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# Modelling the impact of Covid-19 pandemics on health insurance associated services demand

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# Introduction



# Misreporting

Misreporting in data refers to some event responsible for reporting less (under-reporting) or more (over-reporting) than the actual level of data.

- ▶ Many consequences can derive from misreporting: e.g., inferences that emerge from misreported data might be dramatically biased, providing an unrealistic picture of the actual problem.
- ▶ There are many real-world examples where each of the two forms of misreporting are well-documented, i.e., several cancers, STIs, gender-based violence or ADHD.

## Private health insurance and Covid-19 pandemic

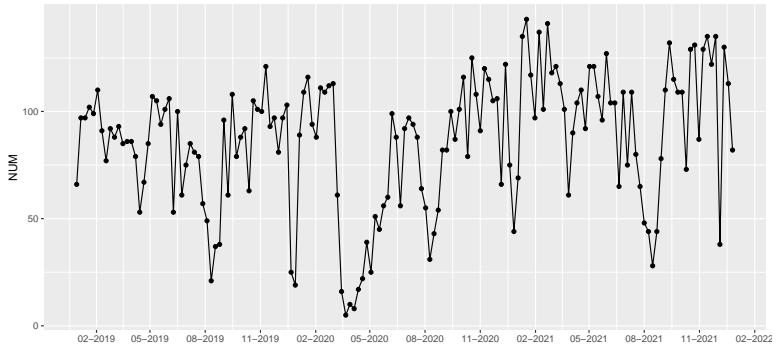
- ▶ More than 25% of the population has this type of coverage in Spain, exceeding 35% in some areas (UNESPA, 2020).
- ▶ The Covid-19 pandemic impacted the companies' claim rates in 2020 and 2021, especially for medical consultations and actions that were delayed for being (apparently) of low priority.
- ▶ **Hypothesis:** The number of company claims dramatically fell during the worst months of the Covid-19 pandemic, and many of the visits that should have been made at that time led to an increase in the number of visits in 2021 and 2022 (over-reporting).
  - ⇒ Before Covid-19, we did not expect over-reporting but generally under-reporting for different health services.

# Data

Data are the weekly number of visits to different health services of a given company from 2019 to 2021.

- ▶ **Geographic areas:** Spanish provinces.
- ▶ **Health services:** Urology, oncology, obstetrics, general medicine, osteopathy and cardiology.
- ▶ **Demographic variables:** Age and gender.

# OBSTETRICS (VISITS) – BARCELONA FEMALES



## The model





## Model I: Considering only under-reporting

Consider a **latent process**  $X_n$  with the following Poisson( $\lambda$ )-INAR(1) structure:

$$X_n = \alpha \circ X_{n-1} + W_n(\lambda),$$

where  $\alpha \in (0, 1)$ .  $\mathbb{E}(X_n) = \mathbb{V}(X_n) = \lambda/(1 - \alpha) = \mu_X$ . The operator  $\circ$  is the binomial thinning such that:  $\alpha \circ X_{n-1} = \sum_{i=1}^{X_{n-1}} Z_i$ , where  $Z_i$  are i.i.d Bernoulli( $\alpha$ ).

Consider the following **observed and potentially under-reported process**  $Y_n$ :  $X_n$  with probability  $1 - \omega$ , or  $q \circ X_n$  with probability  $\omega$ .

## Model II: Considering both under-reporting and over-reporting

Still consider that the latent process  $X_n$  is a  $\text{Poisson}(\lambda)$ -INAR(1) model, but now the processes  $Y_n$  can be misreported:

$$Y_n = \begin{cases} X_n & 1 - \omega, \\ \vartheta \diamond X_n & \omega, \end{cases}$$

where  $\vartheta = (\varphi_1, \varphi_2)$  and  $\vartheta \diamond X_n$  is called the fattening-thinning operator:  $[\vartheta \diamond X_n | X_n = x_n] = \sum_{k=1}^{x_n} W_k$ :

$$\mathbb{P}(W = k | \varphi_1, \varphi_2) = \begin{cases} 1 - \varphi_1 - \varphi_2 & j = 0, \\ \varphi_1 & j = 1, \\ \varphi_2 & j = 2, \\ 0 & \text{otherwise.} \end{cases}$$

## Under-reporting or over-reporting ?

- ▶ To distinguish between under-reporting, no misreporting or over-reporting, the following can easily be computed once the model parameters are estimated:

under-reporting	$\varphi_1 + 2\varphi_2 < 1$
no misreporting	$\varphi_1 + 2\varphi_2 = 1$
over-reporting	$\varphi_1 + 2\varphi_2 > 1$

- ▶ Note that when  $\varphi_2 = 0$ , the model results in the model I, which only accounts for under-reporting.
- ▶ A less flexible version of the operator defined before can be derived if  $W_k \sim \text{Binomial}(2, \varphi)$ .

## Parameter estimation: MoM method

The marginal distribution of the observed (and stationary) process  $Y_n$  is essential to compute the moment-based estimates of the model:

$$Y_n = \begin{cases} \text{Poisson}(\mu_X) & 1 - \omega, \\ \text{Hermite}(\mu_X\varphi_1, \mu_X\varphi_2) & \omega. \end{cases}$$

- 1 Fit the mixture above to obtain estimates of  $\hat{\omega}$ ,  $\hat{\mu}_X$ ,  $\hat{\varphi}_1$  and  $\hat{\varphi}_2$ .
- 2 Use the expression of the ACF,  $\rho_Y(k) = c(\alpha, \lambda, \omega, \varphi_1, \varphi_2)\alpha^k$ , to estimate  $\alpha$ .
- 3 Using  $\hat{\mu}_X$  (step 1) and  $\hat{\alpha}$  (step 2),  $\lambda$  can be easily estimated.

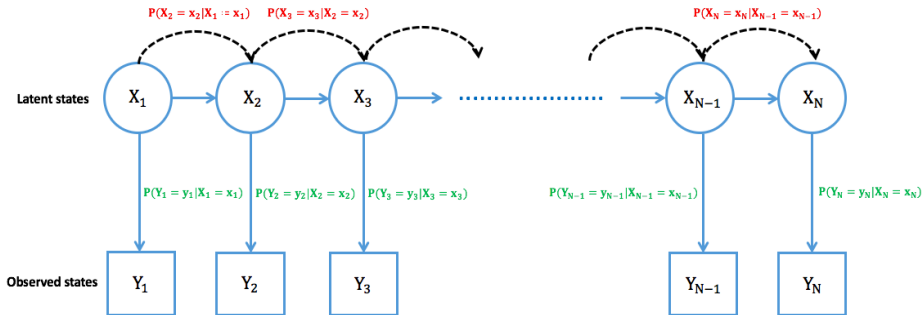
## Parameter estimation: ML method

- ▶ Since the likelihood function (LF) of  $Y_n$  is not directly tractable (HMC with an infinite number of states), it is computed instead with the so-called **forward algorithm**.
- ▶ LF is then computed recursively using the forward probabilities given below:

$$\begin{aligned}\gamma_n(y_{1:n}, x_n) &= \overbrace{\mathbb{P}(Y_n = y_n | X_n = x_n)}^{\text{emission probabilities}} \\ &\times \sum_{x_{n-1} = \frac{y_{n-1}}{2}}^{\infty} \underbrace{\mathbb{P}(X_n = x_n | X_{n-1} = x_{n-1})}_{\text{transition probabilities}} \gamma_{n-1}(y_{1:n-1}, x_{n-1}).\end{aligned}$$

- ▶ Finally:  $\mathbb{P}(Y_{1:N} = y_{1:N}) = \sum_{x_N = \frac{y_N}{2}}^{\infty} \gamma_N(y_{1:N}, x_N).$

# Visual model



## Forward probabilities

- ▶ While the transition probabilities are computed using the cpmf of the  $\text{Poisson}(\lambda)$ -INAR(1), the emission probabilities are computed as below:

$$\mathbb{P}(Y_n = y_n | X_n = x_n) = \begin{cases} 0 & \text{if } x_n < y_n/2, \\ (1 - \omega) + \omega p_n & \text{if } x_n = y_n, \\ \omega p_n & \text{if } x_n > y_n, \\ \omega p_n & \text{if } x_n < y_n, x_n \geq y_n/2. \end{cases}$$

- ▶  $p_n$  can be computed using the following recursive relation:

$$p_n = \frac{1}{n(1 - \varphi_1 - \varphi_2)} [\varphi_1(x_n - (n - 1))p_{n-1} + \varphi_2(2x_n - (n - 2))p_{n-2}].$$

## Preliminary results



## Simulation study

- ▶ We perform a simulation study based on Monte Carlo providing different values for the parameters  $\omega$ ,  $\varphi_1$  and  $\varphi_2$ .
- ▶ In particular, we generate an INAR(1)-Poisson process with  $\alpha = 0.5$  and  $\lambda = 3$ , and different scenarios of over-reporting ( $\omega = (0.3, 0.7)$ ,  $\varphi_1 = (0.2, 0.3)$  and  $\varphi_2 = (0.7, 0.5)$ ) and under-reporting ( $\omega = (0.3, 0.7)$ ,  $\varphi_1 = (0.2, 0.5)$  and  $\varphi_2 = (0.2, 0.1)$ ).
- ▶ We generate  $M = 50$  repetitions of the processes  $X_n$  and  $Y_n$  above, all of length  $n = 200$ .

► **Over-reporting scenario:**  $X_n = 0.5 \circ X_{n-1} + W_n(3)$  and:

$$Y_n = \begin{cases} X_n & 0.3, \\ (0.3, 0.5) \diamond X_n & 0.7. \end{cases}$$

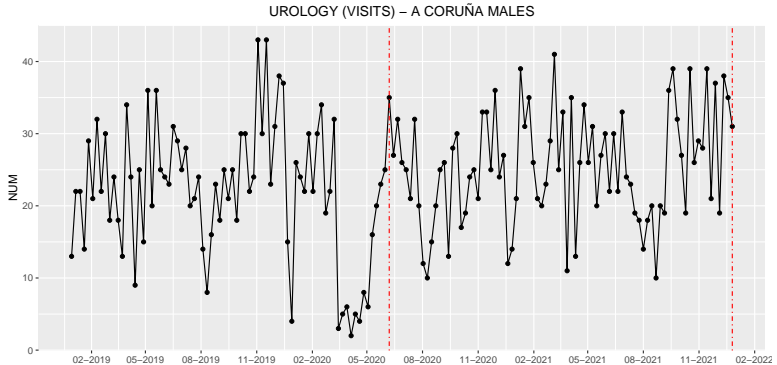
	$\alpha$	$\lambda$	$\omega$	$\varphi_1$	$\varphi_2$
true value	.5	3	.7	.3	.5
mean estimated value	.5172	3.0646	.6761	.2539	.4980
bias	.0172	.0646	-.0239	-.0461	-.0020
coverage (nominal: 0.90)	.9836	.9672	.9672	.6393	.9180

- **Under-reporting scenario:**  $X_n = 0.5 \circ X_{n-1} + W_n(3)$  and:

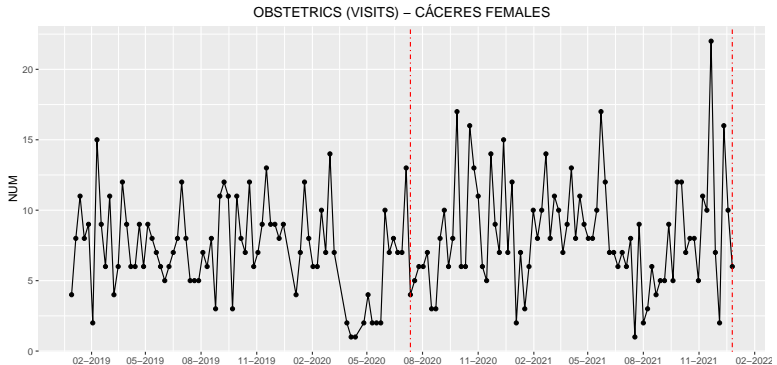
$$Y_n = \begin{cases} X_n & 0.3, \\ (0.2, 0.2) \diamond X_n & 0.7. \end{cases}$$

	$\alpha$	$\lambda$	$\omega$	$\varphi_1$	$\varphi_2$
true value	.5	3	.7	.2	.2
mean estimated value	.4614	3.0228	.5953	.2010	.1892
bias	-.0386	.0228	-.1047	.0010	-.0108
coverage (nominal: 0.90)	.9121	.9890	.8901	.8571	.9121

# Urology (Males) - A Coruña (06-20 to 12-21)



# Obstetrics (Females) - Cáceres (07-20 to 12-21)



	ML point estimate	std. error
A Coruña (urology-males)		
$\alpha$	.3256	.2029
$\lambda$	10.5644	3.1584
$\omega$	.8809	.1020
$\varphi_1$	.1194	.0917
$\varphi_2$	.7988	.1467
Cáceres (obstetrics-females)		
$\alpha$	.2232	.1576
$\lambda$	5.3185	1.2725
$\omega$	.3897	.2446
$\varphi_1$	.1061	.0939
$\varphi_2$	.7105	.1596

- Note that in both cases  $\hat{\varphi}_1 + 2\hat{\varphi}_2 > 1$ , thus over-reporting.

[illegible]