# MATH 110 Summer 2020

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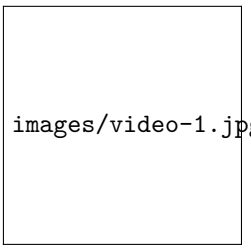


YouTube: https://www.youtube.com/watch?v=Z547IT5Eq-8

This is a collection of the materials (outside of the textbook) we will use in class to guide you on your journey through MATH 110. The textbook used in the course will be Functions Modeling Change 5th edition by Connally, et al.

# Contents

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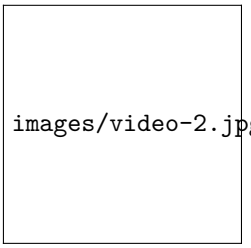
# Chapter 1

# Chapter 1 - Linear Functions & Change

## 1.1 Functions and Function Notation

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### 1.1.1 Worksheet

**1.** The forecast high temperature for the next 7 days in Jacksonville, FL is given by

| Day ($t$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| High Temp ($H$ in $^\circ F$) | 75 | 77 | 83 | 77 | 80 | 78 | 80 |

Is $t$ a function of $H$ or is $H$ a function of $t$?**Solution**. We would say that $H$ a function of $t$ because the temperature depends on the day. We cannot say that $t$ a function of $H$ because the result would have one input that produced 2 outputs. For example 77°F corresponds to two different days ($t = 2$ and $t = 4$).

**2.** If the function $y = f(x)$ represents a child's height ($y$ in inches)as a function of her age ($x$ in years), interpret the statement

$$f(4) = 42.$$

**Solution**. The mathematical statement $f(4) = 42$ indicates that at age 4 years, the child is 42 inches tall.

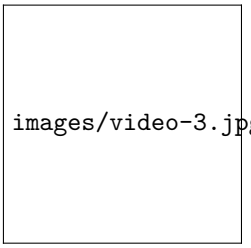
**3.** Complete the following table for the function $g(x) = 3x + 7$.

| $x$ | 0 | 2 | 3 | 5 | 10 |
|---|---|---|---|---|---|
| $g(x)$ | | | | | |

**Solution**. We obtain the missing values by plugging in the values in the top row into the function. For example, $g(0) = 3(0) + 7 = 7$ and $g(2) = 3(2) + 7 = 13$.

| $x$ | 0 | 2 | 3 | 5 | 10 |
|---|---|---|---|---|---|
| $g(x)$ | 7 | 13 | 16 | 22 | 37 |

## 1.2 Rate of Change




YouTube: https://www.youtube.com/watch?v=R-Yn41uAaVc

### 1.2.1 Worksheet

**1.** Let $f(x) = x^2 - 3x + 7$. Compute the average rate of change from $x = 1$ to $x = 4$. Visualize the average rate of change using the line between the two points $(1, f(1))$ and $(4, f(4))$.

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Or create and provide automatic screenshot as `images/interactive-1-preview.png` via the `mbx` script

`www.desmos.com/calculator/ahts9oeusv`

**Hint**. How many units does the line segment go up as you move one unit to the right?**Answer**. 2**Solution**. The average rate of change is $\frac{f(4)-f(1)}{4-1} = \frac{11-5}{3} = 2$. We can see it from the line segment above. As we go to the right one unit, we go up 2 units.

**2.** The total number of Netflix subscribers[1] in a given year between 2010 and 2015 is given by the table below.

| Year | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
|---|---|---|---|---|---|---|
| Subscribers (in millions) | 20.01 | 23.53 | 33.27 | 44.35 | 57.39 | 74.76 |

(a) Determine the rate of change of the number of Netflix subscribers between 2010 and 2013.

(b) Determine the rate of change of the number of Netflix subscribers between 2013 and 2015, and compare to you answer from part (a). What do you think this says about Netflix's growth?

**Answer**.

(a) $8.11\overline{3}$ million subscribers per year

(b) 15.205 million subscribers per year

**Solution**.

(a) Using the rate of change formula we get

$$RoC_{2010 \le x \le 2013} = \frac{44.35 - 20.01}{2013 - 2010} = \frac{24.34}{3} = 8.11\overline{3} \text{ million subscribers per year}$$

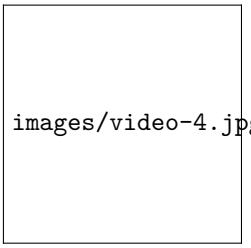
(b) Using the rate of change formula we get

$$RoC_{2013 \le x \le 2015} = \frac{74.76 - 44.35}{2015 - 2013} = \frac{30.41}{2} = 15.205 \text{ million subscribers per year}$$

Comparing to part (a) we see that we get a larger result and hence the average number of subscriptions looks to be increasing faster in the second time interval.

[1] https://www.businessinsider.com/netflix-subscribers-chart-2017-1

## 1.3 Linear Functions




YouTube: https://www.youtube.com/watch?v=jt7nPwcVBzA

### 1.3.1 Worksheet

**1.** After being on Instagram for a while, Sam started keeping track of her followers. Let N represent the number of followers she has on Instagram at time $t$ in months since she started keeping track.

| $t$ (months) | 0 | 1 | 3 | 7 | 10 |
|---|---|---|---|---|---|
| $N$ (number of followers) | 70 | 95 | 145 | 245 | 320 |

(a) Could this function be linear? (Check the rate of change over each interval.)

(b) What is the slope of this linear function?

(c) What does the slope mean in real world terms?

(d) Write a formula for $N$ as a function of $t$. (What is the vertical intercept?)

(e) What does your vertical intercept represent in real world terms?

**Solution**.

(a) Using the rate of change formula we get

$$RoC_{0\leq t\leq 1} = \frac{95-70}{1-0} = 25 \text{ followers per month}$$

$$RoC_{1\leq t\leq 3} = \frac{145-95}{3-1} = 25 \text{ followers per month}$$

$$RoC_{3\leq t\leq 7} = \frac{245-145}{7-3} = 25 \text{ followers per month}$$

$$RoC_{7\leq t\leq 10} = \frac{320-245}{10-7} = 25 \text{ followers per month}$$

From this data, the function looks linear as the rate of change is constant over every interval.

(b) The slope is 25 as the slope is the constant rate of change of a linear function.

(c) The slope represents the change in followers per month. That is, the number of followers increases by 25 per month.

(d) $N = 70 + 25t$

(e) The vertical intercept here is 70 which represents the initial number of followers when Sam started keeping track.

**2.** The enrollment at JU is given by the following graph of a linear function where $S$ represents the number of students, and $t$ represents the number of years since 2000.

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Or create and provide automatic screenshot as `images/interactive-2-preview.png` via the `mbx` script

`www.desmos.com/calculator/pmcraqhvvj`

(a) Using the two given points on the graph, determine the rate of change (slope) of the linear function.

(b) How many students were there in the year 2000?

(c) Determine a formula for $S$ as a function of $t$

**Solution**.

(a) The slope is

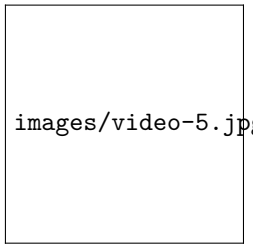
$$m = \frac{2744-2636}{4-1} = 36.$$

(b) If the number of students increases by 36 each year, then going back one year from 2001 to 2000 should result in a decrease in 36 from the 2636 students that were at JU in 2000. Hence, there were 2600 students in 2000.

(c) Using the two pieces of data above

$$S = 2600 + 36t$$

## 1.4 Formulas for Linear Functions




YouTube: https://www.youtube.com/watch?v=DU_5s9fpSz8

### 1.4.1 Worksheet

**1.** Compute a formula for the linear function graphed below.

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`www.desmos.com/calculator/44aftc4bnr`

**Solution**. The slope is

$$\frac{22-13}{5-2} = \frac{9}{3} = 3$$

Using the point-slope form we get

$$y - 13 = 3(x-2) \quad \Rightarrow \quad y = 3x + 7$$

**2.**

(a) Find the equation of a line that is parallel to $y = 4x - 2$ that passes through the point $(2, 5)$.

(b) Find the equation of a line that is perpendicular to $y = 3x + 7$ that passes through the point $(3, 5)$.

**Solution**.

(a) Since the lines are parallel, the new slope is 4. Using the point-slope form we get

$$y - 5 = 4(x-2) \quad \Rightarrow \quad y = 4x - 3$$

(b) Since the lines are perpendicular, the new slope is $\frac{-1}{3}$. Using the point-slope form we get

$$y - 5 = \frac{-1}{3}(x-3) \quad \Rightarrow \quad y = \frac{-1}{3}x + 6$$

**3.** The profit from selling widgets at a widget factory is given by the following table

| $w$ (number of widgets) | 5 | 10 | 20 | 50 | 100 |
|---|---|---|---|---|---|
| $P$ (profit in dollars) | 1 | 7 | 19 | 55 | 115 |

(a) Assuming that profit is linear, find a formula for profit as a function of number of widgets sold.

(b) Why do you think that the vertical intercept (your $b$ value) is negative?

**Solution**.

(a) We can use any two points here, so I'll use $(5, 1)$ and $(10, 7)$.

$$m = \frac{7-1}{10-5} = \frac{6}{5} = 1.2$$

To compute the equation of the line, we use the point-slope form

$$y - 1 = 1.2(x-5) \quad \Rightarrow \quad y = 1.2x - 5$$

(b) When first making a product, there will be initial costs that are only recovered once you start selling your product. Thus, profit will start out negative.

## 1.5 Modeling With Linear Functions

### 1.5.1 Worksheet

**1.** The store Bed, Bath, and Beyond offers two types of coupons, (type A) $5 dollars off a purchase of $15 dollars or more, and a (type B) 20% off coupon.

(a) Model both coupons by linear functions. In this case, discounted price is your dependent variable, and price before the coupon is your independent variable.

(b) Graph both functions and make a recommendation of when to use which coupon. To make it simpler, you can assume that the purchase you are making is $15 or more.

Specify static image with `@preview` attribute,
Or create and provide automatic screenshot as
`images/interactive-4-preview.png` via the `mbx` script

`www.desmos.com/calculator/hurbtg6vyr`

## 1.6 Fitting Linear Functions to Data

### 1.6.1 Worksheet

**1.** blah