Line through the Centroids of Two P-Parallelograms

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Let $P = \text{ppara}(\text{point}(x_1 \quad y_1)\ \text{point}(x_2 \quad y_2)\ \text{point}(x_3 \quad y_3)\ \text{point}(x_4 \quad y_4))$. Let[1]

$$2C_P = \text{vec}(x_{13} \quad y_{13}) = \text{vec}(x_{24} \quad y_{24}).$$

where $x_{13} \equiv x_1 + x_3,\ y_{13} \equiv y_1 + y_3,\ x_{24} \equiv x_2 + x_4,\ y_{24} \equiv y_2 + y_4$. Similarly, let $R = \text{ppara}(\text{point}(x_5 \quad y_5)\ \text{point}(x_6 \quad y_6)\ \text{point}(x_7 \quad y_7)\ \text{point}(x_8 \quad y_8))$. Let

$$2C_R = \text{vec}(x_{57} \quad y_{57}) = \text{vec}(x_{68} \quad y_{68}).$$

where $x_{57} \equiv x_5 + x_7,\ y_{57} \equiv y_5 + y_7,\ x_{68} \equiv x_6 + x_8,\ y_{68} \equiv y_6 + y_8$.

Provided that $C_P \neq C_R$, the line through the centroids of P and R satisfies

$$\text{vec}(x_{13}y_{57} - x_{57}y_{13} \quad 2y_{13} - 2y_{57} \quad 2x_{57} - 2x_{13}) \cdot \text{vec}(1\ x\ y)$$

[1] Recall that the centroid of a parallelogram is the midpoint of either diagonal.