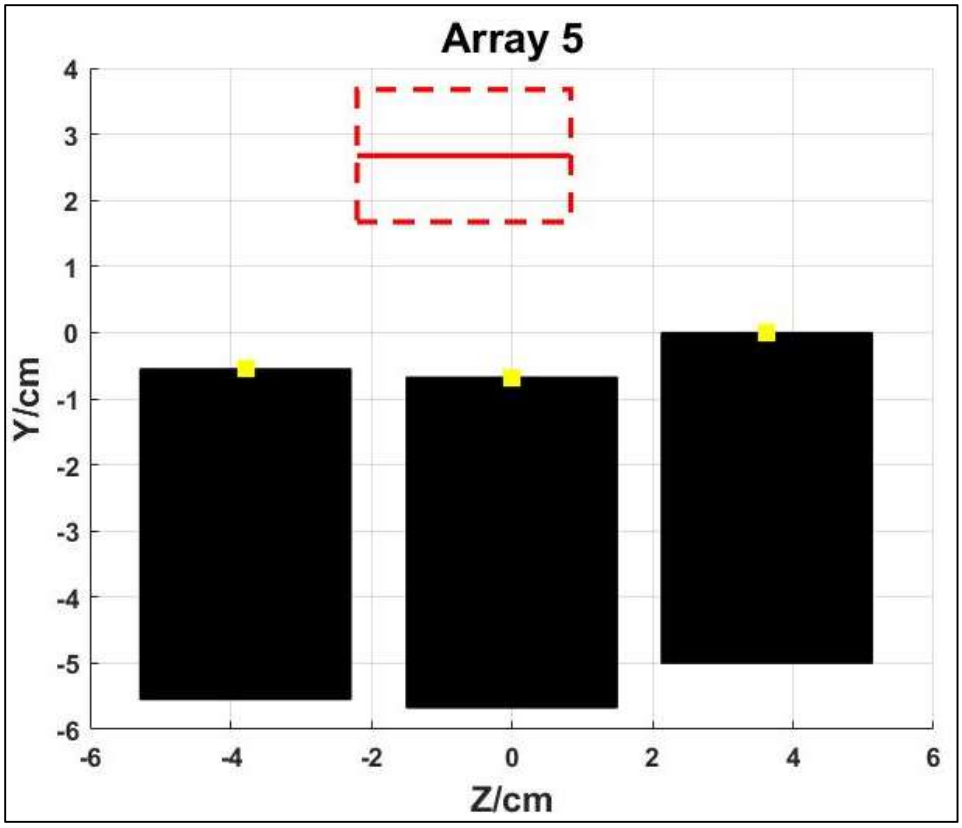

Figure 1.5: Array 5 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 1. The red dotted box is the area over which G_z is approximately constant.

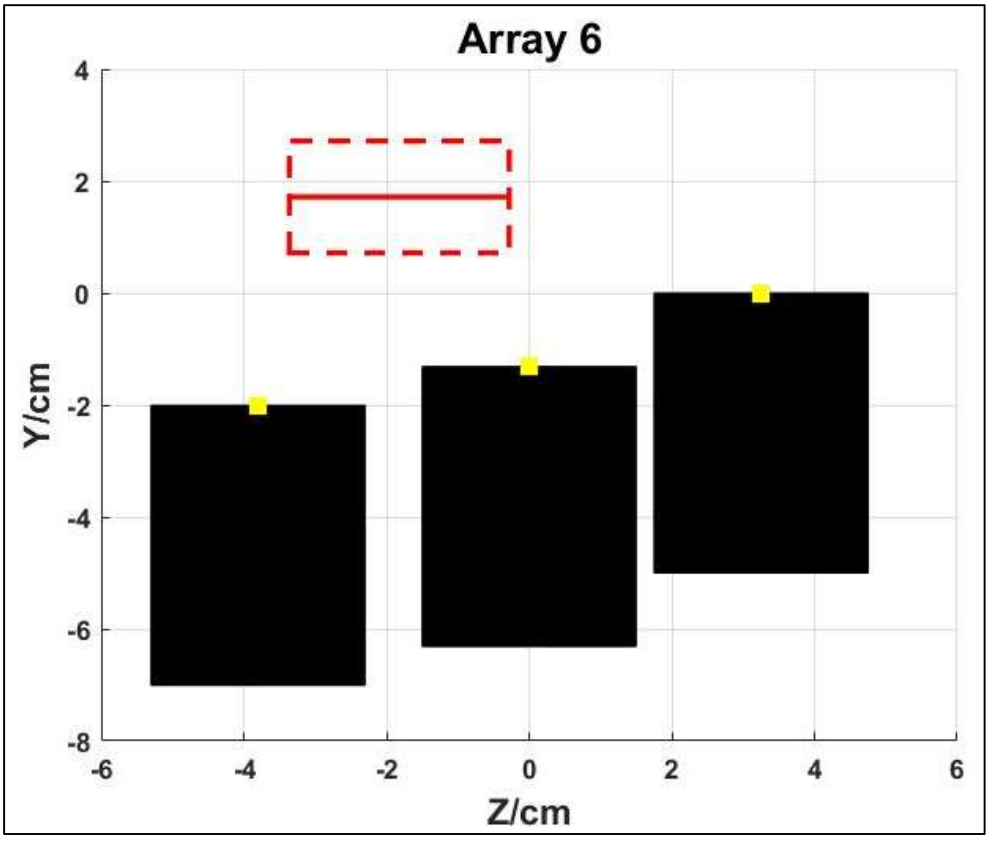

Figure 1.6: Array 6 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 1. The red dotted box is the area over which G_z is approximately constant.

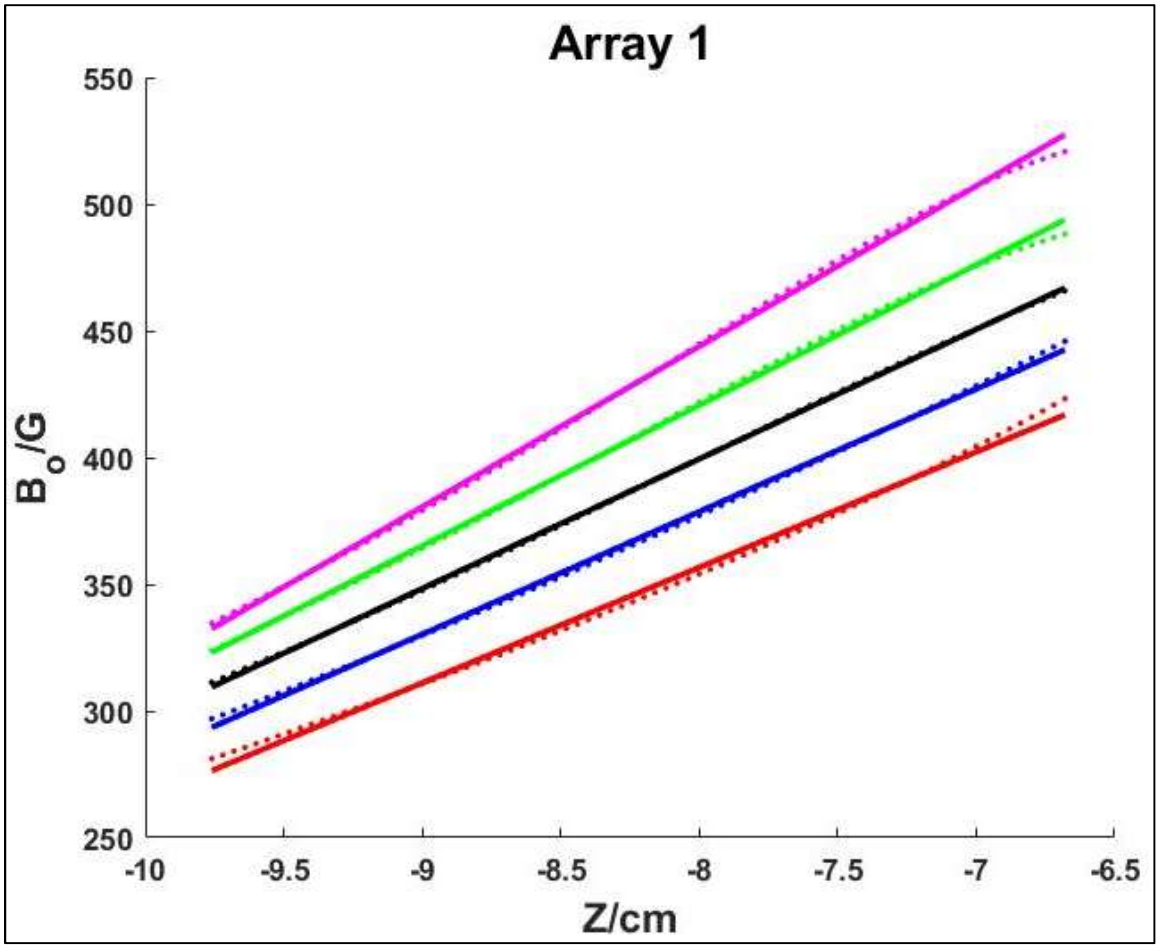

Figure 2.1: B_o of Array 1 as a function of Z for Y = -6.2792 cm, -5.7792 cm, -5.2792 cm, -4.7792 cm, and -4.2792 cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

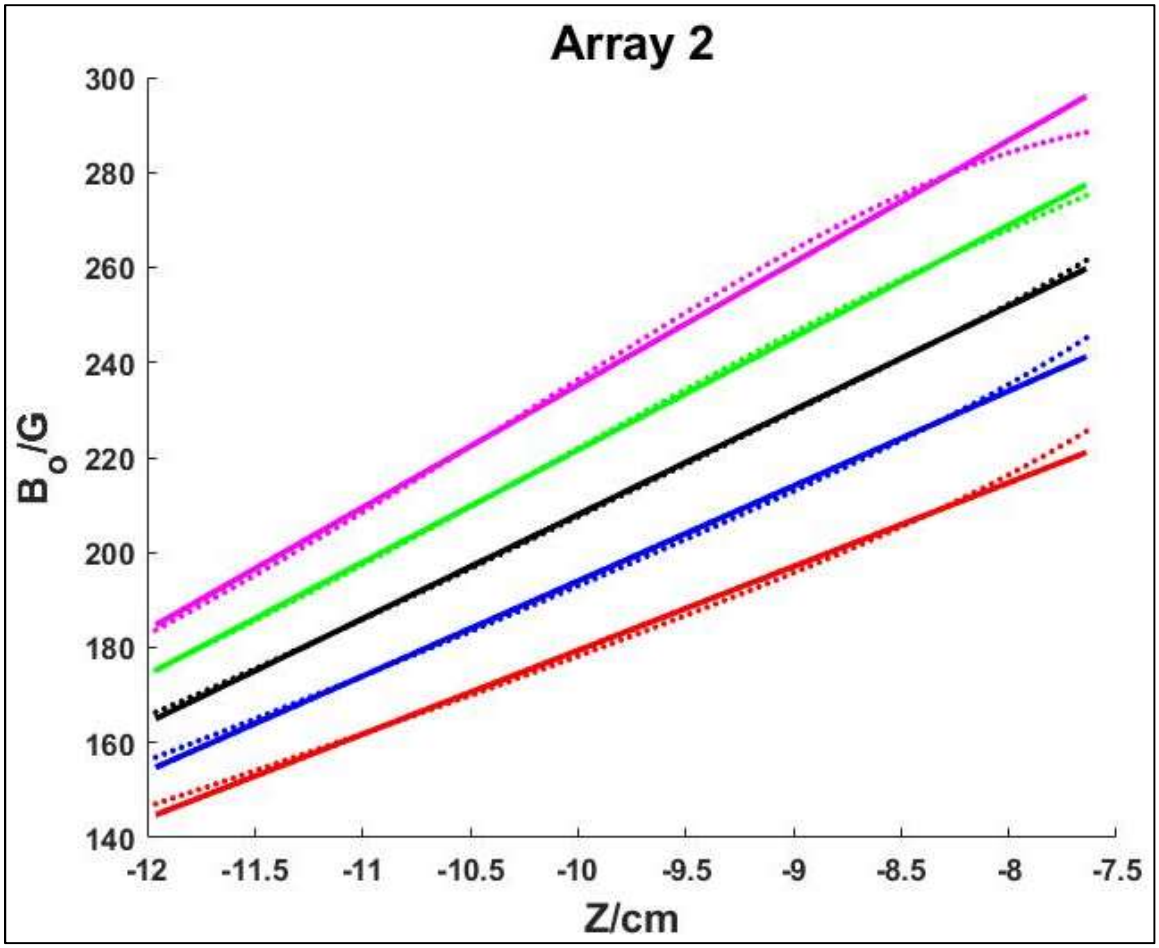

Figure 2.2: B_o of Array 2 as a function of Z for Y = -7.53 cm, -7.03 cm, -6.53 cm, -6.03 cm, and -5.53 cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

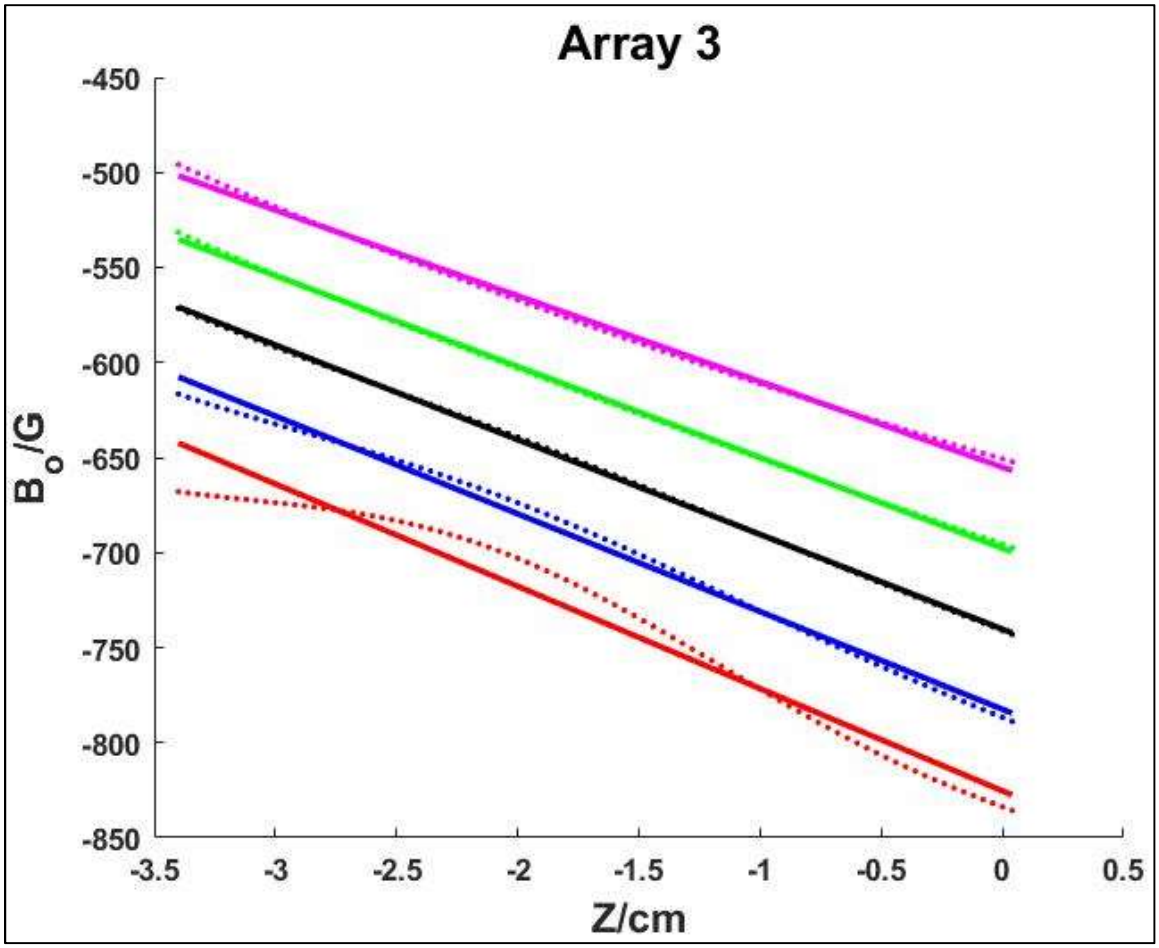

Figure 2.3: B_o of Array 3 as a function of Z for Y = 1.9585 cm, 2.4585 cm, 2.9585 cm, 3.4585 cm, 3.9585 cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

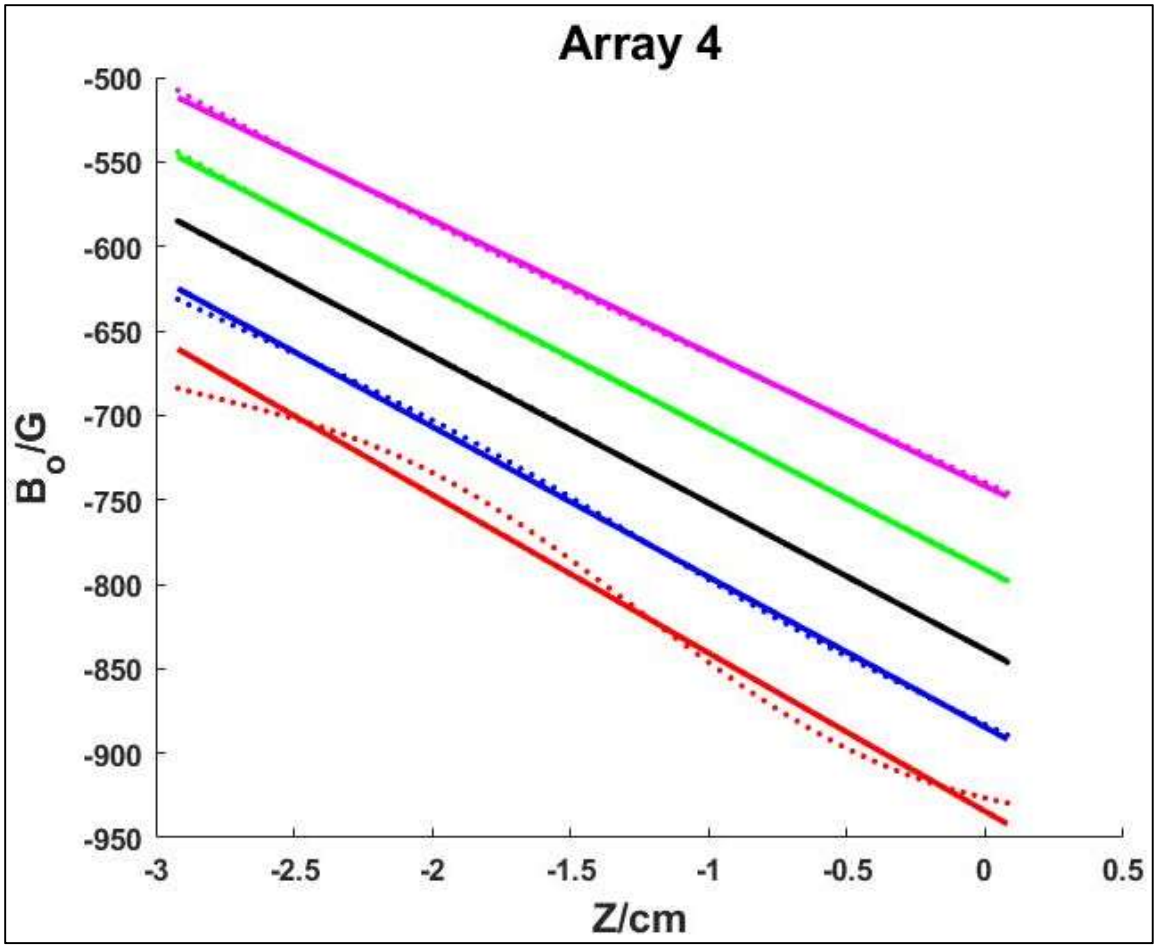

Figure 2.4: B_o of Array 4 as a function of Z for Y = 1.0869 cm, 1.5869 cm, 2.0869 cm, 2.5869 cm, 3.0869 cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

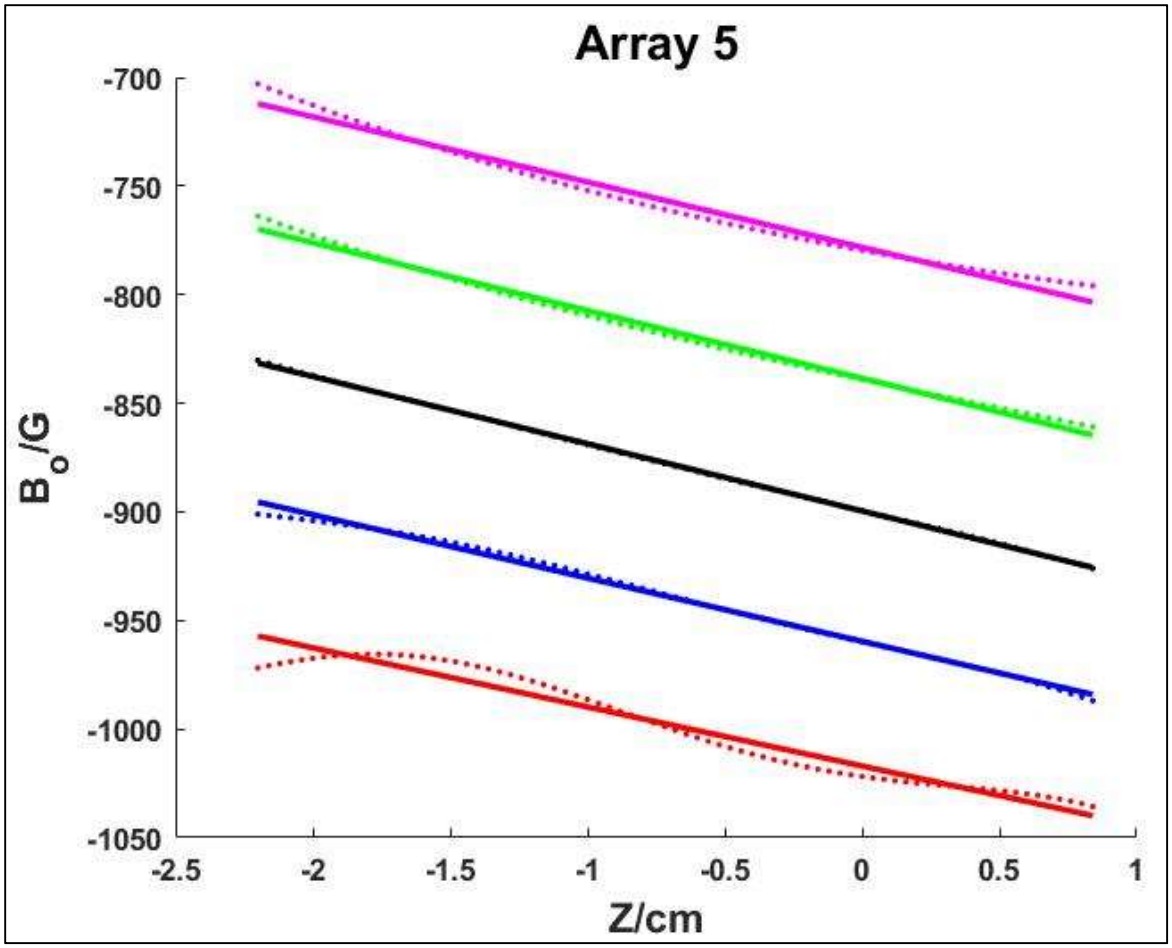

Figure 2.5: B_o of Array 5 as a function of Z for Y =1.6754 cm, 2.1754 cm, 2.6754 cm, 3.1754 cm, and 3.6754 cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

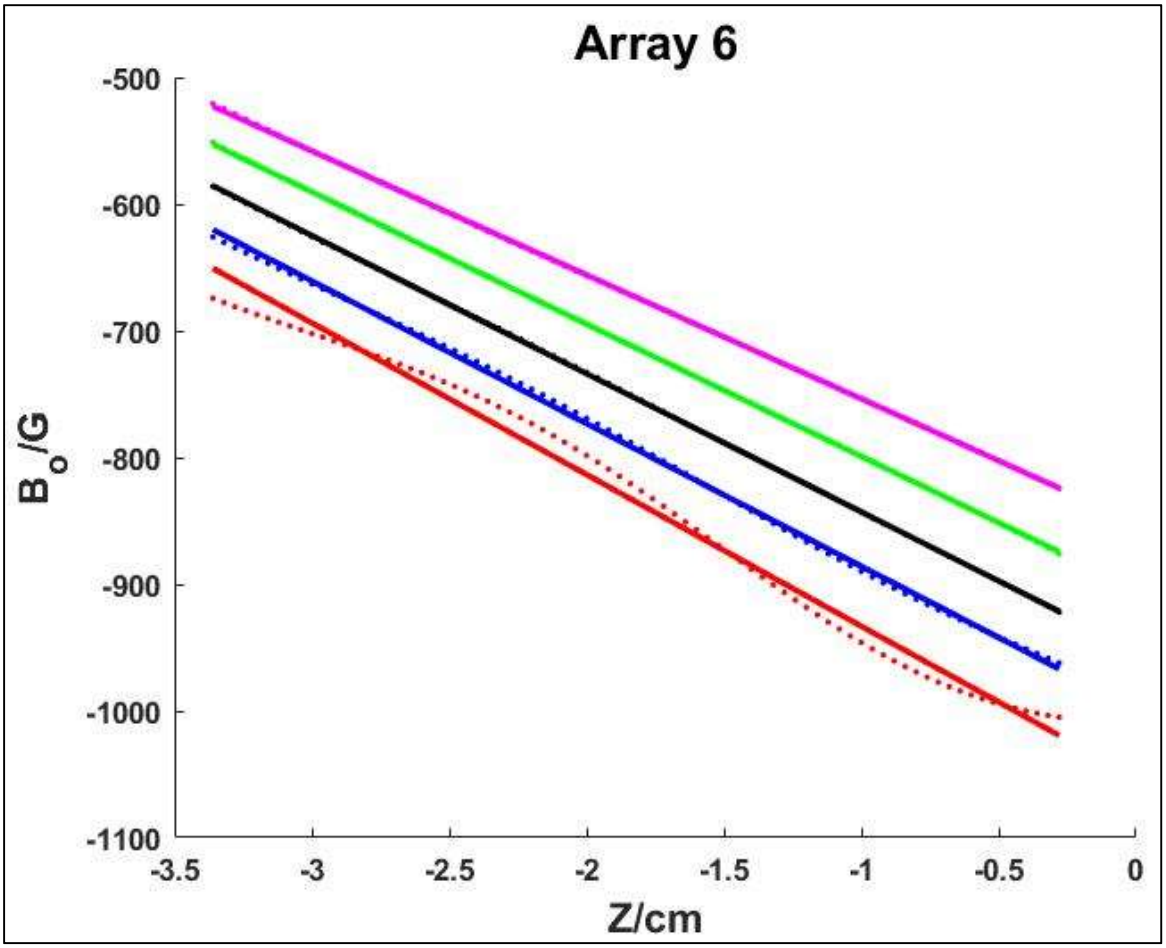

Figure 2.6: B_o of Array 6 as a function of Z for Y = 0.7158 cm, 1.2158 cm, 1.7158 cm, 2.2158 cm, and 2.7158 cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

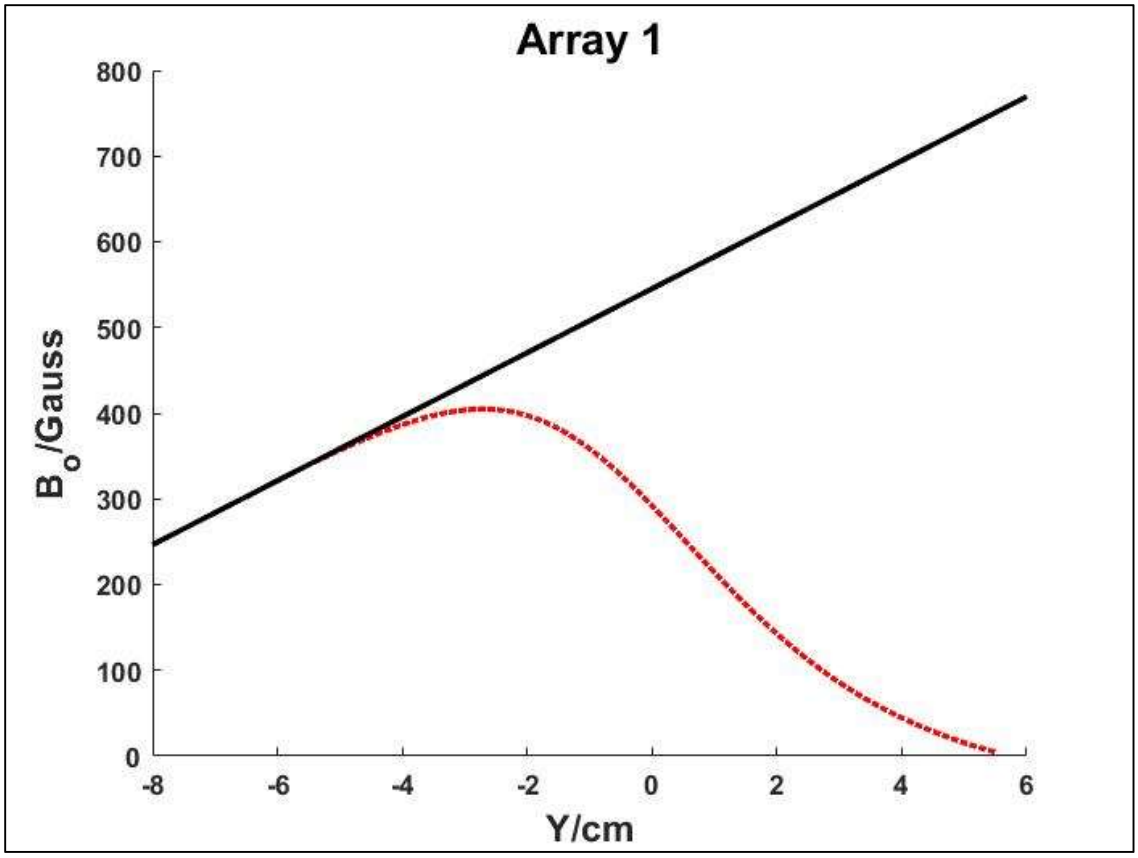

Figure 3.1; B_o of Array 1 as a function of Y for Z = -9 cm (red line), and the best fit line to the area of constant G_y (black line).

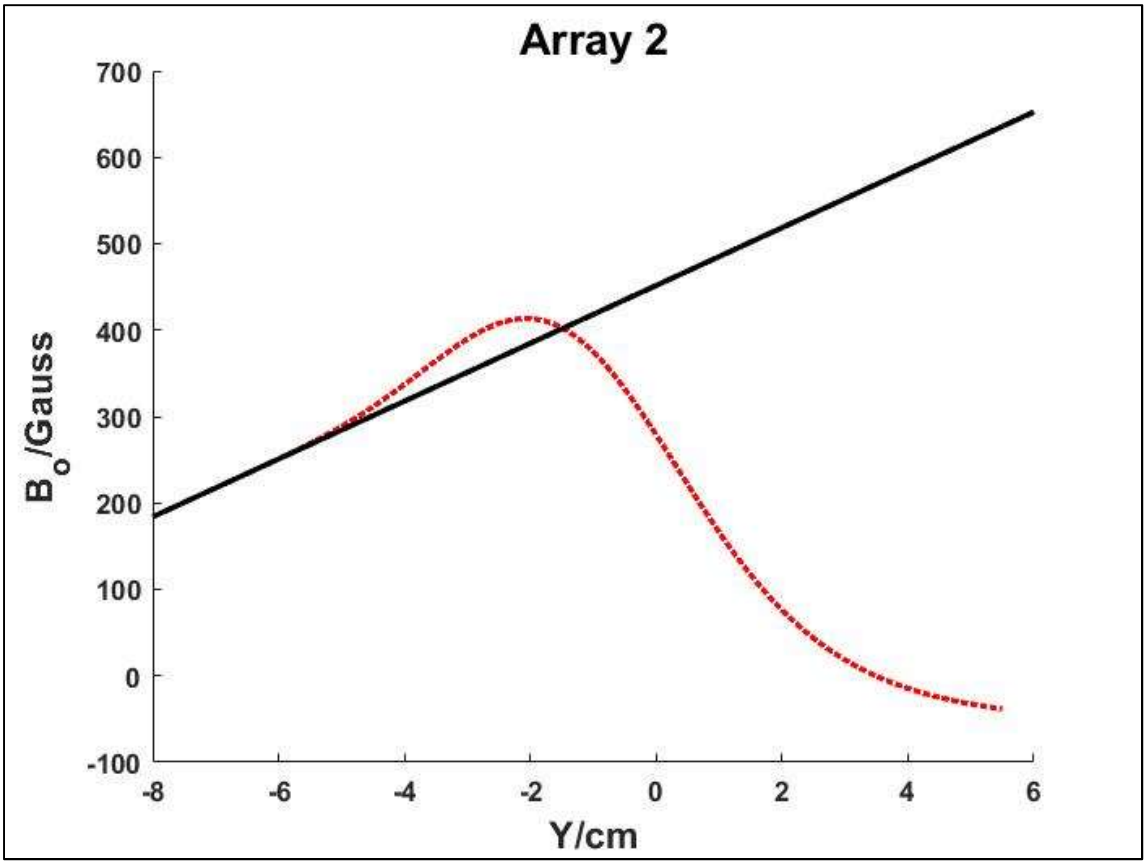

Figure 3.2; B_o of Array 2 as a function of Y for Z = -8.84 cm (red line), and the best fit line to the area of constant G_y (black line).

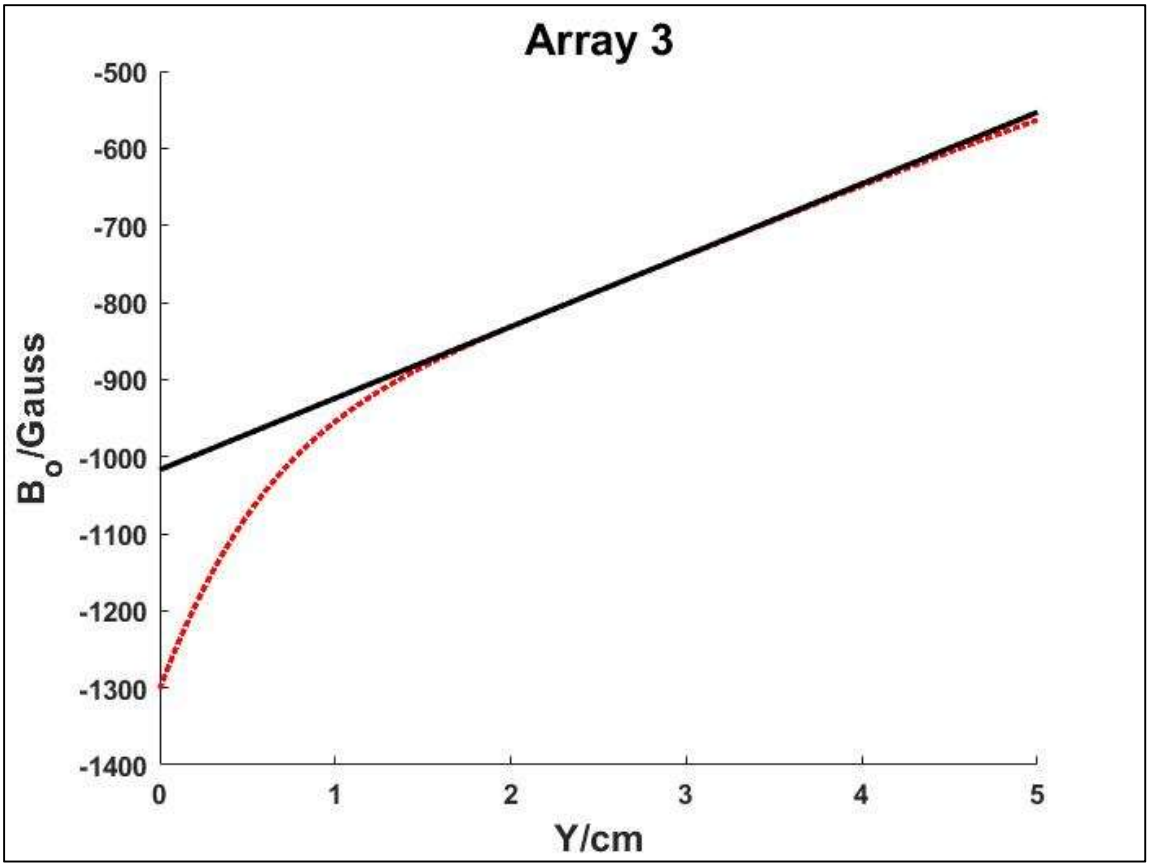

Figure 3.3; B_o of Array 3 as a function of Y for Z = 0.04 cm (red line), and the best fit line to the area of constant G_y (black line).

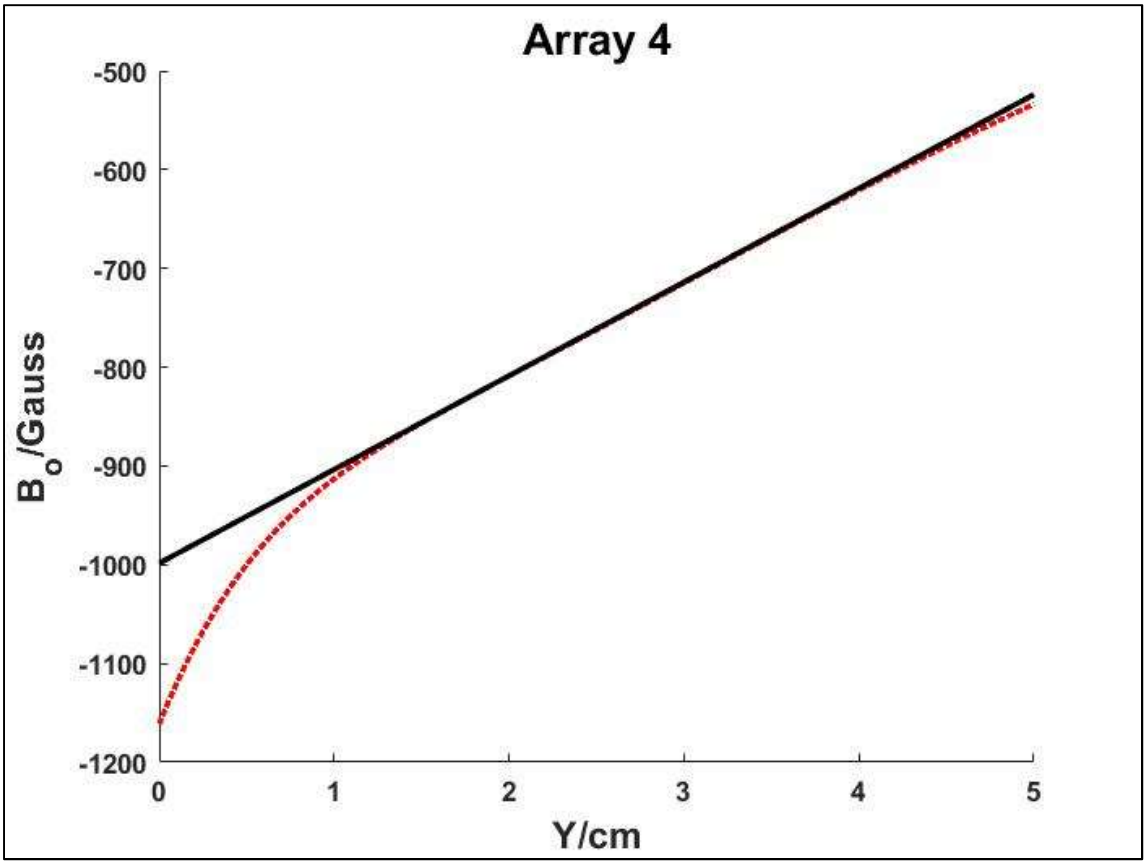

Figure 3.4; B_o of Array 4 as a function of Y for Z = -0.44 cm (red line), and the best fit line to the area of constant G_y (black line).

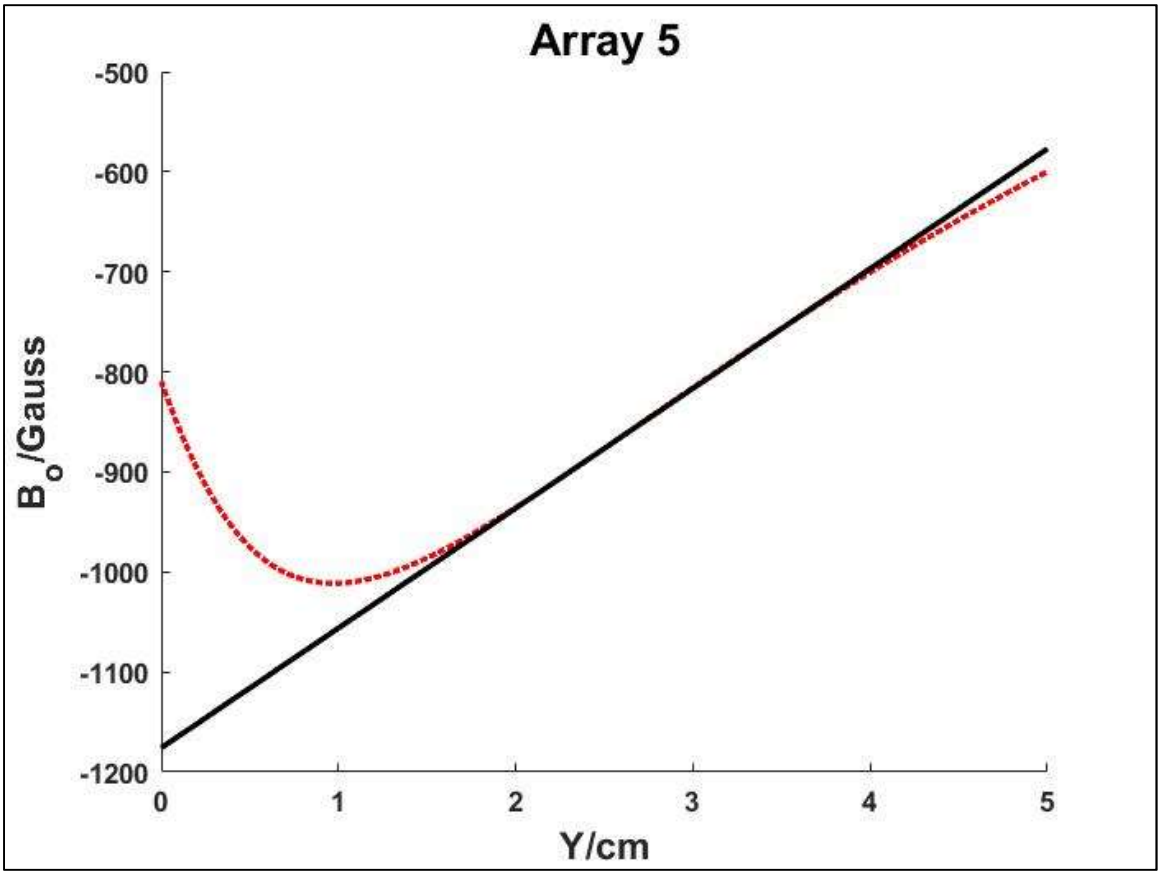

Figure 3.5; B_o of Array 5 as a function of Y for Z = -1.44 cm (red line), and the best fit line to the area of constant G_y (black line).

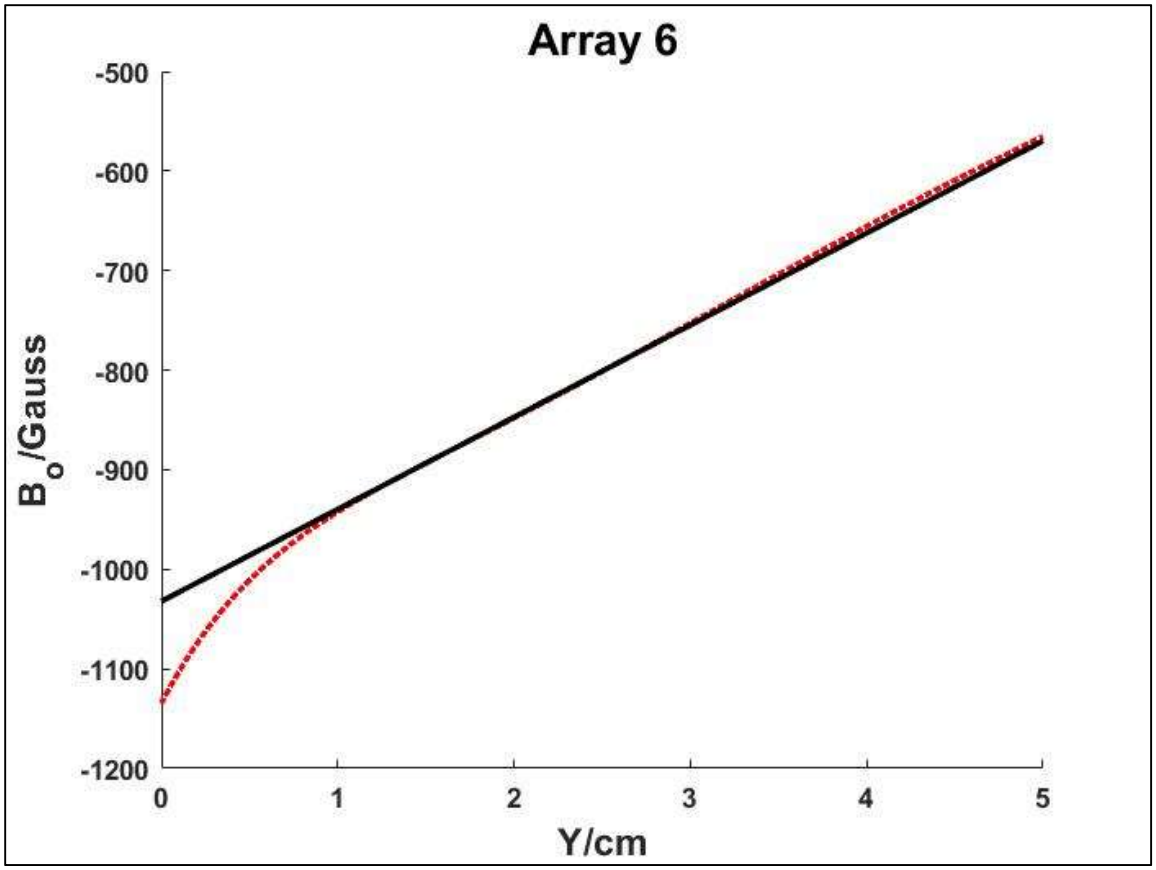

Figure 3.6; B_o of Array 6 as a function of Y for Z = -0.72 cm (red line), and the best fit line to the area of constant G_y (black line).

[7]: https://physics.nist.gov/cgi-bin/cuu/Value?gammap