Lecture 1: Introduction to Trees and Binary Trees

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What is a Tree Data Structure?

A **tree** is a hierarchical data structure composed of elements called **nodes**, with the following characteristics:



- One special node called the root serves as the starting point. Root: A (the topmost node)
- Each node may have **zero or more child nodes**. **Child nodes**: B, C (directly connected to root)
- A node with **no children** is called a **leaf**. **Leaf nodes**: D, E, F (nodes with no children)
- A subtree is any node and all of its descendants. Subtree: B and its children form a subtree
- Nodes with the same parent are **siblings**. **Siblings**: B and C are siblings (share the same parent A)
- Every node (except the root) has exactly one **parent**. **Parent**: A is the parent of B and C; B is the parent of D and E
- The depth of a node is the number of edges from the root to the node. Depth: Node D has depth 2 (A
 → B → D)
- The **height** of a tree is the maximum depth among all nodes. **Height**: The tree has height 3 (levels A → B → D)

Real-World Examples of Trees

- **File systems** in operating systems
- Organizational charts in companies
- Family trees

Why Are Trees Important?

Trees are essential because they represent hierarchical relationships efficiently. Compared to:

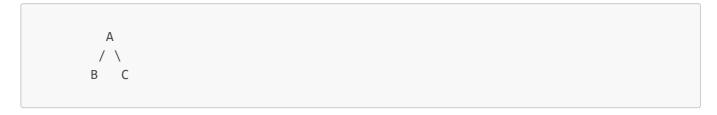
- **Arrays**: Fast indexing, but inefficient for insertions/deletions.
- Linked Lists: Efficient sequential access, but no hierarchy.
- **Trees**: Efficient for representing and traversing hierarchical data.

Types of Trees

1. **General Tree**: No limit on the number of children.

```
A
/ | \
B C D
/ \
E F
```

2. **Binary Tree**: Each node has at most two children.



3. **Full Binary Tree**: Every internal node has exactly two children.

```
A
/\
B C
/\/\
D E F G
```

4. **Complete Binary Tree**: All levels are filled except possibly the last, filled left to right.

```
A
/\
B C
/\ /
D E F
```

5. Balanced Tree: Height difference between left and right subtree is small (e.g., AVL Tree).

```
A / \ B C / D
```

6. **Binary Search Tree (BST)**: Left subtree < root < right subtree.

```
8
/\
3 10
/\\
1 6 14
```

7. **Expression Tree**: Represents mathematical expressions.

```
*
/\
+ -
/\/
a b c d
```

In this week, we will focus on:

- Binary Tree
- Binary Search Tree (BST)

Python Implementation: Binary Tree

```
class Node:
   def __init__(self, data):
       self.data = data
        self.left = None
        self.right = None
class BinaryTree:
   def __init__(self):
        self.root = None
   def insert(self, data):
        if not self.root:
            self.root = Node(data)
        else:
            self._insert(self.root, data)
   def _insert(self, node, data):
       if not node.left:
           node.left = Node(data)
        elif not node.right:
           node.right = Node(data)
        else:
            self._insert(node.left, data) # Simple left-first insertion
   def search(self, node, target):
       if node is None:
            return False
```

```
if node.data == target:
        return True
    return self.search(node.left, target) or self.search(node.right, target)
def delete(self, root, key):
    if root is None:
        return None
    if root.data == key:
        # Case 1: Node with no child
        if not root.left and not root.right:
            return None
        # Case 2: Node with only one child
        if not root.left:
            return root.right
        if not root.right:
            return root.left
        # Case 3: Node with two children - replace with inorder successor
        succ parent = root
        succ = root.right
        while succ.left:
            succ_parent = succ
            succ = succ.left
        if succ_parent != root:
            succ_parent.left = succ.right
        else:
            succ_parent.right = succ.right
        root.data = succ.data
        return root
    root.left = self.delete(root.left, key)
    root.right = self.delete(root.right, key)
    return root
def inorder(self, node):
    if node:
        self.inorder(node.left)
        print(node.data, end=' ')
        self.inorder(node.right)
```

Binary Tree vs. Binary Search Tree

- Binary Tree: No rules on how child nodes are organized.
- Binary Search Tree (BST): Left child < parent < right child.

Python Implementation: BST

```
class BSTNode:
    def __init__(self, data):
        self.data = data
```

```
self.left = None
        self.right = None
class BST:
    def __init__(self):
        self.root = None
    def insert(self, data):
        self.root = self._insert(self.root, data)
    def _insert(self, node, data):
        if node is None:
            return BSTNode(data)
        if data < node.data:</pre>
            node.left = self._insert(node.left, data)
        else:
            node.right = self._insert(node.right, data)
        return node
    def inorder(self, node):
        if node:
            self.inorder(node.left)
            print(node.data, end=' ')
            self.inorder(node.right)
    def search(self, node, target):
        if node is None:
            return False
        if node.data == target:
            return True
        elif target < node.data:</pre>
            return self.search(node.left, target)
        else:
            return self.search(node.right, target)
    def delete(self, node, key):
        if node is None:
            return node
        if key < node.data:</pre>
            node.left = self.delete(node.left, key)
        elif key > node.data:
            node.right = self.delete(node.right, key)
        else:
            if node.left is None:
                return node.right
            elif node.right is None:
                return node.left
            temp = self.findMin(node.right)
            node.data = temp.data
            node.right = self.delete(node.right, temp.data)
        return node
    def findMin(self, node):
        current = node
```

while current.left:
 current = current.left
return current

Lecture 2: Exercises and Assignment

Exercise 1: Tree Traversal

Given a binary tree, implement all three depth-first traversals:

- Inorder
- Preorder
- Postorder

Example Interview Question:

Given a binary tree, return the inorder traversal of its nodes' values.

Exercise 2: Validate BST

Write a function to determine if a given binary tree is a valid BST.

Example Interview Question:

Given a binary tree, check if it is a valid BST.

Exercise 3: Compute the Height of a Binary Tree

Write a function to compute the height of a binary tree.

Example Interview Question:

Given the root of a binary tree, write a function to return its height (the number of nodes along the longest path from the root down to the farthest leaf node).

Assignment for Next Week

- Implement a findMin and findMax method in the BST class.
- Write a function that returns True if the binary tree is balanced, and False otherwise.