Line Amid Two Points

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1 Definition of "Amid"

The follwing defines "amid" with respect to a line and two points. In particular, the existence of a centroid of the two points plays no role.

$$l \equiv \text{line}(c \ a \ b), 0 < a^2 + b^2$$

$$v \equiv \text{vec list } l$$

$$p_1 \equiv \text{point}(x_1 \quad y_1)$$
$$p_2 \equiv \text{point}(x_2 \quad y_2)$$

$$u_1 \equiv \text{vec}(1 \quad x_1 \quad y_1)$$
$$u_2 \equiv \text{vec}(1 \quad x_2 \quad y_2)$$

$$k_1 \equiv v \cdot u_1$$
$$k_2 \equiv v \cdot u_2$$

line l is "amid" points p_1 and $p_2 \Leftrightarrow k_1 = -k_2$.

2 Line Amid Two Points

The line $l, \text{line}(-ax_1 - ax_2 - by_1 - by_2 \quad 2a \quad 2b), 0 < a^2 + b^2$ is amid $\text{point}(x_1 \quad y_1)$ and $\text{point}(x_2 \quad y_2)$.

Proof. Let $k_1 =$

$$\text{vec}(-ax_1 - ax_2 - by_1 - by_2 \quad 2a \quad 2b) \cdot \text{vec}(1 \quad x_1 \quad y_1)$$

$$=$$

$$-ax_1 - ax_2 - by_1 - by_2 + 2ax_1 + 2by_1$$

$$=$$

$$ax_1 - ax_2 + by_1 - by_2$$

And let $k_2 =$

$$\text{vec}(-ax_1 - ax_2 - by_1 - by_2 \quad 2a \quad 2b) \cdot \text{vec}(1 \quad x_2 \quad y_2)$$

$$=$$

$$-ax_1 - ax_2 - by_1 - by_2 + 2ax_2 + 2by_2$$

$$=$$

$$-ax_1 + ax_2 - by_1 + by_2$$

$k_1 = -k_2$. QED