Carnegie Mellon University
Language
Technologies
Institute

11-411/11-611 Natural Language Processing

Sequence Labeling

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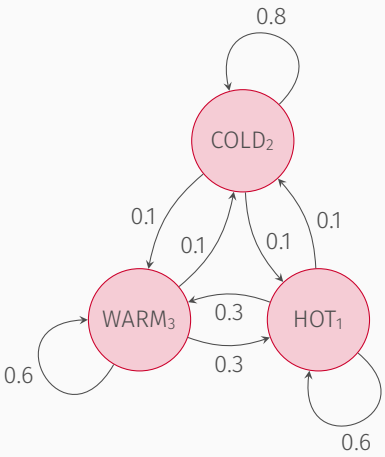
Learning Objectives

At the end of this lecture, you should be able to do the following things:

- Describe HMMs and their relationship to Markov Chains
- Implement the Forward Algorithm (and walk through it with pencil and paper)
- Implement the Viterbi Algorithm (and walk through it with pencil and paper)
- Implement the Baum-Welch Algorithm
- Be able to state the basic properties of Conditional Random Fields
- Be able to describe how RNNs are used for sequence labeling

Markov Chains

Markov Chains Tell Us about the Probabilities of Sequences of Random Variables



The figure to the left represents a Markov Chain.

- States
- Transitions
- Weights (probabilities)

The probability of COLD_2 at the timestep after COLD_2 is 0.8.
The probability of HOT_1 after COLD_2 is 0.1.
The probability of $\text{HOT}_1 \rightarrow \text{WARM}_3 \rightarrow \text{COLD}_2$ is $0.3 \times 0.1 = 0.03$

The Markov Assumption applies.

The Markov Assumption

"When predicting the future, the past doesn't matter—only the present."

in other words

$$p(q_i = a|q_1...q_{i-1}) = P(q_i = a|q_{i-1}) \tag{1}$$

This is the same assumption we made for ngram language modeling.

In Hidden Markov Models, Markov Chains are Hidden

The basic idea of a Hidden Markov Model (or HMM) is like that of a Markov Chain, except that the states are never observed.

- Observations are “emitted” from the hidden states
- So, when seen as a graph, HMMs have two kinds of nodes and two kinds of edges
  - Hidden states and observations
  - Transitional probabilities and emission probabilities
- The hidden states and transitional probabilities represent the latent structure that “sits behind” the observed phenomena

A Formal Defintion of the Hidden Markov Model

$Q = q_1, \ldots, q_N$	a set of N states
$A = a_{1,1}, a_{1,2}, \ldots$	a transitional probability matrix of cells a_{ij}, where each cell is a probability of moving from state i to state j. $\sum_{j=1}^{N} a_{ij} = 1 \; \forall i$
$O = o_1, \ldots, o_T$	a sequence of T observations, each drawn from a vocabulary V.
$B = b_1, \ldots, b_n$	a sequence of observation likelihoods (or emission probabilities). The probability that observation o_t is generated by state q_i.
$\pi = \pi_1, \ldots, \pi_N$	an initial probability distribution over states (the probability that the Markov chain will start in state q_i. Some states q_j may have $p_j = 0$ (meaning they cannot be initial states). $\sum_{i=1}^{N} \pi_i = 1 \; \forall i$

HMMs Assume the Markov Assumption and Output Independence

Like Markov Chains, HMMs require the Markov Assumption:

$$p(q_i|q_1...q_{i-1}) = P(q_i|q_{i-1}) \tag{2}$$

The further assume that the observed outputs depend only upon the state (Output Independence)

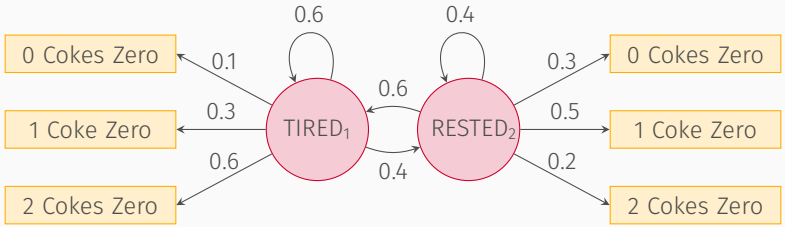
$$P(o_i|q_i, \ldots, q_i, \ldots, q_T, o_1 \ldots, o_i, \ldots o_T) = P(o_i|q_i) \tag{3}$$

Where $q_1, \ldots, q_T$ are the states at each time step and $o_1, \ldots, o_T$ are the outputs at each time step. In other words:

- The preceding or following states do not matter (we assume)
- The preceding or following outputs do not matter (we assume)

The Coke Zero Example

Since I do not drink coffee, I must drink Coke Zero to remain caffeinated. My consumption is related to my exhaustion. Could you build a model to infer my exhaustion from the number of Coke Zero bottles added to my wastebasket each day?



$$\pi = [0.7, 0.3]$$

The Three HMM Problems are Likelihood, Decoding, and Learning

Likelihood Given an HMM $\lambda = (A, B)$ and an observation sequence O, determine the likelihood $P(O|\lambda)$.

Decoding Given an observation sequence O and an HMM $\lambda = (A, B)$, discover the best hidden state sequence Q.

Learning Given an observation sequence O and the set of states in the HMM, learn the HMM parameters A and B.

Computing Likelihood with the Forward Algorithm

Computing Likelihood

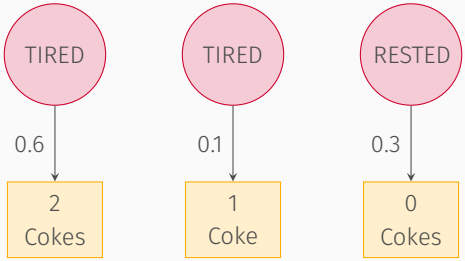
The Likelihood Problem Given an HMM $\lambda = (A, B)$ and an observation sequence O, determine the likelihood $P(O|\lambda)$.

This was easy with a Markov Chain (since we could observe the states): we just followed the path and multiplied the probabilities.

For an HMM, things are not so simple. Lets start with a simpler problem.

What if We Knew Which Days David Was Tired?

To simplify things, let's start off by assuming that we actually knew the sequence of tired/rested days and wanted to predict the probability of a Coke Zero sequence. This is not hard:



$$P(2\ 1\ 0|\text{tired tired rested}) = P(0|\text{tired}) \times P(1|\text{tired}) \times P(1|\text{rested}) \tag{4}$$

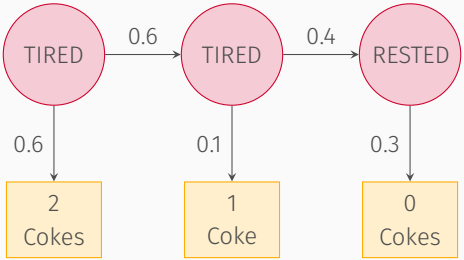
However, We Don't Know Which Days David Was Tired

We don't know the hidden sequence; we can only see the trail of Coke Zero bottles that David leaves behind.

Instead we might compute the probability of Coke events by summing over all possible sequences of tired/rested days, weighted by their probability:

$$P(O, Q) = P(O|Q) \times P(Q) = \prod_{i=1}^{T} P(o_i|q_i) \times \prod_{i=1}^{T} P(q_i|q_{i-1}) \tag{5}$$

Computing Joint Probability of Emission and Transition



$$P(3\ 1\ 0, \text{tired tired rested}) = (0.6 \times 0.1 \times 0.3) \times (0.6 \times 0.4) = 0.00432 \tag{6}$$

Preliminaries to the Forward Algorithm

a_{ij} the **transition probbilities** from previous state q_i to current state q_j

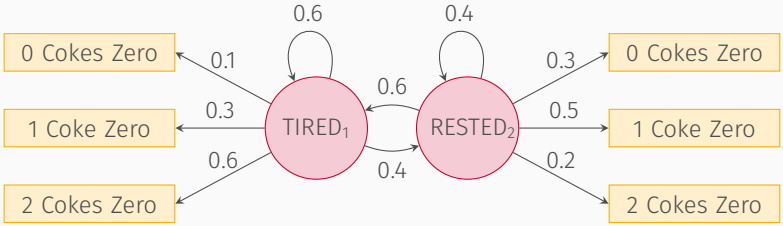
$b_j(o_t)$ the **state observation likelihood** of the observed symbol o_t given the current state j (emission probability)

The Forward Algorithm is a Dynamic Programming Algorithm

1: **function** FORWARD(observations of len T state-graph of len N) **returns** forward-prob
2: create a probability matrix $forward[N, T]$
3: **for each** s from 1 to N **do**
4: $forward[s, 1] \leftarrow \pi_s * b_s(o_1)$ ▷ initialization step
5: **for each** t from 2 to T **do**
6: **for each** s from 1 to N **do**
7: $forward[s, t] \leftarrow \sum_{s'=1}^{N} forward[s', t-1] * a_{s',s} * b_s(o_t)$ ▷ recursion step
8: $forwardprob \leftarrow \sum_{s=1}^{N} forward[s, T]$ ▷ termination step
9: **return** *forwardprob*

A dynamic programming algorithm **memoizes** values that can be used to compute other values rather than recomputing these values multiple times (in this case, in the matrix *forward*). We scan the matrix, computing a value for each cell based on values we have computed before (recursion step).

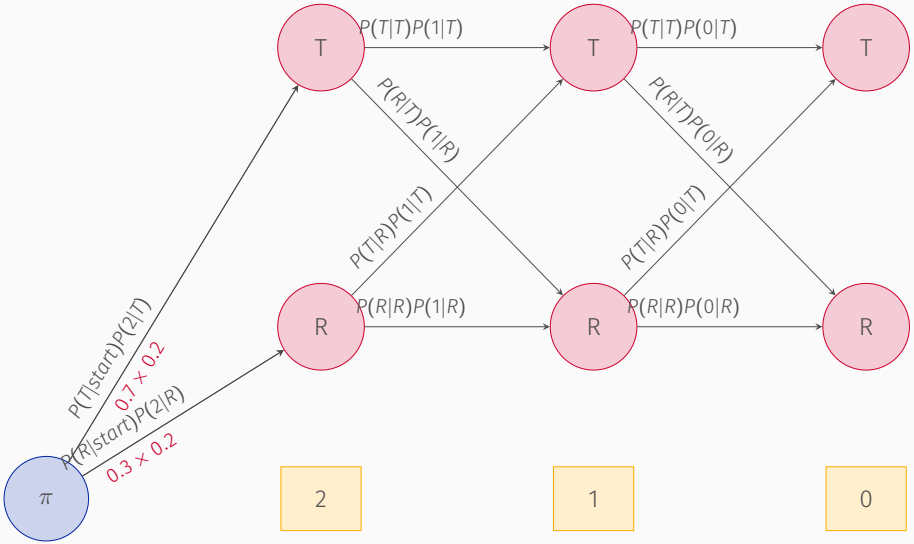
A Reminder: Here Is Our HMM



$$\pi = [0.7, 0.3]$$

Our sequence of observations is [2, 1, 0].

Computing a Forward Trellis



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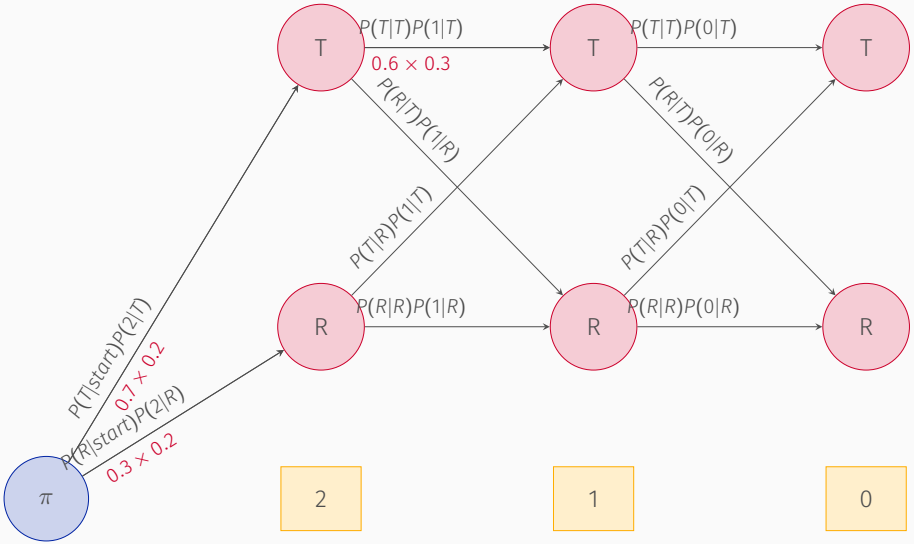

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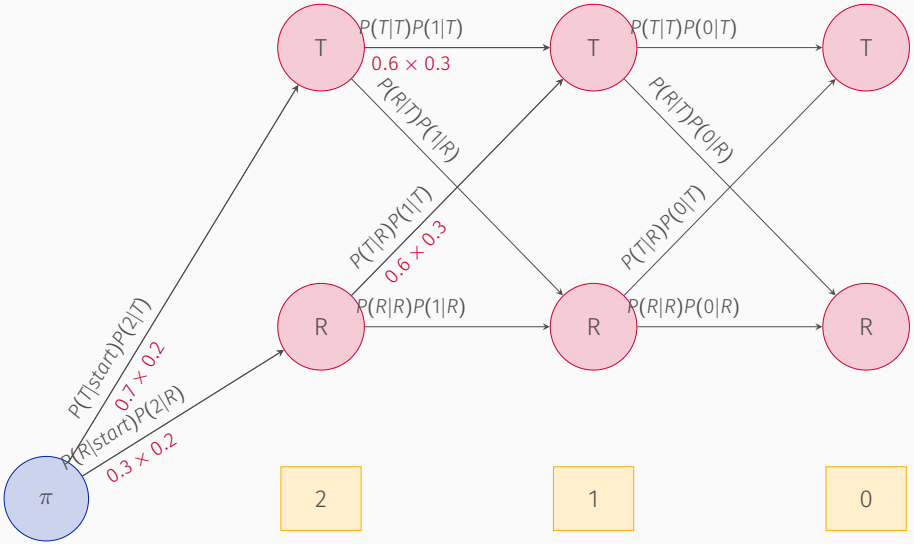

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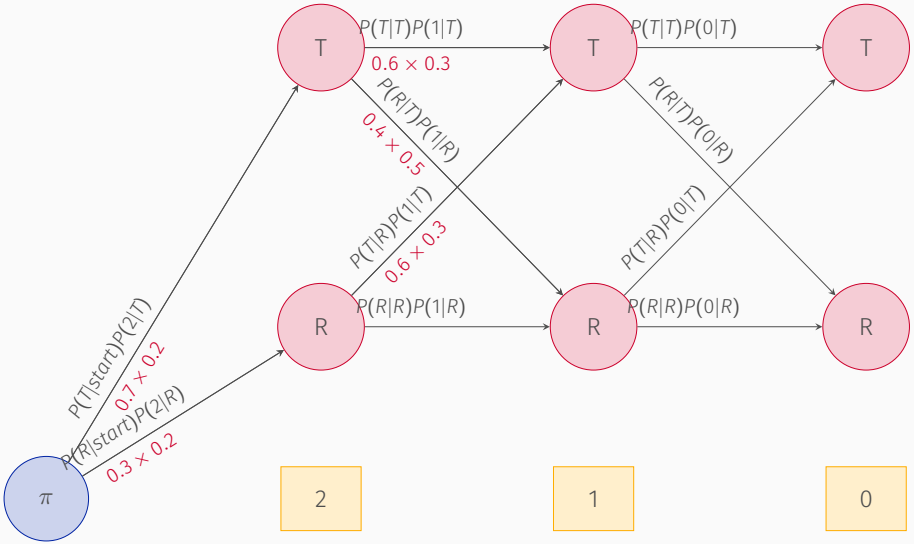

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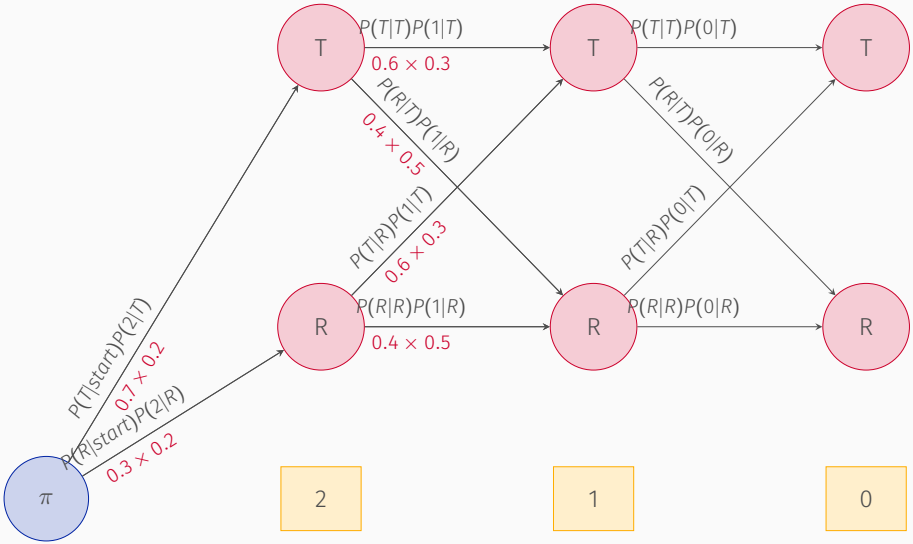

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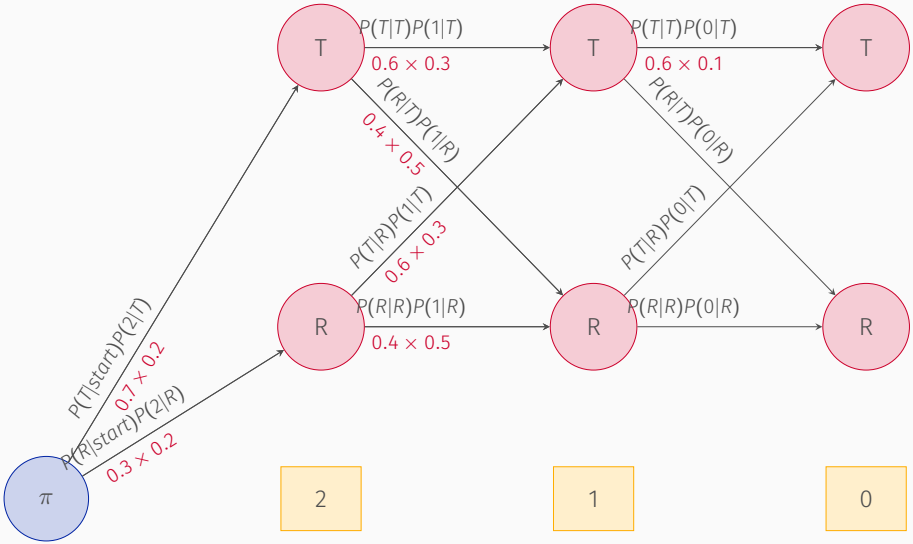

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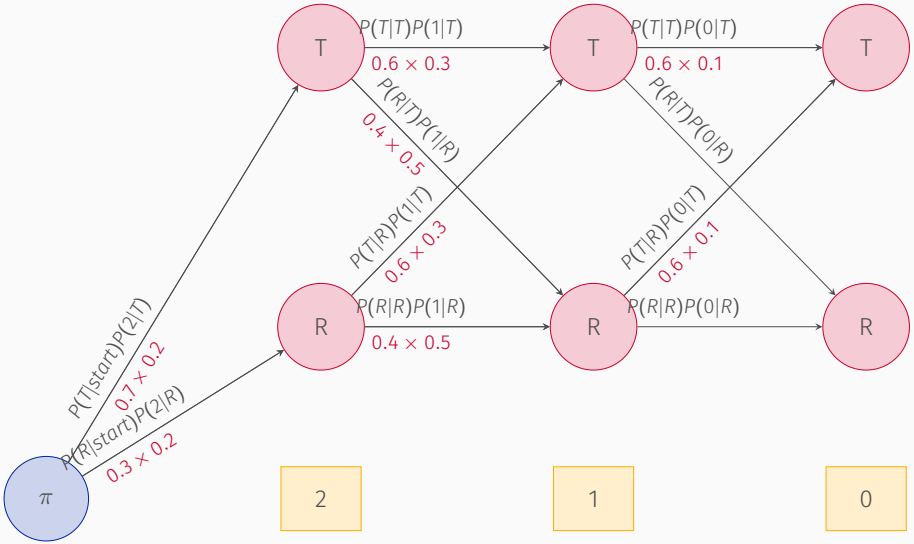

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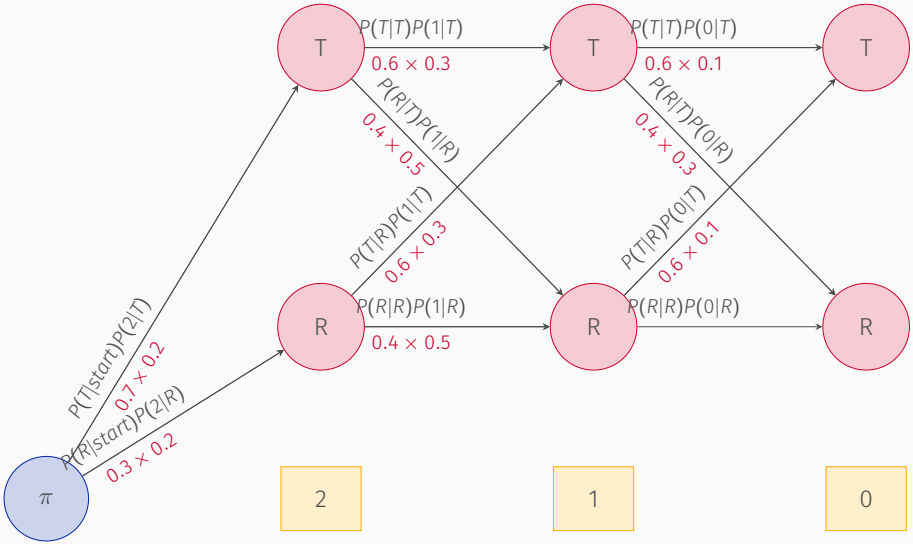

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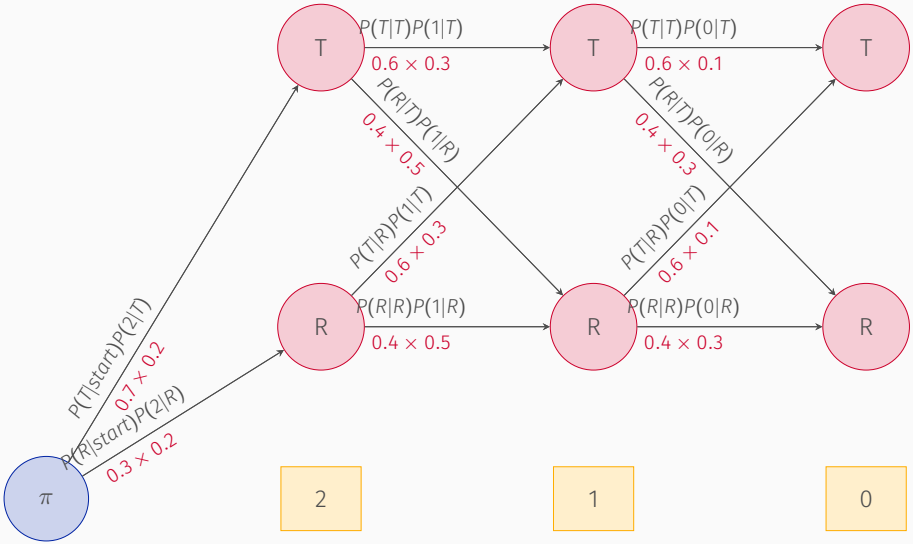

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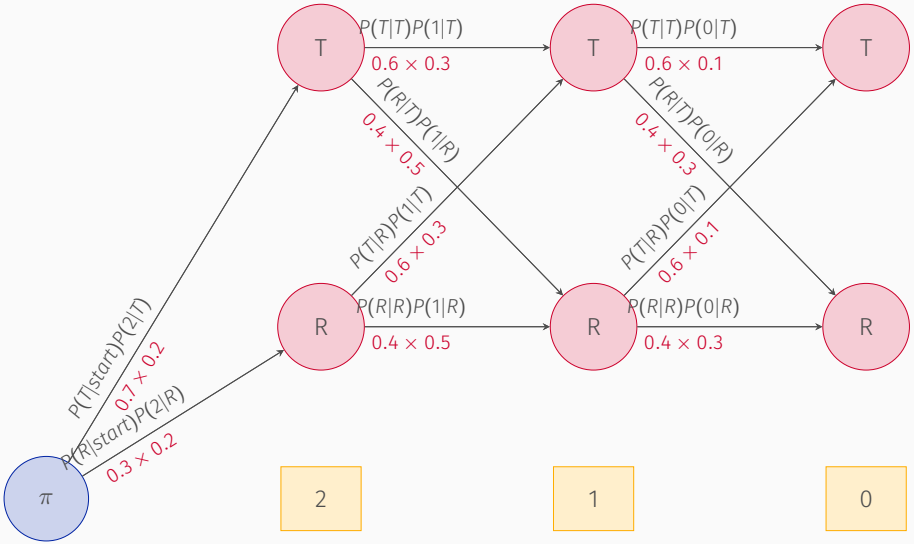


Often, We Want to Decode HMMs

Input A trained HMM and a series of observations
Output A series of labels, corresponding to hidden states of the HMM

This task shows up many times:

- Labeling words according to their parts of speech
- Labeling words according to whether they are at the beginning, otherwise inside of, or outside of a name
- Inferring the sequence of tired and not tired days in the month of your instructor based on his Coke Zero consumption

This is Decoding

More formally, given as input an HMM $\lambda = (A, B)$ and a sequence of observations $O = 0_1 o_2, \ldots, o_T$, find the most probable sequence of states $Q = q_1 q_2, \ldots, q_T$.

Can We Decode with the Forward Algorithm?

In principle, we could use the Forward Algorithm for decoding, but we would have to compute the probability for all possible sequences of states.

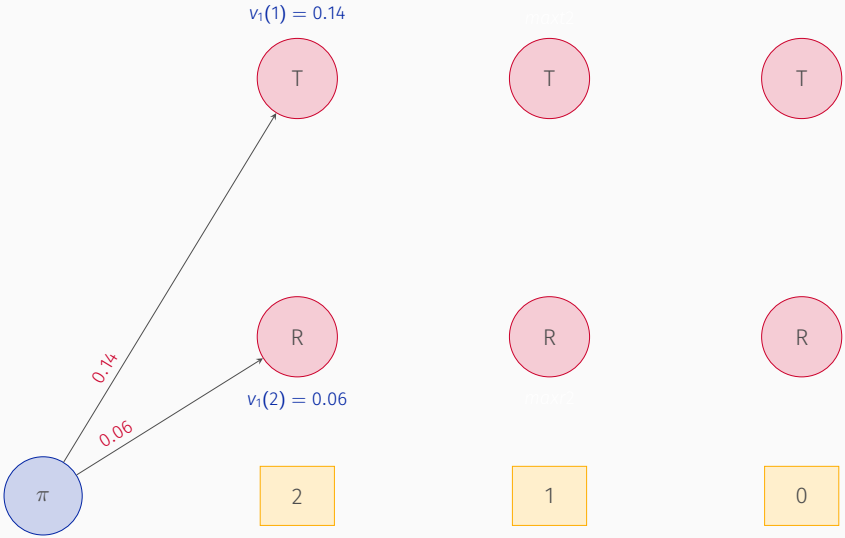
This is computationally infeasible because the set of possible state sequences (e.g. TTT, TRT, TRR, RRR, ...) grows exponentially as the number of states N grows.

Fortunately, there is another way.

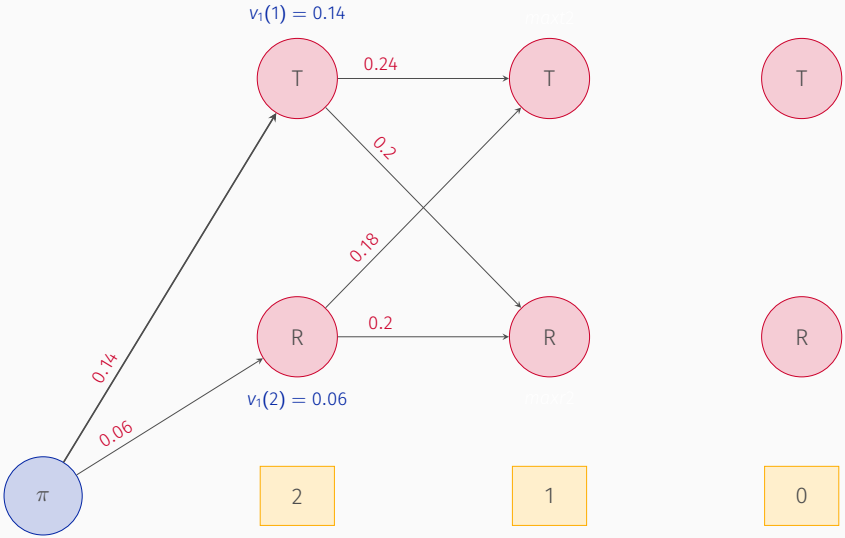
The Viterbi Algorithm Can Be Used to Decode HMMs

1: **function** VITERBI(observations $O = o_1, o_2, \ldots, o_T$, state-graph of length N)
2: $V[N, T] \leftarrow$ empty path probability matrix
3: $B[N, T] \leftarrow$ empty backpointer matrix
4: **for each** $s \in 1..N$ **do**
5: $V[s, 1] \leftarrow \pi_s \cdot b_s(o_1)$
6: $B[s, 1] \leftarrow 0$
7: **for each** $t \in 2..T$ **do**
8: **for each** $s \in 1..N$ **do**
9: $V[s, t] \leftarrow \max_{s'=1}^{N} V[s', t-1] \cdot a_{s',s} \cdot b_s(o_t)$
10: $B[s, t] \leftarrow \text{argmax}_{s'=1}^{N} V[s', t-1] \cdot a_{s',s} \cdot b_s(o_t)$
11: *bestpathprob* $\leftarrow \max_{s=1}^{N} V[s, T]$
12: *bestpathpointer* $\leftarrow \max_{s=1}^{N} V[s, T]$
13: *bestpath* $\leftarrow$ path starting at *bestpathpointer* that follows b to states back in time.
14: **return** *bestpath*, *bestpathprob*

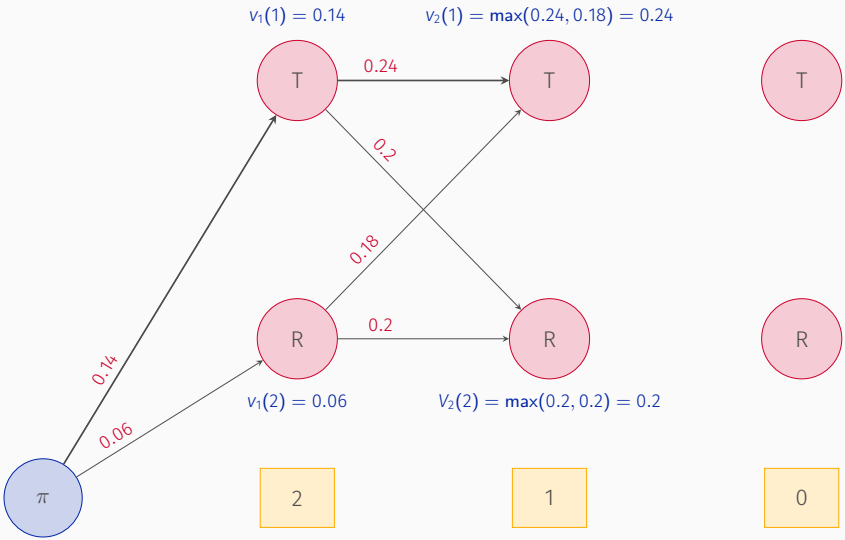
Using Viterbi to Decode an HMM



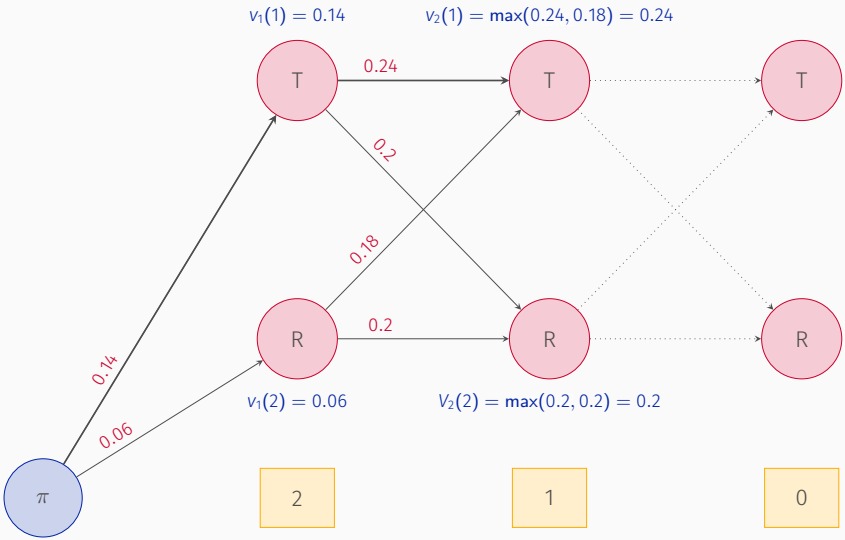
Using Viterbi to Decode an HMM



Using Viterbi to Decode an HMM



Using Viterbi to Decode an HMM



How Do We Train HMMs?

The standard algorithm for training HMMs is the **Forward-Backward** or **Baum-Welch Algorithm**

- Special case of the **Expectation-Maximization Algorithm**
- Trains both A (transition probabilities) and B (emission probabilities)
- Iterative algorithm
  - E-step (expectation) and M-step (maximization)
  - Starts with initial estimate
  - Uses estimate to compute better estimate

input Unlabeled sequences of observations O and a vocabulary of hidden states Q

output An HMM $\lambda = (A, B)$

Training HMMs if Nothing is Hidden

Suppose we know both the sequence of days in which David is tired or rested and the number of Cokes Zero that he consumes each day:

0 rested	3 tired	1 rested
1 tired	2 tired	2 tired
0 rested	0 rested	2 rested

How would your train an HMM?

In a Fantasy World, MLE Can Be Used to Train Fake HMMs

First, compute π from the initial states:

$$\pi_t = 1/3 \; \pi_r = 2/3 \tag{7}$$

The we can compute the matrix A:

$$\begin{aligned} p(tired|tired) &= 1/2 \quad & p(tired|rested) &= 1/6 \\ p(rested|tired) &= 1/3 \quad & p(rested|rested) &= 2/3 \end{aligned}$$

and then the matrix B:

$$\begin{aligned} p(0|tired) &= 0 \quad & p(0|rested) &= 2/5 \\ p(1|tired) &= 1/4 \quad & p(1|rested) &= 1/5 \\ p(2|tired) &= 1/2 \quad & p(2|rested) &= 1/5 \end{aligned}$$

In Real Life, We Don’t Know Enough to Train HMMs with MLE

- We can observe the sequence of events but we don’t necessarily know the sequence of underlying states, even with a lot of Bayesian magic
- Worse, we have no way of estimating the frequency of the hidden states accurately

To Understand Forward-Backward, we Need to Understand Backward Probability

Backward probability β is the probability of seeing the observations from time $t+1$ to the end, given that we are in state i at time t (and given the automaton λ):

$$\beta_t(i) = P(o_{t+1}, o_{t+2}, \ldots, o_T | q_t = i, \lambda) \tag{8}$$

It is computed inductively in a similar manner to the forward algorithm:

1. **Initialization**

$$\beta_T(i) = 1, \ 1 \leq i \leq N$$

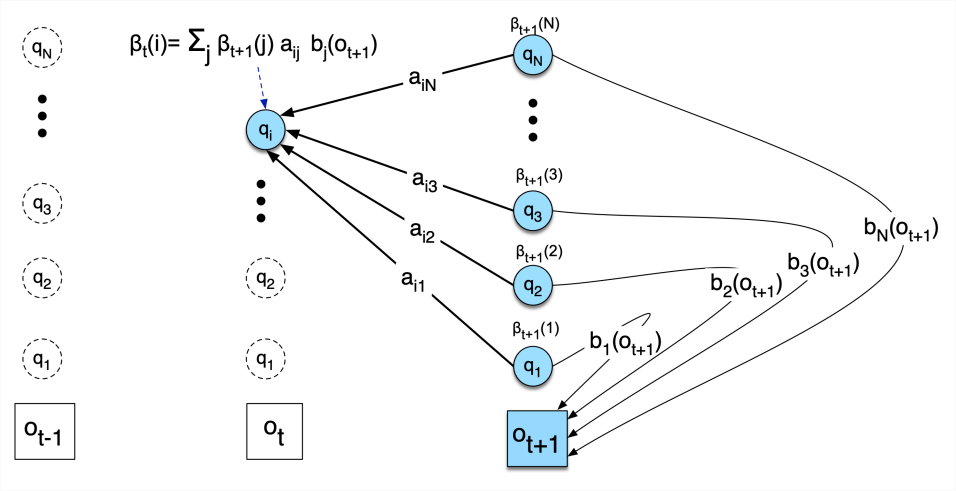
2. **Recursion**

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(o_t + 1) \beta_{t+1}(j), \ 1 \leq i \leq N, \ 1 \leq t < T$$

3. **Termination**

$$P(O|\lambda) = \sum_{j=1}^{N} \pi_j b_j(o_1) \beta_1(j)$$

The Backwards Induction Step



The Forward-Backward Algorithm Can Be Used to Train HMMs

function FORWARD-BACKWARD(Observations $O = o_1, o_2, \ldots, _T$, output vocabulary $V = v_1, v_2, ..., v_n$, hidden states $Q = q_1, q_2, \ldots, q_m$)

initialize A and B

repeat

$$\gamma[j] \leftarrow \frac{\alpha_t[j]\beta_t[j]}{\alpha_T[q_F]} \; \forall t \, \text{and} \, j$$

$$\xi[i,j] \leftarrow \frac{\alpha_t[i]a_{ij}b_j(o_t+1)\beta_{t+1}[j]}{\alpha_T[q_F]} \; \forall t, i, \, \text{and} \, j$$

$$a_{ij} \leftarrow \frac{\sum_{t=1}^{T-1} \xi_t[t,j]}{\sum_{t=1}^{T-1}\sum_{k=1}^{N} \xi_t[i,k]}$$

$$b_j[v_k] \leftarrow \frac{\sum_{t=1 \mathrm{s.t.} O_t = v_k}^{T} \gamma_t[j]}{\sum_{t=1}^{T} \gamma_t[j]}$$

until convergence

return A, B

Conditional Random Fields

Conditional Random Fields are Also Used to Label Sequences

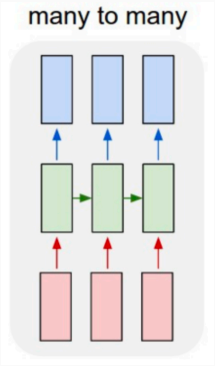
...and they work better than HMMs, but are more complicated to understand.

CRFs are a kind of discriminative undirected probablistic graphic model (whereas HMMs are a kind of generative directved probablistic graphical model).

CRFs are now used **much** more frequently that HMMs, sometimes in conjunction with neural networks like RNNs. I won't have time to talk much about them here, but you can understand them as playing a similar role to HMMs.

Sequence Labeling with RNNs

RNNs Can Also Be Used for Sequence Labeling



An RNN that emits an output (from specified vocabulary) for every input is essentially doing sequence labeling.
In a successful model for NER (named entity recognition, the task of labeling words based on the status as parts of names), bidirectional LSTMs are combined with CNNs (convolutional neural networks), which do feature extraction from words, and CRFs, which actually assign the labels.
We will learn more about NER (and POS tagging) in the next lecture.

Questions?