Appendix: Model Equations, Table A.1, Table A.2, Figure A.1

The model code can allow for multiple sampling areas but for the results reported in this study, the data were treated as representing a single area.

The number of krill of a given age each year is:

$$N_{a,y} = \begin{cases} \bar{R}e^{\varepsilon_y^R} & \text{if } a = 1 \\ \left(N_{a-1,y-1}e^{-Z_{a-1,y-1}}\right) & \text{if } 1 < a < A \\ \left(N_{A-1,y-1}e^{-Z_{A-1,y-1}} + N_{A,y-1}e^{-Z_{A,y-1}}\right) & \text{if } a = A \end{cases} \tag{1}$$

where $N_{a,y}$ is the number of animals of age *a* at the start of year *y*, $\bar{R}$ is the mean recruitment, ε_y^R is the logarithm of the normally-distributed deviation from the mean recruitment in year *y*, calculated as:

$$\varepsilon_y^R \sim \mathrm{N}\left(\frac{-\left(\sigma^R\right)^2}{2}, \left(\sigma^R\right)^2\right),$$

σ^R is the pre-specified recruitment variability, $Z_{a,y}$ is the total mortality rate (currently including emigration and fishing) for animals of age *a* during year *y*, $e^{-Z_{a,y}}$ is the survival rate for animals of age *a* during year *y*, and *A* is the oldest age considered (the plus group)

Survey selectivity *S* by survey *s* for different ages of krill *a* is modeled as either logistic selectivity (a two-parameter form), or double-logistic (a four-parameter form).

Logistic selectivity

$$S_{s,a} = \frac{1}{1 + e^{-\beta_s(a-\alpha_s)}} \tag{2}$$

where β_s and α_s are estimated parameters for survey *s* representing slope and location, respectively, and *a* is age.

Double-logistic selectivity

$$S_{s,a} = e^{-\ln\left(1+e^{-\Lambda_s(a-\Psi_s)}\right)+\ln\left(1-\frac{1}{1+e^{-\varpi_s(a-(\Psi_s+\tau_s))}}\right)} \tag{3}$$

where Λ_s and ϖ_s are parameters for the ascending and descending slopes, respectively, ψ_s and τ_s are location parameters for survey *s*, and *a* is age.

The objective function consists of two likelihood components (length compositions and acoustic biomass) and one penalty function (Beverton-Holt recruitment).

First likelihood component: Survey biomass by survey, area, and year

$$\mathrm{L}_1 = \sum_y \sum_s \left(\frac{\left(\ln\left(I_y^s\right) - \ln\left(\hat{I}_y^s\right)\right)^2}{2\left(\sigma^{I^s}\right)^2} \right) \tag{4}$$

where $\left(\sigma^{I^s}\right)^2$ is calculated from the CV of the biomass index I for survey s as $\ln\left(\mathrm{CV}^2+1\right)$, I_y^s is the total krill biomass calculated from the survey s during year y, $\hat{I}_y^s$ is the model-estimate of survey biomass during year y calculated as $\hat{I}_y^s = \sum_a \left(S_{s,a} W_a N_{a,y}^s \big/ \left(1 + e^{-\ln(q_s)}\right)\right)$, q_s is the catchability of survey s, initially estimated as $\ln\left(q_s\right)$ and constrained to be between 0 and 1 by exponentiating it as a logistic function, W_a is weight at age, $N_{a,y}^s$ is the number of krill of age a during the time of survey s calculated as:

$$N_{a,y}^s \begin{cases} \bar{R} e^{\varepsilon_y^R} & \text{if } a = 1 \\ \left(N_{a-1,y-1} e^{\left(-Z_{a-1,y-1}\right)^\rho} \right) & \text{if } 1 < a < A \\ \left(N_{A-1,y-1} e^{\left(-Z_{a-1,y-1}\right)^\rho} + N_{A,y-1} e^{\left(-Z_{A-1,y-1}\right)^\rho} \right) & \text{if } a = A \end{cases}$$

where ρ is the fraction of the year in which the survey occurred [ρ = (survey month-1)/12, where the first month of the biological year is 1],

Second likelihood component: length compositions by survey, area, and year

$$\mathrm{L}_3 = \sum_y \left(n_y^s \sum_l \left(L_{l,}^s \ln\left(\hat{L}_{l,}^s\right)\right)\right) \tag{5}$$

where n_y^s is the effective sample size of survey s and year y, $L_{l,y}^s$ is the proportion of krill in length bin l during the time of survey s, $\hat{L}_l^s$ is the model-estimate of the proportion of krill in length bin l during the time of survey s calculated as:

$$\hat{L}_{l,y}^s = \frac{N_{a,y}^s e^{\left(-Z_{a-1,y-1}\right)^\rho} S_a \chi_a}{\sum_{h=A_R}^{A} \left(N_{h,y}^s e^{\left(-Z_{h-1,y-1}\right)^\rho} S_h \chi_h \right)}$$

A_R is the minimum age in the samples, χ_a is the age to length transition for age a calculated as:

$$\chi_{l,a} = \frac{\left[\Phi\left(\left((l+1) - L_\infty\left(1-e^{-ka}\right)/\sigma^v\right)\right) - \Phi\left(\left(l - L_\infty\left(1-e^{-ka}\right)/\sigma^v\right)\right)\right]}{\sum_a \left(L_\infty \left(1 - e^{-ka}\right)\right)} \tag{6}$$

where Φ is the cumulative normal distribution, L_∞, K are model-estimated von Bertalanffy parameters, and σ^v is the model-estimated von Bertalanffy standard error.

Penalties $\left(\Lambda_{1-3}\right)$ *: Beverton-Holt recruitment, lognormal recruitment deviations, catchability, steepness*

The first penalty is for deviations from Beverton-Holt recruitment:

$$\Lambda_1 = 0.5\left(\sigma^R\right)^2 \sum_y \left(\ln N_{1,y} - \ln \hat{R}_y + 0.5\left(\left(\sigma^R\right)^2\right)\right)^2 + Y\ \ln\left(\sigma^R\right) \tag{7}$$

where $\hat{R}_y$ is the model-estimate of Beverton-Holt recruitment at the start of year *y* calculated as:

$$\hat{R}_y = \frac{4hR_0B_y}{\left(1-h\right)B_0 + \left(5h-1\right)B_y}$$

h is steepness (the stock productivity at 20% of unfished biomass), R_0 is unfished recruitment penalized for deviations from mean recruitment as $0.5\left(\ln\left(R_0\right) - \ln\left(\bar{R}\right)\right)^2$, B_y is spawning biomass at the start of year *y*, B_0 is unfished spawning biomass, and Y is the number of recruitment years modeled.

A second penalty is for departures from the pre-specified recruitment deviation:

$$\Lambda_2 = \frac{\sum_y \left(\varepsilon_y^R\right)^2 + \ln\left(\sigma^R\right)}{2\left(\sigma^R\right)^2} \tag{8}$$

where ε_y^R is the recruitment deviation during year *y*, and σ^R is the pre-specified recruitment deviation.

A third penalty constrains the estimates for mortality (emigration):

$$\Lambda_3 = \sum_s \left(\frac{\left(M - M^*\right)^2}{2\left(\sigma^M\right)^2}\right) \tag{9}$$

where M^* is the total mortality prior σ^M is the mortality standard deviation prior. In these models that assume fishing mortality is unimportant compared to other factors, $Z = M$.

Table A.1 Prior values assigned to penalties in Appendix Eq. 9.

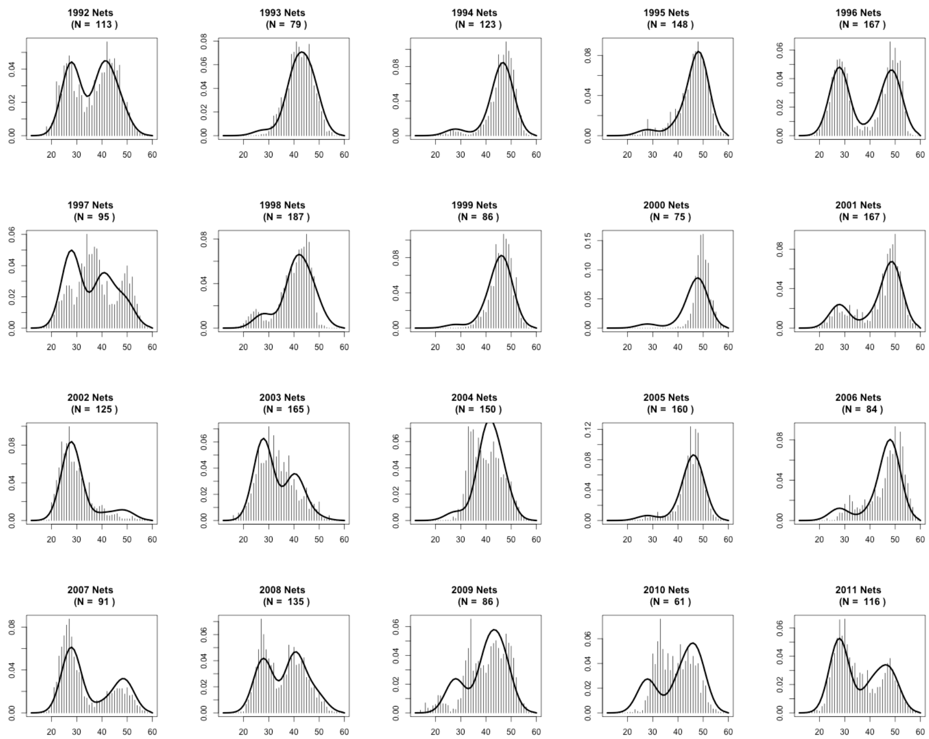
penalty	value
M^*	e^0
σ^M	e^1

Table A.2. Annual biomass indices from nets and acoustic sampling in depth-integrated density (g m^{-2}), total biomass and its standard deviation in the U.S. AMLR study area.

	net biomass			acoustic biomass		
Year	g m^{-2}	mean (1000 t)	SD (1000 t)	g m^{-2}	mean (1000 t)	SD (1000 t)
1992	2.4	304	119			
1993	6.6	826	1340			
1994	3.1	393	365			
1995	1.8	230	140			
1996	10.3	1280	1620	35.5	4440	1150
1997	3.2	399	266	46.5	5820	1630
1998	7.7	958	908	20.7	2590	506
1999	3.5	434	290	7.8	970	307
2000	2.6	327	160	23.6	2950	809
2001	4	501	334	4.1	516	110
2002	5.2	651	429	2.2	272	122
2003	6.1	759	1060	16.6	2070	483
2004	4.2	519	309	3.7	464	137
2005	3	377	206	5.9	739	444
2006	2.6	330	237	9.7	1210	562
2007	3.7	462	218	32.4	4040	1210
2008	7.8	972	645	16.8	2100	879
2009	2.3	291	347	16.1	2010	379
2010	9.8	1220	1030	13.3	1660	222
2011	1.6	205	92.9	13.2	1650	511

Fig. A.1. Fits to annual length-compositions in mm (x-axes) for the "b-ll" configuration with time-constant selectivities for a) actual length-compositions; and b) simulated length-compositions. The number of trawl nets sampled each year (the multinomial sample size) is indicated in the panel titles as "N".

a) fits to actual data



b) fits to simulated data