Line through a Point and the Centroid of a P-Parallelogram

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Consider $x_k, y_k \in \mathbf{Q}$. Provided that quadrilateral

$$P \equiv ([x_1, y_1], [x_2, y_2], [x_3, y_3], [x_4, y_4])$$

is a parallelogram, the centroid of P is[1]

$$C_P = \left[\frac{x_{13}}{2}, \frac{y_{13}}{2}\right] = \left[\frac{x_{24}}{2}, \frac{y_{24}}{2}\right].$$

where $x_{13} \equiv x_1 + x_3,\ y_{13} \equiv y_1 + y_3,\ x_{24} \equiv x_2 + x_4,\ y_{24} \equiv y_2 + y_4$.

Provided that $C_P \neq [x_5, y_5] = R$, the line l_{PR} passing through C_P and R is

$$\mathrm{vec}(\frac{x_{13}}{2} y_5 - x_5 \frac{y_{13}}{2} \quad \frac{y_{13}}{2} - y_5 \quad x_5 - \frac{x_{13}}{2}) \cdot \mathrm{vec}(1\ x\ y) = 0$$

or[2]

$$\mathrm{vec}(x_{13} y_5 - x_5 y_{13} \quad 2y_{13} - y_5 \quad 2x_5 - x_{13}) \cdot \mathrm{vec}(1\ x\ y) = 0$$

[1]Recall that the centroid of the parallelogram is the centroid of the diagonals.

[2]If $x_k, y_k \in \mathbf{Z}$, all elements of the left matrix are in $\mathbf{Z}$.