Constant Gradient 3-Magnet Design: Report 2

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Version 2.3

1. Introduction

This report outlines work done from May 15th-22nd to achieve a constant gradient G_z of various 3-magnet arrays, where G_z is the variation of the Z-component of the magnetic field, B_o, in the Z-direction. This follows from work done by Marble [1], Garcia [2], and Wilbur [3]; Report 1 [4] on this project; and uses the MagnetArray [5] MATLAB program developed for the University of New Brunswick Department of Physics. Various gradients in the range of 15 Gauss/cm to 150 Gauss/cm over a cylindrical volume of about 1 cm diameter and 3-5 cm long are desired to measure how fluids flow through pipes.

2. Optimization.m & Optimization2.m

The MATLAB Optimization and Global Optimization toolboxes allow the user to define a multivariable equation subject to various constraints, and finds values for individual variables to maximize or minimize the function. Optimization.m was written using functions from the Optimization toolbox, but it could not solve for integer values for widths and heights of the magnets, meaning it could not optimize the arrays in terms of magnets already available to the UNB MRI lab. It was also difficult to include constraints to the gradient to ensure G_z was constant. Optimization2.m was written using the Genetic Algorithm (GA) function from the Global Optimization toolbox. The GA function works by using a random number generator to come up with a first generation of several possible solutions to minimize a function, evaluates how much they minimize the function, then adjusts and combines traits of the most-minimizing solutions to create the next generation of possible solutions. The possible solutions end up converging towards the "true" minimum of the function. This type of minimization is favorable over typical calculus minimization for multiparameter functions such as the magnetic field of a 3-magnet array; it is faster for a computer program to execute, and real solutions can be easily obtained, rather than solving for one parameter in terms of the rest.

A constant G_z implies that the second derivative of the magnetic field with respect to Z is 0. The GA function will only find minimums of functions; to force it to find the roots of the second derivative with respect to Z, the absolute value of the second derivative was used. For each parameter in the equation, the GA function also needed an upper and lower bound, shown in Table 1.

Variable	w_1	w_2	w_3	h_1	h_2	h_3	Z_1	Z_2	Z_3	Y_1	Y_2	Y_3	Z_{ga}	Y_{ga}	K
Lower Bound	1	1	1	1	1	1	-5	0	0	-3	-3	0	1	0	1500
Upper Bound	3	3	3	2	2	2	0	0	5	0	0	0	301	4	1500

Table 1: Lower and upper bounds for each parameter optimized by GA. Indices 1, 2, and 3, indicate the parameters of the left, centre, and right magnets respectively. w and h represent widths and heights of the magnets, and $Z_{1\text{-}3}$ and $Y_{1\text{-}3}$ represent the coordinates of the magnets in the YZ plane. Z_{ga} and Y_{ga} are the coordinates above the array the GA algorithm found optimized the function. w_1, w_2, w_3, h_1, h_2, h_3, and Z_{ga} all have indices corresponding to values in separate vectors as their upper and lower bounds. Values that were set (Z_2, Y_3, and K) have the same upper and lower bound.

The three indices the widths could take on corresponded to 1 cm (1), 2 cm (2), and 3 cm (3). The two indices the heights could take corresponded to 3 cm (1) and 5 cm (2). These are widths and heights of magnets already available for use at the UNB MRI lab. To make sure the Y-axis was down the middle of the array and the Z-axis was along the top of the array, Z_2 (the horizontal position of the centre magnet) and Y_3 (the vertical position of the right, highest magnet) were set to 0. K was chosen to be 1500 Gauss/cm.

The Z_{ga} indices correspond to 301 evenly spaced values from -6 cm to 6 cm, increasing by 0.04 for every index. The GA algorithm cannot solve for a range of values, only specific values; mapping Z like this made it easier to find a range of Z with a constant gradient. G_z was evaluated along the vector of possible Z values and compared to G_z evaluated at Z_{ga}. Values with a gradient within 4 Gauss/cm of the gradient at Z_{ga} were retained.

If the range of retained Z-values was greater than 2.8 cm, the magnetic field was calculated over that range for Y values 0.5 cm below the Y value returned by the algorithm, and 0.5 cm above it. Lines were fit to these two data sets, and if the difference in the slopes of the lines was less than 5 Gauss/cm, the data was saved.

Linear constraints supplied to the GA algorithm were:

$$0.5 \cdot w_2 + 0.5 \cdot w_3 - Z_3 \leq 0$$

$$0.5 \cdot w_1 + 0.5 \cdot w_2 + Z_1 \leq 0$$

These ensured the magnets didn't overlap.

Nonlinear constraints supplied to the GA algorithm were:

$$|G_z(Y = Y_{ga} - 0.5\ cm) - G_z(Y = Y_{ga} + 0.5\ cm)| - 5 \leq 0$$

$$|G_z(Z = Z_{ga} - 1.5\ cm) - G_z(Z = Z_{ga} + 1.5\ cm) - 8 \leq 0$$

$$|G_{zgoal} - G_z(Z = Z_{ga}) \leq 0$$

The first two constraints are reiterations of the constant slope conditions. The first one ensures that the variation with respect to Y of G_z is less than 5 Gauss/cm. The second one ensures that G_z is constant, within 8 Gauss/cm. These were included as constraints in the algorithm, as well as conditions to save the data, because the GA algorithm will occasionally violate constraints within a certain tolerance, and would typically find inflection points in B_o, where its 2nd derivative is also zero. The constraints in the GA algorithm found parameters that gave the roots of the 2nd derivative, and the save conditions filtered out any results that were just inflection points and not areas of extended constant G_z.

The third nonlinear constraint determined the gradient magnitude. G_{zgoal} is a predetermined lower bound for G_z. With no lower bound, the algorithm found gradients between 10 Gauss/cm and 20 Gauss/cm. To get a variety of arrays to produce a variety of gradients, G_{zgoal} was adjusted between runs.

It was found by redoing the integral from eq. 2 in [1] that the eq. 3 in [1] giving the magnetic field is wrong; the Z-component has the opposite sign from what it should. This was corrected to come up with these gradients. The equation is now

$$\vec{B} = K \cdot \left[\arctan\left(\frac{z - \frac{w}{2}}{y}\right) - \arctan\left(\frac{z + \frac{w}{2}}{y}\right) - \arctan\left(\frac{z - \frac{w}{2}}{y + h}\right) + \arctan\left(\frac{z + \frac{w}{2}}{y + h}\right)\right] \hat{z}$$

$$+ \frac{K}{2} \cdot \left[\ln\left(\frac{y^2 + \left(z + \frac{w}{2}\right)^2}{y^2 + \left(z - \frac{w}{2}\right)^2}\right) - \ln\left(\frac{(y+h)^2 + \left(z + \frac{w}{2}\right)^2}{(y+h)^2 + \left(z - \frac{w}{2}\right)^2}\right)\right] \hat{y}$$

Where $K = \frac{\mu_o I}{2\pi}$.

In Version 2.2 of this report, the incorrect equations had been used.

3. Data

Array	Magnet	Width/cm	Height/cm	Z/cm	Y/cm	Z range/cm	Y_{ga}/cm
1	Left	2	5	-4.6146	-2.4821	$[-4.20, 0.32]$	3.9839
	Centre	2	3	0	-1.5992		
	Right	2	5	4.4238	0		
2	Left	2	3	-3.8844	-2.6660	$[-4.16, -0.48]$	2.2198
	Centre	2	5	0	-1.7549		
	Right	3	3	3.8338	0		
3	Left	3	3	-3.5323	-2.4297	$[-3.68, -0.28]$	2.4650
	Centre	2	3	0	-1.2737		
	Right	3	5	3.9853	0		

Table 2.1: Arrays generated by the Optimization2.m program. The Z coordinate specifies the positions of the centres of the magnets. The Y coordinate specifies the positions of the tops of the magnets. 'Left', 'Centre', and 'Right' refer to the individual magnets, and subscripts 1, 2, and 3 throughout the paper refer to them respectively. Y_{ga} is the height above the array returned by the GA function that has the most constant G_z. The Z range is the range found by the Optimization2.m program over which G_z is constant.

Array	Magnet	Width/cm	Height/cm	Z/cm	Y/cm	Z range/cm	Y_{ga}/cm
4	Left	1	5	-3.8416	-2.8672	$[-4.24, -1.32]$	1.4132
	Centre	3	5	0	-2.8289		
	Right	3	3	3.4525	0		
5	Left	1	3	-3.3037	-2.6950	$[-4.28, -1.08]$	0.7182
	Centre	3	3	0	-2.4515		
	Right	3	5	3.1679	0		
6	Left	2	3	-3.6908	-2.0209	$[-3.44, -0.68]$	2.6585
	Centre	3	3	0	-1.8521		
	Right	3	5	3.9730	0		
7	Left	2	3	-3.2612	-2.7133	$[-3.48, -0.48]$	1.3389
	Centre	2	5	0	-1.8764		
	Right	2	5	2.8935	0		

Table 2.2: Arrays generated by the Optimization2.m program. The Z coordinate specifies the positions of the centres of the magnets. The Y coordinate specifies the positions of the tops of the magnets. 'Left', 'Centre', and 'Right' refer to the individual magnets, and subscripts 1, 2, and 3 throughout the paper refer to them respectively. Y_{ga} is the height above the array returned by the GA function that has the most constant G_z. The Z range is the range found by the Optimization2.m program over which G_z is constant.

4. Analysis & Discussion

Array	Y/cm	Linear fit
1	2.9839	$-28.91 \cdot Z - 366.4$
	3.4839	$-28.78 \cdot Z - 353.3$
	3.9839	$-28.08 \cdot Z - 339$
	4.4839	$-26.88 \cdot Z - 323.8$
	4.9839	$-25.33 \cdot Z - 308.1$
2	1.2198	$-60.43 \cdot Z - 476.4$
	1.7198	$-60.73 \cdot Z - 466.6$
	2.2198	$-60.85 \cdot Z - 457.9$
	2.7198	$-59.96 \cdot Z - 446.9$
	3.2198	$-57.89 \cdot Z - 432.6$
3	1.4650	$-75.67 \cdot Z - 584.5$
	1.9650	$-73.56 \cdot Z - 563.5$
	2.4650	$-72.24 \cdot Z - 545.7$
	2.9650	$-70.09 \cdot Z - 526.5$
	3.4650	$-66.79 \cdot Z - 504.8$

Table 3.1: Linear fits of B_0 in Gauss as a function of Z. The slopes of the linear fits are G_z, in Gauss/cm. These are plotted in figure 2.

Array	Y/cm	Linear fit
4	0.4132	$-104.1 \cdot Z - 503.5$
	0.9132	$-101.7 \cdot Z - 507.5$
	1.4132	$-100.4 \cdot Z - 516.6$
	1.9132	$-98.82 \cdot Z - 524.3$
	2.4132	$-96.16 \cdot Z - 526.5$
5	-0.2818	$-112.6 \cdot Z - 391.5$
	0.2182	$-108.4 \cdot Z - 404$
	0.7182	$-107.9 \cdot Z - 430.4$
	1.2182	$-108.8 \cdot Z - 460.7$
	1.7182	$-108.8 \cdot Z - 485.5$
6	1.6585	$-68.01 \cdot Z - 550$
	2.1585	$-71.91 \cdot Z - 543.7$
	2.6585	$-73.17 \cdot Z - 532.8$
	3.1585	$-72.06 \cdot Z - 517.2$
	3.6585	$-69.15 \cdot Z - 497.5$
7	0.3389	$-115.6 \cdot Z - 714.1$
	0.8389	$-111.1 \cdot Z - 688.1$
	1.3389	$-109.3 \cdot Z - 671$
	1.8389	$-106.3 \cdot Z - 651.6$
	2.3389	$-100.9 \cdot Z - 625.8$

Table 3.2: Linear fits of B_0 in Gauss as a function of Z. The slopes of the linear fits are G_z in Gauss/cm. Although there is a negative value of Y in array 5, it is still above the left and centre magnets, and its Z range doesn't overlap with the right magnet. These are plotted in figure 2.

Array	Z_{ga}/cm	Linear fit
1	-1.96	$23.9788 \cdot Y - 379.6194$
2	-1.00	$21.2121 \cdot Y - 443.6184$
3	-2.48	$26.9760 \cdot Y - 432.9179$
4	-1.88	$-20.4205 \cdot Y - 298.5394$
5	-3.32	$-42.1436 \cdot Y - 47.8967$
6	-2.36	$28.8801 \cdot Y - 440.2992$
7	-2.92	$21.8537 \cdot Z - 382.6247$

Table 4: Linear fits of B_o in Gauss as a function of Y for the values of Z_{ga} returned by the GA function.

The slopes of the linear fits are G_y in Gauss/cm. These are plotted in figure 3.

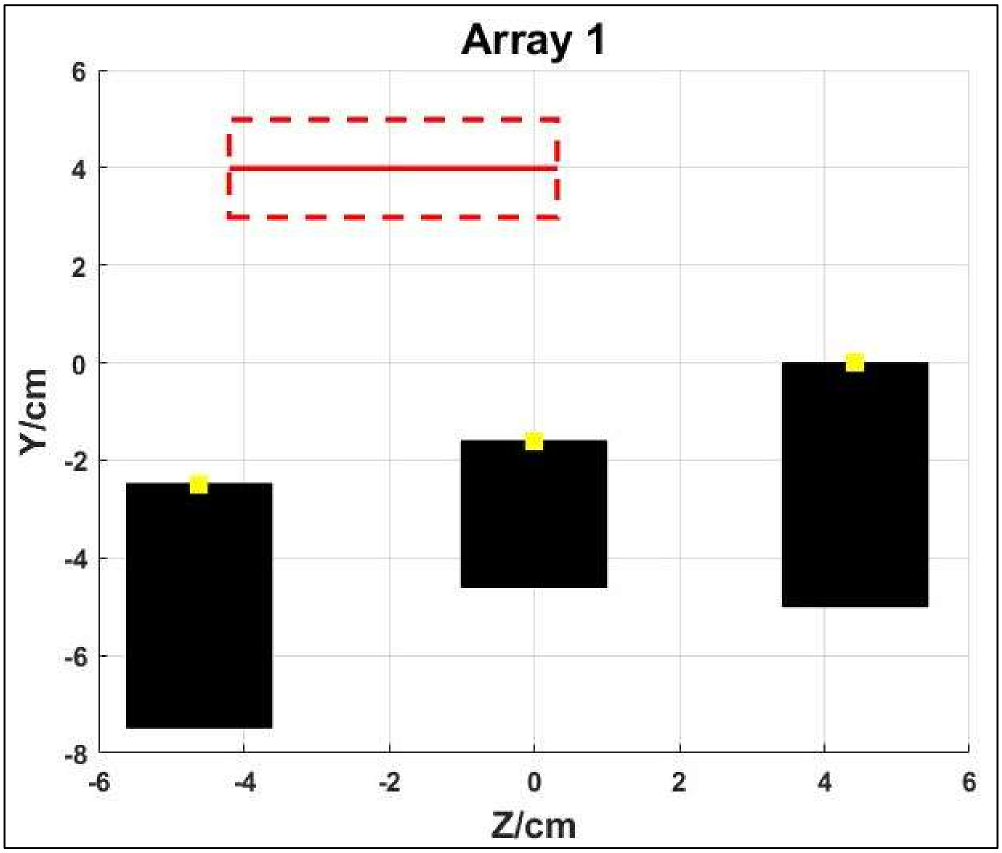

Figure 1.1: Array 1 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 2. The red dotted box is the area over which G_z is approximately constant.

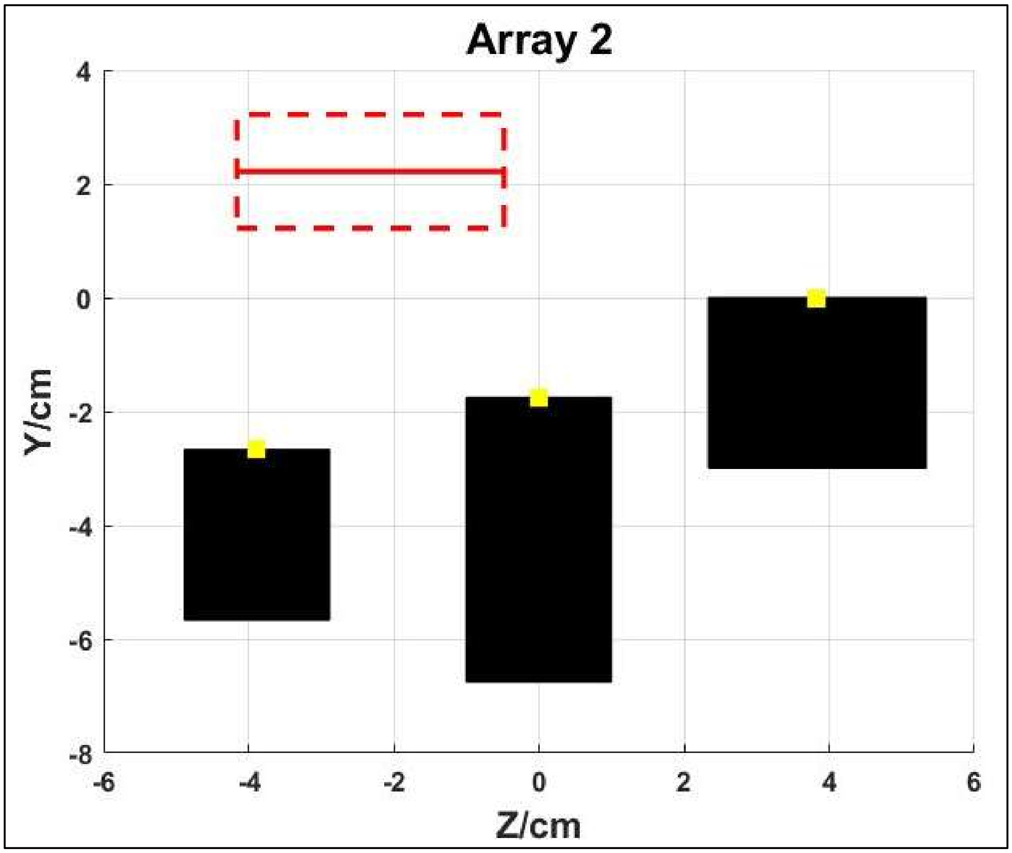

Figure 1.2: Array 2 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 2. The red dotted box is the area over which G_z is approximately constant.

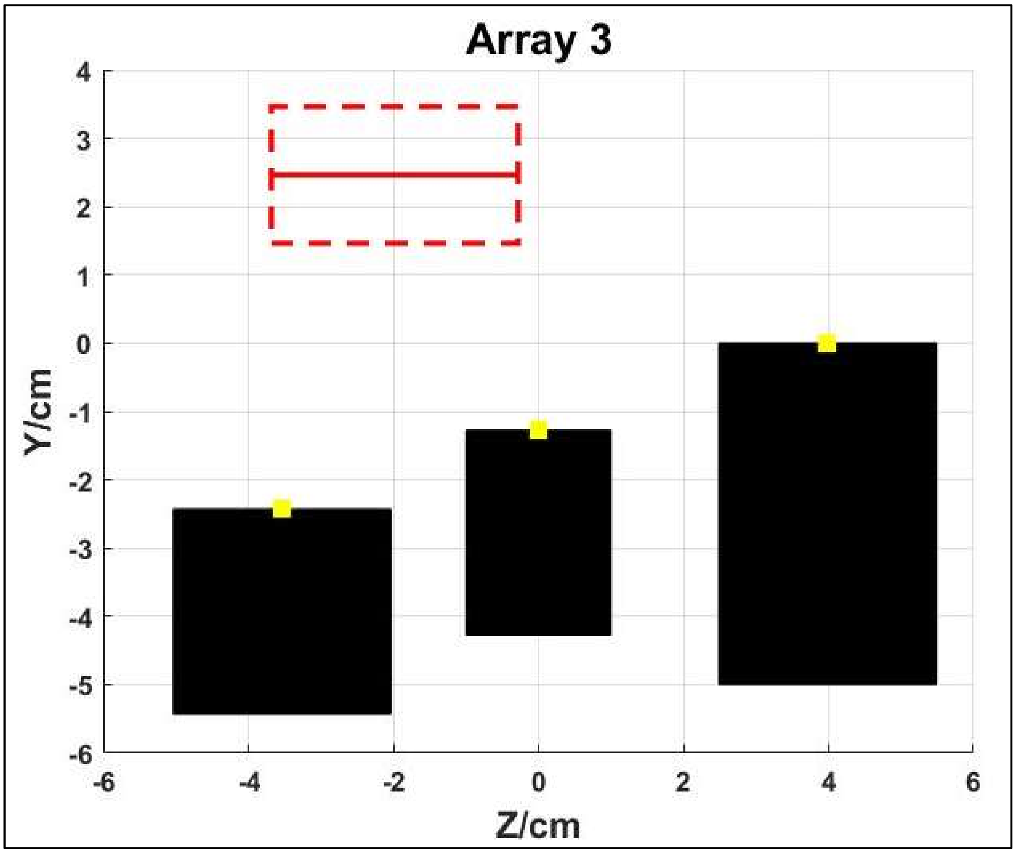

Figure 1.3: Array 3 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 2. The red dotted box is the area over which G_z is approximately constant.

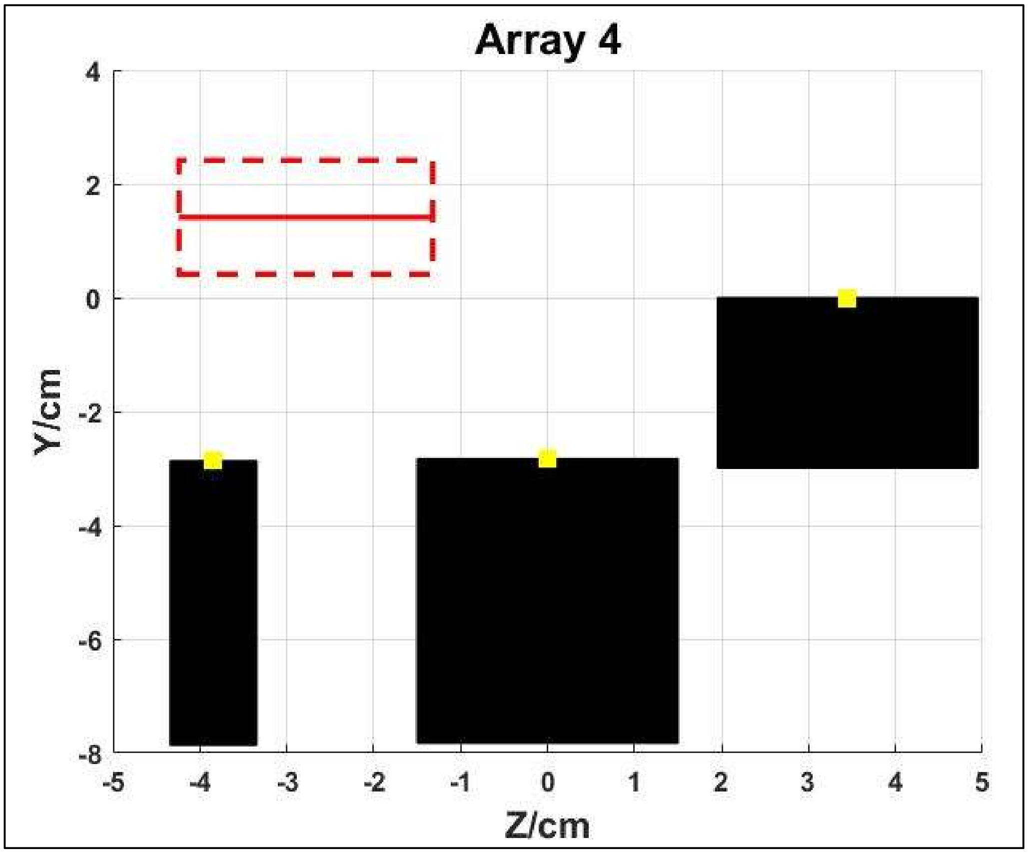

Figure 1.4: Array 4 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 2. The red dotted box is the area over which G_z is approximately constant.

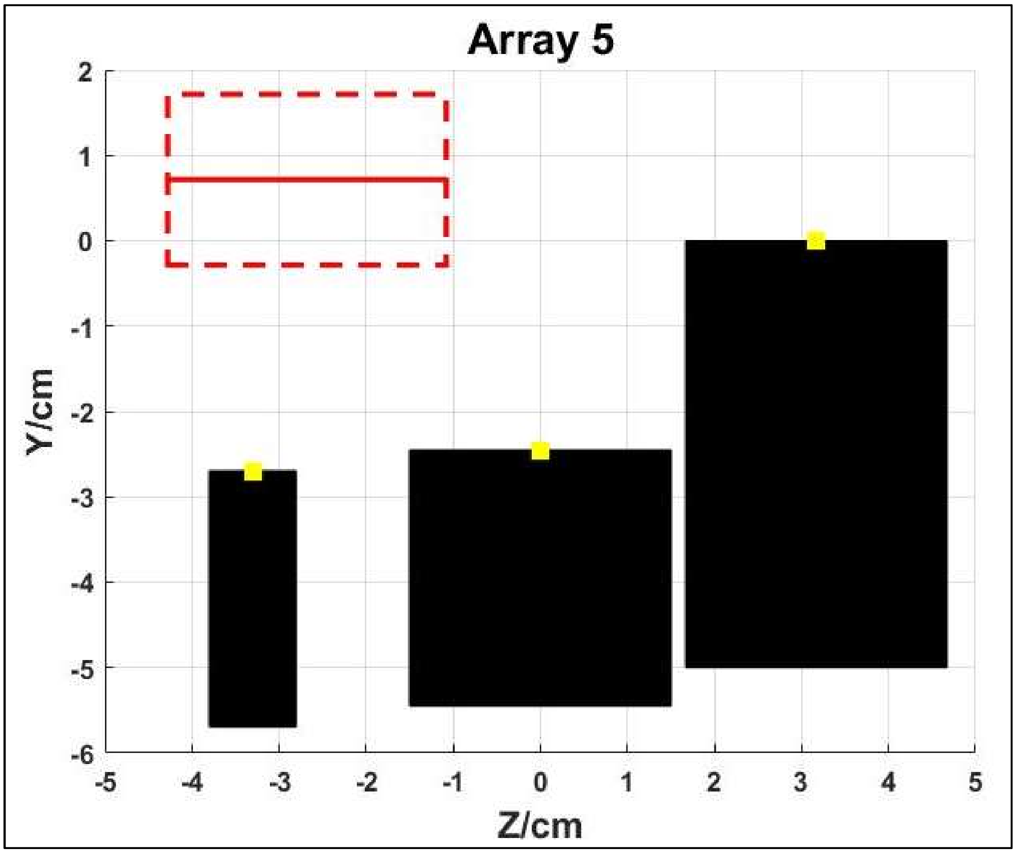

Figure 1.5: Array 5 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 2. The red dotted box is the area over which G_z is approximately constant.

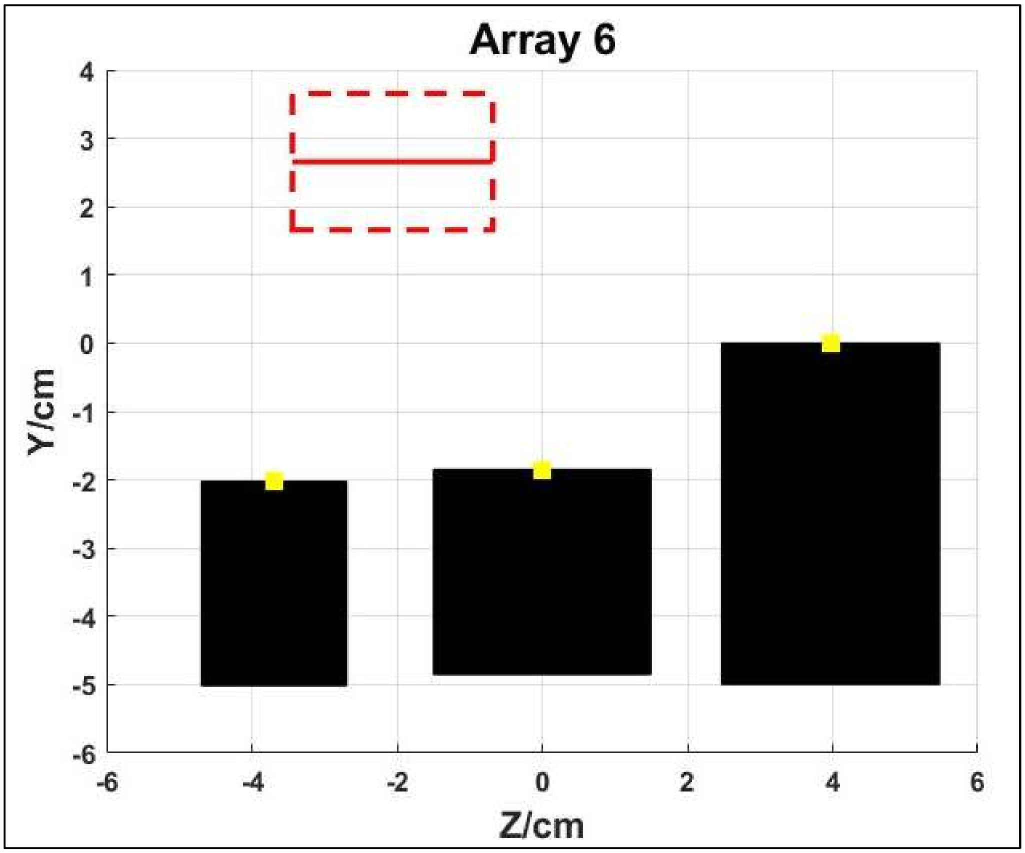

Figure 1.6: Array 6 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 2. The red dotted box is the area over which G_z is approximately constant.

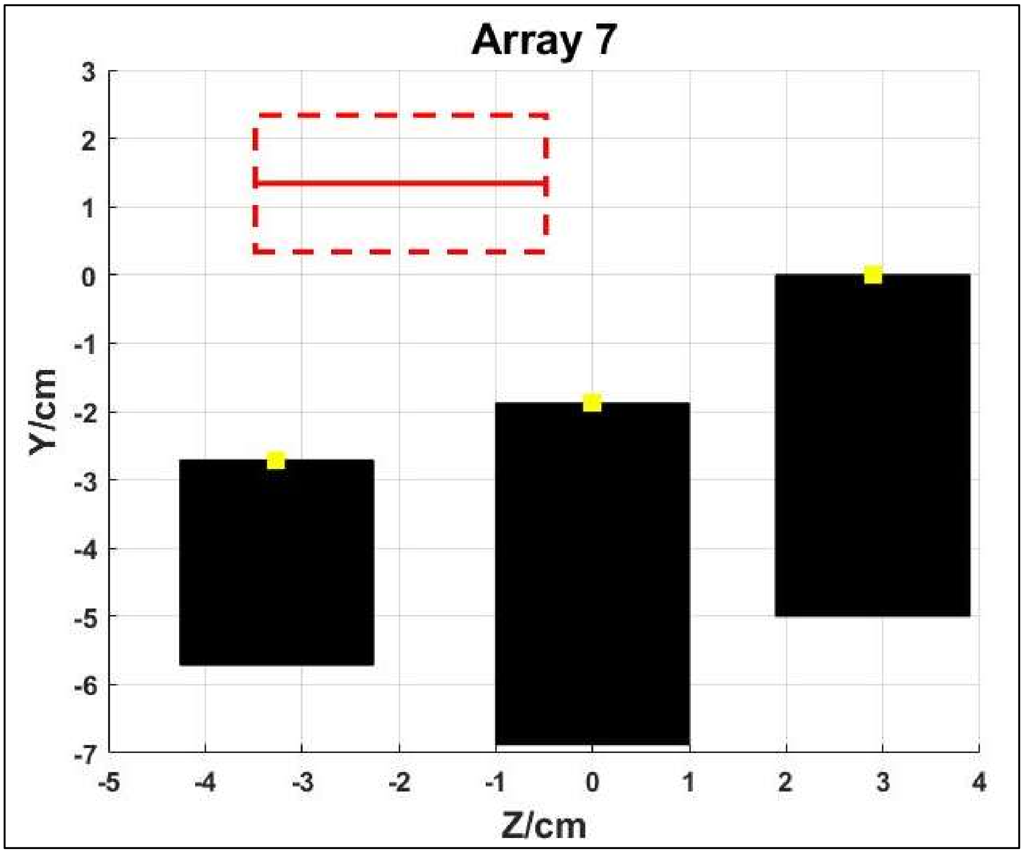

Figure 1.7: Array 7 magnets (black). The Z and Y coordinates of the magnets are measured from the yellow boxes. The red solid line is the height Y_{ga} over the range of Z values specified in Table 2. The red dotted box is the area over which G_z is approximately constant.

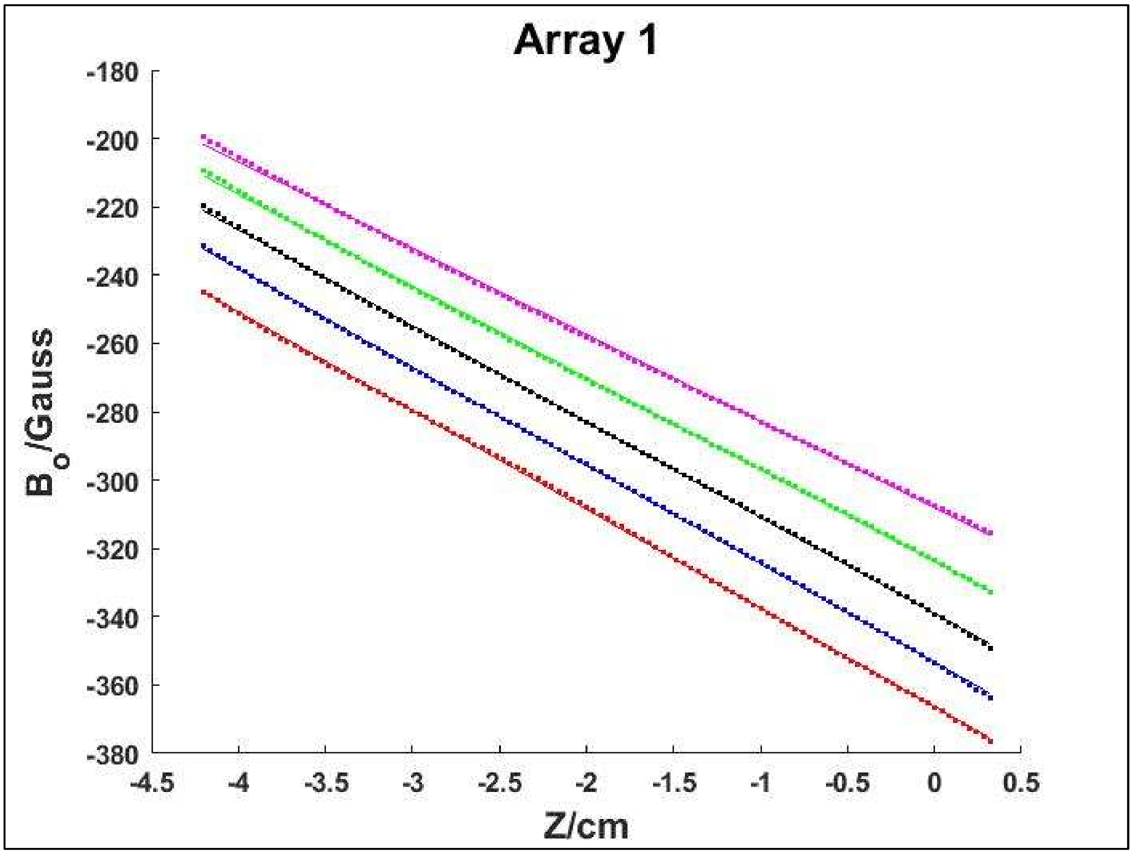

Figure 2.1: B_o of Array 1 as a function of Z for Y = 2.9839 cm, 3.4839 cm, 3.9839 cm, 4.4839 cm, and Y = 4.9839 cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

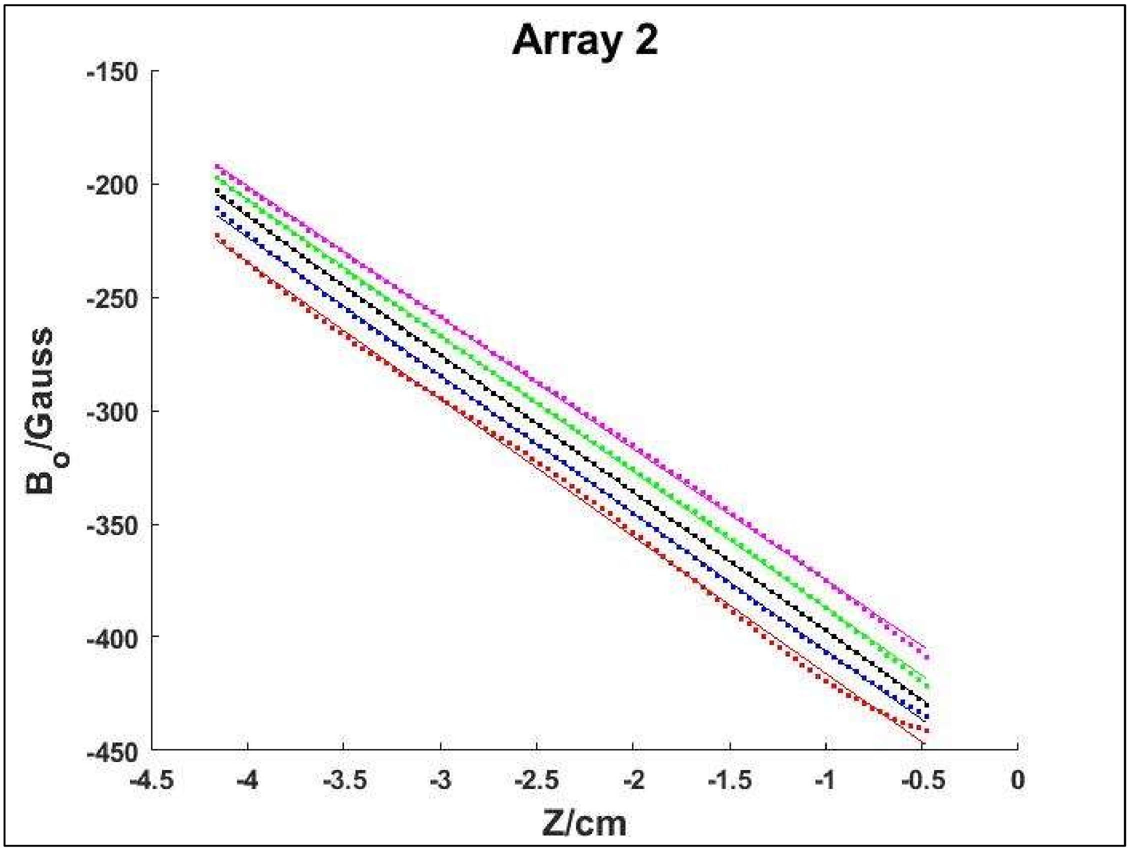

Figure 2.2: B_o of Array 2 as a function of Z for Y = 1.2198cm, 1.7198cm, 2.2198cm, 2.7198cm, and 3.2198cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

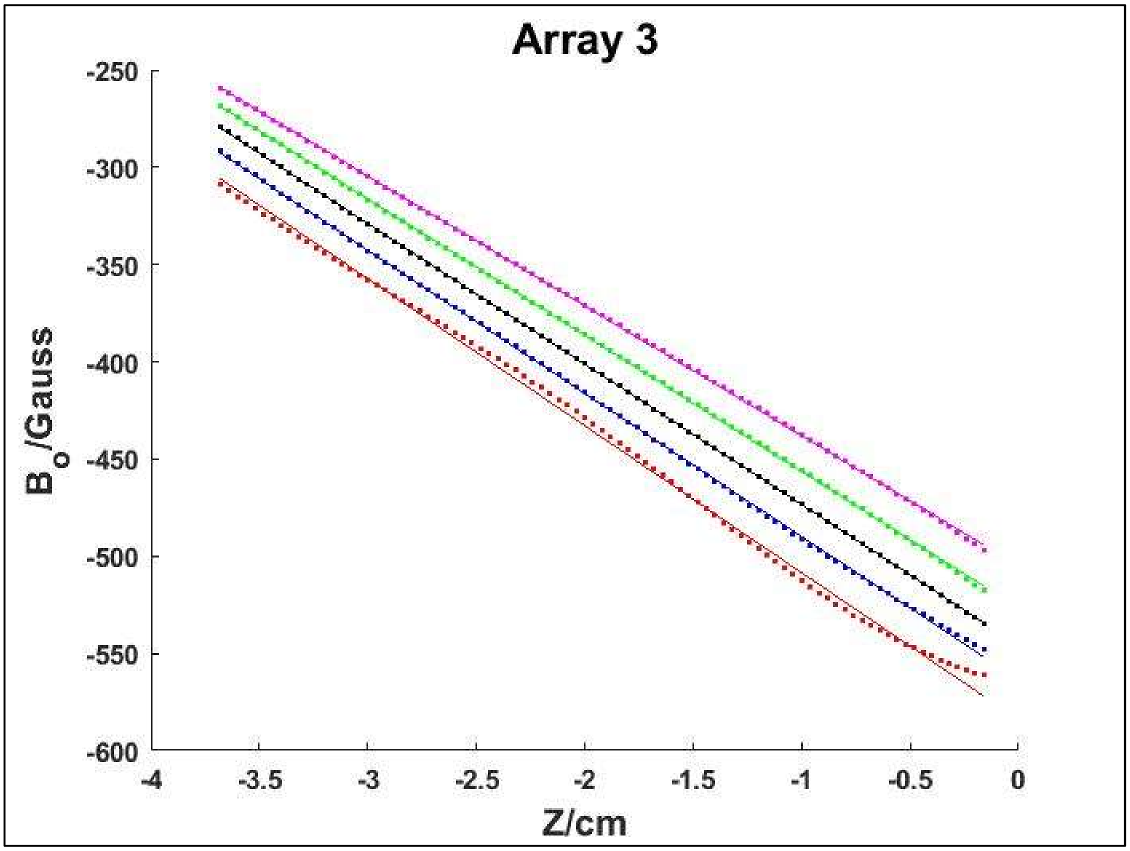

Figure 2.3: B_o of Array 3 as a function of Z for Y = 1.4650 cm, 1.9650 cm, 2.4650 cm, 2.9650 cm, and 3.4650 cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

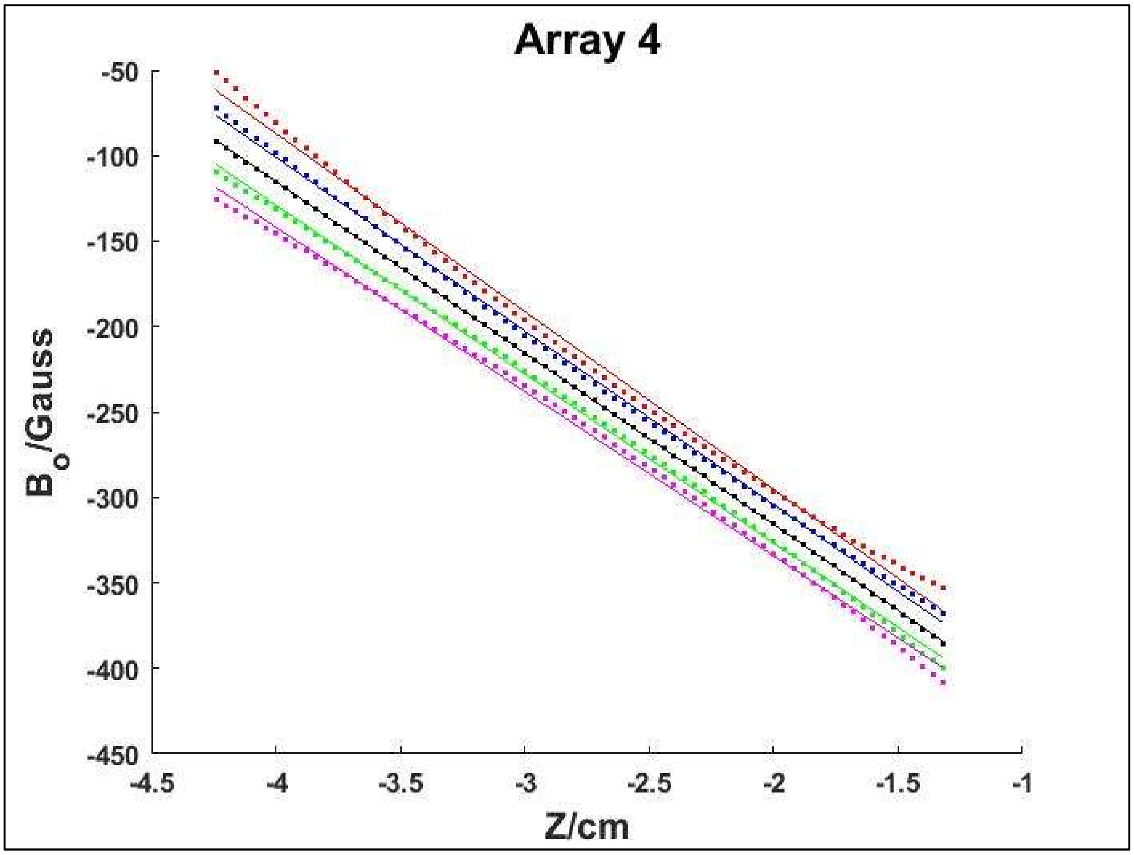

Figure 2.4: B_o of Array 4 as a function of Z for Y = 0.4132 cm, 0.9132 cm, 1.4132 cm, 1.9132 cm, and 2.4132 cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

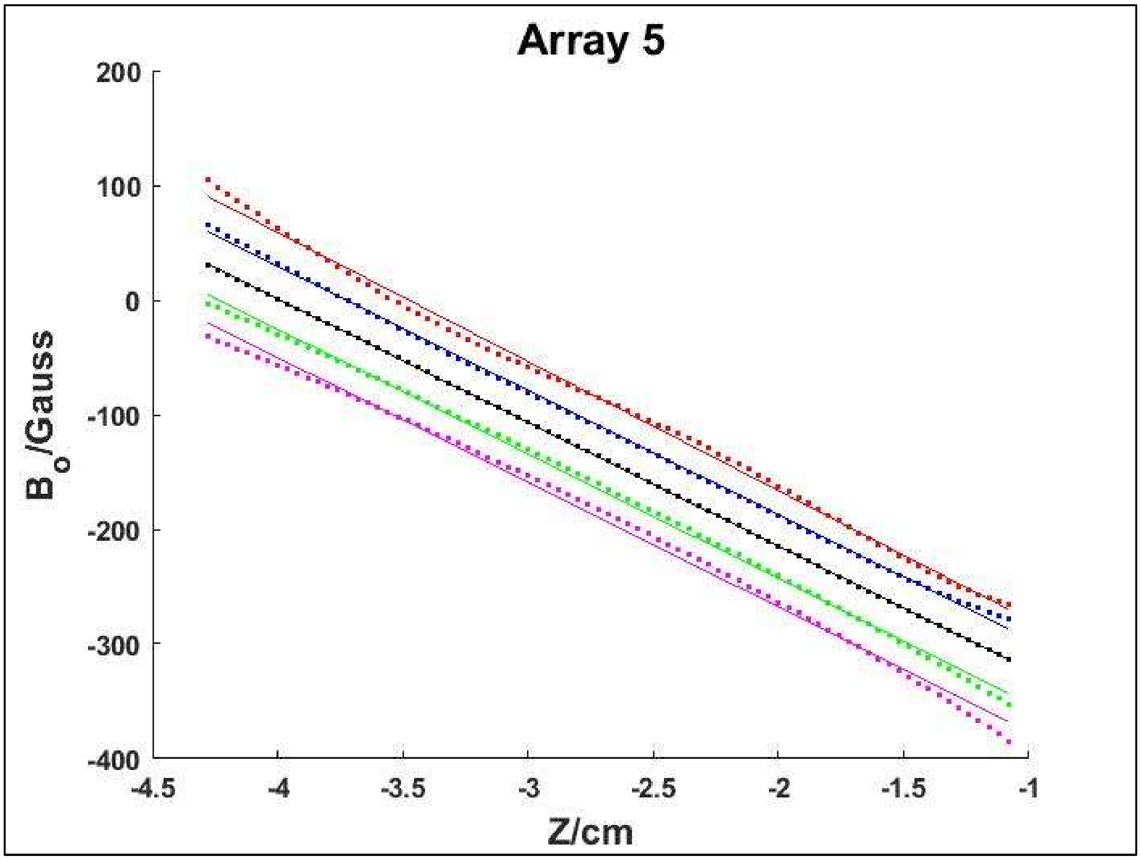

Figure 2.5: B_o of Array 5 as a function of Z for Y = -0.2818 cm, 0.2182 cm, 0.7182 cm, 1.2182 cm, and 1.7182 cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

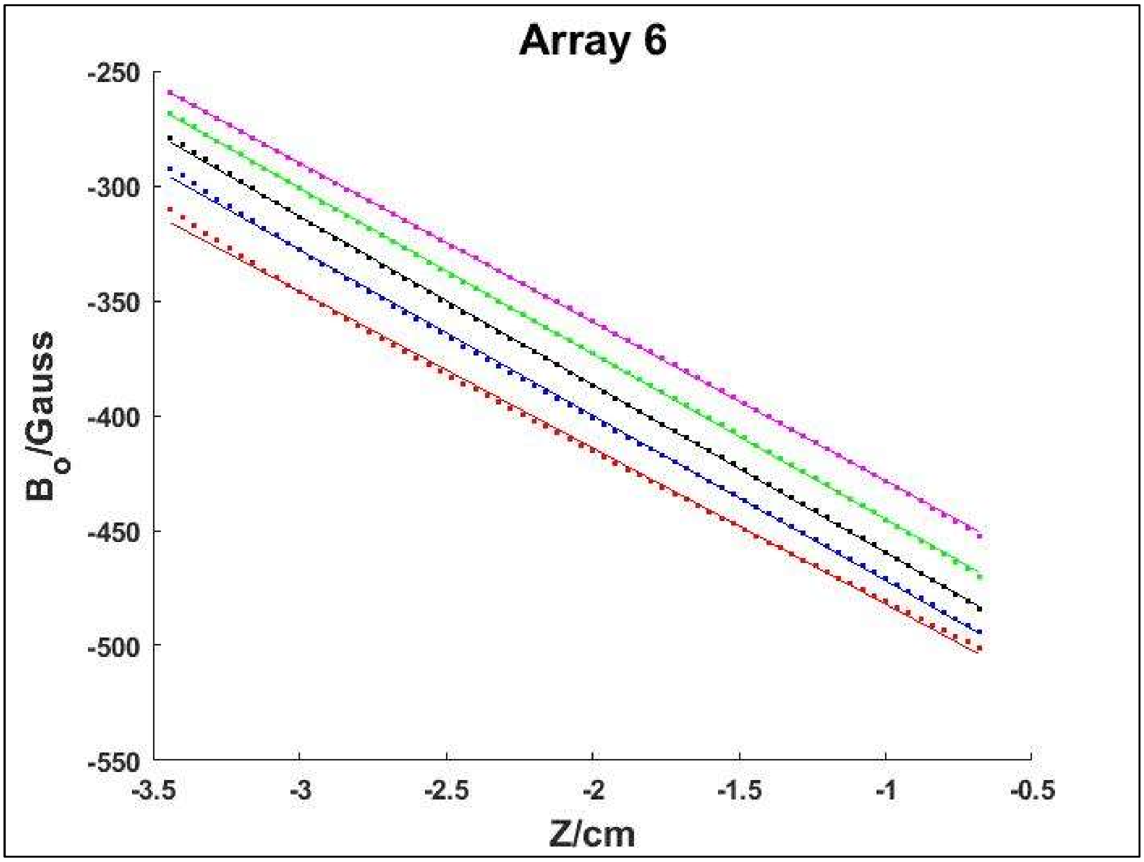

Figure 2.6: B_o of Array 6 as a function of Z for Y = 1.6585 cm, 2.1585 cm, 2.6585 cm, 3.1585 cm, and 3.6585 cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

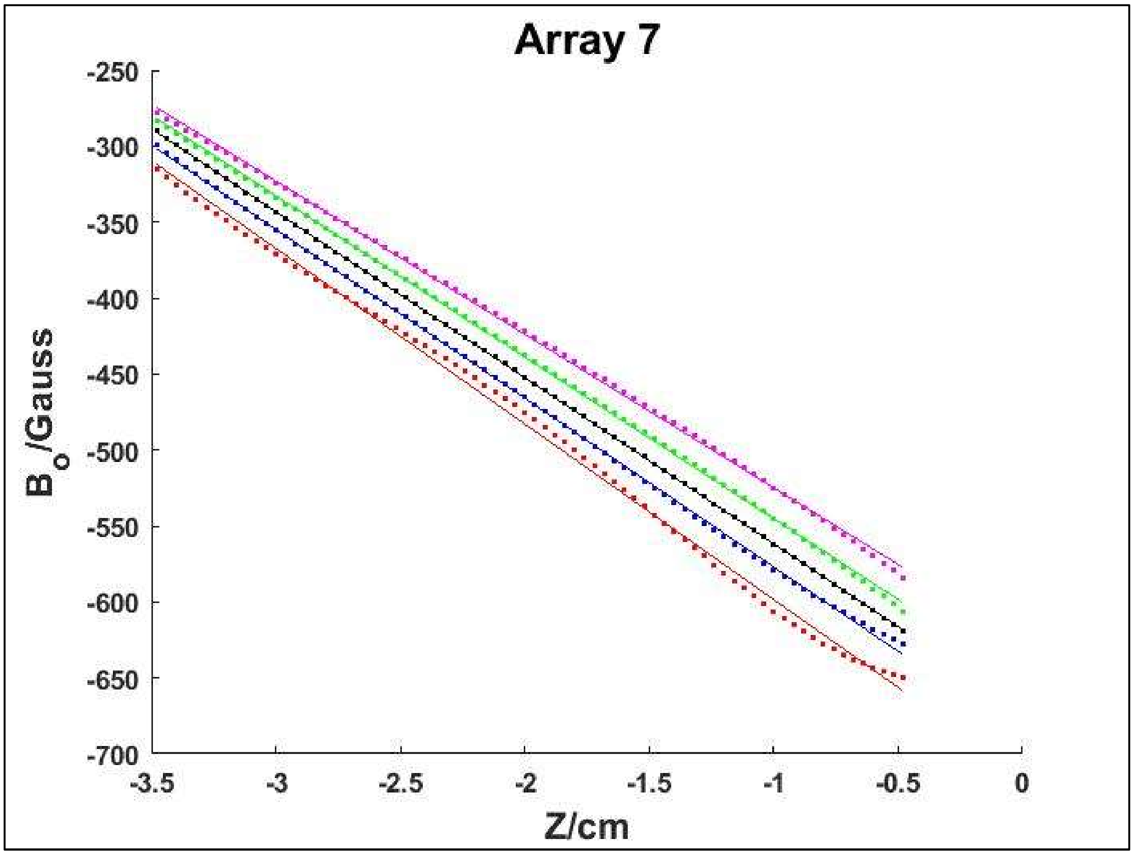

Figure 2.7: B_o of Array 7 as a function of Z for Y = 0.3389 cm, 0.8389 cm, 1.3389 cm, 1.8389 cm, and 2.3389 cm (red, blue, black, green, and pink dotted lines, respectively), and their best fit lines (red, blue, black, green, and pink solid lines, respectively).

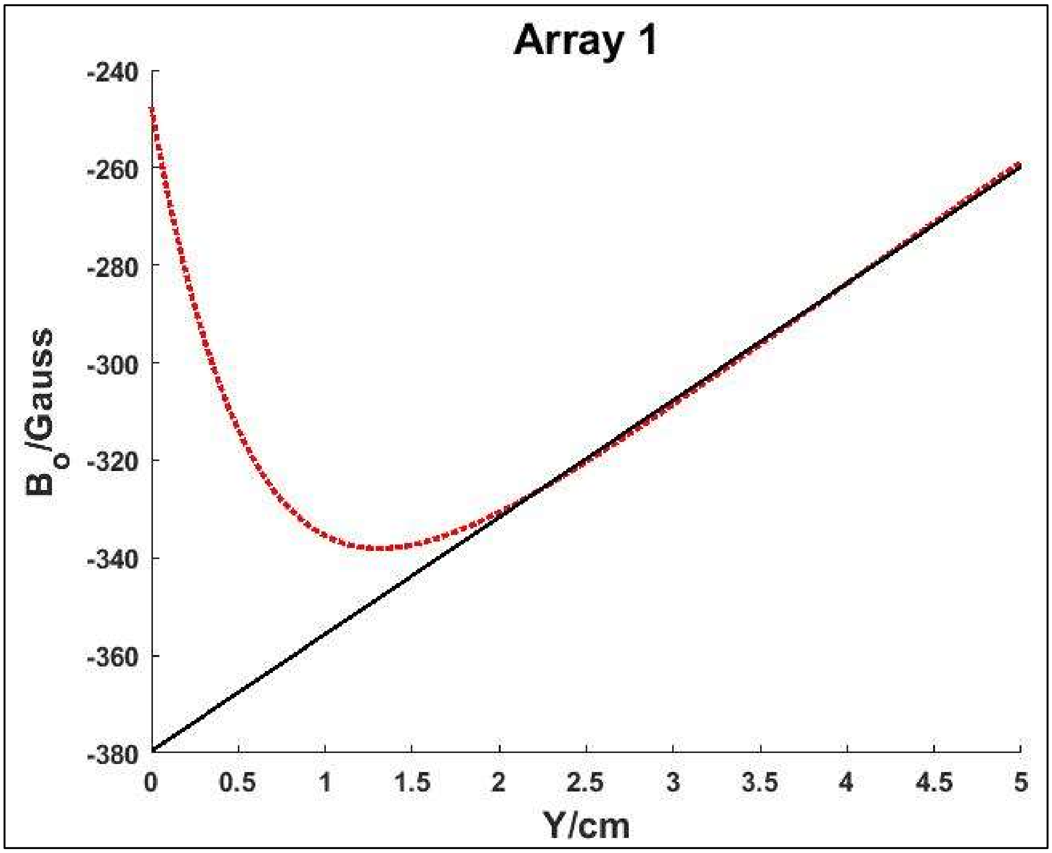

Figure 3.1: B_o of Array 1 as a function of Y for Z = -1.96 cm (red line), and the best fit line to the area of constant G_y (black line).

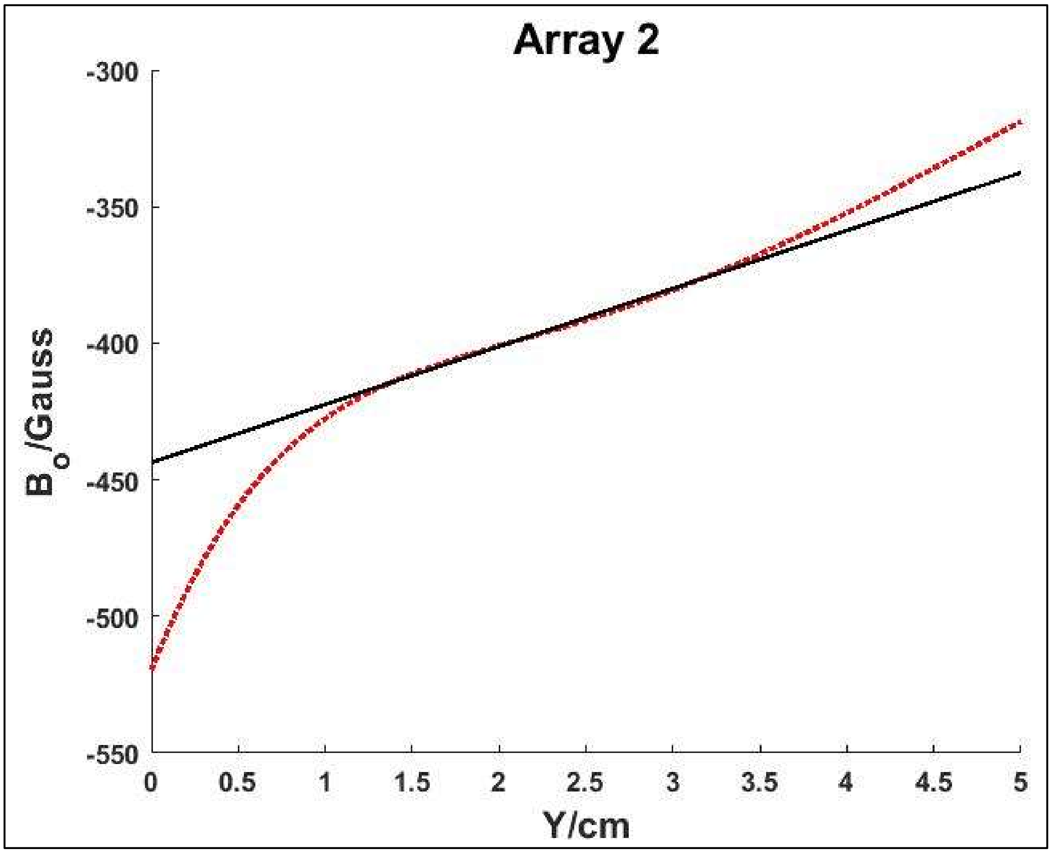

Figure 3.2: B_o of Array 2 as a function of Y for Z = -1.00 cm (red line), and the best fit line to the area of constant G_y (black line).

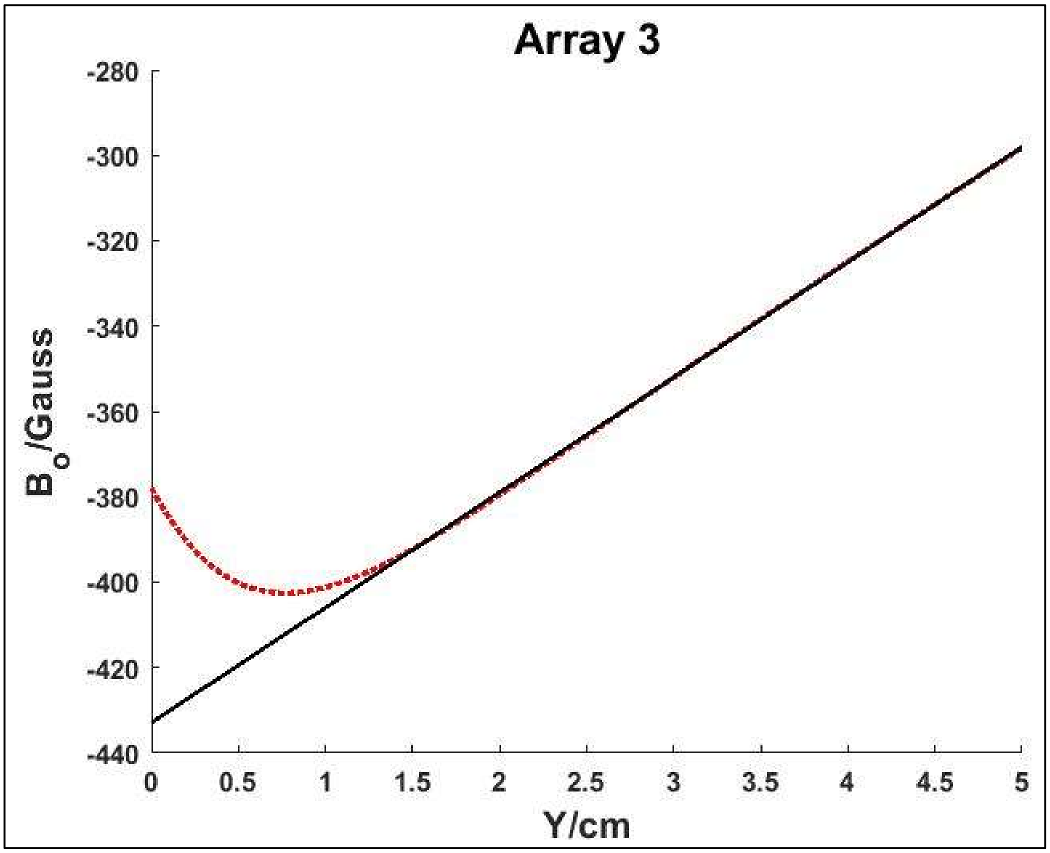

Figure 3.3: B_o of Array 3 as a function of Y for Z = -2.48 cm (red line), and the best fit line to the area of constant G_y (black line).

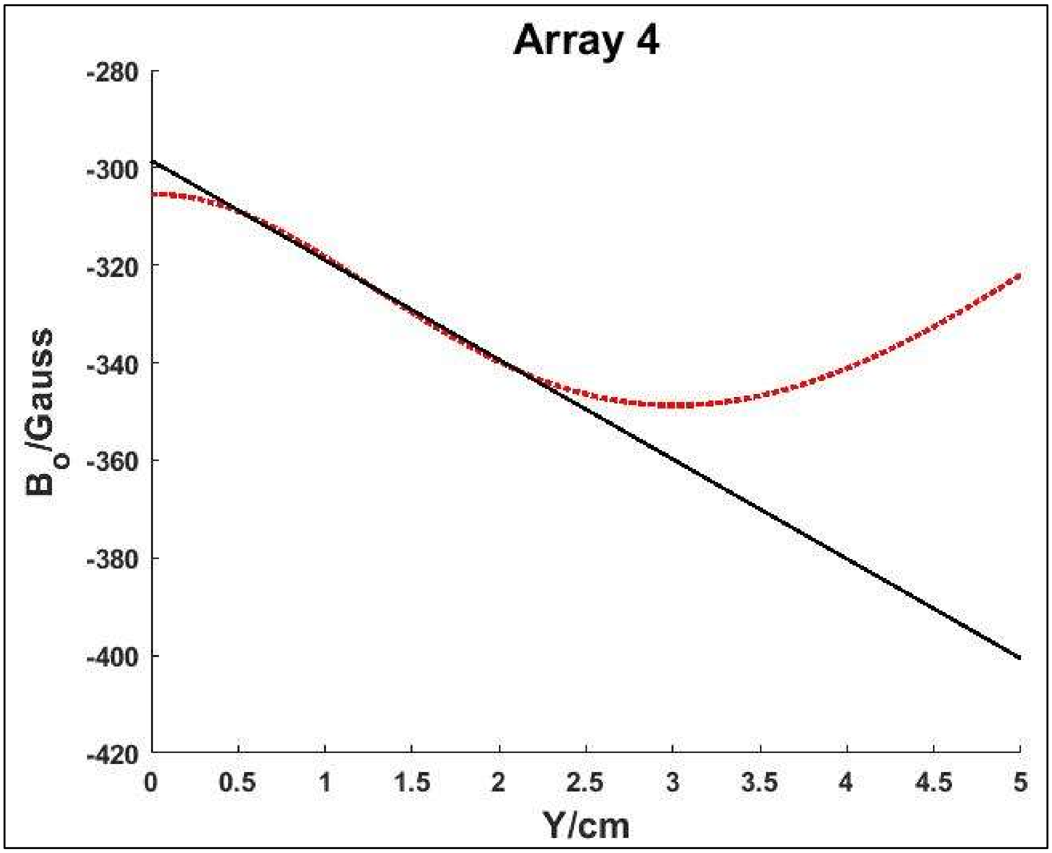

Figure 3.4: B_o of Array 4 as a function of Y for Z = -1.88 cm (red line), and the best fit line to the area of constant G_y (black line).

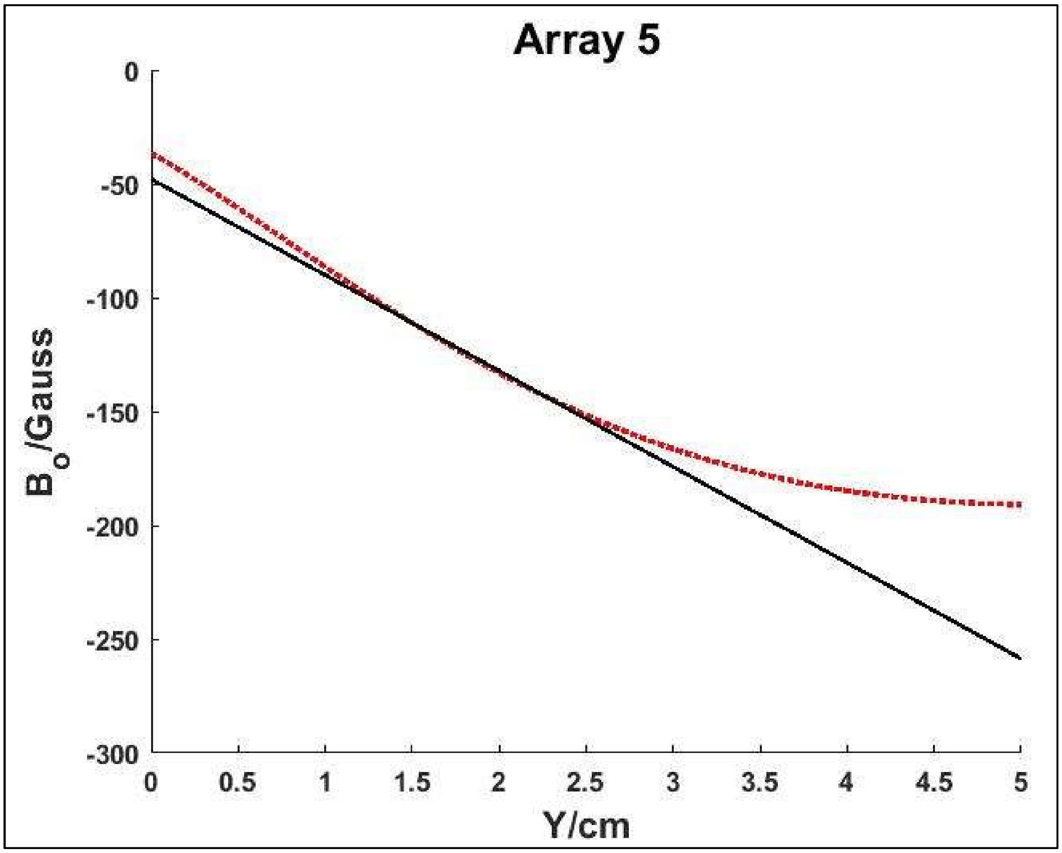

Figure 3.5: B_o of Array 5 as a function of Y for Z = -3.32 cm (red line), and the best fit line to the area of constant G_y (black line).

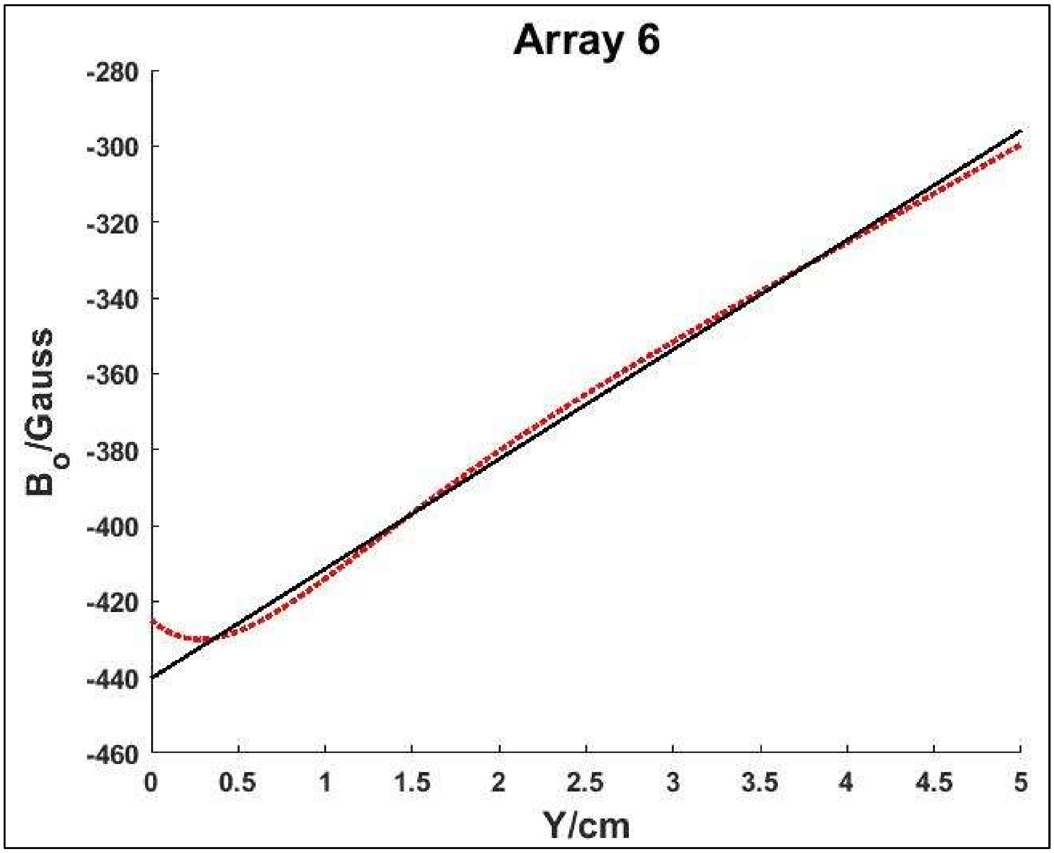

Figure 3.6: B_o of Array 6 as a function of Y for Z = -2.36 cm (red line), and the best fit line to the area of constant G_y (black line).

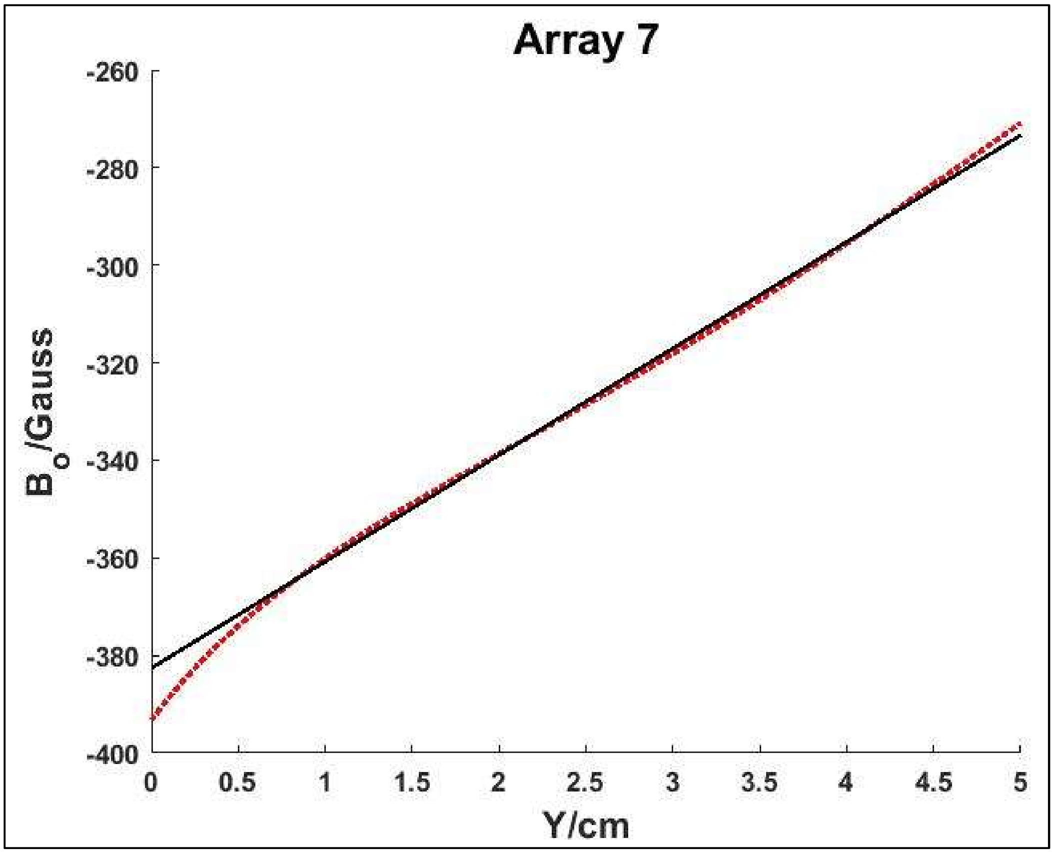

Figure 3.7: B_o of Array 7 as a function of Y for Z = -2.92 cm (red line), and the best fit line to the area of constant G_y (black line).

It is worth noting that the extended constant gradients were all centered over the left side of the arrays, and not the centre. This suggests it may be possible to create an area of extended constant G_z off to the side of the array.

It is also notable that arrays 1, 3, 6, and 7 have an extended constant G_y. These arrays could therefore be used for measurements such as those outlined in [2], as well as for flow measurements parallel to the direction of magnetization.

It took the program a long time to come up with the arrays. The program typically ran less than 200 times to get lower gradients (such as in Array 1), between 2000 and 3000 times for gradients between 50 and 70 Gauss/cm, and upward of 4000 times for gradients above 100 Gauss/cm. Although the GA function could find many solutions that satisfied its own constraints, it usually returned inflection points of B_o and not areas of extended constant G_z. Running the program 1000 times took around 14 minutes; it could take over an hour occasionally to find optimal arrays. Each array returned by the program has a range of at least 1 cm in Y where G_z is approximately constant.

In Report 1, a height termed the "base height" was used to describe heights above the array with an extended constant G_z. It was described as the base height because the heights in Report 1 were found by examining G_z from 0 cm to 5 cm above the array, and the first height with a Z range of 4 cm to 5 cm over which G_z was constant was reported. For this report, the height Y_{ga} found by the GA function was reported, and G_z was compared over a ± 1 cm range in Y from Y_{ga} (as shown in figure 2).

However, the arrays also have a constant G_z beyond the ± 1 cm Y-range, as shown in figures 4-5. B_o for all arrays except Array 5 becomes less linear near the array, but stays almost perfectly linear (and G_z thus stays almost perfectly constant) as distance away from the array increases.

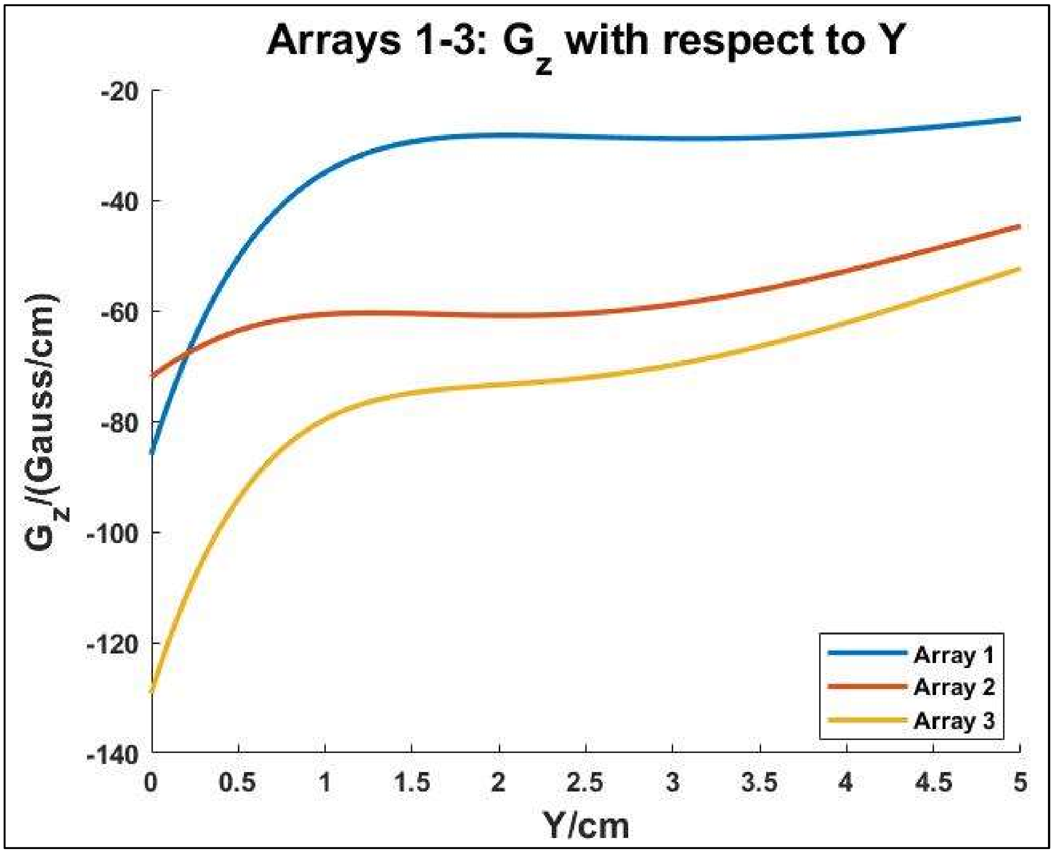

Figure 4: Magnitude of G_z as a function of Y for arrays 1 (blue), 2 (red) and 3 (yellow). These are plotted for the arrays' respective ranges in Z (table 2).

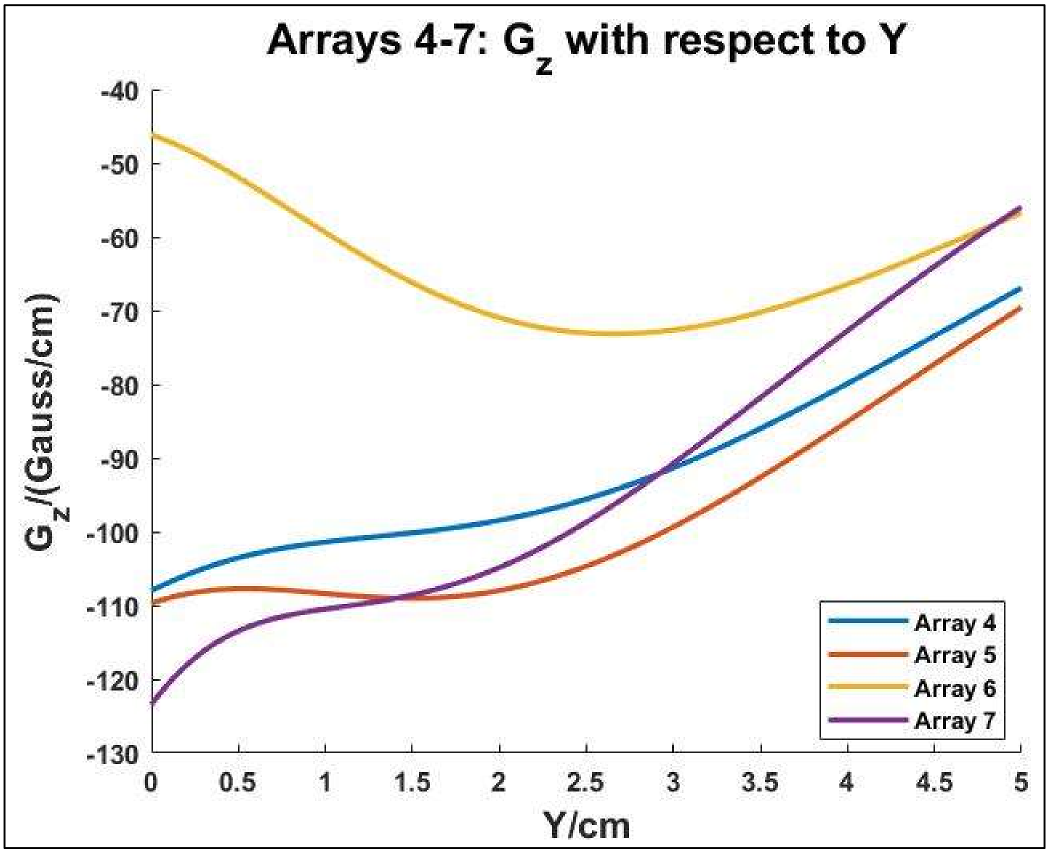

Figure 5: Variation in the magnitude of G_z as a function of Y for arrays 4 (blue), 5 (red), 6 (yellow), and 7 (purple). These are plotted for the arrays' respective ranges in Z (table 2).

5. Conclusion

The MATLAB Optimization Toolbox worked well to find 3-magnet arrays that had a constant extended G_z for suitable ranges of Y and Z. The only downfall was the time it took the program to run; while the program was running, I worked on this report and on analyzing data from previous outputs. Next, I'll adjust the upper and lower bounds for the Z_{ga} and Y_{ga} parameters to work on achieving a constant G_z next to the array, and adjust the nonlinear constraints to try to achieve gradients in the range of 120 Gauss/cm to 150 Gauss/cm.

6. References

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